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Research Article

Optimizing Group Size using Percentile Based Group Acceptance Sampling Plans with Application

Abdullah M. Almarashi[1], **Khushnoor khan**[2*]

[1] Department of Statistics, Faculty of Science, King Abdulaziz University, Jeddah, 21589, Saudi Arabia
[2] Department of Management Sciences, Faculty of Business Administration, MYUniversity, Islamabad, 44000, Pakistan
E-mail: Khushnoorkhan64@gmail.com

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Abstract: The present paper focuses on optimizing the sample size and the acceptance number which are commonly known as design parameters for the group acceptance sampling plan (*GASP*). Design parameters are analyzed under the assumption that the characteristic of interest for the product follows Another Generalized Transmuted-Exponential (*AGTransmuted-Exponential*) distribution. Using different values of the quality levels (*median lifetime*), the values of the Operating Characteristic function is determined. The proposed plan satisfies two types of risks based on producer's point of view and consumer's point of view at varied stipulated quality levels. Optimization of the sample size, group size and acceptance numbers is obtained through Monte Carlo simulation, for which relevant *R* codes were developed. Specific *R* codes are appended for future usage by academia and practitioners from various fields of life. The simulated results of the study are exhibited in the form of tables and explained with relevant examples. Results of the proposed GASP are compared with plan parameters obtained using MLE estimates of *AGTransmuted-Exponential* distribution and also by design parameters obtained using mean as quality level. Results of the study exhibited that Median as a quality parameter resulted in the decrease of group size and acceptance number simultaneously at all quality levels. Easy to follow the methodology of the current paper will open new vistas for applying the proposed GASP to a family of transmuted probability distributions. For illustration purposes, a real data set for fatigue fracture stress is analyzed using MLE estimates of *AGTransmuted-Exponential* distribution to demonstrate the implementation of the proposed sampling plan.

***Keywords*:** optimization, group size, acceptance number, truncation, median lifetime, monte carlo simulation

MSC: 62P30, 62D05, 65C05

1. Introduction

Wide-ranging lot sentencing techniques are used to improve upon the product quality and also to meet the specified standards set by manufacturers and end consumers. To accept or reject a submitted lot the most pivotal statistical quality control tool for lot sentencing is acceptance sampling. Acceptance sampling plan (ASP) envisages the sample size to be used coupled with acceptance and rejection criteria for sentencing individual lots. Numerous authors have worked on ASPs using various probability distributions, for elaborate literature see Aslam et al. [1]. When designing a single

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sampling plan only one element is entered into a tester and tested for possible acceptance or rejection. A single sampling plan usually involves a sample size n and the acceptance number c from a lot of size N. For instance, if from a lot size (N) is 5,000, a random sample size $n = 20$ is selected with acceptance number $c = 2$ this means that if the number of defective items are less than or equal to 2 we accept the lot otherwise reject the lot. However, in practice, testers can host multiple item numbers saving both cost and testing time.

In a group acceptance sampling plan (GASP) several items are placed in a tester at one time, these items are termed as a group and number of items in a group are known as group size. To decide whether to accept or reject a batch of items based on the inspection of sample groups rather than individual items is the basic theme of Group Acceptance Sampling Plans (GASP). GASP aids in decision-making processes, guaranteeing that the quality of the product meets predefined standards while at the same minimizing the inspection effort and cost. GASP are appropriately used in manufacturing and quality control where testing each and every item might be unfeasible or costly.

The application of GASP is significant in many industries e.g. in pharmaceuticals, the quality of products is pivotal, GASP guarantees that batches meet safety standards without the need for extensive testing .In electronics manufacturing, GASP helps maintain high product quality while reducing the cost and time associated with inspection (Dodge and Romig [2]).

The importance of the parameters involved in GASP cannot be overstated. The acceptance number c directly influences the risk of accepting defective batches (producer's risk) and rejecting good ones (consumer's risk). Adjusting the sample size n impacts the precision of the quality assessment and the resources required for inspection. Balancing these parameters helps achieve an optimal trade-off between inspection cost and product quality assurance (Schilling and Neubauer [3]). Latest trends in GASP methodologies have focused on optimizing these parameters to adapt to modern manufacturing practices, enhancing both efficiency and effectiveness.

