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Research Article

Solving Minimum Cost Flow Problem under Neutrosophic Environment Using the Lexicographic Approach

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Abstract: In decision-making, linear programming is one of the most useful models for obtaining the optimal solution. A crucial element of the linear programming (LP) model is the minimum cost flow (MCF). The objective of the MCF is to reduce the transportation cost of a single product across a network with capacity constraints. Recently, neutrosophic set theory has become a strong way to deal with the uncertainty that often comes with trying to optimize things. This manuscript explores how neutrosophic set theory can be applied to the MCF problem which has caught the interest of some researchers. The primary objective of this study is threefold: firstly, to tackle the MCF problem considering the uncertainty of the neutrosophic set, focusing especially on the cost. Secondly, to introduce an innovative lexicographical method tailored for the MCF problem, marking a first in the field of neutrosphic sets. Lastly, to combine this new method with a multi-objective optimization approach, improving the way we solve the MCF problem in various ways at once. This thorough method is meant to lead to more detailed and effective ways of solving optimization problems when there is uncertainty. To show how our method works, we will go through some numerical examples related to the MCF problem with cost defined by neutrosophic numbers.

***Keywords*:** full fuzzy linear programming (FFLP), single-valued triangular neutrosophic (SVTN) numbers, multi-objective problem, minimal cost flow, triangular neutrosophic MCF problem

MSC: 03B52, 03E72, 90C05, 90C70

Abbreviation

NS	Neutrosophic set
MCF	Minimum cost flow
TrNNs	Triangular neutrosophic numbers
MOLP	Multi-objective linear programming
TrpNNs	Trapezoidal neutrosophic numbers
MST	Minimum spanning tree
SVTN	Single-valued triangular neutrosophic

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FFLP	Full fuzzy linear programming
NMFP	Neutrosophic minimal flow problem
MOLP	Multi-objective linear programming
TrNMCF	Triangular neutrosophic minimal flow
MADM	Multi-attribute decision making
SOLP	Single-objective linear programming
C	Maximum capacity of the arc
MO	Multi-objective
LA	Lexicographic approach
RAP	Redundancy allocation problem
H	Terminal node
T	Initial node

1. Introduction

In diverse real-world situations linear programming (LP) problem referred to as linear optimization, aims to maximize (profits) or minimize (cost) a linear function while satisfying a set of linear constraints. LP problem contains a wide range of applications such as optimal resource allocation, production planning, inventory management, production scheduling, bandwidth allocation, network design, and so on. Various researchers have been making significant contributions in the field of optimization problems, such as Hobson et al. [1] have shown that linear programming (LP), a mathematical method, can be effectively used for network problems, including fuel scheduling and contingency analysis. Similarly, Luathep et al. [2] have developed a universal optimization method to tackle mixed transportation network design problems, which are usually complex mathematical problems with balance constraints. Li et al. [3] proposed a step-by-step LP method to determine freight allocation in a multi-mode freight transport network model. In a different approach, Dong et al. [4] introduced a unique version of the maximum concurrent flow problem using LP, known as the triples formulation. They also provided ways to derive a triple solution from an edge-path solution. Fuller and Shanmugham [5] have introduced a new approach to studying rural freight transportation using network flow models, and they have compared this method with LP. Garg and Sharma [6] introduced a problem of RAP in a series system, focusing on multi-objective reliability. They treated the system's reliability and the associated design cost as two separate objectives. In contrast, Garg et al. [7] introduced a methodology for optimizing both system reliability and design cost in a bi-objective reliability redundancy allocation problem for a series-parallel system. This shows the versatility and wide application of LP in solving various network and optimization problems. Addressing all areas of Linear Programming (LP) simultaneously can be quite challenging. Therefore, in this context, we will focus solely on discussing the network flow problem and its sub-areas. Network flow problems contain various sub-areas such as shortest path problem (SPP) [8], maximal flow problem, transportation problem, and minimum cost flow (MCF). Out of these MCF problem is one of the special cases of network flow problem. In the MCF problem, the goal is to determine the least expensive way to transport a specified amount of flow through a network. This network is made up of supply nodes, directed arcs and demand nodes, and each arc has an associated cost and capacity constraint. The task is to find the most cost-effective way to meet the demand from the supply while adhering to the capacity constraints of each arc.

Several researchers are working on the MCF problem. Goldberg et al. [9] present a new approach to solving the MCF problem using successive approximation methods and also Vygen [10] presents a new dual algorithm for the MCF problem using a variation of the best-known strongly polynomial MCF algorithm. Hu et al. [11] developed an algorithm for solving MCF problem with the complementary slackness at each iteration and to find an augmenting path by updating node potential iteratively with a dual approach. Holzhauser et al. [12] present a specialized network simplex algorithm for solving the budget-constrained MCF problem and to solve the MCF problem Ciupala [13] proposed a new method deficit scaling method which is based on a generic pre flow algorithm. then after Ciurea and Ciupala [14] another approach is a sequential and parallel algorithm based on decreasing path algorithms and parallel pre-flow algorithms to solve the MCF problem. We have attempted to explain various approaches to solving the MCF problem in this paragraph. However, we will now explore some applications of the MCF problem, as discussed in the following paragraph.

The various applications of the MCF problem such as Sirivongpaisal [15] introduced a novel approach for addressing the MCF problem by leveraging the stochastic LP technique within the context of supply chain problems and, Ghatee et al. [16] designed a network plan for trainspotting hazardous materials, utilizing the MCF problem under uncertain environments. Furthermore, Grossmann et al. [17] introduced an MCF approach for dynamic assignment trials in networks models with storage devices over time. Additionally, Ghatee and Hashemi [18] tackled the bus network planning problem by employing a generalized MCF problem in uncertain environments and so on.

Based on the above literature survey, the classical Minimum Cost Flow (MCF) problem has various factors like cost, supply, demand, and capacity. In the standard MCF problem, these factors are set and clear. But in real life, these factors can change because of uncertainty. This is why we're introducing a new way to solve the MCF problem when these factors are not certain. There are different theories, like probability theory, vagueness theory, and fuzzy theory, that have been created to solve problems with uncertainty in real-life situations. In 1965, a new concept called the fuzzy set was introduced by, Zadeh [19] to handle uncertain real situations. Utilizing fuzzy logic, Zimmermann [20] introduced a new concept in the field of linear programming in 1978, known as fuzzy linear programming.

Numerous researchers have extensively worked in this area and have developed various methods. Lotfi et al. [21] proposed a method to solve full fuzzy linear programming (FFLP) problems by approximating fuzzy triangular numbers to their nearest symmetric counterparts, thereby transforming the problem into a crisp MOLP problem and subsequently using the LA for optimal solution determination. Similarly, Ezzati et al. [22] proposed an innovative algorithm based on a new LA of triangular fuzzy numbers, which addresses FFLP challenges by transforming them into corresponding MOLP problems and then resolving them using the lexicographic method. In addition, Das et al. [23] introduced an efficient approach for solving FFLP problems, leveraging a novel LA on trapezoidal fuzzy numbers and deriving the method from the auxiliary MOLP model. Following this, Das [24] modified the Das et al. [23] algorithm and proposed a new method using LA for finding the fuzzy optimal solution of FFLP problems with triangular fuzzy numbers. Furthermore, applying a LA, Perez-Canedo and Eduardo [25] introduced a new model for solving fuzzy linear assignment problems involving various fuzzy numbers. Kumar et al. [26] present a network model to solve the SPP in which the arc length is weighted fuzzy. Garg [27] developed an LP model to solve the MCDM problem with unknown attribute weights, leveraging an improved score Function for Interval-Valued Pythagorean Fuzzy Numbers. In contrast, Garg and Singh [28] addressed the issue of interval-valued problems and aggregation operators within the context of a triangular type 2 fuzzy problem. Additionally, Akram et al. [29] introduced the complex picture fuzzy set (CPFS) as a generalized extension of the complex intuitionistic fuzzy set (CIFS) by incorporating a neutral membership degree and Gayen et al. [30] introduce the concept of an anti-fuzzy subgroup and analyze its attributes.

