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Research Article

On Single-Valued Neutrosophic Soft Filter Convergence

Fahad Alsharari

Department of Mathematics, College of Science, Jouf University, Sakaka 72311, Saudi Arabia
E-mail: f.alsharari@ju.edu.sa

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Abstract: The concept of single-valued neutrosophic soft filters plays a crucial role in the study of topological spaces. Extensive research has led to several generalizations of these filters, highlighting their significance in neutrosophic theory. This paper introduces a novel approach to single-valued neutrosophic soft filters, along with the idea of single-valued neutrosophic soft quasi-coincident neighborhood spaces, which are characterized by a unique interaction between the filters and quasi-coincident neighborhood structures. Additionally, we explore advanced neutrosophic theories, focusing on the properties and convergence of single-valued neutrosophic soft filters in soft topological spaces. Finally, we demonstrate the existence of product fuzzy soft filters.

***Keywords*:** single-valued neutrosophic soft set, single-valued neutrosophic soft filters, single-valued neutrosophic soft neighborhood, single-valued neutrosophic soft convergence

MSC: 35A01, 65L10, 65L12, 65L20, 65L70

Abbreviation

n-set	neutrosophic set
svn-set	single-valued neutrosophic set
svns-set	single-valued neutrosophic soft set
svnst	single-valued neutrosophic soft topology
svnsts	single-valued neutrosophic soft topological spaces
svns-filter	single-valued neutrosophic soft filter
$\widetilde{(\pounds, E)}$	the collection of all single-valued neutrosophic soft set
svnsqnc-system	single-valued neutrosophic soft quasi-coincident neighborhood system
svnsqnc-spaces	single-valued neutrosophic soft quasi-coincident neighborhood spaces

1. Introduction

The concept of a filter on a set is a fundamental notion in topology and plays a significant role in the study of topological structures. The foundational theory of filters is discussed comprehensively in [1], while several applications of filter convergence in topological spaces are presented in [2]. The notion of a fuzzy filter was introduced by Höhle

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and Šostak in [3], although similar concepts, with slight variations, had appeared earlier in works such as [4–7]. Recent research has expanded to generalized filters [8–10], exploring their applications in broader contexts. Höhle and Šostak also examined the convergence of fuzzy topological spaces via neighborhood systems of a point in [3]. This paper aims to introduce and explore the concept of fuzzy soft filters, demonstrate some of their properties, and analyze their convergence in fuzzy soft topological spaces. Additionally, the existence of product fuzzy soft filters is established.

Maji et al. [11, 12] extensively explored decision-making problems alongside introducing several new definitions of soft sets. Dey et al. [13, 14] investigated generalized neutrosophic soft multi-attribute group decision-making using the TOPSIS method, as well as neutrosophic soft multi-attribute decision-making based on grey relational projection. The concept of soft sets and soft groups was originally introduced by Aktas and Cağman [15]. Subsequent developments in fuzzy soft sets were carried out by Feng et al. [16], Chen et al. [17], Ali et al. [18], Sun et al. [19], Yang et al. [20], Kharal and Ahmed [21], and Ahmed and Khara [22]. Shabir and Naz [23] provided definitions for soft sets that incorporated separation axioms. The first application of fuzzy soft topology, based on Chang's fuzzy topology [24], was introduced by Tanay and Kandemir [25], who developed key concepts in this area. Pazar Varol and Aygün [26] later defined fuzzy soft topology in the context of Lowen's framework, while Aygünoğlu et al. [27] extended these ideas by defining fuzzy soft topology based on Šostak's work. Additionally, Saber et al. [28] studied single-valued neutrosophic soft topological spaces ($¥$, $\mathscr{T}^{\upsilon}$, $\mathscr{T}^{\mu}$, $\mathscr{T}^{\omega}$) (referred to as svnst-spaces), contributing to the ongoing development of neutrosophic soft topologies.

Smarandache introduced the concept of neutrosophic sets [29], which paved the way for subsequent research on single-valued neutrosophic sets (*svns*) and neutrosophic sets (*ns*) by Wang et al. [30] and Salama et al. [31, 32]. Numerous applications of neutrosophic sets have been explored by various researchers [33–37].

Saber et al. conducted extensive studies in this area, including investigations on Single-Valued Neutrosophic Primal Theory, Single-Valued Neutrosophic Ideals in Šostak's Sense, Single-Valued Neutrosophic Soft Uniform Spaces, as well as the connectedness and stratification of single-valued neutrosophic topological spaces [38–41] The theory of neutrosophic sets is a well-established generalization of fuzzy sets, intuitionistic fuzzy sets, and rough sets, providing a valuable mathematical framework for dealing with uncertainty. This paper introduces the concept of single-valued neutrosophic soft filters, extending previous work by Ridvan et al. [42] and Abbas et al. [43].

Building on this foundation, we explore soft filters by introducing the notions of single-valued neutrosophic soft filters and investigating their convergence properties. Additionally, we study single-valued neutrosophic soft quasi-coincident neighbourhood spaces, highlighting key properties and examining the convergence of neutrosophic soft filters in neutrosophic soft topological spaces.

A neutrosophic set is a powerful and generalized formal framework that extends the classical set, fuzzy set, interval-valued fuzzy set, intuitionistic fuzzy set, and interval intuitionistic fuzzy set, particularly from a philosophical perspective. This framework has diverse applications. For instance, in Geographical Information Systems (GIS), it helps model spatial regions with indeterminate boundaries under conditions of uncertainty (see [44]). Additionally, neutrosophic sets are useful in control engineering, such as in achieving average consensus in multi-agent systems, particularly in scenarios with uncertain topologies, multiple time-varying delays, and random noisy environments (see [45]).

In the analysis, $£$ denotes an initial universe, E is the set of all parameters for X is the set of all *svn-soft set* on $£$ (where $I = [0,\ 1]$) and $I_0 = (0,\ 1]$. $\widetilde{(£,\ E)}$ designates the cluster of all *svn-soft set* on $£$.

A *svns-soft set* $f_A \in \widetilde{(£,\ E)}$ is called a single-valued neutrosophic soft point (*svn-soft point*) if $A = \{e\} \subseteq E$ and $f_A(e)$ is a *svn-soft point* in $£$ i.e., there exists $x \in £$ such that $\pi_{f_A(e)}(x) = t$, $\alpha_{f_A(e)}(x) = s$, $\sigma_{f_A(e)}(x) = k$, $t,\ s,\ k \in \zeta_0$ with $t+s+k \leq 3$ and $\pi_{f_A(e)}(y) = 0$, $\alpha_{f_A(e)}(y) = 1$, $\sigma_{f_A(e)}(y) = 1$ for any $x \neq y$ it is denoted by $e_x^{t,\ s,\ k}$, and the set of all *svn-soft points* in $£$ is denoted by $P_{t,\ s,\ k}\widetilde{(£,\ E)}$. Let $f_A,\ \rho_B \in \widetilde{(£,\ E)}$. Then, f_A is called single-valued neutrosophic soft quasi-coincident (*svnsq-coincident*) with ρ_B, denoted by $f_A\ q\ \rho_B$ if there exist $e \in E$ and $x \in £$ s.t. $\pi_{f_A(e)}(x) + \pi_{\rho_B(e)}(x) > 1$, $\alpha_{f_A(e)}(x) + \alpha_{\rho_B(e)}(x) \leq 1$, $\sigma_{f_A(e)}(x) + \sigma_{\rho_B(e)}(x) \leq 1$. If f_A is not *svnsq-coincident* with ρ_B, that is denoted by $f_A\ \tilde{q}\ \rho_B$.

2. Preliminaries

This section provides an in-depth exploration of the fundamental concepts and methods used in neutrosophic and single-valued neutrosophic (SVN) set theories, laying the foundation for the later development of single-valued neutrosophic soft quasi-coincident neighborhood spaces. As usual, $I^{£}$ denotes the family of all single-valued neutrosophic sets (abbreviated as *SVNS*) on $£$.

Definition 1 [29]. Let $£ \neq \phi$. A neutrosophic set (for short, *ns*) R on $£$ demarcated as:

$$R = \{\langle x,\ \pi_R(x),\ \alpha_R(x),\ \sigma_R(x)\rangle \mid x \in £,\ \pi_R(x),\ \alpha_R(x),\ \sigma_R(x) \in]^-0,\ 1^+\lfloor\},$$

representing the degree of membership where $(\sigma_R(x))$, the degree of falsity membership; $(\alpha_R(x))$ degree of indeterminacy and $\pi_R(x)$ degree of nonmembership; $\forall\ z \in £$ to the set $£$.

Definition 2 Let $£ \neq \phi$ and $R_1,\ R_2 \in I^{£}$ be in the form $R_1 = \{\langle x,\ \pi_{R_1}(x),\ \alpha_{R_1}(x),\ \sigma_{R_1}(x)\rangle \mid x \in £\}$ and $R_2 = \{\langle x,\ \pi_{R_2}(x),\ \alpha_{R_2}(x),\ \sigma_{R_2}(x)\rangle \mid x \in £\}$ on $£$, then

(1) $R_1 \cap R_2$ is an *svn-set* [46], if $\forall\ x \in £$,

$$(R_1 \cap R_2)(x) = min\{\pi_{R_1}(x),\ \pi_{R_2}(x)\},\ \alpha_{R_3}(x) = max\{\alpha_{R_1}(x),\ \alpha_{R_2}(x)\},$$

$$\sigma_{R_3}(x) = max\{\sigma_{R_1}(x),\ \sigma_{R_2}(x)\}.$$

(2) $R_1 \cap R_2$ is an *svn-set* [46], if $\forall\ x \in £$

$$(R_1 \cup R_2)(x) = max\{\pi_{R_1}(x),\ \pi_{R_2}(x)\},\ \alpha_{R_3}(x) = max\{\alpha_{R_1}(x),\ \alpha_{R_2}(x)\},$$

$$\sigma_{R_3}(x) = min\{\sigma_{R_1}(x),\ \sigma_{R_2}(x)\}.$$

(3) $R_1 \subseteq R_2$ [47] for all $x \in £$ defined as:

$$\pi_{R_1}(x) \leq \pi_{R_2}(x),\ \alpha_{R_1}(x) \geq \alpha_{R_2}(x),\ \sigma_{R_1}(x) \geq \sigma_{R_2}(x).$$

(4) The complement of the set R [30] (R^c) defined as next

$$\pi_{R^c}(x) = \sigma_R(x),\ \alpha_{R^c}(x) = 1 - \alpha_R(x),\ \sigma_{R^c}(x) = \pi_R(x).$$

Definition 3 [28] f_A is a *svn-soft set* on $£$ where, $f : E \to I^{£}$; i.e., $f_e = f(e)$ is a *svn-set* on $£$, for every $e \in E$ and $f(e) = \langle 0,\ 1,\ 1\rangle$, if $e \notin E$.

The *svn-set* $f(e)$ is termed as an element of the *svn-soft set* f_A. Thus, a *svn-soft set* f_E on $£$ can be represented by the set of ordered pairs:

$$(f,\ E) = \left\{(e,\ f(e)) \mid e \in E,\ f(e) \in I^{£}\right\} = \left\{(e,\ \langle \pi_f(e),\ \alpha_f(e),\ \sigma_f(e)\rangle) \mid e \in E,\ f(e) \in I^{£}\right\},$$

where $\sigma_f : E \to I$ (σ_f is called as a nonmembership function), $\pi_f : E \to I$ (α_f is called as a membership function), and $\alpha_f : E \to I$ (α_f is called as indeterminacy function) of *svn-soft set*.

A *svn-soft set* f_E on £ is named as a null *svn-soft sets* ($\widetilde{\Phi}$), if $\pi_{f_A}(e) = 0$, $\sigma_{f_A}(e) = 1$ and $\alpha_{f_A}(e) = 1$, $\forall\ e \in E$.

A *svn-soft set* f_E on £ is called absolute *svn-soft set* ($\widetilde{E}$), if $\pi_{f_A}(e) = 1$, $\alpha_{f_A}(e) = 0$ and $\sigma_{f_A}(e) = 0$, for any $e \in E$.

Definition 4 [28] A mapping $\mathscr{T}^{\pi},\ \mathscr{T}^{\alpha},\ \mathscr{T}^{\sigma} : E \to I^{\widetilde{(£,\ E)}}$ is termed to be single-valued neutrosophic soft topology (*svnst*) on £, if it meets the following criteria, for every $e \in E$:

($\mathscr{T}_1$) $\mathscr{T}_e^{\pi}(\widetilde{\Phi}) = \mathscr{T}_e^{\pi}(\widetilde{E}) = 1$ and $\mathscr{T}_e^{\alpha}(\widetilde{\Phi}) = \mathscr{T}_e^{\alpha}(\widetilde{E}) = \mathscr{T}_e^{\sigma}(\widetilde{\Phi}) = \mathscr{T}_e^{\sigma}(\widetilde{E}) = 0$,

($\mathscr{T}_2$) $\mathscr{T}_e^{\pi}(f_A \sqcap \rho_B) \geq \mathscr{T}_e^{\pi}(f_A) \wedge \mathscr{T}_e^{\pi}(\rho_B)$, $\mathscr{T}_e^{\alpha}(f_A \sqcap \rho_B) \leq \mathscr{T}_e^{\alpha}(f_A) \vee \mathscr{T}_e^{\alpha}(\rho_B)$,
$\mathscr{T}_e^{\sigma}(f_A \sqcap \rho_B) \leq \mathscr{T}_e^{\sigma}(f_A) \vee \mathscr{T}_e^{\sigma}(\rho_B)$, $\forall\ f_A,\ \rho_B \in \widetilde{(£,\ E)}$,

($\mathscr{T}_3$) $\mathscr{T}_e^{\pi}(\bigsqcup_{i\in\Gamma}[f_A]_i) \geq \bigwedge_{i\in\Gamma} \mathscr{T}_e^{\pi}([f_A]_i)$, $\mathscr{T}_e^{\alpha}(\bigsqcup_{i\in\Gamma}[f_A]_i) \leq \bigvee_{i\in\Gamma} \tau_e^{\alpha}([f_A]_i)$,
$\mathscr{T}_e^{\sigma}(\bigsqcup_{i\in\Gamma}[f_A]_i) \leq \bigvee_{i\in\Gamma} \mathscr{T}_e^{\sigma}([f_A]_i)$, $\forall\ f_A,\ \rho_B \in \widetilde{(£,\ E)}$.

The quadruple $(£,\ \mathscr{T}^{\pi},\ \mathscr{T}^{\alpha},\ \mathscr{T}^{\sigma})$ is said to be a *svnst-spaces*. Representing the degree of openness ($\mathscr{T}_e^{\pi}(f_A)$), the degree of indeterminacy ($\mathscr{T}_e^{\alpha}(f_A)$), and the degree of non-openness ($\mathscr{T}_e^{\sigma}(f_A)$); of a *svns-set* with respect to that parameter $e \in E$. Occasionally, we will write $\mathscr{T}^{\pi\alpha\sigma}$ for $(\mathscr{T}^{\pi},\ \mathscr{T}^{\alpha},\ \mathscr{T}^{\sigma})$, and it will be no ambiguity.

3. Single-valued neutrosophic soft filters

Within the field of mathematical harmony, Single-Valued Neutrosophic Soft Filters are the result of the combination of neutrosophic logic with soft set theory. These filters provide an integrative structure to deal with uncertainty by serving as crucial bridges between the fields of soft computing and neutrosophic research. Precisely defined, these svn-soft filters reveal their core, from compatibility conditions controlling intersection operations to basic characteristics capturing limit conditions. A central theorem emphasizes the group interaction by giving an orderly approach to combine separate filters into an integrated unit. Through our study of this mathematical setting, the svn-soft filters not only reveal their complexities but also open up fresh prospects for the construction of single-valued neutrosophic soft topologies, showing their important impact on precisely and effectively understanding uncertainty.

We begin it with the following:

Definition 5 A mapping $\mathscr{F}^{\pi},\ \mathscr{F}^{\alpha},\ \mathscr{F}^{\sigma} : E \to I^{\widetilde{(£,\ E)}}$ is termed to be *svn-soft filter* on £, if it meets the following criteria, for every $e \in E$:

(F1) $\mathscr{F}_e^{\pi}(\widetilde{\Phi}) = 0$, $\mathscr{F}_e^{\alpha}(\widetilde{\Phi}) = 1$, $\mathscr{F}_e^{\sigma}(\widetilde{\Phi}) = 1$ and $\mathscr{F}_e^{\pi}(\widetilde{E}) = 1$, $\mathscr{F}_e^{\alpha}(\widetilde{E}) = 0$, $\mathscr{F}_e^{\sigma}(\widetilde{E}) = 0$,

(F2) $\mathscr{F}_e^{\pi}(f_A \sqcap \rho_B) \geq \mathscr{F}_e^{\pi}(f_A) \wedge \mathscr{F}_e^{\pi}(\rho_B)$, $\mathscr{F}_e^{\alpha}(f_A \sqcap \rho_B) \leq \mathscr{F}_e^{\alpha}(f_A) \vee \mathscr{F}_e^{\alpha}(\rho_B)$,
$\mathscr{F}_e^{\sigma}(f_A \sqcap \rho_B) \leq \mathscr{F}_e^{\sigma}(f_A) \vee \mathscr{F}_e^{\sigma}(\rho_B)$, $\forall\ f_A,\ \rho_B \in \widetilde{(£,\ E)}$,

(F3) If $f_A \sqsubseteq \rho_B$, then $\mathscr{F}_e^{\pi}(f_A) \leq \mathscr{F}_e^{\pi}(\rho_B)$, $\mathscr{F}_e^{\alpha}(f_A) \geq \mathscr{F}_e^{\alpha}(\rho_B)$, $\mathscr{F}_e^{\sigma}(f_A) \geq \mathscr{F}_e^{\sigma}(\rho_B)$.

If $\mathscr{F}_E^{\pi\alpha\sigma}$ and $\mathscr{F}_E^{\star\pi\alpha\sigma}$ are *svn-soft filters* on £, then "$\mathscr{F}_E^{\pi\alpha\sigma}$ is finer than $\mathscr{F}_E^{\star\pi\alpha\sigma}$ or ($\mathscr{F}_E^{\star\pi\alpha\sigma}$ is coarser than $\mathscr{F}_E^{\pi\alpha\sigma}$)" denoted by $\mathscr{F}_E^{\pi\alpha\sigma} \sqsubseteq \mathscr{F}_E^{\star\pi\alpha\sigma}$ if and only if

$$\mathscr{F}_e^{\pi}(f_A) \leq \mathscr{F}_e^{\star\pi}(f_A),\ \mathscr{F}_e^{\alpha}(f_A) \geq \mathscr{F}_e^{\star\alpha}(f_A),\ \mathscr{F}_e^{\sigma}(f_A) \geq \mathscr{F}_e^{\star\sigma}(f_A),$$

for any $e \in E$, $f_A \in \widetilde{(£,\ E)}$. Occasionally, we will write $\mathscr{F}^{\pi\alpha\sigma}$ for $(\mathscr{F}^{\pi},\ \mathscr{F}^{\alpha},\ \mathscr{F}^{\sigma})$, and it will be no ambiguity.

The central belongings of *svn-soft filters* are deliberated in the next suggestions:

Theorem 1 Suppose that $\{(\mathscr{F}_j^{\pi\alpha\sigma})_{E_j},\ j \in \Gamma\}$ is an family of svn-soft filter on a set £, then, the mapping $\mathscr{F}^{\pi\alpha\sigma} = \sqcap_{j\in\Gamma}(\mathscr{F}_j^{\pi\alpha\sigma})_{E_j} : E \to I^{\widetilde{(£,\ E)}}$ defined, for every $e \in E$, $f_A \in \widetilde{(£,\ E)}$ by:

$$\mathscr{F}_e^{\pi}(f_A) = \bigwedge_{j\in\Gamma}(\mathscr{F}_j^{\pi})_e(f_A),\ \mathscr{F}_e^{\alpha}(f_A) = \bigvee_{j\in\Gamma}(\mathscr{F}_j^{\alpha})_e(f_A),\ \mathscr{F}_e^{\sigma}(f_A) = \bigvee_{j\in\Gamma}(\mathscr{F}_j^{\sigma})_e(f_A),$$

is a svn-soft filter on £.

Proof. To prove this theorem, the following conditions must be proved:

(F1)

$$\mathscr{F}_e^{\pi}(\widetilde{\Phi}) = \bigwedge_{j\in\Gamma}(\mathscr{F}_j^{\pi})_e(\widetilde{\Phi}) = 0,\ \mathscr{F}_e^{\alpha}(\widetilde{\Phi}) = \bigvee_{j\in\Gamma}(\mathscr{F}_j^{\alpha})_e(\widetilde{\Phi}) = 1,\ \mathscr{F}_e^{\sigma}(\widetilde{\Phi}) = \bigvee_{j\in\Gamma}(\mathscr{F}_j^{\sigma})_e(\widetilde{\Phi}) = 1,$$

and

$$\mathscr{F}_e^{\pi}(\widetilde{E}) = \bigwedge_{j\in\Gamma}(\mathscr{F}_j^{\pi})_e(\widetilde{E}) = 1,\ \mathscr{F}_e^{\alpha}(\widetilde{E}) = \bigvee_{j\in\Gamma}(\mathscr{F}_j^{\alpha})_e(\widetilde{E}) = 0,\ \mathscr{F}_e^{\sigma}(\widetilde{E}) = \bigvee_{j\in\Gamma}(\mathscr{F}_j^{\sigma})_e(\widetilde{E}) = 0.$$

(F2) for all $f_A,\ \rho_B \in \widetilde{(£,\ E)}$, we have

$$\begin{aligned}
\mathscr{F}_e^{\pi}(f_A) \wedge \mathscr{F}_e^{\pi}(\rho_B) &= \bigwedge_{j\in\Gamma}(\mathscr{F}_j^{\pi})_e(f_A) \wedge \bigwedge_{j\in\Gamma}(\mathscr{F}_j^{\pi})_e(\rho_B) \\
&\leq \bigwedge_{j\in\Gamma}((\mathscr{F}_j^{\pi})_e(f_A) \wedge (\mathscr{F}_j^{\pi})_e(\rho_B)) \\
&\leq \bigwedge_{j\in\Gamma}(\mathscr{F}_j^{\pi})_e(f_A \sqcap \rho_B) \\
&= \mathscr{F}_e^{\pi}(f_A \sqcap \rho_B) \\
\mathscr{F}_e^{\alpha}(f_A) \vee \mathscr{F}_e^{\alpha}(\rho_B) &= \bigvee_{j\in\Gamma}(\mathscr{F}_j^{\alpha})_e(f_A) \vee \bigvee_{j\in\Gamma}(\mathscr{F}_j^{\alpha})_e(\rho_B) \\
&\geq \bigvee_{j\in\Gamma}((\mathscr{F}_j^{\alpha})_e(f_A) \vee (\mathscr{F}_j^{\alpha})_e(\rho_B)) \\
&\geq \bigvee_{j\in\Gamma}((\mathscr{F}_j^{\alpha})_e(f_A) \wedge (\mathscr{F}_j^{\alpha})_e(\rho_B)) \\
&\geq \bigwedge_{j\in\Gamma}(\mathscr{F}_j^{\alpha})_e(f_A \sqcap \rho_B) \\
&= \mathscr{F}_e^{\alpha}(f_A \sqcap \rho_B)
\end{aligned}$$

$$\mathscr{F}_e^{\sigma}(f_A)\vee\mathscr{F}_e^{\sigma}(\rho_B)=\bigvee_{j\in\Gamma}(\mathscr{F}_j^{\sigma})_e(f_A)\vee\bigvee_{j\in\Gamma}(\mathscr{F}_j^{\sigma})_e(\rho_B)$$

$$\geq\bigvee_{j\in\Gamma}((\mathscr{F}_j^{\sigma})_e(f_A)\vee(\mathscr{F}_j^{\sigma})_e(\rho_B))$$

$$\geq\bigvee_{j\in\Gamma}((\mathscr{F}_j^{\sigma})_e(f_A)\wedge(\mathscr{F}_j^{\sigma})_e(\rho_B))$$

$$\geq\bigwedge_{j\in\Gamma}(\mathscr{F}_j^{\sigma})_e(f_A\sqcap\rho_B)$$

$$=\mathscr{F}_e^{\sigma}(f_A\sqcap\rho_B).$$

(F3) If $f_A\sqsubseteq\rho_B$, then

$$(\mathscr{F}_j^{\pi})_e(f_A)\leq(\mathscr{F}_j^{\pi})_e(\rho_B),\ (\mathscr{F}_j^{\alpha})_e(f_A)\geq(\mathscr{F}_j^{\alpha})_e(\rho_B),\ (\mathscr{F}_j^{\sigma})_e(f_A)\geq(\mathscr{F}_j^{\sigma})_e(\rho_B),$$

for every $e\in E$, $j\in\Gamma$, and hence

$$\mathscr{F}_e^{\pi}(f_A)=\bigwedge_{j\in\Gamma}(\mathscr{F}_j^{\pi})_e(f_A)\leq\bigwedge_{j\in\Gamma}(\mathscr{F}_j^{\pi})_e(\rho_B)=\mathscr{F}_e^{\pi}(\rho_B)$$

$$\mathscr{F}_e^{\alpha}(f_A)=\bigvee_{j\in\Gamma}(\mathscr{F}_j^{\alpha})_e(f_A)\geq\bigvee_{j\in\Gamma}(\mathscr{F}_j^{\alpha})_e(\rho_B)=\mathscr{F}_e^{\alpha}(\rho_B)$$

$$\mathscr{F}_e^{\sigma}(f_A)=\bigvee_{j\in\Gamma}(\mathscr{F}_j^{\sigma})_e(f_A)\geq\bigvee_{j\in\Gamma}(\mathscr{F}_j^{\sigma})_e(\rho_B)=\mathscr{F}_e^{\sigma}(\rho_B)$$

By proving the three conditions, we have proven the above theorem.

