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Research Article

Mathematical Evaluation and Dynamic Transmissions of a Cervical Cancer Model Using a Fractional Operator

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Abstract: This scientific research investigates the fractional-order compartmental model of cervical cancer. The human papillomavirus (HPV) is responsible for the development of cervical cancer. The Caputo-Fabrizio fractional operator, which includes a section for treatment with antiretroviral drugs, is employed to examine this worldwide phenomenon. The mathematical model of cervical cancer that has been proposed recently is an extension of the integer-order model. The fractional derivative technique is a novel approach for these types of biological models. This paper employs fixed point theory to provide requirements for the existence, uniqueness, and stability of the fractional order cervical cancer model, utilizing the Caputo- Fabrizio operator. We have initially found an approximation solution to a proposed model via the iterative Laplace transform method, which is easily accessible. This approach integrates the Laplace transform technique with a reliable, novel iterative method. We evaluated parameters indicating the advancement of the illness and presented the numeric simulations. The observed outcomes indicate that a fractional factor has a crucial effect on controlling cervical cancer.

***Keywords*:** caputo-fabrizio fractional operator, cervical cancer model, laplace transform method, fixed point theory

MSC: 34A08, 26A33, 92B99, 00A71

1. Introduction

Cervical cancer is a prominent worldwide health concern. Approximately 570,000 new instances of cervical cancer were detected globally in 2018, positioning it as the fourth most prevalent malignancy among women on a global scale. Cervical cancer is a malignancy that develops in the cervix, the lower segment of the uterus that attaches to the vagina. Cervical cancer specifically affects people who possess a cervix, encompassing cisgender women as well as certain transgender individuals. The cervix is an integral component of the female reproductive system and has a significant role in the process of birthing [1, 2]. It is noteworthy that, while other forms of cancer might impact individuals of both genders, cervical cancer specifically pertains to the tissues of the cervix. Cervical cancer mostly arises from the persistent infection of high-risk strains of human papillomavirus (HPV), which is a sexually transmitted infection. HPV,

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or human papillomavirus, is a collection of viruses that have the ability to infect the genital and oral regions of the body. Around 90% of fatalities caused by cervical cancer are concentrated in nations with poor and moderate incomes [3]. Cervical cancer typically progresses gradually, and the initial phases may not manifest significant signs. As the malignancy advances, individuals may have atypical vaginal bleeding, pelvic discomfort, pain during sexual intercourse, and abnormal vaginal discharge. Routine screening examinations, including the pap smear and HPV testing, are essential for promptly identifying cervical abnormalities and cancer. Cervical cancer, when identified at an early stage, can be effectively treated with surgeries, radiation therapy, and occasionally chemotherapy. Vaccination against HPV is a preventative action that can substantially decrease the possibility of finding cervical cancer. The HPV vaccine is commonly given to adolescents prior to the onset of sexual activity. Regular medical examinations, diagnostic tests, and immunization are crucial elements of cervical cancer prevention and timely response. The World Health Organisation (WHO) initiated the "Global Strategy to Accelerate the Elimination of Cervical Cancer" in 2020. The objective of this strategy is to attain a 90Fractional calculus is employed in various disciplines such as engineering, physics, biomedical engineering, earth sciences, control systems, signal processing, and finance. It allows for the simulation and examination of complicated systems, taking into consideration the influence of non-integer dynamics, long-range connections, and memory. The wide-ranging uses of this help to improve comprehension and prediction in multiple fields [4–6]. Computational mathematical models have been employed to simulate the dynamic behavior of infectious diseases in humanity. Historically, mathematical modeling and analysis have been used to study infectious diseases [7, 8]. Numerical mathematical models have been employed to simulate the dynamic behavior of infectious diseases in human beings. Historically, mathematical modeling and analysis have been used to study infectious diseases [9]. The most comprehensive mathematical models demonstrate how individuals navigate through different social sections, effectively describing the specific infectious scenario. The compartmental infection models classify individuals in a community based on their infectious status, and then simulate the general progression of the population over a period of time. These models are extensively utilized in the fields of engineering science, social science, and natural science [10, 11]. The numerical modeling of breakouts acknowledges that ODEs that are nonlinear may provide limited but useful insight into the spreading dynamics and dynamic behaviors of disease transmission. However, these equations are subject to certain limits and limitations regarding the order of the derivatives. Cancer is an intricate and ever-changing biological phenomenon that involves various levels of interactions inside the body. Cancer is intrinsically diverse, since patients display distinct reactions to therapies. Utilizing fractional-order derivatives in modeling provides a robust method for comprehending the intricate dynamics of cancer growth and response to treatment. Fractional calculus enables the integration of memory effects and long-range dependencies into mathematical models. In cancer modeling, the influence of past events and interactions between cells or tissues on future behavior is of utmost importance. The behavior of cancer tissues and cells can be more flexibly described using fractional-order models. By integrating fractional derivatives, models can more effectively reflect the history-dependent characteristics of cancer growth, resulting in more precise forecasts and deeper understanding. In this modern day, there are numerous techniques available for solving differential equations. However, our focus lies on the iterative Laplace transform method (ILTM) for solving the fractional cervical cancer model. This method is a sophisticated combination of the iterative method and the Laplace transform method. The discretization method is more flexible in its application compared to the Laplace transform method, and as a result, it exhibits good performance. This approach offers distinct advantages compared to Laplace transform approaches. The ILTM offers a solution through a fast-converging series that may result in a closed-form solution. The main benefit of this approach lies in its ability to integrate two effective techniques for achieving precise solutions to nonlinear fractional equations [12, 13]. Butt et al. [14] introduced a fractional compartmental model for cervical cancer using the Caputo derivative and examined the dynamic analysis using the nonstandard finite difference approach. A fractal-fractional cervical cancer model was proposed by akgül et al. [15]. Numerical simulations are used to calculate and validate the equilibrium point and basic reproduction number. Zafar et al. [16] examined the fractional-order dynamics of the human papillomavirus and explored how vaccination can help reduce the transmission of HPV infection. The fractional-order mathematical model is simulated using the next-generation approach. Simelane et al. [17] proposed a fractional order model to describe the dynamics of human HPV using the Caputo fractional derivative. Calculate and evaluate numerical solutions to the given fractional model using the Adams-Bashforth-Moulton methodology. Alshammari et al. [18] introduced a fractional model for tumor-immune surveillance. They applied the Laplace residual power series method, a numerical approach, to handle the nonlinearity and complexity of the equations. Mohammadpoor et al. [19] examined the breast cancer model with fractional order in chemotherapy patients using the Caputo-Fabrizio fractional order derivative. Numerical simulations using the Laplace Adomian decomposition method show that the breast cancer dynamics depend on the order of the fractional derivatives. The aim of this study is to provide an explanation for the factors connected with memory by extending the cervical cancer integer-order model into fractional order. By adding fractional operators, the model might be able to better show the complicated processes that happen as cervical cancer grows and spreads, such as tumor invasion, growth, and the body's response to treatment. This may lead to better insights and predictions about disease behavior. This paper's contribution is the novel use of fractional operators to model cervical cancer dynamics. This approach holds promise for enhancing our comprehension of the disease, facilitating clinical choices, and optimizing the results of treatment. To examine HPV transmission among humans, the generated fractional order model is analyzed using the Caputo-Fabrizio fractional operator. Caputo-Fabrizio derivative is a newly created fractional operator includes a nonsingular exponential kernel. Models involving the Caputo-Fabrizio derivative may be computationally more complex and offer greater flexibility and generalizations compared to those using the classical Caputo derivative. The application of the Caputo-Fabrizio derivative to model epidemic dynamics presents a number of benefits, such as its capability to account for non-local behaviors and memory effects. However, its implementation may be constrained by factors like complexity of computation, data accessibility, model validation prerequisites, and interpretability of the model. This allows us to determine how the fractional derivatives affect the behavior of the cervical cancer. Using a widely recognized method, we will conduct a comprehensive numerical analysis to effectively manage the prevalence of HPV in the community. The subsequent sections of this study article are as follows: Section 1 offers a thorough description of the complete introduction to the proposed model. For the purpose of solving the epidemic model, Section 2 provides some foundational fractional-order derivatives. Section 3 discusses the general and fractional forms of the cervical cancer model. The equilibria and basic reproduction number of the fractional cervical cancer model are calculated in Section 4. The discussion on the model's uniqueness and existence may be found in Section 5. Section 6 uses the Picard successive approximation method and the fixed point theory from Banach to look at the stability of the solutions that were found using the iterative Laplace transform method. For mathematical simulations of the fractional cervical cancer model described in Section 7, the proposed method has been applied. In Sections 8 and 9, respectively, the results and conclusion are detailed.

2. Fundamental explanation for the fractional operator

Definition 2.1 [20] For the time fractional Caputo-Fabrizio (CF) fractional differential operator, $\psi(t) \in H^1(0, b)$, where $b > 0$ and $0 < \sigma < 1$, the expression as follows:

$$^{CF}\Delta_t^{\sigma}\psi(\mathrm{t}) = \frac{(2-\sigma)\mathscr{N}(\sigma)}{2(1-\sigma)}\int_0^t \exp\left\{-\frac{\sigma(t-s)}{1-\sigma}\right\}\psi'(\varrho)d\varrho,\ t \geq 0,\ 0 < \sigma < 1. \tag{1}$$

The normalization function $\mathscr{N}(\sigma)$ is dependent on σ, with the condition that $\mathscr{N}(0) = \mathscr{N}(1) = 1$.
Definition 2.2 [20] With order $0 < \sigma < 1$, the Caputo−Fabrizio fractional integral operator is defined as

$$^{CF}\mathscr{I}_t^{\sigma}\psi(t) = \frac{2(1-\sigma)}{(2-\sigma)\mathscr{N}(\sigma)}\psi(t) + \frac{2\sigma}{(2-\sigma)\mathscr{N}(\sigma)}\int_0^t \psi(\varrho)d\varrho. \tag{2}$$

Similar to the conventional Caputo derivative, this new operator produces $^{CF}\Delta_t^{\sigma}\psi(t) = 0$, if $\psi(t)$ is a constant function. The primary benefit of the Caputo-Fabrizio operator over the original Caputo operator is that the new kernel is non-singular at $t = \mathfrak{s}$.

Definition 2.3 [20] For a Caputo-Fabrizio fractional operator of order $0 < \sigma < 1$ and $\vartheta \in \mathbb{N}$, the Laplace transform is defined as:

$$\begin{aligned}\Lambda\left({}^{CF}\Delta_t^{\sigma}\psi(\mathrm{t})\right)(\varnothing) &= \frac{1}{1-\sigma}\Lambda\left(\psi^{(\vartheta+1)}(t)\right)\Lambda\left\{\exp\left(-\frac{\sigma}{1-\sigma}t\right)\right\} \\ &= \frac{\varnothing^{\vartheta+1}\Lambda(\psi(\mathrm{t}))-\varnothing^{\vartheta}\psi(0)-\varnothing^{\vartheta-1}\psi'(0)-\ldots\ldots-\psi^{\vartheta}(0)}{\varnothing+\vartheta(1-\varnothing)}\end{aligned} \tag{3}$$

More precisely, we possess

$$\Lambda\left({}^{CF}\Delta_t^{\sigma}\psi(\mathrm{t})\right)(\varnothing) = \frac{\varnothing\Lambda(\psi(\mathrm{t}))}{\varnothing+\vartheta(1-\varnothing)}, \quad \vartheta = 0. \tag{4}$$

$$\Lambda\left({}^{CF}\Delta_t^{\sigma}\psi(\mathrm{t})\right)(\varnothing) = \frac{\varnothing^2\Lambda(\psi(\mathrm{t}))-\varnothing\psi(0)-\psi'(0)}{\varnothing+\vartheta(1-\varnothing)}, \quad \vartheta = 1. \tag{5}$$

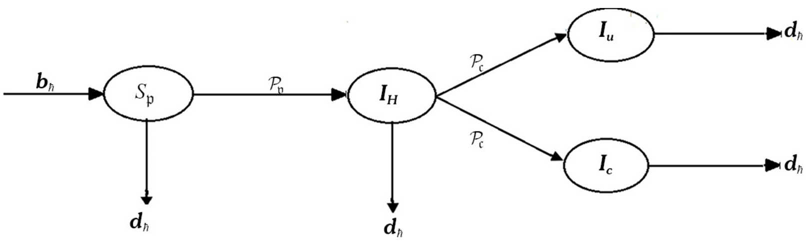

Figure 1. Model of cervical cancer's compartmental structure

3. Cervical cancer model

Utilizing abstract mathematical concepts to represent the potential behavior of a real-world scenario, a mathematical model is an intellectual process. Mathematical models can assist in making informed decisions about a particular process and analyzing functional connections. Furthermore, these models can also be used to predict the qualitative behavior of a system. As previously stated, the importance of mathematical modeling is considered when developing a model to represent the time dynamics of HPV through the described procedures.