Also, there are several researchers have extensively worked in the area of fuzzy MCF problems such as El-Sherbeny [31] introduced a novel algorithm designed to efficiently solve fuzzy MCF problems that incorporate fuzzy time windows, utilizing a polynomial-time algorithm personalized for fuzzy time windows. In a similar vein, Alharbi et al. [32] introduced an interactive methodology aimed at addressing the Multi-objective MCF within a fuzzy environment, where the associated costs are described as trapezoidal fuzzy numbers. And, Khalifa and Edalatanah [33] proposed a novel approach for solving Multi-Objective MCF problems using the fuzzy goal programming technique. They consider objective functions with possibilistic coefficients and employ an alpha-Pareto optimal solution-based scenario. Many researchers have used these theories and their extensions, like fuzzy theory and extended fuzzy theory, to solve problems with uncertainty. In a fuzzy environment, uncertainty is effectively addressed using vague reasoning. However, when confronted with indeterminate and inconsistent information, accurately handling such situations becomes challenging. To tackle this, in 1999, Smarandache [34] introduced the concept of the Neutrosophic Set (NS).

The concept of neutrosophic sets is an advanced mathematical approach that builds upon fuzzy logic to better manage data that is unclear, inconsistent, or incomplete. It focuses on three key measures: the degree to which information is not true (falsity), not determinable (indeterminacy), and true (truth), to improve how we handle uncertain and imprecise data. These three distinct levels of membership fall within a unique range, which is just above 0 and just below 1. However, the membership level of a fuzzy set is confined within the standard range of 0 to 1. The NS is particularly useful in modeling a variety of real-world scenarios. This is because it has the ability to handle information that is incomplete, inconsistent, or uncertain. There are some researchers who work in neutrosophic environments such as Edalatpanah [35] introduced a direct model to solve the LP with TrNNs and Abdel-Basset et al. [36] presented a new method for solving the LP model with TrpNNs. Khan et al. [37] proposed a new approach to solving the MADM problem by using interval-valued neutrosophic environments. Wang et al. [38] introduced SVTN problems for solving the LP problem and also so many researchers [39–42] are worked on SVTN environments. These operators play a crucial role in addressing various practical complexities. We have compiled the significant contributions of various researchers who have introduced these practical applications in Table 1.

Table 1. The significant contributions of various researchers in different NS environments

Authors	Years	Significance
Dey et al. [43]	2019	Dey et al. explore the MST problem in a neutrosophic weighted graph with single-valued neutrosophic arc lengths.
Broumi et al. [44]	2019	To solve the SPP under the triangular and interval-valued TrpNNs.
Khalifa and Kumar [45]	2020	Khalifa and Kumar proposed a new approach to solve the multi-objective assignment problem under an interval-valued TrpNNs environment.
Fallah and Nozari [46]	2021	A neutrosophic programming approach designs a multi-objective problem under uncertainty-resilient biomass supply chain.
Giri and Roy [47]	2022	Solving the MO transportation problem under single-valued trapezoidal neutrosophic numbers.
Adhikary et al. [48]	2024	Solving the MST Problem with trapezoidal neutrosophic numbers representing arc weights.
Dey et al. [49]	2024	To solve the SPP using a Fermatean interval-valued neutrosophic environment.
Gupta et al. [50]	2024	To solve a MO fixed-charge transportation problem with TrpNNs parameters.

1.1 *Motivation and novelties*

Neutrosophic set theory is a recognized method for handling uncertainty in optimization issues. The concept of MCF under the neutrosophic setting has been explored by a handful of scholars. This article proposes a solution method for the MCF problem with neutrosophic parameters, eliminating the need for ranking methods because different ranking methods can lead to varied solutions. This drawback has motivated us to develop a new solution approach that does not depend on any ranking method. Therefore, we use this approach by converting it into an MOLP and then using a lexicographic approach to transform it into a single crisp LP problem. The novelties of this manuscript are as follows:

• Introduce a novel lexicographical approach to the MCF problem, marking a first in the domain of neutrosophic set literature.

• This model facilitates the resolution of new problem sets under the neutrosophic set numbers.

1.2 *Objective*

Existing methods for addressing MCF problems are fraught with numerous challenges, as detailed throughout this manuscript. These issues have inspired the development of an innovative approach that applies neutrosophic logic to MCF. From this perspective, an attempt is being made to extend this article, as there currently exists no method for applying lexicographical approach to the TrNMCF problem. The main objective of this manuscript is as follows:

• To address the MCF problem within the context of neutrosophic uncertainty, where the cost factor is a pivotal element.

• To combine the lexicographical approach with a multi-objective perspective, aiming to optimize the MCF problem on multiple fronts simultaneously.

1.3 *Structure of the manuscript*

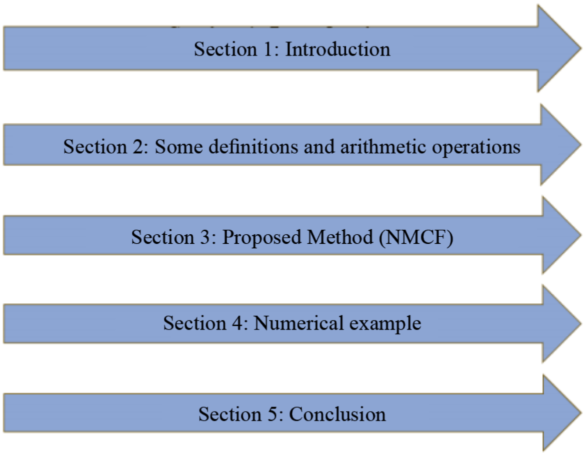

Figure 1. Structure of the manuscript

2. Preliminaries

Definition 1 Neutrosophic Set [51]:

A set $\widetilde{neuE}$ in the universal set K, is said to be a neutrosophic set if $\widetilde{neuE} = \left\{\left(\kappa,\ \left[T_{\widetilde{neuE}}(\kappa),\ I_{\widetilde{neuE}}(\kappa),\ F_{\widetilde{neuE}}(\kappa)\right]\right) :\ \kappa \in K\right\}$ Where $F_{\widetilde{neuE}}(\kappa) : K \to [0,\ 1]$, $I_{\widetilde{neuE}}(\kappa) : K \to [0,\ 1]$, and $T_{\widetilde{neuE}}(\kappa) : K \to [0,\ 1]$ are defined as the falsity membership function $F_{\widetilde{E_\eta}}(\kappa)$, indeterminacy function, $I_{\widetilde{E_\eta}}(\kappa)$ and truth membership functions $T_{\widetilde{E_\eta}}(\kappa)$, of an element κ in $\widetilde{neuE}$ and respectively and $T_{\widetilde{neuE}}(\kappa)$, $I_{\widetilde{neuE}}(\kappa)$, $F_{\widetilde{neuE}}(\kappa)$ satisfy the condition.

$$0 \le T_{\widetilde{neuE}}(\kappa) + I_{\widetilde{neuE}}(\kappa) + F_{\widetilde{neuE}}(\kappa) \le 3$$

Definition 2 Single-valued triangular neutrosophic (SVTN) number [52]:

