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Research Article

Solving Neutrosophic Multi-Dimensional Fixed Charge Transportation Problem

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Abstract: In the decision making problems, transportation problem (TP) is one of the most significant applications that aim to optimize the objective function of products shipped from different sources to different destinations. When an additional charge such as toll tax, parking fee, permit fee and so on are included in the transportation cost then the problem becomes a fixed charge TP. If the shipped products are damageable, they are affected by the type of vehicle and the distance of route. So it is quite evident that the vehicle type and the distance of route play a significant role in TP. In reality, along with sources and destinations, the decision maker (DM) desires to consider the parameters such as the mode of transport and routes of transportation that are used for shipment of the products from sources to destinations in order to minimize the loss of damageable products and to maximize the profit. As a result, mode of transport and routes of transportation are added to TP, then the TP becomes a four-dimensional TP. Sometimes, the data/parameters of the problems may not be always precise due to incomplete information, inadequate data or shortage of evidence. To deal with these obstacles, the parameters of the problem can be represented as the single-valued trapezoidal neutrosophic numbers (SVTrNNs). The neutrosophic data facilitate a reasonable and practicable way for DMs to tackle decision-making problems by managing indeterminacy and providing an effective framework for analysis and synthesis of complex decision scenarios. In view of this, in this paper we have considered the single objective four-dimensional fixed charge transportation problem (4DFCTP) with parameters, supply, demand, conveyance, transportation cost, fixed charge as single-valued trapezoidal neutrosophic numbers and the distance of routes as real numbers. First, the score function is utilized to transform the neutrosophic parameter into its deterministic parameter in order to avoid the negative values for the decision variable. Then the deterministic problem comprises deterministic parameters such as supply, demand, conveyance, transportation cost with fixed charges and distance of routes. Secondly, a novel approach namely, min zero-min cost approach is introduced for finding the optimal solution to the equivalent deterministic problem in polynomial time. The main objective of this paper is to optimize the breakable products and routing plan of vehicles in a way to minimize the total transportation cost with fixed cost of the business organizations using the proposed min zero-min cost approach. To demonstrate the problem's validity and relevance, two numerical examples are solved using our proposed approach. To highlight the proposed approach, comparison of the solution with the LINGO software is performed. The obtained optimal solution from the proposed approach is the same as the LINGO software. At last, conclusions as well as future work related to the study are presented.

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Keywords: four-dimensional fixed charge transportation problem, single-valued trapezoidal neutrosophic numbers, score function, min zero-min cost method

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Abbreviation

DMs	Decision Makers
FS	Fuzzy Sets
IFS	Intuitionistic Fuzzy Set
MF	Membership Function
NMF	Non-Membership Function
NS	Neutrosophic Set
SVNS	Single-Valued Neutrosophic Sets
SVTrNNs	Single-Valued Trapezoidal Neutrosophic Numbers
TP	Transportation Problem
4DFCTP	Four-Dimensional Fixed Charge Transportation Problem
4DTP	Four-Dimensional Transportation Problem

1. Introduction

Transportation problem (TP) also called the two dimensional problem is one of the most well-known decision making problem which targets to optimize the objective function of products transported from various sources to distinct destinations. TP was introduced by Hitchcock [1] in 1941 and then the solution procedure for finding the optimal solution was developed by Koopmans [2] in 1947. During transportation, some additional charges are paid for parking the vehicles, charges in toll plaza for the maintenance of roads in some routes, public contributions for festive occasions, handling fees, voluntary works and so on. These types of additional charges are termed as fixed charges. Generally, a fixed charge is characterized by two forms of costs namely, a variable cost that progressively increases as the number of transported products increases and a fixed charge that is paid when a non-zero quantity of products is shipped from sources to destinations. Tolls at toll plazas, which are based on the type of vehicle and its weight, are an important component of fixed charges incurred while shipping products. When the fixed charge is added to the TP then the problem becomes a fixed charge TP (FCTP). Incorporating toll taxes, parking fees, and permit fees in the transportation cost allows the decision makers for more accurate optimization of transportation routes and schedules and more precise calculation of operational costs, which can be critical in determining pricing strategies. Routes with lower toll taxes or parking fees can be preferred to minimize costs, while understanding permit fees helps in complying with regulatory requirements efficiently. Hirsch and Dantzig [3] were the first to introduce the fixed charge TP and discussed the properties of a general solution to the fixed charge TP. Murty [4] developed an algorithm for ranking basic feasible solutions to obtain the optimal solution for fixed charge TP. Gray [5] developed the decomposition approach for solving fixed charge TP. Schaffer and O'Leary [6], Udatta et al. [7] applied a branch and bound approach to solve fixed charge TP. A simple heuristic algorithm is presented for solving fixed charge TP by Adlakha and Kowalski [8]. Safi [9] developed two algorithms based on order relations to obtain the optimal solution for fixed charge TP. Mollanoori et al. [10] considered a TP with two types of fixed charges. They have utilized Simulated Annealing algorithm and Imperialist competitive algorithm for solving two-stage multi-item fixed charge solid TP (FCSTP). Kartli et al. [11] proposed a new heuristic algorithm for solving fixed charge TP to obtain the optimal solution.

An extension on the TP is based on the typology of dimensions where it can be classified into 2 dimensions, 3 dimensions, 4 dimensions and n dimensions. The three-dimensional TP is also called the solid transportation problem in which three-dimensions are supply, demand and mode of transportation. The three-dimensional TP was introduced by Shell [12]. Pandian and Anuradha [13] proposed an approach for solving solid TP to obtain the optimal solution. The four-dimensional TP is an extension of the three-dimensional TP in which four-dimensional properties such as supply, demand, conveyance and distance of routes are considered in the objective function and constraints, where Giri and Roy proposed the solution approaches to obtain the compromise solutions for this problem. In the real world situations, products are delivered from sources to destinations by different conveyances such as bus, trucks, goods trains, cargo flights, etc. In addition, there may be various routes to transport the products from source to destination. The condition of different roads may vary greatly from good to terrible, smooth to rough with various humps, etc. In these situations, a long smooth path may require less time, whereas a short rough path may require more time to transport the products. Moreover, the cost of transportation can differ depending on the type of conveyance and the route. It is expected that breakability is higher on rough paths than smooth ones. If the product is breakable in transportation, it may depend on the vehicle type, road condition and distance. In this way, route selection is more realistic along with vehicle selection. In addition to source and destination constraints, different modes of transport (bus, train, flight, etc.) and various routes are considered from sources to destinations to minimize the loss of damageable products and maximize the profit. Among these, the four-dimensional TP is a model that attracts entrepreneurs. In the industries, it fits the requirements of the type of vehicles and the distance of routes to transport the products from the supplier to the distributor. Bera et al. [14] were the first to extend the TP into a four-dimensional TP. They employed the generalized reduced gradient technique for solving multi-item four-dimensional TP. Fakhrzad et al. [15] developed meta-heuristic algorithms for solving the fixed charge TP in which there are various routes with various conveyances between sources and destinations. Hameed and Moalla [16] employed a particle swarm optimization algorithm for solving four-dimensional TP. In real situations, the parameters of 4DTP like availability of supply, demand, conveyance, distance of routes and unit transportation cost with fixed charge are not exact due to certain unpredictable factors in traffic delays, conditions on the road and lack of information. In order to deal with this, Zadeh [17] introduced the fuzzy sets (FS) which provide the membership function (MF). Yang and Liu [18] have employed expected, optimistic and pessimistic value criteria for solving FCSTP under type-2 fuzzy environment to determine the optimal solution. Zhang et al. [19] have employed tabu search algorithm for solving fixed charge solid TP (FCSTP) under type-2 fuzzy environment to determine the optimal solution. Jana and Jana [20] developed a generalized reduced gradient technique for solving fixed charge four-dimensional multi-item TP under fuzzy triangular and Gaussian type-2 environment. Bera et al. [21] modified the generalized reduced gradient technique for solving four-dimensional fixed charge TP under type-2 fuzzy environment. Devnath et al. [22] developed a generalized reduced gradient technique for solving two-stage four-dimensional fixed charge TP with multi-items under type-2 fuzzy environment. Devnath et al. [23] modified the generalized reduced gradient technique for solving multi-item two-stage four-dimensional fixed charge TP under fuzzy environment. Aktar et al. [24] developed a generalized reduced gradient technique for solving four-dimensional fixed charge TP under type-2 fuzzy environment. Devnath et al. [25] modified the generalised reduced gradient technique for solving multi-item two stage four-dimensional TP under fuzzy environment. Gazi et al. [26] utilized the Analytic hierarchy process which is a decision-making approach based on the hexagonal fuzzy numbers. Numerous researchers such as Giri and Roy [27], Mondal et al. [28] and Mardanya and Roy [29] have utilized the fuzzy numbers to deal the uncertain data.

In reality, fuzzy numbers might not be suitable for all situations where uncertainty and hesitation coexist. In this case, the intuitionistic fuzzy set (IFS) was introduced by Atanassov [30] to handle both the MF and non-membership function (NMF). The IFS takes into account both the MF and NMF, but it cannot deal with the indeterminacy. Samanta et al. [31] proposed a convex combination approach for solving multi-item multi-objective four-dimensional TP under intuitionistic fuzzy environment. Ghorui et al. [32] utilized the Analytic hierarchy process based on the pentagonal intuitionistic fuzzy numbers. To address these issues, Smarandache [33] introduced the neutrosophic set (NS) which considers both the truth MF and falsity MF along with the indeterminacy MF when making decisions. Wang et al. [34] was the first to analyze the relations and operations over single-valued neutrosophic (SVN) sets. Deli and Subas [35] discussed special forms of SVTrNNs and applied the weighted aggregation operator to solve multi-criteria decision-making. Samanta et al. [36] employed the generalized reduced gradient technique for solving two stage four-dimensional TP under neutrosophic environment. Kar et al. [37] developed the generalized reduced gradient technique for solving multi-item fixed charge four-dimensional TP under neutrosophic environment. Giri and Roy [38] proposed neutrosophic programming and Pythagorean hesitant fuzzy programming approaches for solving neutrosophic multi-objective four-dimensional fixed charge TP. Numerous researchers such as Imran et al. [39], Simic et al. [40] and Senapati et al. [41] have utilized the neutrosophic numbers to deal the uncertain data. Table 1 lists the comparison between the recent literature survey with the present paper.

