Contemporary Mathematics
http://ojs.wiserpub.com/index.php/CM/

UNIVERSAL WISER PUBLISHER

Research Article

Novel Temperature-Based Topological Indices for Certain Convex Polytopes

Sakander Hayat[1*], Muhammad Yasir Hayat Malik[2], Saima Fazal[2]

[1] Mathematical Science, Faculty of Science, Universiti Brunei Darussalam, Jln Tungku Link, Gadong BE1410, Brunei Darussalam
[2] Department of Mathematics, Division of Science and Technology, University of Education, Lahore, Pakistan
E-mails: sakander.hayat@ubd.edu.bn

Received: 30 July 2024; **Revised:** 1 October 2024; **Accepted:** 15 October 2024

Abstract: A topological index is a number that assists in understanding various physical characteristics, chemical reactivities, and boiling activities of a chemical compound by characterizing the whole molecular graph structure. These indices are essential for quantifying different chemical properties of chemical compounds in chemical graph theory. The choice of convex polytopes in this work is an important feature due to its structural adaptability, easy accessibility and astonishing capacity to identify its numerical values. In this paper, we present exact analytical expressions for the general first temperature index, the general second temperature index, the first hyper-temperature index, the second hyper-temperature index, the sum-connectivity temperature index, the product-connectivity temperature index, the reciprocal product-connectivity temperature index, the arithmetic-geometric temperature index and the F-temperature index of convex polytopes.

Keywords: geometrical graph, edge partitioning, convex polytope, temperature index

MSC: 05C92, 05C09, 05C10

1. Introduction

A molecule or chemical compound can be represented graphically by a chemical graph, also known as a molecular graph. Chemical graphs, which depict atoms as vertices and chemical bonds as edges, are used in chemistry to describe the structure of molecules. This graph-based representation simplifies the understanding and analysis of molecular structures and their properties. A numerical quantity determined mathematically from the network structure is called a graphical index. These indices are employed in chemical graph theory to measure a substance's chemical characteristics. Numerical quantities called topological indices can be employed to characterize the attributes of a molecular graph. In order to predict significant physicochemical aspects of chemical compounds, topological indices, which are graph invariants that provide information on the structure of graphs, have shown to be highly helpful in quantitative structure-activity relationships (QSAR) and quantitative structure-property relationships (QSPR).

In 2000, Basak [1] assessed topological indices, including their nature, mutual relatedness, and applicability, whereby they identified 90 topological indices from 3,692 different chemicals by principle component analysis (PCA). They used data from 19,000 number of chemical substances. Asadpour [2] computed some topological indices of nanostructures of

DOI: https://doi.org/10.37256/cm.5420245404

bridge graph in 2012. For the nanostructures of the bridge graph, they calculated the Randić, Zagreb, ABC, and geometric arithmetic indices. In 2016, Kulli [3] investigated the first and second K Banhatti indices, as well as the K hyper-Banhatti indices, of V-phenylenic nanotubes and V-phenylenic nanotorus. Kulli derived results on some multiplicative temperature indices of $HC_5C_7[p, q]$ nanotubes, with inverse sum temperature index of some more nanotubes, also computed the same results for certain networks that include oxide networks and honeycomb networks, see [4–6]. For H-naphtalenic nanotubes, Kulli [7] presented the (a, b)-temperature index in 2019. In this work, the author presented the temperature indices for specific values of a and b for H-naphtalenic nanotubes. Furthermore, Kulli [8, 9] examined the multiplicative (a, b)-temperature indices of certain nanostructures. He introduced the first and second (a, b)-KA and multiplicative (a, b)-KA temperature indices of a chemical graph and computed results for tetrameric 1,3-adamantane and also established multiplicative (a, b)-KA temperature indices. In 2022, Jahanbani et al. [10] described in detail the indices of several OTIS networks' molecular architectures. Zhang et al. [11] in 2022 have determined the analysis of temperature-based topological indices of various COVID-19 drug structures. In this work, they were computed using the analytically closed formulas of particular coronavirus molecular structures, such as ribavirin, sofosbuvir, and oseltamivir, utilizing temperature-based topological indices. Temperature-Sombor and temperature-Nirmala indices were determined by Kulli [12] in 2022 and he also established some properties of these indices.

In order to calculate Kulli temperature indices, Zhang et al. [13] focused on joining SiO_4 in silicate and silicate chain networks in 2022. The physicochemical characteristics of COVID-19 medications are highly correlated with the results of a QSPR analysis of these indices. Kulli [14] in 2023 created some unique temperature indices of oxide and honeycomb networks. The temperature inverse degree and some other variants of temperature indices of an oxide and honeycomb graphs were reported in that study. In 2021, Hayat et al. [15] emphasized on convex polytopes regarding longest-path problems. In this paper, three infinite families of convex polytopes were considered and their property of being Hamilton-connected was established. Moreover, it is shown that not all convex polytopes are Hamilton-connected by building an infinite family of convex polytopes that is not Hamilton-connected. They calculated precise analytical formulas for their detour index by utilizing the Hamilton-connectivity of these graph families.

Turaci [16] identified combinatorial properties of two convex polytopes via eccentricity-based topological indices in 2022. Eccentricity-based topological indices are crucial for QSPR/QSAR research, and numerous works in recent years have examined these values for various graph types. The topological indices of Q_n and R_n, two convex polytopes, are calculated based on their eccentricity. Yoong et al. [17] worked on convex polytopes and the corona product of cycle with the path in 2022. Their main areas of interest were the corona product of cycles with path, convex polytopes D_n and R_n, and edge-irregular reflexive labeling of antiprism.

2. Preliminaries

Now we discuss some basic definitions that are helpful for our work.

$$T_z = \frac{d_x}{\nu - d_x},$$

where $d_z = |y: yz \in E_\zeta|$ is the degree of z. A few keywords and definitions are reviewed below from the body of existing literature.

Definition 1 A temperature-based graphical index of a graph $\zeta = (V_\zeta, E_\zeta)$ is $G_t = \sum_{rs \in E_\zeta} \psi(T_r, T_s)$, where ψ is two-variable symmetric map, and T_r and T_s are the temperature of vertices r and s, respectively. Temperature-based topological indices are a class of molecular descriptors used in chemical graph theory that are defined based on vertices' temperatures.

