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Research Article

Dark-Singular Straddled Optical Solitons for the Dispersive Concatenation Model with Power-Law of Self-Phase Modulation by Tanh-Coth Approach

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Abstract: This paper recovers dark-singular straddled optical solitons for the dispersive concatenation model with power-law of self-phase modulation using the tanh-coth approach. The individual dark or singular solitons are not supported by the model for power-law unless this law collapses to Kerr law as proven with the usage of undetermined coefficients, earlier.

***Keywords*:** solitons, traveling waves, dispersive concatenation, power-law

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1. Introduction

A decade ago, in 2014, an intriguing model was proposed to explore the propagation of solitons through optical fibers [1, 2]. This is a combination of the Sasa-Satsuma equation, the Lakshmanan-Porsezian-Daniel (LPD) model, and the well-known nonlinear Schrödinger's equation (NLSE). This model is now thoroughly examined, and a plethora of results have been reported. Shortly thereafter, a dispersive variant of the concatenation model came int existence during 2015 [3-5]. This is obtained by conjoining dispersive fifth order NLSE, the LPD, and the Schrödinger-Hirota equation (SHE). It is indeed a dispersive concatenation model since it contains fifth order and third-order dispersive effects that come from the fifth order NLSE and the SHE. Subsequently, this model attracted a lot of attention too [6-15].

A few preliminary results for the model have been recovered and reported. One main result is addressing the model by the method of undetermined coefficients that yielded single soliton solutions, save bright and singular solitons. It was

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proved that the model with power-law of self-phase modulation (SPM) does not support dark or singular optical solitons unless this power-law reduces to Kerr law. Hence this paper dives into the tanh-coth approach to retrieve dark-singular straddled optical solitons. The details of the derivation of such straddled solitons are exhibited in the rest of the paper after a quick and succinct introduction to this model.

2. Governing model

The concatenation model with power nonlinearity is formulated as [15]:

$$i\Phi_t + a\Phi_{xx} + b|\Phi|^{2n}\Phi - ic_1\left[\sigma_1\Phi_{xxx} + \sigma_2|\Phi|^{2n}\Phi_x\right] + c_2\left[\sigma_3\Phi_{xxxx} + \sigma_4|\Phi|^{2n}\Phi_{xx}\right.$$

$$+\sigma_5|\Phi|^{2n+2}\Phi + \sigma_6\left|\Phi_x\right|^2\Phi + \sigma_7\Phi_x^2\Phi^* + \left.\sigma_8\Phi_{xx}^*\Phi^2\right] - ic_3\left[\sigma_9\Phi_{xxxxx} + \sigma_{10}|\Phi|^{2n}\Phi_{xxx}\right.$$

$$\left.+\sigma_{11}|\Phi|^{2n+2}\Phi_x + \sigma_{12}\Phi\Phi_x\Phi_{xx}^* + \sigma_{13}\Phi^*\Phi_x\Phi_{xx} + \sigma_{14}\Phi\Phi_x^*\Phi_{xx} + \sigma_{15}\Phi_x^2\Phi_x^*\right] = 0 \tag{1}$$

In equation (1), the independent variables, x and t, stand for the spatial and temporal coordinates, respectively, and the dependent variable, $q(x, t)$, is a complex-valued function that represents the wave amplitude. The linear temporal evolution is represented by the first term, $i = \sqrt{-1}$, and the chromatic dispersion and SPM are represented by the second and third terms, respectively, with their coefficients being a and b. The extension of NLSE to formulate the SHE is represented by the coefficient of c_1. Next, the LPD components and the fifth-order NLSE that incorporates the dispersive impact from the fifth-order dispersion term are represented by the coefficients of c_2 and c_3, respectively. The parameter n represents power-law of SPM and from earlier studies, it is well-known that soliton solutions exist for $0 < n < 2$. Numerical simulations and experimental results also prove this.

3. Travelling wave solution

The solutions of Eq. (1) are assumed to be

$$\Phi(x,\ t) = u(\xi)e^{i\theta(x,\ t)} \tag{2}$$

Where $\xi = (x - \gamma t)$ and the phase $\theta(x,\ t) = -kx + \omega t + \theta_0$. Also, $u\ (\xi)$ is the amplitude components of the wave, γ is its speed, k is the Soliton frequency, ω is its wavenumber and θ_0 is the phase constant. Using Eq. (2) and its derivatives, Eq. (1) transforms to

$$[-i\gamma u' - \omega u] + a\left[u'' - 2iku' - k^2u\right] + bu^{2n+1} - ic_1\left[\sigma_1\left[u''' - 3iku'' - 3k^2u' + ik^3u\right]\right.$$

$$\left.+\sigma_2u^{2n}[u' - kiu]\right] + c_2\left[\sigma_3\left[u'''' - 4iku''' - 6k^2u'' + 4ik^3u' + k^4u\right] + \sigma_4u^{2n}\left[u'' - 2iku' - k^2u\right]\right.$$

$$\left.+\sigma_5u^{2n+3} + \sigma_6\left[u' - kiu\right]^2u + \sigma_7\left[u' - kiu\right]^2u + \sigma_8\left[u'' - 2iku' - k^2u\right]u^2\right]$$

$$-ic_3\left[\sigma_9\left[u''''' - 5iku'''' - 10k^2u''' + 10ik^3u'' + 5k^4u' - ik^5u\right] + \sigma_{10}u^{2n}\left[u''' - 3iku'' - 3k^2u' + ik^3u\right]\right.$$

$$+\sigma_{11}u^{2n+2}\left[u'-kiu\right]+\sigma_{12}u\left[u'-kiu\right]\left[u''-2iku'-k^2u\right]+\sigma_{13}u\left[u'-kiu\right]\left[u''-2iku'-k^2u\right]$$

$$+\sigma_{14}u\left[u'-kiu\right]\left[u''-2iku'-k^2u\right]+\sigma_{15}\left[u'-kiu\right]^3\Big]=0 \quad (3)$$