Life testing sampling procedures are used when the quality characteristics pertain to product lifetime. Usually mean product life or median product life is set as a bench mark for testing the product life times. Ordinarily it is not feasible to observe failure, within available experimental lifetime duration and specially for highly reliable products hence, the researchers resort to truncated life tests. Truncated life times blended with GASP is termed in literature as GASP based on truncated life testing where the lifetime follows some known probability distribution. In short we can say that the number of items placed in a tester are known as group size denoted by r and the number of groups are denoted by g. Hence it can be said that, the number of groups to be determined tantamount to determination of the sample size within the paradigm of such testing. It is not feasible economically or otherwise, to inspect or test every item in the lot of finished product. It is well known that the cost of inspection is directly related to the sample size. To save the cost and time and energy one resorts to select a sample from the lot and tested for a specified time. GASPs are found to be more efficient as compared to the traditional sampling plans in terms of sample size. A rich literature on designing of GASP for various probability distributions is available in the literature. Aslam and Jun [4] used Inverse Rayleigh Distribution plus the Log-Logistic distribution based on the aforementioned premise. Srinivasa Rao [5] worked on the Marshall-Olkin extended Lomax distribution for a GASP based on a truncated life test. A very effective study on Group ASP using Inverse Weibull Distribution using median life as a quality parameter based on truncated life testing was conducted by Singh and Tripathi [6]. Lately Khan and Al garni [7] discussed GASP where the quality parameter median life was used when the product life followed Inverse Weibull Distribution, also Almarashi et al. [8] elaborately discussed the application of GASP when the product life followed Marshall-Olkin Kumaraswamy Exponential (MOKw-E) distribution based on truncated life testing with percentiles as quality levels. To induce more flexibility in the existing distributions generalized transmutations have been introduced in some known life testing distribution for elaborate references, see Owoloko et al. [9]. Both single/double ASPs using transmuted Rayleigh distribution when the duration experiment is truncated at a predetermined time were well demonstrated by Saha et al. [10].

Regarding a new family of distributions generated through generalization coupled with transmutation, one can see Bakouch [11]. A new generator for adding two shape parameters in the existing popular continuous distribution commonly known as "Another family of Generalized Transmuted Distributions -AGT-G" was developed by Merovci et al. [12]. The distribution used in the present study *AGTransmuted-Exponential* originates from the work of Merovci et al. [12]. Since

AGTransmuted-Exponential is a skewed distribution so percentiles will be more suitable as quality parameters as compare to mean.

By exploring the relevant literature one finds no work using percentiles of *AGTransmuted-Exponential* viz-e-viz GASP has been undertaken. In the current paper this gap will be filled by designing GASP using percentiles of *AGTransmuted-Exponential* distribution with special emphasis on the median life as the quality parameter. Present study will focus on obtaining optimum design parameters of GASP such as the sample size '*n*' and the acceptance number '*c*' when the life of the product will assume to follow *AGTransmuted-Exponential* and the duration of experiment is truncated at a predetermined time. The main aim of the current is to design GASP which provides small sample as compared to the existing GASPs.

Format of the Study: Rest of the study is formatted as: In Section 2 envisages basic characteristics-pdf, cdf, and Quantile Function of the *AGTransmuted-Exponential* distribution. In section 3, the GASP design based on *AGTransmuted-Exponential* is developed, particularly using the Quantile Function, together with the relevant equations to optimize average sample numbers (ASN). Section 4 deals with the practical implementation of *AGTransmuted-Exponential*. In Section 5, a comparison of the proposed GASP results obtained using specified parameter values and results using Maximum Likelihood Estimates (MLE) applied to the actual dataset. Finally, a brief conclusion/limitation and future implications are discussed in Section 6.

2. Characteristics of *AGTransmuted-Exponential*

The pdf and cdf of the parent distribution which is exponential are, respectively, given by

$$g(t,\ \psi) = \psi \exp(-\psi t)\ , \quad \psi \text{ and } t > 0, \tag{1}$$

and

$$G(t,\ \psi) = \ 1 - \exp(-\psi t). \tag{2}$$

The cdf and pdf of the *AGTransmuted-Exponential* are adapted from Merovci et al. [12] as:

$$F(t;\ \lambda,\ \alpha,\ \psi) = (1+\lambda)\left[1-\exp(-\alpha\psi t)\right] - \lambda\left[1-\exp(-\alpha\psi t)\right]^2, \quad \alpha,\ t>0,\ |\lambda| \le 1 \tag{3}$$

$$f(t;\ \lambda,\ \alpha,\ \psi) = \alpha\psi\exp(-\alpha\psi t)\left\{1+\lambda-2\lambda\left[1-\exp(-\alpha\psi t)\right]\right\}. \tag{4}$$

The additional two shape parameters are $\alpha > 0$, and $|\lambda| \le 1$.

The Another Generalized Transmuted Class family of distributions can easily be simulated using the following equation by Merovci et al. [12];

$$X_u = G^{-1}\left\{1-\left[\frac{\lambda-1+\sqrt{(1+\lambda)^2-4\lambda U}}{2\lambda}\right]^{\frac{1}{\alpha}}\right\}. \tag{5}$$

Where, U has a Uniform (0, 1) distribution.

