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Research Article

Goal Programming Model Using Data Envelopment Analysis for Human Development Index

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Abstract: United Nations' Human Development Index measures human development and has significant positive correlation with abundance in natural resource. Data Envelopment Analysis is used for composite indicators' development and overcomes weighting techniques' limitations. It has limitations in existence of missing records that Goal Programming can overcome. Accordingly, this paper introduces a new Human Development Index that improves upon the United Nations' Human Development Index by incorporating natural resource abundance and addressing missing data issues through Goal Programming. To address these issues, a model is proposed that combines Data Envelopment Analysis and Goal Programming. The model first estimates missing values and then calculates weights for the Human Development Index using Data Envelopment Analysis, which integrates standardized human development dimensions with natural resource factors. This revised Human Development Index results in new country rankings and is validated through a correlation analysis with the United Nations' Human Development Index and a Wilcoxon Signed-Rank test. The correlation analysis shows strong agreement in rankings despite different weighting methods, while the Wilcoxon test indicates significant differences in median rankings. Our proposed index offers a more comprehensive measure of human development by considering both human development dimensions and natural resources, enhancing the accuracy of the Human Development Index and suggesting areas for future research into additional factors affecting human development, beside others.

Keywords: composite indicators, human development index, data envelopment analysis, goal programming, missing values estimation

MSC: 90C08, 90C29

1. Introduction

Composite Indicators (CIs) like the United Nations' (UN's) Human Development Index (HDI) are vital tools for consolidating complex, multi-dimensional data into a single measure. They help decision-makers rank and compare countries based on various factors related to human development, such as health, education, and standard of living. The HDI aggregates these dimensions to provide a snapshot of development progress. Despite its widespread use, the HDI has faced substantial criticism. Issues include: (1) the use of equal weights for component indices, (2) inconsistent calculation methods, (3) measurement errors due to incomplete or inaccurate data, (4) unrealistic goalposts, especially

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in historical datasets like the 2012 HDI, (5) normalization problems that can lead to distortions when actual values approach minimum thresholds, and (6) the overall reliability of rankings being heavily influenced by the methods used for weighting, normalization, and aggregation [1-9].

Data Envelopment Analysis (DEA) is a non-parametric mathematical programming method used to evaluate the performance of Decision-Making Units (DMUs) by converting multiple inputs into multiple outputs. DEA is praised for its minimal assumptions and empirical orientation, which contrasts with other methods that may be more theoretical. However, DEA has limitations, particularly in handling missing data, which can impair its effectiveness [10-14].

Goal Programming (GP) is another MP technique that is effective in managing missing values and optimizing parameter estimation. This model achieves its goal through absolute deviation minimization; which might not be achieved fully but the closest to the planned goal. One of its goals can be parameters' estimation of regression model, specifically in existence of outliers [15, 16]. Despite the strengths of DEA and GP individually, their combined application to improve HDI calculations has not been thoroughly explored.

This study addresses a key research gap by integrating DEA and GP to develop a revised HDI. The proposed model leverages DEA for calculating weights and evaluating human development dimensions while using GP to handle missing data and optimize parameter estimation. By incorporating natural resource factors alongside human development dimensions, this integrated approach aims to provide a more accurate, robust, and comprehensive measure of human development, thereby addressing the limitations of the UN's HDI and offering enhanced insights for global development assessment.

2. Literature review

2.1 *United nations' human development index*

2.1.1 *United nations' human development index definition and methodology*

The Organisation for Economic Co-operation and Development (OECD) defines CIs as mathematical aggregations of selected individual indicators into a single index. CIs simplify the interpretation of results by reducing the number of indicators without losing critical information, though poorly constructed CIs can produce misleading results. The methodology behind CIs involves subjective decisions related to aggregation and weighting techniques [3, 9, 17].

UN's HDI is one of the most widely recognized CIs. Unlike economic-focused indices, the HDI emphasizes human life quality, measured through the geometric mean of normalized indices across three dimensions: health, education, and standard of living. Health is assessed via life expectancy at birth; education is evaluated through expected years of schooling for children entering school and mean years of schooling for adults aged 25 and older; and standard of living is measured by Gross National Income (GNI) per capita. The Human Development Report (HDR) uses HDI scores to provide national policy recommendations aimed at enhancing human development [7, 8].

In 2016, the HDR noted missing values for the expected years of schooling indicator in ten countries and for mean years of schooling in eleven countries. These missing values were estimated using cross-country regression models [18]. The calculation of the UN's HDI involves developing dimension indices and then aggregating them. This process requires setting goalposts, as detailed in Table 1, to standardize the indicators [18].

Table 1. Goalposts of indicators

Dimension	Indicator	Minimum	Maximum
Health	Life expectancy at birth in years	20	85
Education	Expected years of schooling	0	18
	Mean years of schooling	0	15
Standard of living	GNI per capita reference to Purchasing Power Parity (PPP) dollar in 2011	100	75,000

Source: United Nations Development Programme [18]