From a *svn-soft filter* $f_A::E\to I^{\widetilde{(£,\,E)}}$, we can obtain a single-valued neutrosophic soft topology $\mathscr{T}_{\mathscr{F}}$ on £ as follows:

Theorem 2 Taken that $\mathscr{F}_E^{\pi\alpha\sigma}$ be a svn-soft filter on £ and a map $\mathscr{T}_{\mathscr{F}}^{\pi},\ \mathscr{T}_{\mathscr{F}}^{\alpha},\ \mathscr{T}_{\mathscr{F}}^{\sigma}:E\to I^{\widetilde{(£,\,E)}}$ defined by

$$(\mathscr{T}_{\mathscr{F}}^{\pi})_e(f_A)=\begin{cases}\mathscr{F}_e^{\pi}(f_A), & \text{if } f_A\neq\widetilde{\Phi},\\ 1, & \text{if } f_A=\widetilde{\Phi},\end{cases}$$

$$(\mathscr{T}_{\mathscr{F}}^{\alpha})_e(f_A)=\begin{cases}\mathscr{F}_e^{\alpha}(f_A), & \text{if } f_A\neq\widetilde{\Phi},\\ 0, & \text{if } f_A=\widetilde{\Phi},\end{cases}$$

$$(\mathscr{T}^{\sigma}_{\mathscr{F}})_e(f_A) = \begin{cases} \mathscr{F}^{\sigma}_e(f_A), & \text{if } f_A \neq \widetilde{\Phi}, \\ 0, & \text{if } f_A = \widetilde{\Phi}, \end{cases}$$

then, $(£,\ (\mathscr{T}^{\pi\alpha\sigma}_{\mathscr{F}})_E)$ is a *svnst-spaces*.

Proof. It is straightforward and thus, it is omitted. □

Consider the map $\vartheta : £ \to \mathscr{X}$ between two sets, and the map $\varphi : E \to F$ between two parameters.

Theorem 3 Let $\vartheta_\varphi : \widetilde{(£,\ E)} \to \widetilde{(\mathscr{X},\ F)}$ be a mapping and $\mathscr{F}^{\pi\alpha\sigma}_E$ be a svn-soft filter on $£$. Then, we can define the mapping

$$\vartheta_\varphi(\mathscr{F}^{\pi\alpha\sigma}_e)(\rho_B) = \mathscr{F}^{\pi\alpha\sigma}_e(\vartheta^{-1}_\varphi(\rho_B)),\ \forall\, e \in E\ \rho_B \in \widetilde{(\mathscr{X},\ F)},$$

so that $\vartheta_\varphi(\mathscr{F}^{\pi\alpha\sigma}_E)$ is a svn-soft filter on $\mathscr{X}$.

Proof. To prove this theorem, the following conditions must be proved:

(F1)

$$\vartheta_\varphi(\mathscr{F}^{\pi}_e(\widetilde{\Phi}) = \mathscr{F}^{\pi}_e(\vartheta^{-1}_\varphi(\widetilde{\Phi})) = 0,\ \vartheta_\varphi(\mathscr{F}^{\alpha}_e)(\widetilde{\Phi}) = \mathscr{F}^{\alpha}_e(\vartheta^{-1}_\varphi(\widetilde{\Phi})) = 1,$$

$$\vartheta_\varphi(\mathscr{F}^{\sigma}_e)(\widetilde{\Phi}) = \mathscr{F}^{\sigma}_e(\vartheta^{-1}_\varphi(\widetilde{\Phi})) = 1,$$

and

$$\vartheta_\varphi(\mathscr{F}^{\pi}_e(\widetilde{F}) = \mathscr{F}^{\pi}_e(\vartheta^{-1}_\varphi(\widetilde{F})) = 1,\ \vartheta_\varphi(\mathscr{F}^{\alpha}_e)(\widetilde{F}) = \mathscr{F}^{\alpha}_e(\vartheta^{-1}_\varphi(\widetilde{F})) = 0,$$

$$\vartheta_\varphi(\mathscr{F}^{\sigma}_e)(\widetilde{F}) = \mathscr{F}^{\sigma}_e(\vartheta^{-1}_\varphi(\widetilde{F})) = 0.$$

(F2) For every $f_A,\ \rho_B \in \widetilde{(\mathscr{X},\ F)}$, we obtain

$$\begin{aligned}\vartheta_\varphi(\mathscr{F}^{\pi}_e)(f_A) \wedge \vartheta_\varphi(\mathscr{F}^{\pi}_e)(\rho_B) &= \mathscr{F}^{\pi}_e(\vartheta^{-1}_\varphi(f_A)) \wedge \mathscr{F}^{\pi}_e(\vartheta^{-1}_\varphi(\rho_B)) \\ &\leq \mathscr{F}^{\pi}_e(\vartheta^{-1}_\varphi(f_A) \sqcap \vartheta^{-1}_\varphi(\rho_B)) \\ &= \mathscr{F}^{\pi}_e(\vartheta^{-1}_\varphi(f_A \sqcap \rho_B)) \\ &= \vartheta_\varphi(\mathscr{F}^{\pi}_e)(f_A \sqcap \rho_B).\end{aligned}$$

$$
\begin{aligned}
\vartheta_\varphi(\mathscr{F}_e^\alpha)(f_A)\vee\vartheta_\varphi(\mathscr{F}_e^\alpha)(\rho_B) &= \mathscr{F}_e^\alpha(\vartheta_\varphi^{-1}(f_A))\vee\mathscr{F}_e^\alpha(\vartheta_\varphi^{-1}(\rho_B))\\
&\geq \mathscr{F}_e^\alpha(\vartheta_\varphi^{-1}(f_A)\sqcup\vartheta_\varphi^{-1}(\rho_B))\\
&= \mathscr{F}_e^\alpha(\vartheta_\varphi^{-1}(f_A\sqcup\rho_B))\\
&= \vartheta_\varphi(\mathscr{F}_e^\alpha)(f_A\sqcup\rho_B)\\
&\geq \vartheta_\varphi(\mathscr{F}_e^\alpha)(f_A\sqcap\rho_B).
\end{aligned}
$$

$$
\begin{aligned}
\vartheta_\varphi(\mathscr{F}_e^\sigma)(f_A)\vee\vartheta_\varphi(\mathscr{F}_e^\sigma)(\rho_B) &= \mathscr{F}_e^\sigma(\vartheta_\varphi^{-1}(f_A))\vee\mathscr{F}_e^\sigma(\vartheta_\varphi^{-1}(\rho_B))\\
&\geq \mathscr{F}_e^\sigma(\vartheta_\varphi^{-1}(f_A)\sqcup\vartheta_\varphi^{-1}(\rho_B))\\
&= \mathscr{F}_e^\sigma(\vartheta_\varphi^{-1}(f_A\sqcup\rho_B))\\
&= \vartheta_\varphi(\mathscr{F}_e^\sigma)(f_A\sqcup\rho_B)\\
&\geq \vartheta_\varphi(\mathscr{F}_e^\sigma)(f_A\sqcap\rho_B).
\end{aligned}
$$

(F3) If $f_A\sqsubseteq\rho_B$, then

$$
\vartheta_\varphi(\mathscr{F}_e^\pi)(f_A)=\mathscr{F}_e^\pi(\vartheta_\varphi^{-1}(f_A))\leq\mathscr{F}_e^\pi(\vartheta_\varphi^{-1}(\rho_B))=\vartheta_\varphi(\mathscr{F}_e^\pi)(\rho_B),
$$

$$
\vartheta_\varphi(\mathscr{F}_e^\alpha)(f_A)=\mathscr{F}_e^\alpha(\vartheta_\varphi^{-1}(f_A))\geq\mathscr{F}_e^\alpha(\vartheta_\varphi^{-1}(\rho_B))=\vartheta_\varphi(\mathscr{F}_e^\alpha)(\rho_B),
$$

$$
\vartheta_\varphi(\mathscr{F}_e^\sigma)(f_A)=\mathscr{F}_e^\sigma(\vartheta_\varphi^{-1}(f_A))\geq\mathscr{F}_e^\sigma(\vartheta_\varphi^{-1}(\rho_B))=\vartheta_\varphi(\mathscr{F}_e^\sigma)(\rho_B).
$$

By proving the three conditions, we have proven the above theorem. □

Suppose that $\mathscr{F}_E^{\pi\alpha\sigma}$ and $\mathscr{F}_E^{\star\pi\alpha\sigma}$ are two *svn-soft filters* on £ and $\mathscr{X}$ correspondingly, and $\vartheta_\varphi:\widetilde{(£,\,E)}\to\widetilde{(\mathscr{X},\,F)}$ a mapping. Then, ϑ_φ is called *svn-soft filter map*.

Theorem 4 Make that $\{(\mathscr{F}_j^{\pi\alpha\sigma})_E,\ j\in\Gamma\}$ a family of svn-soft filters on £ fulfills the next case:

(C) If $(f_A)_j\in((\mathscr{F}_j^{\pi\alpha\sigma})_E)^\circ$ for each $j\in\Gamma$, so we get $\sqcap_{j\in\Gamma_0}(f_A)_j\neq\widetilde{\Phi}$ for any finite subset Γ_0 of Γ.

If we defined a mappings

$$
\sqcup_{j\in\Gamma}\mathscr{F}_j^\pi:E\to I^{\widetilde{(£,\,E)}},\ \sqcap_{j\in\Gamma}\mathscr{F}_j^\alpha:E\to I^{\widetilde{(£,\,E)}},\ \sqcap_{j\in\Gamma}\mathscr{F}_j^\sigma:E\to I^{\widetilde{(£,\,E)}},
$$

as next: for all $e\in E$

$$\left(\bigsqcup_{j\in\Gamma}(\mathscr{F}_j^{\pi})\right)_e(\rho_B)=\begin{cases}\bigvee\{\bigwedge_{j\in\Gamma_0}(\mathscr{F}_j^{\pi})_e((\rho_B)_j)\mid\rho_B=\sqcap_{j\in\Gamma_0}(\rho_B)_j\}, & \text{if } (\rho_B)_j\in((\mathscr{F}_j^{\pi})_E)^{\circ},\\ 0, & \text{otherwise,}\end{cases}$$

$$\left(\sqcap_{j\in\Gamma}(\mathscr{F}_j^{\alpha})\right)_e(\rho_B)=\begin{cases}\bigwedge\{\bigvee_{j\in\Gamma_0}(\mathscr{F}_j^{\alpha})_e((\rho_B)_j):\rho_B=\sqcap_{j\in\Gamma_0}(\rho_B)_j\}, & \text{if } (\rho_B)_j\in((\mathscr{F}_j^{\alpha})_E)^{\circ},\\ 1, & \text{otherwise,}\end{cases}$$

$$\left(\sqcap_{j\in\Gamma}(\mathscr{F}_j^{\sigma})\right)_e(\rho_B)=\begin{cases}\bigwedge\{\bigvee_{j\in\Gamma_0}(\mathscr{F}_j^{\sigma})_e((\rho_B)_j):\rho_B=\sqcap_{j\in\Gamma_0}(\rho_B)_j\}, & \text{if } (\rho_B)_j\in((\mathscr{F}_j^{\sigma})_E)^{\circ},\\ 1, & \text{otherwise,}\end{cases}$$

where the supermom $\bigvee$ is taken for any finite index subset Γ_0 of Γ such that $\rho_B=\sqcap_{j\in\Gamma_0}(\rho_B)_j$. Then $\bigsqcup_{j\in\Gamma}(\mathscr{F}_j^{\pi\alpha\sigma})_E$ is the coarsest svn-soft filter finer than $(\mathscr{F}_j^{\pi\alpha\sigma})_E$ for every $j\in\Gamma$.

Proof. Initially; we will show that $\mathscr{H}_E^{\pi\alpha\sigma}=\bigsqcup_{j\in\Gamma}(\mathscr{F}_j^{\pi\alpha\sigma})_E$, [such that, $\mathscr{H}_e^{\pi}=\bigsqcup_{j\in\Gamma}(\mathscr{F}_j^{\pi})_e$, $\mathscr{H}_e^{\alpha}=\sqcap_{j\in\Gamma}(\mathscr{F}_j^{\alpha})_e$, $\mathscr{H}_e^{\sigma}=\sqcap_{j\in\Gamma}(\mathscr{F}_j^{\sigma})_e$] is a *svn-soft filter* on £.

(F1) Obviously, $\mathscr{H}_e^{\pi}(\widetilde{\Phi})=0,\ \mathscr{H}_e^{\alpha}(\widetilde{\Phi})=1,\ \mathscr{H}_e^{\sigma}(\widetilde{\Phi})=1$ and $\mathscr{H}_e^{\pi}(\widetilde{E})=1,\ \mathscr{H}_e^{\alpha}(\widetilde{E})=0,\ \mathscr{H}_e^{\sigma}(\widetilde{E})=0\ \forall,\ e\in E$.

(F2) For any finite index subsets G and N of Γ such that

$$\pi_{f_A}=\sqcap_{i\in G}(\pi_{f_A})_i,\ \alpha_{f_A}=\bigsqcup_{i\in G}(\alpha_{f_A})_i,\ \sigma_{f_A}=\bigsqcup_{i\in G}(\sigma_{f_A})_i,$$

$$\pi_{\rho_B}=\sqcap_{n\in N}(\pi_{\rho_B})_n,\ \alpha_{\rho_B}=\sqcap_{n\in N}(\alpha_{\rho_B})_n,\ \sigma_{\rho_B}=\sqcap_{n\in N}(\sigma_{\rho_B})_n,$$

we have

$$f_A\sqcap\rho_B=[\sqcap_{i\in G}(f_A)_i\sqcap\sqcap_{n\in N}(\rho_B)_n]$$

Furthermore, for all $m\in G\cap N$, put $f_A\sqcap\rho_B=\sqcap_{m\in G\cup N}(h_C)_m$, $C=A\cap B$, where

$$\pi_{(h_C)_m}=\begin{cases}\pi_{(f_A)_m}, & \text{if } n\in G-(G\cap N),\\ \pi_{(\rho_B)_m}, & \text{if } m\in N-(G\cap N),\\ \pi_{(f_A)_m}\cap\pi_{(\rho_B)_m}, & \text{if } m\in G\cap N,\end{cases}$$

$$\alpha_{(h_C)_m}=\begin{cases}\alpha_{(f_A)_m}, & \text{if } m\in G-(G\cap N),\\ \alpha_{(\rho_B)_m}, & \text{if } m\in N-(G\cap N),\\ \alpha_{(f_A)_m}\cup\alpha_{(\rho_B)_m}, & \text{if } m\in G\cap N,\end{cases}$$

$$\sigma_{(h_C)_m}=\begin{cases}\sigma_{(f_A)_m}, & \text{if } m\in G-(G\cap N),\\ \sigma_{(\rho_B)_m}, & \text{if } m\in N-(G\cap N),\\ \sigma_{(f_A)_m}\cup\sigma_{(\rho_B)_m}, & \text{if } m\in G\cap N,\end{cases}$$

which means that

$$\mathscr{H}_e^{\pi}(f_A\sqcap\rho_B)\geq\bigwedge_{m\in G\cap N}(\mathscr{F}_e^{\pi})_m(h_C)_m\geq\bigwedge_{i\in G}(\mathscr{F}_e^{\pi})_i(f_A)_i\wedge\bigwedge_{n\in N}(\mathscr{F}_e^{\pi})_n(\rho_B)_n,$$

$$\mathscr{H}_e^{\alpha}(f_A\sqcap\rho_B)\leq\bigvee_{m\in G\cap N}(\mathscr{F}_e^{\alpha})_m(h_C)_m\leq\bigvee_{i\in G}(\mathscr{F}_e^{\alpha})_i(f_A)_i\vee\bigvee_{n\in N}(\mathscr{F}_e^{\alpha})_n(\rho_B)_n,$$

$$\mathscr{H}_e^{\sigma}(f_A\sqcap\rho_B)\leq\bigvee_{m\in G\cap N}(\mathscr{F}_e^{\sigma})_m(h_C)_m\leq\bigvee_{i\in G}(\mathscr{F}_e^{\sigma})_i(f_A)_i\vee\bigvee_{n\in N}(\mathscr{F}_e^{\sigma})_n(\rho_B)_n.$$

Therefore,

$$\mathscr{H}_e^{\pi}(f_A\sqcap\rho_B)\geq\mathscr{H}_e^{\pi}(f_A)\wedge\mathscr{H}_e^{\pi}(\rho_B),\ \mathscr{H}_e^{\alpha}(f_A\sqcap\rho_B)\leq\mathscr{H}_e^{\alpha}(f_A)\vee\mathscr{H}_e^{\alpha}(\rho_B),$$

$$\mathscr{H}_e^{\sigma}(f_A\sqcap\rho_B)\leq\mathscr{H}_e^{\sigma}(f_A)\vee\mathscr{H}_e^{\sigma}(\rho_B).$$

(F3) Take that $f_A\sqsubseteq\rho_B$, according the definition of $\mathscr{H}$, there exists a finite index set G with

$$\pi_{f_A}=\sqcap_{i\in G}(\pi_{f_A})_i,\ \alpha_{f_A}=\bigsqcup_{i\in G}(\alpha_{f_A})_i,\ \sigma_{f_A}=\bigsqcup_{i\in G}(\sigma_{f_A})_i,$$

therefore

$$\mathscr{H}_e^{\pi}(f_A)\geq\bigwedge_{i\in G}(\mathscr{F}_e^{\pi})_i((f_A)_i),\ \mathscr{H}_e^{\alpha}(f_A)\leq\bigvee_{i\in G}(\mathscr{F}_e^{\alpha})_i((f_A)_i),$$

$$\mathscr{H}_e^{\sigma}(f_A)\leq\bigvee_{i\in G}(\mathscr{F}_e^{\sigma})_i((f_A)_i).$$

On the other hand, since $\rho_B=f_A\sqcup\rho_B=\sqcap_{i\in G}((f_A)_i\sqcup\rho_B)$, then we obtain

$$\mathscr{H}_e^{\pi}(\rho_B) \geq \bigwedge_{i\in G} (\mathscr{F}_e^{\pi})_i((f_A)_i \sqcup \rho_B) \geq \bigwedge_{i\in G} (\mathscr{F}_e^{\pi})_i((f_A)_i),$$

$$\mathscr{H}_e^{\alpha}(\rho_B) \leq \bigvee_{i\in G} (\mathscr{F}_e^{\alpha})_i((f_A)_i \sqcap \rho_B) \leq \bigvee_{i\in G} (\mathscr{F}_e^{\alpha})_i((f_A)_i),$$

$$\mathscr{H}_e^{\sigma}(\rho_B) \leq \bigvee_{i\in G} (\mathscr{F}_e^{\sigma})_i((f_A)_i \sqcap \rho_B) \leq \bigvee_{i\in G} (\mathscr{F}_e^{\sigma})_i((f_A)_i).$$

Hence, $\mathscr{H}_e^{\pi}(\rho_B) \geq \mathscr{H}_e^{\pi}(f_A)$, $\mathscr{H}_e^{\alpha}(f_A) \leq \mathscr{H}_e^{\alpha}(f_A)$ and $\mathscr{H}_e^{\sigma}(\rho_B) \leq \mathscr{H}_e^{\sigma}(f_A)$. Now, we will show that

$$\mathscr{H}_e^{\pi}(f_A) \geq (\mathscr{F}_e^{\pi})_j(f_A),\ \mathscr{H}_e^{\alpha}(f_A) \leq (\mathscr{F}_e^{\alpha})_j(f_A),\ \mathscr{H}_e^{\sigma}(f_A) \leq (\mathscr{F}_e^{\sigma})_j(f_A),$$

for each $j \in \Gamma$ from the next:

If $(\mathscr{F}_e^{\pi})_j(f_A) = 0$, $(\mathscr{F}_e^{\alpha})_j(f_A) = 1$, $(\mathscr{F}_e^{\sigma})_j(f_A) = 1$, then it is trivial.