Eqs. (3) can be decomposed into real and imaginary parts, which are respectively expressed as

$$[-\omega u]+a\left[u''-k^2u\right]+bu^{2n+1}-c_1\left[\sigma_1\left[+3ku''-k^3u\right]+\sigma_2\left[ku^{2n+1}\right]\right]+c_2\left[\sigma_3\left[u''''-6k^2u''+k^4u\right]\right.$$

$$\left.+\sigma_4\left[u''u^{2n}-k^2u^{2n+1}\right]+\sigma_5u^{2n+3}+\left(\sigma_6+\sigma_7\right)\left[u'^2u-k^2u^3\right]+\sigma_8\left[u''u^2-k^2u^3\right]\right]$$

$$-c_3\left[\sigma_9\left[+5ku''''-10k^3u''+k^5u\right]+\sigma_{10}\left[3ku^{2n}u''-k^3u^{2n+1}\right]+\sigma_{11}\left[ku^{2n+3}\right]\right.$$

$$\left.+\left(\sigma_{12}+\sigma_{13}+\sigma_{14}\right)\left[+2kuu'^2+ku^2u''-k^3u^3\right]+\sigma_{15}\left[3u'^2ku-k^3u^3\right]\right]=0 \quad (4)$$

and

$$\left(-\gamma-2ak+3k^2c_1\sigma_1+4c_2\sigma_3k^3-5c_3\sigma_9k^4\right)u'+\left(10c_3\sigma_9k^2-4c_2\sigma_3k-c_1\sigma_1\right)u'''$$

$$+\left(3c_3\sigma_{10}k^2-\sigma_2-2\sigma_4k\right)u^{2n}u'+\left(3c_3\left(\sigma_{12}+\sigma_{13}+\sigma_{14}\right)k^2-2k\left(\sigma_6+\sigma_7+\sigma_8\right)\right.$$

$$\left.+3c_3\sigma_{15}k^2\right)u^2u'-c_3\sigma_9u'''''-c_3\sigma_{10}u'''u^{2n}-c_3\sigma_{11}u'u^{2n+2}-c_3\left(\sigma_{12}+\sigma_{13}+\sigma_{14}\right)u'u''$$

$$-c_3\sigma_{15}u'^3=0 \quad (5)$$

From Eq. (5) the soliton speed is:

$$\gamma=-k\left(2a-3kc_1\sigma_1-4c_2\sigma_3k^2+5c_3\sigma_9k^3\right) \quad (6)$$

whenever

$$c_1\sigma_1=\left(10c_3\sigma_9k^2-4c_2\sigma_3k\right) \quad (7)$$

$$\sigma_2=\left(3c_3\sigma_{10}k^2-2\sigma_4k\right) \quad (8)$$

$$3c_3\left(\sigma_{12}+\sigma_{13}+\sigma_{14}\right)k^2-2k\left(\sigma_6+\sigma_7+\sigma_8\right)+3c_3\sigma_{15}k^2=0 \quad (9)$$

$$\sigma_9=\sigma_{10}=\sigma_{11}=\sigma_{15}=0,\ \left(\sigma_{12}+\sigma_{13}+\sigma_{14}\right)=0,\ \left(\sigma_6+\sigma_7+\sigma_8\right)=0 \quad (10)$$

Eq. (4) can be written as

$$c_2\sigma_3 u'''' + \beta_1 u'' + c_2\sigma_4 u'' u^{2n} + \beta_2 u^{2n+1} + c_2\sigma_5 u^{2n+3} + c_2\left(\sigma_6 + \sigma_7\right) u'^2 u + \beta_3 u + c_2\sigma_8 u'' u^2 = 0 \tag{11}$$

where:

$$\beta_1 = \left(a - 3c_1\sigma_1 k - 6c_2\sigma_3 k^2\right) \tag{12}$$

$$\beta_2 = \left(b - c_1\sigma_2 k - k^2 c_2\sigma_4\right) \tag{13}$$

$$\beta_3 = \left(c_1\sigma_1 k^3 + c_2\sigma_3 k^4 - ak^2 - \omega\right) \tag{14}$$

Setting

$$u = v^{\frac{1}{n}} \tag{15}$$

Eq. (11) is transformed into

$$c_2\sigma_3\left[v^3 v'''' + 4\left(\frac{1-n}{n}\right)v^2 v' v''' + 6\left(\frac{1-n}{n}\right)\left(\frac{1-2n}{n}\right)v v'^2 v'' + 3\left(\frac{1-n}{n}\right)v^2 v''^2 + \left(\frac{1-n}{n}\right)\left(\frac{1-2n}{n}\right)\left(\frac{1-3n}{n}\right)v'^4\right]$$

$$+\beta_1\left[v^3 v'' + \left(\frac{1-n}{n}\right)v^2 v'^2\right] + c_2\sigma_4\left[v^5 v'' + \left(\frac{1-n}{n}\right)v^4 v'^2\right] + \mathrm{n}\beta_2 v^6 + nc_2\sigma_5 v^{\frac{2}{n}+6} + c_2\left(\sigma_6 + \sigma_7\right)\frac{1}{n}v^{\frac{2}{n}+2}v'^2 + n\beta_3 v^4$$

$$+c_2\sigma_8\left[v^{\frac{2}{n}+3}v'' + \left(\frac{1-n}{n}\right)v^{\frac{2}{n}+2}v'^2\right] = 0 \tag{16}$$

For integrability, the coefficient of $v^{\frac{2}{n}+2}$ $v^{\frac{2}{n}+3}$ $v^{\frac{2}{n}+6}$ in Eq. (16) must vanish, thus we obtain the following nonlinear ordinary differential equation:

$$c_2\sigma_3\left[v^3 v'''' + 4M_1 v^2 v' v''' + 6M_1 M_2 v v'^2 v'' + 3M_1 v^2 v''^2 + M_1 M_2 M_3 v'^4\right]$$

$$+\beta_1\left[v^3 v'' + M_1 v^2 v'^2\right] + c_2\sigma_4\left[v^5 v'' + M_1 v^4 v'^2\right] + \mathrm{n}\beta_2 v^6 + n\beta_3 v^4 = 0 \tag{17}$$

where:

$$\sigma_5 = 0,\ \sigma_8 = 0, \tag{18}$$

$$\left(\sigma_6 + \sigma_7\right) = 0 \tag{19}$$

$$M_1 = \left(\frac{1-n}{n}\right) \tag{20}$$

$$M_2 = \left(\frac{1-2n}{n}\right) \tag{21}$$

$$M_3 = \left(\frac{1-3n}{n}\right) \tag{22}$$

4. Tanh-coth method

Assume $v = v(\xi)$, by using the ansatz, [16-17]

$$Y = \tanh(\mu\xi) \tag{23}$$

that leads to the change of variables:

$$\frac{dv}{d\xi} = \mu\left(1-Y^2\right)\frac{dv}{dY}, \tag{24}$$

$$\frac{d^2v}{d\xi^2} = \mu^2\left[-2Y\left(1-Y^2\right)\frac{dv}{dY} + \left(1-Y^2\right)^2\frac{d^2v}{dY^2}\right], \tag{25}$$

For the next step, assume that the solution for Eq. (18) is expressed in the form

$$v(Y) = \sum_{i=0}^{p} a_i Y^i + \sum_{i=1}^{p} b_i Y^{-i}, \tag{26}$$

using the principle of the homogeneous balance method between the nonlinear term $v^3\, v''''$ and the linear term v^6 from Eq. (18), then $3N + N + 4 = 6N$, which gives $N = 2$. Hence, Eq. (26) becomes

$$v(Y) = \left(a_0 + a_1 Y + a_2 Y^2 + \frac{b_1}{Y} + \frac{b_2}{Y^2}\right). \tag{27}$$

Here a_0, a_1, a_2, b_1 and b_2 are constants to be determined.