Temperature-based indices were introduced by Kulli [4] in 2019. Here are some well-known temperature-based topological indices:

• The first hyper-temperature index

For a graph ζ, the first hyper-temperature index $HT_1(\zeta)$ is defined as,

$$HT_1(\zeta) = \sum_{edges} (T_r + T_s)^2$$

where T_r is temperature index of vertex r and T_s is temperature index of vertex s in graph ζ. For a parameter $\kappa \in R$, the general temperature index T_1^{κ} generalizes the first hyper-temperature index.

$$T_1^{\kappa}(\zeta) = \sum_{edges} (T_r + T_s)^{\kappa}$$

we have $HT_1 = T_1^{\kappa}$ with $\kappa = 2$.

• The second hyper-temperature index

For a graph ζ, the second hyper-temperature index $HT_2(\zeta)$ is defined as,

$$HT_2(\zeta) = \sum_{edges} (T_r T_s)^2$$

The second hyper-temperature index for a parameter $\kappa \in R$ is generalized by the general temperature index T_2^{κ}.

$$T_2^{\kappa}(\zeta) = \sum_{edges} (T_r T_s)^{\kappa}$$

we have $HT_2 = T_2^{\kappa}$ with $\kappa = 2$.

• The sum-connectivity temperature index

For a graph ζ, the sum-connectivity temperature index ST is defined as,

$$ST(\zeta) = \sum_{edges} \frac{1}{\sqrt{T_r + T_s}} = \sum_{edges} (T_r + T_s)^{-\frac{1}{2}}$$

The sum-connectivity temperature index is generalized by the general temperature index T_1^{κ} for a parameter $\kappa \in R$.

$$T_1^{\kappa}(\zeta) = \sum_{edges} (T_r + T_s)^{\kappa}$$

we have $ST = T_1^{\kappa}$ with $\kappa = -\frac{1}{2}$.

• The product-connectivity temperature index

The temperature index of product-connectivity for a graph ζ is defined as,

$$PT(\zeta) = \sum_{edges} \frac{1}{\sqrt{T_r T_s}} = \sum_{edges} (T_r T_s)^{-\frac{1}{2}}$$

For a parameter $\kappa \in R$, the general temperature index T_2^{κ} generalizes the product-connectivity temperature index.

$$T_2^{\kappa}(\zeta) = \sum_{edges} (T_r T_s)^{\kappa}$$

we have $PT = T_2^{\kappa}$ with $\kappa = -\frac{1}{2}$.

• The reciprocal product-connectivity temperature index

For a graph ζ, the temperature index of reciprocal product-connectivity RPT is defined as,

$$RPT(\zeta) = \sum_{edges} \sqrt{T_r T_s} = \sum_{edges} (T_r T_s)^{\frac{1}{2}}$$

The reciprocal product-connectivity temperature index is generalized by the second general temperature index T_2^{κ} for a parameter $\kappa \in R$.

$$T_2^{\kappa}(\zeta) = \sum_{edges} (T_r T_s)^{\kappa}$$

Note that, we have $RPT = T_2^{\kappa}$ with $\kappa = \frac{1}{2}$.

• The arithmetic-geometric temperature index

In the case of a graph ζ, the arithmetic-geometric temperature index AGT is defined by,

$$AGT(\zeta) = \sum_{edges} \left(\frac{T_r + T_s}{2\sqrt{T_r T_s}} \right)$$

• The F-temperature index

For a given graph ζ, the F-temperature index can be defined as,

$$FT(\zeta) = \sum_{edges} (T_r^2 + T_s^2)$$

The F-temperature index is generalized by the general temperature index T_{κ} for a parameter $\kappa \in R$.

$$T_{\kappa}(\zeta) = \sum_{edges} (T_r^{\kappa} + T_s^{\kappa})$$

Note that, we have $FT = T_{\kappa}$ with $\kappa = 2$.

Definition 2 In the η-dimensional space $\mathbb{R}^{\eta}$, a convex polytope is a geometric shape defined as the convex hull of a finite set of points in $\mathbb{R}^{\eta}$. In other words, it is the smallest convex set with all of its existing vertices. The edges of convex polytopes are straight line segments, and their boundaries are flat facets.

They are fundamental components of convex geometry and optimization, and they are frequently used in computer science, mathematics, and operational research to represent and resolve issues. Some of the convex polytopes are given as follows:

• The vertex set and the edge set of H_η is $V(H_\eta) = \{v_i, w_i, x_i, y_i, z_i: 1 \le i \le \eta\}$ and

$$E(H_\eta) = \{v_iv_{i-1}, v_iw_i, v_iw_{i-1}, w_iw_{i-1}, w_ix_i, x_ix_{i-1}, x_iy_i, x_iy_{i-1}, y_iy_{i-1}, y_iz_i, z_iz_{i-1}\}$$

respectively. In Figure 3, there is a convex polytope H_η. The convex polytope H_η was introduced by Imran & Siddiqui [18].

• A pair of ordered vertices and edges builds up the graph G_η. The description of its vertex set is as follow: $V(G_\eta) = \{w_i, x_i, y_i, z_i: 1 \le i \le \eta\}$ and edge set of G_η is defined as

$$E(G_\eta) = \{w_iw_{i-1}, w_ix_i, x_ix_{i-1}, x_iy_i, x_iy_{i-1}, y_iy_{i-1}, y_iz_i, z_iz_{i-1}\}.$$

We can see the convex polytope G_η in Figure 4. This family of convex polytope was introduced by Hayat et al. [19].

• The vertex set of R_η consist of three layers of vertices such that, $V(R_\eta) = \{x_i, y_i, z_i: 1 \le i \le \eta\}$ and its edge set R_η is represented by $\{x_ix_{i-1}, x_iy_i, x_iy_{i-1}, y_iy_{i-1}, y_iz_i, z_iz_{i-1}\}$. Figure 5 represents the η-dimensional convex polytope R_η. The family R_η was by Bača [20] in 1992.

• The convex polytope T_η consist of $V(T_\eta) = \{w_i, x_i, y_i, z_i: 1 \le i \le \eta\}$ and

$$E(T_\eta) = \{w_iw_{i-1}, w_ix_i, w_ix_{i-1}, x_ix_{i-1}, x_iy_i, y_iy_{i-1}, y_iz_i, y_iz_{i+1}, z_iz_{i-1}\}$$

as a vertex set and edge set which is shown in Figure 6. The family R_η was by Bača [21] back in 1988.