If $(\mathscr{F}_e^{\pi})_j(f_A) > 0$, $(\mathscr{F}_e^{\alpha})_j(f_A) < 1$, $(\mathscr{F}_e^{\sigma})_j(f_A) < 1$, then for $f_A = f_A \sqcap \widetilde{E}$, we obtain

$$\mathscr{H}_e^{\pi}(f_A) \geq (\mathscr{F}_e^{\pi})_j(f_A) \wedge (\mathscr{F}_e^{\pi})_j(\widetilde{E}) = (\mathscr{F}_e^{\pi})_j(f_A),$$

$$\mathscr{H}_e^{\alpha}(f_A) \leq (\mathscr{F}_e^{\alpha})_j(f_A) \vee (\mathscr{F}_e^{\alpha})_j(\widetilde{E}) = (\mathscr{F}_e^{\alpha})_j(f_A),$$

$$\mathscr{H}_e^{\sigma}(f_A) \leq (\mathscr{F}_e^{\sigma})_j(f_A) \vee (\mathscr{F}_e^{\sigma})_j(\widetilde{E}) = (\mathscr{F}_e^{\sigma})_j(f_A).$$

If $\mathscr{G}_E^{\pi\alpha\sigma} \sqsupseteq (\mathscr{F}_j^{\pi\alpha\sigma})_E$ for each $j \in \Gamma$, we will show that $\mathscr{G}_E^{\pi\alpha\sigma} \sqsupseteq \mathscr{H}_E^{\pi\alpha\sigma}$. By the definition of $\mathscr{H}$, there exists a finite index set G with $f_A = \sqcap_{i\in G}(f_A)_i$ so that

$$\mathscr{H}_e^{\pi}(f_A) \geq \bigwedge_{i\in G} (\mathscr{F}_e^{\pi})_i((f_A)_i),\ \mathscr{H}_e^{\alpha}(f_A) \leq \bigvee_{i\in G} (\mathscr{F}_e^{\alpha})_i((f_A)_i),$$

$$\mathscr{H}_e^{\sigma}(f_A) \leq \bigvee_{i\in G} (\mathscr{F}_e^{\sigma})_i((f_A)_i).$$

On the other hand, since $\mathscr{G}_E^{\pi\alpha\sigma} \sqsupseteq (\mathscr{F}_i^{\pi\alpha\sigma})_E$ for each $i \in G$, then we have

$$\mathscr{G}_e^{\pi}(f_A) \geq \bigwedge_{i\in G} \mathscr{G}_e^{\pi}((f_A)_i) \geq \bigwedge_{i\in G} (\mathscr{F}_e^{\pi})_i((f_A)_i),$$

$$\mathscr{G}_e^{\alpha}(f_A) \leq \bigvee_{i\in G} \mathscr{G}_e^{\alpha}((f_A)_i) \leq \bigvee_{i\in G} (\mathscr{F}_e^{\alpha})_i((f_A)_i),$$

$$\mathscr{G}_e^{\sigma}(f_A) \leq \bigvee_{i\in G} \mathscr{G}_e^{\sigma}((f_A)_i) \leq \bigvee_{i\in G} (\mathscr{F}_e^{\sigma})_i((f_A)_i).$$

Thus, $\mathscr{G}_e^{\pi}(f_A) \geq \mathscr{H}_e^{\pi}(f_A)$, $\mathscr{G}_e^{\alpha}(f_A) \leq \mathscr{H}_e^{\alpha}(f_A)$ and $\mathscr{G}_e^{\sigma}(f_A) \leq \mathscr{H}_e^{\sigma}(f_A)$. □

Theorem 5 Let $\vartheta_\varphi : \widetilde{(£,\, E)} \to \widetilde{(\mathscr{X},\, F)}$ be a mapping and $\mathscr{F}_E^{\pi\alpha\sigma}$ be a svn-soft filter on $\mathscr{X}$. Then, we can define the mapping $\vartheta_\varphi^{-1}(\mathscr{F}_F^{\pi\alpha\sigma}) : E \to I^{\widetilde{(£,\, E)}}$ for all $g \in F$ by:

$$\vartheta_\varphi^{-1}(\mathscr{F}_g^{\pi})(f_A) = \begin{cases} \bigvee\{\mathscr{F}_g^{\pi}(\rho_B) : f_A = \vartheta_\varphi^{-1}(\rho_B)\}, & \text{if } \rho_B \neq \widetilde{\Phi}, \\ 0, & \text{if } \rho_B = \widetilde{\Phi}, \end{cases}$$

$$\vartheta_\varphi^{-1}(\mathscr{F}_g^{\alpha})(f_A) = \begin{cases} \bigwedge\{\mathscr{F}_g^{\alpha}(\rho_B) : f_A = \vartheta_\varphi^{-1}(\rho_B)\}, & \text{if } \rho_B \neq \widetilde{\Phi}, \\ 1, & \text{if } \rho_B = \widetilde{\Phi}, \end{cases}$$

$$\vartheta_\varphi^{-1}(\mathscr{F}_g^{\sigma})(f_A) = \begin{cases} \bigwedge\{\mathscr{F}_g^{\sigma}(\rho_B) : f_A = \vartheta_\varphi^{-1}(\rho_B)\}, & \text{if } \rho_B \neq \widetilde{\Phi}, \\ 1, & \text{if } \rho_B = \widetilde{\Phi}, \end{cases}$$

so that $\vartheta_\varphi^{-1}(\mathscr{F}^{\pi\alpha\sigma})$ is a svn-soft filter on £.

Proof. (F1) It is clear that $\vartheta_\varphi^{-1}(\mathscr{F}_g^{\pi})(\widetilde{\Phi}) = 0$, $\vartheta_\varphi^{-1}(\mathscr{F}_g^{\alpha})(\widetilde{\Phi}) = 1$, $\vartheta_\varphi^{-1}(\mathscr{F}_g^{\sigma})(\widetilde{\Phi}) = 1$ and $\vartheta_\varphi^{-1}(\mathscr{F}_g^{\pi})(\widetilde{E}) = 1$, $\vartheta_\varphi^{-1}(\mathscr{F}_g^{\alpha})(\widetilde{E}) = 0$, $\vartheta_\varphi^{-1}(\mathscr{F}_g^{\sigma})(\widetilde{E}) = 0$ for all $g \in F$.

(F2) It is proved from that:

$$\begin{aligned}
\vartheta_\varphi^{-1}(\mathscr{F}_g^{\pi})(f_A) \wedge \vartheta_\varphi^{-1}(\mathscr{F}_g^{\pi})(\rho_B) &= \left(\bigvee\{\mathscr{F}_g^{\pi}(z_D) : f_A = \vartheta_\varphi^{-1}(z_D)\}\right) \wedge \left(\bigvee\{\mathscr{F}_g^{\pi}(h_C) : \rho_B = \vartheta_\varphi^{-1}(h_C)\}\right) \\
&= \bigvee\{\mathscr{F}_g^{\pi}(z_D) \wedge \mathscr{F}_g^{\pi}(h_C) :_A \sqcap \rho_B = \vartheta_\varphi^{-1}(z_D) \sqcap \vartheta_\varphi^{-1}(h_C)\} \\
&\leq \bigvee\{\mathscr{F}_g^{\pi}(z_D \sqcap h_C) : f_A \sqcap \rho_B = \vartheta_\varphi^{-1}(z_D) \sqcap \vartheta_\varphi^{-1}(h_C)\} \\
&\leq \vartheta_\varphi^{-1}(\mathscr{F}_g^{\pi})(f_A \sqcap \rho_B),
\end{aligned}$$

$$\begin{aligned}
\vartheta_\varphi^{-1}(\mathscr{F}_g^{\alpha})(f_A) \vee \vartheta_\varphi^{-1}(\mathscr{F}_g^{\alpha})(\rho_B) &= \left(\bigwedge\{\mathscr{F}_g^{\alpha}(z_D) : f_A = \vartheta_\varphi^{-1}(z_D)\}\right) \vee \left(\bigwedge\{\mathscr{F}_g^{\alpha}(h_C) : \rho_B = \vartheta_\varphi^{-1}(h_C)\}\right) \\
&= \bigwedge\{\mathscr{F}_g^{\alpha}(z_D) \vee \mathscr{F}_g^{\alpha}(h_C) : f_A \sqcup \rho_B = \vartheta_\varphi^{-1}(z_D) \sqcup \vartheta_\varphi^{-1}(h_C)\} \\
&\geq \bigwedge\{\mathscr{F}_g^{\alpha}(z_D \sqcap h_C) : f_A \sqcap \rho_B = \vartheta_\varphi^{-1}(z_D) \sqcap \vartheta_\varphi^{-1}(h_C)\} \\
&\geq \vartheta_\varphi^{-1}(\mathscr{F}_g^{\alpha})(f_A \sqcap \rho_B),
\end{aligned}$$

$$\vartheta_{\varphi}^{-1}(\mathscr{F}_g^{\sigma})(f_A) \vee \vartheta_{\varphi}^{-1}(\mathscr{F}_g^{\sigma})(\rho_B) = \left(\bigwedge\{\mathscr{F}_g^{\sigma}(z_D) : f_A = \vartheta_{\varphi}^{-1}(z_D)\}\right) \vee \left(\bigwedge\{\mathscr{F}_g^{\sigma}(h_C) : \rho_B = \vartheta_{\varphi}^{-1}(h_C)\}\right)$$

$$= \bigwedge\{\mathscr{F}_g^{\sigma}(z_D) \vee \mathscr{F}_g^{\sigma}(h_C) : f_A \sqcup \rho_B = \vartheta_{\varphi}^{-1}(z_D) \sqcup \vartheta_{\varphi}^{-1}(h_C)\}$$

$$\geq \bigwedge\{\mathscr{F}_g^{\sigma}(z_D \sqcap h_C) : f_A \sqcap \rho_B = \vartheta_{\varphi}^{-1}(z_D) \sqcap \vartheta_{\varphi}^{-1}(h_C)\}$$

$$\geq \vartheta_{\varphi}^{-1}(\mathscr{F}_g^{\sigma})(f_A \sqcap \rho_B).$$

(F3) If $f_A \sqsubseteq \rho_B$, then

$$\vartheta_{\varphi}^{-1}(\mathscr{F}_g^{\pi})(f_A) = \mathscr{F}_g^{\pi}(\vartheta_{\varphi}(f_A)) \leq \mathscr{F}_g^{\pi}(\vartheta_{\varphi}(\rho_B)) = \vartheta_{\varphi}^{-1}(\mathscr{F}_g^{\pi})(\rho_B),$$

$$\vartheta_{\varphi}^{-1}(\mathscr{F}_g^{\alpha})(f_A) = \mathscr{F}_g^{\alpha}(\vartheta_{\varphi}(f_A)) \geq \mathscr{F}_g^{\alpha}(\vartheta_{\varphi}(\rho_B)) = \vartheta_{\varphi}^{-1}(\mathscr{F}_g^{\alpha})(\rho_B),$$

$$\vartheta_{\varphi}^{-1}(\mathscr{F}_g^{\sigma})(f_A) = \mathscr{F}_g^{\sigma}(\vartheta_{\varphi}(f_A)) \geq \mathscr{F}_g^{\sigma}(\vartheta_{\varphi}(\rho_B)) = \vartheta_{\varphi}^{-1}(\mathscr{F}_g^{\sigma})(\rho_B).$$

Theorem 6 Let $\{(\mathscr{F}_j^{\pi\alpha\sigma})_{E_j},\ j \in \Gamma\}$ be a family of svn-soft filters on $£_j$, E_j are parameter sets and $E = \prod_{j\in\Gamma} E_j$. Let $£_j = \prod_{j\in\Gamma} £_j$ be the product space. $\hbar_j : £ \to £_j$, $\varphi_j : E \to E_j$ are the projection maps for each $j \in \Gamma$ and $(\hbar_{\varphi})_j : \widetilde{(£,\ E)} \to \widetilde{(£_j,\ E_j)}$. Then, we can define a maps $\mathscr{F}^{\pi} : E \to I^{\widetilde{(£,\ E)}}$, $\mathscr{F}^{\alpha} : E \to I^{\widetilde{(£,\ E)}}$, $\mathscr{F}^{\sigma} : E \to I^{\widetilde{(£,\ E)}}$ by:

$$\mathscr{F}_e^{\pi}(f_A) = \begin{cases} \bigvee\left\{\bigwedge_{j\in G}(\mathscr{F}_j^{\pi})_e((\rho_B)_j) : f_A = \sqcap_{j\in G}(\hbar_{\varphi}^{-1})_j((\rho_B)_j)\right\}, & \text{if } (\rho_B)_j \in \left((\mathscr{F}_j^{\pi})_{E_j}\right)^{\circ}, \\ 0, & \text{otherwise,} \end{cases}$$

$$\mathscr{F}_e^{\alpha}(f_A) = \begin{cases} \bigwedge\left\{\bigvee_{j\in G}(\mathscr{F}_j^{\alpha})_e((\rho_B)_j) : f_A = \bigsqcup_{j\in G}(\hbar_{\varphi}^{-1})_j((\rho_B)_j)\right\}, & \text{if } (\rho_B)_j \in \left((\mathscr{F}_j^{\alpha})_{E_j}\right)^{\circ}, \\ 1, & \text{otherwise,} \end{cases}$$

$$\mathscr{F}_e^{\sigma}(f_A) = \begin{cases} \bigwedge\left\{\bigvee_{j\in G}(\mathscr{F}_j^{\sigma})_e((\rho_B)_j) : f_A = \bigsqcup_{j\in G}(\hbar_{\varphi}^{-1})_j((\rho_B)_j)\right\}, & \text{if } (\rho_B)_j \in \left((\mathscr{F}_j^{\sigma})_{E_j}\right)^{\circ}, \\ 1, & \text{otherwise,} \end{cases}$$

where the supermom $\bigvee$ is taken for any finite index subset G of Γ such that $\pi_{f_A} = \pi_{\sqcap_{j\in G}(\hbar_{\varphi}^{-1})_j((\rho_B)_j)}$, $\alpha_{f_A} = \alpha_{\bigsqcup_{j\in G}(\hbar_{\varphi}^{-1})_j((\rho_B)_j)}$ and $\sigma_{f_A} = \sigma_{\bigsqcup_{j\in G}(\hbar_{\varphi}^{-1})_j((\rho_B)_j)}$. Then (1) For each $f_A \in \widetilde{(£,\ E)}$, we obtain

$$\mathscr{F}^{\pi}_{e}(f_A) = \bigvee_{j\in\Gamma}(\hbar^{-1}_{\varphi})_j(\mathscr{F}^{\pi}_{j})_{e_j}(f_A),\ \mathscr{F}^{\alpha}_{e}(f_A) = \bigwedge_{j\in\Gamma}(\hbar^{-1}_{\varphi})_j(\mathscr{F}^{\alpha}_{j})_{e_j}(f_A),$$

$$\mathscr{F}^{\sigma}_{e}(f_A) = \bigwedge_{j\in\Gamma}(\hbar^{-1}_{\varphi})_j(\mathscr{F}^{\sigma}_{j})_{e_j}(f_A).$$

(2) $\mathscr{F}^{\pi\alpha\sigma}$ is the coarsest svn-soft filter on £ for which each projection map $(\hbar_{\varphi})_j : \widetilde{(£,\, E)} \to \widetilde{(£_j,\, E_j)}$ is a svn-soft filter mapping.

(3) $\delta_{\varphi} : (\chi,\ \mathscr{H}^{\pi\alpha\sigma}_{F}) \to (£,\ \mathscr{F}^{\pi\alpha\sigma}_{E})$ is a svn-soft filter mapping if and only if for each $j \in \Gamma$, we have $(\hbar_{\varphi})_j \circ \delta_{\varphi} : (\chi,\ \mathscr{H}^{\pi\alpha\sigma}_{F}) \to (£_j,\ (\mathscr{F}^{\pi\alpha\sigma}_{j})_{E_j})$ is a svn-soft filter mapping.

Proof. (1) From Theorem 5, each $(\hbar^{-1}_{\varphi})_j((\mathscr{F}^{\pi\alpha\sigma}_{j})_{E_j})$ is a *svn-soft filter* on $£_j$. Firstly, we will show that $\bigvee_{j\in\Gamma}(\hbar^{-1}_{\varphi})_j$ $((\mathscr{F}^{\pi\alpha\sigma}_{j})_{E_j})$ exists, that is, it satisfies the condition (C) of Theorem 4.

(C) If $(f_A)_j \in (\hbar^{-1}_{\varphi})_j((\mathscr{F}^{\pi\alpha\sigma}_{j})_{E_j})^{\circ}\ \forall\ j \in \Gamma$, there exists $(\rho_B)_j \in \widetilde{(\chi,\, F)}$ with $(f_A)_j = (\hbar^{-1}_{\varphi})_j((\rho_B)_j)$ such that $(\mathscr{F}^{\pi}_{j})_{e_j}((\rho_B)_j) > 0$, $(\mathscr{F}^{\alpha}_{j})_{e_j}((\rho_B)_j) < 1$ and $(\mathscr{F}^{\sigma}_{j})_{e_j}((\rho_B)_j) < 1$. It implies that $(\rho_B)_j \neq \widetilde{\Phi}$, that is, there exists $\kappa_j \in £_j$ with $\pi_{(\rho_B)_j}(\kappa_j) > 0$, $\alpha_{(\rho_B)_j}(\kappa_j) < 1$ and $\sigma_{(\rho_B)_j}(\kappa_j) < 1$. For every finite index subset G of Γ, put

$$\kappa = \begin{cases} \hbar^{-1}_{i}(\kappa_i), \text{ if } \kappa_i \in £_i \text{ for every } i \in G, \\ \hbar^{-1}_{j}(\kappa_j), \text{ if } \kappa_j \in £_j \text{ for every } j \in \Gamma - G. \end{cases}$$

Then, we have

$$\pi_{\bigwedge_{j\in G}(f_e)_j(\kappa)} = \pi_{\bigwedge_{j\in G}(\hbar^{-1}_{\varphi})_j(\rho_{\varphi(e)})_j}(\kappa) = \pi_{\bigwedge_{j\in G}(\rho_{\varphi(e)})_j}(\kappa_j) > 0,$$

$$\alpha_{\bigvee_{j\in G}(f_e)_j(\kappa)} = \alpha_{\bigvee_{j\in G}(\hbar^{-1}_{\varphi})_j(\rho_{\varphi(e)})_j}(\kappa) = \alpha_{\bigvee_{j\in G}(\rho_{\varphi(e)})_j}(\kappa_j) < 1,$$

$$\sigma_{\bigvee_{j\in G}(f_e)_j(\kappa)} = \sigma_{\bigvee_{j\in G}(\hbar^{-1}_{\varphi})_j(\rho_{\varphi(e)})_j}(\kappa) = \sigma_{\bigvee_{j\in G}(\rho_{\varphi(e)})_j}(\kappa_j) < 1.$$

We will show that $\mathscr{F}^{\pi\alpha\sigma} = \bigvee_{j\in\Gamma}(\hbar^{-1}_{\varphi})_j(\mathscr{F}^{\pi\alpha\sigma}_{j})$ By the definition of $\mathscr{F}^{\pi\alpha\sigma}_{E_j}$ there exists a finite index set $G \in \Gamma$ with $\pi_{f_A} = \pi_{\sqcap_{i\in G}(\hbar^{-1}_{\varphi})_i(\rho_B)_i}$, $\alpha_{f_A} = \alpha_{\bigsqcup_{i\in G}(\hbar^{-1}_{\varphi})_j(\rho_B)_i}$ and $\sigma_{f_A} = \sigma_{\bigsqcup_{i\in G}(\hbar^{-1}_{\varphi})_i(\rho_B)_i}$ such that

$$\mathscr{F}^{\pi}_{e}(f_A) \geq \bigwedge_{i\in G}(\mathscr{F}^{\pi}_{\varphi(e)})_i(\rho_{\varphi(e)})_i),\ \mathscr{F}^{\alpha}_{e}(f_A) \leq \bigvee_{i\in G}(\mathscr{F}^{\alpha}_{\varphi(e)})_i((\rho_{\varphi(e)})_i)$$

$$\mathscr{F}^{\sigma}_{e}(f_A) \leq \bigvee_{i\in G}(\mathscr{F}^{\sigma}_{\varphi(e)})_i((\rho_{\varphi(e)})_i),$$

putting $\pi_{(f_A)_i} = \pi_{(\hbar^{-1}_{\varphi})_i(\rho_B)_i}$, $\alpha_{(f_A)_i} = \alpha_{(\hbar^{-1}_{\varphi})_i(\rho_B)_i}$ and $\sigma_{(f_A)_i} = \sigma_{(\hbar^{-1}_{\varphi})_i(\rho_B)_i}$ for each $i \in G$, then for

$$\pi_{f_A} = \pi_{\sqcap_{i \in G}(f_A)_i} = \pi_{\sqcap_{i \in G}(\hbar_\varphi^{-1})_i((\mathscr{F}_i^\pi)_{\varphi(e)})((\rho_B)_i)},$$

$$\alpha_{f_A} = \alpha_{\sqcup_{i \in G}(f_A)_i} = \alpha_{\sqcup_{i \in G}(\hbar_\varphi^{-1})_i((\mathscr{F}_i^\alpha)_{\varphi(e)})((\rho_B)_i)},$$

$$\sigma_{f_A} = \sigma_{\sqcup_{i \in G}(f_A)_i} = \sigma_{\sqcup_{i \in G}(\hbar_\varphi^{-1})_i((\mathscr{F}_i^\sigma)_{\varphi(e)})((\rho_B)_i)},$$

we have

$$\bigvee_{j \in \Gamma} (\hbar_\varphi^{-1})_j(\mathscr{F}_j^\pi)_{e_j}(\rho_B) \geq \bigwedge_{i \in G} (\hbar_\varphi^{-1})_i(\mathscr{F}_i^\pi)_{e_i}((f_A)_i),$$

$$\bigwedge_{j \in \Gamma} (\hbar_\varphi^{-1})_j(\mathscr{F}_j^\alpha)_{e_j}(\rho_B) \leq \bigvee_{i \in G} (\hbar_\varphi^{-1})_i(\mathscr{F}_i^\alpha)_{e_i}((f_A)_i),$$

$$\bigwedge_{j \in \Gamma} (\hbar_\varphi^{-1})_j(\mathscr{F}_j^\sigma)_{e_j}(\rho_B) \leq \bigvee_{i \in G} (\hbar_\varphi^{-1})_i(\mathscr{F}_i^\sigma)_{e_i}((f_A)_i).$$

Hence,

$$\bigvee_{j \in \Gamma} (\hbar_\varphi^{-1})_j(\mathscr{F}_j^\pi)_{e_j} \sqsupseteq \mathscr{F}_e^\pi,\ \bigwedge_{j \in \Gamma} (\hbar_\varphi^{-1})_j(\mathscr{F}_j^\alpha)_{e_j} \sqsubseteq \mathscr{F}_e^\alpha,\ \bigwedge_{j \in \Gamma} (\hbar_\varphi^{-1})_j(\mathscr{F}_j^\sigma)_{e_j} \sqsubseteq \mathscr{F}_e^\sigma.$$

For every finite index set $L \subseteq \Gamma$ with $\pi_{h_C} = \pi_{\sqcap_{l \in L}(f_A)_l}$, $\alpha_{h_C} = \alpha_{\sqcup_{l \in L}(f_A)_l}$ and $\sigma_{h_C} = \sigma_{\sqcup_{l \in L}(f_A)_l}$, we have

$$\bigvee_{j \in \Gamma} (\hbar_\varphi^{-1})_j(\mathscr{F}_j^\pi)_{e_j}(h_C) \geq \bigwedge_{l \in L} (\hbar_\varphi^{-1})_l(\mathscr{F}_l^\pi)_{e_l}((f_A)_l),$$

$$\bigwedge_{j \in \Gamma} (\hbar_\varphi^{-1})_j(\mathscr{F}_j^\alpha)_{e_j}(h_C) \leq \bigvee_{l \in L} (\hbar_\varphi^{-1})_l(\mathscr{F}_l^\alpha)_{e_l}((f_A)_l),$$

$$\bigwedge_{j \in \Gamma} (\hbar_\varphi^{-1})_j(\mathscr{F}_j^\sigma)_{e_j}(h_C) \leq \bigvee_{l \in L} (\hbar_\varphi^{-1})_l(\mathscr{F}_l^\sigma)_{e_l}((f_A)_l),$$

and there exists $(\rho_B)_l \in \widetilde{(\pounds_l,\ E_l)}$ with $(f_A)_l = (\hbar_\varphi^{-1})_l((\rho_B)_l)$ such that

$$\bigwedge_{l\in L}(\hbar_{\varphi}^{-1})_j(\mathscr{F}_l^{\pi})_{e_l}((f_A)_l) \geq \bigwedge_{l\in L}(\mathscr{F}_l^{\pi})_{\varphi(e)}((\rho_B)_l),$$

$$\bigvee_{l\in L}(\hbar_{\varphi}^{-1})_j(\mathscr{F}_l^{\alpha})_{e_l}((f_A)_l) \leq \bigvee_{l\in L}(\mathscr{F}_l^{\alpha})_{\varphi(e)}((\rho_B)_l),$$

$$\bigvee_{l\in L}(\hbar_{\varphi}^{-1})_j(\mathscr{F}_l^{\sigma})_{e_l}((f_A)_l) \leq \bigvee_{l\in L}(\mathscr{F}_l^{\sigma})_{\varphi(e)}((\rho_B)_l).$$