Then, substituting Eq. (27) and its derivatives into Eq. (18), and assuming for simplicity that $a_1 = b_1 = 0$, we obtain the following:

4.1 *Case I*

Let $a_0 = 0$ and $a_2 = b_2$, then

$$4c_2\sigma_3\mu^4\left[2\left(15\left(Y^{12}+\frac{1}{Y^{12}}\right)-30\left(Y^{10}+\frac{1}{Y^{10}}\right)+62\left(Y^{8}+\frac{1}{Y^{8}}\right)-94\left(Y^{6}+\frac{1}{Y^{6}}\right)+113\left(Y^{4}+\frac{1}{Y^{4}}\right)\right.\right.$$

$$\left.-132\left(Y^{2}+\frac{1}{Y^{2}}\right)+132\right)+8M_1\left(6\left(Y^{12}+\frac{1}{Y^{12}}\right)-16\left(Y^{10}+\frac{1}{Y^{10}}\right)+20\left(Y^{8}+\frac{1}{Y^{8}}\right)-16\left(Y^{6}+\frac{1}{Y^{6}}\right)\right.$$

$$-6\left(Y^4+\frac{1}{Y^4}\right)+32\left(Y^2+\frac{1}{Y^2}\right)-40\right)+48M_1M_2\left(3\left(Y^{12}+\frac{1}{Y^{12}}\right)-10\left(Y^{10}+\frac{1}{Y^{10}}\right)+10\left(Y^8+\frac{1}{Y^8}\right)$$

$$-2\left(Y^6+\frac{1}{Y^6}\right)-3\left(Y^4+\frac{1}{Y^4}\right)+12\left(Y^2+\frac{1}{Y^2}\right)-20\right)+3M_1\left(42\left(Y^8+\frac{1}{Y^8}\right)-88\left(Y^6+\frac{1}{Y^6}\right)$$

$$-144\left(Y^2+\frac{1}{Y^2}\right)+111\left(Y^4+\frac{1}{Y^4}\right)-24\left(Y^{10}+\frac{1}{Y^{10}}\right)+9\left(Y^{12}+\frac{1}{Y^{12}}\right)+156)+8M_1M_2M_3$$

$$\left(\left(Y^{12}+\frac{1}{Y^{12}}\right)-4\left(Y^{10}+\frac{1}{Y^{10}}\right)+2\left(Y^8+\frac{1}{Y^8}\right)+12\left(Y^6+\frac{1}{Y^6}\right)-17\left(Y^4+\frac{1}{Y^4}\right)-8\left(Y^2+\frac{1}{Y^2}\right)+28\right)\right)$$

$$+2\beta_1\mu^2\left[\left(11\left(Y^6+\frac{1}{Y^6}\right)-4\left(Y^8+\frac{1}{Y^8}\right)+3\left(Y^{10}+\frac{1}{Y^{10}}\right)-16\left(Y^4+\frac{1}{Y^4}\right)+18\left(Y^2+\frac{1}{Y^2}\right)-24\right)\right.$$

$$\left.+2M_1\left(\left(Y^{10}+\frac{1}{Y^{10}}\right)-2\left(Y^8+\frac{1}{Y^8}\right)+\left(Y^6+\frac{1}{Y^6}\right)-2\left(Y^2+\frac{1}{Y^2}\right)+4\right)\right]$$

$$+n\beta_3\left(\left(Y^8+\frac{1}{Y^8}\right)+4\left(Y^4+\frac{1}{Y^4}\right)+6\right)+n\beta_2a_2^2\left(\left(Y^{12}+\frac{1}{Y^{12}}\right)+6\left(Y^8+\frac{1}{Y^8}\right)+15\left(Y^4+\frac{1}{Y^4}\right)+20\right)=0 \tag{28}$$

A set of algebraic equations (SAE) is obtained:

$$\left(Y^{14}+\frac{1}{Y^{14}}\right):\ 6c_2\sigma_4\mu^2a_2^2=0,\ \text{Then } \sigma_4=0$$

$$\left(Y^{12}+\frac{1}{Y^{12}}\right):\ n\beta_2a_2^2+4c_2\sigma_3\mu^4\left[30+M_1\left(51+8M_2\left(18+M_3\right)\right)\right]=0$$

$$\left(Y^{10}+\frac{1}{Y^{10}}\right):\ 8c_2\sigma_3\mu^2\left[15+2M_1\left(25+4M_2\left(15+M_3\right)\right)\right]-\beta_1\left[3+2M_1\right]=0$$

$$\left(Y^8+\frac{1}{Y^8}\right):\ 2c_2\sigma_3\mu^4\left[124+2M_1\left(143+8M_2\left(30+M_3\right)\right)\right]-4\beta_1\mu^2M_1+3n\beta_2a_2^2=0$$

$$\left(Y^6+\frac{1}{Y^6}\right):\ c_2\sigma_3\mu^2\left[98+8M_1\left(49+12M_2\left(1-M_3\right)\right)\right]-\beta_1M_1=0$$

$$\left(Y^4+\frac{1}{Y^4}\right):\ 4c_2\sigma_3\mu^4\left[226-M_1\left(45+8M_2\left(18+17M_3\right)\right)\right]-32\beta_1\mu^2+15n\beta_2a_2^2=0$$

$$\left(Y^2+\frac{1}{Y^2}\right):\ c_2\sigma_3\mu^2\left[264+16M_1\left(11-4M_2\left(9-M_3\right)\right)\right]-\beta_1\left[9-2M_1\right]=0$$

$$\left(Y^0+\frac{1}{Y^0}\right):\ c_2\sigma_3\mu^4\left[264+M_1\left(148-32M_2\left(30-7M_3\right)\right)\right]-4\beta_1\mu^2\left[3-M_1\right]+20n\beta_2a_2^2=0 \tag{29}$$