The goalposts for the health dimension are set with a minimum value of 20 years for life expectancy at birth, based on historical data, and a maximum value of 85 years, reflecting a realistic aspirational target derived from trends over the past 30 years. For the education dimension, the minimum values for both expected years of schooling and mean years of schooling are set to zero, acknowledging that societies can exist without formal education. The maximum value for expected years of schooling is set at 85 years, which corresponds to the anticipated duration of education equivalent to a master's degree. The mean years of schooling maximum is set at 15 years, reflecting the projected value for 2025. For the standard of living dimension, the minimum value is set at $100, representing a baseline close to the minimum amount of nonmarket production and unmeasured subsistence in economies. The maximum value is set at $75,000, as income levels above this amount show minimal additional benefits to human development and well-being. After setting the goalposts, each indicator is normalized to a scale between 0 and 1. This is achieved using Equation 1 [18]:

$$\text{Dimension index} = \frac{\text{actual value} - \text{minimum value}}{\text{maximum value} - \text{minimum value}} \tag{1}$$

Where, the actual value is the value of the indicator for a particular country, minimum value is the minimum value of the indicator across all countries and maximum value is the maximum value of the indicator across all countries. This linear normalization converts the indicator values into a common scale, allowing for comparability across different dimensions. There are several assumptions accompanied: (1) the minimum and maximum values for each indicator are fixed and known, and (2) the relationship between the raw values and the normalized scale is linear.

As previously noted, the education dimension is assessed using two separate indicators. Consequently, Equation 1 is applied individually to each indicator, and their results are averaged using an arithmetic mean. Additionally, the natural logarithm is applied to the income values and goalposts, reflecting that each dimension index serves as a proxy for its respective capabilities. After computing the dimension indices, Equation 2 is used to determine the UN's HDI by taking the geometric mean of these indices [18]:

$$UN'sHDI = \left(I_{\text{Health}} \cdot I_{\text{Education}} \cdot I_{\text{Income}}\right)^{\frac{1}{3}} \tag{2}$$

Based on the scores obtained per country, the countries are grouped based on cutoff points, as shown in Table 2, highlighting the country's grouping in the first column and their cutoff points in the second column [18].

Table 2. UN's HDI Cutoff Points

Country's grouping	Cutoff point
Very high human development	0.800 and above
High human development	0.700-0.799
Medium human development	0.550-0.699
Low human development	Below 0.550

Source: United nations development programme [18]

2.1.2 *United nations' human development index and effect of natural resources*

In academic and policy discussions, a common argument is that an abundance of natural resources can negatively impact a country's economic growth and development. However, further analysis has challenged this notion, revealing that natural resource wealth has been beneficial in several success stories. For instance, Norway and Chile have experienced economic prosperity due to their natural resources, provided they are managed effectively [6]. Pineda and Rodríguez found that changes in HD, as proxied by the UN's HDI, are positively and significantly correlated with the abundance of natural resources, particularly in non-income dimensions, from 1970 to 2005. This indicates that natural resource abundance may positively influence HD rather than detract from it [6].

Pineda and Rodríguez categorized countries as either net exporters or net importers of natural resources [Net exporters' countries of natural resources are the countries that export more than the average. Net importers' countries of natural resources are the countries that import more than the average]. Their findings showed that net importer countries tend to have higher UN HDI scores and better performance across all components. Changes in life expectancy were similar between both groups of countries, while GDP growth was slower in net exporter countries. Conversely, net exporters showed larger improvements in gross enrollment and literacy rates. Despite lower GDP per capita growth, net exporters had greater progress in other HDI components, indicating that natural resources primarily influence human development through non-income channels [6].

These findings underscore that natural resources can be developmental assets when complemented by effective human and physical capital investments and appropriate policies. When managed well, natural resources can drive sustainable economic development through investments in human capital, volatility management, real exchange rate control, and export diversification [6].

2.1.3 *United nations' human development index's criticism*

The UN's HDI is a CI, retaining certain positive attributes inherent to CIs. CIs offer a simplified understanding of complex phenomena that cannot be captured by multiple indicators alone. They are valuable tools for decision-makers, as they distill complex, multidimensional issues into a single metric, facilitating the ranking of countries [4].

However, CIs face significant critiques, largely due to their subjective construction. These criticisms arise from various factors, including the selection of weights, the choice of functional models, and aggregation mechanisms [2, 4]. Specifically, the UN's HDI has faced criticism for: (1) employing equal weights, (2) differing methodologies for calculating each component index, (3) measurement errors stemming from estimated data sets, lack of census data, and incomplete coverage, (4) unrealistic goalposts in the 2012 HDI dataset, (5) normalization methods that can produce problematic results when values approach the minimum, which is inconsistent with geometric aggregation methods, and (6) limitations in credibility due to the dependence of country rankings on weighting, normalization, and aggregation methods [1, 4, 5, 7].

2.2 *Data envelopment analysis and goal programming model*

2.2.1 *Data envelopment analysis*

DEA is a nonparametric MP method that evaluates the performance of peer entities, known as DMUs, by converting multiple inputs into multiple outputs. DEA determines the efficiency of DMUs, assigning a score of 1 to efficient units and a positive value less than 1 to inefficient ones. DEA is favorably noted for its minimal assumptions and empirical orientation compared to other methods [10, 12-14].

The DEA model is structured with a maximized objective function of F_0; which represents the efficiency of the DMU under investigation. This model is applied N times, once for each DMU.