On the other hand, for $\pi_{h_C} = \pi_{\sqcap_{l\in L}(f_A)_l} = \pi_{\sqcap_{l\in L}(\hbar_{\varphi}^{-1})_l((\rho_B)_l)}$, $\pi_{h_C} = \alpha_{\sqcup_{l\in L}(f_A)_l} = \alpha_{\sqcup_{l\in L}(\hbar_{\varphi}^{-1})_l((\rho_B)_l)}$ and $\sigma_{h_C} = \alpha_{\sqcup_{l\in L}(f_A)_l} = \sigma_{\sqcup_{l\in L}(\hbar_{\varphi}^{-1})_l((\rho_B)_l)}$ we have

$$(\mathscr{F}_e^{\pi})(h_C) \geq \bigwedge_{l\in L}(\mathscr{F}_l^{\pi})_{\varphi(e)}((\rho_B)_l),\ (\mathscr{F}_e^{\alpha})(h_C) \leq \bigvee_{l\in L}(\mathscr{F}_l^{\alpha})_{\varphi(e)}((\rho_B)_l)$$

$$(\mathscr{F}_e^{\sigma})(h_C) \leq \bigvee_{l\in L}(\mathscr{F}_l^{\sigma})_{\varphi(e)}((\rho_B)_l).$$

Then,

$$(\hbar_{\varphi}^{-1})_j(\mathscr{F}_j^{\pi})_{e_j} \sqsubseteq \mathscr{F}_e^{\pi},\ (\hbar_{\varphi}^{-1})_j(\mathscr{F}_j^{\alpha})_{e_j} \sqsupseteq \mathscr{F}_e^{\alpha},\ (\hbar_{\varphi}^{-1})_j(\mathscr{F}_j^{\sigma})_{e_j} \sqsupseteq \mathscr{F}_e^{\sigma},$$

and thus,

$$(\hbar_{\varphi}^{-1})_j(\mathscr{F}_j^{\pi})_{e_j} = \mathscr{F}_e^{\pi},\ (\hbar_{\varphi}^{-1})_j(\mathscr{F}_j^{\alpha})_{E_j} = \mathscr{F}_e^{\alpha},\ (\hbar_{\varphi}^{-1})_j(\mathscr{F}_j^{\sigma})_{e_j} = \mathscr{F}_e^{\sigma}.$$

(2) From (1) above, Theorem 4, and Theorem 5, we get that $\mathscr{F}_E^{\pi\alpha\sigma}$ is a svn-soft filter on £. For each $j \in \Gamma$, and $(\rho_B)_j \in (\widetilde{£_j,\ E_j})$, and by the definition of $\mathscr{F}_E^{\pi\alpha\sigma}$, we then have

$$\mathscr{F}_e^{\pi}((\hbar_{\varphi}^{-1})_j(\rho_B)_j) \geq (\mathscr{F}_j^{\pi})_{\varphi(e)}((\rho_B)_j),\ \mathscr{F}_e^{\alpha}((\hbar_{\varphi}^{-1})_j(\rho_B)_j) \leq (\mathscr{F}_j^{\alpha})_{\varphi(e)}((\rho_B)_j),$$

$$\mathscr{F}_e^{\sigma}((\hbar_{\varphi}^{-1})_j(\rho_B)_j) \leq (\mathscr{F}_j^{\sigma})_{\varphi(e)}((\rho_B)_j).$$

Hence, $(\hbar_{\varphi})_j : (£,\ \mathscr{F}_E^{\pi\alpha\sigma}) \to (£_j,\ \mathscr{F}_{E_j}^{\pi\alpha\sigma})$ is a svn-soft filter mapping.
Let $(\hbar_{\varphi})_j : (£,\ \mathscr{G}_E^{\pi\alpha\sigma}) \to (£_j,\ \mathscr{F}_{E_j}^{\pi\alpha\sigma})$ svn-soft filter mapping for each $j \in \Gamma$, that is,

$$\mathscr{G}_e^{\pi}((\hbar_{\varphi}^{-1})_j((\rho_B)_j)) \geq (\mathscr{F}_j^{\pi})_{\varphi(e)}((\rho_B)_j),\ \mathscr{G}_e^{\alpha}((\hbar_{\varphi}^{-1})_j((\rho_B)_j)) \leq (\mathscr{F}_j^{\alpha})_{\varphi(e)}((\rho_B)_j),$$

$$\mathscr{G}_e^{\sigma}((\hbar_{\varphi}^{-1})_j(\rho_B)_j) \leq (\mathscr{F}_j^{\sigma})_{\varphi(e)}((\rho_B)_j),$$

for all finite index set G with $\pi_{f_A} = \pi_{\sqcap_{i\in G}((\hbar_\varphi^{-1})_i(\rho_B)_i)}$, $\alpha_{f_A} = \alpha_{\sqcup_{i\in G}((\hbar_\varphi^{-1})_i(\rho_B)_i)}$ and $\sigma_{f_A} = \sigma_{\sqcup_{i\in G}((\hbar_\varphi^{-1})_i(\rho_B)_i)}$, and thus,

$$\mathscr{G}_e^\pi(f_A) \geq \bigwedge_{i\in G} \mathscr{G}_e^\pi((\hbar_\varphi^{-1})_i((\rho_B)_i)) \geq \bigwedge_{i\in G} (\mathscr{F}_i^\pi)_{\varphi(e)}((\rho_B)_i),$$

$$\mathscr{G}_e^\alpha(f_A) \leq \bigvee_{i\in G} \mathscr{G}_e^\alpha((\hbar_\varphi^{-1})_i((\rho_B)_i)) \leq \bigvee_{i\in G} (\mathscr{F}_i^\alpha)_{\varphi(e)}((\rho_B)_i),$$

$$\mathscr{G}_e^\sigma(f_A) \leq \bigvee_{i\in G} \mathscr{G}_e^\sigma((\hbar_\varphi^{-1})_i((\rho_B)_i)) \leq \bigvee_{i\in G} (\mathscr{F}_i^\sigma)_{\varphi(e)}((\rho_B)_i),$$

which implies that

$$\mathscr{G}_e^\pi(f_A) \geq \mathscr{F}_e^\pi(f_A),\ \mathscr{G}_e^\alpha(f_A) \leq \mathscr{F}_e^\alpha(f_A),\ \mathscr{G}_e^\sigma(f_A) \leq \mathscr{F}_e^\sigma(f_A),$$

for each $f_A \in \widetilde{(\pounds,\ E)}$.

(3) Necessity of the composition condition is clear since the composition of svn-soft filter mappings is a svn-soft filter mapping.

Conversely, let $\delta_\varphi : (\chi,\ \mathscr{H}_F^{\pi\alpha\sigma}) \to (\pounds,\ \mathscr{F}_E^{\pi\alpha\sigma})$ is just a svn-soft mapping. For every finite index set G with $\pi_{f_A} = \pi_{\sqcap_{i\in G}((\hbar_\varphi^{-1})_i(\rho_B)_i)}$, $\alpha_{f_A} = \alpha_{\sqcup_{i\in G}((\hbar_\varphi^{-1})_i(\rho_B)_i)}$ and $\sigma_{f_A} = \sigma_{\sqcup_{i\in G}((\hbar_\varphi^{-1})_i(\rho_B)_i)}$. Since for each $j \in \Gamma$, we have $((\hbar_\varphi)_j \circ \delta_\varphi) : (\chi,\ \mathscr{H}_F^{\pi\alpha\sigma}) \to (\pounds_j,\ (\mathscr{F}_j^{\pi\alpha\sigma})_{E_j})$ is a svn-soft filter mapping and

$$(\mathscr{F}_j^\pi)_{\varphi(e)}((\rho_B)_j) \leq \mathscr{H}_e^\pi(\delta_\varphi^{-1}((\hbar_\varphi^{-1})_j((\rho_B)_j))),$$

$$(\mathscr{F}_j^\alpha)_{\varphi(e)}((\rho_B)_j) \geq \mathscr{H}_e^\alpha(\delta_\varphi^{-1}((\hbar_\varphi^{-1})_j((\rho_B)_j))),$$

$$(\mathscr{F}_j^\sigma)_{\varphi(e)}((\rho_B)_j) \geq \mathscr{H}_e^\sigma(\delta_\varphi^{-1}((\hbar_\varphi^{-1})_j((\rho_B)_j))).$$

It follows that

$$\mathscr{H}_e^\pi(\delta_\varphi^{-1}((\hbar_\varphi^{-1})_i((\rho_B)_i))) \geq \mathscr{F}_{\varphi(e)}^\pi((\rho_B)_i),$$

$$\mathscr{H}_e^\alpha(\delta_\varphi^{-1}((\hbar_\varphi^{-1})_i((\rho_B)_i))) \leq \mathscr{F}_{\varphi(e)}^\alpha((\rho_B)_i),$$

$$\mathscr{H}_e^\sigma(\delta_\varphi^{-1}((\hbar_\varphi^{-1})_i((\rho_B)_i))) \leq \mathscr{F}_{\varphi(e)}^\sigma((\rho_B)_i).$$

Hence, we have

$$\mathscr{H}_e^{\pi}((\delta_{\varphi}^{-1})(f_A)) \geq \bigwedge_{i\in G} \mathscr{H}_e^{\pi}((\delta_{\varphi}^{-1})((\hbar_{\varphi}^{-1})_i((\rho_B)_i))) \geq \bigwedge_{i\in G} (\mathscr{F}_i^{\pi})_{\varphi(e)}((\rho_B)_i),$$

$$\mathscr{H}_e^{\alpha}((\delta_{\varphi}^{-1})(f_A)) \leq \bigvee_{i\in G} \mathscr{H}_e^{\alpha}((\delta_{\varphi}^{-1})((\hbar_{\varphi}^{-1})_i((\rho_B)_i))) \leq \bigvee_{i\in G} (\mathscr{F}_i^{\alpha})_{\varphi(e)}((\rho_B)_i),$$

$$\mathscr{H}_e^{\sigma}((\delta_{\varphi}^{-1})(f_A)) \leq \bigvee_{i\in G} \mathscr{H}_e^{\sigma}((\delta_{\varphi}^{-1})((\hbar_{\varphi}^{-1})_i((\rho_B)_i))) \leq \bigvee_{i\in G} (\mathscr{F}_i^{\sigma})_{\varphi(e)}((\rho_B)_i).$$

It implies $\mathscr{H}_e^{\pi}(\delta_{\varphi}^{-1}(f_A)) \geq \mathscr{F}_e^{\pi}(f_A)$, $\mathscr{H}_e^{\alpha}(\delta_{\varphi}^{-1}(f_A)) \leq \mathscr{F}_e^{\alpha}(f_A)$ and $\mathscr{H}_e^{\sigma}(\delta_{\varphi}^{-1}(f_A)) \leq \mathscr{F}_e^{\sigma}(f_A)$ for all $f_A \in \widetilde{(\pounds,\ E)}$. Therefore $\delta_{\varphi} : (\chi,\ \mathscr{H}_F^{\pi\alpha\sigma}) \to (\pounds,\ \mathscr{F}_E^{\pi\alpha\sigma})$ is a svn-soft filter mapping. □

Definition 6 Let $\{(\mathscr{F}_j^{\pi\alpha\sigma})_{E_j},\ j\in\Gamma\}$ be a family of *svn-soft filters* on $\pounds_j$, $j\in\Gamma$ and $\pounds = \prod_{j\in\Gamma} \pounds_j$, $E = \prod_{j\in\Gamma} E_j$ are product sets, $\hbar_j : \pounds \to \pounds_j$, $\varphi_j : E \to E_j$ are the projection mappings. The product of *svn-soft filters* is the coarsest *svn-soft filter* on $\pounds$ for which all $(\hbar_{\varphi})_j : (\pounds,\ \mathscr{F}_E^{\pi\alpha\sigma}) \to (\pounds_j,\ (\mathscr{F}_j^{\pi\alpha\sigma})_{E_j})$, $j\in\Gamma$, are *svn-soft filter mappings*.

4. Single-valued neutrosophic soft quasi-coincident neighborhood spaces

Within the field of mathematical harmony, single-valued neutrosophic soft filters (*svns-filters*) are the result of the combination of neutrosophic logic with soft set theory. The focus now switches to single-valued neutrosophic soft quasi-coincident neighborhood Spaces (*svnsqcn-Spaces*), a domain distinguished by its special interaction between single-valued neutrosophic sets and quasi-coincident neighborhood structures, as we continue our exploration of advanced neutrosophic theories.

We begin it with the following:

Definition 7 A mapping $\mathscr{F}^{\pi},\ \mathscr{F}^{\alpha},\ \mathscr{F}^{\sigma} : E \to I^{\widetilde{(\pounds,\ E)}}$ is called *svns-filter* on $\pounds$, if it meets the following criteria, $\forall\ e\in E$:

(F1) $\mathscr{F}_e^{\pi}(\widetilde{\Phi}) = 0,\ \mathscr{F}_e^{\alpha}(\widetilde{\Phi}) = 1,\ \mathscr{F}_e^{\sigma}(\widetilde{\Phi}) = 1$ and $\mathscr{F}_e^{\pi}(\widetilde{E}) = 1,\ \mathscr{F}_e^{\alpha}(\widetilde{E}) = 0, \mathscr{F}_e^{\sigma}(\widetilde{E}) = 0$,

(F2) $\mathscr{F}_e^{\pi}(f_A \sqcap \rho_B) \geq \mathscr{F}_e^{\pi}(f_A) \wedge \mathscr{F}_e^{\pi}(\rho_B)$, $\mathscr{F}_e^{\alpha}(f_A \sqcap \rho_B) \leq \mathscr{F}_e^{\alpha}(f_A) \vee \mathscr{F}_e^{\alpha}(\rho_B)$, $\mathscr{F}_e^{\sigma}(f_A \sqcap \rho_B) \leq \mathscr{F}_e^{\sigma}(f_A) \vee \mathscr{F}_e^{\sigma}(\rho_B)$, $\forall\ f_A,\ \rho_B \in \widetilde{(\pounds,\ E)}$,

(F3) If $f_A \sqsubseteq \rho_B$, then $\mathscr{F}_e^{\pi}(f_A) \leq \mathscr{F}_e^{\pi}(\rho_B)$, $\mathscr{F}_e^{\alpha}(f_A) \geq \mathscr{F}_e^{\alpha}(\rho_B)$, $\mathscr{F}_e^{\sigma}(f_A) \geq \mathscr{F}_e^{\sigma}(\rho_B)$.

If $\mathscr{F}_E^{\pi\alpha\sigma}$ and $\mathscr{F}_E^{\star\pi\alpha\sigma}$ are *svns-filters* on $\pounds$, then $\mathscr{F}_E^{\pi\alpha\sigma}$ is finer than $\mathscr{F}_E^{\star\pi\alpha\sigma}$ or ($\mathscr{F}_E^{\star\pi\alpha\sigma}$ is coarser than $\mathscr{F}_E^{\pi\alpha\sigma}$) denoted by $\mathscr{F}_E^{\pi\alpha\sigma} \sqsubseteq \mathscr{F}_E^{\star\pi\alpha\sigma}$ if and only if

$$\mathscr{F}_e^{\pi}(f_A) \leq \mathscr{F}_e^{\star\pi}(f_A),\ \mathscr{F}_e^{\alpha}(f_A) \geq \mathscr{F}_e^{\star\alpha}(f_A),\ \mathscr{F}_e^{\sigma}(f_A) \geq \mathscr{F}_e^{\star\sigma}(f_A),$$

for each $e\in E$, $f_A \in \widetilde{(\pounds,\ E)}$. Occasionally, we will write $\mathscr{F}^{\pi\alpha\sigma}$ for $(\mathscr{F}^{\pi},\ \mathscr{F}^{\alpha},\ \mathscr{F}^{\sigma})$, and it will be no ambiguity.

The central belongings of *svn-soft filters* are deliberated in the next suggestions:

Theorem 7 Suppos that $\{(\mathscr{F}_j^{\pi\alpha\sigma})_{E_j},\ j\in\Gamma\}$ is a familyollection of svn-soft filter on a set $\pounds$, then, the mapping $\mathscr{F}^{\pi\alpha\sigma} = \sqcap_{j\in\Gamma}(\mathscr{F}_j^{\pi\alpha\sigma})_{E_j} : E \to I^{\widetilde{(\pounds,\ E)}}$ defined, for every $e\in E$, $f_A \in \widetilde{(\pounds,\ E)}$ by:

$$\mathscr{F}_e^{\pi}(f_A) = \bigwedge_{j\in\Gamma} (\mathscr{F}_j^{\pi})_e(f_A),\ \mathscr{F}_e^{\alpha}(f_A) = \bigvee_{j\in\Gamma} (\mathscr{F}_j^{\alpha})_e(f_A),\ \mathscr{F}_e^{\sigma}(f_A) = \bigvee_{j\in\Gamma} (\mathscr{F}_j^{\sigma})_e(f_A),$$

is a svn-soft filter on £.

Proof. To prove this theorem, the following conditions must be proved:

(F1)

$$\mathscr{F}_e^{\pi}(\widetilde{\Phi}) = \bigwedge_{j\in\Gamma}(\mathscr{F}_j^{\pi})_e(\widetilde{\Phi}) = 0,\ \mathscr{F}_e^{\alpha}(\widetilde{\Phi}) = \bigvee_{j\in\Gamma}(\mathscr{F}_j^{\alpha})_e(\widetilde{\Phi}) = 1,\ \mathscr{F}_e^{\sigma}(\widetilde{\Phi}) = \bigvee_{j\in\Gamma}(\mathscr{F}_j^{\sigma})_e(\widetilde{\Phi}) = 1,$$

and

$$\mathscr{F}_e^{\pi}(\widetilde{E}) = \bigwedge_{j\in\Gamma}(\mathscr{F}_j^{\pi})_e(\widetilde{E}) = 1,\ \mathscr{F}_e^{\alpha}(\widetilde{E}) = \bigvee_{j\in\Gamma}(\mathscr{F}_j^{\alpha})_e(\widetilde{E}) = 0,\ \mathscr{F}_e^{\sigma}(\widetilde{E}) = \bigvee_{j\in\Gamma}(\mathscr{F}_j^{\sigma})_e(\widetilde{E}) = 0.$$

(F2) for all $f_A,\ \rho_B \in \widetilde{(£,\ E)}$, we have

$$\mathscr{F}_e^{\pi}(f_A) \wedge \mathscr{F}_e^{\pi}(\rho_B) = \bigwedge_{j\in\Gamma}(\mathscr{F}_j^{\pi})_e(f_A) \wedge \bigwedge_{j\in\Gamma}(\mathscr{F}_j^{\pi})_e(\rho_B) \leq \bigwedge_{j\in\Gamma}((\mathscr{F}_j^{\pi})_e(f_A) \wedge (\mathscr{F}_j^{\pi})_e(\rho_B))$$

$$\leq \bigwedge_{j\in\Gamma}(\mathscr{F}_j^{\pi})_e(f_A \sqcap \rho_B) = \mathscr{F}_e^{\pi}(f_A \sqcap \rho_B),$$

$$\mathscr{F}_e^{\alpha}(f_A) \vee \mathscr{F}_e^{\alpha}(\rho_B) = \bigvee_{j\in\Gamma}(\mathscr{F}_j^{\alpha})_e(f_A) \vee \bigvee_{j\in\Gamma}(\mathscr{F}_j^{\alpha})_e(\rho_B) \geq \bigvee_{j\in\Gamma}((\mathscr{F}_j^{\alpha})_e(f_A) \vee (\mathscr{F}_j^{\alpha})_e(\rho_B))$$

$$\geq \bigvee_{j\in\Gamma}((\mathscr{F}_j^{\alpha})_e(f_A) \wedge (\mathscr{F}_j^{\alpha})_e(\rho_B)) \geq \bigwedge_{j\in\Gamma}(\mathscr{F}_j^{\alpha})_e(f_A \sqcap \rho_B) = \mathscr{F}_e^{\alpha}(f_A \sqcap \rho_B),$$

$$\mathscr{F}_e^{\sigma}(f_A) \vee \mathscr{F}_e^{\sigma}(\rho_B) = \bigvee_{j\in\Gamma}(\mathscr{F}_j^{\sigma})_e(f_A) \vee \bigvee_{j\in\Gamma}(\mathscr{F}_j^{\sigma})_e(\rho_B) \geq \bigvee_{j\in\Gamma}((\mathscr{F}_j^{\sigma})_e(f_A) \vee (\mathscr{F}_j^{\sigma})_e(\rho_B))$$

$$\geq \bigvee_{j\in\Gamma}((\mathscr{F}_j^{\sigma})_e(f_A) \wedge (\mathscr{F}_j^{\sigma})_e(\rho_B)) \geq \bigwedge_{j\in\Gamma}(\mathscr{F}_j^{\sigma})_e(f_A \sqcap \rho_B) = \mathscr{F}_e^{\sigma}(f_A \sqcap \rho_B).$$

(F3) If $f_A \sqsubseteq \rho_B$, then

$$(\mathscr{F}_j^{\pi})_e(f_A) \leq (\mathscr{F}_j^{\pi})_e(\rho_B),\ (\mathscr{F}_j^{\alpha})_e(f_A) \geq (\mathscr{F}_j^{\alpha})_e(\rho_B),\ (\mathscr{F}_j^{\sigma})_e(f_A) \geq (\mathscr{F}_j^{\sigma})_e(\rho_B),$$

for every $e \in E,\ j \in \Gamma$, and hence

$$\mathscr{F}_e^{\pi}(f_A) = \bigwedge_{j\in\Gamma}(\mathscr{F}_j^{\pi})_e(f_A) \leq \bigwedge_{j\in\Gamma}(\mathscr{F}_j^{\pi})_e(\rho_B) = \mathscr{F}_e^{\pi}(\rho_B)$$

$$\mathscr{F}_e^{\alpha}(f_A) = \bigvee_{j\in\Gamma} (\mathscr{F}_j^{\alpha})_e(f_A) \geq \bigvee_{j\in\Gamma} (\mathscr{F}_j^{\alpha})_e(\rho_B) = \mathscr{F}_e^{\alpha}(\rho_B)$$

$$\mathscr{F}_e^{\sigma}(f_A) = \bigvee_{j\in\Gamma} (\mathscr{F}_j^{\sigma})_e(f_A) \geq \bigvee_{j\in\Gamma} (\mathscr{F}_j^{\sigma})_e(\rho_B) = \mathscr{F}_e^{\sigma}(\rho_B)$$

By proving the three conditions, we have proven the above theorem. □

From a *svn-soft filter* $\mathscr{F}_A : E \to I^{\widetilde{(\pounds,\, E)}}$, we can obtain a *svnst* $(\mathscr{T}_{\mathscr{F}}^{\pi},\ \mathscr{T}_{\mathscr{F}}^{\alpha},\ \mathscr{T}_{\mathscr{F}}^{\sigma})$ on £ as follows:

Theorem 8 let $\mathscr{F}_E^{\pi\alpha\sigma}$ be a svn-soft filter on £ and a mappins $\mathscr{T}_{\mathscr{F}}^{\pi} : E \to I^{\widetilde{(\pounds,\, E)}}$, $\mathscr{T}_{\mathscr{F}}^{\alpha} : E \to I^{\widetilde{(\pounds,\, E)}}$, $\mathscr{T}_{\mathscr{F}}^{\sigma} : E \to I^{\widetilde{(\pounds,\, E)}}$ defined by

$$(\mathscr{T}_{\mathscr{F}}^{\pi})_e(f_A) = \begin{cases} \mathscr{F}_e^{\pi}(f_A), & \text{if } f_A \neq \widetilde{\Phi}, \\ 1, & \text{if } f_A = \widetilde{\Phi}, \end{cases}$$

$$(\mathscr{T}_{\mathscr{F}}^{\alpha})_e(f_A) = \begin{cases} \mathscr{F}_e^{\alpha}(f_A), & \text{if } f_A \neq \widetilde{\Phi}, \\ 0, & \text{if } f_A = \widetilde{\Phi}, \end{cases}$$

$$(\mathscr{T}_{\mathscr{F}}^{\sigma})_e(f_A) = \begin{cases} \mathscr{F}_e^{\sigma}(f_A), & \text{if } f_A \neq \widetilde{\Phi}, \\ 0, & \text{if } f_A = \widetilde{\Phi}, \end{cases}$$

then, $(\pounds,\ (\mathscr{T}_{\mathscr{F}}^{\pi\alpha\sigma})_E)$ is a *svnst-spaces*.