Table 1. Comparison between the recent literature survey with the present paper

References	Parameters nature					Dimension			Fixed charge	Methods
	Cr	F	T2F	IF	NF	Two	Three	Four		
Gray [5]	✓					✓			✓	Decomposition
Schaffer and O'Leary [6]	✓					✓			✓	Branch and bound
Udatta et al. [7]	✓					✓			✓	Branch and bound
Adlakha and Kowalski [8]	✓					✓			✓	Heuristic algorithm
Mollanoori et al. [10]	✓						✓		✓	Simulated annealing and imperialist competitive algorithms
Kartli et al. [11]	✓					✓			✓	Heuristic algorithm
Bera et al. [14]	✓							✓		Generalised reduced gradient (GRG) technique
Fakhrzad et al. [15]	✓							✓	✓	Meta-heuristic algorithm
Hameed and Moalla [16]	✓							✓		Particle swarm optimization algorithm
Yang and Liu [18]			✓				✓			Expected, optimistic and pessimistic value criterion
Zhang et al. [19]		✓					✓			Tabu search algorithm
Jana and Jana [20]			✓					✓	✓	GRG technique
Bera et al. [21]			✓					✓	✓	GRG technique
Devnath et al. [22]			✓					✓	✓	GRG technique
Devnath et al. [23]		✓						✓	✓	GRG technique
Aktar et al. [24]			✓					✓	✓	GRG technique
Devnath et al. [25]		✓						✓	✓	GRG technique
Samanta et al. [31]				✓				✓	✓	Convex combination approach
Samanta et al. [36]					✓			✓		GRG technique
Kar et al. [37]					✓			✓	✓	GRG technique
Giri and Roy [38]					✓			✓	✓	Neutrosophic programming, pythagorean hesitant fuzzy programming
Proposed approach					✓			✓	✓	Min zero-min cost approach

Note: Cr-Crisp, F-Fuzzy, T2F-Type-2 fuzzy, IF-Intuitionistic fuzzy, NF-Neutrosophic fuzzy

1.1 *Motivation and contribution*

In recent days, the transportation of products from origin to the destinations by road has played a crucial role due to globalization. While transporting the products through vehicles are of finite capacity plying on the roads from source to destination there may be many available routes connecting the cities. In some routes for maintenance, fixed charges such as toll taxes, parking fees, permit fees, and so on are collected. In real life problems sometimes few/all parameters of the problem are with impreciseness. In this paper, we have considered the single objective four-dimensional fixed charge TP under neutrosophic environment. The parameters such as supply, demand and conveyance are considered as single-valued trapezoidal neutrosophic numbers. The distance of routes in the objective function are represented by real numbers. Based on the literature survey in Table 1 the research gap, motivation and contribution are noted below.

1. It is evident that the work performed so far focuses mostly on obtaining the optimal solution using various softwares such as LINDO, LINGO, GAMS, MATLAB, Python, etc. for solving the four-dimensional fixed charge TP (4DFCTP) under certain environment [14–16], fuzzy environment [18–25], intuitionistic environment [31] and neutrosophic environment [36–38].

2. The fuzzy environment deals only with the membership function whereas the intuitionistic fuzzy environment deals with both the membership and non-membership function. The neutrosophic environment deals with truth, indeterminacy and falsity MF. The neutrosophic numbers are necessary to handle indeterminacy situations, when the DM is under neutral thoughts and he/she is unaware of the decisions. For example, we can consider the statement as "If the supplier 'X' needs to ship the scout drones to the customer 'Y' with minimum toll charge of rupees 20", there is 0.6 chance that 'X' will ship the scout drones with minimum toll charge of rupees 20, it is true, there is 0.5 chance that 'X' will not ship the scout drones with minimum toll charge of rupees 20, it is false and there is 0.2 chance that 'X' may or may not ship the scout drones with minimum toll charge of rupees 20, it is indeterminate. This shows that the neutrosophic number gives an additional importance to represent uncertainty, imprecise, incomplete and inconsistent information which exist in real world. So, it would be more suitable to apply the neutrosophic numbers in the real world applications where the indeterminate information and inconsistent information measures are available.

3. To the best of our knowledge, many researchers have considered the certain and fuzzy environment for solving four-dimensional fixed charge TP whereas solving under intuitionistic and neutrosophic environment receives less attention. Finding the optimal solution to the single objective problem under neutrosophic environment by manual computation is also very rare.

4. This motivates us to propose the novel approach namely, min zero-min cost approach to determine the optimal solution for single objective four-dimensional fixed charge TP under neutrosophic environment. The obtained optimal allotment will help the DM to choose the amount of products to be transported from the source to destination by the suitable vehicle through the suitable route to minimize the loss of damage in the products during transportation, the transportation cost and fixed cost satisfying the availability and requirements of each source and destination of the problem.

5. Obtaining the optimal solution by the proposed approach namely, min zero-min cost approach for solving single objective 4DFCTP under neutrosophic environment which is a novelty in this paper.

6. To validate the proposed approach, comparison of the solution with the LINGO software is performed. The obtained optimal solution using our proposed approach is the same as the LINGO software which demonstrates that our novel approach is one of the computational approaches for the real life problem. Two numerical examples are incorporated to illustrate the applicability of the proposed approach.

The paper is classified into the following categories: Section 2 follows with basic concepts and preliminaries. The mathematical formulation of 4DFCTP under neutrosophic environment is represented in Section 3. Section 4 illustrates the proposed approach to obtain the optimal solution while Section 5 depicts a numerical illustration with results and discussions. In Section 6, a comparison of the proposed approach is illustrated with LINGO software which is performed by Intel(R) Core (TM) i3-7100U CPU @ 2.40 GHz and 4 GB RAM while Section 7 incorporates the final conclusions and future scopes.

2. Preliminaries and essential definitions

In this section, definitions and mathematical formulation are presented. Some fundamental definitions related to the NS [33], SVN sets [34], SVTrNNs, arithmetic operations and score function of SVTrNNs [35] have been outlined.

Definition 2.1 Neutrosophic Set [33]: Let X be a universe discourse. A NS L in X is characterized by a truth MF $A_{\bar{L}^N}(x)$, indeterminacy MF $B_{\bar{L}^N}(x)$ and a falsity MF $C_{\bar{L}^N}(x)$. $A_{\bar{L}^N}(x)$, $B_{\bar{L}^N}(x)$ and $C_{\bar{L}^N}(x)$ are real standard elements of [0, 1]. It can be written as

$$\bar{L}^N = \left\{\langle x,\ A_{\bar{L}^N}(x),\ B_{\bar{L}^N}(x),\ C_{\bar{L}^N}(x)\rangle : x \in X,\ A_{\bar{L}^N}(x),\ B_{\bar{L}^N}(x),\ C_{\bar{L}^N}(x) \in \left[0^-,\ 1^+\right]\right\}$$

There is no restriction on the sum of $A_{\bar{L}^N}(x)$, $B_{\bar{L}^N}(x)$ and $C_{\bar{L}^N}(x)$, so $0^- \leq A_{\bar{L}^N}(x) + B_{\bar{L}^N}(x) + C_{\bar{L}^N}(x) \leq 3^+$.

Definition 2.2 Single-valued neutrosophic sets [34]: A SVNS $\bar{L}^{SVN}$ of a non-empty set X is defined as follows: $\bar{L}^{SVN} = \{\langle x,\ A_{\bar{L}^N}(x),\ B_{\bar{L}^N}(x),\ C_{\bar{L}^N}(x)\rangle : x \in X\}$ where $A_{\bar{L}^N}(x),\ B_{\bar{L}^N}(x),\ C_{\bar{L}^N}(x) \in [0,\ 1]$ for each $x \in X$ and $0 \leq A_{\bar{L}^N}(x) + B_{\bar{L}^N}(x) + C_{\bar{L}^N}(x) \leq 3$.

Definition 2.3 Single-valued trapezoidal neutrosophic number [35]: There are many research papers published on neutrosophic numbers. Depending on the need of the problem, researchers can use triangular neutrosophic numbers, trapezoidal neutrosophic numbers, single-valued triangular neutrosophic numbers, single-valued trapezoidal neutrosophic number and interval valued neutrosophic numbers. In this paper, single-valued trapezoidal neutrosophic numbers have been used.

Let $\sigma_{\tilde{d}^N}$, $\lambda_{\tilde{d}^N}$, $\tau_{\tilde{d}^N} \in [0,\ 1]$ and $p,\ q,\ r,\ s \in \Re$ such that $p \leq q \leq r \leq s$. Then a SVTrNN, $\tilde{d}^N = \left\langle (p,\ q,\ r,\ s);\ \sigma_{\tilde{d}^N},\ \lambda_{\tilde{d}^N},\ \tau_{\tilde{d}^N}\right\rangle$ is a special NS on $\Re$, whose truth membership, indeterminacy membership and falsity membership functions are given below:

$$\mu_{\tilde{d}^N} = \begin{cases} \sigma_{\tilde{d}^N}\left(\dfrac{x-p}{q-p}\right), & p \leq x < q \\ \sigma_{\tilde{d}^N}, & q \leq x \leq r \\ \sigma_{\tilde{d}^N}\left(\dfrac{s-x}{s-r}\right), & r \leq x \leq s \\ 0, & \text{otherwise} \end{cases}$$

$$\delta_{\tilde{d}^N} = \begin{cases} \dfrac{q-x+\lambda_{\tilde{d}^N}(x-p)}{l-k}, & p \leq x < q \\ \lambda_{\tilde{d}^N}, & q \leq x \leq r \\ \dfrac{x-r+\lambda_{\tilde{d}^N}(s-x)}{s-r}, & r \leq x \leq s \\ 0, & \text{otherwise} \end{cases}$$

$$\rho_{\tilde{d}^N} = \begin{cases} \dfrac{q - x + \tau_{\tilde{d}^N}(x-p)}{q-p}, & p \le x < q \\ \tau_{\tilde{d}^N}, & q \le x \le r \\ \dfrac{x - r + \tau_{\tilde{d}^N}(s-x)}{s-r}, & r \le x \le s \\ 1, & \text{otherwise} \end{cases}$$

where $\sigma_{\tilde{d}^N}$, $\lambda_{\tilde{d}^N}$ and $\tau_{\tilde{d}^N}$ denote the maximum truth, minimum indeterminacy, and minimum falsity membership degrees respectively. A SVTrNN $\tilde{d}^N = \langle (p, q, r, s); \sigma_{\tilde{d}^N}, \lambda_{\tilde{d}^N}, \tau_{\tilde{d}^N} \rangle$ may be expressed as an ill-defined quantity of p, which is approximately equal to $[q, r]$.