3. Design of GASP based on *AGTransmuted-Exponential* properties

In the literature of acceptance sampling whenever, if the true median life denoted as m is equal to or if exceeds the specified median life m_o then such a product will be considered as fit and good for consumer's use. According to Aslam and Jun [4] in a GASP r items in a group are tested simultaneously on each different tester for a pre-assigned time t_0. The experiment is truncated if the number of failures in any group exceed the acceptable number c, for the duration of the experiment. Since the inspected items can every time be categorized as good and defective and usually the lot size is infinite, so as a consequence, in such a situation Binomial Distribution is used. It is known that the Binomial Distribution is used when the lot size is large enough ($n / N \leq 10\%$). To further explain the use of the binomial distribution, the reader can see Stephens [13]. Present section will exhibit the procedure for obtaining the design parameters of the GASP. In this study, the median life m of the units will be presumed as a quality parameter, and based on this presumption; we shall construct the GASP. The proposed GASP is characterized by two parameters g and c, commonly known as design parameters, for a given (group size = r). Here we consider an experimental situation where the lifetimes of the test units follow the *AGTransmuted-Exponential* distribution in which a manufacturer states that the specified median life of the units is m_o. It would be apposite to define the termination time t_0 as a multiple of the specified median life m_o (i.e.$t_0 = am_o$). For example, if $a = 0.5$, the experiment time is 50% of the specified median life, or if $a = 3$, it means that the experiment time is thrice the specified median life. A lot under consideration is accepted if $m \geq m_o$ (true median life is greater than or equal to specified median life). Following sequence of steps will be adopted for obtaining the design parameters and for execution of the GASP given in Aslam and Jun [4]:

"Selecting g number of groups and allocating predefining r items to each group so sample size for a lot $n = g \times r$.

Selecting c (acceptance number) for a group with experiment termination time t_0.

Simultaneously experimenting with g groups and recording the number of failures for each group.

Accepting the lot if at most c failures occur in all groups.

Truncate the experiment and reject the lot if more than c failures occur in any group."

We usually employ the quantile function for constructing the acceptance sampling plans when the distribution under study is not symmetrical Shahbaz et al. [14]. Shahbaz et al. [14] elaborately delineated, how to develop ASPs for lot sizes (both finite and infinite). For a given r, two design parameters (g, c) characterize the proposed GASP. We observe from equation (3) which is the cdf of *AGTransmuted-Exponential* and it depends on α, ψ, λ *and* t, and the quantile function of $AGT-E$ can be developed using equation (5).

Performance of any acceptance-sampling plan is measured through the Operating Characteristic Curve (OC) curve. OC curve involves plotting the probability of acceptance P_a versus the probability of defective item p also known as probability of failure. In the present study we will not draw OC curve but dwell on how to get P_a and p. Probability of failure p is worked out using the cdf of the probability distribution under study and the procedure is shown in equation (7) through equation (14). Probability of acceptance P_a is calculated using Operating Characteristic (OC) function which uses probability of failure as an essential ingredient. Hence for a GASP we first derive the equation for the probability of failure p and then work out P_a.

The probability of accepting a lot P_a under the GASP, is also commonly known as Operating Characteristic (OC) function. In any GASP the OC function must satisfy both the producers' risk α and consumer's risk β. For the proposed GASP we have to determining the number of groups g subject to the condition that the consumer's risk does not exceed (0.25, 0.10, 0.05, 0.01). The equation of OC function can be derived as:

$$P_{a(p)} = \left[\sum_{i=0}^{c} \binom{r}{i} p^i (1-p)^{r-i} \right]^g. \tag{6}$$

Where p is the probability that, before the termination time t_0, an item in a group fails. This probability is derived using equation (3) and equation (5):

The p^{th} quantile function of *AGTransmuted-Exponential* is given by equation (5):

$$t_q = G^{-1}\left\{1-\left[\frac{\lambda-1+\sqrt{(1+\lambda)^2-4\lambda U}}{2\lambda}\right]^{\frac{1}{\alpha}}\right\}. \tag{7}$$

U has a Uniform (0, 1) distribution but using U = 0.5, we get the median equation from equation (7) as:

$$t_{0.5} = m = \; G^{-1}\left\{1-\left[\frac{\lambda-1+\sqrt{1+\lambda^2}}{2\lambda}\right]^{\frac{1}{\alpha}}\right\} = \mathrm{G}^{-1}(p). \tag{8}$$

The p^{th} quantile function of the parent exponential distribution is given by:

$$t_p = -\frac{1}{\psi}\log(1-p). \tag{9}$$

The p^{th} quantile function of *AGTransmuted-Exponential* is given by equation (8) and inserting the value of p in equation (9), we get:

$$t_p = m = -\frac{1}{\psi}\log\left\{1-\left\{1-\left[\frac{\lambda-1+\sqrt{1+\lambda^2}}{2\lambda}\right]^{\frac{1}{\alpha}}\right\}\right\}, \tag{10}$$

$$t_p = m = -\frac{1}{\psi\alpha}\log\left[\frac{\lambda-1+\sqrt{1+\lambda^2}}{2\lambda}\right], \tag{11}$$

Let

$$\eta = \log\left[\frac{\lambda-1+\sqrt{1+\lambda^2}}{2\lambda}\right].$$

The GASP can be used to test the hypothesis about the median life of the product. Accepting the lot of the product is equivalent to testing the hypothesis that the true median life m of the product is equal to or greater than the specified median life m_0 of the product. Based on this information the following hypotheses can be formulated:

H_o: Product is good ($m \geq m_0$); H_1: Product is bad ($m \neq m_0$).

According to GASP the null hypothesis is that the lot of the product is accepted when the total number of failures are less than acceptance number c otherwise alternative hypothesis will be supported.

