Contemporary Mathematics
http://ojs.wiserpub.com/index.php/CM/

Research Article

Mathematical Modeling of Velocity Field Induced by the Vortex

I. Kangro[1*], H. Kalis[2], A. Aboltins[3]

[1] Institute of Engineering, Rezekne Academy of Technologies, Atbrīvošanas Aleja 115, LV-4601, Rēzekne, Latvia
[2] Institute of Mathematics and Computer Science, University of Latvia, 19 Blv. Rainis, Riga, LV-1586, Latvia
[3] Faculty of Engineering and Information Technologies, Latvia University of Life Sciences and Technologies, 5 J. Cakstes blvd., Jelgava, LV-3001, Latvia
E-mail: ilmars.kangro@rta.lv

Received: 10 June 2024; **Revised:** 23 July 2024; **Accepted:** 31 July 2024

Abstract: In new technological applications, it is important to use vortex distributions in the area for obtaining large velocity fields. Analogous to electromagnetic and fluid mechanics inductions, vortices are described by the same relationship: the Biot-Savart law. When vortex threads form a vortex track due to fluid mechanics, they induce, analogous to electrical coils on an iron core, a core flow that can be faster than the wind that generates it. In this publication, the velocity field induced in a cylinder using the axially symmetric system of vortex rings and screw vortices and the hydrodynamic flow function in the ideal incompressible fluid are calculated. Also similar problem for mathematical modelling of the heat generation in fluids with alternating current using vortexes is considered. In this paper, it was calculated the distribution of the velocity field and distribution of stream function for ideal incompressible fluid, induced by a different system of finite number of vortex threads: (1) circular vortex lines in a finite cylinder, positioned on its inner, (2) spiral vortex threads, positioned on the inner surface of the finite cylinder or cone, (3) linear vortex lines in the plane channel, positioned on its boundary. An original method was used to calculate the components of the velocity vectors. Such kind of procedure allows calculating the velocity fields inside the domain depending on the arrangement, the intensity, and the radii of vortex lines. In this paper, we have developed a mathematical model for the process in the element of Hurricane Energy Transformer. This element is a central figure in the so-called RKA (ReaktionsKraftAnlage) used on the cars' roofs.

Keywords: channel, circular vortexes, finite cylinder, finite cone, incompressible fluid, spiral vortex threads, velocity field, vector potential, vortexes chain

MSC: 35J57, 35J67, 35J25, 35J15, 35Q35, 35Q80, 65N22, 76W05, 76D99

1. Introduction

In traditional heating systems for dwelling houses fuel is used to warm up the water, which flows through the heating system. The offered mathematical model describes the function of such heating devices in which the water of the heating system is warmed up with the help of alternating current in one mode. The conclusion is that it helps to increase the efficiency of the device (there is no excess loss of heat) and that the device is extremely compact.

DOI: https://doi.org/10.37256/cm.6120255125

The heating of buildings by ecologically clean and compact local devices is interesting and actual problem. One of the modern areas of applications is the effective use of electrical energy produced by alternating current in production of heat energy. This process is ecologically clean, there is no environmental pollution. Though, on the other hand, the aspect of energy is very important: the transformation process is to be organized in such a way that electric energy should be effectively transformed into heat energy. It can be achieved by reducing the amount of elements in the energy transformation process.

For example, if the medium, where heat energy is produced, is at the same time a transmitter in a heating system, then there will be no losses. One such media is undistilled water. It is ecologically harmless and easily available. Water (with a small amount of nonaggressive salts added, if needed) must be a weakly electrically conducting medium (electrolyte). In the last ten years, devices based on this principle have been developed. Compared to the classical devices with heating elements, they are more compact and do not calcify.

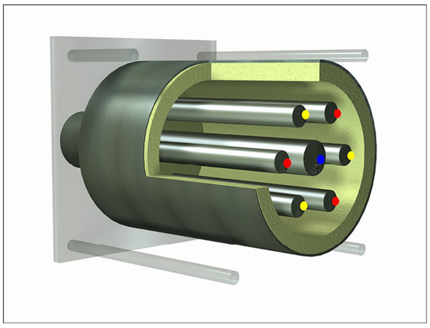

Figure 1. The heat generator

In [1] 2010 presents the mathematical model of one of such devices (see Figure 1). It is an infinite cylinder with metal conductors -electrodes of the forms of bars placed parallel to the cylinder axis in the liquid. For those conductors, the alternating current is connected. Modelled cylinders form electrical heat generators with six circular conductors-electrodes. If this cylinder is placed, for example, in a house heating system together with a small electromotor which rotates water in the entry of cylinder and pumps water through it, we obtain an effective, compact and ecologically clean house heating device. The distribution of electromagnetic fields, forces and temperature has been calculated. The average axially-symmetric motion of electrolyte and temperature distribution in a cylinder has been obtained in dependence on the values of frequencies and arrangement of electrodes.

In new technological applications, it is important to use vortex distributions in the area for obtaining large velocity fields [2]. The effective use of vortex energy in the production of strong velocity fields by different devices is one of the modern areas of applications, developed in the first decade of the 21st. century. Such processes are ecologically clean, there is no environmental pollution.

On the other hand, the aspect of energy is very important: the transformation process should be organized in such way that vortex energy is effectively transformed into mechanical energy. It can be achieved by reducing the amount of elements in the energy transformation process.

This paper, it was calculated the distribution of the velocity field and distribution of stream function for ideal incompressible fluid, induced by a different system of the finite number of vortex threads:

(1) circular vortex lines in a finite cylinder, positioned on its inner,

(2) spiral vortex threads, positioned on the inner surface of the finite cylinder or cone,

(3) linear vortex lines in the plane channel, positioned on its boundary.

An original method was used to calculate the components of the velocity vectors. Such kind of procedure allows calculating the velocity fields inside the domain depending on the arrangement, the intensity, and the radii of vortex lines.

In this paper, we have developed a mathematical model for the process in the element of Hurricane Energy Transformer. This element is a central figure in the so-called RKA (ReaktionsKraftAnlage) used on the cars' roofs for substations reducing the air's drag [3, 4] (see Figure 2), in the area for obtaining large velocity fields [2, 5].

Figure 2. The cars' roofs

People have been dealing with vortices for as long as we can look back. Analogous to electromagnetic and fluid mechanics inductions, vortices are described by the same relationship: the Biot-Savart law. When vortex threads form a vortex track due to fluid mechanics, they induce, analogous to electrical coils on an iron core, a core flow that can be faster than the wind that generates it. In this publication, the velocity field induced in a cylinder using the axially symmetric system of vortex rings and screw vortices and the hydrodynamic flow function in the ideal incompressible fluid are calculated.

In 2004 Bertasius, Buikis and Verzbovicius formulated patent [6] of apparatus and methods for heat generation. Later Buikis and Kalis constructed a mathematical model of this heat generator [7–9].

In this model, the viscous electrically conducting incompressible liquid is located between two infinite coaxial cylinders (rings). The electromagnetic force drives magneto-hydrodynamic flow between the cylinders.

In 2009 is designed a similar generator to [10] and created a mathematical model for generator [1, 11]. In the internal cylinder parallel to the axis are placed metal conductors-electrodes of the forms of bars. For those conductors, the alternating current is connected. The water is a weakly electrically conducting liquid (electrolyte). This is the mathematical model of one device for electrical energy produced by alternating current in the production of heat energy (see Figure 1).

The distribution of electromagnetic fields, forces, 2D magnetohydrodynamic flow, and temperature induced by the system of the alternating electric current or external magnetic field in a conducting cylinder has been calculated using finite difference methods for solving partial differential equations. An original method was used to calculate the mean values of electromagnetic forces.

The second interesting way in the vortexes exploitation in devices was collaboration with inventor Schatz in Germany [12, 13]. In new technological applications, it is important to use vortex distributions for obtaining large values of velocity. The effective use of vortex energy in the production of strong velocity fields by different devices is one of the modern areas of applications, developed during the last decade. Although, on the other hand, the aspect of energy is very important: the transformation process should be organized in such a way that vortex energy is effectively transformed into heat or mechanical energy. In our previous papers [7, 8, 14] we have mathematically modeled the process how transforming the alternating electrical current into heat energy.

The practical aim of this investigation is to try to understand the process in the element of Hurricane Energy Transformer. This element is a central figure in the so-called RKA (German: ReaktionsKraft Anlage, English: Reaction Force Device) used on the cars' roof for substation reducing the airs' drag. On May 14, 2006, Schatz [13] conducted the following experiment: with the passenger car "WW Passat", driving a road section of 553.40 km with the above-mentioned equipment RKA and without this equipment. The results were as follows-when driving with the device RKA consumed an average of 7.33 litres of gasoline per 100 km, while without the device-8.50 litres of gasoline per 100 km. This is all that's done at the practical level in mathematical modeling.

However, several practical and theoretical questions are left unanswered. Devices sometimes have worked with effectiveness higher than 100 percent. Important is that in such a system there are strong vortices and electromagnetic fields or high velocities.

For example, in [6, 10] the alternating electro currency with voltage 380 V is about 1 ampere on 1 cm. Theoretically, the answer may be that we have a contradiction in the macro and micro processes in such devices [15].