Solving SAE (29) gives:

$$\beta_3=0,\ \omega=k^2\left(c_1\sigma_1k+c_2\sigma_3k^3-a\right) \tag{30}$$

$$\epsilon_1=\mu_1=\mp\sqrt{\frac{\beta_1\left[3+2M_1\right]}{8c_2\sigma_3\left[15+2M_1\left(25+4M_2\left(15+M_3\right)\right)\right]}}$$

$$\epsilon_2=\mu_2=\mp\sqrt{\frac{\beta_1\left[9-2M_1\right]}{c_2\sigma_3\left[264+16M_1\left(11-4M_2\left(9-M_3\right)\right)\right]}}$$

$$\epsilon_3=\mu_3=\mp\sqrt{\frac{\beta_1M_1}{c_2\sigma_3\left[98+8M_1\left(49+12M_2\left(1-M_3\right)\right)\right]}}$$

$$\epsilon_4=\mu_4=\mp\sqrt{\frac{-6\beta_1}{c_2\sigma_3\left[168+M_1\left(1{,}520+M_2\left(2{,}688+352M_3\right)\right)\right]}}$$

$$\epsilon_5=\mu_5=\mp\sqrt{\frac{4\left(23M_1-9\right)\beta_1}{c_2\sigma_3\left[2{,}099+M_1\left(5{,}498-M_2\left(8{,}160+656M_3\right)\right)\right]}}$$

$$\epsilon_6=\mu_6=\mp\sqrt{\frac{\left(23+3M_1\right)\beta_1}{4c_2\sigma_3\left[706+M_1\left(-291+M_2\left(144-712M_3\right)\right)\right]}} \tag{31}$$

Family 1

$$\Delta_{1,j}=a_2=b_2=\mp\epsilon_j^2\sqrt{-\frac{4c_2\sigma_3\left[30+M_1\left(51+8M_2\left(18+M_3\right)\right)\right]}{n\beta_2}}$$

$$\Phi_{1,j}(x,\ t)=\Delta_{1,j^n}^{\frac{1}{n}}\left[\tanh^2\left(\epsilon_j(x-\gamma t)\right)+\coth^2\left(\epsilon_j(x-\gamma t)\right)\right]^{\frac{1}{n}}\times\exp\left[i\left(-kx+\omega t+\theta_0\right)\right],$$

$$\text{for } j=1,\ 2,\ 3,\ 4,\ 5,\ 6. \tag{32}$$

Family 2

$$\Delta_{2,j} = a_2 = b_2 = \mp\epsilon_j^2 \sqrt{-\frac{4c_2\sigma_3\left[270 + M_1\left(1189 + M_2\left(2064 + 200M_3\right)\right)\right]}{9n\beta_2}}$$

$$\Phi_{2,j}(x, t) = \Delta_{2,j}^{\frac{1}{n}}\left[\tanh^2\left(\epsilon_j(x - \gamma t)\right) + \coth^2\left(\epsilon_j(x - \gamma t)\right)\right]^{\frac{1}{n}} \times \exp\left[i\left(-kx + \omega t + \theta_0\right),\right.$$

$$\text{for } j = 1, 2, 3, 4, 5, 6. \tag{33}$$

Family 3

$$\Delta_{3,j} = a_2 = b_2 = \mp\epsilon_j^2 \sqrt{\frac{c_2\sigma_3\left[116 + M_1\left(468 + M_2\left(1440 + 144M_3\right)\right)\right]}{7n\beta_2}}$$

$$\Phi_{3,j}(x, t) = \Delta_{3,j}^{\frac{1}{n}}\left[\tanh^2\left(\epsilon_j(x - \gamma t)\right) + \coth^2\left(\epsilon_j(x - \gamma t)\right)\right]^{\frac{1}{n}} \times \exp\left[i\left(-kx + \omega t + \theta_0\right),\right.$$