Raza et al. [21] presented a mathematical model for cervical cancer based on a nonlinear ordinary differential equation. This study specifically examines the patterns and changes in the occurrence and distribution of cervical cancer. In order to accomplish this aim, we shall establish some symbols utilizing variables of state and parameters. Denoted as $\boldsymbol{S}_{\mathrm{p}}(t)$, $\boldsymbol{I}_{\boldsymbol{H}}(t)$, $\boldsymbol{I}_{\mathrm{c}}(t)$, and $\boldsymbol{I}_{\mathrm{u}}(t)$, and the variables are as follows: at any given time t, represent the "susceptible number of women," the "infected number of women with HPV," the "infectious number of HPV females affected by cervical cancer," and the "uninfected number of HPV females unaffected by cervical cancer," respectively. We divided the whole population into four divisions for representation. The expression $\boldsymbol{N}(t) = \boldsymbol{S}_{\mathrm{p}}(t) + \boldsymbol{I}_{\boldsymbol{H}}(t) + \boldsymbol{I}_{\mathrm{c}}(t) + \boldsymbol{I}_{\mathrm{u}}(t)$ represents the time dependent entire compartment. Figure 1 shows that either the state variable $\boldsymbol{I}_{\mathrm{u}}$ or $\boldsymbol{I}_{\mathrm{c}}$ can be eliminated due to the non-coupling effect. Finally, eliminate $\boldsymbol{I}_{\mathrm{u}}$ in the complete compartment expression. Right now, the complete compartment is written as $\mathscr{W}(t) = \boldsymbol{S}_{\mathrm{p}}(t) + \boldsymbol{I}_{\boldsymbol{H}}(t) + \boldsymbol{I}_{\mathrm{c}}(t)$.

$$
\begin{aligned}
\frac{d\boldsymbol{S}_{\mathrm{p}}}{dt} &= b_{\hbar} - \mathscr{P}_{\mathfrak{v}} \boldsymbol{S}_{\mathrm{p}} \boldsymbol{I}_{\boldsymbol{H}} - d_{\hbar} \boldsymbol{S}_{\mathrm{p}} \\
\frac{d\boldsymbol{I}_{\boldsymbol{H}}}{dt} &= \mathscr{P}_{\mathfrak{v}} \boldsymbol{S}_{\mathrm{p}} \boldsymbol{I}_{\boldsymbol{H}} - \mathscr{P}_{\mathrm{c}} \boldsymbol{I}_{\boldsymbol{H}} - d_{\hbar} \boldsymbol{I}_{\boldsymbol{H}}, \\
\frac{d\boldsymbol{I}_{\mathrm{c}}}{dt} &= \mathscr{P}_{\mathrm{c}} \boldsymbol{I}_{\boldsymbol{H}} - d_{\hbar} \boldsymbol{I}_{\mathrm{c}}.
\end{aligned} \tag{6}
$$

Under the primary conditions, however:

$$
\boldsymbol{S}_{\mathrm{p}}(0) \geq 0,\ \boldsymbol{I}_{\boldsymbol{H}}(0) \geq 0,\ \boldsymbol{I}_{c}(0) \geq 0 \tag{7}
$$

The parameter descriptions and numerical values are displayed in Table 1, while Table 2 contains the parameter values.

Table 1. The model's parameters are given

Parameters	Descriptions
$b_{\hbar}$	Human birth rate
$d_{\hbar}$	Human population rate of Death
$\mathscr{P}_{\mathfrak{v}}$	Probability of HPV infection among women
$\mathscr{P}_{\mathrm{c}}$	Cervical cancer risk of death among women

In order to investigate the memory effects and gain a deeper understanding of the spreading of cervical cancer, we enhance the system (6) by including Caputo-Fabrizio fractional derivative. Hence, we suggest a revised cervical cancer model including the Caputo-Fabrizio operator, which is defined as follows:

$$
\begin{aligned}
{}^{CF}\Delta_t^{\sigma} \boldsymbol{S}_{\mathrm{p}}(t) &= b_{\hbar} - \mathscr{P}_{\mathfrak{v}} \boldsymbol{S}_{\mathrm{p}} \boldsymbol{I}_{\boldsymbol{H}} - d_{\hbar} \boldsymbol{S}_{\mathrm{p}}, \\
{}^{CF}\Delta_t^{\sigma} \boldsymbol{I}_{\boldsymbol{H}}(t) &= \mathscr{P}_{\mathfrak{D}} \boldsymbol{S}_{\mathrm{p}} \boldsymbol{I}_{\boldsymbol{H}} - \mathscr{P}_{\mathrm{c}} \boldsymbol{I}_{\boldsymbol{H}} - d_{\hbar} \boldsymbol{I}_{\boldsymbol{H}}, \\
{}^{CF}\Delta_t^{\sigma} \boldsymbol{I}_{\mathrm{c}}(t) &= \mathscr{P}_{\mathrm{c}} \boldsymbol{I}_{\boldsymbol{H}} - d_{\hbar} \boldsymbol{I}_{\mathrm{c}}.
\end{aligned} \tag{8}
$$

with the initial conditions

$$
\boldsymbol{S}_{\boldsymbol{p}}(0) = \boldsymbol{S}_{\boldsymbol{p}}^{0} \geq 0,\ \boldsymbol{I}_{\boldsymbol{H}}(0) = \boldsymbol{I}_{\boldsymbol{H}}^{0} \geq 0,\ \boldsymbol{I}_{\mathrm{c}}(0) = \boldsymbol{I}_{c}^{0} \geq 0. \tag{9}
$$

The operator ${}^{CF}\Delta_t^{\sigma}$ represents a fractional-order derivative using Caputo-Fabrizio operator with an order between 0 and 1 ($0 < \sigma < 1$), for any $t \geq 0$. The biologically feasible zone for model (3) could be represented as

$$\Pi = \left\{ \boldsymbol{S}_{\mathrm{p}}(t),\ \boldsymbol{I}_{\boldsymbol{H}}(t),\ \boldsymbol{I}_{\mathrm{c}}(t) \in \mathrm{R}_{+}^{3} : 0 < \boldsymbol{N}(\mathrm{t}) \leq \frac{b_{\hbar}}{d_{\hat{\hbar}}},\ \boldsymbol{S}_{\mathrm{p}}(t),\ \boldsymbol{I}_{\boldsymbol{H}}(t),\ \boldsymbol{I}_{\mathrm{c}}(t) \geq 0 \right\}. \tag{10}$$

Table 2. The parameters of the suggested model are described

Parameters	Values	Source
$b_{\hbar}$	0.1	[1, 2]
$d_{\hbar}$	0.1	[1, 2]
$\mathscr{P}_{\mathfrak{v}}$	0.6, 1.6	[1, 2]
$\mathscr{P}_{\mathfrak{c}}$	0.7	[1, 2]

4. Qualitative analysis of system

This section focuses on identifying the equilibria and the basic reproduction number of the controlled Caputo-Fabrizio fractional cervical cancer model. Disease-present equilibrium and disease-free equilibrium are the two kinds of equilibrium points. The disease-free equilibrium point of the cervical cancer model is denoted as $\boldsymbol{E}^0\left(\boldsymbol{S}_{\mathrm{p}}^0,\ \boldsymbol{I}_{\boldsymbol{H}}^0,\ \boldsymbol{I}_{\mathrm{c}}^0\right)$ and is determined by equating the right-hand side of model (8) to zero.

$$\begin{aligned}
&b_{\hbar} - \mathscr{P}_{\mathfrak{v}} \boldsymbol{S}_{\mathrm{p}} \boldsymbol{I}_{\boldsymbol{H}} - d_{\hbar} \boldsymbol{S}_{\mathrm{p}} = 0,\\
&\mathscr{P}_{\mathfrak{v}} \boldsymbol{S}_{\mathrm{p}} \boldsymbol{I}_{\boldsymbol{H}} - \mathscr{P}_{\mathfrak{c}} \boldsymbol{I}_{\boldsymbol{H}} - d_{\hbar} \boldsymbol{I}_{\boldsymbol{H}} = 0,\\
&\mathscr{P}_{\mathfrak{c}} \boldsymbol{I}_{\boldsymbol{H}} - d_{\hbar} \boldsymbol{I}_{\mathrm{c}} = 0.
\end{aligned} \tag{11}$$

To get the disease-free equilibrium point of the cervical cancer model, we set $\boldsymbol{S}_{\mathrm{p}} = 0$ and $\boldsymbol{I}_{\boldsymbol{H}} = 0 = \boldsymbol{I}_{\mathrm{c}}$. This yields the following result.

$$\boldsymbol{E}^0\left(\boldsymbol{S}_{p}^0,\ \boldsymbol{I}_{H}^0,\ \boldsymbol{I}_{c}^0\right) = \boldsymbol{E}^0\left(\frac{b_{\hbar}}{d_{\hbar}},\ 0,\ 0\right), \tag{12}$$

and the following expression describes the cervical-present equilibrium point $\boldsymbol{E}^*\left(\boldsymbol{S}_{\mathrm{p}}^*,\ \boldsymbol{I}_{\boldsymbol{H}}^*,\ \boldsymbol{I}_{\mathrm{c}}^*\right)$:

$$\boldsymbol{S}_{\mathrm{p}}^* = \frac{1}{d_{\hbar}}\left[b_{\hbar} - (\mathscr{P}_{\mathfrak{c}} + d_{\hbar}) \boldsymbol{I}_{\boldsymbol{H}}^*\right] > 0, \tag{13}$$

$$\boldsymbol{I}_{\boldsymbol{H}}^* = \frac{d_{\hbar}}{\mathscr{P}_{\mathfrak{v}}}\left[\frac{\mathscr{P}_{\mathfrak{v}}}{\mathscr{P}_{\mathfrak{c}} + d_{\hbar}} - 1\right] > 0, \tag{14}$$

$$\boldsymbol{I}_{\mathrm{c}}^* = \frac{\mathscr{P}_{\mathfrak{c}}}{\mathscr{P}_{\mathfrak{v}}} \boldsymbol{I}_{\boldsymbol{H}}^* > 0, \tag{15}$$

in the feasible zone Π.

The next-generation matrix method carries out the following formulation for the basic reproductive number, $\mathscr{R}_0$, of the system (8):

$$^{CF}\Delta_t^{\sigma}\Gamma = \mathscr{F}(\Gamma) - \mathscr{V}(\Gamma),$$

where $\Gamma = (\boldsymbol{I}_H, \boldsymbol{I}_\mathfrak{c})$,

$$\mathscr{F} = \begin{pmatrix} \mathscr{P}_\mathfrak{v}\boldsymbol{S}_\mathfrak{p}\boldsymbol{I_H} \\ 0 \end{pmatrix}, \ \mathscr{V} = \begin{pmatrix} -\mathscr{P}_\mathfrak{c}\boldsymbol{I_H} - d_\hbar \boldsymbol{I_H} \\ \mathscr{P}_\mathfrak{c}\boldsymbol{I_H} - d_\hbar \boldsymbol{I}_\mathfrak{c} \end{pmatrix}. \tag{16}$$

Jacobian matrix for $\mathscr{F}$ and $\mathscr{V}$ at $\boldsymbol{E}^0\left(\frac{b_\hbar}{d_\hbar}, 0, 0\right)$ is as follows:

$$\mathbb{J}_\mathscr{F}\left(\boldsymbol{E}^0\right) = \begin{pmatrix} \frac{b_\hbar \mathscr{P}_\mathfrak{v}}{d_\hbar} & 0 \\ 0 & 0 \end{pmatrix}, \ \mathbb{J}_\mathscr{V}\left(\boldsymbol{E}^0\right) = \begin{pmatrix} -\mathscr{P}_\mathfrak{c} - d_\hbar & 0 \\ \mathscr{P}_\mathfrak{c} & -d_\hbar \end{pmatrix}.$$

The main Eigen value $\mathscr{R}_0$ of the product matrix $\mathbb{J}_\mathscr{F}\mathbb{J}_\mathscr{V}^{-1}$ is defined as

$$\mathscr{R}_0 = \frac{b_\hbar \mathscr{P}_\mathfrak{v}}{d_\hbar\left(\mathscr{P}_\mathfrak{c} + d_\hbar\right)}. \tag{17}$$

5. Uniqueness and existence of the suggested system

In this part, we establish the uniqueness and existence of the solution to the system through the use of the fixed point theory.