Find the values of w_i and v_j which:

$$\max \frac{\sum_{i=1}^{I} w_i y_{i0}}{\sum_{j=1}^{J} v_j z_{j0}} \tag{3}$$

Subject to:

$$\max \frac{\sum_{i=1}^{I} w_i y_{in}}{\sum_{j=1}^{J} v_j z_{jn}} \leq 1,\ \mathrm{n} = 1,\ 2,\ \ldots,\ \mathrm{N}\ (N\ \text{total constraints per model}) \tag{4}$$

$$w_i,\ v_j \geq 0,\ i = 1,\ 2,\ \ldots,\ I;\ j = 1,\ 2,\ \ldots,\ J \tag{5}$$

Where:

y_{io} is DMU_o's i^{th} output and w_i is the corresponding weight, where $i = 1, 2, ..., I$ and DMU_0 is the DMU under investigation.

z_{io} is DMU_o's j^{th} input and v_i is the corresponding weight, where $j = 1, 2, ..., J$ and DMU_0 is the DMU under investigation.

y_{in} is DMU_n's i^{th} output and w_i is the corresponding weight, where $i = 1, 2, ..., I$ and $n = 1, 2, ..., N$.

z_{jn} is DMU_n's j^{th} input and v_i is the corresponding weight, where $j = 1, 2, ..., J$ and $n = 1, 2, ..., N$.

w_i and v_i are this model's decision variables, where $i = 1, 2, ..., I$ outputs and $j = 1, 2, ..., J$ inputs.

DEA methodology is employed for constructing CIs. It calculates objective weights endogenously, based on the variables in the dataset, thereby highlighting the most significant variables. This approach addresses some of the drawbacks associated with PCA/FA and EW methods [13]. However, DEA encounters difficulties when dealing with missing data due to gaps in input/output coverage or incomplete reporting by DMUs, owing to its nonparametric and multidimensional nature [11]. Consequently, alternative methods are utilized for estimating missing values.

2.2.2 *Goal programming model*

The objective function is designed with deviation variables to be minimized, which allows multiple conflicting goals within the same model. A general formulation for the linear GP model is [15, 19].

Find the values of β_j, e_n^+ and e_n^- which:

$$\text{Minimize}: \ F = \sum_{n=1}^{N}\left(e_n^+ + e_n^-\right) \tag{6}$$

Subject to:

$$\left(\sum_{j=1}^{J}\beta_j x_{jn}\right) + e_n^- - e_n^+ = d_n, \ n = 1, \ 2, \ \ldots, \ N \tag{7}$$

$$e_n^+ * e_n^- = 0, \ n = 1, \ 2, \ \ldots, \ N \tag{8}$$

$$\beta_j \ \text{unrestricted in sign, for } j = 1, \ 2, \ \ldots, \ J \tag{9}$$

$$e_n^-, \ e_n^+ \geq 0, \ \text{for } n = 1, \ 2, \ \ldots, \ N \tag{10}$$

where:

F: The objective function.

x_{jn}: The coefficient associated with variable j in the n^{th} goal, where $j = 1, 2, ..., J$ and $n = 1, 2, ..., N$.

β_j: The j^{th} decision variable, where $j = 1, 2, ..., J$.

d_n: The value of the n^{th} goal, where $n = 1, 2, ..., N$.

e_n^-: Negative deviational variable from the n^{th} goal where $n = 1, 2, ..., N$.

e_n^+: Positive deviational variable from the n^{th} goal, where $n = 1, 2, ..., N$.

Multiple regression is a widely used technique for estimating missing values, but it relies on several assumptions-such as linearity, normality, constant variance, large sample size, and independence-that are not always met. When these assumptions hold, the least squares method is employed for missing value estimation. However, if the assumptions are not satisfied, the results may be unreliable, necessitating the use of alternative methods. GP is one such alternative, which is utilized for estimating regression model parameters. GP has been shown to outperform the least squares method, demonstrating lower Mean Square Error (MSE) across various sample sizes and parameters, and providing better overall model fitting. Additionally, GP is preferred over least squares in the presence of outliers [15, 16, 19].

Assume there are R parameters, $X_r[r = 1, 2, ..., R]$. Among these R parameters $U(U < R)$ parameters have complete values, $X_u[u = 1, 2, ..., U]$, while $(U - R)$ parameters have missing values denoted by $X_1[1 = U + 1, U + 2, ..., R]$. Furthermore, consider a sample of $N(n = 1, 2, ..., N)$ records, where the complete sample consists of $M(m = 1, 2, ..., M)$. The subset of the sample that includes missing values is $A[a = M + 1, M + 2, ..., N]$ and $A = M - N$.

Accordingly, the generally accepted model used in this paper is [15, 19]:

Find the values of β_u, c_m^+ and c_m^- which:

$$\text{Minimize: } F = \sum_{m=1}^{M} \left(c_m^{+} + c_m^{-} \right) \tag{11}$$

Subject to:

$$\left(\sum_{u=1}^{U} \beta_u x_{um} \right) + c_m^{-} - c_m^{+} = x_{lm},\ m = 1,\ 2,\ \ldots,\ M,\ l = U + 1,\ U + 2,\ \ldots,\ R \tag{12}$$

$$c_m^{+} * c_m^{-} = 0,\ m = 1,\ 2,\ \ldots,\ M \tag{13}$$

$$\beta_u \text{ unrestricted in sign, for } u = 1,\ 2,\ \ldots,\ U \tag{14}$$

$$c_m^{-},\ c_m^{+} \geq 0, \text{ for } m = 1,\ 2,\ \ldots,\ M \tag{15}$$

where:

F: The objective function.

x_{um}: The coefficient associated with variable u in the m^{th} goal, where $u = 1, 2, ..., U$ and $m = 1, 2, ..., M$. The dataset M refers to the complete dataset that doesn't include missing values and the dataset N is the full dataset, including both missing and non-missing values.

x_{lm}: The coefficient associated with variable l in the m^{th} goal, where $1 = U + 1, U + 2, ..., R$ and $m = 1, 2, ..., M$.