Following Kim [16], we require a new paradigm beyond materialism including the information field on the theory of Physical vacuum. It is easy to call such science as pseudoscience, but within its framework, it is possible to portray scalar waves [17, 18]. In recent years, there have been several other new approaches: space-time as energy [18]. We should discuss these approaches with an open mind, without a simple rejection.

In [19] 2014 the stability of circular vortices to normal mode perturbations is studied in a multi-layer geostrophic model.

In [20] 2020 study as an oceanographic application the interaction mechanism of a large-scale surface vortex with a smaller vortex/vortices in a 3-layer rotating fluid.

In [21] 2021 this article discusses the possibility of using the Lin-Sidorov-Aristov class of exact solutions and its modification to describe the flows of a fluid with microstructure (with couple stresses). The article presents exact solutions for the described layered, shear and 3-D flow of a micropolar viscous incompressible fluid, generalized classical Couette, Stokes and Poiseuille flows.

The goal of this paper is to develop the mathematical models for a new type of ecologically clean and energetically effective devices [12, 22–25].

Such a type of device firstly was developed by Rechenberg [3] (1988). Now the continuator of the work is one of the authors Schatz. The devices of such type can be considered as the energy source of the new generation. The practical aim of this investigation is to try to understand the process in the element of Hurricane Energy Transformer [12].

This work presents three mathematical models of such devices.

It is well known that the vortex theory begins from the Descartes papers. The three famous Helmholtz vortex theorems are known:

(1) the circulation around different cross-sections in the translation of a vortex tube is constant,

(2) the vortex adheres to the matter, particles that have once formed a vortex line continue to do so,

(3) the circulation of a vortex tube remains constant over time.

First of all, it was investigated the behaviour of the discrete N linear vortex lines with equal intensity Γ, which are in the vertices of the regular rectangle (authors are Helmholtz, Kelvin, and Kirchhoff, see [26–28]).

Helmholtz (1858) proves, that in ideal barotropic fluid, the vortex lines fulfil the conservation law and for the case of two circular vortex lines:

(1) vortexes to approach one other with increasing radii, if the rotation of vortices is opposite,

(2) vortexes move in one direction with different behaviour of their radii, if the rotation is coincide (see [27, 29]). Kirchhoff (1876) proved the equations of N vortexes motions and Kelvin (1867) gave the atom theory of vortexes.

The investigation of contemporary are write in the books [29, 30]: completely are investigated linear vortex lines, vortex sheets, vortex wakes, and vortexes of Karman, but difficulties caused the curves of vortex lines.

Natural hurricanes are known to form over the world's oceans when solar radiation concentrates large amounts of heat on small areas, water vapour is created and a heat flow rises upwards into the atmosphere. Geostrophic winds can cause rotations in this heat flow, which can develop into hurricanes under certain conditions. In hurricanes, nature concentrates kinetic energy equivalent to about 100,000 Hiroshima bombs. The aim was to operate technical hurricanes in machines and systems to generate usable energy.

The properties of air allow it to be regarded as an ideal incompressible fluid. In the late 1980s, Prof. Rechenberg of TU Berlin discovered natural flow-mechanical inductions. This research laid the groundwork for the inventions of Prof. Schatz in Berlin, particularly within the company "EUROVORTEX Energietechnik GmbH" in 2004. Prof. Schatz demonstrated that vortex technologies can be used economically.

The vortex coil concentrator cannot harvest more energy, but the wind influenced by a large, non-rotating surface can be concentrated on a small surface. Therefore, the wind turbine arranged in the core can be built much smaller and thus operated without problems with multiple concentration factors.

Electromagnetic systems, transformers, motors, and generators, as well as "Hurricane or Tornado Energy Transformers", are all described using the same connection: the Biot-Savart induction law. Instead of electrons in electromagnetic systems, molecules in Newtonian fluids are moved. The circulation or intensity γ is analogous to the electric current strength A. This allows the technical reproduction of natural energy concentrations in hurricanes.

The question was whether we could use our understanding of how kinetic energy moves in the atmosphere to accumulate and store kinetic energy for work using similar methods. Ring vortices and screw vortices create a speed parallel to their axis and move at a constant speed in space. This speed increases as the radius of the ring or core decreases. We can manufacture vortex tubes and use them in different ways. These vortex tubes must be created as tube vortices with large circulations Γ and high circumferential speeds. We can generate one or more vortex tubes using technical processes, even in technical facilities. These vortex tubes are crucial for creating vortex engines (referred to as WTW, which stands for Wirbeltriebwerken in German and Vortex Force Device in English) with auxiliary drives requiring strong circulation. Vortex tubes must be generated in vortex generators.

The vortex tubes generated by vortex generators wind up in a WTW to form multi-turn screw vortices that are concentrated in space. High energy densities are created in the WTW. The frequency of the oscillation in a technically formed screw vortex depends directly on the circumferential speed of the inducing vortex tubes. The molecules within the multi-turn screw vortex experience vectorial forces at the centre of gravity. They move in the same direction parallel to each other, and the flow-mechanical resistance decreases in the core flow area. The smaller the pitch angle of the vortex tubes, the greater the induced additional speed. WTWs can also harness flow energy instead of using the rotors of modern windmills.

The technical concept involves creating energetically usable vortex systems inside technical facilities, such as potential vortices, vortex tubes, vortex coils, or screw vortices. These systems create large energy concentrations in technical spaces and can interact with the atmosphere in time-related processes via induced jet flows, as long as the rotations are maintained by an auxiliary energy supply. The first experimental vortex engine was built in Lübben/Spreewald (1998).

In new technological applications, it is important to use vortex distributions to achieve high-velocity values.

2. Mathematical model

Let the cylindrical domain (conus)

$$\Omega_{r,\, z}(\varepsilon) = \{(r,\, z,\, \varphi) : 0 < r < a - \varepsilon z,\, 0 < z < Z,\, 0 < \varphi < 2\pi(M+1)\}\ (0 \le \varepsilon Z < a)$$

contain ideal incompressible fluid, where $a,\, Z$ are the maximal radius and length of the cylinder, M is the number of circulation periods, $r,\, z,\, \varphi$ are the cylindrical coordinates, ε is the nonnegative constant.

If $\varepsilon = 0$, then we have the circular cylinder with the radius a.

Consider the situation when the N discrete circular vortex lines

$$L_i = \{(r,\, z),\ r = a_i,\ z = z_i\},\ \ 0 < z_i < Z,\ \ 0 < a_i < a,\ \}\ i = \overline{1,\, N},$$

with intensity $\Gamma_i \left(\dfrac{m^2}{s}\right)$ and radii $a_i(m)$ are placed in the cylinder.

The vortex creates in the ideal compressible liquid the radial v_r and axial v_z components of the velocity field, which rises to the liquid motion.

Similar can be consider N discreate spiral vortex threads

$$S_i = \{(r,\ z,\ \varphi),\ r = a - \varepsilon t,\ z = bt,\ \varphi = t + i\delta\},\ \ i = \overline{1,\ N},$$

with parameters $\delta = \dfrac{2\pi}{N},\ \ \tau = \dfrac{Z}{2\pi aM},\ \dfrac{2\pi}{N} \le \varphi \le 2\pi(M+1),\ b = a\tau,\ t \in [0,\ 2\pi M].$

Here τ is the rise of the vortex threads, the spiral vortex with $Z = 2\pi,\ a = 1,\ N = 6,\ M = 1,\ \tau = 1,\ \varepsilon = 0.$

In the Figure 3, we can see the circular vortex lines.

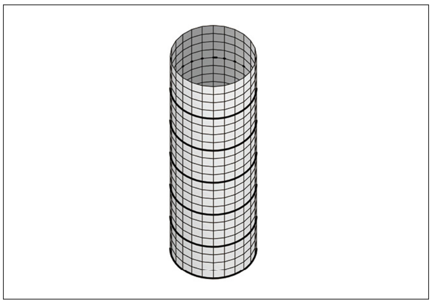

Figure 3. The surface of the cylinder with circular vortexes lines

The spiral vortexes creates in the ideal compressible liquid the radial v_r, axial v_z and azimuthal v_φ components of the velocity field.

The linear vortex lines create in the plane domain-channel

$$\Omega_{x,\ y} = \{(x,\ y) : x \in [0,\ L],\ y \in [0,\ 2],\ z \in (-\infty,\ \infty)\}$$

the $v_x,\ v_y$ components of the velocity field.

The main aim of this work is to analyze the diversity of connection schemes of vortex curves influence the maximal value of velocity.

3. Calculation of the velocity field for the spiral vortexes

The vector potential A is determined from the equations of vortex motion of ideal incompressible fluid [22–28].

$$div\, v = 0,\ \ rot\, v = \Omega,$$

in the following form:

$$\Delta A = -\Omega,$$

where $v = rotA$ and v, Ω the vectors of velocity and vortex fields are, Δ is the Laplace operator.