SVTN number is a special type of SVTN set. A set $\widetilde{E_\eta} = \left\langle \left(\widehat{\varepsilon_{11}},\ \widehat{\varepsilon_{21}},\ \widehat{\varepsilon_{31}}\right),\ (\delta_{11},\ \delta_{21},\ \delta_{31}),\ \left(\widehat{\varpi_{11}},\ \widehat{\varpi_{21}},\ \widehat{\varpi_{31}}\right) \right\rangle$ is called SVTN number if its define a truth functions $T_{\widetilde{E_\eta}}(\kappa)$, indeterminacy function, $I_{\widetilde{E_\eta}}(\kappa)$ and the falsity membership function $F_{\widetilde{E_\eta}}(\kappa)$ of an element κ in $\widetilde{E_\eta}$ is defined as,

$$T_{\widetilde{E_\eta}}(\kappa) = \left\{ \begin{array}{ll} \left(\dfrac{\kappa - \widehat{\varepsilon_{11}}}{\widehat{\varepsilon_{21}} - \widehat{\varepsilon_{11}}}\right) & \widehat{\varepsilon_{11}} \le \kappa < \widehat{\varepsilon_{21}} \\ 1 & \widehat{\varepsilon_{21}} = \kappa \\ \left(\dfrac{\widehat{\varepsilon_{31}} - \kappa}{\widehat{\varepsilon_{31}} - \widehat{\varepsilon_{21}}}\right) & \widehat{\varepsilon_{21}} < \kappa \le \widehat{\varepsilon_{31}} \\ 0 & \text{otherwise} \end{array} \right\},\ I_{\widetilde{E_\eta}}(\kappa) = \left\{ \begin{array}{ll} \left(\dfrac{\delta_{21} - \kappa}{\delta_{21} - \delta_{11}}\right) & \delta_{11} \le \kappa < \delta_{21} \\ 0 & \delta_{21} = \kappa \\ \left(\dfrac{\kappa - \delta_{21}}{\delta_{31} - \delta_{21}}\right) & \delta_{21} < \kappa \le \delta_{31} \\ 1 & \text{otherwise} \end{array} \right\}$$

and

$$F_{\widetilde{E_\eta}}(\kappa)=\begin{cases}\left(\dfrac{\widehat{\varpi_{21}}-\kappa}{\widehat{\varpi_{21}}-\widehat{\varpi_{11}}}\right) & \widehat{\varpi_{11}}\le\kappa<\widehat{\varpi_{21}}\\ 0 & \widehat{\varpi_{21}}=\kappa\\ \left(\dfrac{\kappa-\widehat{\varpi_{21}}}{\widehat{\varpi_{31}}-\widehat{\varpi_{21}}}\right) & \widehat{\varpi_{21}}<\kappa\le\widehat{\varpi_{31}}\\ 1 & \text{otherwise}\end{cases}$$

with the condition $0\le T_{\widetilde{E_\eta}}(\kappa)+I_{\widetilde{E_\eta}}(\kappa)+F_{\widetilde{E_\eta}}(\kappa)\le 3$.

Definition 3 Arithmetic Operations for SVTN number [36]:

Let $\widetilde{E_{\eta_1}}=\left\langle\left(\widehat{\varepsilon_{11}},\ \widehat{\varepsilon_{21}},\ \widehat{\varepsilon_{31}}\right),\ (\delta_{11},\ \delta_{21},\ \delta_{31}),\ \left(\widehat{\varpi_{11}},\ \widehat{\varpi_{21}},\ \widehat{\varpi_{31}}\right)\right\rangle$ and $\mathrm{E}_{\eta_2}=\left\langle\begin{array}{c}\left(\widehat{\varepsilon_{12}},\ \widehat{\varepsilon_{22}},\ \widehat{\varepsilon_{32}}\right),\ (\delta_{12},\ \delta_{22},\ \delta_{32}),\\ \left(\widehat{\varpi_{12}},\ \widehat{\varpi_{22}},\ \widehat{\varpi_{32}}\right)\end{array}\right\rangle$ be two SVTN numbers and $\theta>0$ then

(i) $\mathrm{E}_{\eta_1}\oplus E_{\eta_2}=\left\langle\left(\widehat{\varepsilon_{11}}+\widehat{\varepsilon_{12}},\ \widehat{\varepsilon_{21}}+\widehat{\varepsilon_{22}},\ \widehat{\varepsilon_{31}}+\widehat{\varepsilon_{32}}\right),\ (\delta_{11}+\delta_{12},\ \delta_{21}+\delta_{22},\ \delta_{31}+\delta_{32}),\ \left(\begin{array}{c}\widehat{\varpi_{11}}+\widehat{\varpi_{12}},\ \widehat{\varpi_{21}}\\ +\widehat{\varpi_{22}},\ \widehat{\varpi_{31}}+\widehat{\varpi_{32}}\end{array}\right)\right\rangle$

(ii) $\mathrm{E}_{\eta_1}\otimes E_{\eta_2}=\left\langle\left(\widehat{\varepsilon_{11}}.\widehat{\varepsilon_{12}},\ \widehat{\varepsilon_{21}}.\widehat{\varepsilon_{22}},\ \widehat{\varepsilon_{31}}.\widehat{\varepsilon_{32}}\right),\ (\delta_{11}.\delta_{12},\ \delta_{21}.\delta_{22},\ \delta_{31}.\delta_{32}),\ \left(\widehat{\varpi_{11}}.\widehat{\varpi_{12}},\ \widehat{\varpi_{21}}.\widehat{\varpi_{22}},\ \widehat{\varpi_{31}}.\widehat{\varpi_{32}}\right)\right\rangle$

(iii) $\theta\odot E_{\eta_1}=\left\langle\left(\theta.\widehat{\varepsilon_{11}},\ \theta.\widehat{\varepsilon_{21}},\ \theta\widehat{\varepsilon_{31}}\right),\ (\theta.\delta_{11},\ \theta.\delta_{21},\ \theta.\delta_{31}),\ \left(\theta.\widehat{\varpi_{11}},\ \theta.\widehat{\varpi_{21}},\ \theta.\widehat{\varpi_{31}}\right)\right\rangle$.

Definition 4 Let $\widetilde{E_\eta}=\left\langle\left(\widehat{\varepsilon_{11}},\ \widehat{\varepsilon_{21}},\ \widehat{\varepsilon_{31}}\right),\ (\delta_{11},\ \delta_{21},\ \delta_{31}),\ \left(\widehat{\varpi_{11}},\ \widehat{\varpi_{21}},\ \widehat{\varpi_{31}}\right)\right\rangle$ be an SVTN number and define a functions$M,\ N$ from a set of neutrosophic numbers characterized by a set of the real number $N(\mathbb{R})$ to real line such that each neutrosophic number is such that

$$M\left(\widetilde{E_\eta}\right)=\frac{1}{12}\left[8+\left(\widehat{\varepsilon_{11}}+2.\widehat{\varepsilon_{21}}+\widehat{\varepsilon_{31}}\right)-(\delta_{11}+2.\delta_{21}+\delta_{31})-\left(\widehat{\varpi_{11}}+2.\widehat{\varpi_{21}}+\widehat{\varpi_{31}}\right)\right]$$

And

$$N\left(\widetilde{E_\eta}\right)=\frac{1}{4}\left[\left(\widehat{\varepsilon_{11}}+2.\widehat{\varepsilon_{21}}+\widehat{\varepsilon_{31}}\right)-(\delta_{11}+2.\delta_{21}+\delta_{31})\right]$$

Where the function $M\left(\widetilde{E_\eta}\right)$ and $N\left(\widetilde{E_\eta}\right)$ are known as the score function and the accuracy function respectively.