$$\text{for } j = 1, 2, 3, 4, 5, 6. \tag{34}$$

4.2 *Case II*

Let $a_0 \neq 0$, and $a_2 = b_2$, then

$$
\begin{aligned}
&c_2\sigma_3\mu^4\left[\begin{aligned}
&8a_2\left[\begin{aligned}&a_0^3+3a_0^2a_2\left(15\left(Y^8+\frac{1}{Y^8}\right)-30\left(Y^6+\frac{1}{Y^6}\right)+32\left(Y^4+\frac{1}{Y^4}\right)-34\left(Y^2+\frac{1}{Y^2}\right)+34\right)+\\
&3a_0a_2^2\left(Y^4+2+\frac{1}{Y^4}\right)+a_2^3\left(62\left(Y^8+\frac{1}{Y^8}\right)-94\left(Y^6+\frac{1}{Y^6}\right)+113\left(Y^4+\frac{1}{Y^4}\right)-132\left(Y^2+\frac{1}{Y^2}\right)+132\right)\end{aligned}\right]\\
&-32a_2^2\left[\begin{aligned}&a_0^2\left(-6\left(Y^8+\frac{1}{Y^8}\right)+16\left(Y^6+\frac{1}{Y^6}\right)-8\left(Y^4+\frac{1}{Y^4}\right)-16\left(Y^2+\frac{1}{Y^2}\right)+28\right)\\
&+2a_0a_2\left(-6\left(Y^{10}+\frac{1}{Y^{10}}\right)+16\left(Y^8+\frac{1}{Y^8}\right)-14\left(Y^6+\frac{1}{Y^6}\right)+20\left(Y^2+\frac{1}{Y^2}\right)-32\right)\\
&+a_2^2\left(-6\left(Y^{12}+\frac{1}{Y^{12}}\right)+16\left(Y^{10}+\frac{1}{Y^{10}}\right)-14\left(Y^8+\frac{1}{Y^8}\right)+14\left(Y^4+\frac{1}{Y^4}\right)-16\left(Y^2+\frac{1}{Y^2}\right)+12\right)\end{aligned}\right]\\
&+48M_1M_2a_2^3\left[\begin{aligned}&a_0\left(3\left(Y^{10}+\frac{1}{Y^{10}}\right)-10\left(Y^8+\frac{1}{Y^8}\right)+7\left(Y^6+\frac{1}{Y^6}\right)+8\left(Y^4+\frac{1}{Y^4}\right)-8\left(Y^2+\frac{1}{Y^2}\right)+4\right)\\
&+a_2\left(\begin{aligned}&3\left(Y^{12}+\frac{1}{Y^{12}}\right)-10\left(Y^{10}+\frac{1}{Y^{10}}\right)+10\left(Y^8+\frac{1}{Y^8}\right)\\
&-2\left(Y^6+\frac{1}{Y^6}\right)-2\left(Y^4+\frac{1}{Y^4}\right)+12\left(Y^2+\frac{1}{Y^2}\right)-17\end{aligned}\right)\end{aligned}\right]\\
&+12M_1a_2^2\left[\begin{aligned}&a_0 2\left[9\left(Y^8+\frac{1}{Y^8}\right)-24\left(Y^6+\frac{1}{Y^6}\right)+24\left(Y^4+\frac{1}{Y^4}\right)-40\left(Y^2+\frac{1}{Y^2}\right)+54\right]\\
&+2a_0a_2\left(9\left(Y^{10}+\frac{1}{Y^{10}}\right)-24\left(Y^8+\frac{1}{Y^8}\right)+33\left(Y^6+\frac{1}{Y^6}\right)-64\left(Y^4+\frac{1}{Y^4}\right)+78\left(Y^2+\frac{1}{Y^2}\right)-40\right)\\
&+a_2^2\left(\begin{aligned}&9\left(Y^{12}+\frac{1}{Y^{12}}\right)-24\left(Y^{10}+\frac{1}{Y^{10}}\right)+42\left(Y^8+\frac{1}{Y^8}\right)-88\left(Y^6+\frac{1}{Y^6}\right)\\
&+111\left(Y^4+\frac{1}{Y^4}\right)-106\left(Y^2+\frac{1}{Y^2}\right)+132\end{aligned}\right)\end{aligned}\right]\\
&+16M_1M_2M_3a_2^4\left(\left(Y^{12}+\frac{1}{Y^{12}}\right)-4\left(Y^{10}+\frac{1}{Y^{10}}\right)+2\left(Y^8+\frac{1}{Y^8}\right)+12\left(Y^6+\frac{1}{Y^6}\right)-17\left(Y^4+\frac{1}{Y^4}\right)-8\left(Y^2+\frac{1}{Y^2}\right)+28\right)
\end{aligned}\right]\\
&+4a_2^2\mu^2\beta_1\left[\begin{aligned}&a_0^2\left(\left(Y^6+\frac{1}{Y^6}\right)-2\left(Y^4+\frac{1}{Y^4}\right)-\left(Y^2+\frac{1}{Y^2}\right)+4\right)+2a_0a_2\left(\left(Y^8+\frac{1}{Y^8}\right)-2\left(Y^6+\frac{1}{Y^6}\right)+2\left(Y^2+\frac{1}{Y^2}\right)-2\right)\\
&+a_2^2\left(\left(Y^{10}+\frac{1}{Y^{10}}\right)-2\left(Y^8+\frac{1}{Y^8}\right)+\left(Y^6+\frac{1}{Y^6}\right)-2\left(Y^2+\frac{1}{Y^2}\right)+4\right)\end{aligned}\right]\\
&+n\beta_2\left[\begin{aligned}&a_0^6+6a_0^5a_2\left(Y^2+\frac{1}{Y^2}\right)+15a_0^4a_2^2\left(\left(Y^4+\frac{1}{Y^4}\right)+2\right)+20a_0^3a_2^3\left(\left(Y^6+\frac{1}{Y^6}\right)+3\left(Y^2+\frac{1}{Y^2}\right)\right)\\
&+15a_0^2a_2^4\left(\left(Y^8+\frac{1}{Y^8}\right)+4\left(Y^4+\frac{1}{Y^4}\right)+6\right)+6a_0a_2^5\left(\left(Y^{10}+\frac{1}{Y^{10}}\right)+5\left(Y^6+\frac{1}{Y^6}\right)+10\left(Y^2+\frac{1}{Y^2}\right)\right)\\
&+a_2^6\left(\left(Y^{12}+\frac{1}{Y^{12}}\right)+6\left(Y^8+\frac{1}{Y^8}\right)+15\left(Y^4+\frac{1}{Y^4}\right)+20\right)\end{aligned}\right]=0 \qquad (35)
\end{aligned}
$$

Assume that $a_0 = a_2 = b_2$ in Eq. (35), Set of algebraic equations SAE is obtained

$$\left(Y^{12}+\frac{1}{Y^{12}}\right):\ 4c_2\sigma_3\mu^4\left\lceil 48+36M_2+27M_1+4M_1M_2M_3\right\rceil+n\beta_2a_2^2=0$$

$$\left(Y^{10}+\frac{1}{Y^{10}}\right):\ 4c_2\sigma_3\mu^4\left\lceil 16+6M_1M_2+9M_1+16M_1M_2M_3\right\rceil-4\mu^2\beta_1-3n\beta_2a_2^2=0$$

$$\left(Y^{8}+\frac{1}{Y^{8}}\right):\ 4c_2\sigma_3\mu^4\left\lceil 358-117M_1+8M_1M_2M_3\right\rceil+21n\beta_2a_2^2=0$$

$$\left(Y^{6}+\frac{1}{Y^{6}}\right):\ 8c_2\sigma_3\mu^4\left\lceil -136+30M_1M_2-69M_1+24M_1M_2M_3\right\rceil-8\mu^2\beta_1+30n\beta_2a_2^2=0$$

$$\left(Y^{4}+\frac{1}{Y^{4}}\right):\ 4c_2\sigma_3\mu^4\left\lceil 376+72M_1M_2+21M_1-68M_1M_2M_3\right\rceil+4\mu^2\beta_1+90n\beta_2a_2^2=0$$

$$\left(Y^{2}+\frac{1}{Y^{2}}\right):\ 2c_2\sigma_3\mu^4\left\lceil 808+96M_1M_2+60M_1-64M_1M_2M_3\right\rceil+2\mu^2\beta_1+63n\beta_2a_2^2=0$$

$$\left(Y^{0}+\frac{1}{Y^{0}}\right):\ 4c_2\sigma_3\mu^4\left\lceil 674-78M_1M_2+318M_1+112M_1M_2M_3\right\rceil+16\mu^2\beta_1+140n\beta_2a_2^2=0 \tag{36}$$

Solving SAE (36) gives:

$$\beta_3=0,\ \omega=k^2\left(\mathrm{c}_1\sigma_1\mathrm{k}+\mathrm{c}_2\sigma_3\mathrm{k}^3-\mathrm{a}\right) \tag{37}$$

$$\mu_1=\mp\sqrt{\frac{182\beta_1}{c_2\sigma_3\left\lceil 14,098-M_1\left(15,834-M_2\left(11,634-11,536M_3\right)\right)\right\rceil}}$$

$$\mu_2=\mp\sqrt{\frac{-22\beta_1}{c_2\sigma_3\left\lceil 802+M_1\left(2,568-M_2\left(1,710-1,960M_3\right)\right)\right\rceil}}$$

$$\mu_3=\mp\sqrt{\frac{2\beta_1}{c_2\sigma_3\left\lceil 1,408+M_1\left(153-\left(M_2 24-796M_3\right)\right)\right\rceil}}$$

$$\mu_4=\mp\sqrt{\frac{-40\beta_1}{7c_2\sigma_3\left\lceil 2,566+M_1\left(1,230-M_2\left(354-240M_3\right)\right)\right\rceil}}$$