From equation (2), we deduce that

$$\boldsymbol{S}_\mathfrak{p}(t) = \boldsymbol{S}_\mathfrak{p}(0) + \frac{2(1-\sigma)}{2\sigma\mathscr{N}(\sigma)}\{b_\hbar - \mathscr{P}_\mathfrak{v}\boldsymbol{S}_\mathfrak{p}\boldsymbol{I_H} - d_\hbar\boldsymbol{S}_\mathfrak{p}\} + \frac{2(1-\sigma)}{(2-\sigma)\mathscr{N}(\sigma)}\int_0^t \{b_\hbar - \mathscr{P}_\mathfrak{v}\boldsymbol{S}_\mathfrak{p}\boldsymbol{I_H} - d_\hbar\boldsymbol{S}_\mathfrak{p}\}\, d\varrho,$$

$$\boldsymbol{I_H}(t) = \boldsymbol{I_H}(0) + \frac{2(1-\sigma)}{2\sigma\mathscr{N}(\sigma)}\{\mathscr{P}_\mathfrak{v}\boldsymbol{S}_\mathfrak{p}\boldsymbol{I_H} - \mathscr{P}_\mathfrak{c}\boldsymbol{I_H} - d_\hbar\boldsymbol{I_H}\} + \frac{2(1-\sigma)}{(2-\sigma)\mathscr{N}(\sigma)}\int_0^t \{\mathscr{P}_\mathfrak{v}\boldsymbol{S_p}\boldsymbol{I_H} - \mathscr{P}_\mathfrak{c}\boldsymbol{I_H} - d_\hbar\boldsymbol{I_H}\}\, d\varrho,$$

$$\boldsymbol{I}_\mathfrak{c}(t) = \boldsymbol{I}_\mathfrak{c}(0) + \frac{2(1-\sigma)}{2\sigma\mathscr{N}(\sigma)}\{\mathscr{P}\boldsymbol{I_H} - d_\hbar\boldsymbol{I}_\mathfrak{c}\} + \frac{2(1-\sigma)}{(2-\sigma)\mathscr{N}(\sigma)}\int_0^t \{\mathscr{P}_\mathfrak{c}\boldsymbol{I_H} - d_\hbar\boldsymbol{I}_\mathfrak{c}\}\, d\varrho.$$

We will now examine the subsequent kernels:

$$
\begin{aligned}
&\varphi_1 = b_{\hbar} - \mathscr{P}_{\mathfrak{v}} \boldsymbol{S}_{\mathfrak{p}} \boldsymbol{I}_{\boldsymbol{H}} - d_{\hbar} \boldsymbol{S}_{\mathfrak{p}} \\
&\varphi_2 = \mathscr{P}_{\mathfrak{v}} \boldsymbol{S}_{\mathfrak{p}} \boldsymbol{I}_{\boldsymbol{H}} - \mathscr{P}_{\mathfrak{c}} \boldsymbol{I}_{\boldsymbol{H}} - d_{\hbar} \boldsymbol{I}_{\boldsymbol{H}}, \\
&\varphi_3 = \mathscr{P}_{\mathfrak{c}} \boldsymbol{I}_{\boldsymbol{H}} - d_{\hbar} \boldsymbol{I}_{\mathfrak{c}}.
\end{aligned}
\tag{18}
$$

Theorem 5.1 The kernels φ_1, φ_2 and φ_3 represented in equation (18) satisfy the Lipschitz condition if the subsequent inequality is satisfied.

$$
0 < \Psi_1,\ \Psi_2,\ \Psi_3 < 1
$$

Proof. The functions $\boldsymbol{S}_{\mathfrak{p}}^1$ and $\boldsymbol{S}_{\mathfrak{p}}^2$ correspondence to the kernel φ_1 and the functions $\boldsymbol{I}_{\boldsymbol{H}}^1$ and $\boldsymbol{I}_{\boldsymbol{H}}^2$ correspondence to the kernel φ_2 and the functions $\boldsymbol{I}_{\mathfrak{c}}^1$ and $\boldsymbol{I}_{\mathfrak{c}}^2$ correspondence to the kernel φ_3.

$$
\begin{aligned}
\left\|\varphi_1\left(t,\, \boldsymbol{S}_{\mathfrak{p}}^1(t)\right) - \varphi_1\left(t,\, \boldsymbol{S}_{\mathfrak{p}}^2(t)\right)\right\| &= \left\|\left\{b_{\hbar} - \mathscr{P}_{\mathfrak{v}} \boldsymbol{S}_{\mathfrak{p}}^1(t) \boldsymbol{I}_{\boldsymbol{H}} - d_{\hbar} \boldsymbol{S}_{\mathfrak{p}}^1(t)\right\} - \left\{b_{\hbar} - \mathscr{P}_{\mathfrak{v}} \boldsymbol{S}_{\mathfrak{p}}^2(t) \boldsymbol{I}_{\boldsymbol{H}} - d_{\hbar} \boldsymbol{S}_{\mathfrak{p}}^2(t)\right\}\right\| \\
&= \left\|b_{\hbar} - \mathscr{P}_{\mathfrak{v}} \boldsymbol{S}_{\mathfrak{p}}^1(t) \boldsymbol{I}_{\boldsymbol{H}} - d_{\hbar} \boldsymbol{S}_{\mathfrak{p}}^1(t) - b_{\hbar} + \mathscr{P}_{\mathfrak{v}} \boldsymbol{S}_{\mathfrak{p}}^2(t) \boldsymbol{I}_{\boldsymbol{H}} + d_{\hbar} \boldsymbol{S}_{\mathfrak{p}}^2(t)\right\| \\
&= \left\|-\mathscr{P}_{\mathfrak{v}}\left(\boldsymbol{S}_{\mathfrak{p}}^1(t) - \boldsymbol{S}_{\mathfrak{p}}^2(t)\right) \boldsymbol{I}_{\boldsymbol{H}} - d_{\hbar}\left(\boldsymbol{S}_{\mathfrak{p}}^1(t) - \boldsymbol{S}_{\mathfrak{p}}^2(t)\right)\right\| \\
&= \left\|-\left(\mathscr{P}_{\mathfrak{v}} \boldsymbol{I}_{\boldsymbol{H}} + d_{\hbar}\right)\left(\boldsymbol{S}_{\mathfrak{p}}^1(t) - \boldsymbol{S}_{\mathfrak{p}}^2(t)\right)\right\|
\end{aligned}
$$

$$
\left\|\varphi_1\left(t,\, \boldsymbol{S}_{\mathfrak{p}}^1(t)\right) - \varphi_1\left(t,\, \boldsymbol{S}_{\mathfrak{p}}^2(t)\right)\right\| = \left\|-\left(\mathscr{P}_{\mathfrak{v}} \boldsymbol{I}_{\boldsymbol{H}} + d_{\hbar}\right)\left(\boldsymbol{S}_{\mathfrak{p}}^1(t) - \boldsymbol{S}_{\mathfrak{p}}^2(t)\right)\right\|. \tag{19}
$$

and similarities

$$
\left\|\varphi_2\left(t,\, \boldsymbol{I}_{\boldsymbol{H}}^1(t)\right) - \varphi_2\left(t,\, \boldsymbol{I}_{\boldsymbol{H}}^2(t)\right)\right\| = \left\|\left(\mathscr{P}_{\mathfrak{v}} \boldsymbol{S}_{\mathfrak{p}} - \mathscr{P}_{\mathfrak{c}} - d_{\hbar}\right)\left(\boldsymbol{I}_{\boldsymbol{H}}^1(t) - \boldsymbol{I}_{\boldsymbol{H}}^2(t)\right)\right\|. \tag{20}
$$

$$
\left\|\varphi_3\left(t,\, \boldsymbol{I}_{\mathfrak{c}}^1(t)\right) - \varphi_3\left(t,\, \boldsymbol{I}_{\mathfrak{c}}^2(t)\right)\right\| = \left\|-d_{\hbar}\left(\boldsymbol{I}_{\mathfrak{c}}^1(t) - \boldsymbol{I}_{\mathfrak{c}}^2(t)\right)\right\|. \tag{21}
$$

We now assume

$$
\begin{aligned}
&\left\|\varphi_1\left(t,\ \boldsymbol{S}_{\mathfrak{p}}^1(t)\right)-\varphi_1\left(t,\ \boldsymbol{S}_{\mathfrak{p}}^2(t)\right)\right\|=\left\|-\left(\mathscr{P}_{\mathfrak{v}}\boldsymbol{I}_{\boldsymbol{H}}+d_\hbar\right)\left(\boldsymbol{S}_{\mathfrak{p}}^1(t)-\boldsymbol{S}_{\mathfrak{p}}^2(t)\right)\right\|\\
&\left\|\varphi_1\left(t,\ \boldsymbol{S}_{\mathfrak{p}}^1(t)\right)-\varphi_1\left(t,\ \boldsymbol{S}_{\mathfrak{p}}^2(t)\right)\right\|\leq\left(\mathscr{P}_{\mathfrak{v}}\left\|\boldsymbol{I}_{\boldsymbol{H}}\right\|+d_\hbar\right)\left\|\left(\boldsymbol{S}_{\mathfrak{p}}^1(t)-\boldsymbol{S}_{\mathfrak{p}}^2(t)\right)\right\|\\
&\left\|\varphi_1\left(t,\ \boldsymbol{S}_{\mathfrak{p}}^1(t)\right)-\varphi_1\left(t,\ \boldsymbol{S}_{\mathfrak{p}}^2(t)\right)\right\|\leq\left(\mathscr{P}_{\mathfrak{v}}\mathfrak{i}_H+d_\hbar\right)\left\|\left(\boldsymbol{S}_{\mathfrak{p}}^1(t)-\boldsymbol{S}_{\mathfrak{p}}^2(t)\right)\right\|\\
&\left\|\varphi_1\left(t,\ \boldsymbol{S}_{\mathfrak{p}}^1(t)\right)-\varphi_1\left(t,\ \boldsymbol{S}_{\mathfrak{p}}^2(t)\right)\right\|\leq\Psi_1\left\|\left(\boldsymbol{S}_{\mathfrak{p}}^1(t)-\boldsymbol{S}_{\mathfrak{p}}^2(t)\right)\right\|.
\end{aligned}
\tag{22}
$$

here $\Psi_1=\mathscr{P}_{\mathfrak{v}}\mathfrak{i}_{\boldsymbol{H}}+d_\hbar,\ \mathfrak{i}_{\boldsymbol{H}}=\max_{t\varepsilon\gamma}\left\|\boldsymbol{I}_{\boldsymbol{H}}\right\|$ are bounded function. Likewise we are able to gain

$$
\left\|\varphi_2\left(t,\ \boldsymbol{I}_{\boldsymbol{H}}^1(t)\right)-\varphi_2\left(t,\ \boldsymbol{I}_{\boldsymbol{H}}^2(t)\right)\right\|\leq\Psi_2\left\|\left(\boldsymbol{I}_{\boldsymbol{H}}^1(t)-\boldsymbol{I}_{\boldsymbol{H}}^2(t)\right)\right\|. \tag{23}
$$

$$
\left\|\varphi_3\left(t,\ \boldsymbol{I}_{\mathfrak{c}}^1(t)(t)\right)-\varphi_3\left(t,\ \boldsymbol{I}_{\mathfrak{c}}^2(t)\right)\right\|\leq\Psi_3\left\|\left(\boldsymbol{I}_{\mathfrak{c}}^1(t)-\boldsymbol{I}_{\mathfrak{c}}^2(t)\right)\right\|. \tag{24}
$$

where $\Psi_2=\mathscr{P}_{\mathfrak{v}}\mathfrak{s}_{\mathfrak{p}}-\mathscr{P}_{\mathfrak{c}}-d_\hbar,\ \mathfrak{s}_{\mathfrak{p}}=\max_{t\varepsilon\gamma}\left\|S_{\mathfrak{p}}\right\|$ & $\Psi_3=d_\hbar$.