Proof. The proof of this theory is clear, it is omitted. □

Consider the map $\vartheta : \pounds \to \mathscr{X}$ between two sets, and the map $\varphi : E \to F$ between two parameters.

Theorem 9 Let $\vartheta_{\varphi} : \widetilde{(\pounds,\, E)} \to \widetilde{(\mathscr{X},\, F)}$ be a mapping and $\mathscr{F}_E^{\pi\alpha\sigma}$ be a svn-soft filter on £. Then, we can define the mapping

$$\vartheta_{\varphi}(\mathscr{F}_e^{\pi\alpha\sigma})(\rho_B) = \mathscr{F}_e^{\pi\alpha\sigma}(\vartheta_{\varphi}^{-1}(\rho_B)),\ \forall\, e \in E\ \rho_B \in \widetilde{(\mathscr{X},\, F)},$$

so that $\vartheta_{\varphi}(\mathscr{F}_E^{\pi\alpha\sigma})$ is a svn-soft filter on $\mathscr{X}$.

Proof. To prove this theorem, the following conditions must be proved:

(F1)

$$\vartheta_{\varphi}(\mathscr{F}_e^{\pi}(\widetilde{\Phi}) = \mathscr{F}_e^{\pi}(\vartheta_{\varphi}^{-1}(\widetilde{\Phi})) = 0,\ \vartheta_{\varphi}(\mathscr{F}_e^{\alpha})(\widetilde{\Phi}) = \mathscr{F}_e^{\alpha}(\vartheta_{\varphi}^{-1}(\widetilde{\Phi})) = 1,$$

$$\vartheta_{\varphi}(\mathscr{F}_e^{\sigma})(\widetilde{\Phi}) = \mathscr{F}_e^{\sigma}(\vartheta_{\varphi}^{-1}(\widetilde{\Phi})) = 1,$$

and

$$\vartheta_\varphi(\mathscr{F}_e^\pi(\widetilde{F}) = \mathscr{F}_e^\pi(\vartheta_\varphi^{-1}(\widetilde{F})) = 1,\ \vartheta_\varphi(\mathscr{F}_e^\alpha)(\widetilde{F}) = \mathscr{F}_e^\alpha(\vartheta_\varphi^{-1}(\widetilde{F})) = 0,$$

$$\vartheta_\varphi(\mathscr{F}_e^\sigma)(\widetilde{F}) = \mathscr{F}_e^\sigma(\vartheta_\varphi^{-1}(\widetilde{F})) = 0.$$

(F2) For every $f_A,\ \rho_B \in (\widetilde{\mathscr{X},\ F})$, we obtain

$$\begin{aligned}
\vartheta_\varphi(\mathscr{F}_e^\pi)(f_A) \wedge \vartheta_\varphi(\mathscr{F}_e^\pi)(\rho_B) &= \mathscr{F}_e^\pi(\vartheta_\varphi^{-1}(f_A)) \wedge \mathscr{F}_e^\pi(\vartheta_\varphi^{-1}(\rho_B)) \\
&\leq \mathscr{F}_e^\pi(\vartheta_\varphi^{-1}(f_A) \sqcap \vartheta_\varphi^{-1}(\rho_B)) \\
&= \mathscr{F}_e^\pi(\vartheta_\varphi^{-1}(f_A \sqcap \rho_B)) \\
&= \vartheta_\varphi(\mathscr{F}_e^\pi)(f_A \sqcap \rho_B), \\
\vartheta_\varphi(\mathscr{F}_e^\alpha)(f_A) \vee \vartheta_\varphi(\mathscr{F}_e^\alpha)(\rho_B) &= \mathscr{F}_e^\alpha(\vartheta_\varphi^{-1}(f_A)) \vee \mathscr{F}_e^\alpha(\vartheta_\varphi^{-1}(\rho_B)) \\
&\geq \mathscr{F}_e^\alpha(\vartheta_\varphi^{-1}(f_A) \sqcup \vartheta_\varphi^{-1}(\rho_B)) \\
&= \mathscr{F}_e^\alpha(\vartheta_\varphi^{-1}(f_A \sqcup \rho_B)) \\
&= \vartheta_\varphi(\mathscr{F}_e^\alpha)(f_A \sqcup \rho_B) \\
&\geq \vartheta_\varphi(\mathscr{F}_e^\alpha)(f_A \sqcap \rho_B), \\
\vartheta_\varphi(\mathscr{F}_e^\sigma)(f_A) \vee \vartheta_\varphi(\mathscr{F}_e^\sigma)(\rho_B) &= \mathscr{F}_e^\sigma(\vartheta_\varphi^{-1}(f_A)) \vee \mathscr{F}_e^\sigma(\vartheta_\varphi^{-1}(\rho_B)) \\
&\geq \mathscr{F}_e^\sigma(\vartheta_\varphi^{-1}(f_A) \sqcup \vartheta_\varphi^{-1}(\rho_B)) \\
&= \mathscr{F}_e^\sigma(\vartheta_\varphi^{-1}(f_A \sqcup \rho_B)) \\
&= \vartheta_\varphi(\mathscr{F}_e^\sigma)(f_A \sqcup \rho_B) \\
&\geq \vartheta_\varphi(\mathscr{F}_e^\sigma)(f_A \sqcap \rho_B).
\end{aligned}$$

(F3) If $f_A \sqsubseteq \rho_B$, then

$$\vartheta_\varphi(\mathscr{F}_e^\pi)(f_A) = \mathscr{F}_e^\pi(\vartheta_\varphi^{-1}(f_A)) \leq \mathscr{F}_e^\pi(\vartheta_\varphi^{-1}(\rho_B)) = \vartheta_\varphi(\mathscr{F}_e^\pi)(\rho_B),$$

$$\vartheta_\varphi(\mathscr{F}_e^\alpha)(f_A) = \mathscr{F}_e^\alpha(\vartheta_\varphi^{-1}(f_A)) \geq \mathscr{F}_e^\alpha(\vartheta_\varphi^{-1}(\rho_B)) = \vartheta_\varphi(\mathscr{F}_e^\alpha)(\rho_B),$$

$$\vartheta_\varphi(\mathscr{F}_e^\sigma)(f_A) = \mathscr{F}_e^\sigma(\vartheta_\varphi^{-1}(f_A)) \geq \mathscr{F}_e^\sigma(\vartheta_\varphi^{-1}(\rho_B)) = \vartheta_\varphi(\mathscr{F}_e^\sigma)(\rho_B).$$

By proving the three conditions, we have proven the above theorem. □

Suppose that $\mathscr{F}_E^{\pi\alpha\sigma}$ and $\mathscr{F}_E^{\star\pi\alpha\sigma}$ are two *svn-soft filters* on £ and $\mathscr{X}$ correspondingly, and $\vartheta_\varphi : \widetilde{(£,\, E)} \to \widetilde{(\mathscr{X},\, F)}$ a mapping. Then, ϑ_φ is called *svn-soft filter map*.

Theorem 10 Let $\{(\mathscr{F}_j^{\pi\alpha\sigma})_E,\ j \in \Gamma\}$ be a family of svns-filters on £ satisfying the following condition:

(C) If $(f_A)_j \in ((\mathscr{F}_j^{\pi\alpha\sigma})_E)^\circ$ for each $j \in \Gamma$, then we obtain $\sqcap_{j\in\Gamma_0}(f_A)_j \neq \widetilde{\Phi}$ for evyry subset Γ_0 of Γ.

If we defined a mapping $\bigvee_{j\in\Gamma}\mathscr{F}_j^\pi,\ \bigwedge_{j\in\Gamma}\mathscr{F}_j^\alpha,\ \bigwedge_{j\in\Gamma}\mathscr{F}_j^\sigma : E \to I^{\widetilde{(£,\, E)}}$, as next:

$$\left(\bigvee_{j\in\Gamma}(\mathscr{F}_j^\pi)\right)_e(\rho_B) = \begin{cases} \bigvee\{\bigwedge_{j\in\Gamma_0}(\mathscr{F}_j^\pi)_e((\rho_B)_j) \mid \rho_B = \bigwedge_{j\in\Gamma_0}(\rho_B)_j\}, & \text{if } (\rho_B)_j \in ((\mathscr{F}_j^\pi)_E)^\circ, \\ 0, & \text{otherwise,} \end{cases}$$

$$\left(\bigwedge_{j\in\Gamma}(\mathscr{F}_j^\alpha)\right)_e(\rho_B) = \begin{cases} \bigwedge\{\bigvee_{j\in\Gamma_0}(\mathscr{F}_j^\alpha)_e((\rho_B)_j) : \rho_B = \bigwedge_{j\in\Gamma_0}(\rho_B)_j\}, & \text{if } (\rho_B)_j \in ((\mathscr{F}_j^\alpha)_E)^\circ, \\ 1, & \text{otherwise,} \end{cases}$$

$$\left(\bigwedge_{j\in\Gamma}(\mathscr{F}_j^\sigma)\right)_e(\rho_B) = \begin{cases} \bigwedge\{\bigvee_{j\in\Gamma_0}(\mathscr{F}_j^\sigma)_e((\rho_B)_j) : \rho_B = \bigwedge_{j\in\Gamma_0}(\rho_B)_j\}, & \text{if } (\rho_B)_j \in ((\mathscr{F}_j^\sigma)_E)^\circ, \\ 1, & \text{otherwise,} \end{cases}$$

where the supermom $\bigvee$ is taken for any finite index subset Γ_0 of Γ such that $\rho_B = \bigwedge_{j\in\Gamma_0}(\rho_B)_j$. Then $\bigvee_{j\in\Gamma}(\mathscr{F}_j^{\pi\alpha\sigma})_E$ is the coarsest svns-filter finer than $(\mathscr{F}_j^{\pi\alpha\sigma})_E$ for every $j \in \Gamma$.

Proof. Initially; we will show that

$$\mathscr{H}_e^\pi = \bigvee_{j\in\Gamma}(\mathscr{F}_j^\pi)_e,\ \mathscr{H}_e^\alpha = \bigwedge_{j\in\Gamma}(\mathscr{F}_j^\alpha)_e,\ \mathscr{H}_e^\sigma = \bigwedge_{j\in\Gamma}(\mathscr{F}_j^\sigma)_e,$$

is a *svn-soft filter* on £.

(F1) It's evident that, $\forall,\ e \in E$.

$$\mathscr{H}_e^\pi(\widetilde{\Phi}) = 0,\ \mathscr{H}_e^\alpha(\widetilde{\Phi}) = 1,\ \mathscr{H}_e^\sigma(\widetilde{\Phi}) = 1 and \mathscr{H}_e^\pi(\widetilde{E}) = 1,\ \mathscr{H}_e^\alpha(\widetilde{E}) = 0,\ \mathscr{H}_e^\sigma(\widetilde{E}) = 0.$$

(F2) For any two G and N finite index subsets of Γ such that

$$\pi_{f_A} = \bigwedge_{i \in G} (\pi_{f_A})_i,\ \alpha_{f_A} = \bigvee_{i \in G} (\alpha_{f_A})_i,\ \sigma_{f_A} = \bigvee_{i \in G} (\sigma_{f_A})_i,$$

$$\pi_{\rho_B} = \bigwedge_{n \in N} (\pi_{\rho_B})_n,\ \alpha_{\rho_B} = \bigvee_{n \in N} (\alpha_{\rho_B})_n,\ \sigma_{\rho_B} = \bigvee_{n \in N} (\sigma_{\rho_B})_n,$$

we have

$$f_A \sqcap \rho_B = [\bigwedge_{i \in G} (f_A)_i \sqcap \bigwedge_{n \in N} (\rho_B)_n]$$

Furthermore, for all $m \in G \cap N$, put $f_A \sqcap \rho_B = \bigwedge_{m \in G \cup N} (h_C)_m$, $C = A \cap B$, where

$$\pi_{(h_C)_m} = \begin{cases} \pi_{(f_A)_m}, & \text{if } n \in G - (G \cap N), \\ \pi_{(\rho_B)_m}, & \text{if } m \in N - (G \cap N), \\ \pi_{(f_A)_m} \cap \pi_{(\rho_B)_m}, & \text{if } m \in G \cap N, \end{cases}$$

$$\alpha_{(h_C)_m} = \begin{cases} \alpha_{(f_A)_m}, & \text{if } m \in G - (G \cap N), \\ \alpha_{(\rho_B)_m}, & \text{if } m \in N - (G \cap N), \\ \alpha_{(f_A)_m} \cup \alpha_{(\rho_B)_m}, & \text{if } m \in G \cap N, \end{cases}$$

$$\sigma_{(h_C)_m} = \begin{cases} \sigma_{(f_A)_m}, & \text{if } m \in G - (G \cap N), \\ \sigma_{(\rho_B)_m}, & \text{if } m \in N - (G \cap N), \\ \sigma_{(f_A)_m} \cup \sigma_{(\rho_B)_m}, & \text{if } m \in G \cap N, \end{cases}$$

which means that

$$\mathscr{H}_e^{\pi}(f_A \sqcap \rho_B) \geq \bigwedge_{m \in G \cap N} (\mathscr{F}_e^{\pi})_m (h_C)_m \geq \bigwedge_{i \in G} (\mathscr{F}_e^{\pi})_i (f_A)_i \wedge \bigwedge_{n \in N} (\mathscr{F}_e^{\pi})_n (\rho_B)_n,$$

$$\mathscr{H}_e^{\alpha}(f_A \sqcap \rho_B) \leq \bigvee_{m \in G \cap N} (\mathscr{F}_e^{\alpha})_m (h_C)_m \leq \bigvee_{i \in G} (\mathscr{F}_e^{\alpha})_i (f_A)_i \vee \bigvee_{n \in N} (\mathscr{F}_e^{\alpha})_n (\rho_B)_n,$$

$$\mathscr{H}_e^{\sigma}(f_A \sqcap \rho_B) \leq \bigvee_{m \in G \cap N} (\mathscr{F}_e^{\sigma})_m (h_C)_m \leq \bigvee_{i \in G} (\mathscr{F}_e^{\sigma})_i (f_A)_i \vee \bigvee_{n \in N} (\mathscr{F}_e^{\sigma})_n (\rho_B)_n.$$

Therefore,

$$\mathscr{H}_e^{\pi}(f_A \sqcap \rho_B) \geq \mathscr{H}_e^{\pi}(f_A) \wedge \mathscr{H}_e^{\pi}(\rho_B),\ \mathscr{H}_e^{\alpha}(f_A \sqcap \rho_B) \leq \mathscr{H}_e^{\alpha}(f_A) \vee \mathscr{H}_e^{\alpha}(\rho_B),$$

$$\mathscr{H}_e^{\sigma}(f_A \sqcap \rho_B) \leq \mathscr{H}_e^{\sigma}(f_A) \vee \mathscr{H}_e^{\sigma}(\rho_B).$$

(F3) Take that $f_A \sqsubseteq \rho_B$, according the definition of $\mathscr{H}$, there exists a finite index set G with

$$\pi_{f_A} = \bigwedge_{i\in G} (\pi_{f_A})_i,\ \alpha_{f_A} = \bigvee_{i\in G} (\alpha_{f_A})_i,\ \sigma_{f_A} = \bigvee_{i\in G} (\sigma_{f_A})_i,$$

therefore

$$\mathscr{H}_e^{\pi}(f_A) \geq \bigwedge_{i\in G} (\mathscr{F}_e^{\pi})_i((f_A)_i),\ \mathscr{H}_e^{\alpha}(f_A) \leq \bigvee_{i\in G} (\mathscr{F}_e^{\alpha})_i((f_A)_i),\ \mathscr{H}_e^{\sigma}(f_A) \leq \bigvee_{i\in G} (\mathscr{F}_e^{\sigma})_i((f_A)_i).$$

On the contrary, since $\rho_B = f_A \sqcup \rho_B = \bigwedge_{i\in G} ((f_A)_i \sqcup \rho_B)$, then

$$\mathscr{H}_e^{\pi}(\rho_B) \geq \bigwedge_{i\in G} (\mathscr{F}_e^{\pi})_i((f_A)_i \sqcup \rho_B) \geq \bigwedge_{i\in G} (\mathscr{F}_e^{\pi})_i((f_A)_i),$$

$$\mathscr{H}_e^{\alpha}(\rho_B) \leq \bigvee_{i\in G} (\mathscr{F}_e^{\alpha})_i((f_A)_i \sqcap \rho_B) \leq \bigvee_{i\in G} (\mathscr{F}_e^{\alpha})_i((f_A)_i),$$

$$\mathscr{H}_e^{\sigma}(\rho_B) \leq \bigvee_{i\in G} (\mathscr{F}_e^{\sigma})_i((f_A)_i \sqcap \rho_B) \leq \bigvee_{i\in G} (\mathscr{F}_e^{\sigma})_i((f_A)_i).$$

Hence, $\mathscr{H}_e^{\pi}(\rho_B) \geq \mathscr{H}_e^{\pi}(f_A)$, $\mathscr{H}_e^{\alpha}(f_A) \leq \mathscr{H}_e^{\alpha}(f_A)$ and $\mathscr{H}_e^{\sigma}(\rho_B) \leq \mathscr{H}_e^{\sigma}(f_A)$. Now, we will show that

$$\mathscr{H}_e^{\pi}(f_A) \geq (\mathscr{F}_e^{\pi})_j(f_A),\ \mathscr{H}_e^{\alpha}(f_A) \leq (\mathscr{F}_e^{\alpha})_j(f_A),\ \mathscr{H}_e^{\sigma}(f_A) \leq (\mathscr{F}_e^{\sigma})_j(f_A),$$

for each $j \in \Gamma$ from the next:

If $(\mathscr{F}_e^{\pi})_j(f_A) = 0$, $(\mathscr{F}_e^{\alpha})_j(f_A) = 1$, $(\mathscr{F}_e^{\sigma})_j(f_A) = 1$, then it is trivial.

If $(\mathscr{F}_e^{\pi})_j(f_A) > 0$, $(\mathscr{F}_e^{\alpha})_j(f_A) < 1$, $(\mathscr{F}_e^{\sigma})_j(f_A) < 1$, then for $f_A = f_A \sqcap \widetilde{E}$, we obtain

$$\mathscr{H}_e^{\pi}(f_A) \geq (\mathscr{F}_e^{\pi})_j(f_A) \wedge (\mathscr{F}_e^{\pi})_j(\widetilde{E}) = (\mathscr{F}_e^{\pi})_j(f_A),$$

$$\mathscr{H}_e^{\alpha}(f_A) \leq (\mathscr{F}_e^{\alpha})_j(f_A) \vee (\mathscr{F}_e^{\alpha})_j(\widetilde{E}) = (\mathscr{F}_e^{\alpha})_j(f_A),$$

$$\mathscr{H}_e^{\sigma}(f_A) \leq (\mathscr{F}_e^{\sigma})_j(f_A) \vee (\mathscr{F}_e^{\sigma})_j(\widetilde{E}) = (\mathscr{F}_e^{\sigma})_j(f_A).$$

If $\mathscr{G}_E^{\pi\alpha\sigma} \sqsupseteq (\mathscr{F}_j^{\pi\alpha\sigma})_E$ for each $j \in \Gamma$, we will show that $\mathscr{G}_E^{\pi\alpha\sigma} \sqsupseteq \mathscr{H}_E^{\pi\alpha\sigma}$. By the definition of $\mathscr{H}$, there exists a finite index set G with $f_A = \bigwedge_{i\in G}(f_A)_i$ so that

$$\mathscr{H}_e^{\pi}(f_A) \geq \bigwedge_{i\in G}(\mathscr{F}_e^{\pi})_i((f_A)_i),\ \mathscr{H}_e^{\alpha}(f_A) \leq \bigvee_{i\in G}(\mathscr{F}_e^{\alpha})_i((f_A)_i),\ \mathscr{H}_e^{\sigma}(f_A) \leq \bigvee_{i\in G}(\mathscr{F}_e^{\sigma})_i((f_A)_i).$$

On the contrary, since $\mathscr{G}_E^{\pi\alpha\sigma} \sqsupseteq (\mathscr{F}_i^{\pi\alpha\sigma})_E$ for each $i \in G$, then we have

$$\mathscr{G}_e^{\pi}(f_A) \geq \bigwedge_{i\in G}\mathscr{G}_e^{\pi}((f_A)_i) \geq \bigwedge_{i\in G}(\mathscr{F}_e^{\pi})_i((f_A)_i),$$

$$\mathscr{G}_e^{\alpha}(f_A) \leq \bigvee_{i\in G}\mathscr{G}_e^{\alpha}((f_A)_i) \leq \bigvee_{i\in G}(\mathscr{F}_e^{\alpha})_i((f_A)_i),$$

$$\mathscr{G}_e^{\sigma}(f_A) \leq \bigvee_{i\in G}\mathscr{G}_e^{\sigma}((f_A)_i) \leq \bigvee_{i\in G}(\mathscr{F}_e^{\sigma})_i((f_A)_i).$$

Thus, $\mathscr{G}_e^{\pi}(f_A) \geq \mathscr{H}_e^{\pi}(f_A)$, $\mathscr{G}_e^{\alpha}(f_A) \leq \mathscr{H}_e^{\alpha}(f_A)$ and $\mathscr{G}_e^{\sigma}(f_A) \leq \mathscr{H}_e^{\sigma}(f_A)$. □

Definition 8 A single-valued neutrosophic soft quasi-coincident neighborhood system (*svnsqc-system*) on £ is a set $\mathsf{Q}^{\pi\alpha\sigma} = \{\mathsf{Q}^{\pi\alpha\sigma}_{e_x^{t,\,s,\,k}} : e_x^{t,\,s,\,k} \in P_{t,\,s,\,k}\widetilde{(£,\,E)}\}$ of maps $\mathsf{Q}^{\pi}_{e_x^{t,\,s,\,k}},\ \mathsf{Q}^{\alpha}_{e_x^{t,\,s,\,k}},\ \mathsf{Q}^{\sigma}_{e_x^{t,\,s,\,k}} : E \to I^{\widetilde{(£,\,E)}}$ such that $\forall\ f_A,\ \rho_B \in \widetilde{(£,\,E)}$,

(N1) $(\mathsf{Q}^{\pi}_{e_x^{t,\,s,\,k}},\ \mathsf{Q}^{\alpha}_{e_x^{t,\,s,\,k}},\ \mathsf{Q}^{\sigma}_{e_x^{t,\,s,\,k}})$ is svns-filter on £,

(N2) $(\mathsf{Q}^{\pi}_{e_x^{t,\,s,\,k}})_e(f_A) > 0$, $(\mathsf{Q}^{\alpha}_{e_x^{t,\,s,\,k}})_e(f_A) < 1$, $(\mathsf{Q}^{\sigma}_{e_x^{t,\,s,\,k}})_e(f_A) < 1$ implies $e_x^{t,\,s,\,k}\ q\ f_A$,

(N3)

$$(\mathsf{Q}^{\pi}_{e_x^{t,\,s,\,k}})_e(f_A) = \bigvee_{e_x^{t,\,s,\,k}\ q\ \rho_B,\ \rho_B \sqsubseteq f_A}\left[\bigwedge_{e_y^{t,\,s,\,k}\ q\ \rho_B}(\mathsf{Q}^{\pi}_{e_y^{t,\,s,\,k}})_e(\rho_B)\right],$$

$$(\mathsf{Q}^{\alpha}_{e_x^{t,\,s,\,k}})_e(f_A) = \bigwedge_{e_x^{t,\,s,\,k}\ q\ \rho_B,\ \rho_B \sqsubseteq f_A}\left[\bigvee_{e_y^{t,\,s,\,k}\ q\ \rho_B}(\mathsf{Q}^{\alpha}_{e_y^{t,\,s,\,k}})_e(\rho_B)\right],$$

$$(\mathsf{Q}^{\sigma}_{e_x^{t,\,s,\,k}})_e(f_A) = \bigwedge_{e_x^{t,\,s,\,k}\ q\ \rho_B,\ \rho_B \sqsubseteq f_A}\left[\bigvee_{e_y^{t,\,s,\,k}\ q\ \rho_B}(\mathsf{Q}^{\sigma}_{e_y^{t,\,s,\,k}})_e(\rho_B)\right].$$

The quadruple $(£,\ \mathsf{Q}^{\pi},\ \mathsf{Q}^{\alpha},\ \mathsf{Q}^{\sigma})$ is said to be *svnsqcn-Spaces*.

