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Research Article

Solving Multi-objective Bi-item Capacitated Transportation Problem with Fermatean Fuzzy Multi-Choice Stochastic Mixed Constraints Involving Normal Distribution

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Abstract: The transportation problem (TP) is an important type of linear programming problem. The aim of TP is to optimize the objective function by allocating product shipments from various sources to multiple destinations. When limitations exist during transportation in the number of products transported from a source to destination due to a variety of factors, such as storage limitations, budgetary constraints, and so on, then TP becomes the capacitated transportation problem (CTP). The business entrepreneurs are keen on transporting multiple items with multiple objectives to generate maximum revenue for an organization, which leads to the development of a multi-objective bi-item capacitated transportation problem (MOBICTP). If a supplier's resources significantly increase or decrease, and the demand needs also significantly increase or decrease, the MOBICTP transforms into a MOBICTP with mixed constraints. Most real-world problems have uncertain parameters due to insufficient data, fluctuating market costs for an item, variation in weather conditions, and so on. Such uncertain parameters are addressed using fuzzy, multi-choice, or stochastic programming. In this article, we examined a novel integrated model for a multi-objective bi-item capacitated transportation problem, which includes a fermatean fuzzy multi-choice stochastic mixed constraint that follows normal distribution. First, the Fermatean fuzzy multi-choice stochastic parameter in the constraints is transformed into a multi-choice stochastic parameter in the constraints by using the $(\alpha,\ \beta)$-cut technique and accuracy function. Then the improved chance-constrained method is developed using Newton divided difference interpolation polynomial. The improved chance-constrained method is used to transform the multi-choice stochastic parameter in the constraints into its equivalent deterministic constraints. Secondly, we propose a novel approach, the improved global weighted sum method, which transforms a multi-objective problem into a single objective problem and utilizes Lingo 18.0 software to find the optimal compromise solution to the equivalent deterministic problem. The main aim of this paper is to help business entrepreneurs improve their profit margins by optimizing the quantity of multiple items while minimizing damage costs, labor costs, transportation time, and transportation costs, and maximizing discounts. To show the model's validity and significance, a numerical example is solved using the Lingo 18.0 software. In order to emphasize the proposed method, a comparative analysis is conducted with other existing methods. The final component includes a sensitivity analysis and conclusions with future research directions.

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***Keywords*:** capacitated transportation problem, improved chance constrained method, newton divided difference interpolation polynomial, improved global weighted sum method, normal distribution

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Abbreviation

DMP	Decision Making Problems
FFS	Fermatean Fuzzy Sets
PFS	Pythagorean Fuzzy Sets
TFFN	Triangular Fermatean Fuzzy Number
TIFN	Triangular Intuitionistic Fuzzy Number
MF	Membership Function
NMF	Non-membership Function
TP	Transportation Problem
CTP	Capacitated Transportation Problem
MOBICTP	Multi-objective Bi-item Capacitated Transportation Problem
BICTP	Bi-item Capacitated Transportation Problem
MOCTP	Multi-objective Capacitated Transportation Problem
SP	Stochastic Programming
SA	Sensitivity Analysis
NDD	Newton Divided Difference
ND	Normal Distribution
GWSM	Global Weighted Sum Method
FGP	Fuzzy Goal Programming

1. Introduction

The transportation problem (TP) is a well-known decision-making problem that aims to optimize the objective functions for transporting goods from various origins to specific destinations. In [1], the author proposed the TP, which was subsequently enhanced in [2] with a specialized matrix structure that is more computationally tractable. Various factors, including road safety, storage space limitations, and financial restraints, lead to limitations in the number of items that can be transported from a source to a destination. Then TP transforms into capacitated TP (CTP). In [3], the author proposed the concept of CTP. Several studies [4–6] have done extensive investigations on CTP. In the highly competitive world, it is not always possible to have a single objective for all of the transportation problems. So, to achieve the great benefits, the decision-makers need to consider several objectives to be optimized at the same time, which are conflicting in nature. Then CTP became a multi-objective capacitated transportation problem (MOCTP). In [7], the authors solved the multi-objective transportation problem by using the multi-choice goal programming method. The multi-objective capacitated fractional transportation problem with mixed constraints was developed in [8]. A case study for the multi-objective capacitated transportation problem in a certain and uncertain environment was performed in [9]. In [10], the author proposed an iterative algorithm to solve MOCTP. In [11], the author used the bisection algorithm to solve the multi-objective fractional solid transportation problem with mixed constraints. Because of heavy competition in the market, a transportation service chooses to ship numerous items rather than one. This strategy allows the company to balance any losses incurred from one particular item with the profits from other items. When merchants need to carry several items, they have difficulty determining the precise quantities of each item. This is because they have limited storage space and a restricted budget for transportation costs. In reality, the capacity of supply and destination points fluctuates owing to variations in people, equipment, raw materials, and other factors affecting production and demand. So, some sources aim to provide less than a predetermined quantity of commodities during shortages, while other sources are capable of supplying more than a defined quantity when excess commodities are present. So, under this condition, MOCTP becomes MOCTP with mixed constraints. TP with mixed constraints and solved it using the simple transformation method to determine the solution's feasibility and optimality was introduced in [12]. In [13], the authors developed a time minimization transportation problem with mixed constraints and solved it using a developed algorithm. In the article [14], the author proposed a method to solve mixed constraints cost minimization transportation problem.

In real-world TP, parameters are usually unpredictable because of a variety of factors, such as insufficient input data, market fluctuations, weather conditions, and so on. Additionally, this incorporates an element of uncertainty into the problem. The uncertainty can be addressed with fuzzy/multi-choice/stochastic/fuzzy multi-choice stochastic all together. Initially, in [15], the author introduced decision-making in a fuzzy environment. Fuzzy deals with only the membership function; sometimes in real life, a decision-maker may wish to consider both membership functions and the non-membership function (NMF). In [16], the author proposed Intuitionistic fuzzy systems (IFS), which incorporate both membership functions (MF) and non-membership functions (NMF) with the requirement. The theory of IFS has been considered multiple circumstances when the combined values of MF and NMF exceed 1. In order to minimize such complexities, the concept of the Pythagorean fuzzy set (PFS) [17]; under relaxed conditions, the sum of the squares for the membership as well as the non-membership function must be included in the unit interval. In [18], the authors introduced the notion of Fermatean fuzzy set (FFS) as a substitute for PFS, which is wider than PFS and addresses the limits of PFS. In FFS, the sum of the degrees of membership and non-membership to a cubic set must fall inside the interval values between 0 and 1. For instance, if an expert says that shipping an item from origin to target costs 10 rupees, the probability of the statement being correct is 0.7 and the probability of it being wrong is 0.8. As a result, IFS and PFS are unable to assess this circumstance. Because $0.7+0.8 \geq 1$ and $(0.7)^2+(0.8)^2 \geq 1$. However, it is valid in the context of a Fermatean fuzzy set (i.e.). FFS is more effective at addressing ambiguous information about decision-making issues. There are several studies on real-world applications of the Fermatean fuzzy set [19, 20]. In [21], the author proposed a multi-objective transportation problem in the fermatean fuzzy environment and solved it using the proposed method. In [22], the researcher formulated a multi-objective multi-item solid transportation problem and obtained an optimal solution using fuzzy programming and interval programming. In the real world, decision-makers must choose a single value from a set of parameter values in several decision-making scenarios. This circumstance may be formulated as a mathematical programming model known as a multi-choice programming problem. Techniques of multi-choice programming have become more important in economics, industry, transportation, military applications, and technology. The decision maker assigns numerous alternatives for each multi-choice parameter as a result of fluctuating market prices, supply, and demand. Initially, in [23], the author suggested a multi-choice objective programming technique to solve multi-choice aspiration-level problems and, in [24], introduced an updated version of a multi-choice goal programming technique. Biswal et al.[25] demonstrated the conversion of multi-choice linear programming problems where restrictions are linked to certain multi-choice parameters. Gupta et al.[26] presented a case study that examined the problems posed by uncertain demand and supply in the multi-choice multi-objective capacitated transportation problems. Several factors, including market fluctuations, raw material faults, transportation challenges, unstable consumer preferences, and unpredictable customer demand for goods or services, represent the parameters in real-world problems as random variables. To address these uncertainties, we consider stochastic programming (SP) problems, where random variables follow a specified distribution instead of fixed values. Several researchers have conducted several studies on stochastic TPs [27–30]. The inclusion of the attributes amplifies the persuasive quality of the combination of ambiguity and randomness. Fuzzy stochastic parameters are characterized by their unpredictability in conditions that are both fuzzy and stochastic. In 1979, Kwakernaak [31] introduced the concept of fuzzy stochastic optimization, which involves the use of fuzzy stochastic parameters. In [32], the authors solved the solid transportation problem in a fuzzy stochastic environment using a fuzzy goal programming approach (FGP). In [33], the authors effectively addressed fuzzy stochastic linear fractional programming. Problems by using fuzzy mathematical programming. In [34], the authors developed a bi-objective, bi-item solid transportation problem with inequality fuzzy stochastic constraints that follow normal distribution and solved the problem using a developed technique called the global weighted sum method. For instance, the industry views the raw material production as fuzzy, the raw material demand as stochastic, and the raw material price fluctuation as multi-choice. In reality, from the above-mentioned circumstance, the problem can occur as a combination of uncertainties. So, the combined occurrences of fuzzy, multi-choice, and random circumstances that lead to the formation of combined fuzzy multi-choice stochastic environments. In [35], the authors solved the multi-choice multi-objective fuzzy stochastic transport problem using the weighting mean method. In a fuzzy environment, Nasseri et al. [36] used fuzzy programming techniques to solve a multi-choice stochastic transportation problem.

After optimization, sensitivity analysis (SA) examines the impact of changes in the coefficients of the objective function and the constraints on the ideal level of the objective function. It also determines the valid ranges within which these changes remain effective. In [37], the authors investigated SA for multi-objective waste management through vehicle routing problems. In [38], the researchers investigated SA for a multi-period, two-stage, four-dimensional transportation problem. On objective functions, the SA for the source parameter was performed in [39]. Table 1 represents a comparison between related literature studies and the proposed article.

Table 1. Zone of inhibition of serial diluted *P.* guajava synthesized copper nanoparticles *P.*g-CuNPs.

References	Multi objective			Constraints		Constraints Parameters		
	Capacited	Single item	Multi item	Equality constraints	Mixed constraints	Fuzzy	Multi choice	Stochastic
[9]	✓	✓			✓	✓	✓	
[26]	✓	✓			✓			✓
[30]		✓		✓			✓	✓
[34]			✓	✓		✓		✓
[36]		✓					✓	✓
[40]			✓	✓		✓	✓	
Proposed article	✓		✓		✓	✓	✓	✓

As shown in Table 1, most of the researchers have considered the parameter as a fuzzy multi-choice, fuzzy stochastic, or multi-choice stochastic parameter for a multi-objective transportation problem with mixed constraints, multi-choice fractional TP, and multi-item TP. Based on the literature review in Table 1, the research gap, motivation, and contribution are summarized as follows.

1.1 *Research gap, motivation, and contribution*

In this study, we consider a multi-objective bi-item capacitated transportation problem (MOBICTP). The objectives in this study are multi-objective, involving multiple items and capacity restrictions, with deterministic parameters. The problem constraints are in mixed form, and the parameters in the mixed constraints are in the combination of fuzziness and randomness in nature. The research gap and motivations contribution for this study are given below.

To the great extent of our knowledge, in TP, there are very few articles that contain multiple objectives and multiple items, and the constraints follow normal distribution with the incorporation of capacity limitation, fermatean fuzzy, multi-choice, and stochastic parameters together.

In decision-making problems, multi-objective with multiple items with constraints parameters as fermatean fuzzy multi-choice stochastic are very significant, but these models are very rare in the literature. Transporting multiple items together from one location to another is an important problem in enhancing a country's economic development rate. The purpose of the multi-objective capacitated transportation problem is an area of study and is economically significant because of financial restrictions, concerns about road safety, and storage space that lead to the shipment of multiple items with a predetermined limit for the number of items transported from a source to a destination. Sometimes, the resources of a supplier are significantly increased or decreased, then surplus/shortage in the number of products occurs, and in a similar way, this will happen in demand also so as to enhance the accuracy and reliability of transportation problems. We choose the mixed constraints as probabilistic nature and its parameters as both fermatean fuzzy and multichoice to capture the uncertainty, fluctuations, and variability present in actual transportation systems. Combining fermatean fuzzy, multi-choice, and stochastic environments allows us to understand real-world situations better. This motivates us to investigate the capacitated transportation problem with multiple objectives with multiple items with mixed constraints under a fermatean fuzzy multi-choice stochastic environment.