By employing the method of recursion provided, we get

$$
\begin{cases}
\boldsymbol{S}_{\mathfrak{p}}^n(t)=\frac{2(1-\sigma)}{(2-\sigma)\mathscr{N}(\sigma)}\varphi_1\left(t,\ \boldsymbol{S}_{\mathfrak{p}}^{n-1}(t)\right)+\frac{2\sigma}{(2-\sigma)\mathscr{N}(\sigma)}\int_0^t\varphi_1\left(\varrho,\ \boldsymbol{S}_{\mathfrak{p}}^{n-1}(\varrho)\right)d\varrho,\\
\boldsymbol{I}_{\boldsymbol{H}}^n(t)=\frac{2(1-\sigma)}{(2-\sigma)\mathscr{N}(\sigma)}\varrho_2\left(t,\ \boldsymbol{I}_{\boldsymbol{H}}^{n-1}(t)\right)+\frac{2\sigma}{(2-\sigma)\mathscr{N}(\sigma)}\int_0^t\varphi_2\left(\varrho,\ \boldsymbol{I}_{\boldsymbol{H}}^{n-1}(\varrho)\right)d\varrho,\\
\boldsymbol{I}_{\mathfrak{c}}^n(t)=\frac{2(1-\sigma)}{(2-\sigma)\mathscr{N}(\sigma)}\varphi_3\left(t,\ \boldsymbol{I}_{\mathfrak{c}}^{n-1}(t)\right)+\frac{2\sigma}{(2-\sigma)\mathscr{N}(\sigma)}\int_0^t\varphi_3\left(\varrho,\ \boldsymbol{I}_{\mathfrak{c}}^{n-1}(\varrho)\right)d\varrho.
\end{cases}
\tag{25}
$$

Furthermore, by applying triangular inequality, we can derive

$$
\begin{cases}
\|\mathscr{M}_1^n\| = \left\|\boldsymbol{S}_{\mathrm{p}}^n(t) - \boldsymbol{S}_{\mathrm{p}}^{n-1}(t)\right\| \leq \dfrac{2(1-\sigma)}{(2-\sigma)\mathscr{N}(\sigma)} \left\|\varphi_1\left(t,\, \boldsymbol{S}_{\mathrm{p}}^{n-1}(t)\right) - \varphi_1\left(t,\, \boldsymbol{S}_{\mathrm{p}}^{n-2}(t)\right)\right\| \\[2ex]
\qquad + \dfrac{2\sigma}{(2-\sigma)\mathscr{N}(\sigma)} \left\|\displaystyle\int_0^t \left\{\varphi_1\left(\varrho,\, \boldsymbol{S}_{\mathrm{p}}^{n-1}(\varrho)\right) - \varphi_1\left(\varrho,\, \boldsymbol{S}_{\mathrm{p}}^{n-2}(\varrho)\right)\right\} d\varrho\right\|, \\[2ex]
\|\mathscr{M}_2^n\| = \left\|\boldsymbol{I}_{\boldsymbol{H}}^n(t) - \boldsymbol{I}_{\boldsymbol{H}}^{n-1}(t)\right\| \leq \dfrac{2(1-\sigma)}{(2-\sigma)\mathscr{N}(\sigma)} \left\|\varphi_2\left(t,\, \boldsymbol{I}_{\boldsymbol{H}}^{n-1}(t)\right) - \varphi_2\left(t,\, \boldsymbol{I}_{\boldsymbol{H}}^{n-2}(t)\right)\right\| \\[2ex]
\qquad + \dfrac{2\sigma}{(2-\sigma)\mathscr{N}(\sigma)} \left\|\displaystyle\int_0^t \left\{\varphi_2\left(\varrho,\, \boldsymbol{I}_{\boldsymbol{H}}^{n-1}(\varrho)\right) - \varphi_2\left(\varrho,\, \boldsymbol{I}_{\boldsymbol{H}}^{n-2}(\varrho)\right)\right\} d\varrho\right\|, \\[2ex]
\|\mathscr{M}_3^n\| = \left\|\boldsymbol{I}_{\mathrm{c}}^n(t) - \boldsymbol{I}_{\mathrm{c}}^{n-1}(t)\right\| \leq \dfrac{2(1-\sigma)}{(2-\sigma)\mathscr{N}(\sigma)} \left\|\varphi_3\left(t,\, \boldsymbol{I}_{\mathrm{c}}^{n-1}(t)\right) - \varphi_3\left(t,\, \boldsymbol{I}_{\mathrm{c}}^{n-2}(t)\right)\right\| \\[2ex]
\qquad + \dfrac{2\sigma}{(2-\sigma)\mathscr{N}(\sigma)} \left\|\displaystyle\int_0^t \left\{\varphi_3\left(\varrho,\, \boldsymbol{I}_{\mathrm{c}}^{n-1}(\varrho)\right) - \varphi_3\left(\varrho,\, \boldsymbol{I}_{\mathrm{c}}^{n-2}(\varrho)\right)\right\} d\varrho\right\|.
\end{cases} \tag{26}
$$

$$
\boldsymbol{S}_{\mathrm{p}}^n(t) = \sum_{m=0}^{\infty} \mathscr{M}_1^m(t),\; \boldsymbol{I}_{\boldsymbol{H}}^n(t) = \sum_{m=0}^{\infty} \mathscr{M}_2^m(t),\; \boldsymbol{I}_{\mathrm{c}}^n(t) = \sum_{m=0}^{\infty} \mathscr{M}_3^m(t). \tag{27}
$$

Given that the kernels φ_1, φ_2 and φ_3 satisfy the Lipchitz condition, we can deduce

$$
\begin{cases}
\|\mathscr{M}_1^n\| \leq \dfrac{2(1-\sigma)}{(2-\sigma)\mathscr{N}(\sigma)}\Psi_1 \left\|\boldsymbol{S}_{\mathrm{p}}^{n-1}(t) - \boldsymbol{S}_{\mathrm{p}}^{n-2}(t)\right\| + \dfrac{2\sigma}{(2-\sigma)\mathscr{N}(\sigma)}\Psi_1 \displaystyle\int_0^t \left\|\boldsymbol{S}_{\mathrm{p}}^{n-1}(\varrho) - \boldsymbol{S}_{\mathrm{p}}^{n-2}(\varrho)\right\| d\varrho, \\[2ex]
\|\mathscr{M}_2^n\| \leq \dfrac{2(1-\sigma)}{(2-\sigma)\mathscr{N}(\sigma)}\Psi_2 \left\|\boldsymbol{I}_{\boldsymbol{H}}^{n-1}(t) - \boldsymbol{I}_{\boldsymbol{H}}^{n-2}(t)\right\| + \dfrac{2\sigma}{(2-\sigma)\mathscr{N}(\sigma)}\Psi_2 \displaystyle\int_0^t \left\|\boldsymbol{I}_{\boldsymbol{H}}^{n-1}(\varrho) - \boldsymbol{I}_{\boldsymbol{H}}^{n-2}(\varrho)\right\| d\varrho, \\[2ex]
\|\mathscr{M}_3^n\| \leq \dfrac{2(1-\sigma)}{(2-\sigma)\mathscr{N}(\sigma)}\Psi_3 \left\|\boldsymbol{I}_{\mathrm{c}}^{n-1}(t) - \boldsymbol{I}_{\mathrm{c}}^{n-2}(t)\right\| + \dfrac{2\sigma}{(2-\sigma)\mathscr{N}(\sigma)}\Psi_3 \displaystyle\int_0^t \left\|\boldsymbol{I}_{\mathrm{c}}^{n-1}(\varrho) - \boldsymbol{I}_{\mathrm{c}}^{n-2}(\varrho)\right\| d\varrho.
\end{cases} \tag{28}
$$

Thus proving the result.

5.1 *Existence of the suggested system's solution*

Theorem 5.2 The system has a solution, which is mentioned in (8).

Proof. Based on the formula for recursion and the result of equation (28), the following has been discovered:

$$
\begin{cases}
\|\mathscr{M}_1^n\| \leq \|\boldsymbol{S}_{\mathrm{p}}(0)\| + \left\{ \left(\dfrac{2(1-\sigma)}{(2-\sigma)\mathscr{N}(\sigma)} \Psi_1 \right)^n + \left(\dfrac{2\sigma}{(2-\sigma)\mathscr{N}(\sigma)} \Psi_1^t \right)^n \right\}, \\[2ex]
\|\mathscr{M}_2^n\| \leq \|\boldsymbol{I_H}(0)\| + \left\{ \left(\dfrac{2(1-\sigma)}{(2-\sigma)\mathscr{N}(\sigma)} \Psi_2 \right)^n + \left(\dfrac{2\sigma}{(2-\sigma)\mathscr{N}(\sigma)} \Psi_2^t \right)^n \right\}, \\[2ex]
\|\mathscr{M}_3^n\| \leq \|\boldsymbol{I}_{\mathrm{c}}(0)\| + \left\{ \left(\dfrac{2(1-\sigma)}{(2-\sigma)\mathscr{N}(\sigma)} \Psi_2 \right)^n + \left(\dfrac{2\sigma}{(2-\sigma)\mathscr{N}(\sigma)} \Psi_3^t \right)^n \right\}.
\end{cases} \tag{29}
$$

Consequently, there are equation (29) in existence. Furthermore, we establish that the functions in equation (29) are components of the solution set described in equation (8).

$$
\boldsymbol{S}_{\mathrm{p}}(t) = \boldsymbol{S}_{\mathrm{p}}^n(t) - \theta_{1(n)}(t),\ \boldsymbol{I_H}(t) = \boldsymbol{I_H^n}(t) - \theta_{2(n)}(t),\ \boldsymbol{I}_{\mathrm{c}}(t) = \boldsymbol{I}_{\mathbf{c}}^n(t) - \theta_{3(n)}(t). \tag{30}
$$

The solution's remaining terms are denoted as $\theta_{1(n)}(t)$, $\theta_{2(n)}(t)$ and $\theta_{3(n)}(t)$. Consequently, we gain

$$
\begin{cases}
\boldsymbol{S}_{\mathrm{p}}(t) - \boldsymbol{S}_{\mathrm{p}}^{n-1}(t) = \dfrac{2(1-\sigma)}{(2-\sigma)\mathscr{N}(\sigma)} \varphi_1\left(t,\ \boldsymbol{S}_{\mathrm{p}}(t) - \theta_{1(n)}(t)\right) \\[2ex]
\qquad + \dfrac{2\sigma}{(2-\sigma)\mathscr{N}(\sigma)} \displaystyle\int_0^t \varphi_1\left(\varrho,\ \boldsymbol{S}_{\mathrm{p}}(\varrho) - \theta_{1(n)}(\varrho)\right) d\varrho, \\[2ex]
\boldsymbol{I_H}(t) - \boldsymbol{I_H^{n-1}}(t) = \dfrac{2(1-\sigma)}{(2-\sigma)\mathscr{N}(\sigma)} \varphi_2\left(t,\ \boldsymbol{I_H}(t) - \theta_{2(n)}(t)\right) \\[2ex]
\qquad + \dfrac{2\sigma}{(2-\sigma)\mathscr{N}(\sigma)} \displaystyle\int_0^t \varphi_1\left(\varrho,\ \boldsymbol{I_H}(\varrho) - \theta_{2(n)}(\varrho)\right) d\varrho, \\[2ex]
\boldsymbol{I}_{\mathrm{c}}(t) - \boldsymbol{I}_{\mathbf{c}}^{n-1}(t) = \dfrac{2(1-\sigma)}{(2-\sigma)\mathscr{N}(\sigma)} \varphi_3\left(t,\ \boldsymbol{I}_{\mathrm{c}}(t) - \theta_{3(n)}(t)\right) \\[2ex]
\qquad + \dfrac{2\sigma}{(2-\sigma)\mathscr{N}(\sigma)} \displaystyle\int_0^t \varphi_1\left(\varrho,\ \boldsymbol{I}_{\mathrm{c}}(\varrho) - \theta_{3(n)}(\varrho)\right) d\varrho.
\end{cases} \tag{31}
$$

The norm and the Lipschitz condition have been derived through the application of

$$
\begin{cases}
\boldsymbol{S}_{\mathrm{p}}(t)-\boldsymbol{S}_{\mathrm{p}}(0)-\dfrac{2(1-\sigma)}{(2-\sigma)\mathcal{N}(\sigma)}\varphi_1(t,\,\boldsymbol{S}_{\mathrm{p}}(t))-\dfrac{2\sigma}{(2-\sigma)\mathcal{N}(\sigma)}\displaystyle\int_0^t\varphi_1(\varrho,\,\boldsymbol{S}_{\mathrm{p}}(\varrho))\,d\varrho \\
\leq \left\|\theta_{1(n)}(t)\right\|\left\{1+\left(\dfrac{2(1-\sigma)}{(2-\sigma)\mathcal{N}(\sigma)}\Psi_1\right)+\left(\dfrac{2\sigma}{(2-\sigma)\mathcal{N}(\sigma)}\Psi_1^t\right)\right\}, \\
\boldsymbol{I_H}(t)-\boldsymbol{I_H}(0)-\dfrac{2(1-\sigma)}{(2-\sigma)\mathcal{N}(\sigma)}\varphi_2(t,\,\boldsymbol{I_H}(t))-\dfrac{2\sigma}{(2-\sigma)\mathcal{N}(\sigma)}\displaystyle\int_0^t\varphi_2(\varrho,\,\boldsymbol{I_H}(\varrho))\,d\varrho \\
\leq \left\|\theta_{2(n)}(t)\right\|\left\{1+\left(\dfrac{2(1-\sigma)}{(2-\sigma)\mathcal{N}(\sigma)}\Psi_2\right)+\left(\dfrac{2\sigma}{(2-\sigma)\mathcal{N}(\sigma)}\Psi_2^t\right)\right\}, \\
\boldsymbol{I}_{\mathrm{c}}(t)-\boldsymbol{I}_{\mathrm{c}}(0)-\dfrac{2(1-\sigma)}{(2-\sigma)\mathcal{N}(\sigma)}\varphi_3(t,\,\boldsymbol{I}_{\mathrm{c}}(t))-\dfrac{2\sigma}{(2-\sigma)\mathcal{N}(\sigma)}\displaystyle\int_0^t\varphi_3(\varrho,\,\boldsymbol{I}_{\mathrm{c}}(\varrho))\,d\varrho \\
\leq \left\|\theta_{3(n)}(t)\right\|\left\{1+\left(\dfrac{2(1-\sigma)}{(2-\sigma)\mathcal{N}(\sigma)}\Psi_3\right)+\left(\dfrac{2\sigma}{(2-\sigma)\mathcal{N}(\sigma)}\Psi_3^t\right)\right\}.
\end{cases}
\tag{32}
$$

