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Research Article

Analysis of a Markovian Queue of Single Server Performing in Multi-Phase Subject to Disaster, Recovery and Repair

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Abstract: In this paper, we can study about Markovian single-server queue with servers in distinct phases of disaster and repair. The server can stay in full-active Phase or passive Phase randomly and alternately. The server is set to serve in two Phases, such as the full active Phase and passive Phase. When the server operates in full active Phase, the customers arrive and served in First Come: First Serve (FCFS) queue discipline. However, when the server switches from full active to passive Phase, it can only offer to provide service at a rate lower than that of the server in full active Phase. During passive Phase, customers are not allowed to join the system. The server moves to repair Phase immediately after the last customer is served, irrespective of the Phase. The customers will be restricted from joining the queue in repair Phase. After the repair Phase, the server will enter the full active Phase. The server remains in the system for a random period of time. In the event of a disaster, in full active and passive Phases, all customers in system aredropped out, and system moves to repair Phase. The expressions for the steady state probabilities and some key performance metrics are obtained. A numerical illustration is made to study the effects of server shifting from full active phase to passive phase that affects the customers in the system and the occurrence of disaster which has an adverse effect on the customers in that phase.

Keywords: $M/M/1$ queueing system, server in different phases, disaster and repair

MSC: 60K25, 90B22

1. Introduction

Queueing theory is a branch of applied mathematics that provides methods to predict the performance of systems (Queueing Systems) which keep servers (one or many) for providing service to randomly arriving customers. These methods are aimed at attempting ways to avoid delays in service/customer loss. In the early stages of its development, it emerged as a result of tireless attempts by several investigators to provide mathematical models (later called queueing Models) to find solutions to problems confronted with the operation of telephone systems where calls arrive at a switchboard to get connected for conversation. A. K. Erlang, a Danish mathematician was the pioneer investigator who formulated and solved the first queueing problem of telephone traffic congestion.

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Kendall [1] introduced the notation $P/Q/R/S/V$ to represent a queueing model, considering the basic characteristics (P: Arrival patterns of customers, Q: Service patterns, R: Number of service channels, S: System capacity and V: Queue discipline) of a queueing system. For example, if a queueing model is $M/M/1/N/$FCFS, then it indicates that the customer's arrival is governed by a Poisson process, the service times of customers are modelled to follow an exponential distribution, there are one server to do customer service, the system capacity is N and the discipline of the queue is First-Come: First-Served. Instead of FCFS, some researchers use FIFO to mean equivalently First-In: First-Out.

Applications of queuing theory are numerous but are not limited to, forecasting network congestion and blocking in computer networks, telecommunication, data distribution, high-speed network, production engineering. However, these systems are not reliable and disaster may occur within them due to certain unavoidable circumstances. When there is catastrophe, the system stops operating, and the customers who are waiting in the buffer with the server's activity are lost. Many authors have made a detailed analysis on queueing system. In order to explore the steady-state behaviour, Jain and Sigman [2] analysed non-Markovian queueing systems. It studies on preemptive LIFO queue discipline and disaster occurs due to negative arrivals by adopting the idea of a disaster, they were able to get Pollaczek-Khintchine results for Poisson input and general service queue.

Several authors [3–14] have researched queueing systems that are vulnerable to accidents that happen randomly. Paz and Yechiali [15] investigated the steady-state behaviour of an $M/M/1$ queue operating within a random environment that is subjecting to random disasters. This environment is characterized by an n-phase continuous-time Markov chain. Building upon the queueing model introduced by Paz and Yechiali [15], Udayabaskaran and Dora Pravina [5] have furthered the research by obtaining time-dependent probabilities, providing a dynamic behaviour of the system. Ammar et al. [3] considered queue system with single server and also studied impatient behavior of the customer in multi-phase random environment.

In the literature of queueing theory, certain research works on steady-state analysis of disaster situations in different contexts. The size distribution of queue with geometric arrival and general service process queue with disaster was determined using the queue-theoretic approach by Lee et al. [4]. Ammar and Rajadurai [10] utilised a technique of supplementary variable to find the function generating probability for the queue length of a non-Markovian queue with working breakdown, retrial policy, and disaster. Jain and Singh [16] investigated Markovian feedback queue having disaster and customer impatience in it.

Jain et al. [6] studied $M/M/1$ queuing model with disaster failure with multiple vacation policy. Performance measures are also computed. Perel and Yechiali [14] analysed single and multiple server queues in a two-phase Markovian random environment having impatient customers. Ammar [17] analysed a single server queue with vacations in a multi-phase random environment.

Ramesh and Udayabaskaran [18] analysed a single server queueing system in a doubly stochastic environment makes transitions among N levels controlled by a Random Switch. Demircioglu et al. [7] examined how the occurrence of disaster affects the queuing behaviour in a discrete time single server queuing system with generally independent arrival and service times. The chance of customer loss as result of disaster is derived. Sudhesh et al. [8] considered discrete time Geo/Geo/1 queue with system disaster. Steady-state probabilities of number of customer present in the system are calculated.