Definition 5 [36] A ranking function $M$of neutrosophic number is a function defined from a set of neutrosophic numbers which characterized by a set of real number $N(\mathbb{R})$ to real line such that each neutrosophic number is converted into a real number. Let $\widetilde{E_{\eta_1}}=\left\langle\left(\widehat{\varepsilon_{11}},\ \widehat{\varepsilon_{21}},\ \widehat{\varepsilon_{31}}\right),\ (\delta_{11},\ \delta_{21},\ \delta_{31}),\ \left(\widehat{\varpi_{11}},\ \widehat{\varpi_{21}},\ \widehat{\varpi_{31}}\right)\right\rangle$ and $\mathrm{E}_{\eta_2}=\left\langle\begin{array}{c}\left(\widehat{\varepsilon_{12}},\ \widehat{\varepsilon_{22}},\ \widehat{\varepsilon_{32}}\right),\\ (\delta_{12},\ \delta_{22},\ \delta_{32}),\\ \left(\widehat{\varpi_{12}},\ \widehat{\varpi_{22}},\ \widehat{\varpi_{32}}\right)\end{array}\right\rangle$ be two any SVTN numbers then

1. If $M\left(\widetilde{E_{\eta_1}}\right)>M\left(\widetilde{E_{\eta_2}}\right)$ then $\widetilde{E_{\eta_1}}\succ\widetilde{E_{\eta_2}}$
2. If $M\left(\widetilde{E_{\eta_1}}\right)<M\left(\widetilde{E_{\eta_2}}\right)$ then $\widetilde{E_{\eta_1}}\prec\widetilde{E_{\eta_2}}$
3. If $M\left(\widetilde{E_{\eta_1}}\right)=M\left(\widetilde{E_{\eta_2}}\right)$ and if

(a) $N\left(\widetilde{E_{\eta_1}}\right)>N\left(\widetilde{E_{\eta_2}}\right)$ then $\widetilde{E_{\eta_1}}\succ\widetilde{E_{\eta_2}}$

(b) $N\left(\widetilde{E_{\eta_1}}\right)<N\left(\widetilde{E_{\eta_2}}\right)$ then $\widetilde{E_{\eta_1}}\prec\widetilde{E_{\eta_2}}$

(c) $N\left(\widetilde{E_{\eta_1}}\right) = N\left(\widetilde{E_{\eta_2}}\right)$ then $\widetilde{E_{\eta_1}} \approx \widetilde{E_{\eta_2}}$.

3. Our proposed model

Before we start with a proposed algorithm, we introduce a sub-section i.e., The existing crisp model in MCF problem and the NMFP with neutrosophic cost.

3.1 *Existing crisp model in MCF problem*

Let a directed graph $G^{\eta} = (N_{\eta}, \aleph_{\eta})$ is considered, where $N_{\eta} = \{1, 2, 3, ..., t\}$ is the set of finite nodes and $\aleph_{\eta}$represents the set of arcs. Each arc is denoted by $(q, t) \in \aleph_{\eta}$, accompanied by a flow τ_{qt}, and a cost-per-unit flow Sk_{qt} from arc q to t. The two numbers, p_{qt} and ℓ_{qt} are to be considered as the upper and lower capacity respectively. A number Υ_q, which represents the supply, demand, or transshipment node, is to be assigned to each node $q \in N_{\eta}$. The node q is identified as a transshipment node if Υ_q equals 0. If Υ_q is less than 0, then the node q is identified as a demand node. If Υ_q is greater than 0, then the node q is identified as a supply node.

The general formulation of the mathematical model for the classical MCF problem is to be considered as follows:

$$Mini\ Z = \sum_{(q,\, t)\in \aleph_{\eta}} Sk_{qt}\ \tau_{qt}$$

Subject to

$$\sum_{t:\ (q,\, t)\in \aleph_{\eta}} \tau_{qr} - \sum_{t:\ (t,\, q)\in \aleph_{\eta}} \tau_{tq} = \Upsilon_q,\ \forall q \in N_{\eta}$$

$$0 \le \ell_{qt} \le \tau_{qt} \le p_{qt} \forall (q,\, t) \in \aleph_{\eta}$$

3.2 *Transformation of the crisp model MCF problem into NMFP with neutrosophic cost*

In this section, consider the scenario where we substitute the cost Sk_{qt} convert into a neutrosophic cost per unit flow $\widetilde{Sk^{\eta}_{qt}}$ from arc q to t. Then the general formulation of the mathematical model for the NMFP with neutrosophic cost is to be considered as follows:

$$Mini\ \widetilde{O^{\eta}} \approx \sum_{(q,\, t)\in \aleph_{\eta}} \widetilde{Sk^{\eta}_{qt}}\ \tau_{qt} \tag{1}$$

Subject to constraints

$$\sum_{t:\ (q,\, t)\in \aleph_{\eta}} \tau_{qt} - \sum_{t:\ (t,\, q)\in \aleph_{\eta}} \tau_{tq} = \Upsilon_q,\ \forall q \in N_{\eta} \tag{2}$$

$$0 \le \ell_{qt} \le \tau_{qt} \le p_{qt} \forall (q,\, t) \in \aleph_{\eta} \tag{3}$$

3.3 *Algorithm: A novel approach for finding the NMFP with neutrosophic cost considering as SVTN number for cost parameters*

We considered a directed graph whose arcs denote the neutrosophic cost per unit flow $\widetilde{Sk_{qt}^{\eta}}$ from arc q to t. In this section, our proposed algorithm tends to provide a novel methodology for finding the NMFP with neutrosophic cost considering as SVTN number for cost parameters.

The steps of the algorithm are as follows

Algorithm

Step 1: Here we consider the $\widetilde{Sk_{qt}^{\eta}}$ of the form $\widetilde{Sk_{qt}^{\eta}} = \left\langle \left(\widetilde{Sk_{1qt}^{\eta}}, \widetilde{Sk_{2qt}^{\eta}}, \widetilde{Sk_{3qt}^{\eta}}\right), \left(\widetilde{Sk_{4qt}^{\eta}}, \widetilde{Sk_{5qt}^{\eta}}, \widetilde{Sk_{6qt}^{\eta}}\right), \left(\widetilde{Sk_{7qt}^{\eta}}, \widetilde{Sk_{8qt}^{\eta}},\right.\right.$ $\widetilde{Sk_{9qt}^{\eta}}$. Then the NMFP with neutrosophic cost $\widetilde{Sk_{qt}^{\eta}}$, the objective function in equation (1) will be

$$Mini\widetilde{O^{\eta}} \approx \sum_{(q,\,t)\in\aleph_{\eta}} \left\langle \begin{array}{l} \left(\widetilde{Sk_{1qt}^{\eta}}, \widetilde{Sk_{2qt}^{\eta}}, \widetilde{Sk_{3qt}^{\eta}}\right), \left(\widetilde{Sk_{4qt}^{\eta}}, \widetilde{Sk_{5qt}^{\eta}}, \widetilde{Sk_{6qt}^{\eta}}\right), \\ \left(\widetilde{Sk_{7qt}^{\eta}}, \widetilde{Sk_{8qt}^{\eta}}, \widetilde{Sk_{9qt}^{\eta}}\right) \end{array} \right\rangle \tau_{qt} \tag{4}$$

With subject to constraints equation (2) and (3).

Step 2: Now, the above objective function can be expanded into nine several crisp objective functions as:

$$\left.\begin{array}{l}
Min\left(\widetilde{O_1^{\eta}}\right) = \sum_{(q,\,t)\in\aleph_{\eta}} \widetilde{Sk_{1qt}^{\eta}}\tau_{qt}; Min\left(\widetilde{O_2^{\eta}}\right) = \sum_{(q,\,t)\in\aleph_{\eta}} \widetilde{Sk_{2qt}^{\eta}}\tau_{qt}; \\
Min\left(\widetilde{O_3^{\eta}}\right) = \sum_{(q,\,t)\in\aleph_{\eta}} \widetilde{Sk_{3qt}^{\eta}}\tau_{qt}; Min\left(\widetilde{O_4^{\eta}}\right) = \sum_{(q,\,t)\in\aleph_{\eta}} \widetilde{Sk_{4qt}^{\eta}}\tau_{qt}; \\
Min\left(\widetilde{O_5^{\eta}}\right) = \sum_{(q,\,t)\in\aleph_{\eta}} \widetilde{Sk_{5qt}^{\eta}}\tau_{qt}; Min\left(\widetilde{O_6^{\eta}}\right) = \sum_{(q,\,t)\in\aleph_{\eta}} \widetilde{Sk_{6qt}^{\eta}}\tau_{qt}; \\
Min\left(\widetilde{O_7^{\eta}}\right) = \sum_{(q,\,t)\in\aleph_{\eta}} \widetilde{Sk_{7qt}^{\eta}}\tau_{qt}; Min\left(\widetilde{O_8^{\eta}}\right) = \sum_{(q,\,t)\in\aleph_{\eta}} \widetilde{Sk_{8qt}^{\eta}}\tau_{qt}; \\
Min\left(\widetilde{O_9^{\eta}}\right) = \sum_{(q,\,t)\in\aleph_{\eta}} \widetilde{Sk_{9qt}^{\eta}}.\tau_{qt}
\end{array}\right] \tag{5}$$

With the same subject to constraints equation (2) and (3).