Definition 2.4 Arithmetic Operations on SVTrNNs [35]: Let $\tilde{d}^N = \langle (p, q, r, s); \sigma_{\tilde{d}^N}, \lambda_{\tilde{d}^N}, \tau_{\tilde{d}^N} \rangle$ and $\tilde{g}^N = \left\langle (p', q', r', s'); \sigma_{\tilde{g}^N}, \lambda_{\tilde{g}^N}, \tau_{\tilde{g}^N} \right\rangle$ be two SVTrNNs. The arithmetic operations on $\tilde{d}^N$ and $\tilde{g}^N$ are:

1. $\tilde{d}^N + \tilde{g}^N = \left\langle (p+p', q+q', r+r', s+s'); \sigma_{\tilde{d}^N} \wedge \sigma_{\tilde{g}^N}, \lambda_{\tilde{d}^N} \vee \lambda_{\tilde{g}^N}, \tau_{\tilde{d}^N} \vee \tau_{\tilde{g}^N} \right\rangle$
2. $\tilde{d}^N - \tilde{g}^N = \left\langle (p-p', q-q', r-r', s-s'); \sigma_{\tilde{d}^N} \wedge \sigma_{\tilde{g}^N}, \lambda_{\tilde{d}^N} \vee \lambda_{\tilde{g}^N}, \tau_{\tilde{d}^N} \vee \tau_{\tilde{g}^N} \right\rangle$
3. $\tilde{d}^N \times \tilde{g}^N = \begin{cases} \left\langle (pp', qq', rr', ss'); \sigma_{\tilde{d}^N} \wedge \sigma_{\tilde{g}^N}, \lambda_{\tilde{d}^N} \vee \lambda_{\tilde{g}^N}, \tau_{\tilde{d}^N} \vee \tau_{\tilde{g}^N} \right\rangle, s > 0, s' > 0 \\ \left\langle (ps', qr', qr', ps'); \sigma_{\tilde{d}^N} \wedge \sigma_{\tilde{g}^N}, \lambda_{\tilde{d}^N} \vee \lambda_{\tilde{g}^N}, \tau_{\tilde{d}^N} \vee \tau_{\tilde{g}^N} \right\rangle, s < 0, s' > 0 \\ \left\langle (ss', qq', rr', pp'); \sigma_{\tilde{d}^N} \wedge \sigma_{\tilde{g}^N}, \lambda_{\tilde{d}^N} \vee \lambda_{\tilde{g}^N}, \tau_{\tilde{d}^N} \vee \tau_{\tilde{g}^N} \right\rangle, s < 0, s' < 0 \end{cases}$
4. $\tilde{d}^N / \tilde{g}^N = \begin{cases} \left\langle (p/s', q/r', r/q', s/p'); \sigma_{\tilde{d}^N} \wedge \sigma_{\tilde{g}^N}, \lambda_{\tilde{d}^N} \vee \lambda_{\tilde{g}^N}, \tau_{\tilde{d}^N} \vee \tau_{\tilde{g}^N} \right\rangle, s > 0, s' > 0 \\ \left\langle (s/s', r/r', q/q', p/p'); \sigma_{\tilde{d}^N} \wedge \sigma_{\tilde{g}^N}, \lambda_{\tilde{d}^N} \vee \lambda_{\tilde{g}^N}, \tau_{\tilde{d}^N} \vee \tau_{\tilde{g}^N} \right\rangle, s < 0, s' > 0 \\ \left\langle (s/p', r/q', q/r', p/s'); \sigma_{\tilde{d}^N} \wedge \sigma_{\tilde{g}^N}, \lambda_{\tilde{d}^N} \vee \lambda_{\tilde{g}^N}, \tau_{\tilde{d}^N} \vee \tau_{\tilde{g}^N} \right\rangle, s < 0, s' < 0 \end{cases}$
5. $d\tilde{g}^N = h(x) = \begin{cases} \langle (dp, dq, dr, ds); \sigma_{\tilde{g}^N}, \lambda_{\tilde{g}^N}, \tau_{\tilde{g}^N} \rangle, d > 0 \\ \langle (ds, dr, dq, dp); \sigma_{\tilde{g}^N}, \lambda_{\tilde{g}^N}, \tau_{\tilde{g}^N} \rangle, d < 0 \end{cases}$
6. $\tilde{g}^{N^{-1}} = \left\langle (1/s', 1/r', 1/q', 1/p'); \sigma_{\tilde{g}^N}, \lambda_{\tilde{g}^N}, \tau_{\tilde{g}^N} \right\rangle, \tilde{g}^N \neq 0.$

In the above arithmetic operations, if $s = 0$ and $s' = 0$, then the single-valued trapezoidal neutrosophic numbers $\tilde{d}^N = \langle (p, q, r, 0); \sigma_{\tilde{d}^N}, \lambda_{\tilde{d}^N}, \tau_{\tilde{d}^N} \rangle$ and $\tilde{g}^N = \left\langle (p', q', r', 0); \sigma_{\tilde{g}^N}, \lambda_{\tilde{g}^N}, \tau_{\tilde{d}^N} \right\rangle$ becomes a single-valued triangular neutrosophic numbers $\tilde{d}^N = \langle (p, q, r); \sigma_{\tilde{d}^N}, \lambda_{\tilde{d}^N}, \tau_{\tilde{d}^N} \rangle$ and $\tilde{g}^N = \left\langle (p', q', r'); \sigma_{\tilde{g}^N}, \lambda_{\tilde{g}^N}, \tau_{\tilde{g}^N} \right\rangle$.

Definition 2.5 Score function of SVTrNNs [35]: Let $\tilde{d}^N = \langle (p,\ q,\ r,\ s);\ \sigma_{\tilde{d}^N},\ \lambda_{\tilde{d}^N},\ \tau_{\tilde{d}^N} \rangle$ be a single-valued trapezoidal neutrosophic number then the score function of $\tilde{d}^N$ is

$$S(\tilde{d}^N) = \frac{1}{16}[p+q+r+s]\left(2+\sigma_{\tilde{d}^N} - \lambda_{\tilde{d}^N} - \tau_{\tilde{d}^N}\right)$$

3. Formulation of neutrosophic four-dimensional fixed charge transportation problem (N4DFCTP)

Based on [38], in this paper, we have considered the single objective four-dimensional fixed charge TP under neutrosophic environment where the objective function is taken as cost of transportation along with the fixed charge and the parameters are supply, demand, conveyance and distance of routes. Let there be 'm' sources, 'n' destinations, each are connected by different routes, and in each route, various forms of conveyances are used to shipment the scout drones from sources to destinations. This study considers the four-dimensional fixed charge TP under neutrosophic environment. Here, the parameters of the problem are imprecise and signified by single-valued trapezoidal neutrosophic numbers and the distance of routes as real numbers. The DMs aim is to determine the optimal solution by choosing the appropriate vehicle and right route to transport the scout drones from sources to destinations so that the total transportation cost with fixed charges is minimized. The mathematical formulation of the problem is represented as follows:

$$(G_1)\ \text{Minimize}\ \tilde{Z}^N(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{g}\sum_{r=1}^{h} \tilde{c}^N_{ijkr} x_{ijkr} P_{ijr} + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{g}\sum_{r=1}^{h} \tilde{f}^N_{ijkr} y_{ijkr}$$

$$\text{Subject to:}\ \sum_{j=1}^{n}\sum_{k=1}^{g}\sum_{r=1}^{h} x_{ijkr} \le \tilde{a}^N_i,\ \ i = 1,\ 2,\ \dots,\ m \tag{1}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{g}\sum_{r=1}^{h} x_{ijkr} \le \tilde{b}^N_j,\ \ j = 1,\ 2,\ \dots,\ n \tag{2}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijkr} \le \tilde{e}^N_{kr},\ \ k = 1,\ 2,\ \dots,\ g;\ \ r = 1,\ 2,\ \dots,\ h \tag{3}$$

$$x_{ijkr} \ge 0,\ \ \text{for all}\ \ i,\ j,\ k,\ r \tag{4}$$

$$y_{ijkr} = \begin{cases} 1, & x_{ijkr} > 0 \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

where $\tilde{a}^N_i = (a^1_i,\ a^2_i,\ a^3_i,\ a^4_i;\ a'^1_i,\ a'^2_i,\ a'^3_i)$ for $i = 1,\ 2,\ \dots,\ m$ refers to the single-valued trapezoidal neutrosophic supply at i^{th} origin, $\tilde{b}^N_j = (b^1_j,\ b^2_j,\ b^3_j,\ b^4_j;\ b'^1_j,\ b'^2_j,\ b'^3_j)$ for $j = 1,\ 2,\ \dots,\ n$ refers to the single-valued trapezoidal neutrosophic demand at j^{th} destination and $\tilde{e}^N_{kr} = (e^1_{kr},\ e^2_{kr},\ e^3_{kr},\ e^4_{kr};\ e'^1_{kr},\ e'^2_{kr},\ e'^3_{kr})$ for $k = 1,\ 2,\ \dots,\ g$, $r = 1,\ 2,\ \dots,\ h$ refers to the single-valued trapezoidal neutrosophic capacity at k^{th} conveyance through r^{th} route. $\tilde{c}^N_{ijkr} = (c^1_{ijkr},\ c^2_{ijkr},\ c^3_{ijkr},\ c^4_{ijkr};\ c'^1_{ijkr},\ c'^2_{ijkr},\ c'^3_{ijkr})$

denotes the objective function of single valued trapezoidal neutrosophic transportation cost shipped through r^{th} route by k^{th} conveyance from i^{th} source to j^{th} destination respectively. $\tilde{f}^N_{ijkr} = (f^1_{ijkr},\ f^2_{ijkr},\ f^3_{ijkr},\ f^4_{ijkr};\ f'^1_{ijkr},\ f'^2_{ijkr},\ f'^3_{ijkr})$ refers the single-valued trapezoidal neutrosophic fixed charge shipped through r^{th} route by k^{th} conveyance from i^{th} source to j^{th} destination. P_{ijr} denotes the distance between r^{th} route from i^{th} source to j^{th} destination. x_{ijkr} denotes the scout drones shipped through r^{th} route by k^{th} conveyance from i^{th} source to j^{th} destination. y_{ijkr} denotes the binary variable with a value of 1 if the scout drones were shipped through r^{th} route by k^{th} conveyance from i^{th} source to j^{th} destination and 0 otherwise.

The problem (G_1) satisfies the feasibility condition if $\sum_{i=1}^{m} \tilde{a}^N_i \geq \sum_{j=1}^{n} \tilde{b}^N_j$ and $\sum_{k=1}^{g}\sum_{r=1}^{h} \tilde{e}^N_{kr} \geq \sum_{j=1}^{n} \tilde{b}^N_j$.