Then $m = -\frac{\eta}{\psi\alpha}$ and $\psi = -\frac{\eta}{m\alpha}$. Now substituting $\psi = -\frac{\eta}{m\alpha}$ and $t = \; a * m_o$ in equation (3), we get the probability of failure as:

$$p = (1+\lambda)\ [1-\exp(-\alpha\psi t)] - \lambda\ [1-\exp(-\alpha\psi t)]^2, \tag{12}$$

$$p = (1+\lambda)\left[1-\exp\left(-\alpha\left(-\frac{\eta}{m\alpha}\right)am_0\right)\right] - \lambda\left[1-\exp\left(-\alpha\left(-\frac{\eta}{m\alpha}\right)am_0\right)\right]^2, \tag{13}$$

$$p = (1+\lambda)\left[1-\exp\left(a\eta\left(\frac{m}{m_0}\right)^{-1}\right)\right] - \lambda\left[1-\exp\left(a\eta\left(\frac{m}{m_0}\right)^{-1}\right)\right]^2. \tag{14}$$

For known values of λ, p can be calculated for prespecified values of a and $r_2 = \frac{m}{m_0}$. The ratio of median lifetime to the specified median lifetime $\frac{m}{m_0}$ can express the product's quality level.

$$p_1 = (1+\lambda)[1-\exp(a\eta)] - \lambda[1-\exp(a\eta)]^2\ , \tag{15}$$

$$p_2 = (1+\lambda)\left[1-\exp\left(a\eta\left(\frac{m}{m_0}\right)^{-1}\right)\right] - \lambda\left[1-\exp\left(a\eta\left(\frac{m}{m_0}\right)^{-1}\right)\right]^2. \tag{16}$$

Both equation (15) and equation (16) are extracted from equation (14) as both equations present a different scenario, equation (15) has median lifetime equal to the specified median lifetime, whereas in equation (16) they differ as explained after equation (18). Using the optimization paradigm as stated below the design parameters (g, c) of the proposed GASP can be obtained

Optimize ASN = $n = g \times r$ with following conditions:

$$P_{a(p_1|\frac{m}{m_0}=r_1)} = \left[\sum_{i=0}^{c}\binom{r}{i} p_1^i(1-p_1)^{r-i}\right]^g \leq \beta, \tag{17}$$

$$P_{a(p_2|\frac{m}{m_0}=r_2)} = \left[\sum_{i=0}^{c}\binom{r}{i} p_2^i(1-p_2)^{r-i}\right]^g \geq 1-\alpha.$$

Where α is the producer's risk and β is the consumer's risk and r_1 is the median ratio at the consumer's risk, and r_2 is the median ratio at the producer's risk. The probability of failure to be used in equation (17) and equation (18) are given in equation (15) and equation (16), respectively. The probability of failure (p_1) shown in equation (17) will have the ratio $r_1 = \frac{m}{m_o} = 1$, but in equation (18), $r_2 = \frac{m}{m_o} = (2,\ 4,\ 6,\ 8,\ 10)$.

4. Discussion

Table 1 and 2 exhibit the design parameters of GASP $(g,\ c)$ under *AGTransmuted-Exponential* distribution for two specified values of the transmuted shape parameter λ (0.70 & 1). The GASPs have been worked out for two different choices of r = 5 & 10 with varied values for the median ratio $r_2 = 2(2)10$ at the producer's risk α together with several values of consumer's risk (β = 0.25, 0.10, 0.05, 0.01). Relevant *R* codes for the study are appended in Appendix 'A' to be readily used by academia and practitioners, which they can modify to address their specific needs. From tabulated values, presented in Table 1 and Table 2, it can be easily seen that as consumer's risk β decreases it results in the increase of number of groups. Additionally, as the quality level at each producer's risk r_2 increases, it results in exponential decrease in the number of groups. Nevertheless, after a certain level, even when the number of groups and acceptance number is kept constant. the probability of accepting a lot of 'P_a' increases. The effect of the time termination multiplier a can also be studied from Table 1 and Table 2 for two values (0.5 and 1.0). For instance, consider the case when $r = 10$ and at quality level $r_2 = 4$, as a is increased from 0.5 to 1.0, the number of groups decrease, whereas, for $r_2 = 8$ and 10 as a increased, the number of groups decrease but acceptance number c tend to increase. It is also observed from the results that if an experimenter desires to minimize the total number of units, then a different group size may be suggested. For instance using values from Table 1, with $a = 0.5$, $\beta = 0.25$, $r_2 = 4$, and for $r = 5$, a total of 9 groups or equivalently a total of $(n = g * r) = 9 * 5 = 45$ number of units are required to be placed on the life test. However, keeping the other conditions constant we see than when $r = 10$, we need 4 groups, or a total of $4 * 10 = 40$ items which are required to be used for testing. Therefore, a group size 4 with number of items $r = 10$ would be preferred.