By considering $\lim\limits_{n\to\infty}$ in equation (32), we obtain $\left\|\theta_{\mathrm{i}(n)}\right\|\to 0$ where $\mathrm{i}=1,\ 2,\ 3$ Therefore, we posses

$$
\begin{cases}
\boldsymbol{S}_{\mathrm{p}}(t)=\boldsymbol{S}_{\mathrm{p}}(0)+\dfrac{2(1-\sigma)}{(2-\sigma)\mathcal{N}(\sigma)}\varphi_1(t,\,\boldsymbol{S}_{\mathrm{p}}(t))+\dfrac{2\sigma}{(2-\sigma)\mathcal{N}(\sigma)}\displaystyle\int_0^t\varphi_1(\varrho,\,\boldsymbol{S}_{\mathrm{p}}(\varrho))\,d\varrho, \\
\boldsymbol{I_H}(t)=\boldsymbol{I_H}(0)+\dfrac{2(1-\sigma)}{(2-\sigma)\mathcal{N}(\sigma)}\varphi_2(t,\,\boldsymbol{I_H}(t))+\dfrac{2\sigma}{(2-\sigma)\mathcal{N}(\sigma)}\displaystyle\int_0^t\varphi_2(\varrho,\,\boldsymbol{I_H}(\varrho))\,d\varrho, \\
\boldsymbol{I}_{\mathrm{c}}(t)=\boldsymbol{I}_{\mathrm{c}}(0)+\dfrac{2(1-\sigma)}{(2-\sigma)\mathcal{N}(\sigma)}\varphi_3(t,\,\boldsymbol{I}_{\mathrm{c}}(t))+\dfrac{2\sigma}{(2-\sigma)\mathcal{N}(\sigma)}\displaystyle\int_0^t\varphi_3(\varrho,\,\boldsymbol{I}_{\mathrm{c}}(\varrho))\,d\varrho.
\end{cases}
\tag{33}
$$

In a similar way, when the limit $\lim\limits_{n\to\infty}$ we obtain $\left\|\theta_{\mathrm{i}(n)}(t)\right\|\to 0$ where $\mathrm{i}=1,\ 2,\ 3$ Consequently, a solution exists for (8) exist, similarly to that for equation (33).

5.2 *Suggested system's uniqueness solution*

Theorem 5.3 A distinctive solution exists for the model stated in (8).

Proof. If an alternative solution for system (8) exists, denoted as $\boldsymbol{S}_{\mathrm{p}}^*$, $\boldsymbol{I}_{\boldsymbol{H}}^*$ and $\boldsymbol{I}_{\mathrm{c}}^*$, then we get

$$
\begin{cases}
\boldsymbol{S}_{\mathrm{p}}(t)-\boldsymbol{S}_{\mathrm{p}}^{*}(0)=\dfrac{2(1-\sigma)}{(2-\sigma)\mathcal{N}(\sigma)}\left\{\varphi_1\left(t,\ \boldsymbol{S}_{\mathrm{p}}(t)\right)-\varphi_1\left(t,\ \boldsymbol{S}_{\mathrm{p}}^{*}(t)\right)\right\}\\
\qquad+\dfrac{2\sigma}{(2-\sigma)\mathcal{N}(\sigma)}\displaystyle\int_0^t\left\{\varphi_1\left(\varrho,\ \boldsymbol{S}_{\mathrm{p}}(\varrho)\right)-\varphi_1\left(\varrho,\ \boldsymbol{S}_{\mathrm{p}}^{*}(\varrho)\right)\right\}d\varrho,\\
\boldsymbol{I}_{\boldsymbol{H}}(t)-\boldsymbol{I}_{\boldsymbol{H}}^{*}(0)=\dfrac{2(1-\sigma)}{(2-\sigma)\mathcal{N}(\sigma)}\left\{\varphi_2\left(t,\ \boldsymbol{I}_{\boldsymbol{H}}(t)\right)-\varphi_2\left(t,\ \boldsymbol{I}_{\boldsymbol{H}}^{*}(t)\right)\right\}\\
\qquad+\dfrac{2\sigma}{(2-\sigma)\mathcal{N}(\sigma)}\displaystyle\int_0^t\left\{\varphi_2\left(\varrho,\ \boldsymbol{I}_{\boldsymbol{H}}(\varrho)\right)-\varphi_2\left(\varrho,\ \boldsymbol{I}_{\boldsymbol{H}}^{*}(\varrho)\right)\right\}d\varrho,\\
\boldsymbol{I}_{\mathrm{c}}(t)-\boldsymbol{I}_{\mathrm{c}}^{*}(0)=\dfrac{2(1-\sigma)}{(2-\sigma)\mathcal{N}(\sigma)}\left\{\varphi_3\left(t,\ \boldsymbol{I}_{\mathrm{c}}(t)\right)-\varphi_3\left(t,\ \boldsymbol{I}_{\mathrm{c}}^{*}(t)\right)\right\}\\
\qquad+\dfrac{2\sigma}{(2-\sigma)\mathcal{N}(\sigma)}\displaystyle\int_0^t\left\{\varphi_3\left(\varrho,\ \boldsymbol{I}_{\mathrm{c}}(\varrho)\right)-\varphi_3\left(\varrho,\ \boldsymbol{I}_{\mathrm{c}}^{*}(\varrho)\right)\right\}d\varrho.
\end{cases}
\tag{34}
$$

By applying the norm to equation (34), we get

$$
\begin{cases}
\left\|\boldsymbol{S}_{\mathrm{p}}(t)-\boldsymbol{S}_{\mathrm{p}}^{*}(0)\right\|\leq\dfrac{2(1-\sigma)}{(2-\sigma)\mathcal{N}(\sigma)}\left\|\varphi_1\left(t,\ \boldsymbol{S}_{\mathrm{p}}(t)\right)-\varphi_1\left(t,\ \boldsymbol{S}_{\mathrm{p}}^{*}(t)\right)\right\|\\
\qquad+\dfrac{2\sigma}{(2-\sigma)\mathcal{N}(\sigma)}\displaystyle\int_0^t\left\|\varphi_1\left(\varrho,\ \boldsymbol{S}_{\mathrm{p}}(\varrho)\right)-\varphi_1\left(\varrho,\ \boldsymbol{S}_{\mathrm{p}}^{*}(\varrho)\right)\right\|d\varrho,\\
\left\|\boldsymbol{I}_{\boldsymbol{H}}(t)-\boldsymbol{I}_{\boldsymbol{H}}^{*}(0)\right\|\leq\dfrac{2(1-\sigma)}{(2-\sigma)\mathcal{N}(\sigma)}\left\|\varphi_2\left(t,\ \boldsymbol{I}_{\boldsymbol{H}}(t)\right)-\varphi_2\left(t,\ \boldsymbol{I}_{\boldsymbol{H}}^{*}(t)\right)\right\|\\
\qquad+\dfrac{2\sigma}{(2-\sigma)\mathcal{N}(\sigma)}\displaystyle\int_0^t\left\|\varphi_2\left(\varrho,\ \boldsymbol{I}_{\boldsymbol{H}}(\varrho)\right)-\varphi_2\left(\varrho,\ \boldsymbol{I}_{\boldsymbol{H}}^{*}(\varrho)\right)\right\|d\varrho,\\
\left\|\boldsymbol{I}_{\mathrm{c}}(t)-\boldsymbol{I}_{\mathrm{c}}^{*}(0)\right\|\leq\dfrac{2(1-\sigma)}{(2-\sigma)\mathcal{N}(\sigma)}\left\|\varphi_3\left(t,\ \boldsymbol{I}_{\mathrm{c}}(t)\right)-\varphi_3\left(t,\ \boldsymbol{I}_{\mathrm{c}}^{*}(t)\right)\right\|\\
\qquad+\dfrac{2\sigma}{(2-\sigma)\mathcal{N}(\sigma)}\displaystyle\int_0^t\left\|\varphi_3\left(\varrho,\ \boldsymbol{I}_{\mathrm{c}}(\varrho)\right)-\varphi_3\left(\varrho,\ \boldsymbol{I}_{\mathrm{c}}^{*}(\varrho)\right)\right\|d\varrho.
\end{cases}
\tag{35}
$$

Based on Theorems 5.1 and 5.2, the outcomes reached are as follows:

$$
\begin{cases}
\left\|\boldsymbol{S}_{\mathrm{p}}(t)-\boldsymbol{S}_{\mathrm{p}}^{*}(0)\right\| \leq \dfrac{2(1-\sigma)}{(2-\sigma)\mathscr{N}(\sigma)}\Psi_{1}\left\|\boldsymbol{S}_{\mathrm{p}}(t)-\boldsymbol{S}_{\mathrm{p}}^{*}(t)\right\|+\dfrac{2\sigma}{(2-\sigma)\mathscr{N}(\sigma)}\Psi_{1}^{t}\left\|\boldsymbol{S}_{\mathrm{p}}(\varrho)-\boldsymbol{S}_{\mathrm{p}}^{*}(\varrho)\right\|, \\
\\
\left\|\boldsymbol{I}_{\boldsymbol{H}}(t)-\boldsymbol{I}_{\boldsymbol{H}}^{*}(0)\right\| \leq \dfrac{2(1-\sigma)}{(2-\sigma)\mathscr{N}(\sigma)}\Psi_{2}\left\|\boldsymbol{I}_{\boldsymbol{H}}(t)-\boldsymbol{I}_{\boldsymbol{H}}^{*}(t)\right\|+\dfrac{2\sigma}{(2-\sigma)\mathscr{N}(\sigma)}\Psi_{2}^{t}\left\|\boldsymbol{I}_{\boldsymbol{H}}(\varrho)-\boldsymbol{I}_{\boldsymbol{H}}^{*}(\varrho)\right\|, \\
\\
\left\|\boldsymbol{I}_{\mathrm{c}}(t)-\boldsymbol{I}_{\mathrm{c}}^{*}(0)\right\| \leq \dfrac{2(1-\sigma)}{(2-\sigma)\mathscr{N}(\sigma)}\Psi_{3}\left\|\boldsymbol{I}_{\mathrm{c}}(t)-\boldsymbol{I}_{\mathrm{c}}^{*}(t)\right\|+\dfrac{2\sigma}{(2-\sigma)\mathscr{N}(\sigma)}\Psi_{3}^{t}\left\|\boldsymbol{I}_{\mathrm{c}}(\varrho)-\boldsymbol{I}_{\mathrm{c}}^{*}(\varrho)\right\|.
\end{cases} \tag{36}
$$

The solution algorithms presented in equation (41), satisfy the subsequent inequalities.

$$
\begin{cases}
\left\|\boldsymbol{S}_{\mathrm{p}}(t)-\boldsymbol{S}_{\mathrm{p}}^{*}(t)\right\|\left\{1-\dfrac{2\Psi_{1}}{(2-\sigma)\mathscr{N}(\sigma)}(1-\sigma-t\sigma\right\} \leq 0, \\
\\
\left\|\boldsymbol{I}_{\boldsymbol{H}}(t)-\boldsymbol{I}_{\boldsymbol{H}}^{*}(t)\right\|\left\{1-\dfrac{2\Psi_{2}}{(2-\sigma)\mathscr{N}(\sigma)}(1-\sigma-t\sigma\right\} \leq 0, \text{‘} \\
\\
\left\|\boldsymbol{I}_{\mathrm{c}}(t)-\boldsymbol{I}_{\mathrm{c}}^{*}(t)\right\|\left\{1-\dfrac{2\mathbb{X}_{3}}{(2-\sigma)\mathscr{N}(\sigma)}(1-\sigma-t\sigma\right\} \leq 0.
\end{cases} \tag{37}
$$

The conclusion is derived from the final equation as follows:

$$
\boldsymbol{S}_{\mathrm{p}}(t)=\boldsymbol{S}_{\mathrm{p}}^{*}(t),\ \boldsymbol{I}_{\boldsymbol{H}}(t)=\boldsymbol{I}_{\boldsymbol{H}}^{*}(t),\ \boldsymbol{I}_{\mathrm{c}}(t)=\boldsymbol{I}_{\mathrm{c}}^{*}(t).
$$

6. Stability

This part talks about how to use the fractional cervical cancer model with an “iterative Laplace transform method.” It also talks about the “stability criteria” we set for approximating solutions.