By applying the score function [35] to the problem (G_1), we get

$$(G_2)\ \text{Minimize}\ \Re\left(\tilde{Z}^N(x)\right) = \Re\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{g}\sum_{r=1}^{h}\left(c^1_{ijkr},\ c^2_{ijkr},\ c^3_{ijkr},\ c^4_{ijkr};\ c'^1_{ijkr},\ c'^2_{ijkr},\ c'^3_{ijkr}\right)x_{ijkr}P_{ijr}\right)$$

$$+\Re\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{g}\sum_{r=1}^{h}\left(f^1_{ijkr},\ f^2_{ijkr},\ f^3_{ijkr},\ f^4_{ijkr};\ f'^1_{ijkr},\ f'^2_{ijkr},\ f'^3_{ijkr}\right)y_{ijkr}\right)$$

$$\text{Subject to:}\ \sum_{j=1}^{n}\sum_{k=1}^{g}\sum_{r=1}^{h} x_{ijkr} \leq \Re\left(a^1_i,\ a^2_i,\ a^3_i,\ a^4_i;\ a'^1_i,\ a'^2_i,\ a'^3_i\right),\ \ i = 1,\ 2,\ \ldots,\ m \tag{6}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{g}\sum_{r=1}^{h} x_{ijkr} \leq \Re\left(b^1_j,\ b^2_j,\ b^3_j,\ b^4_j;\ b'^1_j,\ b'^2_j,\ b'^3_j\right),\ \ j = 1,\ 2,\ \ldots,\ n \tag{7}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijkr} \leq \Re\left(e^1_{kr},\ e^2_{kr},\ e^3_{kr},\ e^4_{kr};\ e'^1_{kr},\ e'^2_{kr},\ e'^3_{kr}\right),\ \ k = 1,\ 2,\ \ldots,\ g;\ \ r = 1,\ 2,\ \ldots,\ h \tag{8}$$

$$x_{ijkr} \geq 0,\ \ \text{for all}\ \ i,\ j,\ k,\ r \tag{9}$$

$$y_{ijkr} = \begin{cases} 1, & x_{ijkr} > 0 \\ 0, & \text{otherwise} \end{cases} \tag{10}$$

Now, we extended the theorem based on [13] for the single objective four-dimensional fixed charge TP and then we prove the solution obtained by min zero-min cost approach is an optimal solution to the problem (G_2) in the following theorem.

Theorem 3.1 If $(x^{\circ}_{ijkr},\ i = 1,\ 2,\ \ldots,\ m,\ j = 1,\ 2,\ \ldots,\ n,\ k = 1,\ 2,\ \ldots,\ g,\ r = 1,\ 2,\ \ldots,\ h)$ is an optimal solution of the problem (G_3).

$$(G_3)\ \text{Minimum}\ \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{g}\sum_{r=1}^{h}\left(c_{ijkr}P_{ijr} - u_i - v_j - w_{kr}\right)x_{ijkr} + f_{ijkr}y_{ijkr}$$

Subject to (6) to (10), where u_i, v_j and w_{kr} are any real values, then $(x^{\circ}_{ijkr},\ i = 1,\ 2,\ ...,\ m,\ j = 1,\ 2,\ ...,\ n,\ k = 1,\ 2,\ ...,\ g,\ r = 1,\ 2,\ ...,\ h)$ is an optimal solution to the problem (G_2).

Proof. Clearly, $(x^{\circ}_{ijkr},\ i = 1,\ 2,\ ...,\ m,\ j = 1,\ 2,\ ...,\ n,\ k = 1,\ 2,\ ...,\ g,\ r = 1,\ 2,\ ...,\ h)$ is a feasible solution of (G_2).

Suppose that $(x^{\circ}_{ijkr},\ i = 1,\ 2,\ ...,\ m,\ j = 1,\ 2,\ ...,\ n,\ k = 1,\ 2,\ ...,\ g,\ r = 1,\ 2,\ ...,\ h)$ is not an optimal solution of (G_2).

Then there exists a feasible solution $(l_{ijkr},\ i = 1,\ 2,\ ...,\ m,\ j = 1,\ 2,\ ...,\ n,\ k = 1,\ 2,\ ...,\ g,\ r = 1,\ 2,\ ...,\ h)$ such that

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{g}\sum_{r=1}^{h} c_{ijkr}P_{ijr}l_{ijkr} + f_{ijkr}y_{ijkr} < \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{g}\sum_{r=1}^{h} c_{ijkr}P_{ijr}x^{\circ}_{ijkr} + f_{ijkr}y_{ijkr}$$

Clearly, $(l_{ijkr},\ i = 1,\ 2,\ ...,\ m,\ j = 1,\ 2,\ ...,\ n,\ k = 1,\ 2,\ ...,\ g,\ r = 1,\ 2,\ ...,\ h)$ is also a feasible solution to the problem (G_3).

Now,

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{g}\sum_{r=1}^{h} \left(c_{ijkr}P_{ijr} - u_i - v_j - w_{kr}\right) l_{ijkr} + f_{ijkr}y_{ijkr}$$

$$= \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{g}\sum_{r=1}^{h} c_{ijkr}P_{ijr}l_{ijkr} + f_{ijkr}y_{ijkr} - \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{g}\sum_{r=1}^{h} u_i l_{ijkr} - \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{g}\sum_{r=1}^{h} v_j l_{ijkr} - \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{g}\sum_{r=1}^{h} w_{kr} l_{ijkr}$$

$$< \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{g}\sum_{r=1}^{h} c_{ijkr}P_{ijr}l_{ijkr} + f_{ijkr}y_{ijkr} - \sum_{i=1}^{m} u_i a_i - \sum_{j=1}^{n} v_j b_j - \sum_{k=1}^{g}\sum_{r=1}^{h} w_{kr} e_{kr}$$

$$= \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{g}\sum_{r=1}^{h} \left(c_{ijkr}P_{ijr} - u_i - v_j - w_{kr}\right) x^{\circ}_{ijkr} + f_{ijkr}y_{ijkr}$$

which contradicts $(x^{\circ}_{ijkr},\ i = 1,\ 2,\ ...,\ m,\ j = 1,\ 2,\ ...,\ n,\ k = 1,\ 2,\ ...,\ g,\ r = 1,\ 2,\ ...,\ h)$ is an optimal solution of (G_3).

Therefore, $(x^{\circ}_{ijkr},\ i = 1,\ 2,\ ...,\ m,\ j = 1,\ 2,\ ...,\ n,\ k = 1,\ 2,\ ...,\ g,\ r = 1,\ 2,\ ...,\ h)$ is an optimal solution of (G_2).

Hence the theorem.

The above theorem demonstrates that the solution obtained by min zero-min cost approach is an optimal solution to the problem (G_2) which is applied in the proposed approach. The procedure of the proposed approach is illustrated in the following section.

4. Solution approach

In this section, we have proposed the min zero-min cost approach for obtaining the optimal solution to single objective 4DFCTP under neutrosophic environment. Figure 1 represents the flowchart of the proposed min zero-min cost approach. The procedure of the proposed approach proceeds as follows:

Step 1 Consider the neutrosophic four-dimensional fixed charge TP (G_1). Transform the problem (G_1) into its equivalent deterministic problem (G_2) using the score function [35]. Ensure whether the problem (G_2) is balance or not, if not, make it balance.

Step 2 Construct Supply (S)-Demand (D) table from the problem (G_2) without considering the fixed charges where the rows are supply and columns are demands.

Step 3 Subtract the minimum cost element of each row from all the elements of the row. Ensure that each column contains at least one zero. If not, subtract the minimum cost element of each column from all the elements of the column.

Step 4 Using the zero cost cells, check whether each supply is assigned to its corresponding demands. If it is satisfied, proceed to Step 6 (i), otherwise move to Step 5.

Step 5 In the reduced problem draw the minimum number of horizontal and vertical lines through the supply, demand and conveyance that contains zeros. Subtract the smallest uncovered entry from all uncovered entry and add it to the intersection entry. Repeat the procedure until all the supply/demand/conveyance is assigned to its corresponding demands/conveyances/supplies.

Step 6

(i) Construct Demand (D)-Conveyance (E) table from Step 5 where the rows are demands and columns are conveyances. Repeat step 5 until all the demand is assigned to its corresponding conveyances and then move to Step 6 (ii).

(ii) Construct Conveyance (E)-Supply (S) table from the above step where the rows are conveyances and columns are supply. Repeat Step 5 until all the conveyance is assigned to its corresponding supplies and then move to Step 7.

Step 7 Construct S-D-E table and then choose supply, demand and conveyance with least number of zeros. Next, assign the maximum possible to the zero cell with the least cost. If there is a tie, select one. Repeat the step until all supplies, demands and conveyances are fully utilized.

Step 8 Now represent all the allocations in the problem (G_2). This allocations leads to an optimal solution for the problem (G_2) by Theorem 1. Calculate the neutrosophic optimal solution for the problem (G_1) using the optimal allocations of problem (G_2).

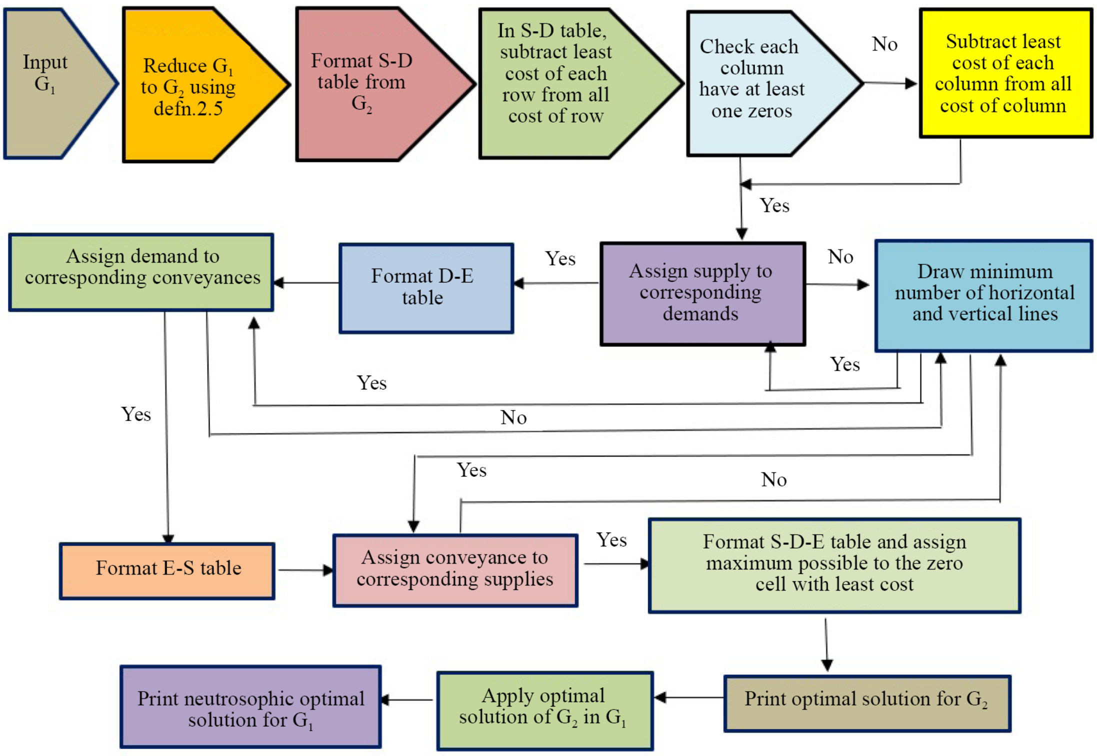

Figure 1. Flowchart of the proposed min zero-min cost approach

The proposed approach for solving the problem (G_1) is demonstrated using two real-life based examples in which for Example 5.1, the data of transportation cost, fixed charge, conveyance, distance of route are considered from [38] and supply and demand are considered due to our preference and for Example 5.2, the data of transportation cost, fixed charge, supply, demand, conveyance, distance of route are considered from [38] in the following section. The primary objective for solving the problem (G_1) is to determine the optimal solution using the proposed approach to transport the products from 'm' sources to 'n' destinations by 'k' conveyances through 'p' routes. In addition to the above, the transportation cost with fixed charges are minimized by choosing the right vehicle and best path in such a way to prevent the loss of damage to products.