• The vertex set of C_η is $V(C_\eta) = \{a_i, b_i, c_i, d_i, e_i: 1 \le i \le \eta\}$ and the edge set is

$$E(C_\eta) = \{a_ia_{i-1}, a_ib_i, b_ib_{i-1}, b_ic_i, b_ic_{i-1}, c_id_i, d_id_{i-1}, d_ie_i, d_ie_{i+1}, e_ie_{i-1}\}.$$

From Figure 7, we have a convex polytope C_η. The convex polytope C_η was discovered by Imran et al. [22].

• A convex polytope D_η is formed by four layers of vertices and their connection with each other. So its vertex set and edge set contain $\{a_i, b_i, c_i, d_i: 1 \le i \le \eta\}$ and $\{a_ia_{i-1}, a_ib_i, a_ib_{i-1}, b_ib_{i-1}, b_ic_i, b_ic_{i-1}, c_id_i, d_id_{i-1}\}$ respectively. We can see Figure 8 for better understanding. The family D_η was by Bača [20] in 1992.

Definition 3 In order to reduce the number of vertices that must be sliced, edge partitioning splits a graph into multiple balanced edge subsets within a specified size. The disjoint partitioning of ζ's edge set into m subsets E_k $(1 \le k \le m)$ is referred to as partitioning. For each $k \ne l$, the partitioning of $E_k \cap E_l = \emptyset$, and $E_k \subseteq E_\zeta$, $\cup_{k\in[m]} E_k = E_\zeta$.

Example 1 Figure 1 displays a graph with 7 vertices and 11 edges. Based on the temperature of each edge's vertices, as shown in Table 1, we can observe that there are five sorts of edges.

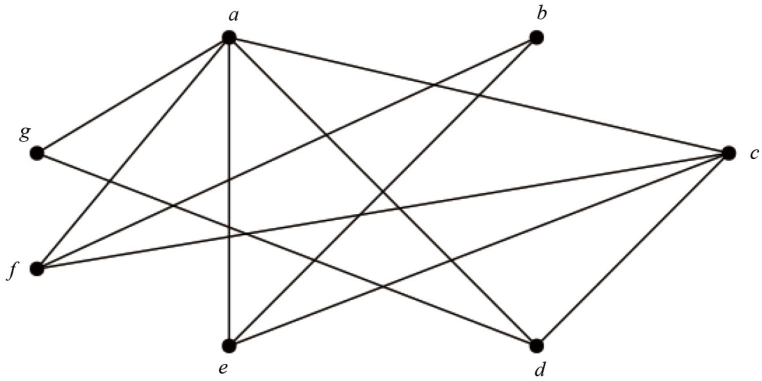

Figure 1. A simple graph

Table 1. Edge Partition

(T_r, T_s), $rs \in E(G)$	Number of edges
$\left(\frac{5}{7-5}, \frac{2}{7-2}\right)$	1
$\left(\frac{5}{7-5}, \frac{3}{7-3}\right)$	3
$\left(\frac{5}{7-5}, \frac{4}{7-4}\right)$	1
$\left(\frac{2}{7-2}, \frac{3}{7-3}\right)$	3
$\left(\frac{4}{7-4}, \frac{3}{7-3}\right)$	3

3. Convex polytopes in mathematical chemistry and temperature-based indices

First, we investigate temperature indices in Section 2 for their applicability in structure-property prediction of physicochemical properties of chemical compounds.

3.1 *Structure-property prediction of physicochemical properties and temperature-based indices*

This subsection investigates the potential applicability of temperature- based indices in structure-property modeling of physicochemical properties of compounds. Regarding test molecules, we consider lower benzenoid hydrocarbons (BHs). Following Gutman and Tošović [23], the representatives of physicochemical properties have been considered as the normal boiling point bp_ρ and standard enthalpy of formation ΔH_f^o. We consider BHs as they represent both cyclic and acyclic structures, as acyclic structures i.e., trees are subgraphs of general graphs. The number of lower BHs that we consider is 22 as the number is sufficiently large enough to validate statistical inferences. Moreover, the experimental data of the chosen test properties i.e., bp_ρ and ΔH_f^o is available publicly. Figure 2 delivers chemical graphs of the lower 22 BHs. The experimental data of both bp_ρ and ΔH_f^o for these BHs was retrieved from the standard NIST repository [24].

Next, we compute all the temperature indices from Section 2 for the graphs of 22 lower BHs and compare those values with the corresponding experimental values of bp_ρ and ΔH_f^o. We compute Pearson's linear correlation coefficient ρ from this data. The data of correlation values is recorded in Table 2.