β_u: The u^{th} decision variable, where $u = 1, 2, ..., U$.

c_m^-: Negative deviational variable from the m^{th} goal, where $m = 1, 2, ..., M$.

c_m^+: Positive deviational variable from the m^{th} goal, where $m = 1, 2, ..., M$.

This model assumes that parameters with missing values are related to all parameters with complete values. Similarly, it applies to all parameters with missing values, with the model being run multiple times based on their quantities. When a parameter with missing values is related to selected parameters with complete values, then $\beta_u = 0$ for the non-related parameters.

The complete dataset (M) is used to estimate the model parameters, noting that (N) represents the full dataset. This leads to the imputation process, where missing values are substituted with estimated values. During imputation, it is assumed that there are relationships between or among the variables and within the model itself. The following equation illustrates how model parameters are used to estimate the missing values in the remaining dataset [15, 16, 20]:

$$\hat{x}_{la} = \left(\sum_{u=1}^{U} \hat{\beta}_u x_{ua} \right),\ l = U + 1,\ U + 2,\ \ldots,\ R \text{ and } a = M + 1,\ M + 2,\ \ldots,\ N \tag{16}$$

where:

x_{ua}: The a^{th} coefficient associated with variable u, where $u = 1, 2, ..., U$ and $a = M + 1, M + 2, ..., N$.

$\hat{x}_{la}$: The a^{th} estimated coefficient associated with variable l, where $1 = U + 1, U + 2, ..., R$, and $a = M + 1, M + 2, ..., N$.

$\hat{\beta}_u$: The u^{th} estimated value for the decision variable β_u, where $u = 1, 2, ..., U$.

3. Methodology

This study presents a GP model that integrates DEA to enhance the calculation of CIs, specifically focusing on HDI. Unlike existing models, this combined approach addresses key limitations of traditional methods. The proposed GP model uniquely contributes to the literature by offering a dual approach: first, it estimates missing values through GP, and second, it calculates HDI weights by combining DEA and GP methods. This integration not only improves accuracy and robustness but also introduces a clear conceptual model that demonstrates how DEA and GP components work together to refine the HDI calculation, providing significant advancements over conventional models. The data used in DEA includes varying numbers of J inputs and I outputs, which often suffer from missing values-a common challenge in DEA applications.

3.1 *Proposed model*

The first objective for this model is to estimate missing values, for which the GP model is employed in this study. Let us assume that there are R inputs, where $X_r[r = 1, 2, ..., R]$. Out of these R inputs, U inputs have complete values, where $X_u[u = 1, 2, ..., U]$, while the remaining $(R - U)$ inputs contain missing values denoted as X_1, where $[1 = U + 1, U + 2, ..., R]$. Additionally, consider a sample of $N(n = 1, 2, ..., N)$ DMUs with M being the complete sample $[m = 1, 2, ..., M]$ and $(M < N)$. The subset of the sample with missing values is A, where $[a = M + 1, M + 2, ..., N]$ and A = N – M.

The GP method is a method used for estimating the values of $\beta_u(u = 1, 2, ..., U)$ using the following model.

Find the values of c_m^+, c_m^- and β_u which:

$$\text{Minimize } F = \sum_{m=1}^{M}\left(c_m^+ + c_m^-\right) \tag{17}$$

Subject to:

$$\left(\sum_{u=1}^{U}\beta_u x_{um}\right) + c_m^- - c_m^+ = x_{lm},\ m = 1,\ 2,\ \ldots,\ M,\ l = U+1,\ U+2,\ \ldots,\ R \tag{18}$$

$$\beta_u \text{ unrestricted in sign, for } u = 1,\ 2,\ \ldots,\ U \tag{19}$$

$$c_m^+ * c_m^- = 0,\ m = 1,\ 2,\ \ldots,\ M \tag{20}$$

$$c_m^+,\ c_m^- \geq 0,\ m = 1,\ 2,\ \ldots,\ M \tag{21}$$

where:

F: The objective function.

x_{um}: The coefficient associated with variable u in the m^{th} goal, where $u = 1, 2, ..., U$ and $m = 1, 2, ..., M$. The dataset refers to the complete dataset that doesn't include missing values and the dataset N is the full dataset, including both missing and non-missing values.

β_u: The u^{th} decision variable, where $u = 1, 2, ..., U$.

x_{lm}: The coefficient associated with variable l in the m^{th} goal, where $1 = U + 1, U + 2, ..., R$ and $m = 1, 2, ..., M$.

c_m^-: Negative deviational variable from the m^{th} goal, where $m = 1, 2, ..., M$.

c_m^+: Positive deviational variable from the m^{th} goal, where $m = 1, 2, ..., M$.

c_m^+, c_m^- and β_u are the model's decision variables.

The objective function is designed with deviation variables to be minimized, which allows multiple conflicting goals within the same model. It is designed to estimate missing values and calculate β_u's efficiently. The derivation involves minimizing deviations from desired values with rationale including improving data accuracy. Key assumptions include linearity in deviations, equal treatment of positive and negative deviations, and the sufficiency of available data.