Table 1. Design parameters of GASP (g, c) under AGTransmutedExponential distribution where median is taken as the quality parameter for specified value of the transmuted shape parameter $\lambda = 0.70$. As the ratio r_2 increases and decreases both design parameters generally, decrease

$\lambda = 0.70$													
		$r = 5$						$r = 10$					
	r_2	$a = 0.5$			$a = 1$			$a = 0.5$			$a = 1$		
		g	c	Pa	g	c	Pa	g	c	Pa	g	c	Pa
	2	-	-	-	-	-	-	-	-	-	-	-	-
	4	9	2	0.9526	7	3	0.97914	3	0.97152	4	0.9726		
0.25	6	9	2	0.9844	3	2	0.96552	2	0.96671	3	0.9814		
	8	9	2	0.9931	3	2	0.98392	2	0.98441	2	0.9532		
	10	3	1	0.9661	1	1	0.95911	1 0.95431	2	0.9731			
	2	-	-	-	-	-	-	-	-	-	-	-	-
	4	78	3	0.9812	12	3	0.96456	3	0.95753	4	0.9592		
0.10	6	14	2	0.9759	4	2	0.95423	2	0.95042	3	0.9631		
	8	14	2	0.9893	4	2	0.97863	2	0.97671	2	0.9532		
	10	4	1	0.9551	4	2	0.98843	2	0.98721	2	0.9731		
	2	-	-	-	-	-	-	-	-	-	-	-	-
	4	101	3	0.9757	15	3	0.955820	4	0.98467	5	0.9853		
0.05	6	18	2	0.9691	15	3	0.98958	3	0.98622	3	0.9631		
	8	18	2	0.9862	5	2	0.97344	2	0.96902	3	0.9856		
	10	18	2	0.9927	5	2 0.98564	2	0.98302	3	0.9934			
	2	-	-	-	-	-	-	-	-	-	-	-	-
	4	155	3	0.9630	146	4	0.983730	4	0.977010	5	0.9791		
0.01	6	27	2	0.9540	23	3	0.983912	3	0.97945	4	0.9867		
	8	27	2	0.9794	7	2	0.96295	2	0.96143	3	0.9785		
	10	27	2	0.9891	7	2	0.97985	2	0.97883	3	0.9901		

Remark: The cells with hyphens (-) indicate that an enormous sample size is needed

Further, Table 2 reports design parameters for the transmuted shape parameter value $\lambda = 1.00$. Our reported values reveal that an increase in the value of transmuted shape parameter for the associated plan results in smaller group size. For instance, with $a = 0.5$, $\beta = 0.25$, $r_2 = 4$, and for $r = 5$, a total of 9 groups or equivalently a total of $9 * 5 = 45$ units are required to be placed on the life test. However, when $r = 10$ using the same conditions as discussed earlier, only 4 groups are required, or equivalently a total of $4 * 10 = 40$ test units are required to be placed on the test. Therefore, in this case, a group of size 4 with a number of items in each group $r = 10$ would be preferred. For the associated plan, it is seen that with an increase in the value of the shape parameter, group sizes somewhat remain constant for higher values of consumer's risk and increase a little for smaller values of consumer's risk. For the considered GASP, under the *AGTransmuted-Exponential* and using median lifetime as the quality parameter, the number of groups decreases, and the OC values (P(a)) usually show an increasing trend when the true median life increases.

Table 2. Design parameters of GASP (g, c) under $AGTransmutedExponential$ distribution where median is taken as the quality parameter for specified value of the transmuted shape parameter $\lambda = 0.70$. As the ratio r_2 increases and decreases both design parameters generally, decrease

		$\lambda = 1.0$											
		$r = 5$						$r = 10$					
	r_2	$a = 0.5$			$a = 1$			$a = 0.5$			$a = 1$		
		g	c	Pa	g	c	Pa	g	c	Pa	g	c	Pa
0.25	2	-	-	-	-	-	-	-	-	-	-	-	-
	4	9	2	0.9529	7	3	0.9793	4	3	0.9717	2	4	0.9728
	6	9	2	0.9845	3	2	0.9656	2	2	0.9668	1	3	0.9815
	8	9	2	0.9931	3	2	0.9840	2	2	0.9845	1	2	0.9535
	10	3	1	0.9663	1	1	0.9593	1	1	0.9545	1	1	0.9733
0.10	2	-	-	-	-	-	-	-	-	-	-	-	-
	4	81	3	0.9822	12	3	0.9670	6	3	0.9609	3	4	0.9624
	6	14	2	0.9776	4	2	0.9569	3	2	0.9536	2	3	0.9657
	8	14	2	0.9900	4	2	0.9800	3	2	0.9782	1	2	0.9650
	10	4	1	0.9573	4	2	0.9892	3	2	0.9881	1	2	0.9747
0.05	2	-	-	-	-	-	-	-	-	-	-	-	-
	4	105	3	0.9770	15	3	0.9589	21	4	0.9855	4	4	0.9502
	6	18	2	0.9712	15	3	0.9903	8	3	0.9874	2	3	0.9657
	8	18	2	0.9872	5	2	0.9751	4	2	0.9711	2	3	0.9868
	10	18	2	0.9932	5	2	0.9865	4	2	0.9842	2	2	0.9501
0.01	2	-	-	-	-	-	-	-	-	-	-	-	-
	4	162	3	0.9647	146	4	0.9852	31	4	0.9787	10	5	0.9812
	6	28	2	0.9556	23	3	0.9852	12	3	0.9812	5	4	0.9879
	8	28	2	0.9802	7	2	0.9653	6	2	0.9570	3	3	0.9803
	10	28	2	0.9895	7	2	0.9812	6	2	0.9764	2	2	0.9501