Suranga Sampath et al. [19] studied repairable single-server queue that is resilient to system failures and working vacations. The performance metrics such as mean and variance of total number of jobs are found. Gupta et al. [20] analyzed an infinite buffer single server queuing model with exponentially distributed service times and negative arrivals. The model also studied how the negative arrivals affect the system performance. Seenivasan et al. [21] considered $M/M/1/N$ model with disaster, restoration and breakdown. Also taken into account of partial breakdown of servers.

In all the multi-phase models, customer can arrive in all the phases, but in the present work, customers arrive only when the servers are in the full active phase. Upon completion of repair phase, the server is restricted to move to full active phase. After some time, due to power consumption, the server changes to passive phase, and no customers are allowed to join during the passive or repair phases. On the other hand, the disasters in server switching to multiple phases and customers being allowed only in one phase are not discussed; this gap motivates the development of this model. It examines how a system behaves when disaster occurs within the multi-phase, and how it recovers and continues to operate.

When a server changes its phase from time to time, it may enter into a passive or repair phase, where it loses contact with the customers for a random time during which the server shuts down all its activity by switching to power saving mode. Paz and Yechiali [15], Udayabaskaran and Dora Pravina [5], Ammar et al. [3], Ammar [17], Ramesh and Udayabaskaran [18] have taken into account random environmental influences on single server queueing systems and analysed the behaviour of the system. There is another piquant situation arising in the operation of a single server at different phases (Active or Passive or Repair), where disasters can occur randomly in the server. This aspect has not been considered so far in literature. In this paper, we remove this pitfall by proposing a queueing model which studies the single server performing in multiple phases subject to disaster, recovery and repair.

The novelty of this work presents a conception of single server queueing models in the presence of self switching server. The result of these queueing models provides a scope for application to mobile communication networks, which has different applications in detecting and transmitting data to the localized node with energy effectiveness.

This paper is planned to be organizedin the following way: 2$^{\text{nd}}$ Section has the model. 3$^{\text{rd}}$ Section contains governing equations for time-independent probabilities of the system. 4$^{\text{th}}$ Section obtains explicit expressions for the steady-state probabilities. 5$^{\text{th}}$ Section has the performance measures. The numerical illustrations are in 6$^{\text{th}}$ Section.

2. Model description

Considering a single server in two phases such as the full active phase and passive phase. Customers arrives based on a Poisson process at the rate of λ to full active phase. Service rates for full active and passive phases are μ_1 and μ_2 such that $\mu_2 < \mu_1$. In full active phase, the server offers service to the available customers on FCFS discipline, and if there is no new customer for service, the server moves to the repair phase with the rate of α. At the maintenance (repair) phase, customers are not allowed to join the system. A random amount of time is spent in maintenance phase, which is distributed exponentially at the mean of $1/\beta$. After the maintenance, the server moves to full active phase and gets ready to serve the arriving customers. When the server is in full active phase, if there is a posibility of being diseased, immediately the server switches to passive phase at the rate of γ and the customers already waiting for service are served with a lesser rate of μ_2. During passive phase, customers are not allowed to join the system, and after serving last customer in the system, it enters the repair phase. The system experiences disasters in both the phases at the rate of $1/\eta_1$ and $1/\eta_2$ respectively. In the event of a disaster in both phases (full active and passive), all customers in system are dropped out and server moves to repair phase.

2.1 *Notations*

1. $\mathscr{P}r(A)$: event A's Probability.
2. $p(i,\ n,\ t)$: Transient Probability of server in state 'i' and 'n' customers in the system.
3. $\pi_{i,\ n} = \lim_{t\to\infty} p(i,\ n,\ t)$: Steady state probability in state 'i' and 'n' customers in the system.
4. $\int_0^t f(u)g(t-u)du : f(t)©g(t)$ Convolution of $f(t)$ and $g(t)$ in Laplace Transform.
5. $\int_0^\infty e^{-st} f(t)dt = f^*\ (s)$: Laplace transform of $f(t)$.
6. $\lim_{t\to\infty} f(t) = \lim_{s\to 0} f^*\ (s)$: Final value theorem in Laplace transform.

3. Governing equations

$U(t)$ describes the phase of server, and $X(t)$ denotes the total customers in the queueing system at time t. Then the joint process $(U(t),\ X(t)|t \geq 0)$ defines a continuous time Markov process. We defined conditional probability function as,

$$p(i,\ n,\ t) = \mathscr{P}r\left[U(t) = i,\ X(t) = n|U(0) = 0,\ X(0) = 0\right],\ \ i = 0,\ 1,\ 2;\ \ n = 0,\ 1,\ 2,\ \ldots$$