Applying the Biot-Savar law [27, 28], we receive the following form of the vector potential created by the vortex thread W_i ($W_i = S_i$ or $W_i = L_i$):

$$A(P)_i = \frac{\Gamma_i}{4\pi} \int_{W_i} \frac{dl}{R(QP)_i}$$

where dl is an element of the curves, $P = P(x, y, z)$ is the fixed point in the liquid, $Q = Q(\xi, \eta, \zeta)$ is the changeable point in the integral

$$R(QP)_i = \sqrt{((z-\zeta)^2 + (x-\xi_i)^2 + (y-\eta_i)^2)}.$$

From cylindrical coordinates $x = r\cos\varphi$, $y = r\sin\varphi$, for the spiral vortexes S_i:

$$\xi_i = a_*(t)\cos(t+i\delta),\ \eta_i = a_*(t)\sin(t+i\delta),\ \zeta = bt,\ (b = a\tau),$$

$$t \in [0,\ 2\pi M]\ (a_*(t) = a - \varepsilon t)$$

and we have the following components of the vector potential:

$$A_{x,\ i} = \frac{\Gamma_i}{4\pi} \int_{S_i} \frac{d\xi}{R_i},\ A_{y,\ i} = \frac{\Gamma_i}{4\pi} \int_{S_i} \frac{d\eta}{R_i},$$

$$A_{z,\ i} = \frac{\Gamma_i}{4\pi} \int_{S_i} \frac{d\zeta}{R_i},$$

where $R_i = R(QP)_i$ (see Figure 4).

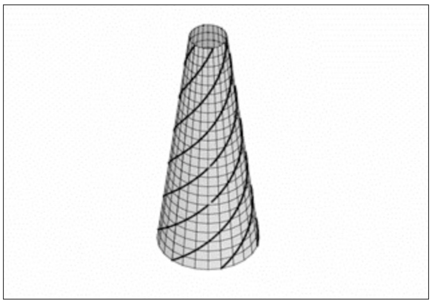

Figure 4. Spiral vortices on the cone with $\varepsilon = 0.1$, $Z = 2\pi$

Therefore

$$d\xi = (-a_*(t)\sin(t+i\delta) - \varepsilon\cos(t+i\delta))dt,\ d\eta = (a_*(t)\cos(t+i\delta) - \varepsilon\sin(t+i\delta))dt,\ d\zeta = bdt,$$

$$R_i = \sqrt{r^2 + a_*(t)^2 - 2a_*(t)r\cos(\varphi - t - i\delta) + (z-bt)^2}$$

and

$$A_{x,\ i} = -\frac{\Gamma_i}{4\pi}\int_0^{2\pi M}\frac{(a_*(t)\sin(t+i\delta) + \varepsilon\cos(t+i\delta))dt}{R_i},$$

$$A_{y,\ i} = \frac{\Gamma_i}{4\pi}\int_0^{2\pi M}\frac{(a_*(t)\cos(t+i\delta) - \varepsilon\sin(t+i\delta))dt}{R_i},$$

$$A_{z,\ i} = \frac{\Gamma_i b}{4\pi}\int_0^{2\pi M}\frac{dt}{R_i}.$$

The vector components of the velocity field (radial, axial, azimuthal) induced by the spiral vortex curves are in the form

$$\begin{cases} v_{r,\ i} = -\dfrac{\partial A_{\varphi,\ i}}{\partial z} + \dfrac{\partial A_{z,\ i}}{r\partial\varphi}, \\[2ex] v_{z,\ i} = \dfrac{1}{r}\dfrac{\partial}{\partial r}(rA_{\varphi,\ i}) - \dfrac{1}{r}\dfrac{\partial A_{r,\ i}}{\partial\varphi}, \\[2ex] v_{\varphi,\ i} = \dfrac{\partial A_{r,\ i}}{\partial z} - \dfrac{\partial A_{z,\ i}}{\partial r}, \end{cases} \tag{1}$$

where

$$A_{r,\ i} = A_{x,\ i}\cos(\varphi) + A_{y,\ i}\sin(\varphi) = \frac{\Gamma_i}{4\pi}\int_0^{2\pi M}\frac{(a_*(t)\sin(\psi(t)) - \varepsilon\cos(\psi(t)))dt}{R_i},$$

$$A_{\varphi,\ i} = -A_{x,\ i}\sin(\varphi) + A_{y,\ i}\cos(\varphi) = \frac{\Gamma_i}{4\pi}\int_0^{2\pi M}\frac{(a_*(t)\cos(\psi(t)) + \varepsilon\sin(\psi(t)))dt}{R_i},$$

$(\psi = \varphi - t - i\delta)$ are the radial and azimuthal components of vector potentials.

Then from the partial derivatives

$$\frac{\partial R_i}{\partial r} = \frac{r - a_*(t)\cos(\psi(t))}{R_i},\ \frac{\partial R_i}{\partial z} = \frac{z-bt}{R_i},\ \frac{\partial R_i}{\partial\varphi} = \frac{a_*(t)r\sin(\psi(t))}{R_i},$$

follows

$$v_{r,\ i}=\frac{\Gamma_i}{4\pi}\int_0^{2\pi M}\frac{1}{R_i^3}[(z-bt)(a_*(t)\cos(\psi(t))+\varepsilon\sin(\psi(t)))-ba_*(t)\sin(\psi(t))]dt, \tag{2}$$

$$v_{z,\ i}=\frac{\Gamma_i}{4\pi}\int_0^{2\pi M}\frac{1}{R_i^3}[a_*(t)(a_*(t)-r\cos(\psi(t)))-\varepsilon r\sin(\psi(t))]dt, \tag{3}$$

$$v_{\varphi,\ i}=\frac{\Gamma_i}{4\pi}\int_0^{2\pi M}\frac{1}{R_i^3}[b(r-a_*(t)\cos(\psi(t)))-(z-bt)(a_*(t)\sin(\psi(t))+\varepsilon\cos(\psi(t)))]dt.$$

For $\varepsilon=0$ and for the symmetrical properties respect to $z=Z/2$ follows that for the all components of velocity $\mathbf{v}_i(r,\ Z/2-z,\ \varphi)=\mathbf{v}_i(r,\ Z/2+z,\ \varphi)$.

If $r=0$, then

$$v_{z,\ i}(0,\ z)=\frac{\Gamma_i}{4\pi}\int_0^{2\pi M}\frac{a_*(t)^2dt}{(a_*(t)^2+(z-bt)^2)^{1.5}} \tag{4}$$

or

$$v_{z,\ i}(0,\ z)=\frac{\Gamma_i\varepsilon^2}{4\pi}\int_{a-2\pi M\varepsilon}^{a}\frac{q^2dq}{R(q)^3},$$

where

$$R(q)=\sqrt{a_1+b_1q+c_1q^2},\ \ a_1=b^2z_0^2,\ \ b_1=-2b^2z_0,\ c_1=\varepsilon^2+b^2,\ \ z_0=a-\frac{z\varepsilon}{b}.$$

Therefore, from [26]:

$$\begin{cases} v_{z,\ i}(0,\ z)=\dfrac{\Gamma_i}{4c_1\pi}\left[\dfrac{d_2a_2-2a_1b_1}{d_1R(a_2)}-\dfrac{d_2a-2a_1b_1}{d_1R(a)}-\right. \\ \left.\dfrac{\varepsilon^2}{\sqrt(c_1)}\ln\dfrac{\sqrt(c_1)R(a_2)+c_1a_2+b_1/2}{\sqrt(c_1)R(a)+c_1a+b_1/2}\right], \end{cases} \tag{5}$$

where

$$a_2=a-2\pi\varepsilon M,\ \ d_1=4b^2z_0^2,\ \ d_2=d_1(\varepsilon^2-b^2).$$

If $\varepsilon=0$, then

$$v_{z,\,i}(0,\,z)=\frac{\Gamma_i M}{2Z}\left[\frac{z}{\sqrt{a^2+z^2}}+\frac{Z-z}{\sqrt{a^2+(Z-z)^2}}\right], \tag{6}$$

and the maximal value of velocity is

$$v_{z,\,i}(0,\,Z/2)=\frac{\Gamma_i M}{2a\sqrt{1+(Z/(2a))^2}} \tag{7}$$

by $z=Z/2$.

The minimal value we have in the form

$$v_{z,\,i}(0,\,0)=v_{z,\,i}(0,\,Z)=\frac{\Gamma_i M}{2a\sqrt{1+(Z/a)^2}} \tag{8}$$

by $z=0$ and $z=Z$.

The averaged value of the axial component of velocity field in the axes of the cylinder $(r=0)$ is

$$v_{av,\,i}=\frac{1}{Z}\int_0^Z v_{z,\,i}(0,\,z)dz. \tag{9}$$

The averaged value for $\varepsilon=0,\ r=0$ is

$$v_{av,\,i}=\frac{\Gamma_i M}{2a}\frac{2}{1+\sqrt{1+(Z/a)^2}}. \tag{10}$$

From Rechenberg [3] $(\varepsilon=0)$ in the middle point of finite vortex spool $(z=Z/2)$ with the length Z the axial component of one vortex thread is

$$v_{max}=\frac{\Gamma_i}{\pi D}ctg(\beta)\sin\left(\arctan\left(\frac{Z}{D}\right)\right), \tag{11}$$

where β is the rise of vortex thread angles ($\beta=\arctan(\tau)$) and $D=2a$ is the diameter of the vortex spool.

For the minimal value of velocty (in the points $z=0$ und $z=Z$) [3]:

$$v_{min}=\frac{\Gamma_i}{2\pi D}ctg(\beta)\sin\left(\arctan\left(\frac{}{Z}a\right)\right). \tag{12}$$

We have equal values of v_{max} from (11) and from (7) using

$$\sin(\arctan(y)) = \frac{y}{\sqrt{1+y^2}}, \; y = \frac{Z}{D}, \;\; ctg(\beta) = \tau^{-1} = \frac{\pi DM}{Z}.$$

The averaged value (10) for $\varepsilon = 0$ is in the following form

$$v_{av} = \frac{\Gamma_i}{\pi D} ctg(\beta) \frac{\alpha}{\alpha a/Z + 1}, \tag{13}$$

where $\alpha = \sin\left(\arctan\left(\frac{}{Z} a\right)\right)$.