$\left[(\mathsf{Q}^{\pi}_{e_x^{t,\,s,\,k}})_e(f_A),\ (\mathsf{Q}^{\alpha}_{e_x^{t,\,s,\,k}})_e(f_A),\ (\mathsf{Q}^{\sigma}_{e_x^{t,\,s,\,k}})_e(f_A)\right]$ can be interpreted as the degree to which f_A is a *svnsqcn* of $e_x^{t,\,s,\,k}$. we will write $\mathsf{Q}^{\pi\alpha\sigma}$ for $(\mathsf{Q}^{\pi},\ \mathsf{Q}^{\alpha},\ \mathsf{Q}^{\sigma})$, and it will be no ambiguity.

An *N-map* between *svnsqcn- spaces* $(£,\ \mathsf{Q}_E^{\pi\alpha\sigma})$ and $(\chi,\ \mathsf{Q}_F^{*\pi\alpha\sigma})$ is a map $\vartheta_\varphi : (£,\ \mathsf{Q}_E^{\pi\alpha\sigma}) \to (\chi,\ \mathsf{Q}_F^{*\pi\alpha\sigma})$ such that

$$(\mathsf{Q}^{\pi}_{e_x^{t,\,s,\,k}})_e(\vartheta_\varphi^{-1}(f_A)) \geq (\mathsf{Q}^{*\pi}_{\varphi(e)^{t,\,s,\,k}_{\vartheta(x)}})_{\varphi(e)}(f_A),$$

$$(\mathsf{Q}^{\alpha}_{e_x^{t,\,s,\,k}})_e(\vartheta_\varphi^{-1}(f_A)) \leq (\mathsf{Q}^{*\alpha}_{\varphi(e)^{t,\,s,\,k}_{\vartheta(x)}})_{\varphi(e)}(f_A),$$

$$(\mathsf{Q}^{\sigma}_{e_x^{t,\,s,\,k}})_e(\vartheta_\varphi^{-1}(f_A)) \leq (\mathsf{Q}^{*\sigma}_{\varphi(e)^{t,\,s,\,k}_{\vartheta(x)}})_{\varphi(e)}(f_A),$$

for all $f_A \in \widetilde{(\chi,\, F)}$, $e \in E$ and for all $e_x^{t,\,s,\,k} \in P_{t,\,s,\,k}\widetilde{(\pounds,\, E)}$.

Theorem 11 Let $(\pounds,\, \mathscr{T}^\pi,\, \mathscr{T}^\alpha,\, \mathscr{T}^\sigma)$ be a *svnst-space* and $e_x^{t,\,s,\,k} \in P_{t,\,s,\,k}\widetilde{(\pounds,\, E)}$. Define a map $\mathsf{Q}^{\pi}_{e_x^{t,\,s,\,k}},\, \mathsf{Q}^{\alpha}_{e_x^{t,\,s,\,k}},\, \mathsf{Q}^{\sigma}_{e_x^{t,\,s,\,k}} : E \to I^{\widetilde{(\pounds,\, E)}}$ as:

$$(\mathsf{Q}^{\mathscr{T}^\pi}_{e_x^{t,\,s,\,k}})_e(f_A) = \begin{cases} \bigvee\{\mathscr{T}^\pi_e(\rho_B) : e_x^{t,\,s,\,k}\, q\, \rho_B,\ \rho_B \sqsubseteq f_A\}, & \text{if } e_x^{t,\,s,\,k}\, q\, f_A, \\ 0, & \text{if } e_x^{t,\,s,\,k}\, \widetilde{q}\, f_A, \end{cases}$$

$$(\mathsf{Q}^{\mathscr{T}^\alpha}_{e_x^{t,\,s,\,k}})_e(f_A) = \begin{cases} \bigwedge\{\mathscr{T}^\alpha_e(\rho_B) : e_x^{t,\,s,\,k}\, q\, \rho_B,\ \rho_B \sqsubseteq f_A\}, & \text{if } e_x^{t,\,s,\,k}\, q\, f_A, \\ 1, & \text{if } e_x^{t,\,s,\,k}\, \widetilde{q}\, f_A, \end{cases}$$

$$(\mathsf{Q}^{\mathscr{T}^\sigma}_{e_x^{t,\,s,\,k}})_e(f_A) = \begin{cases} \bigwedge\{\mathscr{T}^\sigma_e(\rho_B) : e_x^{t,\,s,\,k}\, q\, \rho_B,\ \rho_B \sqsubseteq f_A\}, & \text{if } e_x^{t,\,s,\,k}\, q\, f_A, \\ 1, & \text{if } e_x^{t,\,s,\,k}\, \widetilde{q}\, f_A. \end{cases}$$

Then,

(1) $\mathsf{Q}^{\mathscr{T}^{\pi\alpha\sigma}} = \{\mathsf{Q}^{\mathscr{T}^{\pi\alpha\sigma}}_{e_x^{t,\,s,\,k}} : e_x^{t,\,s,\,k} \in P_{t,\,s,\,k}\widetilde{(\pounds,\, E)}\}$ is a svn-soft quasi-coincident neighborhood system on £,

(2) If $t < t'$, $s > s'$ and $k > k'$ for $t,\ s,\ k,\ t',\ s',\ k' \in I$, then

$$(\mathsf{Q}^{\mathscr{T}^\pi}_{e_x^{t,\,s,\,k}})_e(f_A) \leq (\mathsf{Q}^{\mathscr{T}^\pi}_{e_x^{t',\,s',\,k'}})_e(f_A),\ (\mathsf{Q}^{\mathscr{T}^\alpha}_{e_x^{t,\,s,\,k}})_e(f_A) \geq (\mathsf{Q}^{\mathscr{T}^\alpha}_{e_x^{t',\,s',\,k'}})_e(f_A),\ (\mathsf{Q}^{\mathscr{T}^\sigma}_{e_x^{t,\,s,\,k}})_e(f_A) \geq (\mathsf{Q}^{\mathscr{T}^\sigma}_{e_x^{t',\,s',\,k'}})_e(f_A).$$

Proof. To establish (1), it is necessary to verify all conditions (N1) through (N3).
We begin by confirming (N1). Conditions (F1) and (F3) are straightforward and can be verified easily.
For (F2), assume that there exist $f_A,\ \rho_B \in \widetilde{(\pounds,\, E)}$ such that:

$$(\mathsf{Q}^{\mathscr{T}^\pi}_{e_x^{t,\,s,\,k}})_e(f_A \sqcap \rho_B) \ngeq (\mathsf{Q}^{\mathscr{T}^\pi}_{e_x^{t,\,s,\,k}})_e(f_A) \wedge (\mathsf{Q}^{\mathscr{T}^\pi}_{e_x^{t,\,s,\,k}})_e(\rho_B),$$

$$(\mathsf{Q}^{\mathscr{T}^\alpha}_{e_x^{t,\,s,\,k}})_e(f_A \sqcap \rho_B) \nleq (\mathsf{Q}^{\mathscr{T}^\alpha}_{e_x^{t,\,s,\,k}})_e(f_A) \vee (\mathsf{Q}^{\mathscr{T}^\alpha}_{e_x^{t,\,s,\,k}})_e(\rho_B),$$

$$(\mathsf{Q}^{\mathscr{T}^\sigma}_{e_x^{t,\,s,\,k}})_e(f_A \sqcap \rho_B) \nleq (\mathsf{Q}^{\mathscr{T}^\sigma}_{e_x^{t,\,s,\,k}})_e(f_A) \vee (\mathsf{Q}^{\mathscr{T}^\sigma}_{e_x^{t,\,s,\,k}})_e(\rho_B).$$

According to the definition of $(\mathsf{Q}^{\mathscr{T}^{\pi\alpha\alpha}}_{e_x^{t,\,s,\,k}})_e(f_A)$, there is an $(f_A)_1 \in \widetilde{(£,\,E)}$ with $e_x^{t,\,s,\,k}\; q\; (f_A)_1$, $(f_A)_1 \sqsubseteq f_A$ such that

$$(\mathsf{Q}^{\mathscr{T}^{\pi}}_{e_x^{t,\,s,\,k}})_e(f_A \sqcap \rho_B) \ngeq \mathscr{T}_e^{\pi}((f_A)_1) \wedge (\mathsf{Q}^{\mathscr{T}^{\pi}}_{e_x^{t,\,s,\,k}})_e(\rho_B),$$

$$(\mathsf{Q}^{\mathscr{T}^{\alpha}}_{e_x^{t,\,s,\,k}})_e(f_A \sqcap \rho_B) \nleq \mathscr{T}_e^{\alpha}((f_A)_1) \vee (\mathsf{Q}^{\mathscr{T}^{\alpha}}_{e_x^{t,\,s,\,k}})_e(\rho_B),$$

$$(\mathsf{Q}^{\mathscr{T}^{\sigma}}_{e_x^{t,\,s,\,k}})_e(f_A \sqcap \rho_B) \nleq \mathscr{T}_e^{\sigma}((f_A)_1) \vee (\mathsf{Q}^{\mathscr{T}^{\sigma}}_{e_x^{t,\,s,\,k}})_e(\rho_B).$$

Once more, according to the definition of $(\mathsf{Q}^{\mathscr{T}^{\pi\alpha\alpha}}_{e_x^{t,\,s,\,k}})_e(\rho_B)$, there exists $(\rho_B)_1 \in \widetilde{(£,\,E)}$ with $e_x^{t,\,s,\,k}\; q\; (\rho_B)_1$, $(\rho_B)_1 \sqsubseteq \rho_B$ such that

$$(\mathsf{Q}^{\mathscr{T}^{\pi}}_{e_x^{t,\,s,\,k}})_e(f_A \sqcap \rho_B) \ngeq \mathscr{T}_e^{\pi}((f_A)_1) \wedge \mathscr{T}_e^{\pi}((\rho_B)_1),\; (\mathsf{Q}^{\mathscr{T}^{\alpha}}_{e_x^{t,\,s,\,k}})_e(f_A \sqcap \rho_B) \nleq \mathscr{T}_e^{\alpha}((f_A)_1) \vee \mathscr{T}_e^{\alpha}((\rho_B)_1),$$

$$(\mathsf{Q}^{\mathscr{T}^{\sigma}}_{e_x^{t,\,s,\,k}})_e(f_A \sqcap \rho_B) \nleq \mathscr{T}_e^{\sigma}((f_A)_1) \vee \mathscr{T}_e^{\sigma}((\rho_B)_1).$$

Since $e_x^{t,\,s,\,k}\; q\; [(\rho_B)_1 \sqcap (f_A)_1]$, $[(\rho_B)_1 \sqcap (f_A)_1] \sqsubseteq [\rho_B \sqcap f_A]$, we have

$$(\mathsf{Q}^{\mathscr{T}^{\pi}}_{e_x^{t,\,s,\,k}})_e(f_A \sqcap \rho_B) \geq \mathscr{T}_e^{\pi}((f_A)_1 \sqcap (\rho_B)_1) \geq \mathscr{T}_e^{\pi}((f_A)_1) \wedge \mathscr{T}_e^{\pi}((\rho_B)_1),$$

$$(\mathsf{Q}^{\mathscr{T}^{\alpha}}_{e_x^{t,\,s,\,k}})_e(f_A \sqcap \rho_B) \leq \mathscr{T}_e^{\alpha}((f_A)_1 \sqcap (\rho_B)_1) \leq \mathscr{T}_e^{\alpha}((f_A)_1) \vee \mathscr{T}_e^{\alpha}((\rho_B)_1),$$

$$(\mathsf{Q}^{\mathscr{T}^{\sigma}}_{e_x^{t,\,s,\,k}})_e(f_A \sqcap \rho_B) \leq \mathscr{T}_e^{\sigma}((f_A)_1 \sqcap (\rho_B)_1) \leq \mathscr{T}_e^{\sigma}((f_A)_1) \vee \mathscr{T}_e^{\sigma}((\rho_B)_1).$$

This leads to a contradiction. Therefore, (F2) is valid and consequently, $\mathsf{Q}^{\pi\alpha\sigma}_{e_x^{t,\,s,\,k}}$ is *svns-filter* on £.
Regarding (N2), it follows easily from the definition.
Now concerning (N3), for every $f_A \in \widetilde{(£,\,E)}$ with $e_x^{t,\,s,\,k}\; q\; \rho_B,\; \rho_B \sqsubseteq f_A$, we have

$$\mathscr{T}_e^{\pi}(\rho_B) \leq \bigwedge\{\mathsf{Q}^{\pi}_{e_y^{t',\,s',\,k'}}(\rho_B) \mid e_y^{t',\,s',\,k'}\; q\; \rho_B\} \leq (\mathsf{Q}^{\mathscr{T}^{\pi}}_{e_x^{t,\,s,\,k}})_e(\rho_B) \leq (\mathsf{Q}^{\mathscr{T}^{\pi}}_{e_x^{t,\,s,\,k}})_e(f_A),$$

$$\mathscr{T}_e^{\alpha}(\rho_B) \geq \bigvee\{\mathsf{Q}^{\alpha}_{e_y^{t',\,s',\,k'}}(\rho_B) \mid e_y^{t',\,s',\,k'} q\; \rho_B\} \geq (\mathsf{Q}^{\mathscr{T}^{\alpha}}_{e_x^{t,\,s,\,k}})_e(\rho_B) \geq (\mathsf{Q}^{\mathscr{T}^{\alpha}}_{e_x^{t,\,s,\,k}})_e(f_A),$$

$$\mathscr{T}_e^{\sigma}(\rho_B) \geq \bigvee\{\mathsf{Q}^{\sigma}_{e_y^{t',\,s',\,k'}}(\rho_B) \mid e_y^{t',\,s',\,k'} q\; \rho_B\} \geq (\mathsf{Q}^{\mathscr{T}^{\sigma}}_{e_x^{t,\,s,\,k}})_e(\rho_B) \geq (\mathsf{Q}^{\mathscr{T}^{\sigma}}_{e_x^{t,\,s,\,k}})_e(f_A).$$

Therefore,

$$(\mathsf{Q}^{\mathscr{T}^{\pi}}_{e_x^{t,\,s,\,k}})_e(f_A) = \bigvee_{e_x^{t,\,s,\,k}\,q\,\rho_B,\ \rho_B \sqsubseteq f_A} \mathscr{T}_e^{\pi}(\rho_B)$$

$$\leq \bigvee_{e_x^{t,\,s,\,k}\,q\,\rho_B,\ \rho_B \sqsubseteq f_A} \left[\bigwedge_{e_y^{t',\,s',\,k'}\,q\,\rho_B} (\mathsf{Q}^{\mathscr{T}^{\pi}}_{e_y^{t',\,s',\,k'}})_e(\rho_B) \right] \leq (\mathsf{Q}^{\mathscr{T}^{\pi}}_{e_x^{t,\,s,\,k}})_e(f_A),$$

$$(\mathsf{Q}^{\mathscr{T}^{\alpha}}_{e_x^{t,\,s,\,k}})_e(f_A) = \bigwedge_{e_x^{t,\,s,\,k}\,q\,\rho_B,\ \rho_B \sqsubseteq f_A} \mathscr{T}_e^{\alpha}(\rho_B)$$

$$\geq \bigwedge_{e_x^{t,\,s,\,k}\,q\,\rho_B,\ \rho_B \sqsubseteq f_A} \left[\bigvee_{e_y^{t',\,s',\,k'}\,q\,\rho_B} (\mathsf{Q}^{\mathscr{T}^{\alpha}}_{e_y^{t',\,s',\,k'}})_e(\rho_B) \right] \geq (\mathsf{Q}^{\mathscr{T}^{\alpha}}_{e_x^{t,\,s,\,k}})_e(f_A),$$

$$(\mathsf{Q}^{\mathscr{T}^{\sigma}}_{e_x^{t,\,s,\,k}})_e(f_A) = \bigwedge_{e_x^{t,\,s,\,k}\,q\,\rho_B,\ \rho_B \sqsubseteq f_A} \mathscr{T}_e^{\sigma}(\rho_B)$$

$$\geq \bigwedge_{e_x^{t,\,s,\,k}\,q\,\rho_B,\ \rho_B \sqsubseteq f_A} \left[\bigvee_{e_y^{t',\,s',\,k'}\,q\,\rho_B} (\mathsf{Q}^{\mathscr{T}^{\sigma}}_{e_y^{t',\,s',\,k'}})_e(\rho_B) \right] \geq (\mathsf{Q}^{\mathscr{T}^{\sigma}}_{e_x^{t,\,s,\,k}})_e(f_A).$$

This means that

$$(\mathsf{Q}^{\mathscr{T}^{\pi}}_{e_x^{t,\,s,\,k}})_e(f_A) = \bigvee_{e_x^{t,\,s,\,k}\,q\,\rho_B,\ \rho_B \sqsubseteq f_A} \left[\bigwedge_{e_y^{t',\,s',\,k'}\,q\,\rho_B} (\mathsf{Q}^{\mathscr{T}^{\pi}}_{e_y^{t',\,s',\,k'}})_e(\rho_B) \right],$$

$$(\mathsf{Q}^{\mathscr{T}^{\alpha}}_{e_x^{t,\,s,\,k}})_e(f_A) = \bigwedge_{e_x^{t,\,s,\,k}\,q\,\rho_B,\ \rho_B \sqsubseteq f_A} \left[\bigvee_{e_y^{t',\,s',\,k'}\,q\,\rho_B} (\mathsf{Q}^{\mathscr{T}^{\alpha}}_{e_y^{t',\,s',\,k'}})_e(\rho_B) \right],$$

$$(\mathsf{Q}^{\mathscr{T}^{\sigma}}_{e_x^{t,\,s,\,k}})_e(f_A) = \bigwedge_{e_x^{t,\,s,\,k}\,q\,\rho_B,\ \rho_B \sqsubseteq f_A} \left[\bigvee_{e_y^{t',\,s',\,k'}\,q\,\rho_B} (\mathsf{Q}^{\mathscr{T}^{\sigma}}_{e_y^{t',\,s',\,k'}})_e(\rho_B) \right].$$

Hence, (1) is fulfilled.