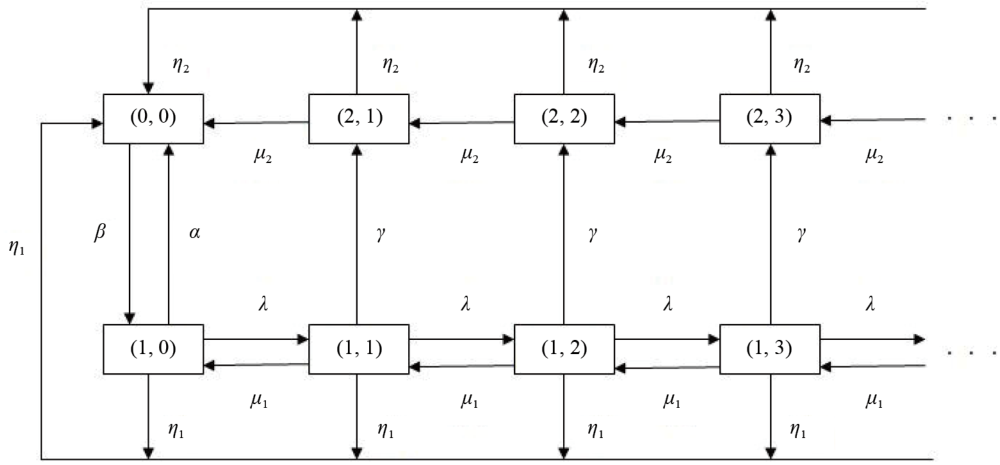

Figure 1. Transition rates diagram

Following notation are used for the convolution of two functions

$$f(t)©g(t) = \int_0^t f(u)g(t-u)du$$

By using probabilistic laws to obtain the following governing equations (in Figure 1).

$$p(0,\ 0,\ t) = e^{-\beta t} + \begin{bmatrix} p(1,\ 0,\ t)\alpha + \mu_2 p(2,\ 1,\ t) + \eta_1 \sum_{n=0}^{\infty} p(1,\ n,\ t) \\ +\eta_2 \sum_{n=1}^{\infty} p(2,\ n,\ t) \end{bmatrix} ©e^{-\beta t} \tag{1}$$

$$p(1,\ 0,\ t) = [\beta p(0,\ 0,\ t) + \mu_1 p(1,\ 1,\ t)]©e^{-(\alpha+\lambda+\eta_1)t} \tag{2}$$

$$p(1,\ n,\ t) = [\lambda p(1,\ n-1,\ t) + \mu_1 p(1,\ n+1,\ t)]©e^{-(\lambda+\mu_1+\eta_1+\gamma)t},\ n = 1,\ 2,\ \cdots \tag{3}$$

$$p(2,\ n,\ t) = [\gamma p(1,\ n,\ t) + \mu_2 p(2,\ n+1,\ t)]©e^{-(\mu_2+\eta_2)t} \tag{4}$$

Denoting the Laplace transform of $p(i,\ n,\ t)$ by

$$p^*(i,\ n,\ s) = \int_0^{\infty} p(i,\ n,\ t)e^{-st}dt \tag{5}$$

And Equations (1)-(4) yields

$$(s+\beta)p^*(0,\ 0,\ s) = 1 + \alpha p^*(1,\ 0,\ s) + \mu_2 p^*(2,\ 1,\ s) + \eta_1 \sum_{n=0}^{\infty} p^*(1,\ n,\ s) + \eta_2 \sum_{n=1}^{\infty} p^*(2,\ n,\ s) \tag{6}$$

$$(s+\alpha+\lambda+\eta_1)\,p^*(1,\,0,\,s)\;=\beta p^*(0,\,0,\,s)+\mu_1 p^*(1,\,1,\,s) \tag{7}$$

$$(s+\lambda+\mu_1+\eta_1+\gamma)\,p^*(1,\,n,\,s)=\lambda p^*(1,\,n-1,\,s)+\mu_1 p^*(1,\,n+1,\,s),\;\; n=1,\,2,\,\ldots \tag{8}$$

$$(s+\mu_2+\eta_2)\,p^*(2,\,n,\,s)=\gamma p^*(1,\,n,\,s)+\mu_2 p^*(2,\,n+1,\,s),\;\; n=1,\,2,\,\cdots \tag{9}$$

4. Steady state solutions

The steady state equations are written by using F.V.T of Laplace transform

$$\pi_{i,\,n}=\lim_{t\to\infty}p(i,\,n,\,t)=\lim_{s\to 0}p^*(i,\,n,\,s)i=1,\,2;\;\; n=0,\,1,\,2,\,\cdots$$

$$\pi_{0,\,0}=\lim_{t\to\infty}p(0,\,0,\,t)=\lim_{s\to 0}p^*(0,\,0,\,s)$$

From Equations (6)-(9) yields

$$\beta\pi_{0,\,0}=\alpha\pi_{1,\,0}+\mu_2\pi_{2,\,1}+\eta_1\sum_{n=0}^{\infty}\pi_{1,\,n}\;+\eta_2\sum_{n=1}^{\infty}\pi_{2,\,n} \tag{10}$$

$$(\lambda+\alpha+\eta_1)\,\pi_{1,\,0}=\beta\pi_{0,\,0}+\mu_1\pi_{1,\,1} \tag{11}$$

$$(\lambda+\mu_1+\eta_1+\gamma)\,\pi_{1,\,n}=\lambda\pi_{1,\,n-1}+\mu_1\pi_{1,\,n+1},\;\; n=1,\,2,\,\cdots \tag{12}$$

$$(\mu_2+\eta_2)\,\pi_{2,\,n}=\gamma\pi_{1,\,n}+\mu_2\pi_{2,\,n+1},\;\; n=1,\,2,\,\cdots \tag{13}$$