In the formulas parameters M and Z are depending:

$$M = \frac{Z}{\tau \pi D}, \;\; \tau = \tan(\beta).$$

Therefore from (4)-(13) for the velocity components $(v_r, \; v_z, \; v_\varphi)$ and for azimuthal component of vector potential A_φ induced by N discrete vortex are

$$v_r = \sum_{i=1}^{N} v_{r,\, i}, \; v_z = \sum_{i=1}^{N} v_{z,\, i}, \; v_\varphi = \sum_{i=1}^{N} v_{\varphi,\, i}, \; A_\varphi = \sum_{i=1}^{N} A_{\varphi,\, i}. \tag{14}$$

Integrals are with the trapezoid formulas calculated.

If the intensity Γ_i of N-spiral vortex S_i is equal Γ, than from (6)-(12) follows:

$$v_z(0, \; Z/2) = \frac{\Gamma NM}{D} \frac{1}{\sqrt{1+(Z/D)^2}}, \tag{15}$$

$$v_z(0, \; 0) = v_z(0, \; Z) = \frac{\Gamma NM}{D} \frac{1}{\sqrt{1+(Z/a)^2}}, \tag{16}$$

$$v_{max} = \frac{\Gamma N}{\pi D} ctg(\beta) \sin(\arctan\left(\frac{H}{D}\right)), \tag{17}$$

$$v_{min} = \frac{\Gamma N}{2\pi D} ctg(\beta) \sin(\arctan\left(\frac{H}{a}\right)), \tag{18}$$

where N-number of vortex threads, $\; H = Z$-the height of the vortex spool (in building synonym of the length) are.

For averaged value of velocity $(\varepsilon = 0)$ we have the formula

$$v_{av} = \frac{\Gamma NM}{D} \frac{2}{1+\sqrt{1+(Z/a)^2}}, \tag{19}$$

or

$$v_{av} = \frac{\Gamma N}{\pi D} \frac{\alpha}{\alpha a/H + 1} ctg(\beta), \tag{20}$$

where $\alpha = \sin\left(\arctan\left(\frac{}{H}a\right)\right)$.

If the averaged value v_{av} is known, then can be calculated from (19) also the dimensionless length $y = \frac{Z}{a}$ in following form

$$y = \frac{2\delta}{\delta^2 - 1},$$

where

$$\delta = \Gamma N ctg(\beta)/(\pi D v_{av}).$$

An example, if $\Gamma = 6.0319\left(\frac{m^2}{s}\right),\ \beta = 10^0(C),\ D = 0.25(m),\ N = 1,\ v_{av} = 30\left(\frac{m}{s}\right),$ then $\delta = 1.452$ and $y = 2.62,\ Z = 0.3275(m)$.

The corresponding formulas (15)-(20) are identical, but from (15), (16) and (19) follows, that the velocity depends on the parameter $M * N$, where $M = \frac{H}{\tau\pi D}$.

From (15), (16) and (19) we can the corresponding multiplicators by $\frac{\Gamma NM}{D}$ calculated (see Table 1).

$$R_1 = \frac{1}{\sqrt{1 + (Z/D)^2}},\ R_2 = \frac{1}{\sqrt{1 + (Z/a)^2}},$$

and

$$R_3 = \frac{2}{1 + \sqrt{1 + (Z/a)^2}}.$$

4. Calculation of the velocity field for the circular vortex lines

For the circular one vortex lines:

$$\xi = a_i \cos\alpha,\ \eta = a_i \sin\alpha,\ \zeta = z_i,\ d\xi = -a_i \sin\alpha d\alpha,$$

$$d\eta = a_i \cos\alpha d\alpha,\ d\zeta = 0$$

and from the axially-symmetric condition follows that by $\varphi = 0$ is $A_{x,\ i} = A_{z,\ i} = 0$ and

$$A_{y,\ i} = A_{\varphi,\ i} = A_i(r,\ z) = \frac{\Gamma_i a_i}{4\pi} I_i,$$

where

$$I_i = \int_0^{2\pi} \frac{\cos\alpha d\alpha}{\sqrt{(z-z_i)^2 + a_i^2 + r^2 - 2a_i r\cos\alpha}}.$$

The integral I_i is equal [28]

$$I_i = \int_0^{\pi/2} \frac{(1-2\sin^2 t)dt}{\sqrt{((z-z_i)^2+(r+a_i)^2)}\sqrt{1-k_i^2\sin^2 t}} = \frac{2}{\sqrt{ra_i}}\left[\left(\frac{2}{k_i}-k_i\right)K(k_i) - \frac{2}{k_i}E(k_i)\right],$$

where

$$t = (\alpha-\pi)/2,\ k_i = 2\sqrt{ar}/c_i, \quad c_i = \sqrt{(a_i+r)^2+(z-z_i)^2},$$

$$K(k) = \int_0^{\pi/2} \frac{dt}{\sqrt{1-k^2\sin^2 t}}$$

is the total elliptical integral of first kind,

$$E(k) = \int_0^{\pi/2} \sqrt{1-k^2\sin^2 t}dt$$

is the total elliptical integral of the second kind.

Therefore the azimuthal component of vector potential A_i induced by a circular vortex line L_i with intensity Γ_i and with radius a_i is

$$A_i(r,\ z) = \frac{\Gamma_i}{2\pi}\sqrt{\frac{a_i}{r}}\left[\left(\frac{2}{k_i}-k_i\right)K(k_i) - \frac{2}{k_i}E(k_i)\right].$$

The vectorial components of the velocity field (the radial and axial components) induced by vortex line L_i are

$$v_{r,\ i} = -\frac{\partial A_i}{\partial z},\ v_{z,\ i} = \frac{1}{r}\frac{\partial}{\partial r}(rA_i). \tag{21}$$

or

$$v_{r,\, i}(r,\, z) = \frac{\Gamma_i}{2\pi r}\frac{z - z_i}{c_i}\left[E(k_i)\frac{a_i^2 + r^2 + (z - z_i)^2}{(a_i - r)^2 + (z - z_i)^2} - K(k_i)\right], \tag{22}$$

$$v_{z,\, i}(r,\, z) = \frac{\Gamma_i}{2\pi c_i}\left[K(k_i) + \frac{a_i^2 - r^2 - (z - z_i)^2}{(a_i - r)^2 + (z - z_i)^2}E(k_i)\right]. \tag{23}$$

If $r = 0$ then

$$v_{z,\, i}(0,\, z) = \frac{\Gamma_i}{2}\frac{a_i^2}{(a_i^2 + (z - z_i)^2)^{1.5}}. \tag{24}$$

This component of vectors have the maximal value $v_{z,\, i} = \dfrac{\Gamma_i}{2a}$ by $z = z_i,\ a_i = a$.
By $z = z_i + Z/2$ we have

$$v_{z,\, i} = \frac{\Gamma_i}{2\sqrt{a^2 + Z^2/4}}\frac{a^2}{a^2 + Z^2/4} < \frac{\Gamma_i}{2\sqrt{a^2 + Z^2/4}} -$$

this is the value of the component of velocity induced by spiral vortex ($\varepsilon = 0$).
If $z = Z/2,\ a_i = a,$ then from (24) follows

$$v_{z,\, i}(0,\, Z/2) = \frac{\Gamma_i}{D}\frac{1}{(1 + ((Z/2 - z_i)/a)^2)^{1.5}}. \tag{25}$$

For the averaged value of the velocity we have

$$v_{av,\, i} = \frac{\Gamma_i}{D}\frac{a}{Z}\left(\frac{(Z - z_i)/a}{\sqrt{1 + ((Z - z_i)/a)^2}} + \frac{z_i/a}{\sqrt{1 + (z_i/a)^2}}\right). \tag{26}$$

If $z_i = Z/2$, then

$$v_{av,\, i} = \frac{\Gamma_i}{D}\frac{1}{\sqrt{1 + (Z/D)^2}}.$$

The summary velocity field $(v_r,\ v_z)$ and the vector potential A_φ induced by N discrete vortex lines we obtain in the form (14). The hydrodynamic stream function $\psi = \psi(r,\, z)$ for velocity components $v_r = -\dfrac{1}{r}\dfrac{\partial \psi}{\partial z},\ v_r = \dfrac{1}{r}\dfrac{\partial \psi}{\partial r},$ from (21) is $\psi(r,\, z) = rA_\varphi(r,\, z)$.