Step 3: First of all, solving the below problem by the LPP method

$$Min\left(\widetilde{O_1^{\eta}}\right) = \sum_{(q,\,t)\in\aleph_{\eta}} \widetilde{Sk_{1qt}^{\eta}}\tau_{qt} \tag{6}$$

With the same subject to constraints equation (2) and (3). The optimal value of the above equation (6) is $\widehat{(Rs_1)}^{*}$

Step 4: Again, calculate the subsequent problem given as below

$$Min\left(\widetilde{O_2^{\eta}}\right) = \sum_{(q,\,t)\in\aleph_{\eta}} \widetilde{Sk_{2qt}^{\eta}}\tau_{qt} \tag{7}$$

Subject to

$$\left.\begin{array}{c}\widehat{(Rs_1)^*} = \sum_{(q,\ t)\in\aleph_\eta} \widetilde{Sk^\eta_{1qt}}\tau_{qt} \\ Also\ with\ the\ constraints\ (2) and (3)\end{array}\right\} \tag{8}$$

The optimal value of the above equation (7) is $\widehat{(Rs_2)^*}$.

Step 5: Now, calculate the subsequent problem given as below

$$Min\left(\widetilde{O^\eta_3}\right) = \sum_{(q,\ t)\in\aleph_\eta} \widetilde{Sk^\eta_{3qt}}\tau_{qt} \tag{9}$$

Subject to

$$\left.\begin{array}{c}\widehat{(Rs_2)^*} = \sum_{(q,\ t)\in\aleph_\eta} \widetilde{Sk^\eta_{2qt}}\tau_{qt} \\ \text{Also with the constraints (8)}\end{array}\right\} \tag{10}$$

The optimal value of the above equation (9) is $\widehat{(Rs_3)^*}$.

Step 6: Again, calculate the subsequent problem given as below

$$Min\left(\widetilde{O^\eta_4}\right) = \sum_{(q,\ t)\in\aleph_\eta} \widetilde{Sk^\eta_{4qt}}\tau_{qt} \tag{11}$$

Subject to

$$\left.\begin{array}{c}\widehat{(Rs_3)^*} = \sum_{(q,\ t)\in\aleph_\eta} \widetilde{Sk^\eta_{3qt}}\tau_{qt} \\ \text{Also with the constraints (10)}\end{array}\right\} \tag{12}$$

The optimal value of the above equation (11) is $\widehat{(Rs_4)^*}$.

Step 7: Again. calculate the subsequent problem given as below

$$Min\left(\widetilde{O^\eta_5}\right) = \sum_{(q,\ t)\in\aleph_\eta} \widetilde{Sk^\eta_{5qt}}\tau_{qt} \tag{13}$$

Subject to

$$\left.\begin{array}{c}\widehat{(Rs_4)^*} = \sum_{(q,\ t)\in\aleph_\eta} \widetilde{Sk^\eta_{4qt}}\tau_{qt} \\ \text{Also with the constraints (12)}\end{array}\right\} \tag{14}$$

The optimal value of the above equation (13) is $\widehat{(Rs_5)}^*$.
Step 8: Again, calculate the subsequent problem given as below

$$Min\left(\widetilde{O_6^\eta}\right) = \sum_{(q,\, t)\in\aleph_\eta} \widetilde{Sk_{6qt}^\eta}\,\tau_{qt} \tag{15}$$

Subject to

$$\left.\begin{array}{c}\widehat{(Rs_5)}^* = \sum_{(q,\, t)\in\aleph_\eta} \widetilde{Sk_{5qt}^\eta}\,\tau_{qt}\\ \text{Also with the constraints (14)}\end{array}\right\} \tag{16}$$

The optimal value of the above equation (15) is $\widehat{(Rs_6)}^*$.
Step 9: Again, calculate the subsequent problem given as below

$$Min\left(\widetilde{O_7^\eta}\right) = \sum_{(q,\, t)\in\aleph_\eta} \widetilde{Sk_{7qt}^\eta}\,\tau_{qt} \tag{17}$$

Subject to

$$\left.\begin{array}{c}\widehat{(Rs_6)}^* = \sum_{(q,\, t)\in\aleph_\eta} \widetilde{Sk_{6qt}^\eta}\,\tau_{qt}\\ \text{Also with the constraints (16)}\end{array}\right\} \tag{18}$$

The optimal value of the above equation (17) is $\widehat{(Rs_7)}^*$.
Step 10: Now, calculate the subsequent problem given as below

$$Min\left(\widetilde{O_8^\eta}\right) = \sum_{(q,\, t)\in\aleph_\eta} \widetilde{Sk_{8qt}^\eta}\,\tau_{qt} \tag{19}$$

Subject to

$$\left.\begin{array}{c}\widehat{(Rs_7)}^* = \sum_{(q,\, t)\in\aleph_\eta} \widetilde{Sk_{7qt}^\eta}\,\tau_{qt}\\ \text{Also with the constraints (18)}\end{array}\right\} \tag{20}$$

The optimal value of the above equation (19) is $\widehat{(Rs_8)}^*$.
Step 11: Again, calculate the subsequent problem given as below

$$Min\left(\widetilde{O_9^\eta}\right) = \sum_{(q,\, t)\in\aleph_\eta} \widetilde{Sk_{9qt}^\eta}.\tau_{qt} \tag{21}$$

Subject to

$$\left.\begin{array}{c}\widehat{(Rs_8)}^* = \sum\limits_{(q,\, t)\in \aleph_\eta} \widetilde{Sk^\eta_{8qt}}\,\tau_{qt} \\ \text{Also with the constraints (20)}\end{array}\right\} \tag{22}$$

The optimal value of the above equation (21) is $\widehat{(Rs_9)}^*$.

Step 12: The final optimal value is in the form of a neutrosophic environment is

$$Min\left(\widetilde{O^n}\right) = \left\langle (\widetilde{O^\eta_1},\, \widetilde{O^\eta_2},\, \widetilde{O^\eta_3}),\, (\widetilde{O^\eta_4},\, \widetilde{O^\eta_5},\, \widetilde{O^\eta_6}),\, (\widetilde{O^\eta_7},\, \widetilde{O^\eta_8},\, \widetilde{O^\eta_1}) \right\rangle$$

The End.

Theorem 1 The optimal solution of equation (4) is the same as the optimal solution of the NMFP with neutrosophic cost equation (21).