6.1 *Iterative Laplace transform procedure*

Consider the cervical cancer model (8) with initial conditions (9). By using the Laplace transform in model (8), we can

$$
\begin{cases}
\dfrac{\delta\Lambda\left(\boldsymbol{S}_{\mathrm{p}}(t)\right)-\boldsymbol{S}_{\mathrm{p}}(0)}{\delta+\sigma(1-\delta)}=\Lambda\left(b_{\hbar}-\mathscr{P}_{\mathfrak{v}}\boldsymbol{S}_{\mathrm{p}}\boldsymbol{I}_{\boldsymbol{H}}-d_{\hbar}\boldsymbol{S}_{\mathrm{p}}\right), \\
\\
\dfrac{\delta\Lambda\left(\boldsymbol{I}_{\boldsymbol{H}}(t)-\boldsymbol{I}_{\boldsymbol{H}}(0)\right.}{\delta+\sigma(1-\delta)}=\Lambda\left(\mathscr{P}_{\mathfrak{v}}\boldsymbol{S}_{\mathrm{p}}\boldsymbol{I}_{\boldsymbol{H}}-\mathscr{P}_{\mathrm{c}}\boldsymbol{I}_{\boldsymbol{H}}-d_{\hbar}\boldsymbol{I}_{\boldsymbol{H}}\right), \\
\\
\dfrac{\delta\Lambda\left(\boldsymbol{I}_{\mathrm{c}}(t)\right)-\boldsymbol{I}_{\mathrm{c}}(0)}{\delta+\sigma(1-\delta)}=\Lambda\left(\mathscr{P}_{\mathrm{c}}\boldsymbol{I}_{\boldsymbol{H}}-d_{\hbar}\boldsymbol{I}_{\mathrm{c}}\right).
\end{cases} \tag{38}
$$

By reorganizing the given information, we obtain

$$
\begin{cases}
\Lambda\left(\boldsymbol{S}_{\mathrm{p}}(t)\right)=\dfrac{\boldsymbol{S}_{\mathrm{p}}(0)}{\delta}+\dfrac{\delta+\sigma(1-\delta)}{\delta}\Lambda\left(b_{\hbar}-\mathscr{P}_{\mathfrak{v}}\boldsymbol{S}_{\mathrm{p}}\boldsymbol{I}_{\boldsymbol{H}}-d_{\hbar}\boldsymbol{S}_{\mathrm{p}}\right), \\[2ex]
\Lambda\left(\boldsymbol{I}_{\boldsymbol{H}}(t)\right)=\dfrac{\boldsymbol{I}_{\boldsymbol{H}}(0)}{\delta}+\dfrac{\delta+\sigma(1-\delta)}{\delta}\Lambda\left(\mathscr{P}_{\mathfrak{v}}\boldsymbol{S}_{\mathrm{p}}\boldsymbol{I}_{\boldsymbol{H}}-\mathscr{P}_{c}\boldsymbol{I}_{\boldsymbol{H}}-d_{\hbar}\boldsymbol{I}_{\boldsymbol{H}}\right), \\[2ex]
\Lambda\left(\boldsymbol{I}_{\mathrm{c}}(t)\right)=\dfrac{\boldsymbol{I}_{\mathrm{c}}(0)}{\delta}+\dfrac{\delta+\sigma(1-\delta)}{\delta}\Lambda\left(\mathscr{P}_{\mathrm{c}}\boldsymbol{I}_{\boldsymbol{H}}-d_{\hbar}\boldsymbol{I}_{\mathrm{c}}\right).
\end{cases}
\tag{39}
$$

Moreover, the inverse Laplace transform of Eq. (39) provides

$$
\begin{cases}
\boldsymbol{S}_{\mathrm{p}}(t)=\boldsymbol{S}_{\mathrm{p}}(0)+\Lambda^{-1}\left[\dfrac{\delta+\sigma(1-\delta)}{\delta}\Lambda\left(b_{\hbar}-\mathscr{P}_{\mathfrak{v}}\boldsymbol{S}_{\mathrm{p}}\boldsymbol{I}_{\boldsymbol{H}}-d_{\hbar}\boldsymbol{S}_{\mathrm{p}}\right)\right], \\[2ex]
\boldsymbol{I}_{\boldsymbol{H}}(t)=\boldsymbol{I}_{\boldsymbol{H}}(0)+\Lambda^{-1}\left[\dfrac{\delta+\sigma(1-\delta)}{\delta}\Lambda\left(\mathscr{P}_{\mathfrak{v}}\boldsymbol{S}_{\mathrm{p}}\boldsymbol{I}_{\boldsymbol{H}}-\mathscr{P}_{\mathrm{c}}\boldsymbol{I}_{\boldsymbol{H}}-d_{\hbar}\boldsymbol{I}_{\boldsymbol{H}}\right)\right], \\[2ex]
\boldsymbol{I}_{\mathrm{c}}(t)=\boldsymbol{I}_{\mathrm{c}}(0)+\Lambda^{-1}\left[\dfrac{\delta+\sigma(1-\delta)}{\delta}\Lambda\left(\mathscr{P}_{\mathrm{c}}\boldsymbol{I}_{\boldsymbol{H}}-d_{\hbar}\boldsymbol{I}_{\mathrm{c}}\right)\right].
\end{cases}
\tag{40}
$$

Using these methods, the infinite series solutions are as follows:

$$
\boldsymbol{S}_{\mathrm{p}}=\sum_{n=0}^{\infty}\boldsymbol{S}_{\mathrm{p}}^{n},\ \boldsymbol{I}_{\boldsymbol{H}}=\sum_{n=0}^{\infty}\boldsymbol{I}_{\boldsymbol{H}}^{n},\ \boldsymbol{I}_{\mathrm{c}}=\sum_{n=0}^{\infty}\boldsymbol{I}_{\mathrm{c}}^{n}. \tag{41}
$$

$\boldsymbol{S}_{\mathrm{p}}\boldsymbol{I}_{\boldsymbol{H}}$ and $\boldsymbol{S}_{\mathrm{p}}\boldsymbol{I}_{\mathrm{c}}$'s nonlinearity can be expressed as

$$
\boldsymbol{S}_{\mathrm{p}}\boldsymbol{I}_{\boldsymbol{H}}=\sum_{n=0}^{\infty}\mathrm{C}^{n},\ \boldsymbol{S}_{\mathrm{p}}\boldsymbol{I}_{\mathrm{c}}=\sum_{n=0}^{\infty}\mathrm{D}^{n}. \tag{42}
$$

The decomposition of C^{n} and D^{n} is as follows:

$$
\mathrm{C}^{n}=\sum_{i=0}^{n}\boldsymbol{S}_{\mathrm{p}}^{n}\sum_{\mathrm{i}=0}^{n}\boldsymbol{I}_{\boldsymbol{H}}^{n}-\sum_{\mathrm{i}=0}^{n-1}\boldsymbol{S}_{\mathrm{p}}^{n}\sum_{\mathrm{i}=0}^{n-1}\boldsymbol{I}_{\boldsymbol{H}}^{n}
$$

$$
\mathrm{D}^{n}=\sum_{\mathrm{i}=0}^{n}\boldsymbol{S}_{\mathrm{p}}^{n}\sum_{\mathrm{i}=0}^{n}\boldsymbol{I}_{\mathrm{c}}^{n}-\sum_{\mathrm{i}=0}^{n-1}\boldsymbol{S}_{\mathrm{p}}^{n}\sum_{\mathrm{i}=0}^{n-1}\boldsymbol{I}_{\mathrm{c}}^{n}
$$

The subsequent recursion formula is obtained by applying initial conditions.

$$\begin{cases} \boldsymbol{S}_{\mathfrak{p}}^{n+1}(t) = \boldsymbol{S}_{\mathfrak{p}}(0) + \Lambda^{-1}\left[\dfrac{\delta+\sigma(1-\delta)}{\delta}\Lambda\left(b_{\hbar} - \mathscr{P}_{\mathfrak{v}}\boldsymbol{S}_{\mathfrak{p}}^{n}\boldsymbol{I}_{\boldsymbol{H}}^{n} - d_{\hbar}\boldsymbol{S}_{\mathfrak{p}}^{n}\right)\right], \\ \boldsymbol{I}_{\boldsymbol{H}}^{n+1}(t) = \boldsymbol{I}_{\boldsymbol{H}}(0) + \Lambda^{-1}\left[\dfrac{\delta+\sigma(1-\delta)}{\delta}\Lambda\left(\mathscr{P}_{\mathfrak{v}}\boldsymbol{S}_{\mathfrak{p}}^{n}\boldsymbol{I}_{\boldsymbol{H}}^{n} - \mathscr{P}_{\mathfrak{c}}\boldsymbol{I}_{\boldsymbol{H}}^{n} - d_{\hbar}\boldsymbol{I}_{\boldsymbol{H}}^{n}\right)\right], \\ \boldsymbol{I}_{\mathfrak{c}}^{n+1}(t) = \boldsymbol{I}_{\mathfrak{c}}(0) + \Lambda^{-1}\left[\dfrac{\delta+\sigma(1-\delta)}{\delta}\Lambda\left(\mathscr{P}_{\mathfrak{c}}\boldsymbol{I}_{\boldsymbol{H}}^{n} - d_{\hbar}\boldsymbol{I}_{\mathfrak{c}}^{n}\right)\right]. \end{cases} \tag{43}$$

7. Analysis of iteration method

Let $\mathbb{X}$ be defined as the self-map of the Banach space denoted by $(\mathbb{Z}, \|\cdot\|)$. In addition, the notation $\mathbb{W}_{n+1} = \mathfrak{g}(\mathscr{J}, \mathbb{W}_n)$ represents an extremely precise recurring procedure. Consider $\mathscr{J}(\mathbb{X})$ to be a set of fixed points on $\mathbb{X}$. In order for $\mathbb{W}_n$ to converge to the point $\mathrm{i} \in \mathscr{J}(\mathbb{X})$, it is necessary for $\mathbb{X}$ to have at least an element. Suppose that $\{z_n \in \mathbb{Z}\}$ and decribe $\mathbb{K}_n = \|z_{n+1} - \mathfrak{g}(\mathbb{X}, z_n)\|$. The iterative method $z_{n+1} = \mathfrak{g}(\mathbb{X}, z_n)$ is considered $\mathbb{X}$ stable if $\lim\limits_{n\to\infty} z^n = \mathrm{i}$. On the contrary, we affirm that the sequence $\{z_n\}$ possesses an upper bound. If all the conditions for $z_{n+1} = \mathbb{X}z_n$ are satisfied, this process is referred to as Picard's iteration, and it exhibits $\mathbb{X}$-stability.

Theorem 7.1 Let $(\mathbb{Z}, \|\cdot\|)$ denote the Banach space, and $\mathbb{X}$ be defined as a satisfactory self-map on $\mathbb{Z}$.

$$\|\mathbb{X}_x - \mathbb{X}_y\| \leq Q\,\|x - \mathbb{X}_x\| + q\|x - y\| \tag{44}$$

$\forall x, y \in \mathbb{Z}$ where $0 \leq Q,\ 0 \leq q < 1$.

Proof. Consider $\mathbb{X}$ to be Picard $\mathbb{X}$-stable. Let us examine the relationship between Equation (43) and Equation (8).

$$\begin{cases} \boldsymbol{S}_{\mathfrak{p}}^{n+1}(t) = \boldsymbol{S}_{\mathfrak{p}}(0) + \Lambda^{-1}\left[\dfrac{s+\sigma(1-s)}{s}\Lambda\left(b_{\hbar} - \mathscr{P}_{\mathfrak{v}}\boldsymbol{S}_{\mathfrak{p}}^{n}\boldsymbol{I}_{\boldsymbol{H}}^{n} - d_{\hbar}\boldsymbol{S}_{\mathfrak{p}}^{n}\right)\right], \\ \boldsymbol{R}_{\mathfrak{q}}^{n+1}(t) = \boldsymbol{R}_{\mathfrak{q}}(0) + \Lambda^{-1}\left[\dfrac{s+\sigma(1-s)}{s}\Lambda\left(\mathscr{P}_{\mathfrak{v}}\boldsymbol{S}_{\mathfrak{p}}^{n}\boldsymbol{I}_{\boldsymbol{H}}^{n} - \mathscr{P}_{\mathfrak{c}}\boldsymbol{I}_{\boldsymbol{H}}^{n} - d_{\hbar}\boldsymbol{I}_{\boldsymbol{H}}^{n}\right)\right], \\ \boldsymbol{S}_{\mathfrak{r}}^{n+1}(t) = \boldsymbol{S}_{\mathfrak{r}}(0) + \Lambda^{-1}\left[\dfrac{s+\sigma(1-s)}{s}\Lambda\left(\mathscr{P}_{\mathfrak{c}}\boldsymbol{I}_{\boldsymbol{H}}^{n} - d_{\hbar}\boldsymbol{I}_{\mathfrak{c}}^{n}\right)\right]. \end{cases} \tag{45}$$

where $\dfrac{\omega+\sigma(1-\omega)}{\omega}$ denotes a Lagrange multiplier in fractional form.