$$\mu_5=\mp\sqrt{\frac{94\beta_1}{c_2\sigma_3\left\lceil 19,504+M_1 6,396-M_2\left(1,560+2,656M_3\right)\right\rceil}}$$

$$\mu_6 = \mp\sqrt{\frac{-31\beta_1}{4c_2\sigma_3\left[5{,}176 + M_1\left(1{,}947 - \left(M_2 522 + 920M_3\right)\right)\right]}}$$

$$\mu_7 = \mp\sqrt{\frac{128\beta_1}{c_2\sigma_3\left[4{,}262 + M_1\left(1{,}578 + M_2\left(606 + 2{,}352M_3\right)\right)\right]}}$$

$$\mu_8 = \mp\sqrt{\frac{41\beta_1}{c_2\sigma_3\left[1{,}480 + M_1\left(438 + M_2\left(348 + 608M_3\right)\right)\right]}}$$

$$\mu_9 = \mp\sqrt{\frac{29\beta_1}{c_2\sigma_3\left[856 + M_1 291 + M_2\left(252 + 412M_3\right)\right]}}$$

$$\mu_{10} = \mp\sqrt{\frac{6\beta_1}{c_2\sigma_3\left[-56 + M_1\left(-24 + M_2\left(60 + 104M_3\right)\right)\right]}} \tag{38}$$

Family 1

$$B_{1,j} = a_2 = b_2 = \mp\mu_j^{\,2}\sqrt{\frac{4c_2\sigma_3\left[48 + M_1\left(27 + M_2\left(36 + 4M_3\right)\right)\right]}{-n\beta_2}}$$

$$\Phi_{1,j}(x,\ t) = B_{1,j}^{\;\frac{1}{n}}\left[1 + \tanh^2\left(\mu_j(x - \gamma t)\right) + \coth^2\left(\mu_j(x - \gamma t)\right)\right]^{\frac{1}{n}} \times \exp\left[i\left(-kx + \omega t + \theta_0\right)\right],$$

$$\text{for}\ j = 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10 \tag{39}$$

Family 2

$$B_{2,j} = a_2 = b_2 = \mp\mu_j^{\,2}\sqrt{\frac{4c_2\sigma_3\left[358 - M_1\left(117 - 8M_2M_3\right)\right]}{-21n\beta_2}}$$

$$\Phi_{2,j}(x,\ t) = B_{2,j}^{\;\frac{1}{n}}\left[1 + \tanh^2\left(\mu_j(x - \gamma t)\right) + \coth^2\left(\mu_j(x - \gamma t)\right)\right]^{\frac{1}{n}} \times \exp\left[i\left(-kx + \omega t + \theta_0\right)\right],$$

$$\text{for}\ j = 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10 \tag{40}$$

Family 3

$$B_{3,j} = a_2 = b_2 = \mp\mu_j^2\sqrt{\frac{2c_2\sigma_3\left[120 + M_1\left(78 - 8M_2\left(3 - M_3\right)\right)\right]}{9n\beta_2}}$$

$$\Phi_{3,j}(x, t) = B_{3,j}^{\frac{1}{n}}\left[1+\tanh^2\left(\mu_j(x-\gamma t)\right)+\coth^2\left(\mu_j(x-\gamma t)\right)\right]^{\frac{1}{n}} \times \exp\left[i\left(-kx+\omega t+\theta_0\right)\right],$$

$$\text{for } j = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 \tag{41}$$

Family 4

$$B_{4,j} = a_2 = b_2 = \mp\mu_j^2\sqrt{\frac{4c_2\sigma_3\left[395+M_1\left(30+M_2\left(78-52M_3\right)\right)\right]}{-87n\beta_2}}$$

$$\Phi_{4,j}(x, t) = B_{4,j}^{\frac{1}{n}}\left[1+\tanh^2\left(\mu_j(x-\gamma t)\right)+\coth^2\left(\mu_j(x-\gamma t)\right)\right]^{\frac{1}{n}} \times \exp\left[i\left(-kx+\omega t+\theta_0\right)\right],$$

$$\text{for } j = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 \tag{42}$$

Family 5

$$B_{5,j} = a_2 = b_2 = \mp\mu_j^2\sqrt{\frac{4c_2\sigma_3\left[824+M_1\left(69+M_2\left(102-48M_3\right)\right)\right]}{-123n\beta_2}}$$

$$\Phi_{5,j}(x, t) = B_5^{\frac{1}{n}}\left[1+\tanh^2\left(\mu_j(x-\gamma t)\right)+\coth^2\left(\mu_j(x-\gamma t)\right)\right]^{\frac{1}{n}} \times \exp\left[i\left(-kx+\omega t+\theta_0\right)\right],$$

$$\text{for } j = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 \tag{43}$$

Family 6

$$B_{6,j} = a_2 = b_2 = \mp\mu_j^2\sqrt{\frac{c_2\sigma_3\left[369+M_1\left(177-M_2\left(27-88M_3\right)\right)\right]}{-16n\beta_2}}$$

$$\Phi_{6,j}(x, t) = B_{6,j}^{\frac{1}{n}}\left[1+\tanh^2\left(\mu_j(x-\gamma t)\right)+\coth^2\left(\mu_j(x-\gamma t)\right)\right]^{\frac{1}{n}} \times \exp\left[i\left(-kx+\omega t+\theta_0\right)\right],$$

$$\text{for } j = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 \tag{44}$$

Family 7

$$B_{7,j} = a_2 = b_2 = \mp\mu_j^2\sqrt{\frac{4c_2\sigma_3\left[240-M_1\left(48-M_2\left(102-44M_3\right)\right)\right]}{-105n\beta_2}}$$

$$\Phi_{7,j}(x, t) = B_{7,j}^{\frac{1}{n}}\left[1+\tanh^2\left(\mu_j(x-\gamma t)\right)+\coth^2\left(\mu_j(x-\gamma t)\right)\right]^{\frac{1}{n}} \times \exp\left[i\left(-kx+\omega t+\theta_0\right)\right],$$

$$\text{for } j = 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10 \tag{45}$$

Family 8

$$B_{8,j} = a_2 = b_2 = \mp\mu_j^2\sqrt{\frac{4c_2\sigma_3\left[672 - M_1\left(39 - M_2\left(126 - 40M_3\right)\right)\right]}{-141n\beta_2}}$$

$$\Phi_{8,j}(x,\ t) = B_{8,j}^{\frac{1}{n}}\left[1 + \tanh^2\left(\mu_j(x-\gamma t)\right) + \coth^2\left(\mu_j(x-\gamma t)\right)\right]^{\frac{1}{n}} \times \exp\left[i\left(-kx + \omega t + \theta_0\right)\right],$$

$$\text{for } j = 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10 \tag{46}$$