The amount of flow through cross-section $[z = z_0,\ 0 < r < a_0]$ is

$$Q(a_0,\, z_0) = \int_0^{a_0}\int_0^{2\pi} v_z(r,\, z_0) r dr\, d\varphi = 2\pi a_0 A_\varphi(a_0,\, z_0) = 2\pi\psi(a_0,\, z_0).$$

The total amount of flow through cross cylindrical domain $[0 < z < Z,\ 0 < r < a_0]$ is

$$Q_t(a_0) = \int_0^Z Q(a_0,\ z)dz = 2\pi \int_0^Z \psi(a_0,\ z)dz.$$

For the circular vortex line, if $z_i/a = 0.2i,\ \ i = \overline{1,\ N},\ N \leq 6$, we can calculated following multiplicators by the factor $\dfrac{\Gamma}{D}$:

$$R_4(Z) = \sum_{i=1}^{N} (1 + ((Z/2 - z_i)/a)^2)^{-1.5}$$

for (25),

$$R_5 = \frac{a}{Z} \sum_{i=1}^{N} \left(\frac{}{(Z - z_i)/a} \sqrt{1 + ((Z - z_i)/a)^2} + \frac{z_i/a}{\sqrt{1 + (z_i/a)^2}} \right)$$

for (26).

An example, if $Z/a = 1.4$ then we can the multiplicators $R_4((0),\ R_4(Z/2),\ R_4(Z),\ R_5$ for the circular vortex lines and $R_1,\ R_2,\ R_3$ for the spiral vortexes by the factor $\dfrac{\Gamma M}{D}$ in the form $R_1 * N,\ \ R_2 * N,\ \ R_3 * N$ calculated (see Table 1).

Table 1. Multiplicators of the velocity for vortexes by $\dfrac{Z}{a} = 1.4$

N	$R_4(0)$	$R_4(Z/2)$	$R_4(Z)$	R_5	R_1	R_2	R_3
1	0.94	0.71	0.26	0.69	0.82	0.58	0.74
2	1.74	1.59	0.62	1.46	1.64	1.16	1.47
3	2.37	2.58	1.09	2.27	2.46	1.74	2.21
4	2.85	3.56	1.72	3.09	3.28	2.32	2.94
5	3.20	4.44	2.52	3.85	4.10	2.91	3.68
6	3.47	5.16	3.47	4.55	4.92	3.48	4.41

In the following calculations, we use the dimensionless form scaling all the lengths to $r_0 = a$ (the inlet radius of the tube), the axial v_z and radial v_r velocity to $v_0 = \dfrac{\Gamma_0}{2\pi r_0}$, the azimuthal components of vector potential A_φ to $A_0 = \dfrac{\Gamma_0}{2\pi}$, the stream function ψ to $\psi_0 = A_0 r_0$ and the total amount of flow Q_t to $Q_0 = \psi_0 r_0$. Here Γ_0 is dimensional scaling of vortex intensity $\Gamma_i,\ i = \overline{1,\ N}$.

5. The flows field induced by linear vortex lines in a channel

Unlike our previous papers [22, 23] here we additionally consider the chain of linear vortexes lines in the plane channel. For symmetry-conditions $\dfrac{\partial v_x}{\partial y}|_{y=1}$ we consider half the plane channel $y \in [0,\ 1]$.

In the plane $y = 0$ we have the slip-conditions $v_x = v_y = 0$ for the velocity vectors of viscous incompressible liquid.

The flow in the the channel is given by a fixed amount of flow through a cross-section of the half channel $Q = \int_0^1 v_x|_{x=0} dy$.

If $L = \infty$, then $v_x = u(y)$, $v_y = 0$ and we have the Poiseuille flow $u = Q(3y - 1.5y^2)$ the solution of Navier-Stokes equation in the channel $\Omega_{x,\, y}$.

The wall $y = 0$ of the channel is placed in a linear chain of vortexes with the axis transfer of the $(x,\, y)$ plane. The one linear vortex line in the point $(x_k,\, y_k)$ create the following components of velocity:

$$v_x = -\frac{\Gamma_k}{2\pi}\frac{y-y_k}{R^2},\; v_y = \frac{\Gamma_k}{2\pi}\frac{x-x_k}{R^2}, \tag{27}$$

where $R^2 = (x-x_k)^2 + (y-y_k)^2$.

In the centre of this point-wise vortex, the velocity field is infinite therefore we consider the vortex line with finite cross section the circle with radius a. In this case the expressions (27) are valid when $R \geq a$, but for $R = a$ we have

$$v_x = -\frac{\Gamma_k}{2\pi a^2}(y-y_k),\; v_y = \frac{\Gamma_k}{2\pi a^2}(x-x_k). \tag{28}$$

6. Some numerical results and discussion

6.1 *The flow in the channel*

We consider the channel with finite length $L = 2.5$, the Poiseuille flow with $Q = 3$ and three wise of the chain of vortexes:

(1) the main chain with coordinates and radius of the linear vortex

$$x_k = 0.2 + (k-1)0.4,\; y_k = 2a,\; k = 1,\, 2,\, 3,\, 4,\, 5,\, 6,\; a = 0.05, \tag{29}$$

rotate clockwise with the intensity Γ_1,

(2) the second chain with coordinates and radius of the linear vortex

$$x_k = 0.4 + (k-1)0.4,\; y_k = 2a_1,\; k = 1,\, 2,\, 3,\, 4,\, 5,\; a_1 = 0.025, \tag{30}$$

rotate opposite clockwise with the intensity Γ_2,

(3) the thread chain with coordinates and radius of the linear vortex

$$x_k = 0.3 + (k-1)0.4,\; y_k = 2a + a_1,\; k = 1,\, 2,\, 3,\, 4,\, 5,\; a = 0.05,\; a_1 = 0.025, \tag{31}$$

rotate opposite clockwise with the intensity Γ_3.

For the pointwise vortexes line (29) outside the channel $(y_k = -0.025)$ with $\Gamma_1 = -6$ we have following results: $mV = 5.9895$, $mX = 1.00$, $mY = 0$.

For the Karman chain [28] of vortexes (preliminary vortexes line and (30) with $y_k = -0.05$, $\Gamma_2 = 6$) we have $mV = 3.9790$, $mX = 0.20$, $mY = 0$.

In following Table 2 can see the amount (Q), maximal value of velocity u, (mV) with the coordinates $(mX,\, mY)$ depending of the vortex intensity Γ_1, Γ_2, Γ_3.

Table 2. The dependence of flow velocity from the intensity of the vortexes

Γ_1	Γ_2	Γ_3	Q	mV	mX	mY
0	0	0	3.00	4.500	0.00	1.00
-6	3	3	3.97	18.19	2.20	0.15
-6	4	4	3.46	22.90	0.30	0.10
-6	3	0	4.62	18.36	0.20	0.15
-6	2	2	4.49	18.63	2.20	0.15
-6	1	1	5.00	19.08	2.20	o.15
-6	1	0	5.22	19.14	0.20	0.15
-6	0	1	5.30	19.47	2.20	0.15
-6	0	0	5.52	19.86	1.00	0.15

6.2 *The circular vortexes lines*

As the basis for the calculations of N circular vortex lines $L_i,\ i = \overline{1,\ N}$ are $N \leq 6$ chosen, which are arranged in the axial direction at the points with the following dimensionless coordinates $(z_i = 0.2i,\ r_i = a_i),\ i = \overline{1,\ N}$.

The dimensionless radius of the circular vortex lines a_i is considered in three forms (the sequence $a = [a_1,\ a_2,\ a_3,\ a_4,\ a_5,\ a_6]$) :

(1) the constant sequence(radius of the cylinder) $a_c = [1,\ 1,\ 1,\ 1,\ 1,\ 1]$,

(2) the monotonous increasing sequence $a_{in} = [0.75,\ 0.80,\ 0.85,\ 0.90,\ 0.95,\ 1.0]$,

(3) the monotonous decreasing sequence $a_d = [1.0,\ 0.95,\ 0.90,\ 0.85,\ 0.80,\ 0.75]$.

All results of numerical experiments to solve this problem were obtained at the Institute of Mathematics and Computer Science of the University of Latvia, using the MAPLE, and MATLAB software packages.

The results of numerical experiments for dimensionless values $v_r,\ v_z,\ \psi,\ Q_t$ was obtained of different dimensionless intensity of vortex lines $\tilde{\Gamma}_i = \dfrac{\Gamma_i}{2\pi\Gamma_0} = \pm\,6;\ \pm\,3;\ \pm\,2;\ 1;\ 0.5,$ and $l = Z/r_0 = 2,\ a_0 = 0.7$.

The summary intensity of absolute values is equal to 6. The velocity field is calculated in the uniform grid $(n_r \times n_z)$ by the steps $h_1 = h_2 = 0.1$ in the $r,\ z$ directions.

The numerical results show that the velocity field is induced by circular vortex lines is concentrated inside the cylinder. The results depend on the arrangement and the radius of vortex lines a_i.

Typical results of calculations are: the dimensionless velocity field and the distribution of stream function in the cylinder. We can see the velocity formation depends on the arrangement of vortices lines with coordinates $zj = [z_1,\ z_2,\ z_3,\ z_4,\ z_5,\ z_6]$, and of the radii a_i.

If $\tilde{\Gamma}_i > 0$ then all vortices move in the positive direction of Oz axis $(v_z > 0)$, but the radii of vortex lines to stay a different way (for $v_r < 0$ the radius is decreasing and for $v_r > 0$ the radius is increasing).