Now (2), for $t < t'$, $s > s'$ and $k > k'$ with $t,\ s,\ k,\ t',\ s',\ k' \in I$, $f_A \in \widetilde{(£,\ E)}$, since

$$\{\rho_B \in \widetilde{(£,\ E)} \mid e_x^{t,\,s,\,k} q\,\rho_B,\ \rho_B \sqsubseteq f_A\} \sqsubseteq \{h_C \in \widetilde{(£,\ E)} : e_x^{t,\,s,\,k}\,q\,h_C,\ h_C \sqsubseteq f_A\},$$

then we have

$$(\mathsf{Q}^{\mathscr{T}v^{\pi}}_{e_x^{t,\,s,\,k}})_e(f_A) \leq (\mathsf{Q}^{\mathscr{T}^{\pi}}_{e_x^{t',\,s',\,k'}})_e(f_A),\ (\mathsf{Q}^{\mathscr{T}^{\alpha}}_{e_x^{t,\,s,\,k}})_e(f_A) \geq (\mathsf{Q}^{\mathscr{T}^{\alpha}}_{e_x^{t',\,s',\,k'}})_e(f_A),$$

$$(\mathsf{Q}^{\mathscr{T}^{\sigma}}_{e_x^{t,\,s,\,k}})_e(f_A) \geq (\mathsf{Q}^{\mathscr{T}^{\sigma}}_{e_x^{t',\,s',\,k'}})_e(f_A).$$

□

Theorem 12 Consider $\mathsf{Q}^{\pi\alpha\sigma} = \{\mathsf{Q}^{\pi\alpha\sigma}_{e_x^{t,\,s,\,k}} \mid e_x^{t,\,s,\,k} \in P_{t,\,s,\,k}\widetilde{(£,\,E)}\}$ as a family of $\mathsf{Q}^{\pi}_{e_x^{t,\,s,\,k}},\ \mathsf{Q}^{\alpha}_{e_x^{t,\,s,\,k}},\ \mathsf{Q}^{\sigma}_{e_x^{t,\,s,\,k}} : E \to I^{\widetilde{(£,\,E)}}$ fulfilling (N1) and (N2) of Definition 7. We define a map $\mathscr{T}^{\pi},\ \mathscr{T}^{\alpha},\ \mathscr{T}^{\sigma} : E \to I^{\widetilde{(£,\,E)}}$ as follows:

$$(\mathscr{T}^{\pi}_{\mathsf{Q}})_e(f_A) = \begin{cases} \bigwedge\{(\mathsf{Q}^{\pi}_{e_x^{t,\,s,\,k}})_e(f_A) \mid e_x^{t,\,s,\,k} q\ f_A\}, & \text{if } f_A \neq \widetilde{\Phi}, \\ 1, & \text{if } f_A = \widetilde{\Phi}, \end{cases}$$

$$(\mathscr{T}^{\alpha}_{\mathsf{Q}})_e(f_A) = \begin{cases} \bigvee\{(\mathsf{Q}^{\alpha}_{e_x^{t,\,s,\,k}})_e(f_A) \mid e_x^{t,\,s,\,k} q\ f_A\}, & \text{if } f_A \neq \widetilde{\Phi}, \\ 0, & \text{if } f_A = \widetilde{\Phi}, \end{cases}$$

$$(\mathscr{T}^{\sigma}_{\mathsf{Q}})_e(f_A) = \begin{cases} \bigvee\{(\mathsf{Q}^{\sigma}_{e_x^{t,\,s,\,k}})_e(f_A) \mid e_x^{t,\,s,\,k} q\ f_A\}, & \text{if } f_A \neq \widetilde{\Phi}, \\ 0, & \text{if } f_A = \widetilde{\Phi}. \end{cases}$$

Thus, the following properties hold:

(1) $(\mathscr{T}^{\pi\alpha\sigma}_{\mathsf{Q}})_E$ is a svnst on £, (2) If $\mathsf{Q}^{\pi\sigma\sigma}_E$ is a svnsqcn-system on £, then $\mathsf{Q}^{\mathscr{T}^{\pi\alpha\sigma}_{\mathsf{Q}}}_{e_x^{t,\,s,\,k}} = \mathsf{Q}^{\pi\alpha\sigma}_{e_x^{t,\,s,\,k}}$ for all $e_x^{t,\,s,\,k} \in P_{t,\,s,\,k}\widetilde{(£,\,E)}$, (3) If $\mathsf{Q}^{\pi\sigma\sigma}_E$ and $\mathsf{Q}^{*\pi\sigma\sigma}_E$ are svnsqcn-systems on £ such that $(\mathscr{T}^{\pi\alpha\sigma}_{\mathsf{Q}})_E = (\mathscr{T}^{*\pi\alpha\sigma}_{\mathsf{Q}})_E$ then $\mathsf{Q}^{\pi\sigma\sigma}_E = \mathsf{Q}^{*\pi\sigma\sigma}_E$.

Proof. In order to demonstrate (1), we must establish all conditions ($\mathscr{T}_1$)-($\mathscr{T}_3$).

($\mathscr{T}_1$) It's simple and therefore, not included.

($\mathscr{T}_2$) For $f_A,\ \rho_B \in \widetilde{(£,\,E)}$ we have

$$\begin{aligned}
(\mathscr{T}^{\pi}_{\mathsf{Q}})_e(f_A \sqcap \rho_B) =& \bigwedge\{(\mathsf{Q}^{\pi}_{e_x^{t,\,s,\,k}})_e(f_A \sqcap \rho_B) \mid e_x^{t,\,s,\,k} q\ (f_A \sqcap \rho_B)\} \\
\geq& \bigwedge\{(\mathsf{Q}^{\pi}_{e_x^{t,\,s,\,k}})_e(f_A) \wedge (\mathsf{Q}^{\pi}_{e_x^{t,\,s,\,k}})_e(\rho_B) : e_x^{t,\,s,\,k}\ q\ (f_A \sqcap \rho_B)\} \\
=& \left[\bigwedge\{(\mathsf{Q}^{\pi}_{e_x^{t,\,s,\,k}})_e(f_A) \mid e_x^{t,\,s,\,k} q\ (f_A \sqcap \rho_B)\}\right] \wedge \left[\bigwedge\{(\mathsf{Q}^{\pi}_{e_x^{t,\,s,\,k}})_e(\rho_B) \mid e_x^{t,\,s,\,k} q\ (f_A \sqcap \rho_B)\}\right] \\
\geq& \left[\bigwedge\{(\mathsf{Q}^{\pi}_{e_x^{t,\,s,\,k}})_e(f_A) \mid e_x^{t,\,s,\,k} q\ (f_A)\}\right] \wedge \left[\bigwedge\{(\mathsf{Q}^{\pi}_{e_x^{t,\,s,\,k}})_e(\rho_B) \mid e_x^{t,\,s,\,k} q\ (\rho_B)\}\right] \\
=& (\mathscr{T}^{\pi}_{\mathsf{Q}})_e(f_A) \wedge (\mathscr{T}^{\pi}_{\mathsf{Q}})_e(\rho_B),
\end{aligned}$$

$$
\begin{aligned}
(\mathscr{T}^{\alpha}_{\mathsf{Q}})_e(f_A \sqcap \rho_B) &= \bigvee\{(\mathsf{Q}^{\alpha}_{e_x^{t,\, s,\, k}})_e(f_A \sqcap \rho_B) \mid e_x^{t,\, s,\, k} q\ (f_A \sqcap \rho_B)\}\\
&\leq \bigvee\{(\mathsf{Q}^{\alpha}_{e_x^{t,\, s,\, k}})_e(f_A) \vee (\mathsf{Q}^{\alpha}_{e_x^{t,\, s,\, k}})_e(\rho_B) \mid e_x^{t,\, s,\, k} q\ (f_A \sqcup \rho_B)\}\\
&= \left[\bigvee\{(\mathsf{Q}^{\alpha}_{e_x^{t,\, s,\, k}})_e(f_A) \mid e_x^{t,\, s,\, k} q\ (f_A \sqcup \rho_B)\}\right] \vee \left[\bigvee\{(\mathsf{Q}^{\alpha}_{e_x^{t,\, s,\, k}})_e(\rho_B) \mid e_x^{t,\, s,\, k} q\ (f_A \sqcup \rho_B)\}\right]\\
&\leq \left[\bigvee\{(\mathsf{Q}^{\alpha}_{e_x^{t,\, s,\, k}})_e(f_A) \mid e_x^{t,\, s,\, k} q\ (f_A)\}\right] \vee \left[\bigvee\{(\mathsf{Q}^{\alpha}_{e_x^{t,\, s,\, k}})_e(\rho_B) \mid e_x^{t,\, s,\, k} q\ (\rho_B)\}\right]\\
&= (\mathscr{T}^{\alpha}_{\mathsf{Q}})_e(f_A) \vee (\mathscr{T}^{\alpha}_{\mathsf{Q}})_e(\rho_B).
\end{aligned}
$$

$$
\begin{aligned}
(\mathscr{T}^{\sigma}_{\mathsf{Q}})_e(f_A \sqcap \rho_B) &= \bigvee\{(\mathsf{Q}^{\sigma}_{e_x^{t,\, s,\, k}})_e(f_A \sqcap \rho_B) \mid e_x^{t,\, s,\, k} q\ (f_A \sqcap \rho_B)\}\\
&\leq \bigvee\{(\mathsf{Q}^{\sigma}_{e_x^{t,\, s,\, k}})_e(f_A) \vee (\mathsf{Q}^{\sigma}_{e_x^{t,\, s,\, k}})_e(\rho_B) \mid e_x^{t,\, s,\, k} q\ (f_A \sqcup \rho_B)\}\\
&= \left[\bigvee\{(\mathsf{Q}^{\sigma}_{e_x^{t,\, s,\, k}})_e(f_A) \mid e_x^{t,\, s,\, k} q\ (f_A \sqcup \rho_B)\}\right] \vee \left[\bigvee\{(\mathsf{Q}^{\sigma}_{e_x^{t,\, s,\, k}})_e(\rho_B) \mid e_x^{t,\, s,\, k} q\ (f_A \sqcup \rho_B)\}\right]\\
&\leq \left[\bigvee\{(\mathsf{Q}^{\sigma}_{e_x^{t,\, s,\, k}})_e(f_A) \mid e_x^{t,\, s,\, k} q\ (f_A)\}\right] \vee \left[\bigvee\{(\mathsf{Q}^{\sigma}_{e_x^{t,\, s,\, k}})_e(\rho_B) \mid e_x^{t,\, s,\, k} q\ (\rho_B)\}\right]\\
&= (\mathscr{T}^{\sigma}_{\mathsf{Q}})_e(f_A) \vee (\mathscr{T}^{\sigma}_{\mathsf{Q}})_e(\rho_B).
\end{aligned}
$$

($\mathscr{T}_3$) Since $(\mathsf{Q}^{\pi\alpha\sigma}_{e_x^{t,\, s,\, k}})_e\left(\bigsqcup_{j\in\Gamma}(\rho_B)_j\right) \geq \bigwedge_{j\in\Gamma}(\mathsf{Q}^{\pi\alpha\sigma}_{e_x^{t,\, s,\, k}})_e((\rho_B)_{j)}$, then

$$
\begin{aligned}
(\mathscr{T}^{\pi}_{\mathsf{Q}})_e\left(\bigsqcup_{j\in\Gamma}(\rho_B)_j\right) &= \bigwedge\left\{(\mathsf{Q}^{\pi}_{e_x^{t,\, s,\, k}})_e\left(\bigsqcup_{j\in\Gamma}(\rho_B)_j\right) \mid e_x^{t,\, s,\, k} q\ \left(\bigsqcup_{j\in\Gamma}(\rho_B)_j\right)\right\}\\
&\geq \bigwedge\left\{\bigwedge_{j\in\Gamma}(\mathsf{Q}^{\pi}_{e_x^{t,\, s,\, k}})_e((\rho_B)_j) \mid e_x^{t,\, s,\, k} q\ ((\rho_B)_j)\right\}\\
&= \bigwedge_{j\in\Gamma}\left\{\bigwedge(\mathsf{Q}^{\pi}_{e_x^{t,\, s,\, k}})_e((\rho_B)_j) \mid e_x^{t,\, s,\, k} q\ ((\rho_B)_j)\right\}\\
&= \bigwedge_{j\in\Gamma}((\mathscr{T}^{\pi}_{\mathsf{Q}})_e((\rho_B)_j).
\end{aligned}
$$

Similarly, it can be obtained

$$(\mathscr{T}_{\mathsf{Q}}^{\alpha})_e\left(\bigsqcup_{j\in\Gamma}(\rho_B)_j\right)\leq\bigvee_{j\in\Gamma}((\mathscr{T}_{\mathsf{Q}}^{\alpha})_e((\rho_B)_j),\ (\mathscr{T}_{\mathsf{Q}}^{\sigma})_e\left(\bigsqcup_{j\in\Gamma}(\rho_B)_j\right)\leq\bigvee_{j\in\Gamma}((\mathscr{T}_{\mathsf{Q}}^{\sigma})_e((\rho_B)_j).$$

Thus, $(\mathscr{T}_{\mathsf{Q}}^{\pi\alpha\sigma})_E$ is a *svnst* on £.

For (2), it is proved by (N3) and theorem 5, so that

$$\left(\mathsf{Q}_{e_x^{t,\ s,\ k}}^{\mathscr{T}_{\mathsf{Q}}^{\pi}}\right)_e(f_A)=\bigvee_{e_x^{t,\ s,\ k}q\,\rho_B,\ \rho_B\sqsubseteq f_A}(\mathscr{T}_{\mathsf{Q}}^{\pi})_e(\rho_B)=\bigvee_{e_x^{t,\ s,\ k}q\,\rho_B,\ \rho_B\sqsubseteq f_A}\bigwedge\left\{(\mathsf{Q}_{e_y^{t',\ s',\ k'}}^{\pi})_e(\rho_B)\mid e_y^{t',\ s',\ k'}q\,\rho_B\right\}$$

$$=\left(\mathsf{Q}_{e_x^{t,\ s,\ k}}^{\pi}\right)_e(f_A),$$

Similarly, it can be obtained $\left(\mathsf{Q}_{e_x^{t,\ s,\ k}}^{\mathscr{T}_{\mathsf{Q}}^{\alpha}}\right)_e(f_A)=\left(\mathsf{Q}_{e_x^{t,\ s,\ k}}^{\alpha}\right)_e(f_A)$, $\left(\mathsf{Q}_{e_x^{t,\ s,\ k}}^{\mathscr{T}_{\mathsf{Q}}^{\sigma}}\right)_e(f_A)=\left(\mathsf{Q}_{e_x^{t,\ s,\ k}}^{\sigma}\right)_e(f_A)$. Hence, $\mathsf{Q}_{e_x^{t,\ s,\ k}}^{\mathscr{T}_{\mathsf{Q}}^{\pi\alpha\sigma}}$ $=\mathsf{Q}_{e_x^{t,\ s,\ k}}^{\pi\alpha\sigma}$.

For (3), Similar to the proof of (2). □

Theorem 13 Let $(£,\ \mathscr{T}^{\pi},\ \mathscr{T}^{\alpha},\ \mathscr{T}^{\sigma})$ be a svnst-space and $\mathsf{Q}_E^{\mathscr{T}^{\pi\alpha\sigma}}$ a svnsqcn- system on $(£,\ \mathscr{T}^{\pi},\ \mathscr{T}^{\alpha},\ \mathscr{T}^{\sigma})$. Then $(\mathscr{T}^{\pi\alpha\sigma})_E=(\mathscr{T}^{\mathsf{Q}^{\mathscr{T}^{\pi\alpha\sigma}}})_E$.

Proof. Since

$$\left(\mathsf{Q}_{e_x^{t,\ s,\ k}}^{\mathscr{T}^{\pi}}\right)_e(f_A)=\bigvee_{e_x^{t,\ s,\ k}q\,\rho_B,\ \rho_B\sqsubseteq f_A}\mathscr{T}_e^{\pi}(\rho_B)\geq\mathscr{T}_e^{\pi}(f_A),$$

$$\left(\mathsf{Q}_{e_x^{t,\ s,\ k}}^{\mathscr{T}^{\alpha}}\right)_e(f_A)=\bigwedge_{e_x^{t,\ s,\ k}q\,\rho_B,\ \rho_B\sqsubseteq f_A}\mathscr{T}_e^{\alpha}(\rho_B)\leq\mathscr{T}_e^{\alpha}(f_A),$$

$$\left(\mathsf{Q}_{e_x^{t,\ s,\ k}}^{\mathscr{T}^{\sigma}}\right)_e(f_A)=\bigwedge_{e_x^{t,\ s,\ k}q\,\rho_B,\ \rho_B\sqsubseteq f_A}\mathscr{T}_e^{\sigma}(\rho_B)\leq\mathscr{T}_e^{\sigma}(f_A),$$

for all $e\in E$, $e_x^{t,\ s,\ k}\ q\ f_A$, then we have

$$\bigwedge_{e_x^{t,\ s,\ k}q\,f_A}\left(\mathsf{Q}_{e_x^{t,\ s,\ k}}^{\pi}\right)_e(f_A)\geq\mathscr{T}_e^{\pi}(f_A),\quad\bigvee_{e_x^{t,\ s,\ k}\,q\,f_A}\left(\mathsf{Q}_{e_x^{t,\ s,\ k}}^{\alpha}\right)_e(f_A)\leq\mathscr{T}_e^{\alpha}(f_A),$$

$$\bigvee_{e_x^{t,\ s,\ k}\,q\,f_A}\left(\mathsf{Q}_{e_x^{t,\ s,\ k}}^{\sigma}\right)_e(f_A)\leq\mathscr{T}_e^{\sigma}(f_A).$$

Hence, $(\mathscr{T}^{\pi\alpha\sigma})_E\sqsubseteq((\mathscr{T}^{\mathsf{Q}^{\mathscr{T}^{\pi\alpha\sigma}}})_E$.

On the other hand, let's assume there is an $f_A \in \widetilde{(£, E)}$ with $e \in E$ such that

$$\mathscr{T}_e^{\pi}(f_A) \ngeq (\mathscr{T}^{\mathsf{Q}^{\mathscr{T}^{\pi}}})_e(f_A),\ \mathscr{T}_e^{\alpha}(f_A) \nleq (\mathscr{T}^{\mathsf{Q}^{\mathscr{T}^{\alpha}}})_e(f_A),\ \mathscr{T}_e^{\sigma}(f_A) \nleq (\mathscr{T}^{\mathsf{Q}^{\mathscr{T}^{\sigma}}})_e(f_A).$$

For each $e_x^{t,\, s,\, k} \in P_{t,\, s,\, k}\widetilde{(£, E)}\}$ with $e_x^{t,\, s,\, k}\ q\ f_A$. If $e_x^{t,\, s,\, k}\ q\ (\rho_B)_{e_x^{t,\, s,\, k}}$, $(\rho_B)_{e_x^{t,\, s,\, k}} \sqsubseteq f_A$, then $f_A = \bigvee_{e_x^{t,\, s,\, k}\ q\ f_A} (\rho_B)_{e_x^{t,\, s,\, k}}$. That is,

$$\mathscr{T}_e^{\pi}(f_A) = \mathscr{T}_e^{\pi}\left(\bigvee_{e_x^{t,\, s,\, k} q\, f_A} (\rho_B)_{e_x^{t,\, s,\, k}}\right) \geq \bigwedge_{e_x^{t,\, s,\, k}\, q\, f_A} \mathscr{T}_e^{\pi}\left((\rho_B)_{e_x^{t,\, s,\, k}}\right),$$

$$\mathscr{T}_e^{\alpha}(f_A) = \mathscr{T}_e^{\alpha}\left(\bigvee_{e_x^{t,\, s,\, k} q\, f_A} (\rho_B)_{e_x^{t,\, s,\, k}}\right) \leq \bigvee_{e_x^{t,\, s,\, k}\, q\, f_A} \mathscr{T}_e^{\alpha}\left((\rho_B)_{e_x^{t,\, s,\, k}}\right),$$

$$\mathscr{T}_e^{\sigma}(f_A) = \mathscr{T}_e^{\sigma}\left(\bigvee_{e_x^{t,\, s,\, k} q\, f_A} (\rho_B)_{e_x^{t,\, s,\, k}}\right) \leq \bigvee_{e_x^{t,\, s,\, k}\, q\, f_A} \mathscr{T}_e^{\sigma}\left((\rho_B)_{e_x^{t,\, s,\, k}}\right),$$

which means that

$$\bigwedge_{e_x^{t,\, s,\, k} q\, f_A} \mathscr{T}_e^{\pi}\left((\rho_B)_{e_x^{t,\, s,\, k}}\right) \ngeq \left(\mathscr{T}^{\mathsf{Q}^{\mathscr{T}^{\pi}}}\right)_e(f_A) = \bigwedge_{e_x^{t,\, s,\, k}\, q\, f_A} \left(\mathsf{Q}^{\mathscr{T}^{\pi}}_{e_x^{t,\, s,\, k}}\right)_e(f_A),$$

$$\bigvee_{e_x^{t,\, s,\, k} q\, f_A} \mathscr{T}_e^{\alpha}\left((\rho_B)_{e_x^{t,\, s,\, k}}\right) \nleq \left(\mathscr{T}^{\mathsf{Q}^{\mathscr{T}^{\alpha}}}\right))_e(f_A) = \bigvee_{e_x^{t,\, s,\, k} q\, f_A} \left(\mathsf{Q}^{\mathscr{T}^{\alpha}}_{e_x^{t,\, s,\, k}}\right)_e(f_A),$$

$$\bigvee_{e_x^{t,\, s,\, k}\, q\, f_A} \mathscr{T}_e^{\sigma}\left((\rho_B)_{e_x^{t,\, s,\, k}}\right) \nleq \left(\mathscr{T}^{\mathsf{Q}^{\mathscr{T}^{\sigma}}}\right)_e(f_A) = \bigvee_{e_x^{t,\, s,\, k}\, q\, f_A} \left(\mathsf{Q}^{\mathscr{T}^{\sigma}}_{e_x^{t,\, s,\, k}}\right)_e(f_A).$$

This leads to a contradiction. Consequently, $(\mathscr{T}^{\pi\alpha\sigma})_E \sqsupseteq \left(\mathscr{T}^{\mathsf{Q}^{\mathscr{T}^{\pi\alpha\sigma}}}\right)_E$ □

Theorem 14 Let $(£,\ (\mathsf{Q}_1^{\pi\sigma\sigma})_E)$ and $(\chi,\ (\mathsf{Q}_2^{\pi\sigma\sigma})_F)$ be two svnsqcn-spaces. A map $\vartheta_{\varphi} : (£,\ (\mathsf{Q}_1^{\pi\sigma\sigma})_E) \to (\chi,\ (\mathsf{Q}_2^{\pi\sigma\sigma})_F)$ is N-map iff $\vartheta_{\varphi} : \left(£,\ \left(\mathscr{T}^{\mathsf{Q}_1^{\pi\alpha\sigma}}\right)_E\right) \to \left(\chi,\ \left(\mathscr{T}^{\mathsf{Q}_2^{\pi\sigma\sigma}}\right)_F\right)$ is svns-continuous.