To solve from (10)-(13), we define Probability Generating Function as

$$G_1(u)=\sum_{n=0}^{\infty}\pi(1,\,n)u^n \tag{14}$$

$$G_2(u)=\sum_{n=1}^{\infty}\pi(2,\,n)u^n \tag{15}$$

In Equation (12), multiply both sides by u^n and summing up from 1 to ∞, we get

$$(\lambda+\mu_1+\eta_1+\gamma)\sum_{n=1}^{\infty}\pi_{1,\,n}u^n=\lambda\sum_{n=1}^{\infty}\pi_{1,\,n-1}u^n+\mu_1\sum_{n=1}^{\infty}\pi_{1,\,n+1}u^n$$

By using (11), (14) and (15), after simplifying we get

$$G_1(u) = \frac{\beta u \pi_{0,\,0} - [(\alpha - \mu_1 - \gamma)\, u + \mu_1]\, \pi_{1,\,0}}{(\lambda + \mu_1 + \eta_1 + \gamma)\, u - \lambda u^2 - \mu_1} \tag{16}$$

For $|u| < 1$, the function $G_1(u)$ converges and hence the right hand side's numerator of Equation (16) should be null at all zeros in $|u| < 1$ of the denominator of Equation (16). The zeros of the denominator are given by

$$r_1 = \frac{(\lambda + \mu_1 + \eta_1 + \gamma) - \sqrt{(\lambda + \mu_1 + \eta_1 + \gamma)^2 - 4\lambda\mu_1}}{2\lambda} \tag{17}$$

$$r_2 = \frac{(\lambda + \mu_1 + \eta_1 + \gamma) + \sqrt{(\lambda + \mu_1 + \eta_1 + \gamma)^2 - 4\lambda\mu_1}}{2\lambda} \tag{18}$$

Since the roots r_1 and r_2 are real and distinct. For stability, we assume $\mu_1 + \mu_2 > \lambda$:
Some of the properties of r_1 and r_2 are

$$0 < r_1 < 1 < r_2, \;\; r_1 + r_2 = \frac{\lambda + \mu_1 + \eta_1 + \gamma}{\lambda}, \;\; r_1 r_2 = \frac{\mu_1}{\lambda}$$

$$(\lambda + \mu_1 + \eta_1 + \gamma)u - \lambda u^2 - \mu_1 = \lambda(u - r_1)(r_2 - u)$$

There is exactly one zero, namely r_1 lying in $|u| < 1$. So the numerator of the right hand Equation (16) vanishes at r_1 ant it cannot vanish at r_2. Consequently, we get

$$\pi_{1,\,0} = \frac{\beta r_1 \pi_{0,\,0}}{(\alpha - \mu_1 - \gamma) r_1 + \mu_1} \tag{19}$$

Replacing u by 1 in Equation (16) can be rewritten as

$$G_1(u) = \frac{r_2 \pi_{1,\,0}}{r_2 - u} \tag{20}$$

Expanding $G_1(u)$ in the region of convergence $|u| < 1$, we get

$$\sum_{n=0}^{\infty} \pi_{1,\,n} u^n = \pi_{1,\,0} \sum_{n=0}^{\infty} \frac{u^n}{r_2^n} = \pi_{1,\,0} \sum_{n=0}^{\infty} \frac{1}{r_2^n} u^n \tag{21}$$

Equating coefficient of u^n on both sides of (21), we get

$$\pi_{1,\,n} = \frac{1}{r_2^n} \pi_{1,\,0} = r_2^{-n} \pi_{1,\,0}, \;\; n = 0,\; 1,\; 2,\; \cdots \tag{22}$$

$$\pi_{1,\ n} = \frac{r_2}{r_2 - 1}\pi_{1,\ 0}$$

Now, multiplying both sides of Equation (13) by u^n, summing up from 1 to ∞, we get

$$G_2(u) = \frac{\gamma u G_1(u) - \gamma u \pi_{1,\ 0} - \mu_2 u \pi_{2,\ 1}}{(\mu_2 + \eta_2)u - \mu_2} \tag{23}$$