Family 9

$$B_{9,j} = a_2 = b_2 = \mp\mu_j^2\sqrt{\frac{c_2\sigma_3\left[65 + M_1\left(21 + M_2\left(21 + 104M_3\right)\right)\right]}{-50n\beta_2}}$$

$$\Phi_{9,j}(x,\ t) = B_{9,j}^{\frac{1}{n}}\left[1 + \tanh^2\left(\mu_j(x-\gamma t)\right) + \coth^2\left(\mu_j(x-\gamma t)\right)\right]^{\frac{1}{n}} \times \exp\left[i\left(-kx + \omega t + \theta_0\right)\right],$$

$$\text{for } j = 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10 \tag{47}$$

Family 10

$$B_{10,j} = a_2 = b_2 = \mp\mu_j^2\sqrt{\frac{c_2\sigma_3\left[432 + M_1\left(39 + M_2\left(24 - 4M_3\right)\right)\right]}{9n\beta_2}}$$

$$\Phi_{10,j}(x,\ t) = B_{10,j}^{\frac{1}{n}}\left[1 + \tanh^2\left(\mu_j(x-\gamma t)\right) + \coth^2\left(\mu_j(x-\gamma t)\right)\right]^{\frac{1}{n}} \times \exp\left[i\left(-kx + \omega t + \theta_0\right)\right]$$

$$\text{for } j = 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10 \tag{48}$$

Family 11

$$B_{11,j} = a_2 = b_2 = \mp\mu_j{}^2\sqrt{\frac{2c_2\sigma_3\left[415 - M_1\left(117 - M_2\left(183 - 192M_3\right)\right)\right]}{-55n\beta_2}}$$

$$\Phi_{11,j}(x,\ t) = B_{11,j}^{\frac{1}{n}}\left[1 + \tanh^2\left(\mu_j(x-\gamma t)\right) + \coth^2\left(\mu_j(x-\gamma t)\right)\right]^{\frac{1}{n}} \times \exp\left[i\left(-kx + \omega t + \theta_0\right)\right],$$

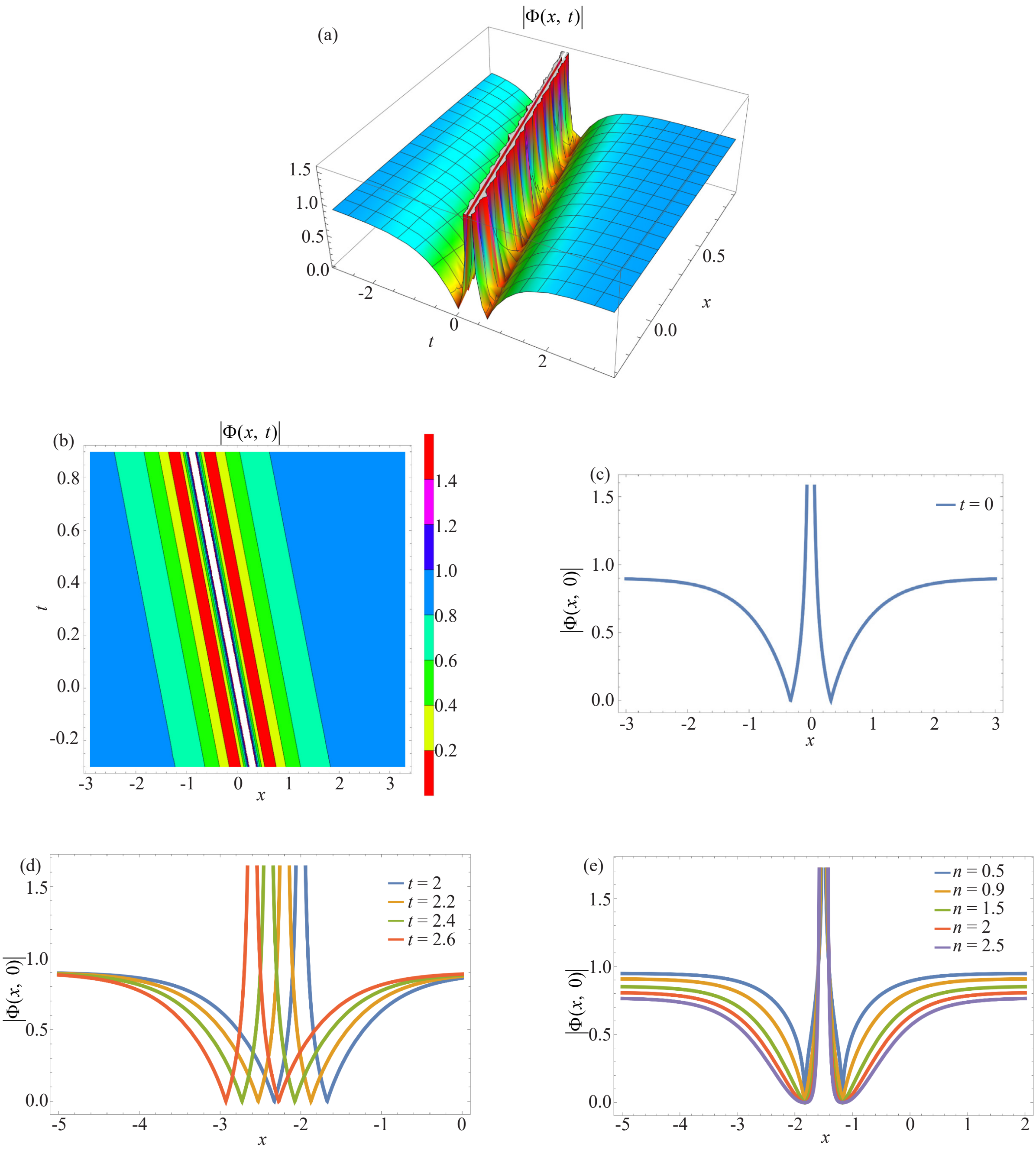
$$\text{for } j = 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10 \tag{49}$$

Family 12

$$B_{12,j} = a_2 = b_2 = \mp\mu_j^2 \sqrt{\frac{2c_2\sigma_3\left[1,279 - M_1\left(39 - M_2\left(153 + 184M_3\right)\right)\right]}{-91n\beta_2}}$$

$$\Phi_{12,j}(x,\ t) = B_{12,j}^{\frac{1}{n}}\left[1 + \tanh^2\left(\mu_j(x - \gamma t)\right) + \coth^2\left(\mu_j(x - \gamma t)\right)\right]^{\frac{1}{n}} \times \exp\left[i\left(-kx + \omega t + \theta_0\right)\right],$$

$$\text{for } j = 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10 \tag{50}$$