Proof. Since for all $f_A \in \widetilde{(\chi, F)}$, $e_x^{t,\, s,\, k} \in P_{t,\, s,\, k}\widetilde{(£, E)}$

$$e_x^{t,\, s,\, k}\ q\ \vartheta_{\varphi}^{-1}(f_A)\ iff\ (\vartheta_{\varphi}(e_x^{t,\, s,\, k}) = \varphi(e)^{t,\, s,\, k}_{\vartheta(x)})\ q\ f_A$$

and

$$\left\{\varphi(e)_y^{t,\,s,\,k} \in P_{t,\,s,\,k}\widetilde{(\chi,\,F)} \mid \varphi(e)_y^{t,\,s,\,k} q f_A\right\} \sqsupseteq \varphi(e)_{\vartheta(x)}^{t,\,s,\,k} \in P_{t,\,s,\,k}\widetilde{(\chi,\,F)} : e_x^{t,\,s,\,k} \in P_{t,\,s,\,k}\widetilde{(\chi,\,F)},\ \varphi(e)_{\vartheta(x)}^{t,\,s,\,k}) q f_A\}.$$

Then, we have

$$\begin{aligned}
\left(\mathscr{T}^{\mathsf{Q}_2^\pi}\right)_{\varphi(e)}(f_A) &= \bigwedge\left\{\left(\left(\mathsf{Q}_2^\pi\right)_{\varphi(e)_y^{t,\,s,\,k}}\right)_{\varphi(e)}(f_A) \mid \varphi(e)_y^{t,\,s,\,k} q\, f_A\right\} \\
&\leq \bigwedge\left\{\left(\left(\mathsf{Q}_2^\pi\right)_{\varphi(e)_{\vartheta(x)}^{t,\,s,\,k}}\right)_{\varphi(e)}(f_A) \mid \varphi(e)_{\vartheta(x)}^{t,\,s,\,k} q\, f_A\right\} \\
&\leq \bigwedge\left\{\left(\left(\mathsf{Q}_1^\pi\right)_{e_x^{t,\,s,\,k}}\right)_e(\vartheta_\varphi^{-1}(f_A)) \mid e_x^{t,\,s,\,k} q\, \vartheta_\varphi^{-1}(f_A)\right\} \\
&= \left(\mathscr{T}^{\mathsf{Q}_1^\pi}\right)_e\left(\vartheta_\varphi^{-1}(f_A)\right),
\end{aligned}$$

$$\begin{aligned}
\left(\mathscr{T}^{\mathsf{Q}_2^\alpha}\right)_{\varphi(e)}(f_A) &= \bigvee\left\{\left(\left(\mathsf{Q}_2^\alpha\right)_{\varphi(e)_y^{t,\,s,\,k}}\right)_{\varphi(e)}(f_A) \mid \varphi(e)_y^{t,\,s,\,k} q\, f_A\right\} \\
&\geq \bigvee\left\{\left(\left(\mathsf{Q}_2^\alpha\right)_{\varphi(e)_{\vartheta(x)}^{t,\,s,\,k}}\right)_{\varphi(e)}(f_A) \mid \varphi(e)_{\vartheta(x)}^{t,\,s,\,k} q\, f_A\right\} \\
&\geq \bigvee\left\{\left(\left(\mathsf{Q}_1^\alpha\right)_{e_x^{t,\,s,\,k}}\right)_e(\vartheta_\varphi^{-1}(f_A)) \mid e_x^{t,\,s,\,k} q\, \vartheta_\varphi^{-1}(f_A)\right\} \\
&= \left(\mathscr{T}^{\mathsf{Q}_1^\alpha}\right)_e\left(\vartheta_\varphi^{-1}(f_A)\right).
\end{aligned}$$

Similarly, it can be obtained $(\mathscr{T}^{\mathsf{Q}_2^\sigma})_{\varphi(e)}(f_A) \geq (\mathscr{T}^{\mathsf{Q}_1^\sigma})_e(\vartheta_\varphi^{-1}(f_A))$.
Therefore, $\vartheta_\varphi : (\pounds,\ (\mathscr{T}^{\mathsf{Q}_1^{\pi\alpha\sigma}})_E) \to (\chi,\ (\mathscr{T}^{\mathsf{Q}_2^{\pi\sigma\sigma}})_F)$ is svns-continuous.
Conversely, since for all $f_A \in \widetilde{(\chi,\,F)}$,

$$\left[(\mathscr{T}^{\mathsf{Q}_2^\pi})_{\varphi(e)}(f_A) \leq (\mathscr{T}^{\mathsf{Q}_1^\pi})_e(\vartheta_\varphi^{-1}(f_A)),\ \left[(\mathscr{T}^{\mathsf{Q}_2^\alpha})_{\varphi(e)}(f_A) \geq (\mathscr{T}^{\mathsf{Q}_1^\alpha})_e(\vartheta_\varphi^{-1}(f_A)),\ \left[(\mathscr{T}^{\mathsf{Q}_2^\sigma})_{\varphi(e)}(f_A) \geq (\mathscr{T}^{\mathsf{Q}_1^\sigma})_e(\vartheta_\varphi^{-1}(f_A)),\right.\right.\right.$$

and $(\mathsf{Q}_1^{\pi\alpha\sigma})_E = \left(\mathsf{Q}^{\mathscr{T}^{\mathsf{Q}_1^{\pi\alpha\sigma}}}\right)_E$, $(\mathsf{Q}_2^{\pi\alpha\sigma})_F = \left(\mathsf{Q}^{\mathscr{T}^{\mathsf{Q}_2^{\pi\alpha\sigma}}}\right)_F$, then we have

$$\left((Q_2^{\pi})_{\varphi(e)^{t,\,s,\,k}_{\vartheta(x)}}\right)_{\varphi(e)}(f_A)=\bigwedge\left\{(\mathscr{T}^{Q_2^{\pi}})_{\varphi(e)}(\rho_B):\varphi(e)^{t,\,s,\,k}_{\vartheta(x)}\,q\,\rho_B,\ \rho_B\sqsubseteq f_A\right\}$$

$$\leq\bigwedge\left\{(\mathscr{T}^{Q_2^{\pi}})_{\varphi(e)}(\rho_B):e_x^{t,\,s,\,k}\,q\,\vartheta_{\varphi}^{-1}(\rho_B),\ \vartheta_{\varphi}^{-1}(\rho_B)\sqsubseteq\vartheta_{\varphi}^{-1}(f_A)\right\}$$

$$\leq\bigwedge\left\{(\mathscr{T}^{Q_1^{\pi}})_{e}(\vartheta_{\varphi}^{-1}(\rho_B)):e_x^{t,\,s,\,k}\,q\,\vartheta_{\varphi}^{-1}(\rho_B),\ \vartheta_{\varphi}^{-1}(\rho_B)\sqsubseteq\vartheta_{\varphi}^{-1}(f_A)\right\}$$

$$\leq\left((Q_1^{\pi})_{e_x^{t,\,s,\,k}}\right)_{e}\left(\vartheta_{\varphi}^{-1}(f_A)\right).$$

Similarly, it can be obtained

$$\left((Q_2^{\alpha})_{\varphi(e)^{t,\,s,\,k}_{\vartheta(x)}}\right)_{\varphi(e)}(f_A)\geq\left((Q_1^{\alpha})_{e_x^{t,\,s,\,k}}\right)_{e}\left(\vartheta_{\varphi}^{-1}(f_A)\right),$$

and

$$\left((Q_2^{\sigma})_{\varphi(e)^{t,\,s,\,k}_{\vartheta(x)}}\right)_{\varphi(e)}(f_A)\geq\left((Q_1^{\sigma})_{e_x^{t,\,s,\,k}}\right)_{e}\left(\vartheta_{\varphi}^{-1}(f_A)\right).$$

□

Theorem 15 Let $(\pounds,\ \mathscr{T}_E^{\pi\sigma\sigma})$ and $(\chi,\ \mathscr{T}_F^{*\pi\sigma\sigma})$ be two svnst-spaces. A mapping $\vartheta_{\varphi}:(\pounds,\ \mathscr{T}_E^{\pi\sigma\sigma})\to(\chi,\ \mathscr{T}_F^{*\pi\sigma\sigma})$ is svns-continuous iff $\vartheta_{\varphi}:(\pounds,\ \mathscr{T}_E^{Q^{\pi\sigma\sigma}})\to(\chi,\ \mathscr{T}_F^{*\pi\sigma\sigma})$ is N-map.

Proof. Similar to the proof of Theorem 8. □

5. Single-valued neutrosophic soft filter convergence

The convergence and properties of single-valued neutrosophic soft filters were examined in the previous section with respect to neutrosophic soft quasi-coincident neighborhood spaces. In this section, we delve deeper into the complexities of Single-Valued Neutrosophic Soft Quasi-Coincident Neighborhood Spaces, expanding on the concepts introduced earlier.

Focusing on svns-filter structures, we provide a detailed analysis of cluster points, limit points, and convergence criteria. This exploration not only advances our understanding of neutrosophic soft systems but also opens up new avenues for both theoretical research and practical applications

Definition 9 Let $(\pounds,\ \mathscr{T}^{\pi},\ \mathscr{T}^{\alpha},\ \mathscr{T}^{\sigma})$ be a *svnst-space*, $\mathscr{F}_E^{\pi\alpha\sigma}$ a *svn-soft filter*, $f_A,\ \rho_B\in\widetilde{(\pounds,\ E)}$ and $e_x^{t,\,s,\,k}\in P_{t,\,s,\,k}\widetilde{(\pounds,\ E)}$.

(1) $e_x^{t,\,s,\,k}$ is called a single-valued neutrosophic soft cluster point (for short, *svns-cluster point*) of $\mathscr{F}_E^{\pi\alpha\sigma}$, indicated by $\mathscr{F}_E^{\pi\alpha\sigma}\to e_x^{t,\,s,\,k}$ if for any $\rho_B\in\left(Q^{\pi\alpha\sigma}_{e_x^{t,\,s,\,k}}\right)_E^{\circ}$ and $f_A\in(\mathscr{F}_E^{\pi\alpha\sigma})^{\circ}$, we have $f_A\sqcap\rho_B\neq\widetilde{\Phi}$.

(2) $e_x^{t,\, s,\, k}$ is called a single-valued neutrosophic soft limit point (for short, *svns-limit point*) of $\mathscr{F}_E^{\pi\alpha\sigma}$, indicated by $\mathscr{F}_E^{\pi\alpha\sigma} \leftrightarrow e_x^{t,\, s,\, k}$ if for all $\left(\mathsf{Q}^{\pi\alpha\sigma}_{e_x^{t,\, s,\, k}}\right)_E \sqsubseteq \mathscr{F}_E^{\pi\alpha\sigma}$.

We denote

$$cls_{\mathscr{T}_E^{\pi\alpha\sigma}}(\mathscr{F}_E^{\pi\alpha\sigma}) = \bigsqcup \left\{ e_x^{t,\, s,\, k} \in \widetilde{(\pounds,\, E)} : e_x^{t,\, s,\, k}\ \textit{svns-cluster point of}\ \mathscr{F}_E^{\pi\alpha\sigma} \right\},$$

$$lim_{\mathscr{T}_E^{\pi\alpha\sigma}}(\mathscr{F}_E^{\pi\alpha\sigma}) = \bigsqcup \left\{ e_x^{t,\, s,\, k} \in \widetilde{(\pounds,\, E)} : e_x^{t,\, s,\, k}\ \textit{svns-limit point of}\ \mathscr{F}_E^{\pi\alpha\sigma} \right\}.$$

Theorem 16 Let $(\pounds,\ \mathscr{T}^{\pi},\ \mathscr{T}^{\alpha},\ \mathscr{T}^{\sigma})$ be a svnst-space and $\mathscr{F}_E^{\pi\alpha\sigma}$, $\mathscr{H}_E^{\pi\alpha\sigma}$ are two svns-filters on £ such that $\mathscr{F}_E^{\pi\alpha\sigma}$ is coarser than $\mathscr{H}_E^{\pi\alpha\sigma}$. Then the following properties hold.

(1) $\mathscr{F}_E^{\pi\alpha\sigma} \leftrightarrow e_x^{t,\, s,\, k} \Rightarrow \mathscr{F}_E^{\pi\alpha\sigma} \to e_x^{t,\, s,\, k}$.

(2) $lim_{\mathbf{T}_E^{\pi\alpha\sigma}}(\mathscr{F}_E^{\pi\alpha\sigma}) \sqsubseteq cls_{\mathbf{T}_E^{\pi\alpha\sigma}}(\mathscr{F}_E^{\pi\alpha\sigma})$.

(3) $\mathscr{F}_E^{\pi\alpha\sigma} \to e_x^{t,\, s,\, k}$, $e_x^{t',\, s',\, k'} \sqsubseteq e_x^{t,\, s,\, k} \Rightarrow \mathscr{F}_E^{\pi\alpha\sigma} \to e_x^{t',\, s',\, k'}$.

(4) $\mathscr{F}_E^{\pi\alpha\sigma} \leftrightarrow e_x^{t,\, s,\, k}$, $e_x^{t',s',k'} \sqsubseteq e_x^{t,\, s,\, k} \Rightarrow \mathscr{F}_E^{\pi\alpha\sigma} \leftrightarrow e_x^{t',\, s',\, k'}$.

(5) $\mathscr{F}_E^{\pi\alpha\sigma} \to e_x^{t,\, s,\, k} \Leftrightarrow e_x^{t,\, s,\, k} \sqsubseteq cls_{\mathscr{T}_E^{\pi\alpha\sigma}}(\mathscr{F}_E^{\pi\alpha\sigma})$.

(6) $\mathscr{F}_E^{\pi\alpha\sigma} \leftrightarrow e_x^{t,\, s,\, k} \Leftrightarrow e_x^{t,\, s,\, k} \sqsubseteq lim_{\mathscr{T}_E^{\pi\alpha\sigma}}(\mathscr{F}_E^{\pi\alpha\sigma})$.

(7) $\mathscr{F}_E^{\pi\alpha\sigma} \leftrightarrow e_x^{t,\, s,\, k} \Rightarrow \mathscr{H}_E^{\pi\alpha\sigma} \leftrightarrow e_x^{t,\, s,\, k}$.

(8) $lim_{\mathscr{T}_E^{\pi\alpha\sigma}}(\mathscr{F}_E^{\pi\alpha\sigma}) \sqsubseteq lim_{\mathscr{T}_E^{\pi\alpha\sigma}}(\mathscr{H}_E^{\pi\alpha\sigma})$.

(9) $\mathscr{H}_E^{\pi\alpha\sigma} \to e_x^{t,\, s,\, k} \Rightarrow \mathscr{F}_E^{\pi\alpha\sigma} \to e_x^{t,\, s,\, k}$.

(10) $cls_{\mathscr{T}_E^{\pi\alpha\sigma}}(\mathscr{H}_E^{\pi\alpha\sigma}) \sqsubseteq cls_{\mathscr{T}_E^{\pi\alpha\sigma}}(\mathscr{F}_E^{\pi\alpha\sigma})$.

Proof. (1) For each $\rho_B \in \left(\mathsf{Q}^{\pi\alpha\sigma}_{e_x^{t,\, s,\, k}}\right)_E^{\circ}$, $f_A \in (\mathscr{F}_E^{\pi\alpha\sigma})^{\circ}$, since $\left(\mathsf{Q}^{\pi\alpha\sigma}_{e_x^{t,\, s,\, k}}\right)_E \sqsubseteq \mathscr{F}_E^{\pi\alpha\sigma}$, we obtain $\rho_B \in (\mathscr{F}_E^{\pi\alpha\sigma})^{\circ}$. Thus,

$$\mathscr{F}^{\pi}(f_A \sqcap \rho_B) > 0,\ \mathscr{F}^{\alpha}(f_A \sqcup \rho_B) < 1,\ \mathscr{F}^{\sigma}(f_A \sqcup \rho_B) < 1.$$

This leads to that $f_A \sqcap \rho_B \neq \widetilde{(\Phi)}$.

(2) From (1), it is obvious.

(3) Since $e_x^{t',\, s',\, k'} \sqsubseteq e_x^{t,\, s,\, k}$, and by using Theorem 5(2), we obtain

$$\left(\mathsf{Q}^{\mathscr{T}^{\pi}}_{e_x^{t',\, s',\, k'}}\right)_e(f_A) \leq \left(\mathsf{Q}^{\mathscr{T}^{\pi}}_{e_x^{t,\, s,\, k}}\right)_e(f_A),\ \left(\mathsf{Q}^{\mathscr{T}^{\alpha}}_{e_x^{t',\, s',\, k'}}\right)_e(f_A) \geq \left(\mathsf{Q}^{\mathscr{T}^{\alpha}}_{e_x^{t,\, s,\, k}}\right)_e(f_A),\ \left(\mathsf{Q}^{\mathscr{T}^{\sigma}}_{e_x^{t',\, s',\, k'}}\right)_e(f_A) \geq \left(\mathsf{Q}^{\mathscr{T}^{\sigma}}_{e_x^{t,\, s,\, k}}\right)_e(f_A).$$

For every $\rho_B \in \left(\mathsf{Q}^{\pi\alpha\sigma}_{e_x^{t',\, s',\, k'}}\right)_E^{\circ}$, we obtain $\rho_B \in \left(\mathsf{Q}^{\pi\alpha\sigma}_{e_x^{t,\, s,\, k}}\right)_E^{\circ}$. Since, $\mathscr{F}_E^{\pi\alpha\sigma} \to e_x^{t,\, s,\, k}$, for every $f_A \in (\mathscr{F}_e^{\pi\alpha\sigma})^{\circ}$, we obtain $f_A \sqcap \rho_B \neq \widetilde{(\Phi)}$. Hence, $\mathscr{F}_E^{\pi\alpha\sigma} \to e_x^{t',\, s',\, k'}$.

(4) Since $\mathscr{F}_E^{\pi\alpha\sigma} \leftrightarrow e_x^{t,\, s,\, k}$, $\left(\mathsf{Q}^{\pi\alpha\sigma}_{e_x^{t,\, s,\, k}}\right)_E \sqsubseteq \mathscr{F}_E^{\pi\alpha\sigma}$. Since $e_x^{t',\, s',\, k'} \sqsubseteq e_x^{t,\, s,\, k}$, by using Theorem 5(2),

$$\left(\mathsf{Q}^{\mathscr{T}^{\pi}}_{e_x^{t',\, s',\, k'}}\right)_e(f_A) \leq \left(\mathsf{Q}^{\mathscr{T}^{\pi}}_{e_x^{t,\, s,\, k}}\right)_e(f_A),\ \left(\mathsf{Q}^{\mathscr{T}^{\alpha}}_{e_x^{t',\, s',\, k'}}\right)_e(f_A) \geq \left(\mathsf{Q}^{\mathscr{T}^{\alpha}}_{e_x^{t,\, s,\, k}}\right)_e(f_A),$$

$$\left(\mathsf{Q}^{\mathscr{T}^{\sigma}}_{e_x^{t',\, s',\, k'}}\right)_e(f_A) \geq \left(\mathsf{Q}^{\mathscr{T}^{\sigma}}_{e_x^{t,\, s,\, k}}\right)_e(f_A).$$

Therefor, $\left(\mathsf{Q}^{\pi\alpha\sigma}_{e_x^{t',\, s',\, k'}}\right)_E \sqsubseteq \left(\mathsf{Q}^{\pi\alpha\sigma}_{e_x^{t,\, s,\, k}}\right)_E \sqsubseteq \mathscr{F}_E^{\pi\alpha\sigma}$. Hence, $\mathscr{F}_E^{\pi\alpha\sigma} \leftrightarrow e_x^{t',\, s',\, k'}$.

(5) If $e_x^{t,\, s,\, k} \sqsubseteq cls_{\mathscr{T}_E^{\pi\alpha\sigma}}(\mathscr{F}_E^{\pi\alpha\sigma})$, for all $\rho_B \in \left(\mathsf{Q}^{\pi\alpha\sigma}_{e_x^{t,\, s,\, k}}\right)_E^{\circ}$ according to the definition of $\left(\mathsf{Q}^{\pi\alpha\sigma}_{e_x^{t,\, s,\, k}}\right)_E$, there exists $h_C \in \widetilde{(\pounds,\, E)}$ such that $e_x^{t,\, s,\, k}\ q\ h_C$, $h_C \sqsubseteq \rho_B$ and

$$\left(\mathsf{Q}^{\pi}_{e_x^{t,\, s,\, k}}\right)_e(\rho_B) \geq \mathscr{T}_e^{\alpha}(h_C) > 0,\ \left(\mathsf{Q}^{\alpha}_{e_x^{t,\, s,\, k}}\right)_e(\rho_B) \leq \mathscr{T}_e^{\alpha}(h_C) < 1,\ \left(\mathsf{Q}^{\sigma}_{e_x^{t,\, s,\, k}}\right)_e(\rho_B) \leq \mathscr{T}_e^{\sigma}(h_C) < 1.$$

This leads to that $h_C\ q\ cls_{\mathscr{T}_e^{\pi\alpha\sigma}}(\mathscr{F}_e^{\pi\alpha\sigma})$.

According to the definition of $cls_{\mathscr{T}_e^{\pi\alpha\sigma}}(\mathscr{F}_e^{\pi\alpha\sigma})$, there exists a *svns-cluster point* $e_x^{t,\, s,\, k} \in P_{t,\, s,\, k}\widetilde{(\pounds,\, E)}$ of $\mathscr{F}_E^{\pi\alpha\sigma}$ such that $e_x^{t,\, s,\, k} q\ h_C$ implies that $h_C\ q\ cls_{\mathscr{T}_e^{\pi\alpha\sigma}}(\mathscr{F}_e^{\pi\alpha\sigma})$. Hence, $e_x^{t,\, s,\, k}\ q\ h_C$, $h_C \sqsubseteq \rho_B$ and

$$\left(\mathsf{Q}^{\pi}_{e_x^{t',\, s',\, k'}}\right)_e(\rho_B) \geq \mathscr{T}_e^{\pi}(h_C) > 0,\ \left(\mathsf{Q}^{\alpha}_{e_x^{t',\, s',\, k'}}\right)_e(\rho_B) \leq \mathscr{T}_e^{\alpha}(h_C) < 1,$$

$$\left(\mathsf{Q}^{\sigma}_{e_x^{t',\, s',\, k'}}\right)_e(\rho_B) \leq \mathscr{T}_e^{\sigma}(h_C) < 1.$$

Thus, $\rho_B \in \left(\mathsf{Q}^{\pi\alpha\sigma}_{e_x^{t',\, s',\, k'}}\right)_E^{\circ}$ and $e_x^{t',\, s',\, k'}$ is a *svns-cluster point* of $\mathscr{F}_E^{\pi\alpha\sigma}$. Thus, $\forall\, f_A \in (\mathscr{F}_E^{\pi\alpha\sigma})^{\circ}$, $f_A \sqcap \rho_B \neq \widetilde{\Phi}$. Therefore, $\mathscr{F}_E^{\pi\alpha\sigma} \to e_x^{t,\, s,\, k}$.

The converse is obvious.

(6) It is comparable to (5).

(7) It is simply proved from $\left(\mathsf{Q}^{\pi\alpha\sigma}_{e_x^{t,\, s,\, k}}\right)_e \sqsubseteq \mathscr{F}_e^{\pi\alpha\sigma} \sqsubseteq \mathscr{H}_e^{\pi\alpha\sigma}$.

(8) From (7), it is clear.

(9) For all $\rho_B \in \left(\mathsf{Q}^{\pi\alpha\sigma}_{e_x^{t,\, s,\, k}}\right)_E^{\circ}$ and $f_A \in (\mathscr{F}_E^{\pi\alpha\sigma})^{\circ}$ since $\mathscr{F}_e^{\pi\alpha\sigma} \sqsubseteq \mathscr{H}_e^{\pi\alpha\sigma}$, we have $f_A \in (\mathscr{H}_e^{\pi\alpha\sigma})_E^{\circ}$. Since $\mathscr{H}_E^{\pi\alpha\sigma} \to e_x^{t,\, s,\, k}$, $f_A \sqcap \rho_B \neq \widetilde{\Phi}$. That is, $\mathscr{F}_E^{\pi\alpha\sigma} \to e_x^{t,\, s,\, k}$.

(10) It is comparable to (9). □

6. Conclusions

In conclusion, this paper explored the complex domain of Single-Valued Neutrosophic Soft Quasi-Coincident Neighborhood Spaces and clarified the convergence procedures within this novel framework. We developed basic theorems that clarify the connections between finer and coarser filters by investigating svns-filters, cluster points, and limit points. These theorems offer important new insights into the patterns of convergence of neutrosophic soft systems. The newly presented ideas of svns-limit and svns-cluster points proved to be crucial in describing the behavior of these systems.

The results of this research open up new possibilities for practical applications in several fields and strengthen the theoretical foundations of neutrosophic soft systems. Novel approaches to computational intelligence and decision support systems are made possible by the established theorems and insights, which offer scholars and practitioners a sophisticated knowledge of convergence in Single-Valued Neutrosophic Soft Quasi-Coincident Neighborhood Spaces. In summary, the work described here represents a major advancement in the field of neutrosophic soft structures, providing a useful foundation for future study.

7. Discussion of future work

Boundedness in topological spaces (see [48]) is a well-established concept that plays a critical role in topological analysis. It is well known that the collection of bounded sets forms an ideal, a concept further generalized by the notion of bornology, which essentially represents an ideal in this context. In fuzzy set theory, this generalization is extended through fuzzy bornology (see [49–51]). Building on these ideas, the following concepts could be explored in the context of single-valued neutrosophic topological spaces:

(a) The collection of bounded single-valued neutrosophic soft sets;

(b) The concept of boundedness within neutrosophic soft topological spaces.

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Conflict of interest

The author declares that there is no conflict of interest regarding the publication of this article.

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