5. Numerical example

In this section, we have taken two real-life based examples in which Example 1 is taken from [38] which demonstrates that the scout drones are transported from two supplies to two destinations by two vehicles through two routes. Example 2 is taken from [38] and considered as unbalanced problem which demonstrates that the vaccines are transported from two supplies to two destinations by two vehicles through two routes. In these problems, the parameters such as transportation cost, supply at sources, demand at destinations and capacity at conveyances are considered as fully single-valued trapezoidal neutrosophic numbers and distance of routes in the objective function as real numbers.

Example 5.1

The Aerial Drobotics-Agriculture Drone Sprayer, Namakkal, Tamil Nadu, India supplies scout drones from two origins in Madurai (S_1) and Chennai (S_2) to two destinations located in Coimbatore (D_1) and Udumalpet (D_2). The Aerial Drobotics transports these scout drones through two different routes such as NH-83 (P_1) and NH-79 (P_2) by two forms of conveyance such as trucks (E_1) and vans (E_2). On these NH roads, two different toll stations collect toll taxes for different types of vehicles which are termed as fixed charges (F_1) and (F_2). Here the goal of Aerial Drobotics is to minimize the transportation cost by shipping the scout drones from the two origins to the two destinations by the two types of conveyances through the two various routes. Due to the state of the market, changes in the climate, scout drone conditions, vehicle conditions, the parameters such as transportation cost $(\tilde{c}^N_{ijkr})$, fixed charge $(\tilde{f}^N_{ijkr})$, supply $(\tilde{a}^N_i)$, demand $(\tilde{b}^N_j)$ and conveyance $(\tilde{e}^N_{kr})$ are all SVTrNNs. The distance of routes (P_{ijr}) are represented in real numbers. Table 2 shows the neutrosophic data of transportation cost $(\tilde{c}^N_{ijkr})$ with fixed charge $(\tilde{f}^N_{ijkr})$ and distance of routes (P_{ijr}).

Table 2. Neutrosophic transportation cost $(\tilde{c}^N_{ijkr})$, fixed charge $(\tilde{f}^N_{ijkr})$ and distance of routes (P_{ijr})

D_j	D_1				D_2				Supply
S_i/E_{kr}	E_{11}	E_{21}	E_{12}	E_{22}	E_{11}	E_{21}	E_{12}	E_{22}	
S_1	$\tilde{c}^N_{1111}(P_{111}) + \tilde{f}^N_{1111}$	$\tilde{c}^N_{1121}(P_{111}) + \tilde{f}^N_{1121}$	$\tilde{c}^N_{1112}(P_{112}) + \tilde{f}^N_{1112}$	$\tilde{c}^N_{1122}(P_{112}) + \tilde{f}^N_{1122}$	$\tilde{c}^N_{1211}(P_{121}) + \tilde{f}^N_{1211}$	$\tilde{c}^N_{1221}(P_{121}) + \tilde{f}^N_{1221}$	$\tilde{c}^N_{1212}(P_{122}) + \tilde{f}^N_{1212}$	$\tilde{c}^N_{1222}(P_{122}) + \tilde{f}^N_{1222}$	$\tilde{a}^N_1$
S_2	$\tilde{c}^N_{2111}(P_{211}) + \tilde{f}^N_{2111}$	$\tilde{c}^N_{2121}(P_{211}) + \tilde{f}^N_{2121}$	$\tilde{c}^N_{2112}(P_{212}) + \tilde{f}^N_{2112}$	$\tilde{c}^N_{2122}(P_{212}) + \tilde{f}^N_{2122}$	$\tilde{c}^N_{2211}(P_{221}) + \tilde{f}^N_{2211}$	$\tilde{c}^N_{2221}(P_{221}) + \tilde{f}^N_{2221}$	$\tilde{c}^N_{2212}(P_{222}) + \tilde{f}^N_{2212}$	$\tilde{c}^N_{2222}(P_{222}) + \tilde{f}^N_{2222}$	$\tilde{a}^N_2$
Conveyance	$\tilde{e}^N_{11}$	$\tilde{e}^N_{21}$	$\tilde{e}^N_{12}$	$\tilde{e}^N_{22}$	$\tilde{e}^N_{11}$	$\tilde{e}^N_{21}$	$\tilde{e}^N_{12}$	$\tilde{e}^N_{22}$	
Demand	$\tilde{b}^N_1$				$\tilde{b}^N_2$				

Supplies: $\tilde{a}^N_1 = (14,\ 16,\ 21,\ 23;\ 0.7,\ 0.5,\ 0.3)$ and $\tilde{a}^N_2 = (7,\ 9,\ 12,\ 15;\ 0.8,\ 0.2,\ 0.1)$.

Demands: $\tilde{b}^N_1 = (14,\ 17,\ 21,\ 28;\ 0.8,\ 0.2,\ 0.6)$ and $\tilde{b}^N_2 = (5,\ 9,\ 11,\ 13;\ 0.8,\ 0.2,\ 0.1)$.

Conveyances: $\tilde{e}^N_{11} = (3,\ 5,\ 6,\ 8;\ 0.7,\ 0.3,\ 0.2)$, $\tilde{e}^N_{12} = (4,\ 5,\ 6,\ 7;\ 0.9,\ 0.1,\ 0.2)$, $\tilde{e}^N_{21} = (4,\ 6,\ 7,\ 9;\ 0.8,\ 0.2,\ 0.2)$ and $\tilde{e}^N_{22} = (5,\ 7,\ 8,\ 10;\ 0.8,\ 0.1,\ 0.2)$.

Distance of routes: $P_{111} = 12$, $P_{112} = 34$, $P_{121} = 18$, $P_{122} = 56$, $P_{211} = 45$, $P_{212} = 16$, $P_{221} = 37$ and $P_{222} = 48$.

Transportation cost:
$\tilde{c}^N_{1111} = (3, 5, 6, 8; 0.9, 0.3, 0.2)$; $\tilde{c}^N_{1211} = (5, 6, 7, 8; 0.9, 0.1, 0.2)$; $\tilde{c}^N_{1121} = (4, 5, 6, 7; 0.8, 0.2, 0.1)$;
$\tilde{c}^N_{1221} = (2, 5, 6, 9; 0.7, 0.3, 0.1)$; $\tilde{c}^N_{1112} = (2, 4, 6, 9; 0.7, 0.2, 0.1)$; $\tilde{c}^N_{1212} = (4, 6, 9, 11; 0.9, 0.2, 0.1)$;
$\tilde{c}^N_{1122} = (5, 7, 9, 11; 0.8, 0.3, 0.2)$; $\tilde{c}^N_{1222} = (4, 6, 9, 11; 0.9, 0.2, 0.1)$; $\tilde{c}^N_{2111} = (5, 9, 11, 13; 0.8, 0.2, 0.1)$;
$\tilde{c}^N_{2211} = (7, 9, 11, 13; 0.9, 0.1, 0.3)$; $\tilde{c}^N_{2121} = (2, 3, 5, 7; 0.7, 0.2, 0.1)$; $\tilde{c}^N_{2221} = (3, 4, 5, 6; 0.7, 0.1, 0.3)$;
$\tilde{c}^N_{2112} = (6, 7, 8, 9; 0.8, 0.3, 0.1)$; $\tilde{c}^N_{2212} = (6, 8, 10, 12; 0.7, 0.3, 0.2)$; $\tilde{c}^N_{2122} = (4, 6, 8, 10; 0.9, 0.1, 0.2)$;
$\tilde{c}^N_{2222} = (4, 5, 6, 8; 0.8, 0.1, 0.3)$.
Fixed charge:
$\tilde{f}^N_{1111} = (2, 3, 4, 5; 0.9, 0.2, 0.1)$; $\tilde{f}^N_{1211} = (5, 6, 7, 8; 0.7, 0.3, 0.1)$; $\tilde{f}^N_{1121} = (4, 5, 6, 8; 0.8, 0.2, 0.2)$;
$\tilde{f}^N_{1221} = (3, 4, 5, 6; 0.7, 0.1, 0.1)$; $\tilde{f}^N_{1112} = (3, 5, 7, 8; 0.7, 0.2, 0.3)$; $\tilde{f}^N_{1212} = (6, 7, 8, 9; 0.7, 0.2, 0.2)$;
$\tilde{f}^N_{1122} = (3, 4, 5, 6; 0.8, 0.1, 0.2)$; $\tilde{f}^N_{1222} = (4, 5, 6, 7; 0.8, 0.3, 0.1)$; $\tilde{f}^N_{2111} = (6, 7, 8, 10; 0.8, 0.1, 0.1)$;
$\tilde{f}^N_{2211} = (3, 5, 7, 10; 0.9, 0.1, 0.1)$; $\tilde{f}^N_{2121} = (4, 5, 7, 9; 0.7, 0.2, 0.1)$; $\tilde{f}^N_{2221} = (4, 5, 7, 9; 0.8, 0.2, 0.1)$;
$\tilde{f}^N_{2112} = (5, 7, 8, 10; 0.7, 0.2, 0.1)$; $\tilde{f}^N_{2212} = (5, 6, 7, 9; 0.9, 0.2, 0.1)$; $\tilde{f}^N_{2122} = (4, 6, 7, 8; 0.8, 0.2, 0.3)$;
$\tilde{f}^N_{2222} = (5, 6, 7, 9; 0.8, 0.3, 0.2)$.

Using Step 1, transform the problem (G_1) into its equivalent deterministic problem (G_2) using the score function are given as follows (Table 3):

Table 3. Deterministic c_{ijkr} and f_{ijkr} using score function

	D_1				D_2				Supply
	E_{11}	E_{21}	E_{12}	E_{22}	E_{11}	E_{21}	E_{12}	E_{22}	
S_1	3(12) + 2	3(12) + 3	3(34) + 3	5(34) + 3	4(18) + 4	3(18) + 3	5(56) + 4	5(56) + 3	9
S_2	6(45) + 5	3(45) + 4	5(16) + 5	5(16) + 4	6(37) + 4	3(37) + 4	5(48) + 4	3(48) + 4	7
Conveyance	3	4	4	5	3	4	4	5	
Demand	10				6				

As in Step 1, the reduced deterministic problem (G_2) is balanced. By Step 2, the S-D table is constructed without considering the fixed charges is shown below (Table 4).

Table 4. S-D table

	D_1				D_2				Supply
	E_{11}	E_{21}	E_{12}	E_{22}	E_{11}	E_{21}	E_{12}	E_{22}	
S_1	36	36	102	170	72	54	280	280	9
S_2	270	135	80	80	222	111	240	144	7
Demand	10				6				

Using Step 3, by subtracting the least cost element of each row from all the elements of row and by subtracting the least cost element of each column from all the elements of column is given below (Table 5).