As a result of this motivation, the major contribution of this study is presented as follows:

In this present study, a multi-objective bi-item capacitated transportation problem is formulated with multiple objectives in which damage and labor cost are linear objectives, transportation time, transportation cost, and discounts are fractional objectives, which are in deterministic nature, and the mixed constraints, which are in fermatean fuzzy multi-choice stochastic nature.

The formulation provides information on the problem's mixed constraint parameters as fermatean fuzzy multi-choice random variables, which is a novel development in this area of study.

We convert Fermatean fuzzy multi-choice stochastic parameter to interval multi-choice stochastic parameter using the (α, β)-cut method. Then, using the accuracy function, the reduced problem is transformed into a deterministic multi-choice parameter. We developed an improved chance constrained technique based on the transformation technique [34], which is used to transform multi-choice stochastic supply and demand constraints into deterministic supply and demand constraints.

Based on the global weighted sum method, we proposed the improved Global Weighted Sum Method, which transforms any multi-objective optimization (combining both maximization and minimization) objectives into a single objective optimization problem. In this study, we used the proposed improved Global Weighted Sum Method (GWSM) to convert the transformed deterministic multi-objective problem to a single objective problem, which we then solved using Lingo 18.0 software to obtain the MOBICTP's optimal compromise solution.

The remaining sections of the article are organized as follows: Section 2 contains necessary definitions. Section 3 presents a mathematical formulation of the multi-objective bi-item capacitated transportation problem with fermatean fuzzy multi-choice stochastic constraint. Section 4 describes the equivalence deterministic model for this problem. Section 5 explains the solution approach. Section 6 discusses the formulated model's numerical example. Section 7 provides the conclusions and recommendations for further research.

2. Preliminaries

Basic definitions of fermatean fuzzy sets and fermatean fuzzy numbers are to be found in [18], and we presented the essential definition and notations associated with the uncertainty theory.

Definition 2.1 Triangular Fermatean fuzzy number [41]: A triangular Fermatean fuzzy number (TFFN) $\tilde{A}^F = \langle (a_1,\ a_2,\ a_3);\ \mu_{\tilde{A}^F}, \gamma_{\tilde{A}^F} \rangle$ is a Fermatean fuzzy set (FFS) with the membership function $\mu_{\tilde{A}^F}(x)$ and non-membership function $\gamma_{\tilde{A}^F}(x)$ given as:

$$\mu_{\tilde{A}^F}(x) = \begin{cases} \dfrac{\mu_{\tilde{A}^F}(x-a_1)}{a_2-a_1}, & a_1 \leq x < a_2 \\[2ex] \mu_{\tilde{A}^F}, & x = a_2 \\[2ex] \dfrac{\mu_{\tilde{A}^F}(a_3-x)}{a_3-a_2}, & a_2 < x \leq a_3 \\[2ex] 0, & otherwise \end{cases}$$

$$\gamma_{\tilde{A}^F}(x) = \begin{cases} \dfrac{a_2 - x + \gamma_{\tilde{A}^F}(x - a_1)}{a_2 - a_1}, & a_1 \le x < a_2 \\ \gamma_{\tilde{A}^F}, & x = a_2 \\ \dfrac{x - a_2 + \gamma_{\tilde{A}^F}(a_3 - x)}{a_3 - a_2}, & a_2 < x \le a_3 \\ 1, & otherwise \end{cases}$$

The values $\mu_{\tilde{A}^F}$ and $\gamma_{\tilde{A}^F}$ describe the maximum value of membership function $\mu_{\tilde{A}^F}(x)$ and minimum value of non-membership function $\gamma_{\tilde{A}^F}(x)$, respectively, such that $\mu_{\tilde{A}^F} \in [0,\ 1]$, $\gamma_{\tilde{A}^F} \in [0,\ 1]$ and $0 \le (\mu_{\tilde{A}^F}(x))^3 + (\gamma_{\tilde{A}^F}(x))^3 \le 1$.
By taking $\mu_{\tilde{A}^F}$= 1 and $\gamma_{\tilde{A}^F}$= 0, TFFN assumes the form $\tilde{A}^F = \langle (a_1,\ a_2,\ a_3);\ (\bar{a}_1,\ a_2,\ \bar{a}_3) \rangle$whose membership $\mu_{\tilde{A}^F}(x)$ and non-membership functions $\gamma_{\tilde{A}^F}$ can be represented by

$$\mu_{\tilde{A}^F}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x < a_2 \\ 1, & x = a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 < x \le a_3 \\ 0, & otherwise \end{cases}$$

$$\gamma_{\tilde{A}^F}(x) = \begin{cases} \dfrac{a_2 - x}{a_2 - a_1}, & a_1 \le x < a_2 \\ 0, & x = a_2 \\ \dfrac{x - a_2}{a_3 - a_2}, & a_2 < x \le a_3 \\ 1, & otherwise \end{cases}$$

where $\bar{a}_1 \le a_1 \le a_2 \le a_3 \le \bar{a}_3$.

The following $(\alpha,\ \beta)$-cut and accuracy functions of a TFFN have been established as per the Definitions 2.2 and 2.3, which are based on definition in [42] for Triangular Intuitionistic Fuzzy Number (TIFN).

Definition 2.2 $(\alpha,\ \beta)$-cut of a Triangular Fermatean fuzzy number: Let $(\alpha,\ \beta)$-cut of a Triangular Fermatean fuzzy number$\tilde{A}^F = \langle (a_1,\ a_2,\ a_3);\ (\bar{a}_1,\ a_2,\ \bar{a}_3) \rangle$is the set of all x whose degree of membership is greater than or equal to α and degree of non-membership is less than or equal to β. That is defined by $\tilde{A}^F_{(\alpha,\ \beta)} = \{x : \mu_{\tilde{A}^F} \geq \alpha,\ \gamma_{\tilde{A}^F} \leq \beta,\ (\alpha + \beta) \leq 1 :$ $x \in X$ Now, $\mu_{\tilde{A}^F} \geq \alpha$, which implies that $\dfrac{x - a_1}{a_2 - a_1} \geq \alpha,\ \dfrac{a_3 - x}{a_3 - a_2} \leq \alpha$.

And $x \geq a_1 + \alpha(a_2 - a_1),\ x \leq a_3 - \alpha(a_3 - a_2)$.

Therefore, the α-cut of $\tilde{A}^F$ is $[a_1 + \alpha(a_2 - a_1),\ a_3 - \alpha(a_3 - a_2)]$.

Again $\gamma_{\tilde{A}^F} \geq \beta$, which implies that $\dfrac{a_2 - x}{a_2 - \bar{a}_1} \geq \beta,\ \dfrac{x - a_2}{\bar{a}_3 - \bar{a}_1} \leq \beta$.

And $x \geq a_2 - \beta(a_2 - \bar{a}_1),\ x \leq a_2 + \beta(\bar{a}_3 - a_2)$.

Therefore, the β-cut of $\tilde{A}^F$ is $[a_2 - \beta(a_2 - \bar{a}_1),\ a_2 + \beta(\bar{a}_3 - a_2)]$.

Denoting α-cut of $\tilde{A}^F$ as $[\mu^l_{\tilde{A}^F},\ \mu^u_{\tilde{A}^F}] = [a_1 + \alpha(a_2 - a_1),\ a_3 - \alpha(a_3 - a_2)]$ and β-cut of $\tilde{A}^F$ as $[\gamma^l_{\tilde{A}^F},\ \gamma^u_{\tilde{A}^F}] = [a_2 - \beta(a_2 - \bar{a}_1),\ a_2 + \beta(\bar{a}_3 - a_2)]$.

Definition 2.3 Accuracy function of a Triangular Fermatean fuzzy number:Consider a TFFN $\tilde{A}^F = (a_1,\ a_2,\ a_3;\ \bar{a}_1,\ a_2,\ \bar{a}_3)$. Then, the accuracy function is $M(\tilde{A}^F) :\ \bar{X}(\tilde{A}^F) \rightarrow \mathbb{R}$,where $\bar{X}(\tilde{A}^F)$is the collection of all interval valued fermatean fuzzy sets obtained from $(\alpha,\ \beta)$-cut.This accuracy function is defined in terms of the interval valued fermatean fuzzy set $\bar{X}(\tilde{A}^F) = \left\langle [\mu^l_{\tilde{A}^F},\ \mu^u_{\tilde{A}^F}];\ [\gamma^l_{\tilde{A}^F},\ \gamma^u_{\tilde{A}^F}] \right\rangle$ as $M(\tilde{A}^F) = \dfrac{(\mu^l_{\tilde{A}^F} + \mu^u_{\tilde{A}^F} + \gamma^l_{\tilde{A}^F} + \gamma^u_{\tilde{A}^F})}{2}$.

3. Mathematical formulation

This section presents notations, mathematical formulation of multi-objective multi-item capacitated transportation problem (MOBICTP), mathematical formulation of MOBICTP with fermatean fuzzy multi-choice stochastic mixed constraints, and mathematical formulation of MOBICTP with multi-choice stochastic mixed constraints via the improved chance constraint method. A list of the notations employed in the model is given below:

3.1 *Notations*

i, index for sources $(i = 1,\ 2,\ ...,\ m)$.

j, index for destinations $(j = 1,\ 2,\ ...,\ n)$.

p, index for items $(p = 1,\ 2)$.

x^p_{ij} is the variable for p^{th} item that indicates the unknown amount carried from origin to destination.

d^p_{ij} is the damage cost for p^{th} item arises during transportation.

l^p_{ij} is the labor cost for p^{th} item arises during transportation.

t^{pa}_{ij} is the actual transportation time for p^{th} item arises during transporting $x^p_{ij} \geq 0$ units.

t^{ps}_{ij} is the standard transportation time for p^{th} item arises during transporting $x^p_{ij} \geq 0$ units.

c^{pa}_{ij} actual transportation cost for p^{th} item arises during transporting $x^p_{ij} \geq 0$ units.

c^{ps}_{ij} the standard transportation cost for p^{th} item arises during transporting $x^p_{ij} \geq 0$ units.

d^{ps}_{ij} the discount cost that was given for p^{th} item during transportation.

$(\tilde{a}^{p(1)}_i,\ \tilde{a}^{p(2)}_i,\ ....,\ \tilde{a}^{p(k_i)}_i)$ is the fuzzy multi choice parameter of total amount of the product available for p^{th} item at i^{th} source.

$(\tilde{b}^{p(1)}_j,\ \tilde{b}^{p(2)}_j,\ ....,\ \tilde{b}^{p(k_j)}_j)$ is the fuzzy multi choice demand of total products with p^{th} item at j^{th} destination.

$\tilde{\beta}^p_i$ is the fuzzy probability for supply constraints for p^{th} item.

$\tilde{\gamma}^p_j$ is the fuzzy probability for demand constraints for p^{th} item.

r^p_{ij} is the maximum amount of quantity for p^{th} item to be transported from i^{th} source to j^{th} destination.

$F_{a^p_i}(S_{a^p_i})$ is the interpolating polynomial of supply constraints for p^{th} item.

$F_{b^p_j}(S_{b^p_j})$ is the interpolating polynomial of demand constraints for p^{th} item.

$\mu[F_{a^p_i}(S_{a^p_i})]$ is the mean of interpolating polynomial of supply constraints for p^{th} item.

$\sigma^2[F_{a^p_i}(S_{a^p_i})]$ is the variance of interpolating polynomial of supply constraints for p^{th} item.

$\mu[F_{b^p_j}(S_{b^p_j})]$ is the mean of interpolating polynomial of demand constraints for p^{th} item.

$\sigma^2[F_{b_j^p}(S_{b_j^p})]$ is the variance of interpolating polynomial of demand constraints for p^{th} item.