Moreover, this model assumes that each input variable with missing values is related to all inputs variables with complete data. If an input variable with missing values is only related to a subset of the complete inputs then $\beta_u = 0$ for non-related inputs. After estimating $\beta_u[u = 1, 2, ..., U]$ for each $x_1[1 = U + 1, U + 2, ..., R]$, then the missing values of these input variables are estimated using the below equation, which applies to the dataset $M + 1, \ldots, N$. This imputation process takes place, which occurs via substituting a missing value with a particular one. Imputation provides assumptions about relationships among or between the variables; along with the relationships in the analytic model itself. Accordingly, those model parameters are used to estimate the missing values of the remaining datasets through the below equation. This dataset includes the x values corresponding to the missing data.

$$\hat{x}_{la} = \left(\sum_{u=1}^{U} \hat{\beta}_u x_{ua}\right),\ l = U+1,\ U+2,\ \ldots,\ R \text{ and } a = M+1,\ M+2,\ \ldots,\ N \tag{22}$$

where:

x_{ua} The a^{th} coefficient associated with variable u, where $u = 1, 2, ..., U$ and $a = M + 1, M + 2, ..., N$.

$\hat{x}_{la}$ The a^{th} coefficient, to be estimated, associated with variable l, where $1 = U + 1, U + 2, ..., R$ and $a = M + 1, M + 2, ..., N$.

$\hat{\beta}_u$ The u^{th} estimated value for the decision variable β_u, where $u = 1, 2, ..., U$.

The data is now prepared with complete values for all inputs. A similar approach is then applied to estimate the missing values for outputs, resulting in a dataset with complete values for both inputs and outputs.

Moving to the second objective of this model, it is crucial to note that DEA operates under the premise of evaluating a set of N DMUs [10]. Consequently, the DEA model is applied N times, once for each DMUn, where $n = 1, 2, ..., N$, to calculate a matrix of different weights [13]. This process results in a calculated weight for each input variable, v_j, and to each output variable, w_j, for every DMU. The matrix of weights derived from DEA has several limitations. First, it lacks a consistent basis for comparing and ranking different DMUs, as each DMU has its own unique set of weights. Second, the weights for each DMU do not sum to one, making it difficult to compare them with weights obtained from other methods. Third, the constraint that weights must be non-negative means that a DMU might be deemed efficient even if it performs well in only one input/output, leading to many DMUs being classified as efficient. This results in low discriminating power and leaves little scope for identifying performance improvements [13].

The proposed model addresses these limitations by using a GP approach to consolidate the DEA process. Instead of applying the DEA model individually for each DMU, which would be done N times, this approach integrates all DMUs into a single model. The GP model sets an efficiency aspiration level of 1 for each constraint related to every DMU, representing the highest possible efficiency. This model's advantage lies in its ability to provide a solution that is as close as possible to this aspiration level.

Accordingly, the second objective is to maximize $\dfrac{\sum_{i=1}^{I} w_i y_{in}}{\sum_{j=1}^{J} v_j z_{jn}}$ for each DMU which reflects its efficiency with a maximum value of 1. Therefore, the GP model is formulated and the equation below is applied N times, once for each DMUn, where $n = 1, 2, ..., N$. Additionally, two nonnegative decision variables s_n^-, s_n^+ are introduced to address issues of infeasibility or non-optimality by capturing negative and positive deviations. This leads to the development of constraints aimed at maximizing efficiency for each DMU as follows:

$$\frac{\sum_{i=1}^{I} w_i y_{in}}{\sum_{j=1}^{J} v_j z_{jn}} + s_n^- - s_n^+ = 1;\ n = 1,\ 2,\ \ldots,\ N \tag{23}$$

where:

z_{jn}: DMU_n's j^{th} standardized input and v_j is the corresponding weight, where $j = 1, 2, ..., J$ and $n = 1, 2, ..., N$.

y_{in}: DMU_n's i^{th} standardized output and w_i is the corresponding weight, where $i = 1, 2, ..., I$, $n = 1, 2, ..., N$.

s_n^+: Positive deviational variable, where $n = 1, 2, ..., N$.

s_n^-: Negative deviational variable, where $n = 1, 2, ..., N$.

This model also includes constraints to prevent zero weights, as shown below, to ensure that the effects of all inputs and outputs are considered. Additionally, constraints are incorporated to ensure that the sum of the weights for inputs and outputs equals one for each DMU. This approach facilitates comparability between the weights obtained from this model and those derived from other weighting method(s):

$$w_i,\ v_j \geq \varepsilon;\ i = 1,\ 2,\ \ldots,\ I;\ j = 1,\ 2,\ \ldots,\ J \tag{24}$$

$$\sum_{i=1}^{I} w_i = 1 \tag{25}$$

$$\sum_{j=1}^{J} v_j = 1 \tag{26}$$

where:

v_i: The weight corresponding to x_{jn} which is 's DMU$_n$'s j^{th} standardized input, where i = 1, 2, ..., I and n = 1, 2, ..., N.

w_i: The weight corresponding to y_{jn} which is 's DMU$_n$'s i^{th} standardized output and, where i = 1, 2, ..., I and n = 1, 2, ..., N.

ε: Positive small value.

Finally, the proposed goal programming model is designed to calculate the weights required to determine the proposed HDI using the DEA approach; as detailed below.