Theorem 7.2 Suppose a self-mapping $\mathbb{X}$ is defined as:

$$
\begin{cases}
\mathbb{X}\left(\boldsymbol{S}_{\mathfrak{p}}^{n}(t)\right)=\boldsymbol{S}_{\mathfrak{p}}^{n+1}(t)=\boldsymbol{S}_{\mathfrak{p}}(0)+\Lambda^{-1}\left[\dfrac{\delta+\sigma(1-\delta)}{\delta}\Lambda\left(b_{\hbar}-\mathscr{P}_{\mathfrak{v}}\boldsymbol{S}_{\mathfrak{p}}^{n}\boldsymbol{I}_{\boldsymbol{H}}^{n}-d_{\hbar}\boldsymbol{S}_{\mathfrak{p}}^{n}\right)\right], \\
\mathbb{X}\left(\boldsymbol{I}_{\boldsymbol{H}}^{n}(t)\right)=\boldsymbol{I}_{\boldsymbol{H}}^{n+1}(t)=\boldsymbol{I}_{\boldsymbol{H}}(0)+\Lambda^{-1}\left[\dfrac{\delta+\sigma(1-\delta)}{\delta}\Lambda\left(\mathscr{P}_{\mathfrak{v}}\boldsymbol{S}_{\mathfrak{p}}^{n}\boldsymbol{I}_{\boldsymbol{H}}^{n}-\mathscr{P}_{\mathfrak{c}}\boldsymbol{I}_{\boldsymbol{H}}^{n}-d_{\hbar}\boldsymbol{I}_{\boldsymbol{H}}^{n}\right)\right], \\
\mathbb{X}\left(\boldsymbol{I}_{\mathfrak{c}}^{n}(t)\right)=\boldsymbol{I}_{\mathfrak{c}}^{n+1}(t)=\boldsymbol{I}_{\mathfrak{c}}(0)+\Lambda^{-1}\left[\dfrac{\delta+\sigma(1-\delta)}{\delta}\Lambda\left(\mathscr{P}_{\mathfrak{c}}\boldsymbol{I}_{\boldsymbol{H}}^{n}-d_{\hbar}\boldsymbol{I}_{\mathfrak{c}}^{n}\right)\right].
\end{cases}
\tag{46}
$$

is $\mathbb{X}$ stable in $\mathbb{L}^{1}(\mathfrak{a},\ \mathfrak{b})$ if

$$
\begin{cases}
\left(\mathscr{P}_{\mathfrak{v}}\mathbb{E}_{1}\mathbb{h}_{1}(\sigma)+\mathscr{P}_{\mathfrak{v}}\mathbb{E}_{2}\mathbb{h}_{2}(\sigma)+d_{\hbar}\mathbb{h}_{3}(\sigma)\right)<1, \\
\left(\mathscr{P}_{\mathfrak{v}}\mathbb{E}_{1}\mathbb{h}_{4}(\sigma)+\mathscr{P}_{\mathfrak{v}}\mathbb{E}_{2}\mathbb{h}_{5}(\sigma)+\mathscr{P}_{\mathfrak{c}}\mathbb{h}_{6}(\sigma)+d_{\hbar}\mathbb{h}_{7}(\sigma)\right)<1, \\
\left(\mathscr{P}_{\mathfrak{c}}\mathbb{h}_{8}(\sigma)+d_{\hbar}\mathbb{h}_{9}(\sigma)\right)<1.
\end{cases}
\tag{47}
$$

Proof. We will now illustrate that $\mathbb{X}$ possesses a fixed point. Consequently, we evaluate the subsequent for each $(\mathfrak{m},\ \mathfrak{n})\in\mathbb{N}\times\mathbb{N}$.

$$
\begin{cases}
\mathbb{X}(\boldsymbol{S}_{\mathfrak{p}}^{n}(t))-\mathbb{X}(\boldsymbol{S}_{\mathfrak{p}}^{m}(t))=\Lambda^{-1}\left[\dfrac{\delta+\sigma(1-\delta)}{\delta}\Lambda\left(b_{\hbar}-\mathscr{P}_{\mathfrak{v}}\boldsymbol{S}_{\mathfrak{p}}^{n}\boldsymbol{I}_{\boldsymbol{H}}^{n}-d_{\hbar}\boldsymbol{S}_{\mathfrak{p}}^{n}\right)\right] \\
\qquad -\Lambda^{-1}\left[\dfrac{\delta+\sigma(1-\delta)}{\delta}\Lambda\left(b_{\hbar}-\mathscr{P}_{\mathfrak{v}}\boldsymbol{S}_{\mathfrak{p}}^{m}\boldsymbol{I}_{\boldsymbol{H}}^{m}-d_{\hbar}\boldsymbol{S}_{\mathfrak{p}}^{m}\right)\right], \\
\mathbb{X}(\boldsymbol{I}_{\boldsymbol{H}}^{n}(t))-\mathbb{X}(\boldsymbol{I}_{\boldsymbol{H}}^{m}(t))=\Lambda^{-1}\left[\dfrac{\delta+\sigma(1-\delta)}{\delta}\Lambda\left(\mathscr{P}_{\mathfrak{v}}\boldsymbol{S}_{\mathfrak{p}}^{n}\boldsymbol{I}_{\boldsymbol{H}}^{n}-\mathscr{P}_{\mathfrak{c}}\boldsymbol{I}_{\boldsymbol{H}}^{n}-d_{\hbar}\boldsymbol{I}_{\boldsymbol{H}}^{n}\right)\right] \\
\qquad -\Lambda^{-1}\left[\dfrac{\delta+\sigma(1-\delta)}{\delta}\Lambda\left(\mathscr{P}_{\mathfrak{v}}\boldsymbol{S}_{\mathfrak{p}}^{m}\boldsymbol{I}_{\boldsymbol{H}}^{m}-\mathscr{P}_{\mathfrak{c}}\boldsymbol{I}_{\boldsymbol{H}}^{m}-d_{\hbar}\boldsymbol{I}_{\boldsymbol{H}}^{m}\right)\right], \\
\mathbb{X}(\boldsymbol{I}_{\mathfrak{c}}^{n}(t))-\mathbb{X}(\boldsymbol{I}_{\mathfrak{c}}^{m}(t))=\Lambda^{-1}\left[\dfrac{\delta+\sigma(1-\delta)}{\delta}\Lambda\left(\mathscr{P}_{\mathfrak{c}}\boldsymbol{I}_{\boldsymbol{H}}^{n}-d_{\hbar}\boldsymbol{I}_{\mathfrak{c}}^{n}\right)\right] \\
\qquad -\Lambda^{-1}\left[\dfrac{\delta+\sigma(1-\delta)}{\delta}\Lambda\left(\mathscr{P}_{\mathfrak{c}}\boldsymbol{I}_{\boldsymbol{H}}^{m}-d_{\hbar}\boldsymbol{I}_{\mathfrak{c}}^{m}\right)\right].
\end{cases}
\tag{48}
$$

By taking the norm to equation (48) and without sacrificing generality, we get

$$
\begin{cases}
\left\|\mathbb{X}(\boldsymbol{S}_{\mathfrak{p}}^{n}(t))-\mathbb{X}(\boldsymbol{S}_{\mathfrak{p}}^{m}(t))\right\|=\left\|\begin{array}{l}\Lambda^{-1}\left[\dfrac{\delta+\sigma(1-\delta)}{\delta}\Lambda\left(b_{\hbar}-\mathscr{P}_{\mathfrak{v}}\boldsymbol{S}_{\mathfrak{p}}^{n}\boldsymbol{I}_{\boldsymbol{H}}^{n}-d_{\hbar}\boldsymbol{S}_{\mathfrak{p}}^{n}\right)\right]\\[2ex] -\Lambda^{-1}\left[\dfrac{\delta+\sigma(1-\delta)}{\delta}\Lambda\left(b_{\hbar}-\mathscr{P}_{\mathfrak{v}}\boldsymbol{S}_{\mathfrak{p}}^{m}\boldsymbol{I}_{\boldsymbol{H}}^{m}-d_{\hbar}\boldsymbol{S}_{\mathfrak{p}}^{m}\right)\right]\end{array}\right\|,\\[6ex]
\|\mathbb{X}(\boldsymbol{I}_{\boldsymbol{H}}^{n}(t))-\mathbb{X}(\boldsymbol{I}_{\boldsymbol{H}}^{m}(t))\|=\left\|\begin{array}{l}\Lambda^{-1}\left[\dfrac{\delta+\sigma(1-\delta)}{\delta}\Lambda\left(\mathscr{P}_{\mathfrak{v}}\boldsymbol{S}_{\mathfrak{p}}^{n}\boldsymbol{I}_{\boldsymbol{H}}^{n}-\mathscr{P}_{\mathfrak{c}}\boldsymbol{I}_{\boldsymbol{H}}^{n}-d_{\hbar}\boldsymbol{I}_{\boldsymbol{H}}^{n}\right)\right]\\[2ex] -\Lambda^{-1}\left[\dfrac{\delta+\sigma(1-\delta)}{\delta}\Lambda\left(\mathscr{P}_{\mathfrak{v}}\boldsymbol{S}_{\mathfrak{p}}^{n}\boldsymbol{I}_{\boldsymbol{H}}^{m}-\mathscr{P}_{\mathfrak{c}}\boldsymbol{I}_{\boldsymbol{H}}^{m}-d_{\hbar}\boldsymbol{I}_{\boldsymbol{H}}^{m}\right)\right]\end{array}\right\|,\\[6ex]
\|\mathbb{X}(\boldsymbol{I}_{c}^{n}(t))-\mathbb{X}(\boldsymbol{I}_{c}^{m}(t))\|=\left\|\begin{array}{l}\Lambda^{-1}\left[\dfrac{\delta+\sigma(1-\delta)}{\delta}\Lambda\left(\mathscr{P}_{\mathfrak{c}}\boldsymbol{I}_{\boldsymbol{H}}^{n}-d_{\hbar}\boldsymbol{I}_{c}^{n}\right)\right]\\[2ex] -\Lambda^{-1}\left[\dfrac{\delta+\sigma(1-\delta)}{\delta}\Lambda\left(\mathscr{P}_{\mathfrak{c}}\boldsymbol{I}_{\boldsymbol{H}}^{m}-d_{\hbar}\boldsymbol{I}_{\mathfrak{c}}^{m}\right)\right]\end{array}\right\|.
\end{cases}
\tag{49}
$$

By applying the triangular inequality and subsequently simplifying equation (49), we obtain

$$
\begin{cases}
\left\|\mathbb{X}\left(\boldsymbol{S}_{\mathfrak{p}}^{n}(t)\right)-\mathbb{X}\left(\boldsymbol{S}_{\mathfrak{p}}^{m}(t)\right)\right\|\leq\Lambda^{-1}\left[\dfrac{\delta+\sigma(1-\delta)}{\delta}\Lambda\left(\begin{array}{c}\left\|-\mathscr{P}_{\mathfrak{v}}\boldsymbol{S}_{\mathfrak{p}}^{n}\left(\boldsymbol{I}_{\boldsymbol{H}}^{n}-\boldsymbol{I}_{\boldsymbol{H}}^{m}\right)\right\|+\\ \left\|-\mathscr{P}_{\mathfrak{v}}\boldsymbol{I}_{\boldsymbol{H}}^{m}\left(\boldsymbol{S}_{\mathfrak{p}}^{n}-\boldsymbol{S}_{\mathfrak{p}}^{m}\right)\right\|+\left\|-d_{\hbar}\left(\boldsymbol{S}_{\mathfrak{p}}^{n}-\boldsymbol{S}_{\mathfrak{p}}^{m}\right)\right\|\end{array}\right)\right],\\[4ex]
\left\|\mathbb{X}\left(\boldsymbol{I}_{\boldsymbol{H}}^{n}(t)\right)-\mathbb{X}\left(\boldsymbol{I}_{\boldsymbol{H}}^{m}(t)\right)\right\|\leq\Lambda^{-1}\left[\dfrac{\delta+\sigma(1-\delta)}{\delta}\Lambda\left(\begin{array}{c}\left\|\mathscr{P}_{\mathfrak{v}}\boldsymbol{S}_{\mathfrak{p}}^{n}\left(\boldsymbol{I}_{\boldsymbol{H}}^{n}-\boldsymbol{I}_{\boldsymbol{H}}^{m}\right)\right\|+\left\|\mathscr{P}_{\mathfrak{v}}\boldsymbol{I}_{\boldsymbol{H}}^{m}\left(\boldsymbol{S}_{\mathfrak{p}}^{n}-\boldsymbol{S}_{\mathfrak{p}}^{m}\right)\right\|\\ +\left\|-\mathscr{P}_{\mathfrak{c}}\left(\boldsymbol{I}_{\boldsymbol{H}}^{n}-\boldsymbol{I}_{\boldsymbol{H}}^{m}\right)\right\|+\left\|-d_{\hbar}\left(\boldsymbol{I}_{\boldsymbol{H}}^{n}-\boldsymbol{I}_{\boldsymbol{H}}^{m}\right)\right\|\end{array}\right)\right],\\[4ex]
\left\|\mathbb{X}\left(\boldsymbol{I}_{\mathfrak{c}}^{n}(t)\right)-\mathbb{X}\left(\boldsymbol{I}_{\mathfrak{c}}^{m}(t)\right)\right\|\leq\Lambda^{-1}\left[\dfrac{\delta+\sigma(1-\delta)}{\delta}\Lambda\left(\left\|\mathscr{P}_{\mathfrak{c}}\left(\boldsymbol{I}_{\boldsymbol{H}}^{n}-\boldsymbol{I}_{\boldsymbol{H}}^{m}\right)\right\|+\left\|-d_{\hbar}\left(\boldsymbol{I}_{\mathfrak{c}}^{n}-\boldsymbol{I}_{\mathfrak{c}}^{m}\right)\right\|\right)\right].
\end{cases}
\tag{50}
$$