3.2 *Formulation of multi objective bi-item capacitated transportation problem (MOBICTP)*

In real-life Decision Making problems (DMP), there are many conflicting objectives to be considered and optimized simultaneously. To achieve the purpose of a decision making, the optimization problem not only focuses on minimizing transportation costs but also to optimize other objective functions simultaneously, such as labor cost, delivery time, damage cost, and discount costs. Many practical situations may require addressing simultaneous linear and fractional objectives together. In this study, labor cost and damage cost are considered linear objectives, whereas delivery time, transportation costs, and discount costs are considered fractional objective functions, which are used as performance metrics for analyzing financial aspects of transportation enterprises and managing transportation situations. The maintenance of favorable ratios between significant parameters related to the transportation of goods from specific origins to different destinations will help in maximizing profit. So, we formulated the MOBICTP with both linear and fractional objectives, along with mixed constraints in which two different products are sent from the source to the destination. Based on this assumption, the mixed-constraint mathematical formulation for MOBICTP is as follows:

$$(G)\ Minimize\left(Z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p=1}^{2} d_{ij}^p x_{ij}^p, \qquad Z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p=1}^{2} l_{ij}^p x_{ij}^p\right) \tag{1}$$

$$Minimize\left(Z_3 = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p=1}^{2} t_{ij}^{pa} x_{ij}^p}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p=1}^{2} t_{ij}^{ps} x_{ij}^p}, \qquad Z_4 = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p=1}^{2} c_{ij}^{pa} x_{ij}^p}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p=1}^{2} c_{ij}^{ps} x_{ij}^p}\right) \tag{2}$$

$$Maximize\left(Z_5 = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p=1}^{2} d_{ij}^{ps} x_{ij}^p}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p=1}^{2} c_{ij}^{pa} x_{ij}^p}\right) \tag{3}$$

$$Subject\ to: \sum_{j=1}^{n} x_{ij}^p \le a_i^p,\ i \in I_1 \tag{4}$$

$$\sum_{j=1}^{n} x_{ij}^p = a_i^p,\ i \in I_2 \tag{5}$$

$$\sum_{j=1}^{n} x_{ij}^p \ge a_i^p,\ i \in I_3 \tag{6}$$

$$\sum_{i=1}^{m} x_{ij}^p \ge b_j^p,\ j \in J_1 \tag{7}$$

$$\sum_{i=1}^{m} x_{ij}^p = b_j^p,\ j \in J_2 \tag{8}$$

$$\sum_{i=1}^{m} x_{ij}^{p} \leq b_{j}^{p},\ j \in J_3 \tag{9}$$

$$0 \leq x_{ij}^{p} \leq r_{ij}^{p},\ i = \{I_1 \cup I_2 \cup I_3\} = 1,\ 2,\ ...,\ m,\ j = \{J_1 \cup J_2 \cup J_3\} = 1,\ 2,\ ...,\ n,\ \ p = 1,\ 2 \tag{10}$$

3.3 *Formulation for multi-objective bi-item capacitated transportation problem (MOBICTP) with fuzzy multi choice stochastic mixed constraints*

In real-world problems, the data of input parameters are in the nature of uncertainty and multiple choices. Sometimes, the parameters can be the combination of uncertainty and multiple choices, like fermatean fuzzy, stochastic, and multi-choices all together. If the parameters that are uncertain in fermatean fuzzy, multi-choice, and stochastic in nature are referred to as fermatean fuzzy multi-choice stochastic parameters. All parameters in the constraints of this study are regarded as fermatean fuzzy multi-choice stochastic parameters. These parameters can be considered in accordance with numerous probability distributions; however, in our formulation, we use the normal distribution.

The mathematical formulation of MOBICTP with fermatean fuzzy multi-choice stochastic mixed constraints is presented below:

$$(G1)\ Minimize\left(Z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p=1}^{2} d_{ij}^{p} x_{ij}^{p}, \qquad Z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p=1}^{2} l_{ij}^{p} x_{ij}^{p}\right)$$

$$Minimize\left(Z_3 = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p=1}^{2} t_{ij}^{pa} x_{ij}^{p}}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p=1}^{2} t_{ij}^{ps} x_{ij}^{p}}, \qquad Z_4 = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p=1}^{2} c_{ij}^{pa} x_{ij}^{p}}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p=1}^{2} c_{ij}^{ps} x_{ij}^{p}}\right)$$

$$Maximize\left(Z_5 = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p=1}^{2} d_{ij}^{ps} x_{ij}^{p}}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p=1}^{2} c_{ij}^{pa} x_{ij}^{p}}\right)$$

$$Subject\ to: P\left(\sum_{j=1}^{n} x_{ij}^{p} \leq \tilde{a}_{i}^{p(1)},\ \tilde{a}_{i}^{p(2)},\ ...,\ \tilde{a}_{i}^{p(k_i)}\right) \geq \tilde{\beta}_{i}^{p},\ i \in I_1$$

$$P\left(\sum_{j=1}^{n} x_{ij}^{p} = \tilde{a}_{i}^{p(1)},\ \tilde{a}_{i}^{p(2)},\ ...,\ \tilde{a}_{i}^{p(k_i)}\right) \geq \tilde{\beta}_{i}^{p},\ i \in I_2$$

$$P\left(\sum_{j=1}^{n} x_{ij}^{p} \geq \tilde{a}_{i}^{p(1)}, \tilde{a}_{i}^{p(2)},\ ...,\ \tilde{a}_{i}^{p(k_i)}\right) \geq \tilde{\beta}_{i}^{p},\ i \in I_3$$

$$P\left(\sum_{i=1}^{m} x_{ij}^{p} \geq \tilde{b}_{j}^{p(1)},\ \tilde{b}_{j}^{p(2)},\ \ldots,\ \tilde{b}_{j}^{p(k_j)}\right) \geq \tilde{\gamma}_{j}^{p},\ j \in J_1 \tag{11}$$

$$P\left(\sum_{i=1}^{m} x_{ij}^{p} = \tilde{b}_{j}^{p(1)}, \tilde{b}_{j}^{p(2)},\ \ldots,\ \tilde{b}_{j}^{p(k_j)}\right) \geq \tilde{\gamma}_{j}^{p},\ j \in J_2 \tag{12}$$

$$P\left(\sum_{i=1}^{m} x_{ij}^{p} \leq \tilde{b}_{j}^{p(1)},\ \tilde{b}_{j}^{p(2)},\ \ldots,\ \tilde{b}_{j}^{p(k_j)}\right) \geq \tilde{\gamma}_{j}^{p},\ j \in J_3 \tag{13}$$

$$0 \leq x_{ij}^{p} \leq r_{ij}^{p},\ i = \{I_1 \cup I_2 \cup I_3\} = 1,\ 2,\ \ldots,\ m,\ j = \{J_1 \cup J_2 \cup J_3\} \tag{14}$$

$$= 1,\ 2,\ \ldots,\ n,\ k_i = 2,\ 3,\ \ldots,\ k_j = 2,\ 3,\ \ldots,\ p = 1,\ 2$$

The DMP stated above, which uses fermatean fuzzy multi-choice stochastic constraints, will need more computational effort to calculate the objectives while considering the constraints directly. In order to simplify the computation process and to minimize time, it is necessary to convert fermatean fuzzy data into deterministic data by using results of definitions 2.2 and 2.3. Thus, MOBICTP with multi-choice stochastic mixed constraints is obtained. The following section will discuss the process of converting multi-choice parameters into optimal choices in MOBICTP with multi-choice stochastic mixed constraints.

3.4 *Formulation for MOBICTP with multi choice stochastic mixed constraints via Newton's divided difference interpolating polynomial*

To convert the multi-choice parameter into the optimal choice, we use Newton's Divided Difference (NDD) interpolation as on [29] which is one of the numerical approximation methods. For each multi-choice parameter, an integer variable is introduced such that the interpolating polynomial is formulated. Due to the fact that the above problem has λ number of choices for the supply parameter and number of choices for the demands, the integer variables $F_{a_i^p}^{k}(S_{a_i^p})$, $(k = 0,\ 1,\ \ldots,\ \lambda - 1)$, $F_{b_j^p}^{t}(S_{b_j^p})$, $(t = 0,\ 1,\ \ldots,\ \eta - 1)$ are introduced, respectively. For each multi-choice parameter, a different order of divided difference is generated based on the available alternatives.

Table 2. Divided difference table

$S_{a_i^p}^{k}$	$F_{a_i^p}(S_{a_i^p}^{k})$	1^{st} Divided difference	2^{st} Divided difference	3^{st} Divided difference
0	$S_{a_i^p}^{1}$			
		$f[S_{a_i^p}^{0}, S_{a_i^p}^{1}]$		
1	$S_{a_i^p}^{2}$		$f[S_{a_i^p}^{0}, S_{a_i^p}^{1}, S_{a_i^p}^{2}]$	
		$f[S_{a_i^p}^{1}, S_{a_i^p}^{2}]$		$f[S_{a_i^p}^{3}, S_{a_i^p}^{1}, S_{a_i^p}^{2}, S_{a_i^p}^{3}]$
2	$S_{a_i^p}^{3}$		$f[S_{a_i^p}^{1}, S_{a_i^p}^{2}, S_{a_i^p}^{3}]$	
		$f[S_{a_i^p}^{2}, S_{a_i^p}^{3}]$		
3	$S_{a_i^p}^{4}$			

Table 2 displays the various orders of divided differences. The Newton's divided Difference interpolating polynomial for the supply parameter in Eq.(18) is formulated using Table 2.

$$F_{a_i^p}(S_{a_i^p}) = f[S^0_{a_i^p}] + (S_{a_i^p} - S^0_{a_i^p})f[S^0_{a_i^p},\ S^1_{a_i^p}] + (S_{a_i^p} - S^0_{a_i^p})(S_{a_i^p} - S^1_{a_i^p})f[S^0_{a_i^p},\ S^1_{a_i^p},\ S^2_{a_i^p}] + \dots \tag{15}$$

$$+ (S_{a_i^p} - S^0_{a_i^p})(S_{a_i^p} - S^1_{a_i^p})\dots(S_{a_i^p} - S^{\lambda-1}_{a_i^p})f[S^0_{a_i^p},\ S^1_{a_i^p},\ \dots,\ S^{\lambda-1}_{a_i^p}]$$

$$F_{a_i^p}(S_{a_i^p}) = a_i^{p1} + (S_{a_i^p} - S^0_{a_i^p})(a_i^{p2} - a_i^{p1}) + (S_{a_i^p} - S^0_{a_i^p})(S_{a_i^p} - S^1_{a_i^p})\left(\frac{a_i^{p3} - 2a_i^{p2} + a_i^{p1}}{S^2_{a_i^p} - S^0_{a_i^p}}\right) + \dots \tag{16}$$

$$+ \left(\sum_{h=1}^{\lambda} \frac{a_i^{ph}}{\prod_{h\neq l+1,\ l=0}^{\lambda-1} S^{h-1}_{a_i^p} - S^l_{a_i^p}}\right)$$

Where $f[S^0_{a_i^p},\ S^1_{a_i^p}] = (a_i^{p2} - a_i^{p1})$ and $f[S^0_{a_i^p},\ S^1_{a_i^p},\ S^2_{a_i^p}] = \dfrac{a_i^{p3} - 2a_i^{p2} + a_i^{p1}}{S^2_{a_i^p} - S^0_{a_i^p}}$.

In the same way, we can obtain the NDD interpolating polynomials for the demand given below.

$$F_{b_j^p}(S_{b_j^p}) = b_j^{p1} + (S_{b_j^p} - S^0_{b_j^p})(b_j^{p2} - b_j^{p1}) + (S_{b_j^p} - S^0_{b_j^p})(S_{b_j^p} - S^1_{b_j^p})\frac{b_j^{p3} - 2b_j^{p2} + b_j^{p1}}{S^2_{b_j^p} - S^0_{b_j^p}} + \dots \tag{17}$$

$$+ \left(\sum_{h=1}^{\eta} \frac{b_j^{ph}}{\prod_{h\neq l+1,\ l=0}^{\eta-1} S^{h-1}_{b_j^p} - S^l_{b_j^p}}\right).$$

Now, the multi-choice parameters in the problem MOBICTP with multi-choice stochastic mixed constraints are replaced by its interpolating polynomial. So, the mathematical model of MOBICTP with multi-choice stochastic mixed constraints with interpolating polynomial (G2) is defined as follows:

$$(G2)\ Minimize\left(Z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p=1}^{2} d_{ij}^p x_{ij}^p, \qquad Z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p=1}^{2} l_{ij}^p x_{ij}^p\right)$$

$$Minimize\left(Z_3 = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p=1}^{2} t_{ij}^{pa} x_{ij}^p}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p=1}^{2} t_{ij}^{ps} x_{ij}^p}, \qquad Z_4 = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p=1}^{2} c_{ij}^{pa} x_{ij}^p}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p=1}^{2} c_{ij}^{ps} x_{ij}^p}\right)$$

$$Maximize\left(Z_5 = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p=1}^{2} d_{ij}^{ps} x_{ij}^p}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p=1}^{2} c_{ij}^{pa} x_{ij}^p}\right)$$

$$Subject\ to: P\left(\sum_{j=1}^{n} x_{ij}^{p} \leq F_{a_i^p}(S_{a_i^p})\right) \geq \beta_i^p,\ i \in I_1 \tag{18}$$

$$P\left(\sum_{j=1}^{n} x_{ij}^{p} = F_{a_i^p}(S_{a_i^p})\right) \geq \beta_i^p,\ i \in I_2 \tag{19}$$

$$P\left(\sum_{j=1}^{n} x_{ij}^{p} \geq F_{a_i^p}(S_{a_i^p})\right) \geq \beta_i^p,\ i \in I_3 \tag{20}$$

$$P\left(\sum_{i=1}^{m} x_{ij}^{p} \geq F_{b_j^p}(S_{b_j^p})\right) \geq \gamma_j^p,\ j \in J_1 \tag{21}$$

$$P\left(\sum_{i=1}^{m} x_{ij}^{p} = F_{b_j^p}(S_{b_j^p})\right) \geq \gamma_j^p,\ j \in J_2 \tag{22}$$

$$P\left(\sum_{i=1}^{m} x_{ij}^{p} \leq F_{b_j^p}(S_{b_j^p})\right) \geq \gamma_j^p,\ j \in J_3 \tag{23}$$

$$0 \leq x_{ij}^{p} \leq r_{ij}^{p},\ i = \{I_1 \cup I_2 \cup I_3\} = 1,\ 2,\ ...,\ m,\ j = \{J_1 \cup J_2 \cup J_3\} = 1,\ 2,\ ...,\ n,\ p = 1,\ 2 \tag{24}$$

Thus, we obtained a mathematical formulation for MOBICTP with multi-choice stochastic mixed constraints via NDD interpolating polynomial. To save time and simplify the computation procedure, the conversion of random parameters with NDD interpolating polynomials into deterministic parameters is essential. So, the following section discusses the conversion of stochastic constraints with the NDD interpolating polynomial into deterministic constraints.