Find the values of s_n^+, s_n^-, w_i and v_j which:

$$\text{Minimize } F = \sum_{n=1}^{N} s_n^- \tag{27}$$

Subject to:

$$\frac{\sum_{i=1}^{I} w_i y_{in}}{\sum_{j=1}^{J} v_j z_{jn}} + s_n^- - s_n^+ = 1;\ n = 1,\ 2,\ \ldots,\ N \tag{28}$$

$$w_i,\ v_j \geq \varepsilon;\ i = 1,\ 2,\ \ldots,\ I;\ j = 1,\ 2,\ \ldots,\ J \tag{29}$$

$$\sum_{i=1}^{I} w_i = 1 \tag{30}$$

$$\sum_{j=1}^{J} v_j = 1 \tag{31}$$

$$s_n^+ * s_n^- = 0,\ n = 1,\ 2,\ \ldots,\ N \tag{32}$$

$$s_n^+,\ s_n^- \geq 0,\ n = 1,\ 2,\ \ldots,\ N \tag{33}$$

where:

F: the objective function,

z_{jn}: DMU$_n$'s j^{th} standardized input and v_j is the corresponding weight, where j = 1, 2, ..., J and n = 1, 2, ..., N.

y_{in}: DMU$_n$'s i^{th} standardized output and w_i is the corresponding weight, where i = 1, 2, ..., I and n = 1, 2, ..., N.

ε: Positive small value.

s_n^-: Negative deviational variable, where n = 1, 2, ..., N.

s_n^+: Positive deviational variable, where n = 1, 2, ..., N.

s_n^+, s_n^-, w_i and v_j are the model's decision variables.

Weights have now been calculated using the GP model, accounting for both standardized inputs and outputs. These weights are now prepared for use in calculating the proposed HDI. The proposed HDI is determined as a weighted average of the standardized outputs, as follows:

$$\text{New proposed HDI } = \sum_{i=1}^{I} \widehat{w}_i y_{in}, \; n = 1, \; 2, \; \ldots, \; N \tag{34}$$

Where:

y_{in}: DMU$_n$'s i^{th} standardized output and $\widehat{w}_i$ is the corresponding estimated weight, where i = 1, 2, ..., I and n = 1, 2, ..., N.

3.2 *Data and results*

Step 1-"Selection of Outputs and Inputs": This model includes specified inputs and outputs. For inputs, natural resources are considered, typically measured by the export share of primary products, as commonly referenced by scholars. These primary products include ores, food, fuel, metals, and agricultural raw materials. However, this selection primarily reflects reliance on natural resources rather than measures of resource abundance. To address this, the model adopts net exports of natural resources, as defined by Pineda and Rodríguez. This measure aims to mitigate two consumption-related issues: (1) Increased consumption driven by income growth may skew the relationship between net exports of natural resources and income, and (2) A reduction in natural resource exports coupled with an increase in imports often corresponds to a rise in capital endowment. To counteract these effects, additional inputs are introduced, including natural resource imports by labor force. This adjustment helps capture the indirect impact of natural resources on human development [6].

The input data for 189 countries are sourced from the World Bank's Development Indicators for Natural Resources, which include 43 records with complete data missing, 14 records with missing labor force data, and 4 records with missing fuel export data [21]. The World Bank provides comprehensive datasets that are generally considered reliable and robust. However, potential limitations include variations in data collection methods across countries, differences in reporting standards, and updates that may not be timely. These factors can affect the accuracy and consistency of the data.

For outputs, the study uses four standardized HDI indices that cover health, education, and standard of living. Health is measured by life expectancy at birth; education is assessed through expected years of schooling for children starting school and mean years of schooling for adults aged 25 and older; and the standard of living is measured by GNI per capita [18, 22]. The UN's HDI data are widely used but have been criticized for measurement errors, such as inconsistencies in data collection and normalization processes. This can impact the comparability and reliability of the HDI scores across different countries and over time.

By acknowledging the potential limitations of data for inputs and outputs, the study aims to provide a more accurate and reliable assessment of the proposed HDI model.

Step 2-"Missing Values Estimation through Median Imputation Method": For the 43 missing records, the median imputation method was employed. This method was applied after classifying the countries based on income according to the World Bank classification, which divides countries into four groups: High, Upper-Middle, Lower-Middle, and Low [23]. The same approach was used for estimating the 14 missing records related to the labor force.

Step 3-"Missing Values Estimation using GP Model": The inputs and outputs are prepared for the proposed model. However, there are 4 records with missing data related to fuel exports. The proposed model, addressing its first objective, will be applied to estimate the missing values for these 4 fuel export records.

Step 4-"Weights Calculation for the Proposed HDI using DEA Approach and GP Model": The data is now complete and ready for the new proposed GP model to address its second objective. This objective involves calculating the weights for the proposed HDI using both the DEA approach and the GP model, incorporating both inputs and outputs.

Step 5-"Calculating the Proposed HDI": The weights are now prepared for calculating the proposed HDI, which is determined by taking a weighted average of the standardized outputs.

An analysis of the differences between the UN's HDI rankings and those derived from the proposed model, which incorporates the effect of natural resources, revealed some notable shifts. For instance, Norway, which ranked first in the UN's HDI, dropped to the second batch of countries in the proposed model, whereas Switzerland and Australia maintained their top positions. Conversely, Canada improved from the second batch to the top of the list in the new model. Additionally, the rankings of ten countries-Finland, Belgium, Panama, Bosnia and Herzegovina, Paraguay, Indonesia, Afghanistan, Yemen, Sierra Leone, and Niger-remained unchanged.