Table 5. Reduced S-D table

	D_1				D_2				Supply
	E_{11}	E_{21}	E_{12}	E_{22}	E_{11}	E_{21}	E_{12}	E_{22}	
S_1	0	0	66	134	18	0	226	226	9
S_2	190	55	0	0	124	13	142	46	7
Demand		10				6			

Using the zero cost cells, each supply is less than the sum of the demands ($9 < 16$ and $7 < 10$). From Table 5, it is clear that each supply is assigned to its corresponding demands, so we move to the next step. As in Step 6 (i), the D-E table is constructed from Table 5 is shown below (Table 6).

Table 6. D-E table

	S_1				S_2				Demand
	E_{11}	E_{21}	E_{12}	E_{22}	E_{11}	E_{21}	E_{12}	E_{22}	
D_1	0	0	66	134	190	55	0	0	10
D_2	18	0	226	226	124	13	142	46	6
Conveyance	3	4	4	5	3	4	4	5	

From Table 6, using the zero cost cells, the demand of second row exceeds the conveyance of second column. So, we cover the zeros with one horizontal line (D_1) and one vertical line (E_{21}). Then by subtracting the smallest uncovered element (i.e., 18) from all the uncovered element and add the smallest uncovered element to the intersection element is shown below (Table 7).

Table 7. Reduced D-E table

	S_1				S_2				Demand
	E_{11}	E_{21}	E_{12}	E_{22}	E_{11}	E_{21}	E_{12}	E_{22}	
D_1	0	18	66	134	190	73	0	0	10
D_2	0	0	208	208	106	13	124	28	6
Conveyance	3	4	4	5	3	4	4	5	

Using the zero cost cells, each demand is less than the sum of the conveyances. From Table 7, it is clear that each demand is assigned to its corresponding conveyances, so we move to the next step. By Step 6 (ii), the E-S table is constructed from Table 7 is shown below (Table 8).

Table 8. E-S table

	D_1				D_2				Demand
	E_{11}	E_{21}	E_{12}	E_{22}	E_{11}	E_{21}	E_{12}	E_{22}	
S_1	0	18	66	134	0	0	208	208	9
S_2	190	73	0	0	106	13	124	28	7
Conveyance	3	4	4	5	3	4	4	5	

Using the zero cost cells, each conveyance is less than the sum of the supplies. From the above table, it is clear that each conveyance is assigned to its corresponding supplies, so we move to the next step. By Step 7, the S-D-E table is constructed and the obtained optimal allotment table is shown below (Table 9).

Table 9. S-D-E table

	D_1				D_2				Supply
	E_{11}	E_{21}	E_{12}	E_{22}	E_{11}	E_{21}	E_{12}	E_{22}	
S_1	0 **(1)**	18	66 **(2)**	134	0 **(2)**	0 **(4)**	208	208	9
S_2	190	73	0 **(2)**	0 **(5)**	106	13	124	28	7
Conveyance	3	4	4	5	3	4	4	5	
Demand	10				6				

As in Step 8, to obtain the optimal transportation cost of the problem (G_2) we have allocated the optimal allocations of Table 9 in the problem (G_2) is shown in Table 10.

Table 10. Optimal allotment table

	D_1 (Coimbatore)				D_2 (Udumalpet)				Supply
	E_{11}	E_{21}	E_{12}	E_{22}	E_{11}	E_{21}	E_{12}	E_{22}	
	(Truck and NH-83)	(Van and NH-83)	(Truck and NH-79)	(Van and NH-79)	(Truck and NH-83)	(Van and NH-83)	(Truck and NH-79)	(Van and NH-79)	
S_1 (Madurai)	3(12) + 2 **(1)**	3(12) + 3	3(34) + 3 **(2)**	5(34) + 3	4(18) + 4 **(2)**	3(18) + 3 **(4)**	5(56) + 4	5(56) + 3	9
S_2 (Chennai)	6(45) + 5	3(45) + 4	5(16) + 5 **(2)**	5(16) + 4 **(5)**	6(37) + 4	3(37) + 4	5(48) + 4	3(48) + 4	7
Conveyance	3	4	4	5	3	4	4	5	
Demand	10				6				

Results and discussions

Nowadays, the shipment of products from sources to the destinations by road is getting more important due to globalization. While transporting the products through vehicles are of finite capacity plying on the roads from source to destination there may be many available routes connecting the cities. In some routes for maintenance, fixed charges such as toll taxes, parking fees, permit fees, and so on are collected. Now the DM needs to decide the amount of products to be transported from various sources to different destinations by the suitable vehicle through the suitable route to minimize the loss of damage in the products, the transportation cost and fixed cost satisfying the availability and requirements of each source and destination of the problem. From Table 10, the optimal allotments for Example 1 are picturized as network diagram in Figure 2.

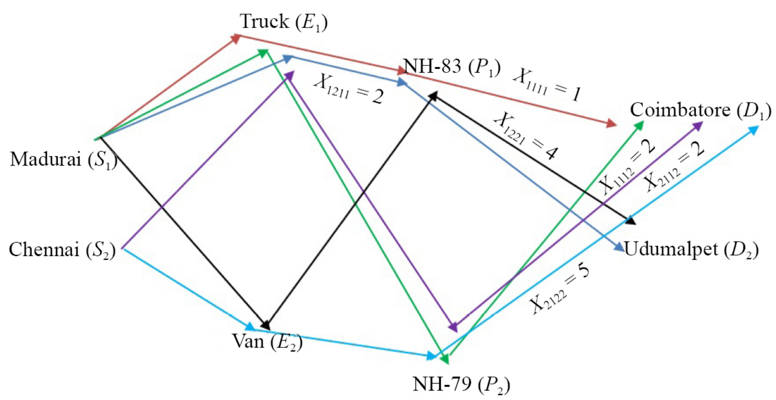

Figure 2. Network diagram of optimal allotment for Example 1

From Figure 2, it is clear that the number of scout drones transported from the source by different vehicles through various routes to reach the destination: $S_1 \to E_1 \to P_1 \to D_1$ is 1 (i.e., $x_{1111} = 1$); $S_1 \to E_1 \to P_1 \to D_2$ is 2 (i.e., $x_{1112} = 2$); $S_1 \to E_2 \to P_1 \to D_1$ is 2 (i.e., $x_{1211} = 2$); $S_1 \to E_2 \to P_2 \to D_1$ is 4 (i.e., $x_{1221} = 4$), $S_2 \to E_1 \to P_1 \to D_2$ is 2 (i.e., $x_{2112} = 2$); $S_2 \to E_1 \to P_2 \to D_2$ is 5 (i.e., $x_{2122} = 5$). From Table 10 and Figure 2, it is observed that 3 scout drones are shipped from source to destination by truck through NH-83, 4 scout drones by truck through NH-79, 4 scout drones by van through NH-83 and 5 scout drones by van through NH-79. From this analysis, we conclude that the DM transported the maximum number of scout drones by the vehicle (van) through route (NH-79) which results in minimal loss of damage in products at minimal cost with fixed charge. Finally, the optimal transportation cost with fixed charge is 1,181 and the neutrosophic optimal transportation cost with fixed charge is (79, 120, 150, 191; 0.7, 0.3, 0.3).

Example 5.2

An Integrated Vaccine Park (IVP) in Chengalpet, Tamil Nadu, India supplies two different types of vaccines, namely Covaxin and Covishield from two origins in Chennai (S_1) and Vellore (S_2) to two destinations located in Trichy (D_1) and Coonoor (D_2). IVP transports these vaccines through two different routes such as NH-38 (P_1) and NH-45 (P_2) by two forms of conveyances such as refrigerated trucks (E_1) and refrigerated vans (E_2). On these NH roads, two different toll stations collect toll taxes for different types of vehicles which are termed as fixed charges (F_1) and (F_2). Here the goal of IVP is to minimize the transportation cost by shipping the vaccines from the two origins to the two destinations by the two types of conveyances through the two various routes. Due to the state of the market, changes in the climate, scout drone conditions, vehicle conditions, the parameters such as transportation cost ($\tilde{c}^N_{ijkr}$), fixed charge ($\tilde{f}^N_{ijkr}$), supply ($\tilde{a}^N_i$), demand ($\tilde{b}^N_j$) and conveyance ($\tilde{e}^N_{kr}$) are all SVTrNNs. The distance of routes (P_{ijr}) are represented in real numbers. Table 11 shows the neutrosophic data of transportation cost ($\tilde{c}^N_{ijkr}$) with fixed charge ($\tilde{f}^N_{ijkr}$) and distance of routes (P_{ijr}).

Table 11. Neutrosophic transportation cost ($\tilde{c}^N_{ijkr}$), fixed charge ($\tilde{f}^N_{ijkr}$) and distance of routes (P_{ijr})

D_j	D_1				D_2				Supply
S_i/E_{kr}	E_{11}	E_{21}	E_{12}	E_{22}	E_{11}	E_{21}	E_{12}	E_{22}	
S_1	$\tilde{c}^N_{1111}(P_{111}) + \tilde{f}^N_{1111}$	$\tilde{c}^N_{1121}(P_{111}) + \tilde{f}^N_{1121}$	$\tilde{c}^N_{1112}(P_{112}) + \tilde{f}^N_{1112}$	$\tilde{c}^N_{1122}(P_{112}) + \tilde{f}^N_{1122}$	$\tilde{c}^N_{1211}(P_{121}) + \tilde{f}^N_{1211}$	$\tilde{c}^N_{1221}(P_{121}) + \tilde{f}^N_{1221}$	$\tilde{c}^N_{1212}(P_{122}) + \tilde{f}^N_{1212}$	$\tilde{c}^N_{1222}(P_{122}) + \tilde{f}^N_{1222}$	$\tilde{a}^N_1$
S_2	$\tilde{c}^N_{2111}(P_{211}) + \tilde{f}^N_{2111}$	$\tilde{c}^N_{2121}(P_{211}) + \tilde{f}^N_{2121}$	$\tilde{c}^N_{2112}(P_{212}) + \tilde{f}^N_{2112}$	$\tilde{c}^N_{2122}(P_{212}) + \tilde{f}^N_{2122}$	$\tilde{c}^N_{2211}(P_{221}) + \tilde{f}^N_{2211}$	$\tilde{c}^N_{2221}(P_{221}) + \tilde{f}^N_{2221}$	$\tilde{c}^N_{2212}(P_{222}) + \tilde{f}^N_{2212}$	$\tilde{c}^N_{2222}(P_{222}) + \tilde{f}^N_{2222}$	$\tilde{a}^N_2$
Conveyance	$\tilde{e}^N_{11}$	$\tilde{e}^N_{21}$	$\tilde{e}^N_{12}$	$\tilde{e}^N_{22}$	$\tilde{e}^N_{11}$	$\tilde{e}^N_{21}$	$\tilde{e}^N_{12}$	$\tilde{e}^N_{22}$	
Demand	$\tilde{b}^N_1$				$\tilde{b}^N_2$				

Supplies: $\tilde{a}^N_1 = (5,\ 7,\ 9,\ 11;\ 0.9,\ 0.1,\ 0.1)$ and $\tilde{a}^N_2 = (6,\ 8,\ 10,\ 12;\ 0.8,\ 0.1,\ 0.2)$.