Proof. Let τ^*_{qt} be the optimal solution of equation (21) and $\tau^\propto_{qt}$ be another solution of NMFP with neutrosophic cost of equation (4). Then by using concept of above proposed method we obtained that the solution of the equation (21) is the least value among the all problems from equation (4) to equation (19). Now using the optimality condition, τ^*_{qt} is optimal value of the problem equation (6) and from the feasibility condition, $\tau^\propto_{qt}$ is the feasible solution of the equation (6). Therefore, it can be written as-

$$\widehat{(Rs_1)}^* = \sum_{(q,\, t)\in \aleph_\eta} \widetilde{Sk^\eta_{1qt}}.\tau^*_{qt} \le \sum_{(q,\, t)\in \aleph_\eta} \widetilde{Sk^\eta_{1qt}}.\tau^\propto_{qt}$$

Moreover, due to optimality condition of τ^*_{qt} and feasibility condition of $\tau^\propto_{qt}$ for equation 7 and 8 can be written as

$$\widehat{(Rs_2)}^* = \sum_{(q,\, t)\in \aleph_\eta} \widetilde{Sk^\eta_{2qt}}.\tau^*_{qt} \le \sum_{(q,\, t)\in \aleph_\eta} \widetilde{Sk^\eta_{2qt}}.\tau^\propto_{qt}$$

Similarly,

$$\widehat{(Rs_3)}^* = \sum_{(q,\, t)\in \aleph_\eta} \widetilde{Sk^\eta_{3qt}}.\tau^*_{qt} \le \sum_{(q,\, t)\in \aleph_\eta} \widetilde{Sk^\eta_{3qt}}.\tau^\propto_{qt}$$

$$\widehat{(Rs_4)}^* = \sum_{(q,\, t)\in \aleph_\eta} \widetilde{Sk^\eta_{4qt}}.\tau^*_{qt} \le \sum_{(q,\, t)\in \aleph_\eta} \widetilde{Sk^\eta_{4qt}}.\tau^\propto_{qt}$$

$$\widehat{(Rs_5)}^* = \sum_{(q,\, t)\in \aleph_\eta} \widetilde{Sk^\eta_{5qt}}.\tau^*_{qt} \le \sum_{(q,\, t)\in \aleph_\eta} \widetilde{Sk^\eta_{5qt}}.\tau^\propto_{qt}$$

$$\widehat{(Rs_6)}^* = \sum_{(q,\, t)\in \aleph_\eta} \widetilde{Sk^\eta_{6qt}}.\tau^*_{qt} \le \sum_{(q,\, t)\in \aleph_\eta} \widetilde{Sk^\eta_{6qt}}.\tau^\propto_{qt}$$

$$\widehat{(Rs_7)}^* = \sum_{(q,\, t)\in \aleph_\eta} \widetilde{Sk^\eta_{7qt}}.\tau^*_{qt} \le \sum_{(q,\, t)\in \aleph_\eta} \widetilde{Sk^\eta_{7qt}}.\tau^\propto_{qt}$$

$$\widehat{(Rs_9)}^* = \sum_{(q,\,t)\in\aleph_\eta} \widetilde{Sk^\eta_{9qt}}.\tau^*_{qt} \leq \sum_{(q,\,t)\in\aleph_\eta} \widetilde{Sk^\eta_{9qt}}.\tau^\propto_{qt}$$

$$\widehat{(Rs_8)}^* = \sum_{(q,\,t)\in\aleph_\eta} Sk^\eta_{8qt}.\tau^*_{qt} \leq \sum_{(q,\,t)\in\aleph_\eta} \widetilde{Sk^\eta_{8qt}}.\tau^\propto_{qt}$$

Hence,

$$\left\langle \begin{array}{l} \left(\widehat{(Rs_1)}^*,\ \widehat{(Rs_2)}^*,\ \widehat{(Rs_3)}^*\right), \\ \left(\widehat{(Rs_4)}^*,\ \widehat{(Rs_5)}^*,\ \widehat{(Rs_6)}^*\right), \\ \left(\widehat{(Rs_7)}^*,\ \widehat{(Rs_8)}^*,\ \widehat{(Rs_9)}^*\right) \end{array} \right\rangle \leq \left\langle \begin{array}{l} \sum_{(q,\,t)\in\aleph_\eta} Sk^\eta_{1qt}\tau^\propto_{qt},\ \sum_{(q,\,t)\in\aleph_\eta} Sk^\eta_{2qt}\tau^\propto_{qt},\ \sum_{(q,\,t)\in\aleph_\eta} Sk^\eta_{3qt}\tau^\propto_{qt}, \\ \sum_{(q,\,t)\in\aleph_\eta} Sk^\eta_{4qt}\tau^\propto_{qt},\ \sum_{(q,\,t)\in\aleph_\eta} Sk^\eta_{5qt}\tau^\propto_{qt},\ \sum_{(q,\,t)\in\aleph_\eta} Sk^\eta_{6qt}\tau^\propto_{qt}, \\ \sum_{(q,\,t)\in\aleph_\eta} Sk^\eta_{7qt}\tau^\propto_{qt},\ \sum_{(q,\,t)\in\aleph_\eta} Sk^\eta_{8qt}\tau^\propto_{qt},\ \sum_{(q,\,t)\in\aleph_\eta} Sk^\eta_{9qt}\tau^\propto_{qt} \end{array} \right\rangle$$

To illustrate our proposed algorithm, we consider an example, a flow network shown in see Figure 2.

4. Numerical example

Now let us consider an example, a flow network presented by Ghatee et al [16]. (see Figure 2) with fourteen arcs and seven nodes. The cost value of each arc is presented by triangular neutrosophic numbers. In this example, to determine the flow of network with the least cost.

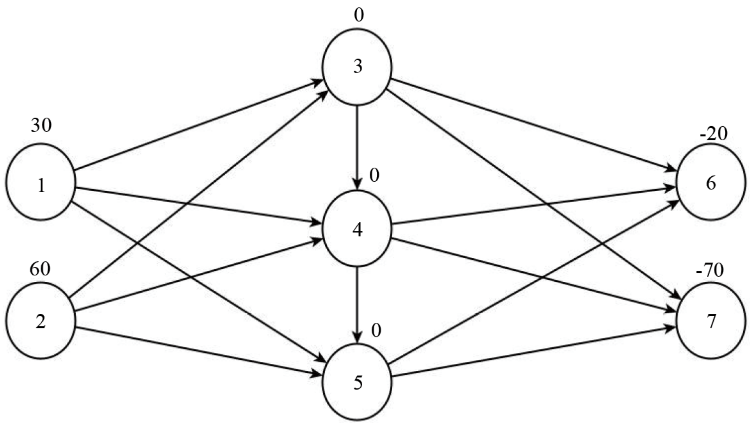

Figure 2. The network consider for Solving MCF problem under neiutrosophic environment

Table 2. Consider the below data for Solving NMFP under neiutrosophic environment

T	H	Neutrosophic arc cost	C	T	H	Neutrosophic arc cost	C
1	3	<(1, 4, 7), (1, 3, 5), (3.5, 6, 7.5)>	40	3	6	<(1, 5, 8), (1.5, 4.5, 7.5), (4, 6.5, 9)>	100
1	4	<(0.5, 2.5, 4.5), (1, 2, 3), (1.5, 3.5, 5.5)>	10	3	7	<(1, 5, 8), (1.5, 3, 6.5), (4, 7, 9)>	90
1	5	<(1, 3, 5), (0.5, 1.5, 3.5), (3, 4, 6)>	15	4	5	<(10, 15, 20), (14, 16, 22), (12, 15, 19)>	15
2	3	<(1, 2, 3), (0.5, 1.5, 2.5), (1.5, 2.5, 3.5)>	50	4	6	<(20, 25, 30), (24, 26, 32), (22, 25, 29)>	30
2	4	<(1, 1.5, 4), (0.5, 1, 2.5), (2.25, 3, 4.25)>	80	4	7	<(15, 20, 25), (19, 21, 27), (17, 20, 24)>	20
2	5	<(1.5, 2.5, 3.5), (1, 1.5, 3), (2, 3, 4)>	40	5	6	<(13, 18, 23), (17, 19, 25), (15, 18, 22)>	15
3	4	<(2, 4, 6), (1.5, 2.5, 4.5), (3, 5, 7)>	20	5	7	<(15, 20, 25), (19, 21, 27), (17, 20, 24)>	100

Solution:

Step 1: To solve the problem, first of all, transform the above network into a mathematical model named equation (23)

$$
\begin{aligned}
Min\left(\widetilde{O^{\eta}}\right) = & \langle(1,\ 4,\ 7),\ (1,\ 3,\ 5),\ (3.5,\ 6,\ 7.5)\rangle\,\tau_{13} + \langle(0.5,\ 2.5,\ 4.5),\ (1,\ 2,\ 3),(1.5,\ 3.5,\ 5.5)\rangle\,\tau_{14} \\
& + \langle(1,\ 3,\ 5),(0.5,\ 1.5,\ 3.5),(3,\ 4,\ 6)\rangle\,\tau_{15} + \langle(1,\ 2,\ 3),\ (0.5,\ 1.5,\ 2.5),\ (1.5,\ 2.5,\ 3.5)\rangle\,\tau_{23} \\
& + \langle(1,\ 1.5,\ 4),\ (0.5,\ 1,\ 2.5),\ (2.25,\ 3,\ 4.25)\rangle\,\tau_{24} + \langle(1.5,\ 2.5,\ 3.5),\ (1,\ 1.5,\ 3),\ (2,\ 3,\ 4)\rangle\,\tau_{25} \\
& + \langle(2,\ 4,\ 6),\ (1.5,\ 2.5,\ 4.5),\ (3,\ 5,\ 7)\rangle\,\tau_{34} + \langle(1,\ 5,\ 8),\ (1.5,\ 4.5,\ 7.5),\ (4,\ 6.5,\ 9)\rangle\,\tau_{36} \\
& + \langle(1,\ 5,\ 8),\ (1.5,\ 3,\ 6.5),\ (4,\ 7,\ 9)\rangle\,\tau_{37} + \langle(10,\ 15,\ 20),\ (14,\ 16,\ 22),\ (12,\ 15,\ 19)\rangle\,\tau_{45} \\
& + \langle(20,\ 25,\ 30),\ (24,\ 26,\ 32),\ (22,\ 25,\ 29)\rangle\,\tau_{46} + \langle(15,\ 20,\ 25),\ (19,\ 21,\ 27),\ (17,\ 20,\ 24)\rangle\,\tau_{47} \\
& + \langle(13,\ 18,\ 23),\ (17,\ 19,\ 25),\ (15,\ 18,\ 22)\rangle\,\tau_{56} + \langle(15,\ 20,\ 25),\ (19,\ 21,\ 27),\ (17,\ 20,\ 24)\rangle\,\tau_{57}. \quad (23)
\end{aligned}
$$

Subject to constraints:

$$
\left.
\begin{array}{l}
\tau_{15}+\tau_{14}+\tau_{13}=30;\ 0\le\tau_{14}\le10;\ \tau_{25}+\tau_{24}+\tau_{23}=60;\ 0\le\tau_{15}\le15 \\
0\le\tau_{13}\le40;\ 0\le\tau_{34}\le20;\ \tau_{37}+\tau_{36}+\tau_{34}-\tau_{23}-\tau_{13}=0;\ 0\le\tau_{36}\le100; \\
0\le\tau_{23}\le50,\ -\tau_{36}-\tau_{46}-\tau_{56}=-20;\ 0\le\tau_{25}\le40;\ 0\le\tau_{24}\le80; \\
0\le\tau_{37}\le90;\ \tau_{45}+\tau_{46}+\tau_{47}-\tau_{14}-\tau_{24}-\tau_{34}=0;\ 0\le\tau_{56}\le15 \\
0\le\tau_{45}\le15,\ -\tau_{37}-\tau_{47}-\tau_{57}=-70;\ 0\le\tau_{46}\le30,\ -\tau_{37}-\tau_{47}-\tau_{57}=-70; \\
\tau_{57}+\tau_{56}-\tau_{45}-\tau_{25}-\tau_{15}=0;\ 0\le\tau_{47}\le20;\ 0\le\tau_{57}\le100
\end{array}
\right\} \quad (24)
$$

Step 2:

$$Min\left(\widetilde{O_1^{\eta}}\right) = 1\tau_{13} + 0.5\tau_{14} + 1\tau_{15} + 1\tau_{23} + 1\tau_{24} + 1.5\tau_{25} + 2\tau_{34} + 1\tau_{36} + 1\tau_{37} + 10\tau_{45}$$

$$+ 20\tau_{46} + 15\tau_{47} + 13\tau_{56} + 15\tau_{57}.$$

$$Min\left(\widetilde{O_2^{\eta}}\right) = 4\tau_{13} + 2.5\tau_{14} + 3\tau_{15} + 2\tau_{23} + 1.5\tau_{24} + 2.5\tau_{25} + 4\tau_{34} + 5\tau_{36} + 5\tau_{37} + 15\tau_{45}$$

$$+ 25\tau_{46} + 20\tau_{47} + 18\tau_{56} + 20\tau_{57}.$$

$$Min\left(\widetilde{O_3^{\eta}}\right) = 7\tau_{13} + 4.5\tau_{14} + 5\tau_{15} + 3\tau_{23} + 4\tau_{24} + 3.5\tau_{25} + 6\tau_{34} + 8\tau_{36} + 8\tau_{37} + 20\tau_{45}$$

$$+ 30\tau_{46} + 25\tau_{47} + 23\tau_{56} + 25\tau_{57}.$$

$$Min\left(\widetilde{O_4^{\eta}}\right) = 1\tau_{13} + 1\tau_{14} + 0.5\tau_{15} + 0.5\tau_{23} + 0.5\tau_{24} + 1\tau_{25} + 1.5\tau_{34} + 1.5\tau_{36} + 1.5\tau_{37}$$

$$+ 14\tau_{45} + 24\tau_{46} + 19\tau_{47} + 17\tau_{56} + 19\tau_{57}.$$

$$Min\left(\widetilde{O_5^{\eta}}\right) = 3\tau_{13} + 2\tau_{14} + 1.5\tau_{15} + 1.5\tau_{23} + 1\tau_{24} + 1.5\tau_{25} + 2.5\tau_{34} + 4.5\tau_{36} + 3\tau_{37}$$

$$+ 16\tau_{45} + 26\tau_{46} + 21\tau_{47} + 19\tau_{56} + 21\tau_{57}.$$

$$Min\left(\widetilde{O_6^{\eta}}\right) = 5\tau_{13} + 3\tau_{14} + 3.5\tau_{15} + 2.5\tau_{23} + 2.5\tau_{24} + 3\tau_{25} + 4.5\tau_{34} + 7.5\tau_{36} + 6.5\tau_{37}$$

$$+ 22\tau_{45} + 32\tau_{46} + 27\tau_{47} + 25\tau_{56} + 27\tau_{57}.$$

$$Min\left(\widetilde{O_7^{\eta}}\right) = 3.5\tau_{13} + 1.5\tau_{14} + 3\tau_{15} + 1.5\tau_{23} + 2.25\tau_{24} + 2\tau_{25} + 3\tau_{34} + 4\tau_{36} + 4\tau_{37}$$

$$+ 12\tau_{45} + 22\tau_{46} + 17\tau_{47} + 15\tau_{56} + 17\tau_{57}.$$

$$Min\left(\widetilde{O_8^{\eta}}\right) = 6\tau_{13} + 3.5\tau_{14} + 4\tau_{15} + 2.5\tau_{23} + 3\tau_{24} + 3\tau_{25} + 5\tau_{34} + 6.5\tau_{36} + 7\tau_{37} + 15\tau_{45}$$

$$+ 25\tau_{46} + 20\tau_{47} + 18\tau_{56} + 20\tau_{57}.$$

$$Min\left(\widetilde{O_9^{\eta}}\right) = 7.5\tau_{13} + 5.5\tau_{14} + 6\tau_{15} + 3.5\tau_{23} + 4.25\tau_{24} + 4\tau_{25} + 7\tau_{34} + 9\tau_{36} + 9\tau_{37}$$