4. The proposed human development index validations and discussion of findings

4.1 *The proposed human development index validation*

The proposed HDI, like other composite indicators, involves subjective calculations, particularly in its aggregation and normalization techniques. These methods significantly impact the final CI calculations, making it crucial to assess subjectivity to evaluate the reliability of the CI effectively [24].

4.1.1 *Correlation between the proposed human development index and united nations' human development index ranks*

As previously noted, one criterion used to compare the proposed HDI rankings with those of the UN is Spearman's Rank Correlation Coefficient ρ. This coefficient is used to assess whether the HDI rankings are significantly affected by the weighting technique used.

The hypotheses tested is H_0: $\rho = 0$ versus H_1: $\rho \neq 0$; comparing the proposed HDI ranks to the UN's HDI ones; with $\rho = 0.949$. The value of ρ indicates a strong positive correlation between the two sets of rankings. The p-value for this test is less than 0.001, demonstrating a high degree of correlation despite the different weighting methods.

These results suggest that the new method for calculating HDI does not substantially alter the rankings. However, the new method incorporates both human development dimensions and natural resources, offering several advantages over the UN's HDI methodology. These advantages address some of the limitations inherent in the UN's HDI calculations and are detailed below

4.1.2 *Wilcoxon signed rank test*

The second test applied is the Wilcoxon Signed Rank test, a non-parametric method used to assess hypothesis about the median; i.e.: H_0: $\mu_d = 0$ against H_1: $\mu_d \neq 0$; such that μ_d reflects the deviation average between the paired sample two sets [25]. This test produced a zero p-value, which rejects H_0; i.e.: there is a difference in the median for the paired sample.

This demonstrates that the new approach yields distinct HDI values and rankings by considering not only the outputs, such as human development dimensions, but also the inputs, such as natural resources. Additionally, the proposed methodology has several advantages, which are outlined below. These advantages address and overcome some of the limitations associated with the UN's HDI calculations.

4.2 *The proposed human development index advantages and limitations*

4.2.1 *The proposed human development index advantages*

The new methodology offers several advantages:

1. Equitable Treatment of All Countries: The approach maintains the primary benefit of DEA by ensuring that each country's CI value is maximized.

2. Endogenous and Automatic Weight Calculation: The methodology calculates weights directly from the data, eliminating the need for prior weight information and reducing subjectivity. This approach considers the effects of natural resource variables and automates the weight calculation process.

3. Emphasis on Sub-Indicators: By deriving equal or unequal weights from the data, the methodology enhances the impact of sub-indicators on decision-making.

4. Comparable and Non-Zero Weights: The weights for sub-indicators sum to one, allowing for straightforward comparison with other weighting methods. The methodology also introduces an endogenous weight bound.

5. Consideration of Natural Resources: Unlike the UN's HDI, which uses three normalized human dimension sub-indicators, the proposed HDI includes four, accounting for the impact of natural resources.

6. Minimization of Absolute Residuals: The methodology uses the GP model for missing value estimation, which has proven to be more accurate than the least-squares method, especially in the presence of outliers.

4.2.2 *The proposed human development index limitations*

The new methodology also has several limitations:

1. DEA Model-Related Issues: The efficiency of the DEA approach can be highly sensitive to variations in the sample and the selection of inputs and outputs.

2. Nonlinear Model-Related Issues: The proposed model incorporates nonlinear goals, as represented in Equation 28, necessitating the use of nonlinear solving algorithms.

3. Goal Deviational Variables-Related Issues: The model involves a large number of goal deviational variables, especially with an increased sample size, such as a larger number of countries. This results in greater model complexity and longer processing times.

5. Conclusion

CIs are widely employed for performance monitoring and policy analysis [3, 9]. The UN's HDI is a prominent CI, calculated using the geometric mean of selected normalized indices [7, 8]. The UN's HDI demonstrates a significant positive correlation with natural resource abundance, particularly in non-income-related dimensions [6]. However, it faces criticism for measurement errors, biases in international data, and the arbitrary nature of its weighting and aggregation methods [4].

To address these criticisms, many researchers have applied DEA, a data-oriented approach that transforms multiple inputs into multiple outputs to evaluate the performance of peer entities, known as DMUs [10, 12-14]. DEA faces challenges with missing data, which can be mitigated using GP models, particularly for estimating regression parameters [11, 15].

This paper introduces a proposed GP model utilizing the DEA approach to calculate a revised HDI. This model defines outputs and inputs based on average achievements in key human development dimensions and natural resources, respectively.

Given the subjectivity inherent in different calculation procedures, such as normalization and aggregation, the reliability of the proposed HDI is assessed through various methods. These include correlation analysis between the proposed HDI rankings and the UN's HDI rankings, and the Wilcoxon Signed Rank Test. The correlation analysis shows a high level of correlation in rankings despite different weighting techniques, while the Wilcoxon Signed Rank Test reveals significant differences in median rankings for paired samples.

The model's advantages include its equitable treatment of all countries, automatic and endogenous weight calculation, emphasis on sub-indicator influences, comparability of non-zero weights, consideration of natural resources' impact, and minimization of the sum of absolute residuals. However, the model also has limitations related to DEA issues, nonlinear aspects, and goal deviation variables. These limitations are addressed and explored as potential areas for future research.

6. Points for future research

1. The current model uses natural resources as inputs, but exploring additional inputs correlated with the HD-defined outputs could provide further insights. These additional inputs might influence the HDI calculation and impact the resulting rankings.