The function $G_2(u)$ converges and hence the right hand side's numerator of Equation (23) should be null at the zero in $|u| < 1$ of the denominator of Equation (23). The zero of the denominator is given by

$$u_1 = \frac{\mu_2}{\mu_2 + \eta_2}$$

There is exactly one zero, namely u_1 lying in $|u| < 1$. So the numerator of the right hand Equation (23) vanishes at u_1, we get

$$\mu_2 \pi_{2,\ 1} = \gamma G_1(u_1) - \gamma \pi_{1,\ 0} \tag{24}$$

Equation (24) is substituted in Equation (23) and on simplifying,

$$G_2(u) = \frac{\gamma u\left[G_1(u) - G_1(u_1)\right]}{(\mu_2 + \eta_2)(u - u_1)} \tag{25}$$

$$G_2(u) = \frac{\gamma}{\mu_2 + \eta_2} \sum_{n=1}^{\infty} \sum_{j=n}^{\infty} \pi_{1,\ j} u_1^{j-n} u^n \tag{26}$$

Equating Coefficient of u^n on both sides, we get

$$\pi_{2,\ n} = \frac{\gamma}{\mu_2 + \eta_2} \sum_{j=n}^{\infty} \pi_{1,\ j} u_1^{j-n}, \ \ n = 1,\ 2,\ \cdots \tag{27}$$

By putting $u = 1$ in Equation (25), we get

$$G_2(1) = \frac{\gamma}{\eta_2}\left[G_1(1) - G_1(u_1)\right] \tag{28}$$

where $u_1 = \dfrac{\mu_2}{\mu_2 + \eta_2}$.

Substituting and simplifying we get (28) as

$$G_2(1) = \frac{\gamma}{\eta_2} \sum_{n=0}^{\infty} \pi_{1,\, n} \left(1 - u_1^n\right) \tag{29}$$

By total probability law, we have

$$\pi_{0,\, 0} + G_1(1) + G_2(1) = 1 \tag{30}$$

Equations (20) and (29) are substituted in Equation (30), on simplifying we get

$$\pi_{0,\, 0} = \left[1 + \frac{r_2}{r_2 - 1} \frac{\beta r_1}{(\alpha - \mu_1 - \gamma) r_1 + \mu_1} \left[1 + \frac{\gamma}{\eta_2} \sum_{n=0}^{\infty} (1 - u_1^n)\right]\right]^{-1} \tag{31}$$

explicitly we get all the steady state probabilities in Equations (22), (27), (31).

5. Performance measures

Few important measures of system performance such as average of customers in fully active phase and passive phase, average number of customers, number of times the server switches from full active phase to passive Phase, from repair phase to passive phase and effective arrival rate are derived.

5.1 *Average number of customers in fully active phase*

Let average number of customers in full active Phase is denoted by E_F.

$$E_F = \sum_{n=0}^{\infty} n \pi_{1,\, n} = \pi_{1,\, 0} \sum_{n=0}^{\infty} n r_2^{-n}$$

$$E_F = \pi_{1,\, 0} \frac{1}{r_2} \left(\frac{r_2}{r_2 - 1}\right)^2 \tag{32}$$

5.2 *Average number of customers in passive phase*

Let E_P denote average number of customers in passive phase.

$$E_P = \sum_{n=1}^{\infty} n \pi_{2,\, n} = \sum_{n=1}^{\infty} n \frac{\gamma}{\mu_2 + \eta_2} \sum_{j=n}^{\infty} \pi_{1,\, j} u_1^{j-n}$$

$$E_P = \frac{\gamma}{\mu_2 + \eta_2} \sum_{n=1}^{\infty} \sum_{j=n}^{\infty} n \pi_{1,\, j} u_1^{j-n} \tag{33}$$

5.3 *Average number of customers in the system*

Let E_S denote average customers in the system.

$$E_S = E_F + E_P = \pi_{1,\,0}\frac{1}{r_2}\left(\frac{r_2}{r_2-1}\right)^2 + \frac{\gamma}{\mu_2+\eta_2}\sum_{n=1}^{\infty}\sum_{j=n}^{\infty} n\pi_{1,\,j}u_1^{j-n} \tag{34}$$

5.4 *Average number of times the server switches from full active phase to passive phase*

Let E_{FP} denotes expected number of times the server switching from full active to passive phase.

$$E_{FP} = \gamma\sum_{n=1}^{\infty}\pi_{1,\,n} = \gamma\sum_{n=1}^{\infty} r_2^{-n}\pi_{1,\,0} \tag{35}$$

5.5 *Average number of times the server switches from repair phase to full active phase*

Let E_{RF} denotes expected number of times the server switching from Repair phase to full active phase per unit time.

$$E_{RF} = \beta\pi_{0,\,0} = \beta\left[1 + \frac{r_2}{r_2-1}\frac{\beta r_1}{(\alpha-\mu_1-\gamma)r_1+\mu_1}\left[1+\frac{\gamma}{\eta_2}\sum_{n=0}^{\infty}(1-u_1^n)\right]\right]^{-1} \tag{36}$$

5.6 *Effective arrival rate*

Let E_A be effective arrival rate, it is defined as total arrival when server is available. The server is available either in full active or Passive phase but here customers are allowed to join only in full active phase.