4. Equivalent deterministic formulation

The constraints in the mathematical model (G2) are stochastic parameters with NDD interpolating polynomials. Such constraints cannot be resolved directly due to their complexity and time-consuming nature. Hence, stochastic uncertainty with NDD interpolating polynomials will be eliminated by using the improved chance-constraint programming method based on a normal distribution. This technique allows for deviations from the constraints up to a predetermined probability level. In order to address this problem, it is necessary to use the following theorems. The theorem aims to convert the stochastic mixed constraints with NDD interpolating polynomial into deterministic mixed constraints of a multi-objective bi-item capacited transportation problem, which is expressed as follows:

Theorem 4.1 If $F_{a_i^p}(S_{a_i^p})$, $i = 1,\ 2,\ ...,\ m,\ p = 1,\ 2$, are interpolating polynomial with the normal distribution then $P\left(\sum_{j=1}^{n} x_{ij}^{p} \leq F_{a_i^p}(S_{a_i^p})\right) \geq \beta_i^p$, is equivalent to $\sum_{j=1}^{n} x_{ij}^{p} \leq \phi^{-1}(1-\beta_i^p)\sqrt{\sigma^2\left(F_{a_i^p}(S_{a_i^p})\right)} + \mu\left(F_{a_i^p}(S_{a_i^p})\right)$, the two known parameters, $\mu\left(F_{a_i^p}(S_{a_i^p})\right)$, $\sigma^2\left(F_{a_i^p}(S_{a_i^p})\right)$ are the mean and variance of interpolating polynomial $F_{a_i^p}(S_{a_i^p})$ respectively follows normal distribution, β_i^p is the probability for supply constraints at item p.

Proof. It is assumed that, $i = 1, 2, \ldots, n$, $p = 1, 2$, interpolating polynomial follows normal distribution (ND) with mean and variance respectively.

Let us consider stochastic supply constraints as

$$P\left(\sum_{j=1}^{n} x_{ij}^{p} \leq F_{a_i^p}(S_{a_i^p})\right) \geq \beta_i^p \tag{25}$$

$$1 - P\left(F_{a_i^p}(S_{a_i^p})\right) \leq \sum_{j=1}^{n} x_{ij}^{p}\Bigg) \geq \beta_i^p \tag{26}$$

$$1 - P\left[\frac{F_{a_i^p}(S_{a_i^p}) - \mu\left(F_{a_i^p}(S_{a_i^p})\right)}{\sqrt{\sigma^2\left(F_{a_i^p}(S_{a_i^p})\right)}} \leq \frac{\sum_{j=1}^{n} x_{ij}^{p} - \mu\left(F_{a_i^p}(S_{a_i^p})\right)}{\sqrt{\sigma^2\left(F_{a_i^p}(S_{a_i^p})\right)}}\right] \geq \beta_i^p \tag{27}$$

We know that, $P(z \leq x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{\left(\frac{-(z)^2}{2}\right)} dz = \phi(X)$ is the cumulative distribution function of one-dimensional standard normal variable. Hence the probabilistic constraints are represented as

$$1 - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\sum_{j=1}^{n} x_{ij}^{p} - \mu\left(F_{a_i^p}(S_{a_i^p})\right)}{\sqrt{\sigma^2\left(F_{a_i^p}(S_{a_i^p})\right)}}} e^{\frac{-\left(\frac{(F_{a_i^p}(S_{a_i^p} - \mu(F_{a_i^p}(S_{a_i^p}))}{\sqrt{\sigma^2\left(F_{a_i^p}(S_{a_i^p})\right)}}\right)^2}{2}} dz \geq \beta_i^p \tag{28}$$

$$1 - \phi\left[\frac{\sum_{j=1}^{n} x_{ij}^{p} - \mu\left(F_{a_i^p}(S_{a_i^p})\right)}{\sqrt{\sigma^2\left(F_{a_i^p}(S_{a_i^p})\right)}}\right] \geq \beta_i^p$$

On rearranging, we obtain

$$\phi\left[\frac{\sum_{j=1}^{n} x_{ij}^{p} - \mu\left(F_{a_i^p}(S_{a_i^p})\right)}{\sqrt{\sigma^2\left(F_{a_i^p}(S_{a_i^p})\right)}}\right] \geq 1 - \beta_i^p$$

$$\left[\frac{\sum_{j=1}^{n} x_{ij}^{p} - \mu\left(F_{a_i^p}(S_{a_i^p})\right)}{\sqrt{\sigma^2\left(F_{a_i^p}(S_{a_i^p})\right)}}\right] \geq \phi^{-1}(1 - \beta_i^p)$$

$$\sum_{j=1}^{n} x_{ij}^{p} \leq \phi^{-1}(1 - \beta_i^p)\sqrt{\sigma^2\left(F_{a_i^p}(S_{a_i^p})\right)} + \mu\left(F_{a_i^p}(S_{a_i^p})\right) \tag{29}$$

Where the $\mu\left(F_{a_i^p}(S_{a_i^p})\right)$ and $\sigma^2\left(F_{a_i^p}(S_{a_i^p})\right)$ is calculated by using the following equations:

$$\mu\left[F_{a_i^p}(S_{a_i^p})\right] = \mu\left[a_i^{p1} + (S_{a_i^p} - S^0_{a_i^p})(a_i^{p2} - a_i^{p1}) + (S_{a_i^p} - S^0_{a_i^p})(S_{a_i^p} - S^1_{a_i^p})\frac{a_i^{p3} - 2a_i^{p2} + a_i^{p1}}{S^2_{a_i^p} - S^0_{a_i^p}}\right.$$

$$\left. + \ldots + \left(\sum_{h=1}^{\lambda} \frac{a_i^{ph}}{\prod_{h\neq l+1,\ l=0}^{\lambda-1} S^{h-1}_{a_i^p} - S^l_{a_i^p}}\right)\right]$$

$$\mu\left[F_{a_i^p}(S_{a_i^p})\right] = \mu(a_i^{p1}) + (S_{a_i^p} - S^0_{a_i^p})(\mu(a_i^{p2}) - \mu(a_i^{p1})) + (S_{a_i^p} - S^0_{a_i^p})(S_{a_i^p} - S^1_{a_i^p})\frac{\mu(a_i^{p3}) - 2\mu(a_i^{p2}) + \mu(a_i^{p1})}{S^2_{a_i^p} - S^0_{a_i^p}}$$

$$+ \ldots + \left(\sum_{h=1}^{\lambda} \frac{\mu(a_i^{ph})}{\prod_{h\neq l+1,\ l=0}^{\lambda-1} S^{h-1}_{a_i^p} - S^l_{a_i^p}}\right)\Bigg] \tag{30}$$

$$\sigma^2\left[F_{a_i^p}(S_{a_i^p})\right] = \sigma^2\left[a_i^{p1} + (S_{a_i^p} - S^0_{a_i^p})(a_i^{p2} - a_i^{p1}) + (S_{a_i^p} - S^0_{a_i^p})(S_{a_i^p} - S^1_{a_i^p})\frac{a_i^{p3} - 2a_i^{p2} + a_i^{p1}}{S^2_{a_i^p} - S^0_{a_i^p}} + \ldots\right.$$

$$\left. + \left(\sum_{h=1}^{\lambda} \frac{a_i^{ph}}{\prod_{h\neq l+1,\ l=0}^{\lambda-1} S^{h-1}_{a_i^p} - S^l_{a_i^p}}\right)\right]$$

$$\sigma^2\left[F_{a_i^p}(S_{a_i^p})\right] = \sigma^2(a_i^{p1}) + (S_{a_i^p} - S^0_{a_i^p})^2(\sigma^2(a_i^{p2}) - \sigma^2(a_i^{p1})) + (S_{a_i^p} - S^0_{a_i^p})^2(S_{a_i^p} - S^1_{a_i^p})^2\frac{\sigma^2(a_i^{p3}) + 4\sigma^2(a_i^{p2}) + \sigma^2(a_i^{p1})}{S^2_{a_i^p} - S^0_{a_i^p}{}^2}$$

$$+ \ldots + \left(\sum_{h=1}^{\lambda} \frac{\sigma^2(a_i^{ph})}{\prod_{h\neq l+1,\ l=0}^{\lambda-1} S^{h-1}_{a_i^p} - S^l_{a_i^p}}\right)\Bigg] \tag{31}$$

Thus, the mean and variance of the $F_{a_i^p}(S_{a_i^p})$ are given in Eqs. (33) and (34). Similarly, Eqs. (33) and (34) can be used to get the mean and variance of $F_{b_j^p}(S_{b_j^p})$.

Here, Z-Score Percentile ND Table is used to find $\phi^{-1}(1-\beta_i^p)$.

In the similar manner, by following the same procedure for the demand constraints $P\left(\sum_{i=1}^m x_{ij}^p \leq F^t_{b_j^p}(S_{b_j^p})\right) \geq \gamma_i^p$, we can obtain the equivalent deterministic constraint is as follows.

$$\sum_{i=1}^{m} x_{ij}^p \leq \phi^{-1}(1-\gamma_j^p)\sqrt{\sigma^2\left(F_{b_j^p}(S_{b_j^p})\right)} + \mu\left(F_{b_j^p}(S_{b_j^p})\right) \tag{32}$$

Where $\mu\left(F_{b_j^p}(S_{b_j^p})\right)$, $\sigma^2\left(F_{b_j^p}(S_{b_j^p})\right)$ are the mean and variance of interpolating polynomial $F_{b_j^p}(S_{b_j^p})$ respectively follows normal distribution. γ_j^p is the probability for demand constraints at item p.

Theorem 4.2 If $F_{a_i^p}(S_{a_i^p})$, $i = 1, 2, ..., m$, $p = 1, 2$, are interpolating polynomial with the normal distribution then $P\left(\sum_{j=1}^{n} x_{ij}^p \geq F_{a_i^p}(S_{a_i^p})\right) \geq \beta_i^p$, is equivalent to $\sum_{j=1}^{n} x_{ij}^p \geq \phi^{-1}(\beta_i^p)\sqrt{\sigma^2\left(F_{a_i^p}(S_{a_i^p})\right)} + \mu\left(F_{a_i^p}(S_{a_i^p})\right)$, the two known parameters, $\mu\left(F_{a_i^p}(S_{a_i^p})\right)$, $\sigma^2\left(F_{a_i^p}(S_{a_i^p})\right)$ are the mean and variance of interpolating polynomial $F_{a_i^p}(S_{a_i^p})$ respectively follows normal distribution, β_i^p is the probability for supply constraints at item p.