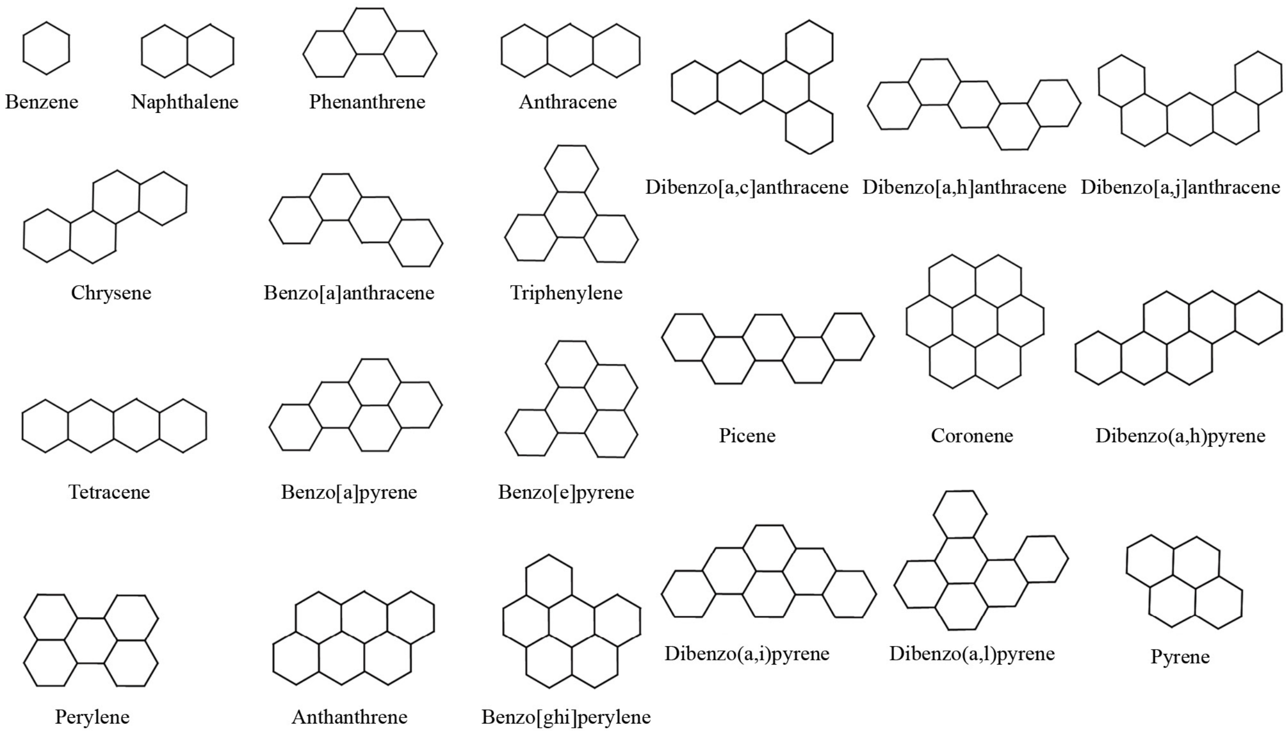

Figure 2. Chemical graphs of the 22 lower BHs considered in this testing

Table 2. Correlation coefficients $\rho(\Delta H_f^o)$ (resp. $\rho(bp_\rho)$) between temperature graphical indices and ΔH_f^o (resp. $b.p.$) for 22 lower BHs

Temperature index	$\rho(bp_\rho)$	$\rho(\Delta H_f^o)$
$HT_1 = T_1^\beta$ with $\beta = 2$	-0.9400	-0.9239
$HT_2 = T_2^\beta$ with $\beta = 2$	-0.8030	-0.7671
$ST = T_1^\beta$ with $\beta = -\frac{1}{2}$	0.9852	0.9432
$PT = T_2^\beta$ with $\beta = -\frac{1}{2}$	0.9832	0.9490
$RPT = T_2^\beta$ with $\beta = \frac{1}{2}$	0.6722	0.4709
AGT	0.9960	0.9456
T_1^β with $\beta = \frac{1}{2}$	0.9928	0.9231
T_1^β with $\beta = 1$	0.6753	0.4776
T_1^β with $\beta = -1$	0.9690	0.9325
T_1^β with $\beta = -2$	0.9304	0.9009
T_2^β with $\beta = 1$	-0.9342	-0.9180
T_2^β with $\beta = -1$	0.9306	0.9009
T_2^β with $\beta = -2$	0.8557	0.8337
$FT = T_\beta$ with $\beta = 2$	-0.9451	-0.9291
T_β with $\beta = 1$	0.6753	0.4776
T_β with $\beta = -1$	0.9695	0.9329
T_β with $\beta = -2$	0.9310	0.9009

Notice that most of the indices produce correlation values higher than 0.9, thus delivering a strong predictive potential with the physicochemical properties of BHs. In light of Table 2, the best performance has been delivered by the arithmetic-geometric temperature index i.e., AGT. We investigate the AGT further and put forward the most appropriate regression models which are, in fact, linear. Here we present those regression models with detailed statistical indicators i.e., the determination coefficient r^2 and standard error of fit s:

$$bp = -13.848_{\pm 20.5239} + 20.754_{\pm 0.8656} AGT, \quad r^2 = 0.9921, \ \rho = 0.996, \ s = 11.7708$$

$$\Delta H_f^o = 20.2_{\pm 44.266} + 11.636_{\pm 1.8668} AGT, \quad r^2 = 0.8942, \ \rho = 0.9456, \ s = 25.3872.$$

This suggests that the AGT index besides other temperature indices deserve further attention in QSPR studies. Moroever, Hayat et al. [25] (resp. Hayat & Liu [26]) deliver strong applicative potentials of temperature-based indices for predicting thermodynamic properties (resp. the total π-electron energy) of BHs.

Next, we present applicability of convex polytopes in different areas of mathematical chemistry.

3.2 *Convex polytopes in mathematical chemistry*

This subsection studies the potential applicability of convex polytopes or, in general, convex polytopes in mathematical and analytical chemistry. Diudea et al. in their seminal book [27] studied polytopes and other graph theoretical models to study molecular topology. It emphasizes the connection between molecular structure and graph theory, using polytopes as geometric representations of molecules. Buck and Litherland [28] discuss how polytopes can be used to model the 3D structure of molecules. The geometry of convex polytopes plays a significant role in understanding the stability and symmetry of molecular configurations. This fact was, in fact, studied in detail by Zhou and Du [29] who investigated the computation of molecular symmetry and stability using convex polytopes. The symmetry properties of polytopes are applied to the molecular structures to predict stability. For more applications of geometrical shapes such as polyhedra and other graphical objects in chemistry, we refer the reader to the work by Balaban [30].

This motivates us to consider a detailed computational analysis of temperature-based graphical indicecs for convex polytopes.

4. Temperature indices of convex polytopes

In this section, we calculate the temperature indices of convex polytopes of different categories by using edge partitioning.

4.1 *Computational results of temperature indices of convex polytope H_η*

The topological indices of the convex polytope H_η with dimension η, which are dependent on temperature, are calculated in this section. Firstly, we introduce the graph of convex polytope H_η in Figure 3. For any arbitrary vertices r and s of H_η, we denote T_r for the temperature index of vertex r and T_s is for the temperature index of vertex s then the edge partitioning of the given graph is represented by (T_r, T_s), and it is further explained in Table 3 that follows.

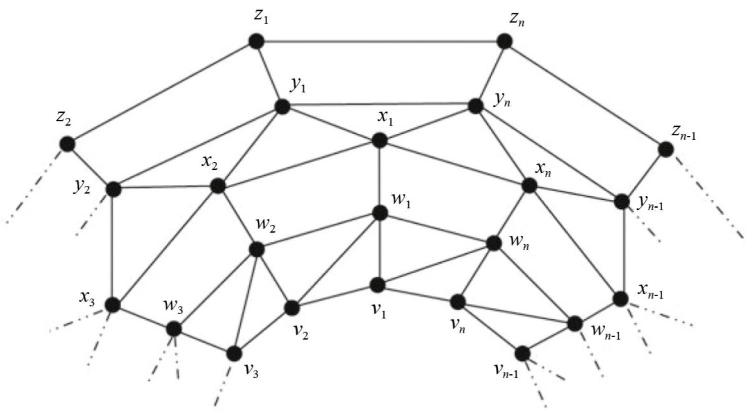

Figure 3. The convex polytope H_η in η dimensions

Table 3. The edge partition of convex polytope H_η

$(T_r, T_s) \setminus rs \in E(G)$	Number of edges
$\left(\frac{4}{5g-4}, \frac{4}{5g-4}\right)$	g
$\left(\frac{4}{5g-4}, \frac{5}{5g-5}\right)$	$2g$
$\left(\frac{5}{5g-5}, \frac{5}{5g-5}\right)$	$6g$
$\left(\frac{5}{5g-5}, \frac{3}{5g-3}\right)$	g
$\left(\frac{3}{5g-3}, \frac{3}{5g-3}\right)$	g