2. The proposed model includes nonlinear goal constraints, which introduce computational complexity. Linear transformations could simplify these constraints, making the model easier to solve compared to nonlinear approaches.

3. Conducting simulation studies could help assess the reliability of the proposed model by varying the number of

inputs and outputs, exploring different correlations between them, and using different sample sizes.

4. The new proposed HDI can be calculated using the geometric mean of the standardized outputs, addressing the limitations associated with using the weighted arithmetic mean.

5. Additional validation methods, such as sensitivity analysis and robustness checks, could provide a more comprehensive evaluation of the proposed HDI.

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Conflict of interest

There is no conflict of interest for this study.

References

[1] Aguña CG, Kovacevic M. Uncertainty and sensitivity analysis of the human development index. *Human Development Research Paper*. 2011; 47(2010): 2010.

[2] European Commission. *Competence centre on composite indicators and scoreboards: online platforms*. 2018. Available from: https://composite-indicators.jrc.ec.europa.eu/?q=10-step-guide%2Fstep-8-sensitivity-analysis [Accessed 25th March 2024].

[3] Hudrliková L. Composite indicators as a useful tool for international comparison: the europe 2020 example. *Prague Economic Papers*. 2013; 22(4): 459-473.

[4] Kovacevic M. Review of HDI critiques and potential improvements. *Human Development Research Paper*. 2010; 33: 1-44.

[5] Pinar M, Stengos T, Yazgan ME. Measuring human development in the MENA region. *Emerging Markets Finance and Trade*. 2015; 51(6): 1179-1192.

[6] Pineda J, Rodríguez F. *Curse or Blessing?: Natural Resources and Human Development*. New York: United Nations Development Programme; 2010.

[7] Sayed H, Hamed R, Hosny SH, Abdelhamid AH. Avoiding ranking contradictions in the human development index using goal programming. *Social Indicators Research*. 2017; 138(2): 405-442.

[8] United Nations Development Programme. *Human development reports*. 2016. Available from: http://hdr.undp.org/en/content/human-development-index-hdi [Accessed 25th March 2024].

[9] Zhou P, Ang BW, Poh KL. A mathematical programming approach to constructing composite indicators. *Ecological Economics*. 2007; 62(2): 291-297.

[10] Cooper WW, Seiford LM, Zhu J. *Handbook on Data Envelopment Analysis*. Springer New York Dordrecht Heidelberg London; 2011.

[11] Kuosmanen T. *Modeling Blank Data Entries in Data Envelopment Analysis*. Department of Social Sciences, Wageningen University; 2002.

[12] Martín-Gamboa M, Iribarren D. Coupled life cycle thinking and data envelopment analysis for quantitative sustainability improvement. In: Ren J. (eds.) *Methods in sustainability science*. Elsevier; 2021. p.295-320.

[13] Sayed H, Hamed R, Ramadan MAG, Hosny S. Using meta-goal programming for a new human development indicator with distinguishable country ranks. *Social Indicators Research*. 2015; 123(1): 1-27.

[14] Thanassoulis E, Portela MC, Despic O. Data envelopment analysis: the mathematical programming approach to efficiency analysis. In: Fried HO, Lovell CAK, Schmidt SS. (eds.) *The measurement of productive efficiency and productivity growth*. Oxford University Press; 2008. p.251-420.

[15] Ahmad MH, Adnan R, Lau CK, Daud ZM. Comparing least-squares and goal programming estimates of linear

regression parameter. *Matematika: Malaysian Journal of Industrial and Applied Mathematics*. 2005; 21: 101-112.

[16] Schniederjans M. *Goal Programming Methodology and Applications*. Springer Science & Business Media; 1995.

[17] Organisation for Economic Co-Operation and Development OECD. *Handbook on constructing composite indicators: methodology and user guide*. 2008. Available from: https://www.oecd.org/sdd/42495745.pdf [Accessed 25th March 2024].

[18] United Nations Development Programme, Technical Notes. *Human Development Report 2016*. Human Development for Everyone; 2016.

[19] Hussain JN, Ali BK. *Goal Programming to Estimate the Parameters of Multiple Linear Regression with High Dimensional Data*. 2nd Conference on New Advances on Science and Metascience; 2019.

[20] Center for Behavioral Health Statistics and Quality, Substance Abuse and Mental Health Services Administration. *Methods for Handling Missing Item Values in Regression Models Using the National Survey on Drug Use and Health (NSDUH): NSDUH Methodological Report*. Rockville, MD; 2018.

[21] World Bank, Data Bank. *World development indicators*. 2017. Available from: https://databank.worldbank.org/source/world-development-indicators [Accessed 25th March 2024].

[22] Anand S, Sen A. The income component of the human development index. *Journal of Human Development*. 2000; 1(1): 83-106.

[23] World Bank. *Classifying countries by income*. World Development Indicators. Available from: https://datatopics.worldbank.org/world-development-indicators/stories/the-classification-of-countries-by-income.html [Accessed 25th March 2024].

[24] Ahmed A. *Enhancement of the Human Development Index Ranking Using Two-Stage Data Envelopment Analysis*. Unpublished M.Sc. Degree in Statistics, Department of Statistics, Faculty of Economics and Political Science, Cairo University; 2007.

[25] Imam A, Mohammed U, Moses Abanyam C. On consistency and limitation of paired t-test, sign and wilcoxon sign rank test. *IOSR Journal of Mathematics*. 2014; 10(1): 1-6.