We obtain for the dimensionless values of $v_r \in [v_{r.min},\ v_{r.max}],\ vz_{max},\ \psi_{max},\ Q_t$ for $zj = [0.2,\ 0.4,\ 0.6,\ 0.8,\ 1.0,\ 1.2]$ and for different radius of vortex lines a_i and sequence of intensity $gj = [g_1,\ g_2,\ g_3,\ g_4,\ g_5,\ g_6]$ the following results:

1. The radii are constant $a_c = [1,\ 1,\ 1,\ 1,\ 1,\ 1]$

(1) The intensity of the one vortex lines L_3 is $\tilde{\Gamma}_3 = 6,\ N = 1 : v_r \in (-5.9,\ 5.9),\ vz_{max} = 18.85,\ \psi_{max} = 3.25, v_r = 0$ if $z = z_3 = 0.6$ and $v_r > 0$ if $z > z_3$, therefore the radius of vortex increased [27];

(2) The intensity of the one vortex line L_3 is $\tilde{\Gamma}_3 = -6,\ N = 1$ (the opposite direction): $v_r \in (-5.9,\ 5.9),\ vz_{min} = -18.85,\ \psi_{min} = -3.25,$ the vortex move in the negative direction of Oz axes $(v_z < 0),\ v_r = 0$ if $z = z_3 = 0.6$ and $v_r > 0$ if $z < z_3$, therefore the radius of vortex also increased [27];

(3) The intensity of the two vortex lines $L_3,\ L_4$ are $\tilde{\Gamma}_3 = 3,\ \tilde{\Gamma}_4 = 3,\ N = 2 : v_r \in (-5.7,\ 5.7),\ vz_{max} = 18.57,\ \psi_{max} = 3.17,$ the vortexes move in the positive direction of Oz axes $(v_z > 0),\ v_r = 0$ if $z = (z_3 + z_4)/2 = 0.7$ and $v_r(a_0,\ z_3) = -2.46,\ v_r(a_0,\ z_4) = 4.37,$ therefore the radius of the first vortex lines L_3 decreased, but for the second vortex lines L_4 increased and the first vortex can be moved through the second vortex [27];

(4) The intensity of the two vortex lines $L_3,\ L_4$ are $\tilde{\Gamma}_3 = -3,\ \tilde{\Gamma}_4 = 3,\ N = 2 : v_r \in (-2.9,\ 0.64),\ v_z \in (-3.0,\ 3.0),$ $\psi \in (-0.32,\ 0.32),\ v_z = 0$ if $z = 0.7$ and $v_z(a_0,\ z_3) = -1.72,\ v_z(a_0,\ z_4) = 2.76,$ therefore the first vortex moves in the negative direction, but the second to the positive direction of Oz axes and the radii of the vortexes decreased (this case is not in [27] considered);

(5) The intensity of the two vortex lines $L_3,\ L_4$ are $\tilde{\Gamma}_3 = 3,\ \tilde{\Gamma}_4 = -3,\ N = 2 : v_r \in (-0.64,\ 2.9),\ v_z \in (-3.0,\ 3.0),$ $\psi \in (-0.32,\ 0.32),\ v_z = 0$ if $z = 0.7$ and $v_z(a_0,\ z_3) = 1.72,\ v_z(a_0,\ z_4) = -2.76,$ the first vortex move to the positive direction, but the second to the negative direction of Oz axes and the radii of the vortexes increased [27];

(6) The intensity of the three vortex lines $L_1,\ L_3,\ L_5$ are $\tilde{\Gamma}_1 = 2,\ \tilde{\Gamma}_3 = 2,\ \tilde{\Gamma}_5 = 2,\ N = 3 : v_r \in (-4.1,\ 4.1),$ $vz_{max} = 16.34,\ \psi_{max} = 2.63,\ v_r = 0$ if $z = z_3 = 0.6$ and $v_z(a_0,\ z_1) = 15.92,\ v_z(a_0,\ z_3) = 16.16,\ v_z(a_0,\ z_5) = 15.92,$ $v_r(a_0,\ z_1) = -3.8,\ v_r(a_0,\ z_5) = 1.6,$ the vortexes move in the positive direction of the Oz axis and the radius of the first vortex decreased, but of the third vortex increased;

(7) The intensity of the three vortex lines $L_1,\ L_3,\ L_5$ are $\tilde{\Gamma}_1 = -2,\ \tilde{\Gamma}_3 = 2,\ \tilde{\Gamma}_5 = -2,\ N = 3 : v_r \in (-1.6,\ 1.6),$ $vz_{min} = -5.83,\ \psi_{min} = -0.74,\ v_r = 0$ if $z = z_3 = 0.6,\ z = 0.1,\ z = 1.1$ and $v_z(a_0,\ z_1) = -5.67,\ v_z(a_0,\ z_3) = -2.42,$ $v_z(a_0,\ z_5) = -3.56,\ v_r(a_0,\ z_1) = -0.77,\ v_r(a_0,\ z_5) = 0.77,$ the vortexes move in the negative direction of the Oz axis and the radius of the first vortex decreased, but of the third vortex increased;

(8) The intensity of the three vortex lines $L_1,\ L_3,\ L_5$ are $\tilde{\Gamma}_1 = 2,\ \tilde{\Gamma}_3 = -2,\ \tilde{\Gamma}_5 = 2,\ N = 3 : v_r \in (-1.6,\ 1.6),\ v_{z.max} =$ $5.83,\ \psi_{max} = 0.74,\ v_r = 0$ if $z = z_3 = 0.6$ and $v_z(a_0,\ z_1) = 5.67,\ v_z(a_0,\ z_3) = 1.97,\ v_z(a_0,\ z_5) = 5.67,\ v_r(a_0,\ z_1) = 0.77,$ $v_r(a_0,\ z_5) = -0.77,$ the vortexes move in the positive direction of Oz axis and the radius of the first vortex increased, but of the third vortex decreased;

(9) The intensity of the three vortex lines $L_1,\ L_3,\ L_5$ are $\tilde{\Gamma}_1 = -2,\ \tilde{\Gamma}_3 = 2,\ \tilde{\Gamma}_5 = 2,\ N = 3 : v_r \in (-4.9,\ 2.6),$ $v_z \in (-1.75,\ 11.1),\ \psi \in (-0.10,\ 1.45),\ v_r = 0$ if $z = 0.9$ and $v_z(a_0,\ z_1) = -0.64,\ v_z(a_0,\ z_3) = 8.28,\ v_z(a_0,\ z_5) = 10.89,$ $v_r(a_0,\ z_1) = -3.17,\ v_r(a_0,\ z_3) = -3.95,\ v_r(a_0,\ z_5) = 0.77,$ the two vortexes $L_3,\ L_5$ move in the positive direction, but the first in the negative direction of the Oz axis and the radii of the two vortexes $L_1,\ L_3$ are decreased, but of the third vortex increased;

(10) The intensity of the three vortex lines $L_1,\ L_3,\ L_5$ are $\tilde{\Gamma}_1 = 2,\ \tilde{\Gamma}_3 = 2,\ \tilde{\Gamma}_5 = -2,\ N = 3 : v_r \in (-2.6,\ 4.9),$ $v_z \in (-1.75,\ 11.1),\ \psi \in (-0.10,\ 1.45),\ v_r = 0$ if $z = 0.3$ and $v_z(a_0,\ z_1) = 10.89,\ v_z(a_0,\ z_3) = 8.28,\ v_z(a_0,\ z_5) = -0.64,$ $v_r(a_0,\ z_1) = -0.77,\ v_r(a_0,\ z_3) = 3.95,\ v_r(a_0,\ z_5) = 3.17,$ the two vortexes $L_1,\ L_3$ move in the positive direction, but the vortex L_5 in the negative direction of the Oz axis and the radii of the two vortexes $L_3,\ L_5$ are increased, but of the third vortex L_1 decreased;

(11) The intensity of the three vortex lines $L_1,\ L_3,\ L_5$ are $\tilde{\Gamma}_1 = -2,\ \tilde{\Gamma}_3 = -2,\ \tilde{\Gamma}_5 = 2,\ N = 3 : v_r \in (-4.9,\ 2.6),$ $v_z \in (-11.1,\ 1.75),\ \psi \in (-1.45,\ 0.10),\ v_r = 0$ if $z = 0.3$ and $v_z(a_0,\ z_1) = -10.89,\ v_z(a_0,\ z_3) = -8.28,\ v_z(a_0,\ z_5) = 0.64,$ $v_r(a_0,\ z_1) = 0.77,\ v_r(a_0,\ z_3) = -3.95,\ v_r(a_0,\ z_5) = -3.17,$ the two vortexes $L_1,\ L_3$ move in the negative direction, but the third in the positive direction of the Oz axis and the radii of the two vortexes $L_3,\ L_5$ are decreased, but of the first vortex increased.