Application: Complete data with the exact times of failure used in Nadarajah and Kotz [15] for the life of fatigue fracture of Kevlar 373/epoxy that are subject to constant pressure at the 90% stress level until all had failed is considered to demonstrate the implementation of the proposed GASP:

0.0251, 0.0886, 0.0891, 0.2501, 0.3113, 0.3451, 0.4763, 0.5650, 0.5671, 0.6566, 0.6748, 0.6751, 0.6753, 0.7696, 0.8375, 0.8391, 0.8425, 0.8645, 0.8851, 0.9113, 0.9120, 0.9836, 1.0483, 1.0596, 1.0773, 1.1733, 1.2570, 1.2766, 1.2985, 1.3211, 1.3503, 1.3551, 1.4595, 1.4880, 1.5728, 1.5733, 1.7083, 1.7263, 1.7460, 1.7630, 1.7746, 1.8275, 1.8375, 1.8503, 1.8808, 1.8878, 1.8881, 1.9316, 1.9558, 2.0048, 2.0408, 2.0903, 2.1093, 2.1330, 2.2100, 2.2460, 2.2878, 2.3203, 2.3470, 2.3513, 2.4951, 2.5260, 2.9911, 3.0256, 3.2678, 3.4045, 3.4846, 3.7433, 3.7455, 3.9143, 4.8073, 5.4005, 5.4435, 5.5295, 6.5541, 9.0960

The first step will be to verify that whether the *AGTransmuted-Exponential* distribution fits the data well or not. The maximum likelihood method is used to estimate the unknown parameters of the distribution.

To fit a sample data to the given pdf and estimating the MLE parameters, we followed these steps:

Defining the Likelihood Function.

$$L(\lambda,\ \alpha,\ \psi) = \prod_{i=1}^{n} f(t_i;\ \lambda,\ \alpha,\ \psi)$$

Defining the Log-Likelihood Function.

$$l(\lambda,\ \alpha,\ \psi) = \sum_{i=1}^{n} \log f(t_i;\ \lambda,\ \alpha,\ \psi)$$

$$l(\lambda,\ \alpha,\ \psi) = \sum_{i=1}^{n} \log[\alpha\psi \exp(-\alpha\psi t)\{1+\lambda-2\lambda\,[1-\exp(-\alpha\psi t)]\}], \tag{18}$$

Use optimization to find the MLE estimates. By providing initial guesses for the parameters $\lambda,\ \alpha,\ \psi$ optimizing the negative log-likelihood function using *optim* function in R-software.

MLE estimates for *AGTransmuted-Exponential* distribution were $\widehat{\lambda} = 0.730,\ \widehat{\alpha} = 1.193,\ \widehat{\psi} = 0.759$. To compare the two distribution models, we consider criteria like Kolmogorov-Smirnov (K-S) statistics, -2l, AIC (Akaike information criterion), and AICC (corrected Akaike information criterion) and BIC (Bayesian Information Criterion) for the data set. The better distribution corresponds to smaller KS, -2l, AIC, AIcc, and BIC values. Values in Table 3 indicate that the *AGTransmuted-Exponential* leads to a better fit than the exponential distribution. The Kolmogorov-Smirnov (K-S) test revealed that the maximum difference between the actual data and the fitted AGTransmuted-Exponential model in Table 4 is 0.095 with a p-value of 0.471. Therefore, the given data set can be analyzed using AGTransmuted-Exponential distribution in Table 5. The fitted distribution function and density function of the considered distribution *AGTransmuted-Exponential* and the P-P and hazard rate plots are plotted in Figures 1 and 2, respectively, for the data set.

Table 3. Model comparison and relevant information criteria are exhibited and it is witnessed that *AGTransmuted-Exponential* Distribution fits the data better than the Exponential Distribution

Models	K-S test	-2l	AIC	AIcc	BIC
AGTransmuted-Exponential Distribution	0.0900	241.522	243.999	239.146	252.636
Exp Distribution	0.4135	250.331	255.118	257.001	257.123

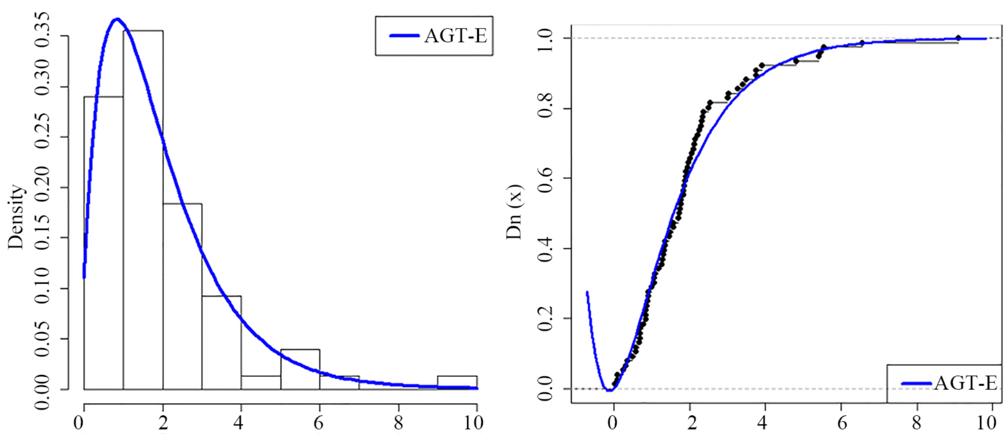

Figure 1. The fitted distribution function and density function of the considered distribution AGTransmuted-Exponential to the observed data