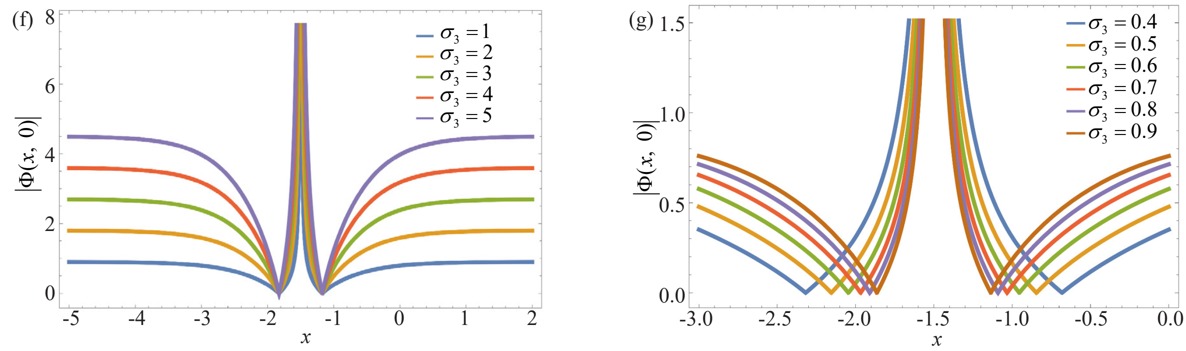


Figure 1. Profile of a dark-singular straddled optical soliton (a) Surface plot; (b) Contour plot; (c) 2D plot setting the time variable: $t = 0$; (d) 2D plot with the time variable t set at various intervals; (e) 2D plot with the effect of power nonlinearity; (f) 2D plot with the effect of higher order dispertion; (g) 2D plot with the effect of nonlinear dispertion

5. Results and discussion

In this section, we present and discuss the results obtained from the analysis of the dark-singular straddled optical soliton solution described by the complex-valued solution (39). The evolution of this solution is illustrated through various subfigures in Figure 1, where we explore the impacts of different parameters, including time (t), power nonlinearity (n), higher-order dispersion (σ_3), and nonlinear dispersion (σ_4). The detailed analysis of these parameters provides a comprehensive understanding of the behavior and characteristics of the dark-singular straddled optical soliton solution. Figure 1 (a) presents a surface plot of the dark-singular straddled optical soliton solution. This plot illustrates the three-dimensional representation of the soliton's amplitude over time and space. The surface plot clearly shows the evolution of the soliton, highlighting the regions of maximum and minimum amplitude. The visualization helps in understanding the overall structure and dynamics of the soliton, emphasizing the interaction between dark and singular components. Figure 1 (b) depicts a contour plot of the dark-singular straddled optical soliton solution. This plot provides a two-dimensional representation, where contour lines represent the amplitude levels of the soliton. The contour plot is particularly useful for identifying the spatial distribution of the soliton's amplitude and observing the changes in its shape and intensity over time. The contour lines reveal the presence of distinct dark and singular regions within the soliton structure. The 2D plots in Figures 1 (c) to Figures 1 (g) offer detailed insights into the specific effects of various parameters on the dark-singular straddled optical soliton solution. These plots present the soliton's amplitude as a function of space at different instances and parameter settings. Figures 1 (c) and Figures 1 (d) show the 2D plots of the soliton's amplitude at different time variables: $t = 0.2, 2.2, 2.4, 2.6$. The evolution of the soliton over time reveals the dynamic nature of the dark and singular components. The soliton undergoes periodic changes in amplitude, with the dark region appearing as a dip in amplitude and the singular region characterized by sharp peaks. Figure 1 (e) addresses the effect of power nonlinearity (n) on the dark-singular straddled optical soliton solution. By varying the power nonlinearity variables: $n = 0.5, 0.9, 1.5, 2, 2.5$, we observe significant changes in the soliton's amplitude and shape. As the power nonlinearity increases, the amplitude of the soliton becomes more pronounced, indicating a stronger interaction between the dark and singular components. Figure 1 (f) illustrates the impact of higher-order dispersion (σ_3) on the soliton solution. The higher-order dispersion variables are set to: $\sigma_3 = 1, 2, 3, 4, 5$. The results show that higher-order dispersion affects the spreading and localization of the soliton. Increased dispersion leads to broader soliton profiles, whereas lower dispersion maintains a more localized structure. Figure 1 (g) explores the effect of nonlinear dispersion (σ_4) on the dark-singular straddled optical soliton solution. The nonlinear dispersion variables are: $\sigma_4 = 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$. The plots demonstrate that nonlinear dispersion significantly influences the soliton's stability and amplitude modulation. Higher nonlinear dispersion values enhance the soliton's robustness and intensity. The dark-singular straddled optical soliton solution is also analyzed with respect to several key parameter variables: $k = 1$, $a = 1$, $b = 1$, $c_1 = 1$, $c_2 = 1$, $c_3 = 1$, $\sigma_1 = 1$, $\sigma_2 = 1$, $\sigma_9 = 1$. These parameters are critical in defining the soliton's characteristics and behavior. The comprehensive analysis of these parameters provides insights into how different physical factors contribute to the formation and evolution of the dark-singular straddled optical soliton. In addition to the surface, contour, and 2D plots, Figure 1 also includes the modulus of the dark-singular straddled optical soliton solution. The modulus representation emphasizes the amplitude of the soliton, providing a clear visualization of its intensity distribution. The modulus plot is particularly useful for identifying regions of maximum and minimum amplitude, and for understanding the overall energy distribution within the soliton. The detailed analysis presented in Figure 1 highlights the complex and dynamic nature of the dark-singular straddled optical soliton solution. By examining the effects of time, power nonlinearity, higher-order dispersion, and nonlinear dispersion, we gain a comprehensive understanding of how these factors influence the soliton's behavior. The surface plot, contour plot, and 2D plots provide valuable insights into the soliton's structure, evolution, and stability. The findings from this study contribute to the broader understanding of optical solitons and their potential applications in photonics and optical communications.

6. Conclusion

The dispersive concatenation model was integrated in the current research, exhibiting power-law nonlinearity that led to dark-singular straddled optical solitons. Tanh-Coth integration technique has enabled this retrieval. The current paper paves way for further investigations in this avenue. This model will be addressed further along with additional issues such as bifurcation analysis, numerical studies using Laplace-Adomian decomposition and variational iteration approach, studying the model with differential group delay and with dispersion-flattened fibers and several many features [18-23]. The results of such research activities will be disseminated that would yield wider perspective to this model. Those results are currently awaited.

Conflicts of interest

The authors declare no competing financial interest.

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