We calculate temperature indices in the following theorems based on this edge partitioning of H_η:

Theorem 1 The convex polytope H_η has general first temperature index

$$g\left(\frac{8}{5g-4}\right)^{\kappa}+2g\left(\frac{9g-8}{5g^2-9g+4}\right)^{\kappa}+6g\left(\frac{2}{g-1}\right)^{\kappa}+g\left(\frac{8g-6}{5g^2-8g+3}\right)^{\kappa}+g\left(\frac{6}{5g-3}\right)^{\kappa}.$$

Proof. Let H_η be a convex polytope. By definition, we have

$$T_1^{\kappa}(H_\eta)=\sum_{edges}(T_r+T_s)^{\kappa}.$$

Now by using edge partitioning in the Table 3, we deduce

$$T_1^{\kappa}(H_\eta) = |E_1|\left(\frac{4}{5g-4}+\frac{4}{5g-4}\right)^{\kappa} + |E_2|\left(\frac{4}{5g-4}+\frac{5}{5g-5}\right)^{\kappa}$$

$$+|E_3|\left(\frac{5}{5g-5}+\frac{5}{5g-5}\right)^{\kappa} + |E_4|\left(\frac{5}{5g-5}+\frac{3}{5g-3}\right)^{\kappa} + |E_5|\left(\frac{3}{5g-3}+\frac{3}{5g-3}\right)^{\kappa}$$

$$T_1^{\kappa}(H_\eta) = g\left(\frac{8}{5g-4}\right)^{\kappa} + 2g\left(\frac{45g-40}{25g^2-45g+20}\right)^{\kappa} + 6g\left(\frac{10}{5g-5}\right)^{\kappa}$$

$$+g\left(\frac{40g-30}{25g^2-40g+15}\right)^{\kappa} + g\left(\frac{6}{5g-3}\right)^{\kappa}$$

$$T_1^{\kappa}(H_\eta) = g\left(\frac{8}{5g-4}\right)^{\kappa} + 2g\left(\frac{9g-8}{5g^2-9g+4}\right)^{\kappa} + 6g\left(\frac{2}{g-1}\right)^{\kappa}$$

$$+g\left(\frac{8g-6}{5g^2-8g+3}\right)^{\kappa} + g\left(\frac{6}{5g-3}\right)^{\kappa} \tag{1}$$

□

Corollary 1 The first hyper-temperature index of convex polytope H_η is

$$\frac{64g}{(5g-4)^2} + \frac{162g^3-288g^2+128g}{(5g^2-9g+4)^2} + \frac{24g}{(g-1)^2} + \frac{64g^3-96g^2+36g}{(5g^2-8g+3)^2} + \frac{36g}{(5g-3)^2}.$$

Corollary 2 The sum-connectivity temperature index of convex polytope H_η is

$$g\sqrt{\frac{5g-4}{8}} + 2g\sqrt{\frac{5g^2-9g+4}{9g-8}} + 3g\sqrt{2(g-1)} + g\sqrt{\frac{5g^2-8g+3}{8g-6}} + g\sqrt{\frac{5g-3}{6}}.$$

Proof. For $\kappa = 2$ & $\kappa = -\frac{1}{2}$ in Equation 1, then we get the above results respectively. □

Theorem 2 The convex polytope H_η has general second temperature index

$$g\left(\frac{4}{5g-4}\right)^{2\kappa} + 2g\left(\frac{4}{5g^2-9g+4}\right)^{\kappa} + 6g\left(\frac{1}{g-1}\right)^{2\kappa} + g\left(\frac{3}{5g^2-8g+3}\right)^{\kappa} + g\left(\frac{3}{5g-3}\right)^{2\kappa}.$$

Proof. Let H_η be a convex polytope. By definition, we have

$$T_2^{\kappa}(H_\eta) = \sum_{edges} (T_r T_s)^{\kappa}.$$

then by using edge partitioning in the Table 3, we deduce

$$T_2^{\kappa}(H_\eta) = |E_1|\left(\frac{4}{5g-4}\times\frac{4}{5g-4}\right)^{\kappa} + |E_2|\left(\frac{4}{5g-4}\times\frac{5}{5g-5}\right)^{\kappa} +$$

$$|E_3|\left(\frac{5}{5g-5}\times\frac{5}{5g-5}\right)^{\kappa} + |E_4|\left(\frac{5}{5g-5}\times\frac{3}{5g-3}\right)^{\kappa} + |E_5|\left(\frac{3}{5g-3}\times\frac{3}{5g-3}\right)^{\kappa}$$

$$T_2^{\kappa}(H_\eta) = g\left(\frac{4}{5g-4}\right)^{2\kappa} + 2g\left(\frac{20}{25g^2-45g+20}\right)^{\kappa} + 6g\left(\frac{5}{5g-5}\right)^{2\kappa} +$$

$$g\left(\frac{15}{25g^2-40g+15}\right)^{\kappa} + g\left(\frac{3}{5g-3}\right)^{2\kappa}$$

$$T_2^{\kappa}(H_\eta) = g\left(\frac{4}{5g-4}\right)^{2\kappa} + 2g\left(\frac{4}{5g^2-9g+4}\right)^{\kappa} + 6g\left(\frac{1}{g-1}\right)^{2\kappa} +$$

$$g\left(\frac{3}{5g^2-8g+3}\right)^{\kappa} + g\left(\frac{3}{5g-3}\right)^{2\kappa} \tag{2}$$

□

Corollary 3 The second hyper-temperature index of convex polytope H_η is

$$\frac{256g}{(5g-4)^4} + \frac{32g}{(5g^2-9g+4)^2} + \frac{6g}{(g-1)^4} + \frac{9g}{(5g^2-8g+3)^2} + \frac{81g}{(5g-3)^4}.$$

Corollary 4 The product-connectivity temperature index of convex polytope H_η is

$$\frac{107}{12}g^2 - 8g + g\sqrt{5g^2-9g+4} + \frac{g}{\sqrt{3}}\sqrt{5g^2-8g+3}.$$