Therefore,

$$\pi_{0,\,0} + \sum_{n=0}^{\infty}\pi_{1,\,n} + \sum_{n=1}^{\infty}\pi_{2,\,n} = 1$$

$$\sum_{n=0}^{\infty}\pi_{1,\,n} = 1 - \pi_{0,\,0} - \sum_{n=1}^{\infty}\pi_{2,\,n}$$

$$E_A = \left[1 - \pi_{0,\,0} - \sum_{n=1}^{\infty}\pi_{2,\,n}\right]\lambda$$

$$E_A = \left[1 - \left[1 + \frac{r_2}{r_2-1}\frac{\beta r_1}{(\alpha-\mu_1-\gamma)r_1+\mu_1}\left[1+\frac{\gamma}{\eta_2}\sum_{n=0}^{\infty}(1-u_1^n)\right]\right]^{-1} - \frac{\gamma}{\mu_2+\eta_2}\sum_{j=n}^{\infty}\pi_{1,\,j}u_1^{j-n}\right]\lambda \tag{37}$$

6. Numerical illustration

6.1 *Stationary probabilities*

Assuming $N = 5$ and the parameters of the system with the following values:

Such as $\lambda = 1$; $\alpha = 0.5$; $\beta = 0.8$; $\gamma = 1.1$; $\mu_1 = 2$; $\mu_2 = 1$; $\eta_1 = 0.4$; $\eta_2 = 0.2$. Arrived the steady-state probabilities by using (19), (22), (27) and (31), probability distribution is given in Table 1.

Table 1. Steady state probability distribution

(i, j)	π(i, j)	(i, j)	π(i, j)
(0, 0)	0.097	(2, 1)	0.0160
(1, 0)	0.0554	(2, 2)	0.0040
(1, 1)	0.0138	(2, 3)	0.0010
(1, 2)	0.0034	(2, 4)	0.0002
(1, 3)	0.0008	(2, 5)	0.0000
(1, 4)	0.0002		
(1, 5)	0.0000		

6.2 *Stationary mean number of times the server switches from full active phase to passive phase against γ*

On fixing $\lambda = 1$; $\alpha = 0.5$; $\beta = 0.8$; $\mu_1 = 2$; $\mu_2 = 1$; $\eta_1 = 0.4$; $\eta_2 = 0.2$; and varying γ from 0.1 to 3.0. The average number of customers in Full active Phase to passive Phase is listed out in Table 2 and shown in Figure 2.

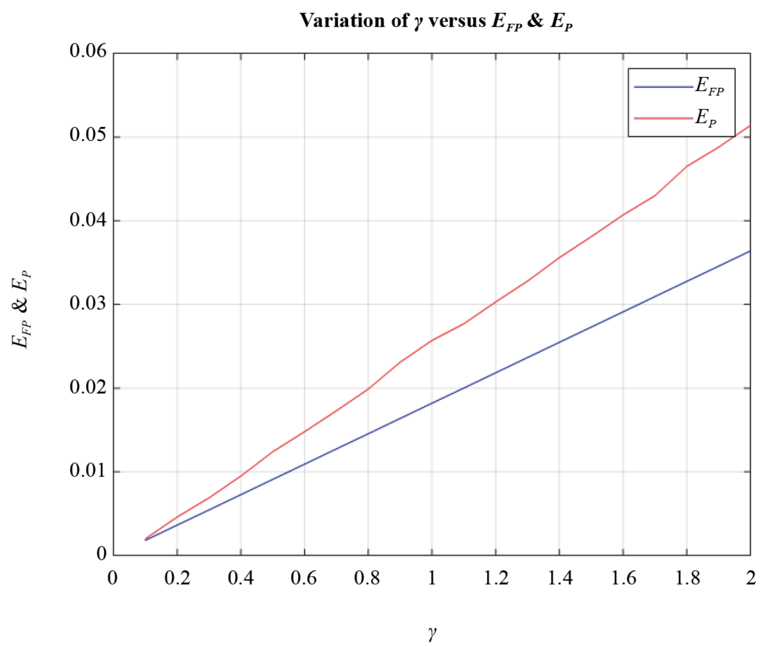

Figure 2. E_{FP} and E_P as a function γ

From Table 2 and Figure 2, mean number of times the server changing from full active to passive phase increases as rate of server switching to passive phase (γ) increases and the average number of customers in passive phase increases as the rate of server switching to passive phase (γ) increases.

Table 2. Variation of average number of times the server switches from full active phase to passive phase and variation of average number of customers in passive phase against γ

γ	E_{FP}	E_P	γ	E_{FP}	E_P	γ	E_{FP}	E_P
0.1	0.00182	0.0020	1.1	0.02002	0.0277	2.1	0.03822	0.0539
0.2	0.00364	0.0046	1.2	0.02184	0.0303	2.2	0.04004	0.0560
0.3	0.00546	0.0069	1.3	0.02366	0.0328	2.3	0.04186	0.0589
0.4	0.00728	0.0095	1.4	0.02548	0.0356	2.4	0.04368	0.0615
0.5	0.0091	0.0124	1.5	0.0273	0.0381	2.5	0.0455	0.0638
0.6	0.01092	0.0148	1.6	0.02912	0.0407	2.6	0.04732	0.0664
0.7	0.01274	0.0173	1.7	0.03094	0.0430	2.7	0.04914	0.0696
0.8	0.01456	0.0199	1.8	0.03276	0.0465	2.8	0.05096	0.0722
0.9	0.01638	0.0231	1.9	0.03458	0.0488	2.9	0.05278	0.0747
1.0	0.0182	0.0257	2.0	0.03640	0.0514	3.0	0.0546	0.0761