Demands: $\tilde{b}^N_1 = (4,\ 6,\ 8,\ 9;\ 0.7,\ 0.1,\ 0.2)$ and $\tilde{b}^N_2 = (5,\ 6,\ 7,\ 8;\ 0.9,\ 0.2,\ 0.1)$.

Conveyances: $\tilde{e}^N_{11} = (3,\ 5,\ 6,\ 8;\ 0.7,\ 0.3,\ 0.2)$, $\tilde{e}^N_{12} = (4,\ 5,\ 6,\ 7;\ 0.9,\ 0.1,\ 0.2)$, $\tilde{e}^N_{21} = (4,\ 6,\ 7,\ 9;\ 0.8,\ 0.2,\ 0.2)$ and $\tilde{e}^N_{22} = (5,\ 7,\ 8,\ 10;\ 0.8,\ 0.1,\ 0.2)$.

Distance of routes: $P_{111} = 12$, $P_{112} = 34$, $P_{121} = 18$, $P_{122} = 56$, $P_{211} = 45$, $P_{212} = 16$, $P_{221} = 37$ and $P_{222} = 48$.

Transportation cost:

$\tilde{c}^N_{1111} = (3,\ 5,\ 6,\ 8;\ 0.9,\ 0.3,\ 0.2)$; $\tilde{c}^N_{1211} = (5,\ 6,\ 7,\ 8;\ 0.9,\ 0.1,\ 0.2)$; $\tilde{c}^N_{1121} = (4,\ 5,\ 6,\ 7;\ 0.8,\ 0.2,\ 0.1)$; $\tilde{c}^N_{1221} = (2,\ 5,\ 6,\ 9;\ 0.7,\ 0.3,\ 0.1)$; $\tilde{c}^N_{1112} = (2,\ 4,\ 6,\ 9;\ 0.7,\ 0.2,\ 0.1)$; $\tilde{c}^N_{1212} = (4,\ 6,\ 9,\ 11;\ 0.9,\ 0.2,\ 0.1)$; $\tilde{c}^N_{1122} = (5,\ 7,\ 9,\ 11;\ 0.8,\ 0.3,\ 0.2)$; $\tilde{c}^N_{1222} = (4,\ 6,\ 9,\ 11;\ 0.9,\ 0.2,\ 0.1)$; $\tilde{c}^N_{2111} = (5,\ 9,\ 11,\ 13;\ 0.8,\ 0.2,\ 0.1)$; $\tilde{c}^N_{2211} = (7,\ 9,\ 11,\ 13;\ 0.9,\ 0.1,\ 0.3)$; $\tilde{c}^N_{2121} = (2,\ 3,\ 5,\ 7;\ 0.7,\ 0.2,\ 0.1)$; $\tilde{c}^N_{2221} = (3,\ 4,\ 5,\ 6;\ 0.7,\ 0.1,\ 0.3)$; $\tilde{c}^N_{2112} = (6,\ 7,\ 8,\ 9;\ 0.8,\ 0.3,\ 0.1)$; $\tilde{c}^N_{2212} = (6,\ 8,\ 10,\ 12;\ 0.7,\ 0.3,\ 0.2)$; $\tilde{c}^N_{2122} = (4,\ 6,\ 8,\ 10;\ 0.9,\ 0.1,\ 0.2)$; $\tilde{c}^N_{2222} = (4,\ 5,\ 6,\ 8;\ 0.8,\ 0.1,\ 0.3)$.

Fixed charge:

$\tilde{f}^N_{1111} = (2,\ 3,\ 4,\ 5;\ 0.9,\ 0.2,\ 0.1)$; $\tilde{f}^N_{1211} = (5,\ 6,\ 7,\ 8;\ 0.7,\ 0.3,\ 0.1)$; $\tilde{f}^N_{1121} = (4,\ 5,\ 6,\ 8;\ 0.8,\ 0.2,\ 0.2)$; $\tilde{f}^N_{1221} = (3,\ 4,\ 5,\ 6;\ 0.7,\ 0.1,\ 0.1)$; $\tilde{f}^N_{1112} = (3,\ 5,\ 7,\ 8;\ 0.7,\ 0.2,\ 0.3)$; $\tilde{f}^N_{1212} = (6,\ 7,\ 8,\ 9;\ 0.7,\ 0.2,\ 0.2)$; $\tilde{f}^N_{1122} = (3,\ 4,\ 5,\ 6;\ 0.8,\ 0.1,\ 0.2)$; $\tilde{f}^N_{1222} = (4,\ 5,\ 6,\ 7;\ 0.8,\ 0.3,\ 0.1)$; $\tilde{f}^N_{2111} = (6,\ 7,\ 8,\ 10;\ 0.8,\ 0.1,\ 0.1)$; $\tilde{f}^N_{2211} = (3,\ 5,\ 7,\ 10;\ 0.9,\ 0.1,\ 0.1)$; $\tilde{f}^N_{2121} = (4,\ 5,\ 7,\ 9;\ 0.7,\ 0.2,\ 0.1)$; $\tilde{f}^N_{2221} = (4,\ 5,\ 7,\ 9;\ 0.8,\ 0.2,\ 0.1)$; $\tilde{f}^N_{2112} = (5,\ 7,\ 8,\ 10;\ 0.7,\ 0.2,\ 0.1)$; $\tilde{f}^N_{2212} = (5,\ 6,\ 7,\ 9;\ 0.9,\ 0.2,\ 0.1)$; $\tilde{f}^N_{2122} = (4,\ 6,\ 7,\ 8;\ 0.8,\ 0.2,\ 0.3)$; $\tilde{f}^N_{2222} = (5,\ 6,\ 7,\ 9;\ 0.8,\ 0.3,\ 0.2)$.

Using Step 1, transform the problem (G_1) into its equivalent deterministic problem (G_2) using the score function are given as follows (Table 12):

Table 12. Deterministic c_{ijkr} and f_{ijkr} using score function

	D_1				D_2				Supply
	E_{11}	E_{21}	E_{12}	E_{22}	E_{11}	E_{21}	E_{12}	E_{22}	
S_1	3(12) + 2	3(12) + 3	3(34) + 3	5(34) + 3	4(18) + 4	3(18) + 3	5(56) + 4	5(56) + 3	5
S_2	6(45) + 5	3(45) + 4	5(16) + 5	5(16) + 4	6(37) + 4	3(37) + 4	5(48) + 4	3(48) + 4	6
Conveyance	3	4	4	5	3	4	4	5	
Demand	4				4				

As in Step 1, the reduced deterministic problem is unbalanced. So, balance it (Table 13).

Table 13. Deterministic c_{ijkr} and f_{ijkr} using score function

	D_1				D_2				D_3				Supply
	E_{11}	E_{21}	E_{12}	E_{22}	E_{11}	E_{21}	E_{12}	E_{22}	E_{11}	E_{21}	E_{12}	E_{22}	
S_1	3(12) + 2	3(12) + 3	3(34) + 3	5(34) + 3	4(18) + 4	3(18) + 3	5(56) + 4	5(56) + 3	0	0	0	0	5
S_2	6(45) + 5	3(45) + 4	5(16) + 5	5(16) + 4	6(37) + 4	3(37) + 4	5(48) + 4	3(48) + 4	0	0	0	0	6
S_3	0	0	0	0	0	0	0	0	0	0	0	0	5
Conveyance	3	4	4	5	3	4	4	5	3	4	4	5	
Demand	4				4				8				

By Step 2, the S-D table is constructed without considering the fixed charges is shown below (Table 14).

Table 14. S-D table

	D_1				D_2				D_3				Supply
	E_{11}	E_{21}	E_{12}	E_{22}	E_{11}	E_{21}	E_{12}	E_{22}	E_{11}	E_{21}	E_{12}	E_{22}	
S_1	36	36	102	170	72	54	280	280	0	0	0	0	5
S_2	270	135	80	80	222	111	240	144	0	0	0	0	6
S_3	0	0	0	0	0	0	0	0	0	0	0	0	5
Demand	4				4				8				

Using the zero cost cells, each supply is less than the sum of the demands. From Table 14, it is clear that each supply is assigned to its corresponding demands, so we move to the next step. As in Step 6 (i), the D-E table is constructed from Table 14 is shown below (Table 15).

Table 15. D-E table

	S_1				S_2				S_3				Demand
	E_{11}	E_{21}	E_{12}	E_{22}	E_{11}	E_{21}	E_{12}	E_{22}	E_{11}	E_{21}	E_{12}	E_{22}	
D_1	36	36	102	170	270	135	80	80	0	0	0	0	4
D_2	72	54	280	280	222	111	240	144	0	0	0	0	4
D_3	0	0	0	0	0	0	0	0	0	0	0	0	8
Conveyance	3	4	4	5	3	4	4	5	3	4	4	5	

Using the zero cost cells, each demand is less than the sum of the conveyances. From Table 15, it is clear that each demand is assigned to its corresponding conveyances, so we move to the next step. By Step 6 (ii), the E-S table is constructed from Table 15 is shown below (Table 16).

Table 16. E-S table

	D_1				D_2				D_3				Supply
	E_{11}	E_{21}	E_{12}	E_{22}	E_{11}	E_{21}	E_{12}	E_{22}	E_{11}	E_{21}	E_{12}	E_{22}	
S_1	36	36	102	170	72	54	280	280	0	0	0	0	4
S_2	270	135	80	80	222	111	240	144	0	0	0	0	4
S_3	0	0	0	0	0	0	0	0	0	0	0	0	8
Conveyance	3	4	4	5	3	4	4	5	3	4	4	5	

Using the zero cost cells, each conveyance is less than the sum of the supplies. From Table 16, it is clear that each conveyance is assigned to its corresponding supplies, so we move to the next step. By Step 7, the S-D-E table is constructed and the obtained optimal allotment table is shown below Table 17.

Table 17. S-D-E table

	D_1				D_2				D_3				Supply
	E_{11}	E_{21}	E_{12}	E_{22}	E_{11}	E_{21}	E_{12}	E_{22}	E_{11}	E_{21}	E_{12}	E_{22}	
S_1	36	36 **(3)**	102	170	72	54	280	280	0	0	0 **(2)**	0	5
S_2	270	135	80	80	222	111	240	144	0 **(3)**	0	0 **(2)**	0 **(1)**	6
S_3	0	0 **(1)**	0	80	0	0	0	0 **(4)**	0	0	0	0	5
Conveyance	3	4	4	5	3	4	4	5	3	4	4	5	
Demand	4				4				8				

As in Step 8, to obtain the optimal transportation cost of the problem (G_2) we have allocated the optimal allocations of Table 17 in the problem (G_2) is shown in Table 18.