Corollary 5 The reciprocal product-connectivity temperature index of convex polytope H_η is

$$\frac{4g}{5g-4} + \frac{4g}{\sqrt{5g^2-9g+4}} + \frac{6g}{g-1} + \frac{\sqrt{3}g}{\sqrt{5g^2-8g+3}} + \frac{3g}{5g-3}.$$

Proof. For $\kappa = 2, -\frac{1}{2}$ & $\kappa = \frac{1}{2}$ in Equation 2, then we get the above results respectively. □

Theorem 3 The arithmetic-geometric temperature index of convex polytope H_η is

$$8g + \frac{9g^2-8g}{2\sqrt{(5g^2-9g+4)}} + \frac{4g^2-3g}{\sqrt{(15g^2-24g+9)}}.$$

Proof. Let H_η be a convex polytope. By definition, we have

$$AGT(H_\eta) = \sum_{edges} \left(\frac{T_r + T_s}{2\sqrt{T_r T_s}} \right).$$

then by using edge partitioning in the Table 3, we deduce

$$AGT(H_\eta) = |E_1| \left(\frac{\frac{4}{5g-4} + \frac{4}{5g-4}}{2\sqrt{\left(\frac{4}{5g-4}\right)\left(\frac{4}{5g-4}\right)}} \right) + |E_2| \left(\frac{\frac{4}{5g-4} + \frac{5}{5g-5}}{2\sqrt{\left(\frac{4}{5g-4}\right)\left(\frac{5}{5g-5}\right)}} \right) +$$

$$|E_3| \left(\frac{\frac{5}{5g-5} + \frac{5}{5g-5}}{2\sqrt{\left(\frac{5}{5g-5}\right)\left(\frac{5}{5g-5}\right)}} \right) + |E_4| \left(\frac{\frac{5}{5g-5} + \frac{3}{5g-3}}{2\sqrt{\left(\frac{5}{5g-5}\right)\left(\frac{3}{5g-3}\right)}} \right) + |E_5| \left(\frac{\frac{3}{5g-3} + \frac{3}{5g-3}}{2\sqrt{\left(\frac{3}{5g-3}\right)\left(\frac{3}{5g-3}\right)}} \right)$$

$$AGT(H_\eta) = g \left(\frac{\frac{8}{5g-4}}{2\sqrt{\left(\frac{4}{5g-4}\right)^2}} \right) + 2g \left(\frac{\frac{45g-40}{(5g-4)(5g-5)}}{2\sqrt{\frac{20}{(5g-4)(5g-5)}}} \right) + 6g \left(\frac{\frac{10}{5g-5}}{2\sqrt{\left(\frac{5}{5g-5}\right)^2}} \right) +$$

$$g \left(\frac{\frac{40g-30}{(5g-3)(5g-5)}}{2\sqrt{\frac{15}{(5g-3)(5g-5)}}} \right) + g \left(\frac{\frac{6}{5g-3}}{2\sqrt{\left(\frac{3}{5g-3}\right)^2}} \right)$$

$$AGT(H_\eta) = 8g + \frac{9g^2 - 8g}{2\sqrt{(5g^2 - 9g + 4)}} + \frac{4g^2 - 3g}{\sqrt{(15g^2 - 24g + 9)}}.$$

□

Theorem 4 The general temperature index of convex polytope H_η is

$$4g\left(\frac{4}{5g-4}\right)^\kappa + 15g\left(\frac{1}{g-1}\right)^\kappa + 3g\left(\frac{3}{5g-3}\right)^\kappa.$$

Proof. Let H_η be a convex polytope. By definition, we have

$$T_\kappa(H_\eta) = \sum_{edges} (T_r^\kappa + T_s^\kappa).$$

then by using edge partitioning in the Table 3, we deduce

$$T_\kappa(H_\eta) = |E_1|\left(\left(\frac{4}{5g-4}\right)^\kappa + \left(\frac{4}{5g-4}\right)^\kappa\right) + |E_2|\left(\left(\frac{4}{5g-4}\right)^\kappa + \left(\frac{5}{5g-5}\right)^\kappa\right) +$$

$$|E_3|\left(\left(\frac{5}{5g-5}\right)^\kappa + \left(\frac{5}{5g-5}\right)^\kappa\right) + |E_4|\left(\left(\frac{5}{5g-5}\right)^\kappa + \left(\frac{3}{5g-3}\right)^\kappa\right) +$$

$$|E_5|\left(\left(\frac{3}{5g-3}\right)^\kappa + \left(\frac{3}{5g-3}\right)^\kappa\right)$$

$$T_\kappa(H_\eta) = g\left(2\left(\frac{4}{5g-4}\right)^\kappa\right) + 2g\left(\left(\frac{4}{5g-4}\right)^\kappa + \left(\frac{5}{5g-5}\right)^\kappa\right) +$$

$$6g\left(2\left(\frac{5}{5g-5}\right)^\kappa\right) + g\left(\left(\frac{5}{5g-5}\right)^\kappa + \left(\frac{3}{5g-3}\right)^\kappa\right) + g\left(2\left(\frac{3}{5g-3}\right)^\kappa\right)$$

$$T_\kappa(H_\eta) = 4g\left(\frac{4}{5g-4}\right)^\kappa + 15g\left(\frac{1}{g-1}\right)^\kappa + 3g\left(\frac{3}{5g-3}\right)^\kappa \tag{3}$$

□

Corollary 6 The F-temperature index of convex polytope H_η is

$$\frac{64g}{(5g-4)^2} + \frac{15g}{(g-1)^2} + \frac{27g}{(5g-3)^2}.$$

Proof. Put $\kappa = 2$ in Equation 3, then we get the required result. □

4.2 *Computational results of temperature indices on η-dimensional convex polytope G_η*

The temperature-based topological indices of the convex polytope G_η with dimension η are calculated in this section. Firstly, we introduce the graph of convex polytope G_η in Figure 4.

For any arbitrary vertices r and s of G_η, we denote T_r for temperature index of vertex r and T_s is for temperature index of vertex s then the edge partitioning of the given graph is represented by (T_r, T_s), and it is further explained in the Table 4 that follows.