2. The radii are increasing a_{in}

(1) The non-uniform distribution of intensity $gj = [2,\ 2,\ 1,\ 0.5,\ 0.5,\ 0],\ N = 5 : v_r \in (-12.7,\ 7.4),\ v_{z.max} = 21.15,$ $\psi_{max} = 4.6,\ Q_t = 28.34,\ v_r = 0$ if $z = 0.3,$ the radius of the first vortex decreased but increased the radii of the last four vortexes;

(2) The distribution of intensity $gj = [2,\ 2,\ 2,\ 0,\ 0,\ 0],\ N = 3 : v_r \in (-13.3,\ 10.02),\ v_{z.max} = 22.23,\ \psi_{max} =$ $4.8,\ Q_t = 28.69,\ v_r = 0$ if $z = 0.3,$ the radius of the first vortex decreased but increased the radii of the last vortex;

(3) The distribution of intensity $gj = [0,\ 0,\ 3,\ 3,\ 0,\ 0],\ N = 2 : v_r \in (-10.2,\ 9.4),\ v_{z.max} = 21.14,\ \psi_{max} = 4.64,\ Q_t =$ $29.20,\ v_r = 0$ if $z = 0.7,$ the radius of the first vortex decreased but increased the radii of the last vortex;

(4) The intensity of first vortex lines $gj = [6,\ 0,\ 0,\ 0,\ 0,\ 0],\ N = 1 : v_r \in (-19.2,\ 19.2),\ v_{z.max} = 25.13,\ \psi_{max} =$ $6.47,\ Q_t = 27.10,\ v_r = 0$ if $z = 0.2,$ the radius of the vortex increased;

(5) The intensity of second vortex lines $gj = [0,\ 6,\ 0,\ 0,\ 0,\ 0],\ N = 1 : v_r \in (-15.0,\ 15.0),\ v_{z.max} = 23.56,$ $\psi_{max} = 5.69,\ Q_t = 29.28,\ v_r = 0$ if $z = 0.4,$ the radius of the vortex increased;

(6) The intensity of third vortex lines $gj = [0,\ 0,\ 6,\ 0,\ 0,\ 0],\ N = 1 : v_r \in (-11.8,\ 11.8),\ v_{z.max} = 22.18,\ \psi_{max} =$ $5.11,\ Q_t = 29.69,\ v_r = 0$ if $z = 0.3,$ the radius of the vortex increased;

(7) The intensity of fourth vortex lines $gj = [0, 0, 0, 6, 0, 0]$, $N = 1 : v_r \in (-9.6, 9.6)$, $v_{z.max} = 20.94$, $\psi_{max} = 4.66$, $Q_t = 28.72$, $v_r = 0$ if $z = 0.3$, the radius of the vortex increased.

3. The uniform distribution of intensity $gj = [1, 1, 1, 1, 1, 1]$

(1) Radii of vortex lines are constant (the sequence $a_c : v_r \in (-4.5, 4.5)$, $v_{z.max} = 16.21$, $\psi_{max} = 3.14$, $Q_t = 25.12$, $v_r = 0$ if $z = 0.7$, the radii of the first three vortexes decreased, but of the last three vortexes increased;

(2) Radii of vortex lines are $a_{in} : v_r \in (-8.4, 4.9)$, $v_{z.max} = 17.98$, $\psi_{max} = 3.52$, $Q_t = 27.36$, $v_r = 0$ if $z = 0.8$, the radii of the first three vortexes decreased but of the last three vortexes increased;

(3) Radii of vortex lines are $a_d : v_r \in (-4.9, 8.4)$, $v_{z.max} = 17.98$, $\psi_{max} = 3.52$, $Q_t = 27.36$, $v_r = 0$ if $z = 0.5$, the radii of the first two vortexes decreased but increased the radii of the last four vortexes.

4. The distribution of intensity $gj = [2, 2, 0.5, 0.5, 0.5, 0.5]$

(1) Radii of vortex lines are $a_{in} : v_r \in (-12.3, 6.9)$, $v_{z.max} = 20.19$, $\psi_{max} = 4.4$, $Q_t = 27.77$, $v_r = 0$ if $z = 0.3$, the radius of the first vortex decreased but increased the radii of the last five vortexes;

(2) Radii of vortex lines are $a_d : v_r \in (-5.7, 5.6)$, $v_{z.max} = 17.30$, $\psi_{max} = 3.4$, $Q_t = 26.01$, $v_r = 0$ if $z = 0.4$, the radius of the first vortex decreased but increased the radii of the last five vortexes.

5. The distribution of intensity $gj = [0.5, 0.5, 0.5, 0.5, 2, 2]$

(1) Radii of vortex lines are $a_{in} : v_r \in (-5.6, 5.8)$, $v_{z.max} = 17.30$, $\psi_{max} = 3.4$, $Q_t = 26.01$, $v_r = 0$ if $z = 1.0$, the radii of the first four vortexes decreased but increased the radius of the last vortex;

(2) Radii of vortex lines are $a_d : v_r \in (-6.8, 12.3)$, $v_{z.max} = 20.19$, $\psi_{max} = 4.4$, $Q_t = 27.77$, $v_r = 0$ if $z = 1.1$, the radii of the first five vortexes decreased, but those of the last vortex increased.

6. The distribution of intensity $gj = [0.5, 0.5, 2, 2, 0.5, 0.5]$

(1) Radii vortex lines are $a_{in} : v_r \in (-7.4, 6.6)$, $v_{z.max} = 19.47$, $\psi_{max} = 4.0$, $Q_t = 28.28$, $v_r = 0$ if $z = 0.7$, the radii of the first two vortexes decreased but increased the radii of the last four vortexes;

(2) Radii of vortex lines are $a_d : v_r \in (-6.6, 7.4)$, $v_{z.max} = 19.47$, $\psi_{max} = 4.0$, $Q_t = 28.28$, $v_r = 0$ if $z = 0.7$, the radii of the first two vortexes decreased but increased the radii of the last four vortexes.

6.3 *The spiral vortexes in the cylinder (ε = 0)*

We consider $N \leq 6$ spiral vortexes S_i, $i = \overline{1, N}$, where started from the points $(a, 0, i2\pi/N)$ at the cylinder. The dimensionless radius of the cylinder a is equal to 1.

All results of the numerical experiments are for the dimensionless values $A_\varphi(a_0, z, \varphi)$, $v_z(0, z)$, $Q(z)$, Q_t and parametern $l = Z/a = 0.5; 1; 1.5; 2; 3$, $a_0 = 0.7$ obtain. The summary intensity of absolute values is equal to 6.

The azimuthal components of the vector potential are in the uniform grid $(N_z \times N_\varphi)$ by the steps $h_z = l/N_z$, $h_\varphi = 2\pi/N_\varphi$, $(N_z = N_\varphi = 30)$ in the r, φ direction calculed. The component $A_\varphi(z, \varphi)$, $(r = a_0)$ using the trapezoid formula is calculated. We obtain typical results of calculations: the dimensionless velocity field and the distribution of the azimuthal component of the velocity $(r = a_0)$ in the cylinder. The velocity formation depends on the length l of the cylinder. The maximum of the azimuthal components of vector potentials A_{max} is depending of the intensity parameter $g_i = \tilde{\Gamma}_i$. We obtain the dimensionless values of $v_{z.max}$, Q_{max}, A_{max}, Q_t depends on different sequence of intensity $gj = [g_1, g_2, g_3, g_4, g_5, g_6]$.

In the following Table 3 can see the maximum of the azimuthal components of vector potentials A_{max} depending of the vortex intensity $gj = [g_1, g_2, g_3, g_4, g_5, g_6]$ for the length $l = 1.5$ of the cylinder, by $v_{z.max} = 15.08$, $Q_{max} = 24.98$, $Q_t = 33.20$.

Table 3. The dependence of the maximum of the azimuthal components of vector potentials A_{max} from the intensity of the vortexes by $l = 1.5$

g_1	g_2	g_3	g_4	g_5	g_6	A_{max}	A_φ form
1	1	1	1	1	1	5.68	uniform
2	2	1	0.5	0.5	0	5.68	nonuniform
2	2	2	0	0	0	5.75	oscillate
2	1	1	1	1	0	6.14	nonuniform
1.5	1.5	1.5	1.5	0	0	5.68	oscillate
3	3	0	0	08	0	5.83	nonuniform
6	0	0	0	0	0	8.54	nonuniform

For the length $l = 1.5$, the distribution of the intensity $gj = [6,\ 0,\ 0,\ 0,\ 0,\ 0]$, $N = 1$, $b = 0$, $A_{max} = 8.40$, $v_{z.max} = 18.85$, $Q_{max} = 36.95$, $Q_t = 24.54$, the distribution A_φ is uniform in the φ direction (this is the velocity field induced by the circular vortex line $(z_i = 0)$).

In following Table 4 can see the dimensionless values of $v_{z.max}$, Q_{max}, A_{max}, Q_t depending on the vortex intensity $gj = [g_1,\ g_2,\ g_3,\ g_4,\ g_5,\ g_6]$ and on the length l of the cylinder.

If $N_\varphi = N_z = M = 50$, then for $gj = [2,\ 2,\ 1,\ 0.5,\ 0.5,\ 0]$, $l = 2$, then $A_{max} = 6.0388$, $v_{z.max} = 13.3286$, $Q_{max} = 21.4262$, $Q_t = 37.6017$.

Table 4. The dependence of the dimensionless values of $v_{z.max}$, Q_{max}, A_{max}, Q_t from the intensity of the vortexes by different lengths

g_1	g_2	g_3	g_4	g_5	g_6	l	A_{max}	$v_{z.max}$	Q_{max}	Q_t
6	0	0	0	0	0	1	8.42	18.29	34.25	16.28
2	2	1	0.5	0.5	0	3	5.11	10.46	16.36	43.03
2	2	1	0.5	0.5	0	2	6.04	13.33	21.43	37.60
2	2	1	0.5	0.5	0	1	7.35	16.86	29.39	26.65
2	2	1	0.5	0.5	0	0.5	8.11	18.29	34.25	16.28

6.4 *The spiral vortexes in the cones ($\varepsilon \neq 0$)*

In this case, we have some results for the behaviour of spiral vortexes.