Table 18. Optimal allotment table

	D_1 (Trichy)				D_2 (Coonoor)				D_3 (Coimbatore)				
	E_{11}	E_{21}	E_{12}	E_{22}	E_{11}	E_{21}	E_{12}	E_{22}	E_{11}	E_{21}	E_{12}	E_{22}	Supply
	(Truck and NH-83)	(Van and NH-83)	(Truck and NH-79)	(Van and NH-79)	(Truck and NH-83)	(Van and NH-83)	(Truck and NH-79)	(Van and NH-79)	(Truck and NH-83)	(Van and NH-83)	(Truck and NH-79)	(Van and NH-79)	
S_1 (Chennai)	3(12) + 2	3(12) + 3 **(3)**	3(34) + 3	5(34) + 3	4(18) + 4	3(18) + 3	5(56) + 4	5(56) + 3	0	0	0 **(2)**	0	5
S_2 (Vellore)	6(45) + 5	3(45) + 4	5(16) + 5	5(16) + 4	6(37) + 4	3(37) + 4	5(48) + 4	3(48) + 4	0 **(3)**	0	0 **(2)**	0 **(1)**	6
S_3 (Salem)	0	0 **(1)**	0	0	0	0	0	0 **(4)**	0	0	0	0	5
Conveyance	3	4	4	5	3	4	4	5	3	4	4	5	
Demand	4				4				8				

Results and discussions

From Table 18, the optimal allotments for Example 2 is picturized as network diagram in Figure 3.

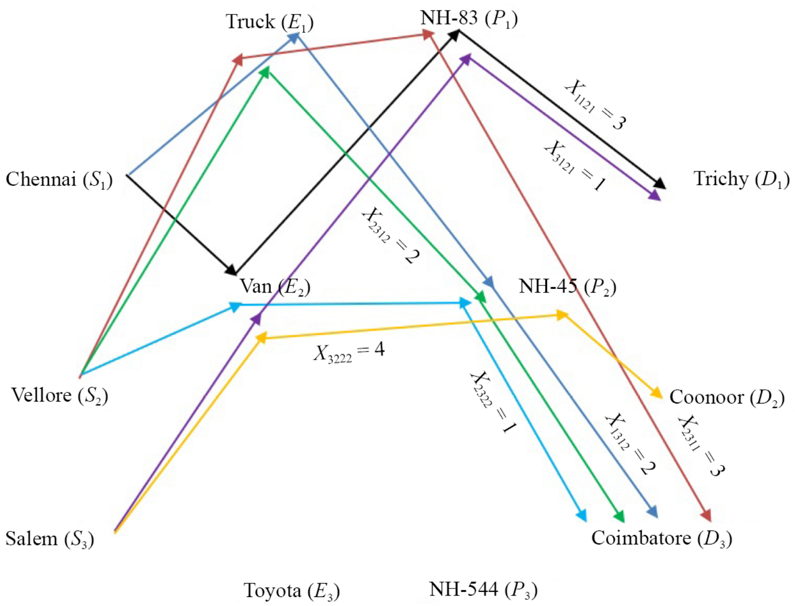

Figure 3. Network diagram of optimal allotment for Example 2

From Figure 3, it is clear that the number of vaccines transported from the source by the different vehicles through various routes to reach the destination is: $S_1 \to E_1 \to P_2 \to D_1$ is 3 (i.e., $x_{1121} = 3$); $S_1 \to E_3 \to P_1 \to D_2$ is 2 (i.e., $x_{1312} = 2$); $S_2 \to E_3 \to P_1 \to D_1$ is 3 (i.e., $x_{2311} = 3$); $S_2 \to E_3 \to P_1 \to D_2$ is 2 (i.e., $x_{2312} = 2$); $S_2 \to E_3 \to P_2 \to D_2$ is 1 (i.e., $x_{2322} = 1$); $S_3 \to E_1 \to P_2 \to D_1$ is 1 (i.e., $x_{3121} = 1$); $S_3 \to E_2 \to P_2 \to D_2$ is 4 (i.e., $x_{3222} = 4$). From Table

18 and Figure 3, it is observed that 3 vaccines are shipped from source to destination by refrigerated truck through NH-38, 4 vaccines by refrigerated truck through NH-45, 4 vaccines by refrigerated van through NH-38 and 5 vaccines by refrigerated van through NH-45. From this, we conclude that the DM transported the maximum number of vaccines by the vehicle (refrigerated van) through route (NH-45) which results in minimal loss of damage in products at minimal cost with fixed charge. Finally, the optimal transportation cost with fixed charge is 111 and the neutrosophic optimal transportation cost with fixed charge is (16, 20, 24, 29; 0.8, 0.2, 0.2).

6. Comparative study

To check the efficiency of the proposed approach, we have compared the Examples 1 and 2 using the LINGO software which is performed with Intel(R) Core (TM) i3-7100U CPU @ 2.40 GHz and 4 GB RAM is shown in Table 19.

Table 19. Neutrosophic optimal solution of the proposed approach and the LINGO software

		Methods	
		Proposed approach	LINGO software
Example 1	Values of decision variable	$x_{1111}=1, x_{1112}=2, x_{1211}=2, x_{1221}=4, x_{2112}=2, x_{2122}=5$	$x_{1111}=1, x_{1112}=2, x_{1211}=2, x_{1221}=4, x_{2112}=2, x_{2122}=5$
	Neutrosophic optimal solution	(79, 120, 150, 191; 0.7, 0.3, 0.3)	(79, 120, 150, 191; 0.7, 0.3, 0.3)
Example 2	Values of decision variable	$x_{1121}=3, x_{1312}=2, x_{2311}=3, x_{2312}=2, x_{2322}=1, x_{3121}=1, x_{3222}=4$	$x_{1121}=3, x_{1312}=2, x_{2311}=3, x_{2312}=2, x_{2322}=1, x_{3121}=1, x_{3222}=4$
	Neutrosophic optimal solution	(16, 20, 24, 29; 0.8, 0.2, 0.2)	(16, 20, 24, 29; 0.8, 0.2, 0.2)

For better understanding, the obtained optimal solution for Examples 1 and 2 are compared between the proposed approach and the LINGO software is shown graphically in Figures 4 and 5 respectively.

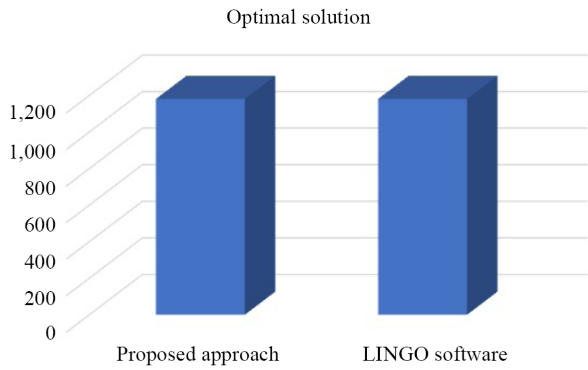

Figure 4. Comparison between the proposed approach and the LINGO software for Example 1

From Table 19, Figures 4 and 5, it is clear that the obtained optimal solution for Examples 1 and 2 using the proposed approach provides the same result as the LINGO software. During transportation product damageability plays a crucial role. The rates of product damage depend on the vehicle type, route condition between source and destination and so on. While transporting the products the decision makers is always keen on minimizing the transportation cost by selecting a suitable mode of transport and routes which will optimize the product damage during transportation. Our proposed approach is one of the manual computational approaches which will help the DM in choosing the suitable vehicle and path to minimize the total transportation cost.

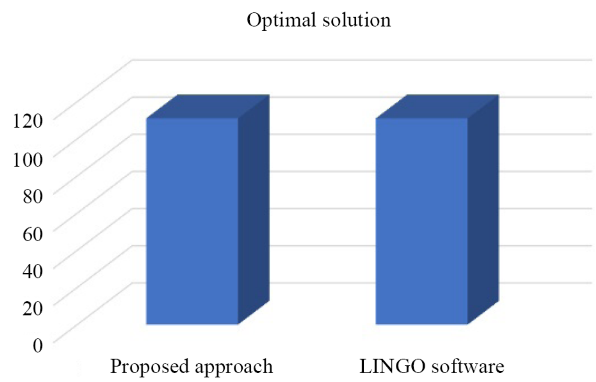

Figure 5. Comparison between the proposed approach and the LINGO software for Example 2

7. Conclusions and future scopes

In this article, we have considered the single objective four-dimensional fixed charge TP under neutrosophic environment. Here all the parameters other than the distance of routes are considered as single-valued trapezoidal neutrosophic numbers. The single-valued trapezoidal neutrosophic number gives an additional possibility to represent uncertainty, imprecise, incomplete and inconsistent information which exist in real world. In this article, we have tried to focus on computational approach, min zero-min cost approach for obtaining optimal solution to the problem. The score function of SVTrNNs is utilized to transform the neutrosophic 4DFCTP into its equivalent deterministic 4DFCTP to prevent negative values for the decision variable. The reduced deterministic problem is then solved using the proposed min zero-min cost approach to determine the optimal solution. Therefore, the proposed approach is to optimize the damageable products and routing plan of vehicles in a way to minimize the total transportation cost with fixed charge of the business organizations. The optimal solution obtained from the proposed approach is the same as the LINGO software that validates its effectiveness. This approach can also be applied to other applications of neutrosophic TPs. The solution obtained using this approach is helpful for the DMs to manage indeterminacy, analyze and synthesize complex decision scenarios by solving decision-making problems. This problem will help the DM by selecting a suitable vehicle and appropriate route for transporting the products from sources to destinations to minimize the transportation costs with fixed charges and maximize profits in the economic and business sectors. The proposed approach limitations include the time complexity in handling higher dimensional problems. Our future research will focus on such issues. This study has a formulation of four-dimensional fixed charge TP under neutrosophic environment, where the single objective that is transportation cost is considered. In real-world situations, DMs meet with multiple conflicting objectives which provides them with a comprehensive set of profits. So, we must understand the presence of numerous criteria that can improve multi-criteria decision-making problems. For the future perspective in this topic from our point of view, researchers may extend to neutrosophic four-dimensional fixed charge fractional TP, neutrosophic multi-item multi-objective four-dimensional fixed charge TP, neutrosophic multi-stage multi-objective four-dimensional fixed charge TP and neutrosophic multi-level four-dimensional multi-item fixed charge TP. In spite of this, one can formulate the other types of neutrosophic numbers. The proposed approach have been illustrated with two sources, two demands by two conveyances through two routes. In the future, these can be illustrated for large data sets. This approach is not limited to transportation problems, it can be applied to the decision-making problems such as assignment problem and travelling salesman problem.

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Conflict of interest

The authors declare that they have no conflicts of interest.

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