Table 4. Edge Partition of Convex Polytope G_η

$(T_r, T_s) \setminus rs \in E(G)$	Number of edges
$\left(\frac{3}{4g-3}, \frac{3}{4g-3}\right)$	$2g$
$\left(\frac{3}{4g-3}, \frac{5}{4g-5}\right)$	$2g$
$\left(\frac{5}{4g-5}, \frac{5}{4g-5}\right)$	$4g$

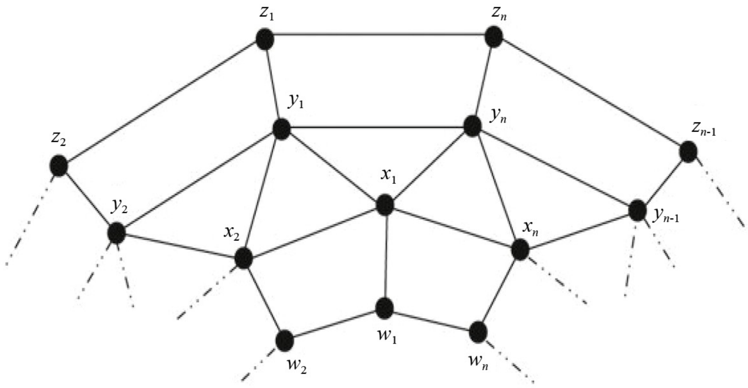

Figure 4. The convex polytope G_η in η dimension

We calculate temperature indices in the following theorems based on this edge partitioning of G_η:

Theorem 5 The convex polytope G_η has the general first temperature index

$$2g\left(\frac{6}{4g-3}\right)^{\kappa}+2g\left(\frac{32g-30}{16g^2-32g+15}\right)^{\kappa}+4g\left(\frac{10}{4g-5}\right)^{\kappa}.$$

Proof. Let G_η be a convex polytope. By definition, we have

$$T_1^{\kappa}(G_\eta)=\sum_{edges}(T_r+T_s)^{\kappa}.$$

then by using edge partitioning in the Table 4, we deduce

$$T_1^{\kappa}(G_\eta)=|E_1|\left(\frac{3}{4g-3}+\frac{3}{4g-3}\right)^{\kappa}+|E_2|\left(\frac{3}{4g-3}+\frac{5}{4g-5}\right)^{\kappa}+|E_3|\left(\frac{5}{4g-5}+\frac{5}{4g-5}\right)^{\kappa}$$

$$T_1^{\kappa}(G_\eta)=2g\left(\frac{6}{4g-3}\right)^{\kappa}+2g\left(\frac{32g-30}{16g^2-32g+15}\right)^{\kappa}+4g\left(\frac{10}{4g-5}\right)^{\kappa} \tag{4}$$

□

Corollary 7 The first hyper-temperature index of convex polytope G_η is

$$\frac{72g}{(4g-3)^2}+\left(\frac{2048g^3-3840g^2+1800g}{(16g^2-32g+15)^2}\right)+\frac{400g}{(4g-5)^2}.$$

Corollary 8 The sum-connectivity temperature index of convex polytope G_η is

$$\frac{g}{\sqrt{3}}\sqrt{8g-6}+g\sqrt{\frac{32g^2-64g+30}{16g-15}}+\frac{4g}{\sqrt{10}}\sqrt{4g-5}.$$

Proof. Put $\kappa=2$ & $kappa=-\frac{1}{2}$ in Equation 4, then we get the required results respectively. □

Theorem 6 The convex polytope G_η has the general second temperature index

$$2g\left(\frac{3}{4g-3}\right)^{2\kappa}+2g\left(\frac{15}{16g^2-32g+15}\right)^{\kappa}+4g\left(\frac{5}{4g-5}\right)^{2\kappa}.$$

Proof. Let G_η be a convex polytope graph. By definition, we have

$$T_2^{\kappa}(G_\eta)=\sum_{edges}(T_rT_s)^{\kappa}.$$

then by using edge partitioning in the Table 4, we deduce

$$T_2^{\kappa}(G_\eta)=|E_1|\left(\frac{3}{4g-3}\times\frac{3}{4g-3}\right)^{\kappa}+|E_2|\left(\frac{3}{4g-3}\times\frac{5}{4g-5}\right)^{\kappa}+|E_3|\left(\frac{5}{4g-5}\times\frac{5}{4g-5}\right)^{\kappa}$$

$$T_2^{\kappa}(G_\eta)=2g\left(\frac{3}{4g-3}\right)^{2\kappa}+2g\left(\frac{15}{16g^2-32g+15}\right)^{\kappa}+4g\left(\frac{5}{4g-5}\right)^{2\kappa} \tag{5}$$

□

Corollary 9 The second hyper-temperature index of convex polytope G_η is

$$\frac{162g}{(4g-3)^4}+\frac{450g}{(16g^2-32g+15)^2}+\frac{2500g}{(4g-5)^4}.$$

Corollary 10 The product-connectivity temperature index of convex polytope G_η is

$$\frac{88}{15}g^2-6g+\frac{2g}{\sqrt{15}}\sqrt{16g^2-32g+15}.$$

Corollary 11 The reciprocal product-connectivity temperature index of convex polytope G_η is

$$\frac{6g}{4g-3}+\frac{2\sqrt{15}g}{\sqrt{16g^2-32g+15}}+\frac{20g}{4g-5}.$$

Proof. Put $\kappa=2,\ -\frac{1}{2}$ & $\frac{1}{2}$ in Equation 5, then we get the required results respectively. □

Theorem 7 The arithmetic-geometric temperature index of convex polytope G_η is

$$6g+\frac{32g^2-30g}{\sqrt{15(16g^2-32g+15)}}.$$

Proof. Let G_η be a convex polytope graph. By definition, we have

$$AGT(G_\eta)=\sum_{edges}\left(\frac{T_r+T_s}{2\sqrt{T_rT_s}}\right).$$

then by using edge partitioning in the Table 4, we deduce

$$AGT(G_\eta)=|E_1|\left(\frac{\frac{3}{4g-3}+\frac{3}{4g-3}}{2\sqrt{\left(\frac{3}{4g-3}\right)\left(\frac{3}{4g-3}\right)}}\right)+|E_2|\left(\frac{\frac{3}{4g-3}+\frac{5}{4g-5}}{2\sqrt{\left(\frac{3}{4g-3}\right)\left(\frac{5}{4g-5}\right)}}\right)+|E_3|\left(\frac{\frac{5}{4g-5}+\frac{5}{4g-5}}{2\sqrt{\left(\frac{5}{4g-5}\right)\left(\frac{5}{4g-5}\right)}}\right)$$