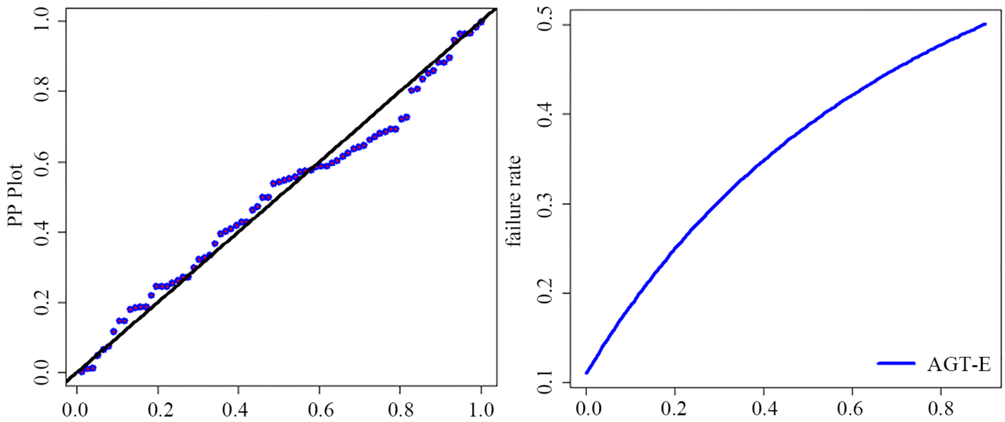

Figure 2. The P-P plots shows how good the AGTransmutedExponential distribution fits the observed data as the blue data dots are very close to the black diagonal line. The plot on the right panel shows the hazard rate

Table 4. Design parameters of GASP $(g,\ c)$ under *AGTransmuted-Exponential* distribution where mean is taken as the quality parameter for MLE value of the transmuted shape parameter $\widehat{\lambda} = 0.730$. As the ratio r_2 increases and $\beta = 0.25$ decreases both design parameters decrease

		$\widehat{\lambda} = 0.730$											
		$r = 5$						$r = 10$					
	r_2	$a = 0.5$			$a = 1$			$a = 0.5$			$a = 1$		
		g	c	Pa	g	c	Pa	g	c	Pa	g	c	Pa
	2	-	-	-	-	-	-	-	-	-	-	-	-
	4	14	3	0.9834	3	3	0.9615	3	4	0.9854			
0.25	6	4	2	0.9770	1	2	0.9647	1	2	0.9501	3	4	0.9547
	8	4	2	0.9896	1	2	0.9830	1	2	0.9755	3	4	0.9854
	10	4	2	0.9944	1	2	0.9905	1	2	0.9863	2	3	0.9713

Table 5. Design parameters of GASP $(g,\ c)$ under *AGTransmuted-Exponential* distribution where Median is taken as the quality parameter for MLE value of the transmuted shape parameter $\widehat{\lambda} = 0.730$. As the ratio r_2 increases and $\beta = 0.25$ decreases both design parameters decrease

		$\widehat{\lambda} = 0.730$											
		$r = 5$						$r = 10$					
	r_2	$a = 0.5$			$a = 1$			$a = 0.5$			$a = 1$		
		g	c	Pa	g	c	Pa	g	c	Pa	g	c	Pa
	2	-	-	-	-	-	-	-	-	-	-	-	-
	4	9	2	0.9529	7	3	0.9793	4	3	0.9717	2	4	0.9728
0.25	6	9	2	0.9845	3	2	0.9656	2	2	0.9668	1	3	0.9815
	8	9	2	0.9931	3	2	0.9840	2	2	0.9845	1	2	0.9535
	10	3	1	0.9663	1	1	0.9593	1	1	0.9545	1	1	0.9733