Proof. It is assumed that, $i = 1, 2, \ldots, n$, $p = 1, 2$, interpolating polynomial follows normal distribution with mean and variance respectively. Let us consider stochastic supply constraints as

$$P\left(\sum_{j=1}^{n} x_{ij}^p \geq F_{a_i^p}(S_{a_i^p})\right) \geq \beta_i^p \tag{33}$$

$$P\left(F_{a_i^p}(S_{a_i^p})\right) \leq \sum_{j=1}^{n} x_{ij}^p\Bigg) \geq \beta_i^p$$

$$P\left[\frac{F_{a_i^p}(S_{a_i^p}) - \mu\left(F_{a_i^p}(S_{a_i^p})\right)}{\sqrt{\sigma^2\left(F_{a_i^p}(S_{a_i^p})\right)}} \leq \frac{\sum_{j=1}^{n} x_{ij}^p - \mu\left(F_{a_i^p}(S_{a_i^p})\right)}{\sqrt{\sigma^2\left(F_{a_i^p}(S_{a_i^p})\right)}}\right] \geq \beta_i^p$$

Hence the probabilistic constraints are represented as

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\sum_{j=1}^{n} x_{ij}^p - \mu\left(F_{a_i^p}(S_{a_i^p})\right)}{\sqrt{\sigma^2\left(F_{a_i^p}(S_{a_i^p})\right)}}} e^{\left(\frac{-\left(\frac{F_{a_i^p}(S_{a_i^p}) - \mu\left(F_{a_i^p}(S_{a_i^p})\right)}{\sqrt{\sigma^2\left(F_{a_i^p}(S_{a_i^p})\right)}}\right)^2}{2}\right)} dz \geq \beta_i^p$$

The above integral can be represented as:

$$\phi\left[\frac{\sum_{j=1}^{n} x_{ij}^p - \mu\left(F_{a_i^p}(S_{a_i^p})\right)}{\sqrt{\sigma^2\left(F_{a_i^p}(S_{a_i^p})\right)}}\right] \geq \beta_i^p \tag{34}$$

$$\left[\frac{\sum_{j=1}^{n} x_{ij}^p - \mu\left(F_{a_i^p}(S_{a_i^p})\right)}{\sqrt{\sigma^2\left(F_{a_i^p}(S_{a_i^p})\right)}}\right] \geq \phi^{-1}(\beta_i^p)$$

$$\sum_{j=1}^{n} x_{ij}^p \geq \phi^{-1}(\beta_i^p)\sqrt{\sigma^2\left(F_{a_i^p}(S_{a_i^p})\right)} + \mu\left(F_{a_i^p}(S_{a_i^p})\right) \tag{35}$$

In the similar manner, by following the same procedure for the demand constraints $P\left(\sum_{i=1}^{m} x_{ij}^{p} \geq F_{b_j^p}(S_{b_j^p})\right) \geq \gamma_i^p$ we can obtain the equivalent deterministic constraint is as follows.

$$\sum_{i=1}^{m} x_{ij}^{p} \geq \phi^{-1}(\gamma_j^p)\sqrt{\sigma^2\left(F_{b_j^p}(S_{b_j^p})\right)} + \mu\left(F_{b_j^p}(S_{b_j^p})\right) \tag{36}$$

Where $\mu\left(F_{b_j^p}(S_{b_j^p})\right)$, $\sigma^2\left(F_{b_j^p}(S_{b_j^p})\right)$ are the mean and variance of interpolating polynomial $F_{b_j^p}(S_{b_j^p})$ respectively follows normal distribution. γ_j^p is the probability for demand constraints at item p.

Remarks Let $P\left(\sum_{j=1}^{n} x_{ij}^{p} = F_{a_i^p}(S_{a_i^p})\right) \geq \beta_i^p$, $i = 1, 2, \ldots, m$ and $P\left(\sum_{i=1}^{m} x_{ij}^{p} = F_{b_j^p}(S_{b_j^p})\right) \geq \gamma_j^p$, $j = 1, 2, \ldots, n$ be the supply and demand of equality type constraints. Then chance equality to inequality type for the supply constraint as $P\left(\sum_{j=1}^{n} x_{ij}^{p} \leq F_{a_i^p}(S_{a_i^p})\right) \geq \beta_i^p$ and $P\left(\sum_{j=1}^{n} x_{ij}^{p} \geq F_{a_i^p}(S_{a_i^p})\right) \geq \beta_i^p$, for the demand constraint as $P\left(\sum_{i=1}^{m} x_{ij}^{p} \leq F_{b_j^p}(S_{b_j^p})\right) \geq \gamma_j^p$ and $P\left(\sum_{i=1}^{m} x_{ij}^{p} \geq F_{b_j^p}(S_{b_j^p})\right) \geq \gamma_j^p$. By choosing the probability value as 0.5 in the inequality constraint of supply and demand follows normal distribution, we obtain equal deterministic value for both supply and demand inequality types of constraints. So, the deterministic values of equality constraint for the supply as $\sum_{j=1}^{n} x_{ij}^{p} = \phi^{-1}(1-\beta_i^p)\sqrt{\sigma^2\left(F_{a_i^p}(S_{a_i^p})\right)} + \mu\left(F_{a_i^p}(S_{a_i^p})\right)$ or $\sum_{j=1}^{n} x_{ij}^{p} = \phi^{-1}(\beta_i^p)\sqrt{\sigma^2\left(F_{a_i^p}(S_{a_i^p})\right)} + \mu\left(F_{a_i^p}(S_{a_i^p})\right)$ and the demand as $\sum_{i=1}^{m} x_{ij}^{p} = \phi^{-1}(1-\gamma_j^p)\sqrt{\sigma^2\left(F_{b_j^p}(S_{b_j^p})\right)} + \mu\left(F_{b_j^p}(S_{b_j^p})\right)$ or $\sum_{i=1}^{m} x_{ij}^{p} = \phi^{-1}(\gamma_i^p)\sqrt{\sigma^2\left(F_{b_j^p}(S_{b_j^p})\right)} + \mu\left(F_{b_j^p}(S_{b_j^p})\right)$.

4.1 *Equivalent deterministic formulation of model (G2)*

In model (G2), the interpolating polynomial of supply $F_{a_i^p}(S_{a_i^p})$ and demand $F_{b_j^p}(S_{b_j^p})$ follows normal distribution. By using results of Theorem (4.1) and Theorem (4.2), we convert the stochastic supply and demand constraint with NDD interpolating polynomial into an equivalent deterministic constraint. Now, the model (G2) is constructed with equations (32), (35), (38) and (39) and is formulated as follows:

$$(R1)\ Minimize\left(Z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p=1}^{2} d_{ij}^{p} x_{ij}^{p}, \qquad Z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p=1}^{2} l_{ij}^{p} x_{ij}^{p}\right)$$

$$Minimize\left(Z_3 = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p=1}^{2} t_{ij}^{pa} x_{ij}^{p}}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p=1}^{2} t_{ij}^{ps} x_{ij}^{p}}, \qquad Z_4 = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p=1}^{2} c_{ij}^{pa} x_{ij}^{p}}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p=1}^{2} c_{ij}^{ps} x_{ij}^{p}}\right)$$

$$Maximize\left(Z_5 = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p=1}^{2} d_{ij}^{ps} x_{ij}^{p}}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p=1}^{2} c_{ij}^{pa} x_{ij}^{p}}\right)$$

$$Subject\ to: \sum_{j=1}^{n} x_{ij}^{p} \leq \phi^{-1}(1-\beta_i^p)\sigma^2\left(F_{a_i^p}(S_{a_i^p})\right) + \mu\left(F_{a_i^p}(S_{a_i^p})\right),\ i \in I_1 \tag{37}$$

$$\sum_{j=1}^{n} x_{ij}^{p} = \phi^{-1}(1-\beta_i^p)\sigma^2\left(F_{a_i^p}(S_{a_i^p})\right)+\mu\left(F_{a_i^p}(S_{a_i^p})\right),\ i \in I_2 \tag{38}$$

$$\sum_{j=1}^{n} x_{ij}^{p} \geq \phi^{-1}(\beta_i^p)\sigma^2\left(F_{a_i^p}(S_{a_i^p})\right)+\mu\left(F_{a_i^p}(S_{a_i^p})\right),\ i \in I_3 \tag{39}$$

$$\sum_{i=1}^{m} x_{ij}^{p} \geq \phi^{-1}(\gamma_j^p)\sigma^2\left(F_{b_j^p}(S_{b_j^p})\right)+\mu\left(F_{b_j^p}(S_{b_j^p})\right),\ j \in J_1 \tag{40}$$

$$\sum_{i=1}^{m} x_{ij}^{p} = \phi^{-1}(\gamma_j^p)\sigma^2\left(F_{b_j^p}(S_{b_j^p})\right)+\mu\left(F_{b_j^p}(S_{b_j^p})\right),\ j \in J_2 \tag{41}$$

$$\sum_{i=1}^{m} x_{ij}^{p} \leq \phi^{-1}(1-\gamma_j^p)\sigma^2\left(F_{b_j^p}(S_{b_j^p})\right)+\mu\left(F_{b_j^p}(S_{b_j^p})\right),\ j \in J_3 \tag{42}$$

$$0 \leq x_{ij}^{p} \leq r_{ij}^{p},\ i = \{I_1 \cup I_2 \cup I_3\} = 1,\ 2,\ \ldots,\ m,\ j = \{J_1 \cup J_2 \cup J_3\} = 1,\ 2,\ \ldots,\ n,\ p = 1,\ 2 \tag{43}$$

At present, a deterministic model (R1) has been attained with five objectives. Then, the reduced multi-objective model (R1) is transformed into single objective model by using the improved global weighted sum method and the model is solved using Lingo 18.0 software to find the optimal compromise solution. Now, the following section will discuss the solution approach for solving MOBICTP with fuzzy multi-choice stochastic mixed constraints following normal distribution.

5. Solution approach

This section presents the solution approach for solving MOBICTP with fermatean fuzzy multi-choice stochastic mixed constraints, which adheres to the normal distribution:

Step 1 Formulate the model (G2) from the model (G1).

(i) Transform the triangular fermatean fuzzy multi-choice stochastic parameters of normal distribution into an equivalent interval-valued fermatean fuzzy multi-choice stochastic constraints using the (α, β)-cut technique definition 2.2.

(ii) Using accuracy function definition 2.3, the step 1 (i) interval-valued fermatean fuzzy multi-choice stochastic constraints are reduced to an equivalent multi-choice stochastic constraint by choosing any alpha and beta value between 0 and 1.

Step 2 Transform the model (G2) into an equivalent deterministic model (R1) using the Eqs. (32), (35), (38) and (39).

Step 3 Convert the model (R1) into an equivalent single objective bi-item capacitated transportation problem by utilizing the proposed improved global weighted sum method as follows.

5.1 *Improved global weighted sum method*

The procedure for transforming the multi-objective optimization problem to the single objective optimization problem using the improved global weighted sum method is given as follows:

Step 1 Select a single objective at a time together with all the constraints and ignore other objectives. Solve the multi-objective optimization problem as a single objective optimization problem to find the optimal solutions for minimization objective Z_k^* (Z_1^*, Z_2^*, Z_3^*, ..., Z_l^*) and maximization objective $\bar{Z}_r$ ($\bar{Z}_1$, $\bar{Z}_2$, $\bar{Z}_3$, ..., $\bar{Z}_s$).

Step 2 Take the optimum solution obtained in Step a as the ideal solution for both minimization objective (Z_1^*, Z_2^*, Z_3^*, ..., Z_l^*) and maximization objective ($\bar{Z}_1$, $\bar{Z}_2$, $\bar{Z}_3$, ..., $\bar{Z}_s$).

Step 3 By using Step 2, construct the following auxiliary problem (D):

$$Minimize\ D = \left[\sum_{k=1}^{l} W_k \left(\frac{(Z_k(x) - Z_k^*)}{Z_k^*}\right)^p\right]^{\frac{1}{p}} + \left[\sum_{r=1}^{s} W_r \left(\frac{(\bar{Z}_r - Z_r(x))}{\bar{Z}_r}\right)^p\right]^{\frac{1}{p}}$$

Subject to constraints (28-34) and $D \geq 0$.

Where $1 \leq p \leq \infty$, the usual value of p is 2, W_k is the weight of the k^{th} objective function, and W_r is the weight of the r^{th} objective function should satisfy the conditions $\sum_{k=1}^{l} W_k + \sum_{r=1}^{s} W_r = 1$ and $W_k,\ W_r > 0$.

Many real-life decision-making problems have the combination of both maximization and minimization objectives. Based on the global weighted sum method [34], we have proposed the improved global weighted sum method, which is used to transform any multi-objective optimization problem (both maximization and minimization objectives) to a single objective optimization problem. This method will help the decision-maker to balance the situation because the weights and priority ranking are used for the objective functions.

Step 4 The reduced single objective bi-item capacitated transportation problem obtained from step 3 is solved by utilizing Lingo 18.0 software to find the optimal compromise solution.

The methodology for solving the MOBICTP fermatean fuzzy multi-choice stochastic mixed constraints following the normal distribution is shown in the Figure 1 below.