Since the solution that was obtained fulfills the same role, we assume that

$$
\left\|\boldsymbol{S}_{\mathfrak{p}}^{n}(t)-\boldsymbol{S}_{\mathfrak{p}}^{m}(t)\right\|=\left\|\boldsymbol{I}_{\boldsymbol{H}}^{n}(t)-\boldsymbol{I}_{\boldsymbol{H}}^{m}(t)\right\|=\left\|\boldsymbol{I}_{\mathfrak{c}}^{n}(t)-\boldsymbol{I}_{\mathfrak{c}}^{m}(t)\right\| \tag{51}
$$

By substituting this into equation (51), the subsequent relationship is obtained

$$\begin{cases} \left\|\mathbb{X}\left(\boldsymbol{S}_{\mathfrak{p}}^{n}(t)\right)-\mathbb{X}\left(\boldsymbol{S}_{\mathfrak{p}}^{m}(t)\right)\right\| \leq \Lambda^{-1}\left[\dfrac{\delta+\sigma(1-\delta)}{\delta}\Lambda\begin{pmatrix} \left\|-\mathscr{P}_{\mathfrak{v}}\boldsymbol{S}_{\mathfrak{p}}^{n}\left(\boldsymbol{S}_{\mathfrak{p}}^{n}-\boldsymbol{S}_{\mathfrak{p}}^{m}\right)\right\|+ \\ \left\|-\mathscr{P}_{\mathfrak{v}}\boldsymbol{I}_{\boldsymbol{H}}^{m}\left(\boldsymbol{S}_{\mathfrak{p}}^{n}-\boldsymbol{S}_{\mathfrak{p}}^{m}\right)\right\|+\left\|-d_{\hbar}\left(\boldsymbol{S}_{\mathfrak{p}}^{n}-\boldsymbol{S}_{\mathfrak{p}}^{m}\right)\right\| \end{pmatrix}\right], \\ \\ \left\|\mathbb{X}\left(\boldsymbol{I}_{\boldsymbol{H}}^{n}(t)\right)-\mathbb{X}\left(\boldsymbol{I}_{\boldsymbol{H}}^{m}(t)\right)\right\| \leq \Lambda^{-1}\left[\dfrac{\delta+\sigma(1-\delta)}{\delta}\Lambda\begin{pmatrix} \left\|\mathscr{P}_{\mathfrak{v}}\boldsymbol{S}_{\mathfrak{p}}^{n}\left(\boldsymbol{I}_{\boldsymbol{H}}^{n}-\boldsymbol{I}_{\boldsymbol{H}}^{m}\right)\right\|+\left\|\mathscr{P}_{\mathfrak{v}}\boldsymbol{I}_{\boldsymbol{H}}^{m}\left(\boldsymbol{I}_{\boldsymbol{H}}^{n}-\boldsymbol{I}_{\boldsymbol{H}}^{m}\right)\right\| \\ +\left\|-\mathscr{P}_{c}\left(\boldsymbol{I}_{\boldsymbol{H}}^{n}-\boldsymbol{I}_{\boldsymbol{H}}^{m}\right)\right\|+\left\|-d_{\hbar}\left(\boldsymbol{I}_{\boldsymbol{H}}^{n}-\boldsymbol{I}_{\boldsymbol{H}}^{m}\right)\right\| \end{pmatrix}\right], \\ \\ \left\|\mathbb{X}\left(\boldsymbol{I}_{\mathfrak{c}}^{n}(t)\right)-\mathbb{X}\left(\boldsymbol{I}_{\mathfrak{c}}^{m}(t)\right)\right\| \leq \Lambda^{-1}\left[\dfrac{\delta+\sigma(1-\delta)}{\delta}\Lambda\left(\left\|\mathscr{P}_{\mathfrak{c}}\left(\boldsymbol{I}_{\mathfrak{c}}^{n}-\boldsymbol{I}_{\mathfrak{c}}^{m}\right)\right\|+\left\|-d_{\hbar}\left(\boldsymbol{I}_{\mathfrak{c}}^{n}-\boldsymbol{I}_{\mathfrak{c}}^{m}\right)\right\|\right)\right]. \end{cases} \tag{52}$$

Furthermore, by considering the sequences $\boldsymbol{S}_{\mathfrak{p}}^{n}$, $\boldsymbol{I}_{\boldsymbol{H}}^{n}$ and $\boldsymbol{I}_{\mathfrak{c}}^{n}$ we can derive three alternate positive constants, as these sequences converge and so are limited. For any value of t, denoted as $\mathbb{E}_1$, $\mathbb{E}_2$ and $\mathbb{E}_3$.

$$\left\|\boldsymbol{S}_{\mathfrak{p}}^{n}\right\|<\mathbb{E}_1,\ \left\|\boldsymbol{I}_{\boldsymbol{H}}^{n}\right\|<\mathbb{E}_2,\ \left\|\boldsymbol{I}_{\mathfrak{c}}^{n}\right\|<\mathbb{E}_3. \tag{53}$$

Let us now examine equations (52) and (53), resulting in

$$\begin{cases} \left\|\mathbb{X}\left(\boldsymbol{S}_{\mathfrak{p}}^{n}(t)\right)-\mathbb{X}\left(\boldsymbol{S}_{\mathfrak{p}}^{m}(t)\right)\right\| \leq \left(\mathscr{P}_{\mathfrak{v}}\mathbb{E}_1\mathbb{h}_1(\sigma)+\mathscr{P}_{\mathfrak{v}}\mathbb{E}_2\mathbb{h}_2(\sigma)+d_{\hbar}\mathbb{h}_3(\sigma)\right), \\ \\ \left\|\mathbb{X}\left(\boldsymbol{I}_{\boldsymbol{H}}^{n}(t)\right)-\mathbb{X}\left(\boldsymbol{I}_{\boldsymbol{H}}^{m}(t)\right)\right\| \leq \left(\mathscr{P}_{\mathfrak{v}}\mathbb{E}_1\mathbb{h}_4(\sigma)+\mathscr{P}_{\mathfrak{v}}\mathbb{E}_2\mathbb{h}_5(\sigma)+\mathscr{P}_{\mathfrak{c}}\mathbb{h}_6(\sigma)+d_{\hbar}\mathbb{h}_7(\sigma)\right), \\ \\ \left\|\mathbb{X}\left(\boldsymbol{I}_{\mathfrak{c}}^{n}(t)\right)-\mathbb{X}\left(\boldsymbol{I}_{\mathfrak{c}}^{m}(t)\right)\right\| \leq \left(\mathscr{P}_{\mathfrak{c}}\mathbb{h}_8(\sigma)+d_{\hbar}\mathbb{h}_9(\sigma)\right). \end{cases} \tag{54}$$

The functions $\mathbb{h}_{\mathfrak{i}}(\sigma)$, where $\mathfrak{i}=1,\ 2,\ 3,\ \ldots\ldots,\ 9$ from $\Lambda^{-1}\left[\Lambda\dfrac{\delta+\sigma(1-\delta)}{\delta}\right]$.

Therefore, there is a fixed point in the $\mathbb{X}$ mapping. We subsequently establish that $\mathbb{X}$ fulfils all the criteria outlined in Theorem 7.1, discussed earlier. Assuming that equations (53) and (54) are valid.

$$\Xi=(0,\ 0,\ 0)$$

$$\Xi=\begin{cases} \left(\mathscr{P}_{\mathfrak{v}}\mathbb{E}_1\mathbb{h}_1(\sigma)+\mathscr{P}_{\mathfrak{v}}\mathbb{E}_2\mathbb{h}_2(\sigma)+d_{\hbar}\mathbb{h}_3(\sigma)\right)<1 \\ \\ \left(\mathscr{P}_{\mathfrak{v}}\mathbb{E}_1\mathbb{h}_4(\sigma)+\mathscr{P}_{\mathfrak{v}}\mathbb{E}_2\mathbb{h}_5(\sigma)+\mathscr{P}_{\mathfrak{c}}\mathbb{h}_6(\sigma)+d_{\hbar}\mathbb{h}_7(\sigma)\right)<1 \\ \\ \left(\mathscr{P}_{\mathfrak{c}}\mathbb{h}_8(\sigma)+d_{\hbar}\mathbb{h}_9(\sigma)\right)<1 \end{cases} \tag{55}$$

$\mathbb{X}$ fulfils all the criteria stated in Theorem 7.2. Therefore, $\mathbb{X}$ is stable under Picard iterations.

8. Numerical results and discussion

The Caputo-Fabrizio operator is a practical instrument for analysing diseases, as determined by a comparison of analytical methods and mathematical outcomes. A number of fractional orders of cervical cancer transmission are examined. In this part, we numerically simulate the Caputo-Fabrizio cervical cancer model (8) under several values of the fractional order $\sigma \in (0,\ 1)$, considering the initial conditions (9). The mathematical analysis of the non-linear model contending that the human papillomavirus (HPV) causes cervical cancer is complete. Multiple numeric calculations are carried out using the parameter values to see how the fractional derivative affects the therapy compartments (see Table 2). This is done to see how the parameters change the cervical cancer model. The numeric outcomes of the model for various fractional values that align with the stable state point are generated using the fractional derivative. The essential values of the biological parameters are given. The fractional cervical cancer model (8) is solved through the iterative Laplace transform method, which provides a three-term approximation.

It was noted that the number of sensitive individuals $(\boldsymbol{S_p}(t))$consistently decreased as the fractional parameter's value dropped from 1. Nevertheless, the quantity of individuals with HPV infection $(\boldsymbol{I_H}(t))$ and those who are capable of transmitting HPV $(\boldsymbol{I_c}(t))$ in relation to cervical cancer also decreases due to a decrease in the fractional value. Therefore, the human papillomavirus (HPV) responsible for cervical cancer decreases gradually and may be neutralised through a reduction in the value of the fractional index. By setting the fractional order to 1, the obtained results are consistent with those obtained when the integer order is used. Consistent with the steady state point, the fractional derivative is utilised to generate the model's numerical results for various fractional values. By applying the iterative Laplace transform method to the fractional cervical cancer model, it can be inferred that the variables' behavior in the considered area can be accurately predicted. The simulations illustrate how differences in regard significantly influence the dynamics of the model. The non-integer order has minimal impact on the motion of cervical cancer spreading in the human body.

These demonstrate that a specific fractional operator, including the Caputo-Fabrizio operator, reduces noise and enhances the accuracy of predictions. In addition, Caputo-Fabrizio possesses hybrid characteristics that enable him to effectively capture intricate models and generate significant estimations.

9. Conclusion

This work employed iterative Laplace transform technique to examine fractional-order mathematical model for cervical cancer in the sense of Caputo-Fabrizio operator. The focus was on understanding the dynamics of both indirect and direct transmission of the disease. Furthermore, the Banach theorem has been utilised to establish the uniqueness, stability, and existence of stable solutions. The wide range of solutions generated by this efficient technology demonstrate a noteworthy congruence in reducing the harmful impact of cervical cancer on the human body across varying durations and eliminating a mortality-inducing factor. By removing phases and adding extra components, the effectiveness of this method can be greatly enhanced. We utilised a stochastic set of parameters in the evolution of the aforementioned model for the pandemic of cervical cancer. Moving forward, it may be possible to generate a sample of the dynamical framework's potential behaviors through simulations employing alternative parameter value configurations. Furthermore, apart from the specific case of cervical cancer, the techniques outlined in this article can be applied to various models of epidemics.

The expected outcome suggests that reducing the fractional values will enhance the success of the solution in comparison to the classical derivative. The aforementioned simulations demonstrate how alterations in value affect the behavior of the model. The simulation allows for an accurate prediction of the long-term prospects of individuals with cervical cancer. Consequently, a study of this nature becomes more significant for the purpose of decision-making and implementing control techniques.

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Conflict of interest

The authors declare there is no conflict of interest at any point with reference to research findings.

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