The design parameters for the proposed GASP using median lifetime as the quality parameter are determined using fitted parametric values for consumers' risk at $\beta = 0.25$ for $r_2 = 2,4,6,8,10$ and are shown in Table 5. But the same design parameters using mean are exhibited in Table 4. It is quite evident by comparing the values of Table 4 and Table 5 that using the Median as a quality parameter resulted in the decrease of group size and acceptance number simultaneously at all quality levels, which was expected since, in an exponential distribution median should be less than mean. The pattern of the MLE plan parameters given in Table 5 follows the same pattern as depicted in Table 1 and Table 2; hence instead of using specific values of the transmuted shape parameter of the *AGTransmuted-Exponential* distribution, MLE estimates can also be used. To further elaborate the implementation of the proposed plan, suppose experiment is run for 2000 cycles by taking r = 5 units in each group and is concerned with identifying whether the median life of the failures exceeds the specified life. It is known in advance that the consumer's risk is 25% when the true median life of the failures is m_o = 4000 cycles, and the producer's risk is 5% when the true median life of the failures is 16000 cycles. Thus, we have λ= 0.730, m = 2000 cycles, a = 0.5 (2000/4000), r = 5, β = 0.25, r_1 = 1, α = 0.05 and r_2 =16000/2000= 4. Hence, keeping in view the given information, the design parameters from Table 5 can be extracted, with group size (g = 9) and acceptance number (c = 2). These design parameter values suggest that 5 units should be allocated to each of the 9 groups so that the total random sample should consist of 9*5 = 45 units, and if less than 2 units fail in each such group prior to completion of 2000 cycles, then it is statistically ensured that median life of failures is larger than the specified life. Therefore, the lot under sentencing will be accepted as good.

5. Conclusion

Table 6. Five Information Criteria

Information Criteria	Mathematical Representation
Kolmogorov-Smirnov (K-S) Statistic	The K-S statistic is: $D = \max_{1\leq i\leq n} \mid (X_{(i)} - \frac{i-1}{n} \mid$ and $D^{+} = \max_{1\leq i\leq n} \mid (X_{(i)} - \frac{i}{n} \mid$ Where $D measures the maximum deviation between$ the empirical cumulative distribution function and the theoretical distribution from below, while D^{+} $measures the deviation from above$. The overall K-S statistic is the maximum of these two: $D_{ks} = max(D,\ D^{+})$
Log-Likelihood (-2L)	$-2l = \ -2\sum_{i=1}^{n} \log L_i$ Where L_i is the likelihood of the observed data given the model parameter for observation i
Akaike Information Criterion (AIC)	$AIC = \ -2l + 2k$ l is the log-likelihood of the model k is the number of estimated parameters of the model
Corrected Akaike Information Criterion (AICC)	$AICc = AIC + \frac{2k(k+1)}{n-k-1}$ n is the number of observations k is the number of estimated parameters of the model
Bayesian Information Criterion (BIC)	$BIC = -2l + k\ log(n)$ l is the log-likelihood of the model k is the number of estimated parameters of the model n is the number of observations

In this paper, GASP is proposed for *AGTransmuted-Exponential* distribution based on truncated life tests under the assumption that the median lifetime is a quality parameter. Design parameters for various sampling plans are obtained under specific consumers' and producers' risks simultaneously using different quality levels. Proposed plans are explained with the help of examples and tables. A real data set to illustrate the implementation of the proposed plan is analyzed. We also observe that in the present study, group size increases as the consumer's risk decreases. But when median is used a quality parameter instead of mean it is observed that the group sizes decrease to some extent at quality level '4'. Also with the increase in the sample units (r) from 5 to 10 within a group we see that as the consumer's risk increases the group size and acceptance numbers exponentially decrease. As a future direction, one can develop GASPs when the product lifetimes follow Bayesian and E-Bayesian *AGTransmuted-Exponential* distributions. Table 6 displays the formulae for calculating the five criteria.

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Conflict of interest

The authors declare no competing financial interest.

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Appendix A

Relevant R Codes

```
g = seq (1, 5,000, 1); c = c (0, 1, 2, 3, 4, 5); lp2 = double (length(g));
lp1 = double (length(g)); lp21 = double (length(c)); lp22 = double (length(c));
lp23 = double (length(c)); lp24 = double (length(c)); G1 = double (length(c));
G2 = double (length(c)); G3 = double (length(c)); G4 = double (length(c));
p = function (ratio, a, lam) {
nu =log ((lam-1+sqrt(1+lam^2))/(2*lam))
y = (((1+lam)*(1-exp(a*nu*(1/ratio))))-(lam*((1-exp(a*nu*(1/ratio)))^2)))
}
p2 = round (p (c (2, 4, 6, 8, 10), 0.5, 0.7), 4); p2;
p1 = round (p (1, 0.5, 0.7), 4); p1
for (i in 1: length (c)){
for (j in 1: length (g)){
lp2 [j] = (pbinom (c [i], 5, p2 [2]))^j ### changing the values of r and p2###
lp1 [j] = (pbinom (c [i], 5, p1))^j
}
G1 [i] = min (which (lp2 > = 0.95 & lp1 < 0.25)); lp21 [i] = round (lp2 [G1 [i]], 4);
G2 [i] = min (which (lp2 > = 0.95 & lp1 < 0.10)); lp22 [i] = round (lp2 [G2 [i]], 4);
G3 [i] = min (which (lp2 > = 0.95 & lp1 < 0.05)); lp23 [i] = round (lp2 [G3 [i]], 4);
G4 [i] = min (which (lp2 > = 0.95 & lp1 < 0.01)); lp24 [i] = round (lp2 [G4 [i]], 4);
}
cbind(G1, lp21, G2, lp22, G3, lp23, G4, lp24, c) ### read the smallest numbers
```