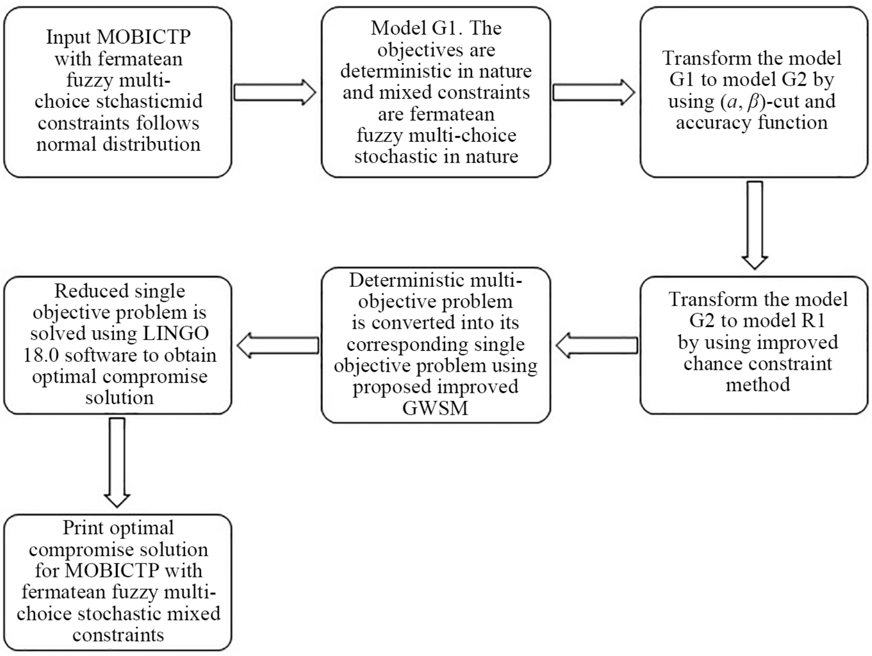

Figure 1. Flowchart for the solution approach

The consecutive numerical example will help to demonstrate the formulated model. In the numerical example, the values of item-1 in Tables 3-7 are taken from [9], and the values of item-2, supply and demand, are considered according to our preferences.

6. Numerical example

This case study delves into the transportation challenges faced by a startup dairy company in Tamil Nadu, India, that manufactures and markets two types of packed items, raw milk (Item 1) and flavored milk (Item 2), from its three manufacturing points situated at three different places, namely: Chennai (O_1), Kanchipuram (O_2), and Chengalpattu (O_3), of Tamil Nadu, by truck to the three retail stores situated at Ranipet (D_1), Tiruvannamalai (D_2), and Puducherry (D_3) of Tamil Nadu. The raw milk and flavored milk are perishable products. So, the dairy company faces many challenges during transport. The difficulties during transportation are presented as follows: (i) damage in packets, spoilage of the product; (ii) managing labor resources; (iii) time taken for transportation; (iv) cost of transportation; (v) discounts; and so on. By considering the above difficulties as objective functions, the company can have multifaceted benefits like enhancing productivity, reducing overhead expenses, maintaining product freshness, cost savings, and so on, with its fixed capacity limit for item 1 and item 2. The company's administrator plans on starting the transport of items in the next month. In order to begin his project, he must gather all the primary records that are essential. But the administrator is unable to get the required data because of unpredictable human and environmental factors, as shown below. The requirements for certain items change from one region to another. It is observed that the supply and demand may not be known previously due to different unpredictable factors such as weather, season, different social activities, etc., and again, due to fluctuations in the market, the actual value of the parameters cannot always be determined because of these environment's uncertainty and multiple choices; as a result, the supply and demand are regarded as triangular fermatean fuzzy multichoice random parameters. After analyzing the past date, it is finalized that the supply and the demand are following a triangular fermatean fuzzy multi-choice normal random variable with known mean, variance, and probabilities as fermatean triangular fuzzy numbers. As a result, for the supplier $\tilde{a}_i^{pk}$, the probability of having the required quantity of goods for item p is represented by $\tilde{\beta}_i^p$. Products' demand is also unpredictability in nature. It is caused by inaccurate demand forecasts, fluctuations in demand, or unexpected supply delays. Thus, the probability of an anticipated demand for item p in cities $\tilde{b}_j^{pk}$ is represented by $\tilde{\gamma}_j^p$. The damage cost, labor cost, transportation time, transportation cost, and discount from origin to destination for two items are given in Tables 3-7. The triangular fermatean fuzzy multi-choice stochastic constraint parameters, and the entrepreneurs may choose the probability depending on their perception, are given in Tables 8 and 9. They follow a normal distribution as $\left\{N((\mu(\tilde{a}_i^{pk}); \sigma^2(\tilde{a}_i^{pk})),\ p(\tilde{a}_i^k)),\ N((\mu(\tilde{b}_j^{pk});\ \sigma^2(\tilde{b}_j^{pk})),\ p(\tilde{b}_j^k))\right\}$, they are all expressed as triangular fermatean fuzzy numbers. It is important to consider how to maximize profit while minimizing expenditures.

Consequently, the entrepreneur aims to maximize its profit by minimizing damage cost, labor cost, transportation time, transportation cost, and maximizing discount benefits.

Table 3. Damage cost d_{ij}^p

item-1					item-2				
	D_1	D_2	D_3	supply		D_1	D_2	D_3	supply
O_1	8	9	11	$\leq \tilde{a}_1^1$	O_1	10	8	10	$\leq \tilde{a}_1^2$
O_2	13	14	17	$= \tilde{a}_2^1$	O_2	11	15	18	$= \tilde{a}_2^2$
O_3	12	13	18	$\geq \tilde{a}_3^1$	O_3	13	12	20	$\geq \tilde{a}_3^2$
demand	$\geq \tilde{b}_1^1$	$= \tilde{b}_2^1$	$\leq \tilde{b}_3^1$		demand	$\geq \tilde{b}_1^2$	$= \tilde{b}_2^2$	$\leq \tilde{b}_3^2$	

Table 4. Labor cost l_{ij}^{p}

item-1					item-2				
	D_1	D_2	D_3	supply		D_1	D_2	D_3	supply
O_1	14	15	13	$\leq \tilde{a}_1^1$	O_1	12	17	14	$\leq \tilde{a}_1^2$
O_2	19	18	15	$= \tilde{a}_2^1$	O_2	20	16	13	$= \tilde{a}_2^2$
O_3	18	17	15	$\geq \tilde{a}_3^1$	O_3	17	18	16	$\geq \tilde{a}_3^2$
demand	$\geq \tilde{b}_1^1$	$= \tilde{b}_2^1$	$\leq \tilde{b}_3^1$		demand	$\geq \tilde{b}_1^2$	$= \tilde{b}_2^2$	$\leq \tilde{b}_3^2$	

Table 5. Transportation time $\frac{t_{ij}^{pa}}{t_{ij}^{ps}}$

item-1					item-2				
	D_1	D_2	D_3	supply		D_1	D_2	D_3	supply
O_1	$\frac{30}{21}$	$\frac{35}{25}$	$\frac{35}{25}$	$\leq \tilde{a}_1^1$	O_1	$\frac{29}{20}$	$\frac{33}{26}$	$\frac{33}{26}$	$\leq \tilde{a}_1^2$
O_2	$\frac{41}{35}$	$\frac{41}{41}$	$\frac{21}{10}$	$= \tilde{a}_2^1$	O_2	$\frac{40}{36}$	$\frac{42}{42}$	$\frac{21}{11}$	$= \tilde{a}_2^2$
O_3	$\frac{41}{25}$	$\frac{21}{15}$	$\frac{51}{51}$	$\geq \tilde{a}_3^1$	O_3	$\frac{40}{26}$	$\frac{20}{16}$	$\frac{50}{50}$	$\geq \tilde{a}_3^2$
demand	$\geq \tilde{b}_1^1$	$= \tilde{b}_2^1$	$\leq \tilde{b}_3^1$		demand	$\geq \tilde{b}_1^2$	$= \tilde{b}_2^2$	$\leq \tilde{b}_3^2$	

Table 6. Transportation cost $\frac{c_{ij}^{pa}}{c_{ij}^{ps}}$

item-1					item-2				
	D_1	D_2	D_3	supply		D_1	D_2	D_3	supply
O_1	$\frac{27}{25}$	$\frac{27}{22}$	$\frac{27}{25}$	$\leq \tilde{a}_1^1$	O_1	$\frac{28}{26}$	$\frac{28}{24}$	$\frac{28}{26}$	$\leq \tilde{a}_1^2$
O_2	$\frac{30}{25}$	$\frac{28}{25}$	$\frac{30}{26}$	$= \tilde{a}_2^1$	O_2	$\frac{32}{26}$	$\frac{30}{26}$	$\frac{32}{25}$	$= \tilde{a}_2^2$
O_3	$\frac{28}{28}$	$\frac{32}{26}$	$\frac{33}{30}$	$\geq \tilde{a}_3^1$	O_3	$\frac{40}{30}$	$\frac{20}{25}$	$\frac{50}{32}$	$\geq \tilde{a}_3^2$
demand	$\geq \tilde{b}_1^1$	$= \tilde{b}_2^1$	$\leq \tilde{b}_3^1$		demand	$\geq \tilde{b}_1^2$	$= \tilde{b}_2^2$	$\leq \tilde{b}_3^2$	

Table 7. Discount cost $\frac{d_{ij}^{ps}}{c_{ij}^{pa}}$

item-1					item-2				
	D_1	D_2	D_3	supply		D_1	D_2	D_3	supply
O_1	$\frac{3}{27}$	$\frac{4}{27}$	$\frac{3}{27}$	$\leq \tilde{a}_1^1$	O_1	$\frac{4}{28}$	$\frac{5}{28}$	$\frac{4}{28}$	$\leq \tilde{a}_1^2$
O_2	$\frac{4}{30}$	$\frac{3}{28}$	$\frac{4}{26}$	$= \tilde{a}_2^1$	O_2	$\frac{5}{32}$	$\frac{4}{30}$	$\frac{5}{32}$	$= \tilde{a}_2^2$
O_3	$\frac{3}{28}$	$\frac{4}{32}$	$\frac{5}{33}$	$\geq \tilde{a}_3^1$	O_3	$\frac{4}{40}$	$\frac{5}{20}$	$\frac{8}{50}$	$\geq \tilde{a}_3^2$
demand	$\geq \tilde{b}_1^1$	$= \tilde{b}_2^1$	$\leq \tilde{b}_3^1$		demand	$\geq \tilde{b}_1^2$	$= \tilde{b}_2^2$	$\leq \tilde{b}_3^2$	

The parameters of stochastic constraints that follow normal distribution are triangular fermatean fuzzy multi-choice mean, variance, and triangular fermatean fuzzy stochastic for supply and demand, respectively, given in Tables 8 and 9.

Table 8. Supply: Mean, variance, probabilities

supply	Item-1			Item-2		
	$\tilde{a}_1^1$	$\tilde{a}_2^1$	$\tilde{a}_3^1$	$\tilde{a}_1^2$	$\tilde{a}_2^2$	$\tilde{a}_3^2$
$\mu(\tilde{a}_i^1)$	(4, 6, 12); (3, 6, 14)	(4, 7, 12); (3, 7, 14)	(6, 10, 14); (3, 10, 15)	(4, 6, 12); (3, 6, 14)	(4, 6, 12); (3, 6, 14)	(4, 6, 12); (3, 6, 14)
$\mu(\tilde{a}_i^2)$	(1, 4, 7); (0, 4, 8)	(1, 2, 5); (0, 2, 12)	(4, 5, 10); (3, 5, 12)	(1, 4, 7); (0, 4, 8)	(1, 2, 5); (0, 2, 12)	(4, 4, 10); (3, 4, 12)
$\mu(\tilde{a}_i^3)$	(4, 8, 12); (3, 8, 18)	(4, 8, 12); (3, 8, 15)	(6, 12, 14); (4, 12, 16)	(4, 5, 10); (3, 5, 12)	(4, 7, 8); (3, 7, 15)	(6, 11, 14); (3, 11, 15)
$\sigma^2(\tilde{a}_i^1)$	(1, 3, 5); (1, 3, 7)	(1, 2, 5); (1, 2, 11)	(1, 1, 3); (0, 1, 4)	(1, 3, 5); (1, 3, 7)	(1, 2, 3); (0, 2, 11)	(1, 1, 3); (0, 1, 4)
$\sigma^2(\tilde{a}_i^2)$	(1, 4, 5); (0, 4, 8)	(1, 2, 5); (0, 2, 12)	(1, 2, 3); (0, 2, 11)	(1, 4, 5); (0, 4, 8)	(1, 2, 5); (0, 2, 11)	(1, 2, 3); (0, 2, 11)
$\sigma^2(\tilde{a}_i^3)$	(1, 4, 7); (0, 4, 7)	(1, 2, 3); (0, 2, 11)	(1, 2, 5); (0, 2, 12)	(1, 4, 7); (0, 4, 8)	(1, 3, 5); (1, 3, 7)	(1, 2, 5); (0, 2, 11)
$P(\tilde{a}_i)$	(0.3, 0.4, 0.5); (0, 0.4, 0.6)	(0.2, 0.25, 0.3); (0.1, 0.25, 0.4)	(0.3, 0.4, 0.5); (0, 0.4, 0.6)	(0.2, 0.3, 0.4); (0.1, 0.3, 0.5)	(0.2, 0.25, 0.3); (0.1, 0.25, 0.4)	(0.3, 0.5, 0.6); (0.2, 0.5, 0.8)