6.3 *Average number of times the server switches from repair phase to full active phase against β*

On fixing $\lambda = 1$; $\alpha = 0.5$; $\gamma = 1.1$; $\mu_1 = 2$; $\mu_2 = 1$; $\eta_1 = 0.4$; $\eta_2 = 0.2$ and varying β from 0.1 to 3.0. The average customers in the repair phase to full active phase is listed out in Table 3 and shown in Figure 3.

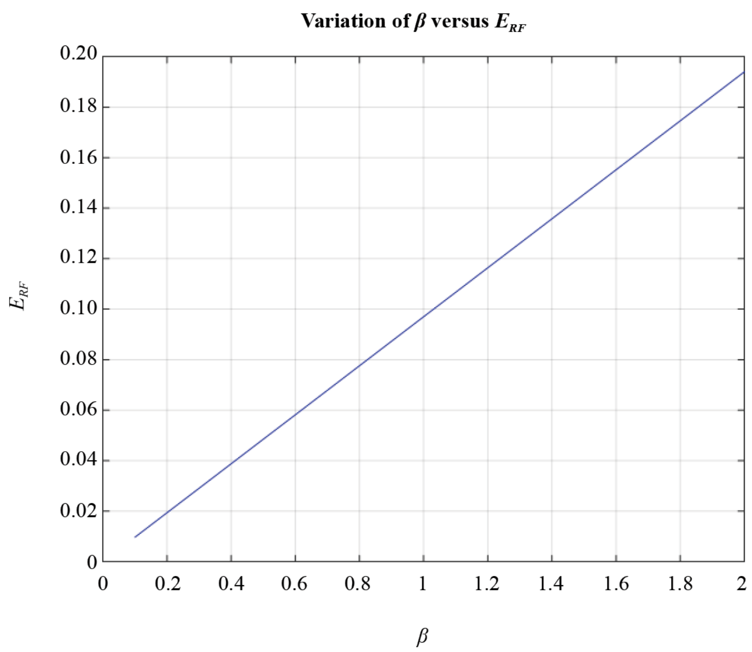

Figure 3. E_{RF} as a function of β

From Table 3 and Figure 3, mean number of times server changing from repair phase to full active phase increases as rate of server switching to full active mode after the repairing phase (β) increases.

Table 3. Variation of average number of times the server switches from repair phase to full active phase against β

β	E_{RF}	β	E_{RF}	β	E_{RF}
0.1	0.0097	1.1	0.1067	2.1	0.2037
0.2	0.0194	1.2	0.1164	2.2	0.2134
0.3	0.0291	1.3	0.1261	2.3	0.2231
0.4	0.0388	1.4	0.1358	2.4	0.2328
0.5	0.0485	1.5	0.1455	2.5	0.2425
0.6	0.0582	1.6	0.1552	2.6	0.2522
0.7	0.0679	1.7	0.1649	2.7	0.2619
0.8	0.0776	1.8	0.1746	2.8	0.2716
0.9	0.0873	1.9	0.1843	2.9	0.2813
1.0	0.0970	2.0	0.1940	3.0	0.2910

6.4 *Mean number of customers in full active phase against* η_1

On fixing $\lambda = 1$; $\alpha = 0.5$; $\beta = 0.8$; $\gamma = 1.1$; $\mu_1 = 2$; $\mu_2 = 1$; $\eta_2 = 0.2$; and varying η_1 from 0.1 to 3.0. The average customers in full active phase is shown in the following Table 4 and shown in Figure 4.

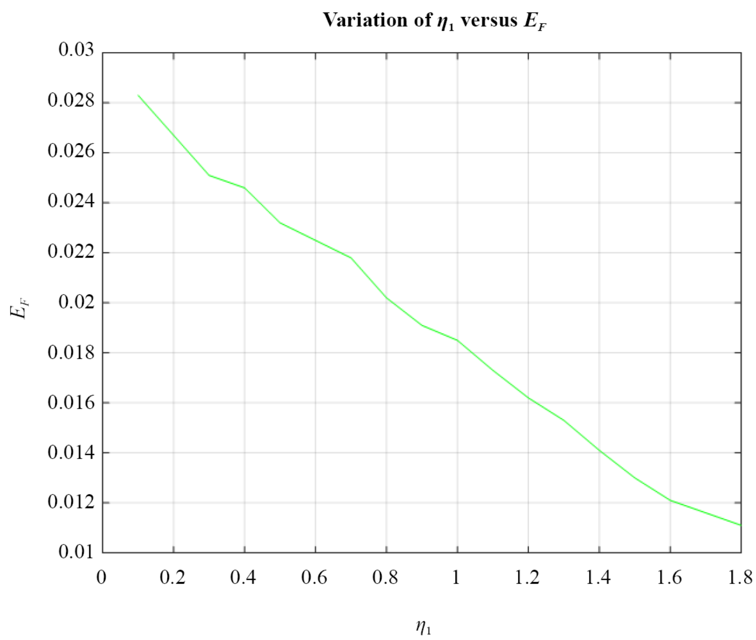

Figure 4. E_F as a function of η_1

From Table 4 and Figure 4, average number of customers in full active phase grows as rate of disaster occurring in that phase (η_1) lowers.