1. If $\Gamma = 6.0319\ (m^2/s)$, $N = 1$, $\beta = 10^0\ (C)\ (\tau = tg\ (\beta) = 0.1763)$, $a = 0.125\ (m)$, $Z \in [0.1,\ 1.0]\ (m)$, then from the formulas (14, 18) can be the values M; $V_1\ (\varepsilon = 0)$; $V_2(\varepsilon = 0.001)$; $V_3(\varepsilon = 0.002)$; $V_4(\varepsilon = -0.002)\ (m/s)$ calculated (see the Table 5).

Table 5. The velocity v_{av} by $a = 0.125$

Z	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
M	0.72	1.44	2.17	2.89	3.61	4.33	5.06	5.78	6.50	7.22
V_1	15.3	24.1	29.0	32.0	34.0	35.4	36.5	37.3	37.9	38.5
V_2	15.5	24.6	29.7	32.7	34.8	36.2	37.3	38.2	38.8	39.4
V_3	15.7	25.1	30.3	33.5	35.6	37.1	38.2	39.1	39.8	40.4
V_4	14.9	23.3	27.9	30.7	32.6	33.9	34.9	35.7	36.3	36.8

For V_2 and V_3 the radii by $Z = 1$ decreased from $a = 0.125\ (m)$ with $0.080\ (m)$ and $0.034\ (m)$, but for V_4 the radius increased with $0.216\ (m)$.

2. If $a = 0.25$ (m), then similar from the formulas (19, 20) can be the values M; $V_1(\varepsilon = 0)$; $V_2(\varepsilon = 0.004)$; $V_3(\varepsilon = 0.008)$; $V_4(\varepsilon = -0.008)$ (m/s) calculated (see the Table 6).

Table 6. The velocity v_{av} by $a = 0.25$

Z	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
M	0.36	0.72	1.08	1.44	1.80	2.17	2.53	2.89	3.25	3.61
V_1	4.19	7.64	10.2	12.1	13.5	14.5	15.4	16.0	16.6	17.0
V_2	4.27	7.86	10.6	12.6	14.0	15.2	16.0	16.7	17.3	17.8
V_3	4.34	8.10	11.0	13.1	14.6	15.9	16.8	17.6	18.2	18.7
V_4	4.06	7.23	9.54	11.2	12.4	13.4	14.1	14.7	15.2	15.6

For V_2 and V_3 the radii by $Z = 1$ decreased from $a = 0.25$ (m) with 0.16 (m) and 0.07 (m), but for V_4 the radius increased with 0.43 (m).

7. Conclusions

• Velocity fields of ideal compressible fluid influenced by a curved vortex field in a finite cylinder, finite cone and channel are investigated.

• Numerical results show that the maximum axial velocity and the total amount of flow depends on the connection method of producers of vortex energy.

• The maximal velocity is developed in the case of non-uniform distribution of vortex intensity and smaller radius of vortex lines.

• The maximal value of the velocity induced by the spiral vortexes is in the middle of the cylinder.

• The behaviour of vortex lines in the ideal incompressible flow depends on the number and the orientation of the vortex.

• The realization of circular vortices inside the pipe at the surface accelerates the flow speed inside the pipe if they rotate clockwise the flow depends on the values of parameters Re, Γ, A and the inflow mode in the pipe.

• The calculations are related to specific applications for velocity field in energy, induced by vortex curves.

• The numerical investigation made it possible to determine many specific vortex properties without expensive physical experiments.

• In the future, using the created mathematical models, wind and sea wave energy should be used to create different types of vortex related to specific applications for velocity fields and transform them into practical devices.

Conflict of interest

The authors declare no competing financial interest.

References

[1] Buikis A, Kalis H, Gedroics A. Mathematical modelling of 2D magnetohydrodynamics and temperature fields, induced by bar-type conductors in a cylinder. *Magnetohydrodynamics*. 2010; 46(1): 41-58.

[2] Widnall SE. The structure and dynamics of vortex filaments. *Annual Review of Fluid Mechanics*. 1975; 7(1): 141-166.

[3] Rechenberg I. *Entwicklung, Bau und Betrieb einer neuartigen Windkraftanlage mit Wirbelschraubens Konzentrator Projekt "BERWIAN"*. Germany: BMFT d., Bundesrepublik Deutschland; 1988.

[4] Kalis H, Kangro I. *Effective Finite Difference and Conservative Averaging Methods for Solving Problems of Mathematical Physics*. Latvia: Rezekne Academy of Technologies; 2021.

[5] Tu JY, Yeoh GH, Liu CQ, Tao Y. *Computational Fluid Dynamics, A Practical Approach*. Amsterdam: Elsevier; 2008.

[6] Veržbovičs P, Buikis A, Bertašius A. *Apparatus and Method for Heat Generation*. Latvian Patent No. 13168. Riga: Latvia Patent Office; 2004.

[7] Buikis A, Kalis H. Creation of temperature field in a finite cylinder by alternated electromagnetic force. In: Buikis A, Ciegis R, Fitt AD. (eds.) *Progress in Industrial Mathematics at ECMI 2002*. Heidelberg: Springer; 2004. p.247-251.

[8] Buikis A, Kalis H. Numerical modelling of heat and magnetohydrodynamic flows in a finite cylinder. *Computational Methods in Applied Mathematics*. 2002; 2(3): 243-259.

[9] Buikis A, Kalis H. The vortex formation in horizontal finite cylinder by alternating electric current. *Mathematical Modelling and Analysis*. 2005; 10(1): 9-18.

[10] Buikis A, Kursitis J. *Devices for Heat Generation*. Latvian Patent No. 13168. Riga: Latvia Patent Office; 2009.

[11] Buikis A, Kalis H, Buligins L. Mathematical modelling of alternating electromagnetic and hydrodynamic fields, induced by bar-type conductors in a cylinder. *Mathematical Modelling and Analysis*. 2009; 14(1): 1-9.

[12] Schatz J. *Mini-Tornado Mit Wirbeleffekten. Neue Universale Energielösungen*. Riga, Latvia: Riga Technical University Press; 2011. p.247-290.

[13] Schatz J. *Verfahren Und Vorrichtungen Zur Herstellung Und Wirtschaflichen Nutzung Er Neuerbarer Energietrager in Newtonischen Fluiden*. Berlin: Deutsches Patent-und Markenamt; 2014.

[14] Buikis A, Kalis H. Flow and temperature calculations of electrolyte for a finite cylinder in the alternating field of finite number circular wires. *Magnetohydrodynamics*. 2004; 40(1): 77-90.

[15] Greene B. *Das Elegante Universum*. Berlin: Siedler Verlag; 2000.

[16] Kim WH. Era for new paradigm. *Fluid Mechanics Open Access*. 2012; 1(1): 1000e105.

[17] Meyl K. Scalar waves theory and experiments. *Journal of Scientific Exploration*. 2001; 15(2): 199-205.

[18] Meyl K. About vortex physics and vortex losses. *Journal of Vortex Science and Technology*. 2012; 1: 1-10. Available from: https://doi.org/10.4303/jvst/235563.

[19] Carton X, Sokolovskiy M, Menesguen C, Agular A, Meunier T. Vortex stability in a multi-layer quasi-geostrophic model: Application to mediterranean water eddies. *Fluid Dynamics Research*. 2014; 46(6): 061401.

[20] Sokolovskiy MA, Carton XJ, Filyushkin BN. Mathematical modelling of vortex interaction using a three-layer quasigeotrophic model. Part 2: Finite-core-vortex approach and oceanographic application. *Mathematics*. 2020; 8(8): 1267

[21] Baranovkii ES, Burmasheva NV, Prosviryakov EY. Exact solution to the navier-stokes equations with couple stresses. *Symmetry*. 2021; 13: 1355. Available from: https://doi.org/10.3390/sym13081355.

[22] Buikis A, Kalis H, Schatz J. Velocity field in the ideal fluid, induced by vortex curves in a finite cylinder and cone. *WSEAS Transactions on Environment and Development*. 2005; 1(2-4): 26-32.

[23] Buikis A, Kalis H, Schatz J. Calculation of the velocity field for ideal fluid, induced by vortex curves in a finite cylinder. *Proceedings of WSEAS International Conference on Environment, Ecosystems and Development*. Venice, Italy: WSEAS Transactions on Environment and Development; 2006. p.255-260.

[24] Buikis A, Kalis H, Schatz J. Numerical computation of flows field caused by vortices in finite cylinder, cone and channel. In: *WSEAS Transactions on Mathematics*. Greece: World Scientific and Engineering Academy and Society; 2008.

[25] Buikis A. Vortexes in the physical science and technology. *Fluid Mechanics: Open Access*. 2015; 2(1): 1000109.

[26] Gradshtein IS, Rizjik LM. *The Tables of Integrals, Sums, Series and Products*. USA: Academic Press; 1962.

[27] Milne-Thomson JM. *Theoretical Hydrodynamics*. USA: Dover Publications; 1960.

[28] Kochin NE, Kibelj IA, Roze NV. *Theoretical Hydrodynamics*. USA: John Wiley and Sons; 1964.

[29] Boricov AV, Mamajev IS, Sokolovskij MA. *Fundamental and Application Problems of Vortexes Theory*. USA: Springer; 1997.

[30] Korn G, Korn TM. *Mathematical Handbook for Scientists and Engineers*. USA:Dover Publications; 1961.