Table 9. Demand: Supply: Mean, variance, probabilities

supply	Item-1			Item-2		
	$\tilde{b}_1^1$	$\tilde{b}_2^1$	$\tilde{b}_3^1$	$\tilde{b}_1^2$	$\tilde{b}_2^2$	$\tilde{b}_3^2$
$\mu(\tilde{b}_j^1)$	(1, 6, 7); (0, 6, 9)	(4, 4, 10); (3, 4, 12)	(5, 10, 13); (3, 10, 14)	(1, 4, 7); (0, 4, 8)	(4, 7, 8); (3, 7, 15)	(6, 10, 14); (3, 10, 15)
$\mu(\tilde{b}_j^2)$	(1, 2, 3); (0, 2, 11)	(1, 1, 3); (0, 1, 4)	(4, 4, 10); (3, 4, 10)	(1, 1, 3); (0, 1, 4)	(1, 4, 5); (0, 4, 8)	(4, 5, 10); (3, 5, 10)
$\mu(\tilde{b}_j^3)$	((4, 5, 10); (3, 5, 12)	(4, 7, 8); (3, 7, 15)	(6, 11, 14); (3, 11, 15)	(1, 6, 7); (0, 6, 9)	(4, 7, 12); (3, 7, 14)	(6, 12, 14); (4, 12, 16)
$\sigma^2(\tilde{b}_j^1)$	(1, 3, 5); (1, 3, 7)	(1, 2, 3); (0, 2, 11)	(1, 1, 3); (0, 1, 4)	(1, 3, 5); (1, 3, 7)	(1, 2, 3); (0, 2, 11)	(1, 1, 3); (0, 1, 4)
$\sigma^2(\tilde{b}_j^2)$	(1, 4, 5); (0, 4, 8)	(1, 2, 5); (0, 2, 11)	(1, 2, 3); (0, 2, 11)	(1, 4, 5); (0, 4, 8)	(1, 2, 5); (0, 2, 11)	(1, 2, 3); (0, 2, 11)
$\sigma^2(\tilde{b}_j^3)$	(1, 4, 7); (0, 4, 8)	(1, 3, 5); (1, 3, 7)	(1, 2, 5); (0, 2, 11)	(1, 4, 7); (0, 4, 8)	(1, 3, 5); (1, 3, 7)	(1, 2, 5); (0, 2, 11)
$P(\tilde{b}_j)$	(0.3, 0.45, 0.5); (0, 0.45, 0.6)	(0.2, 0.25, 0.3); (0.1, 0.25, 0.4)	(0.2, 0.39, 0.4); (0.1, 0.39, 0.6)	(0.2, 0.35, 0.4); (0.1, 0.35, 0.5)	(0.2, 0.25, 0.3); (0.1, 0.25, 0.4)	(0.2, 0.5, 0.6); (0.1, 0.5, 0.8)

The fixed capacity limit on each route for item 1 and item 2 are given below,

$$0 \leq x_{11}^p \leq 6, \quad 0 \leq x_{12}^p \leq 7, \quad 0 \leq x_{13}^p \leq 13$$

$$0 \leq x_{21}^p \leq 6, \quad 0 \leq x_{22}^p \leq 2, \quad 0 \leq x_{23}^p \leq 13$$

$$0 \leq x_{31}^p \leq 4, \quad 0 \leq x_{32}^p \leq 7, \quad 0 \leq x_{33}^p \leq 14$$

By using step 1(i), formulate the $(\alpha,\ \beta)$-cut representation for model (G2) by Definition 2.2. We showed the detailed calculation of $(\alpha,\ \beta)$-cut representation for supply $\tilde{a}_1^1$, which is shown in Table 10, and in the same manner, we can calculate $(\alpha,\ \beta)$-cut for all other supply and demand.

Table 10. $(\alpha,\ \beta)$-cut for $\tilde{a}_1^1$

$(\alpha,\ \beta)$-cut for $\tilde{a}_1^1 = \{N((\mu(\tilde{a}_1^{11},\ \tilde{a}_1^{12},\ \tilde{a}_1^{13});\ \sigma^2(\tilde{a}_1^{11},\ \tilde{a}_1^{12},\ \tilde{a}_1^{13}),\ p(\tilde{a}_1^1)))\}$	
$\mu(\tilde{a}_1^{11}) = [4+2\alpha,\ 12-6\alpha],\ [6-3\beta,\ 6+8\beta]$	$\sigma^2(\tilde{a}_1^{11}) = [1+2\alpha,\ 5-2\alpha],\ [3-2\beta,\ 3+4\beta]$
$\mu(\tilde{a}_1^{12}) = [1+3\alpha,\ 7-3\alpha],\ [4-7\beta,\ 4+4\beta]$	$\sigma^2(\tilde{a}_1^{12}) = [1+3\alpha,\ 5-3\alpha],\ [4-4\beta,\ 4+4\beta]$
$\mu(\tilde{a}_1^{13}) = [8+4\alpha,\ 12-4\alpha],\ [8-5\beta,\ 8+10\beta]$	$\sigma^2(\tilde{a}_1^{13}) = [1+3\alpha,\ 7-3\alpha],\ [4-4\beta,\ 4+4\beta]$
$P(\tilde{a}_1) = [0.3+0.1\alpha,\ 0.5-0.1\alpha], [0.4-0.4\beta,\ 0.4+0.2\beta]$	

By step 1 (ii), choose any alpha and beta value between 0 and 1. Using Definition 2.3 with $\alpha = 0$ and $\beta = 0$, we have reduced the alpha and beta cut of $\tilde{a}_1^1$ value into deterministic value as $\tilde{a}_1^1 = \left\{N((\mu(14,\ 8,\ 16);\ \sigma^2(6,\ 7,\ 8),\ p(0.80)))\right\}$. In the same manner, we can compute for other supply and demand values as shown in Table 11.

Table 11. Supply and demand values using $(\alpha,\ \beta)$-cut

Item-1	Item-2
$\tilde{a}_1^1 = N(\mu(14,\ 8,\ 16),\ \sigma^2(6,\ 7,\ 8));\ p(\tilde{a}_1^1) = 0.80$	$\tilde{a}_1^2 = N(\mu(12,\ 8,\ 14),\ \sigma^2(4,\ 5,\ 6));\ p(\tilde{a}_1^1) = 0.85$
$\tilde{a}_2^1 = N(\mu(15,\ 5,\ 17),\ \sigma^2(4,\ 5,\ 6));\ p(\tilde{a}_2^1) = 0.50$	$\tilde{a}_2^2 = N(\mu(14,\ 5,\ 16),\ \sigma^2(6,\ 7,\ 8));\ p(\tilde{a}_2^1) = 0.50$
$\tilde{a}_3^1 = N(\mu(20,\ 12,\ 22),\ \sigma^2(3,\ 4,\ 5));\ p(\tilde{a}_3^1) = 0.85$	$\tilde{a}_3^2 = N(\mu(19,\ 11,\ 21),\ \sigma^2(5,\ 6,\ 7));\ p(\tilde{a}_3^1) = 0.69$
$\tilde{b}_1^1 = N(\mu(10,\ 4,\ 12),\ \sigma^2(6,\ 7,\ 8));\ p(\tilde{b}_1^1) = 0.60$	$\tilde{b}_1^2 = N(\mu(8,\ 3,\ 10),\ \sigma^2(7,\ 8,\ 9));\ p(\tilde{b}_1^2) = 0.65$
$\tilde{b}_2^1 = N(\mu(11,\ 3,\ 13),\ \sigma^2(4,\ 5,\ 6));\ p(\tilde{b}_2^1) = 0.50$	$\tilde{b}_2^2 = N(\mu(13,\ 7,\ 15),\ \sigma^2(6,\ 7,\ 8));\ p(\tilde{b}_2^2) = 0.50$
$\tilde{b}_3^1 = N(\mu(19,\ 11,\ 21),\ \sigma^2(3,\ 4,\ 5));\ p(\tilde{b}_3^1) = 0.95$	$\tilde{b}_3^2 = N(\mu(20,\ 12,\ 22),\ \sigma^2(5,\ 6,\ 7));\ p(\tilde{b}_3^2) = 0.90$

By using step 2, we have reduced the constraints in problem (G2) into constraints in problem (R1), as shown in Table 12.

Table 12. Deterministic constraints for Item-1 and Item-2

Item-1	Item-2
$\tilde{a}_1^1 = \phi^{-1}(1-0.8) * \sqrt{\sigma^2(6,\ 7,\ 8)} + \mu(14,\ 8,\ 16)$	$\tilde{a}_1^2 = \phi^{-1}(1-0.85) * \sqrt{\sigma^2(4,\ 5,\ 6)} + \mu(12,\ 8,\ 14)$
$\tilde{a}_2^1 = \phi^{-1}(0.5) * \sqrt{\sigma^2(4,\ 5,\ 6)} + \mu(15,\ 5,\ 17)$	$\tilde{a}_2^2 = \phi^{-1}(0.5) * \sqrt{\sigma^2(6,\ 7,\ 8)} + \mu(14,\ 5,\ 16)$
$\tilde{a}_3^1 = \phi^{-1}(0.65) * \sqrt{\sigma^2(3,\ 4,\ 5)} + \mu(20,\ 12,\ 22)$	$\tilde{a}_3^2 = \phi^{-1}(0.69) * \sqrt{\sigma^2(5,\ 6,\ 7)} + \mu(19,\ 11,\ 21)$
$\tilde{b}_1^1 = \phi^{-1}(0.60) * \sqrt{\sigma^2(6,\ 7,\ 8)} + \mu(10,\ 4,\ 12)$	$\tilde{b}_1^2 = \phi^{-1}(0.65) * \sqrt{\sigma^2(7,\ 8,\ 9)} + \mu(8,\ 3,\ 10)$
$\tilde{b}_2^1 = \phi^{-1}(0.5) * \sqrt{\sigma^2(4,\ 5,\ 6)} + \mu(11,\ 3,\ 13)$	$\tilde{b}_2^2 = \phi^{-1}(0.5) * \sqrt{\sigma^2(6,\ 7,\ 8)} + \mu(13,\ 7,\ 15)$
$\tilde{b}_3^1 = \phi^{-1}(1-0.95) * \sqrt{\sigma^2(3,\ 4,\ 5)} + \mu(19,\ 11,\ 21)$	$\tilde{b}_3^2 = \phi^{-1}(1-0.90) * \sqrt{\sigma^2(5,\ 6,\ 7)} + \mu(20,\ 12,\ 22)$

To deal with the multi-choice stochastic constraints, we use the improved chance constraint method to obtain the equivalent deterministic constraints, which are shown in Table 13.