Table 4. Variation of average number of customers in full active phase against η_1

η_1	E_F	η_1	E_F	η_1	E_F
0.1	0.0283	1.1	0.0185	2.1	0.0137
0.2	0.0267	1.2	0.0178	2.2	0.0134
0.3	0.0251	1.3	0.0173	2.3	0.0130
0.4	0.0246	1.4	0.0167	2.4	0.0127
0.5	0.0232	1.5	0.0162	2.5	0.0124
0.6	0.0218	1.6	0.0157	2.6	0.0121
0.7	0.0215	1.7	0.0153	2.7	0.0119
0.8	0.0202	1.8	0.0145	2.8	0.0116
0.9	0.019	1.9	0.0144	2.9	0.0114
1.0	0.0191	2.0	0.0141	3.0	0.0111

6.5 *Mean number of customers in passive phase against η_2*

We next fix $\lambda = 1$; $\alpha = 0.5$; $\beta = 0.8$; $\gamma = 1.1$; $\mu_1 = 2$; $\mu_2 = 1$; $\eta_1 = 0.4$; and varying η_2 from 0.1 to 3.0. The average number of customers in passive phase is in Table 5 and shown in Figure 5.

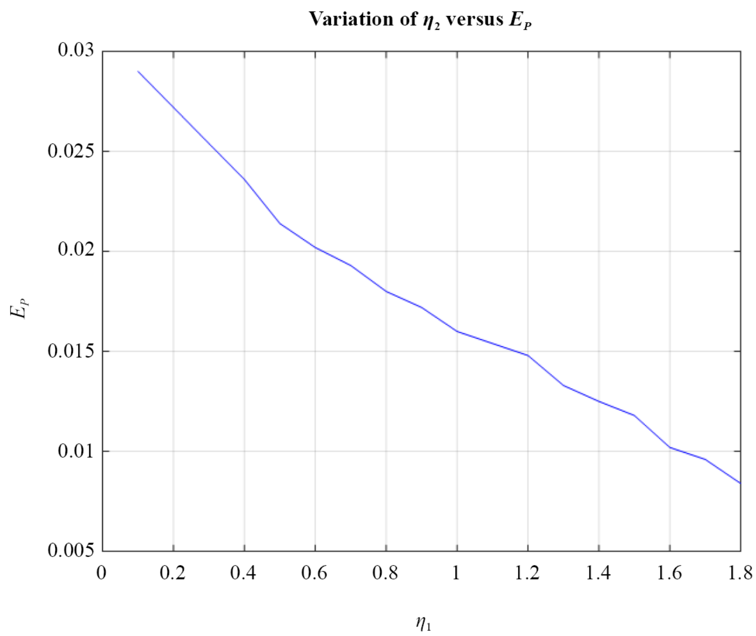

Figure 5. E_P as a function of η_2

From Table 5 and Figure 5, as the rate of disasters happening in that phase (η_2) lowers, the typical amount of customers in the passive phase grows.

Table 5. Variation of average number of customers in passive phase against η_2

η_2	E_P	η_2	E_P	η_2	E_P
0.1	0.037	1.1	0.0154	2.1	0.01
0.2	0.0272	1.2	0.0148	2.2	0.0096
0.3	0.0254	1.3	0.0142	2.3	0.0095
0.4	0.0236	1.4	0.0133	2.4	0.0093
0.5	0.0214	1.5	0.013	2.5	0.0089
0.6	0.0202	1.6	0.0125	2.6	0.0088
0.7	0.0193	1.7	0.012	2.7	0.0084
0.8	0.018	1.8	0.0118	2.8	0.0083
0.9	0.0172	1.9	0.0114	2.9	0.0082
1.0	0.016	2.0	0.0102	3.0	0.0075

7. Conclusions

Here, a steady-state analysis for a single server queue having servers in distinct phases of disaster, repair and recovery were performed. The server serving the customers in different phases, and if there is a disaster in any phase and if the customers are dropped out the system, the server will immediately relocate to repair phase. Some performance matrices are obtained related to the server and customers. The numerical illustration shows the effects of server shifting from full active phase to passive phase, effects on its customers and the occurrence of disaster which has an adverse effect on the customers in that phase. In future, this model can be studied by assuming that the server begins to work in a different environment with a different service rate, when the server returns to work after repair phase.

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Conflict of interest

The authors have no conflict of interest either wholly or partially in the content of the article.

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