$$+ 19\tau_{45} + 29\tau_{46} + 24\tau_{47} + 22\tau_{56} + 24\tau_{57}.$$

Subject to constraints:
Along with the same constraints equation (24).
Step 3: First of all, solving the below problem by usual method

$$Min\left(\widetilde{O_1^{\eta}}\right) = 1\tau_{13} + 0.5\tau_{14} + 1\tau_{15} + 1\tau_{23} + 1\tau_{24} + 1.5\tau_{25} + 2\tau_{34} + 1\tau_{36} + 1\tau_{37} + 10\tau_{45}$$

$$+ 20\tau_{46} + 15\tau_{47} + 13\tau_{56} + 15\tau_{57}. \tag{25}$$

With the same subject to constraints equation (24). The optimal value of the above equation (25) is $\widehat{(Rs_1)^*} = 305$.
Step 4: Again, calculate the subsequent problem given as below

$$Min\left(\widetilde{O_2^{\eta}}\right) = 4\tau_{13} + 2.5\tau_{14} + 3\tau_{15} + 2\tau_{23} + 1.5\tau_{24} + 2.5\tau_{25} + 4\tau_{34} + 5\tau_{36} + 5\tau_{37} + 15\tau_{45}$$

$$+ 25\tau_{46} + 20\tau_{47} + 18\tau_{56} + 20\tau_{57}. \tag{26}$$

Subject to

$$\left.\begin{array}{l} \widehat{(Rs_1)^*} = 1\tau_{13} + 0.5\tau_{14} + 1\tau_{15} + 1\tau_{23} + 1\tau_{24} + 1.5\tau_{25} + 2\tau_{34} + 1\tau_{36} + 1\tau_{37} + 10\tau_{45} \\ \qquad\qquad +20\tau_{46} + 15\tau_{47} + 13\tau_{56} + 15\tau_{57} = 305 \\ \text{Also with the constraints equation (24)} \end{array}\right\} \tag{27}$$

The optimal value of the above equation (26) is $\widehat{(Rs_2)^*} = 825$.
Step 5: Again, calculate the subsequent problem given as below

$$Min\left(\widetilde{O_3^{\eta}}\right) = 7\tau_{13} + 4.5\tau_{14} + 5\tau_{15} + 3\tau_{23} + 4\tau_{24} + 3.5\tau_{25} + 6\tau_{34} + 8\tau_{36} + 8\tau_{37} + 20\tau_{45}$$

$$+ 30\tau_{46} + 25\tau_{47} + 23\tau_{56} + 25\tau_{57}. \tag{28}$$

Subject to

$$\left.\begin{array}{l}\widehat{(Rs_2)}^* = 4\tau_{13}+2.5\tau_{14}+3\tau_{15}+2\tau_{23}+1.5\tau_{24}+2.5\tau_{25}+4\tau_{34}+5\tau_{36}+5\tau_{37}+15\tau_{45} \\ \qquad +25\tau_{46}+20\tau_{47}+18\tau_{56}+20\tau_{57}. = 825 \\ \text{Also with the constraints equation (27)}\end{array}\right\} \tag{29}$$

The optimal value of the above equation (28) is $\widehat{(Rs_3)}^* = 1,265$.

Step 6: Again, calculate the subsequent problem given as below

$$Min\left(\widetilde{O_4^{\eta}}\right) = 1\tau_{13}+1\tau_{14}+0.5\tau_{15}+0.5\tau_{23}+0.5\tau_{24}+1\tau_{25}+1.5\tau_{34}+1.5\tau_{36}+1.5\tau_{37}+14\tau_{45}$$

$$+24\tau_{46}+19\tau_{47}+17\tau_{56}+19\tau_{57}. \tag{30}$$

Subject to

$$\left.\begin{array}{l}\widehat{(Rs_3)}^* = 7\tau_{13}+4.5\tau_{14}+5\tau_{15}+3\tau_{23}+4\tau_{24}+3.5\tau_{25}+6\tau_{34}+8\tau_{36}+8\tau_{37}+20\tau_{45} \\ \qquad +30\tau_{46}+25\tau_{47}+23\tau_{56}+25\tau_{57} = 1,265 \\ \text{Also with the constraints equation (29)}\end{array}\right\} \tag{31}$$

The optimal value of the above equation (30) is $\widehat{(Rs_4)}^* = 355$.

Similarly proceed from step 6 to step 10, we get the solution $\widehat{(Rs_9)}^* = 1,380$.

Now, The optimal value of the above is $\widehat{(Rs_9)}^* = 1,380$ and $\tau_{13} = 30$, $\tau_{14} = 0$, $\tau_{15} = 0$, $\tau_{23} = 50$, $\tau_{24} = 0$, $\tau_{25} = 10$, $\tau_{34} = 0$, $\tau_{36} = 10$, $\tau_{37} = 70$, $\tau_{45} = 0$, $\tau_{46} = 0$, $\tau_{47} = 0$, $\tau_{56} = 10$, $\tau_{57} = 0$.

Hence, the solution $\langle(305,\ 825,\ 1,265),\ (355,\ 625,\ 1,085),\ (670,\ 1,070,\ 1,380)\rangle$ and $\tau_{13} = 30$, $\tau_{14} = 0$, $\tau_{15} = 0$, $\tau_{23} = 50$, $\tau_{24} = 0$, $\tau_{25} = 10$, $\tau_{34} = 0$, $\tau_{36} = 10$, $\tau_{37} = 70$, $\tau_{45} = 0$, $\tau_{46} = 0$, $\tau_{47} = 0$, $\tau_{56} = 10$, $\tau_{57} = 0$.

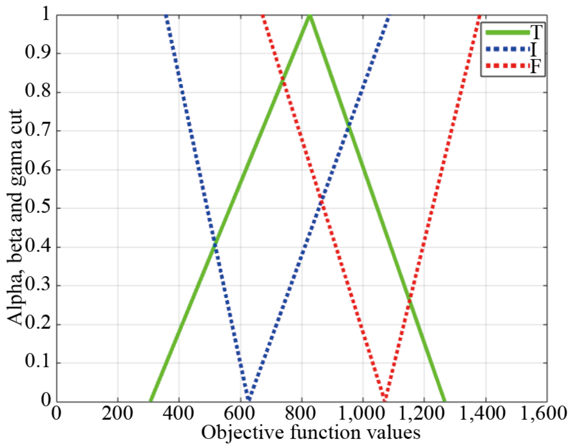

Figure 3. The graphical representation of $\langle(305,\ 825,\ 1,265),\ (355,\ 625,\ 1,085),\ (670,\ 1,070,\ 1,380)\rangle$

Advantage and limitations of proposed model:

In this article, we introduce a novel approach for solving the TrNMCF using lexicographic approach, and the proposed model offers several significant advantages of the proposed model are given as below.

• According to our knowledge, this could be the first approach for solving the TrNMCF problem using Lexicographic approach.

• Lexicographic approach can be estimated with significant level of accuracy and simple algorithm can be developed as well.

• This proposed model represents easy and reality efficiently than existing model, because we consider all aspects (i.e., the falsity, indeterminacy, and truthness degree) of the decision-making process.

Limitation of proposed model the number of constraints is larger than the original problem because we use the lexicographic approach to convert the multi-objective LP problem.

5. Conclusion

The Minimum Cost Flow (MCF) problem seeks the optimal flow through a network at minimal cost. It has various factors like cost, supply, demand, and capacity. The goal is to determine the least expensive way to transport a specified amount of flow through an MCF problem. Our novel approach employs Neutrosophic cost, treating cost parameters as triangular neutrosophic numbers while maintaining crisp decision variables and capacities. Focusing on lexicographic methods, this active research field has practical implications. We illustrate our method by solving an MCF problem with neutrosophic cost using LINGO 18.0 software.

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Confilict of interest

The authors declare no competing financial interest.

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