Table 13. Deterministic constraints using improved chance constraint method for Item-1 and Item-2

Item-1	Item-2
$\tilde{a}_1^1=\phi^{-1}(1-0.8)*\sqrt{6+13s_1^2+((8+4*7+6)/4)s_1^2(s_1^2-1)}$	$\tilde{a}_1^2=\phi^{-1}(1-0.85)*\sqrt{4+9u_1^2+((6+4*5+4)/4)u_1^2(u_1^2-1)}$
$+14-6s_1+((16-2*8+14)/4)s_1(s_1-1)$	$+12-4u_1+((14-2*8+12)/2)u_1(u_1-1)$
$\tilde{a}_2^1=\phi^{-1}(0.5)*\sqrt{4+9s_2^2+((6+4*5+4)/4)s_2^2(s_2^2-1)}$	$\tilde{a}_2^2=\phi^{-1}(0.5)*\sqrt{6+13u_2^2+((8+4*7+6)/4)u_2^2(u_2^2-1)}$
$+15-10s_2+((17-2*5+15)/2)s_2(s_2-1)$	$+14-9u_2+((16-2*5+14)/2)u_2(u_2-1)$
$\tilde{a}_3^1=\phi^{-1}(0.65)*\sqrt{3+7s_3^2+((5+4*4+3)/4)s_3^2(s_3^2-1)}$	$\tilde{a}_3^2=\phi^{-1}(0.69)*\sqrt{5+11u_3^2+((7+4*6+5)/4)u_3^2(u_3^2-1)}$
$+20-8s_3+((22-2*12+20)/2)s_3(s_3-1)$	$+19-8s_1+((21-2*12+19)/2)u_3(u_3-1)$
$\tilde{b}_1^1=\phi^{-1}(0.60)*\sqrt{6+13t_1^2+((8+4*7+6)/4)t_1^2(t_1^2-1)}$	$\tilde{b}_1^2=\phi^{-1}(0.65)*\sqrt{7+15v_1^2+((9+4*8+7)/4)v_1^2(v_1^2-1)}$
$+10-6t_1+((12-2*8+10)/2)t_1(t_1-1)$	$+8-5s_1+((10-2*3+8)/2)v_1(v_1-1)$
$\tilde{b}_2^1=\phi^{-1}(0.5)*\sqrt{4+9t_2^2+((6+4*5+4)/4)t_2^2(t_2^2-1)}$	$\tilde{b}_2^2=\phi^{-1}(0.5)*\sqrt{6+13v_2^2+((8+4*7+6)/4)v_2^2(v_2^2-1)}$
$+11-8t_2+((13-2*3+11)/2)t_2(t_2-1)$	$+13-6v_2+((15-2*7+13)/2)v_2(v_2-1)$
$\tilde{b}_3^1=\phi^{-1}(1-0.95)*\sqrt{3+7t_3^2+((5+4*4+3)/4)t_3^2(t_3^2-1)}$	$\tilde{b}_3^2=\phi^{-1}(1-0.90)*\sqrt{5+11v_3^2+((7+4*6+5)/4)v_3^2(v_3^2-1)}$
$+19-8t_3+((21-2*11+19)/2)t_3(t_3-1)$	$+20-8v_3+((22-2*12+20)/2)v_3(v_3-1)$

where $0\le x_{11}^p\le 6,\quad 0\le x_{12}^p\le 7,\quad 0\le x_{13}^p\le 13,\quad 0\le x_{21}^p\le 6,\quad 0\le x_{22}^p\le 2,\quad 0\le x_{23}^p\le 13,\quad 0\le x_{31}^p\le 4,\quad 0\le x_{32}^p\le 7,$ $0\le x_{33}^p\le 14$, $i=1,\ 2,\ 3,\ s_i,\ t_i,\ u_i,\ v_i\in z^+$.

Now, the deterministic MOBICTP with mixed constraint is obtained. The reduced problem is not possible to solve directly. Although there are several methods in the literature that can be used to reduce a multi-objective optimization problem to a single objective problem. In this paper, we used the improved global weighted sum method based on [34] to convert a multi-objective problem to a single objective problem because it is simple for implementation. By using our improved Global Weighted Sum Method, the decision-maker can adjust the weights according to their priorities, and it is applicable to various ranges of fields. Thus, the improved Global Weighted Sum Method is an effective tool for simplifying the complicated decision-making problems that involve the combination of minimum and maximum objectives.

To get started with the improved global weighted sum method, in MOBICTP one objective should be taken at a time; according to the limitations, other objectives should be ignored. Use Lingo 18.0 software for optimal solutions to each objective. The optimum solutions for the objectives are $Z_1^*=546.9940$, $Z_2^*=656.6437$, $Z_3^*=1.0707$, $Z_4^*=1.0993$ and $Z_5^*=0.1754$. The optimal solutions obtained above are considered the ideal solution. Construct the mathematical model (R1) with equal weights of 0.2 assigned to each objective in the following manner:

$$(R2)Minimize\ D=\left[0.2\left(\frac{(Z_1(x)-546.9940)}{546.9940}\right)^2+0.2\left(\frac{(Z_2(x)-656.6437)}{656.6437}\right)^2+0.2\left(\frac{(Z_3(x)-1.0707)}{1.0707}\right)^2\right.$$

$$\left.+0.2\left(\frac{(Z_4(x)-1.0993)}{1.0993}\right)^2+\left[0.2\left(\frac{(0.1754-Z_5(x))}{0.1754}\right)^2\right]\right]^{\frac{1}{2}}$$

and subject to (28-34).

Now, by step 4, the Lingo 18.0 software is employed to solve the model (R2). The obtained compromise solution is $Z_1=576.1964$, $Z_2=671.0344$, $Z_3=1.1085$, $Z_4=1.6720$, $Z_5=0.1571$, quantities to be shipped for item-1 are $x_{11}^1=0$, $x_{12}^1=0$, $x_{13}^1=0$, $x_{21}^1=5$, $x_{22}^1=0$, $x_{23}^1=0$, $x_{31}^1=1.4063$, $x_{32}^1=3$, $x_{33}^1=9.5186$, quantities to be shipped for item-2 are $x_{11}^2=0$, $x_{12}^2=0$, $x_{13}^2=0$, $x_{21}^2=3.5826$, $x_{22}^2=0.3964$, $x_{23}^2=1.0209$, $x_{31}^2=2.2383$, $x_{32}^2=6.9182$, $x_{33}^2=5.8106$. As a result, the obtained compromise solution is quite beneficial for the decision-makers to select the most effective way to transport multiple items in the considered complex environment.

6.1 *Results and discussions with comparative analysis*

Nowadays, the transportation of multiple items from origin to destination plays an important role in industrialization. This study presents the construction of a MOBICTP model that incorporates triangular fermatean fuzzy multi-choice stochastic mixed constraints involving normal distributions. In certain circumstances, such as insufficient information regarding transportation or fluctuations in market value, it is appropriate to consider all constraint parameters in the MOBICTP as uncertain. A practical illustration is presented for model R1. In addition, our improved GWSM reduces R1 to a single objective BICTP. The reduced problem is then solved using the Lingo 18.0 software to get the optimal compromise solution, with $Z_1 = 576.1964$, $Z_2 = 671.0344$, $Z_3 = 1.1085$, $Z_4 = 1.6720$, $Z_5 = 0.1571$. To evaluate the effectiveness of the improved GWSM, we have done a comparative study with the existing FGP method [9] and Global Criteria Method [43] for solving the R1 model. The outcomes of this comparison were presented as follows in Table 14.

Table 14. Comparative analysis

Methods	Z_1	Z_2	Z_3	Z_4	Z_5
[9]	828.3192	925.7508	1.1183	1.0922	0.1505
[43]	576.1964	671.0344	1.1085	1.1672	0.1571
Proposed improved GWSM	576.1964	671.0344	1.1085	1.1672	0.1571

In order to understand in a better way, the optimal compromise solution of the problem obtained by using the proposed method is compared with existing methods, and it is shown as a graphical representation in Figure 2.

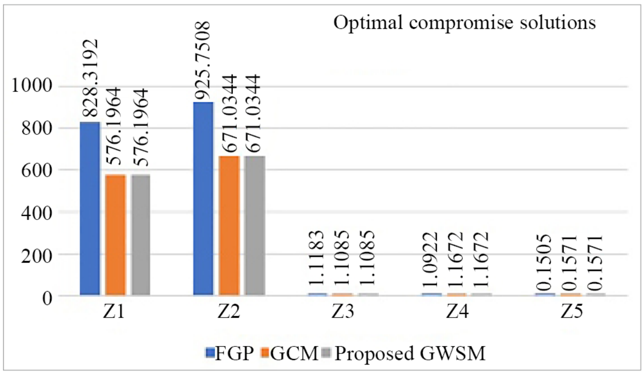

Figure 2. Comparison between the proposed method and existing methods

From Table 14 and Figure 2, it is evident that the improved GWSM gave the same results when compared with the Global Criteria Method [43] and better results for Z_1, Z_2, Z_3 and Z_5 , and when compared with, the FGP method [9]. It can be seen in terms of resultant outcomes. The identification of the most appropriate strategy is mainly based on the decision-maker.

6.2 *Sensitivity analysis*

In optimization problems, sensitivity analysis is a necessary and important phase to explain and understand what happens when the values of the objective functions are changed. Based on [38], the sensitivity analysis involves changing a specified parameter by a specific percentage while keeping other factors constant. This method is repeated until the core variables are stable. We proceed with the sensitivity analysis for the optimal compromise solution obtained using the improved GWSM. Table 15 shows the ranges of all the parameters in the objective functions, as well as how they affect the objective values. In sensitivity analysis, all the parameters in the objective functions are altered by a certain percentage (increase/decrease), while supply and demand parameters are kept constant at their original levels. This procedure is repeated until the fundamental variables remain steady, although their values may have altered.

Table 15. Change of all parameters in objective function

$d^{p}_{ij}, l^{p}_{ij}, t^{pa}_{ij}, t^{ps}_{ij}, c^{pa}_{ij}, c^{ps}_{ij}, d^{ps}_{ij}$	-5%	-3%	-1%	0%	1%	3%	5%
Z_1	581.551	578.4746	580.6618	576.1964	580.6367	592.1344	603.6322
Z_2	660.5007	663.5745	670.611	671.0344	676.5056	689.9018	703.29
Z_3	1.1	1.1002	1.104	1.1085	1.1087	1.1087	1.1087
Z_4	1.1565	1.167	1.1679	1.1672	1.1677	1.1677	1.1677
Z_5	0.1513	0.1565	0.15658	0.1571	0.1569	0.1569	0.1569

For our better understanding, the comparison of sensitivity analysis for change of rate of all parameters in objective function is represented graphically in Figure 3.

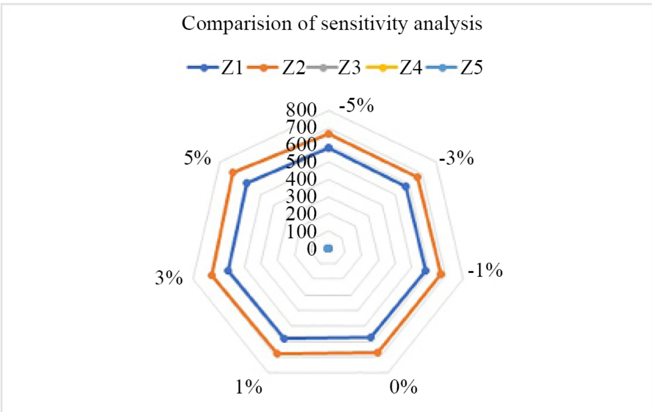

Figure 3. Comparison of sensitivity analysis for change of rate of all parameters in objective function

Table 15 and Figure 3 show the effect of all parameters of objective functions. From analyzing Table 15, it is found that if the range of all parameters increases, then the values of transportation time, transportation cost, and discount cost are changed in one time, but damage cost and labor cost are increased for all increasing parameters. When all objective parameters decrease, then labor cost, transportation time, transportation cost, and discount cost decrease, but the damage cost increases. DM can identify the optimal parametric values to improve the facility in all aspects by examining the impact of a variation of parameters depending on economic, social, and environmental sustainability.

7. Conclusions

This article presents the formulation of the MOBICTP model with multiple objectives and multiple items that incorporate both linear and fractional in objective functions with mixed constraints that include fermatean fuzzy multi-choice stochastic constraint parameters. This model is mainly designed to capture real-world problems, including ambiguity, multiple choices, and randomness. To reduce the complexity of the combined uncertainties, we use the (α, β)-cut technique to handle fermatean fuzzy as in the model (G2), and the improved chance constrained technique is used to handle multi-choice stochastic as in the model (R1). The improved GWSM is utilized to reduce the deterministic model (R1) to model (R2). Because an improved GWSM approach offers simplicity, flexibility in weight assignment, compatibility, and unique capability to address practical challenges in several domains such as TPs, agriculture, economics, and industry. The reduced model is solved by Lingo 18.0 software to get the optimal compromise solution, and comparative analysis is done with the Global Criteria Method and the FGP Method. The obtained optimal compromise solution using the proposed method is similar to the Global Criteria Method and better than the FGP method. A sensitivity analysis was conducted on the preferred strategy to determine an acceptable range of variables and assess their impact on objective values. In reality, most of the shipping companies transport multiple products together to improve their profit. Our developed model will be very helpful for the shipping companies to ship their multiple products while optimizing their objectives. This study aims to serve as a complete guide for researchers and industrial experts who are involved with handling multiple commodities between different locations and to handle complicated uncertain data in supply and demand of transportation problems.

However, our study has some limitations. Specifically, various routes, alternative modes of transportation, and fixed charges are not included in this study. While we are not yet considering the process of deterioration, it is important to remember that the transportation of perishable goods frequently meets this common challenge. We failed to determine the deterioration rate in this case, but certain perishable things are delivered, and we didn't use any preservation method that may help to avoid deterioration. To overcome these limitations, in our further research, we wish to investigate MOBICTP situations by considering the objective functions with uncertainties with preservation technology and different modes of transportation. Also, the investigation of bi-level solid optimization models including fuzzy multi-choice stochastic parameters is now a significant area of interest; real-world case studies on MOBICTP with mixed constraints that include fermatean fuzzy multi-choice random parameters are an essential area of focus for future studies.

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Conflict of interest

The authors declare no competing financial interest.

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