$$AGT(G_\eta)=2g\left(\frac{\frac{6}{4g-3}}{2\sqrt{\left(\frac{3}{4g-3}\right)^2}}\right)+2g\left(\frac{\frac{32g-30}{16g^2-32g+15}}{2\sqrt{\frac{15}{16g^2-32g+15}}}\right)+4g\left(\frac{\frac{10}{4g-5}}{2\sqrt{\left(\frac{5}{4g-5}\right)^2}}\right)$$

$$AGT(G_\eta)=6g+\frac{32g^2-30g}{\sqrt{15(16g^2-32g+15)}}.$$

□

Theorem 8 The general temperature index of convex polytope G_η is

$$6g\left(\frac{3}{4g-3}\right)^\kappa+10g\left(\frac{5}{4g-5}\right)^\kappa.$$

Proof. Let G_η be a convex polytope. By definition, we have

$$T_\kappa(G_\eta)=\sum_{edges}(T_r^\kappa+T_s^\kappa).$$

then by using edge partitioning in the Table 4, we deduce

$$T_\kappa(G_\eta) = |E_1|\left(\left(\frac{3}{4g-3}\right)^\kappa + \left(\frac{3}{4g-3}\right)^\kappa\right) + |E_2|\left(\left(\frac{3}{4g-3}\right)^\kappa + \left(\frac{5}{4g-5}\right)^\kappa\right) +$$

$$|E_3|\left(\left(\frac{5}{4g-5}\right)^\kappa + \left(\frac{5}{4g-5}\right)^\kappa\right)$$

$$T_\kappa(G_\eta) = 2g\left(2\left(\frac{3}{4g-3}\right)^\kappa\right) + 2g\left(\left(\frac{3}{4g-3}\right)^\kappa + \left(\frac{5}{4g-5}\right)^\kappa\right) + 4g\left(2\left(\frac{5}{4g-5}\right)^\kappa\right)$$

$$T_\kappa(G_\eta) = 6g\left(\frac{3}{4g-3}\right)^\kappa + 10g\left(\frac{5}{4g-5}\right)^\kappa \tag{6}$$

□

Corollary 12 The F-temperature index of convex polytope G_η is

$$\frac{54g}{(4g-3)^2} + \frac{250g}{(4g-5)^2}.$$

Proof. Put $\kappa = 2$ in Equation 6, then we get the above result. □

4.3 *Computational results of temperature indices on η-dimensional convex polytope R_η*

The temperature-based topological indices of the convex polytope R_η with dimension η are calculated in this section. Firstly we introduce the graph of convex polytope R_η in Figure 5.

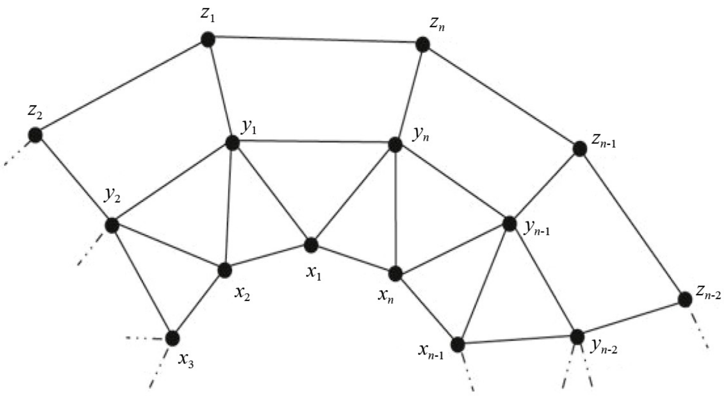

Figure 5. The convex polytope R_η in η dimensions

For any arbitrary vertices r and s of R_η we denote T_r for temperature index of vertex r and T_s is for temperature index of vertex s then the edge partitioning of the given graph is represented by (T_r, T_s), and it is further explained in the Table 5 that follows.

Table 5. Edge Partition of Convex Polytope R_η

$(T_r, T_s) \setminus rs \in E(G)$	Number of edges
$\left(\frac{3}{3g-3}, \frac{3}{3g-3}\right)$	g
$\left(\frac{3}{3g-3}, \frac{5}{3g-5}\right)$	g
$\left(\frac{5}{3g-5}, \frac{5}{3g-5}\right)$	g
$\left(\frac{5}{3g-5}, \frac{4}{3g-4}\right)$	$2g$
$\left(\frac{4}{3g-4}, \frac{4}{3g-4}\right)$	g

We calculate temperature indices in the following theorems based on this edge partitioning of R_η:

Theorem 9 The convex polytope R_η has general first temperature index

$$g\left(\frac{2}{g-1}\right)^{\kappa}+g\left(\frac{8g-10}{3g^2-8g+5}\right)^{\kappa}+g\left(\frac{10}{3g-5}\right)^{\kappa}+2g\left(\frac{27g-40}{9g^2-27g+20}\right)^{\kappa}+g\left(\frac{8}{3g-4}\right)^{\kappa}.$$

Proof. Let R_η be a convex polytope. By definition, we have

$$T_1^{\kappa}(R_\eta)=\sum_{edges}(T_r+T_s)^{\kappa}.$$

then by using edge partitioning in the Table 5, we deduce

$$T_1^{\kappa}(R_\eta)=|E_1|\left(\frac{3}{3g-3}+\frac{3}{3g-3}\right)^{\kappa}+|E_2|\left(\frac{3}{3g-3}+\frac{5}{3g-5}\right)^{\kappa}+|E_3|\left(\frac{5}{3g-5}+\frac{5}{3g-5}\right)^{\kappa}+$$

$$|E_4|\left(\frac{5}{3g-5}+\frac{4}{3g-4}\right)^{\kappa}+|E_5|\left(\frac{4}{3g-4}+\frac{4}{3g-4}\right)^{\kappa}$$

$$T_1^{\kappa}(R_\eta)=g\left(\frac{6}{3g-3}\right)^{\kappa}+g\left(\frac{24g-30}{9g^2-24g+15}\right)^{\kappa}+g\left(\frac{10}{3g-5}\right)^{\kappa}+$$

$$2g\left(\frac{27g-40}{9g^2-27g+20}\right)^{\kappa}+g\left(\frac{8}{3g-4}\right)^{\kappa}$$

$$T_1^{\kappa}(R_\eta)=g\left(\frac{2}{g-1}\right)^{\kappa}+g\left(\frac{8g-10}{3g^2-8g+5}\right)^{\kappa}+g\left(\frac{10}{3g-5}\right)^{\kappa}+2g\left(\frac{27g-40}{9g^2-27g+20}\right)^{\kappa}+g\left(\frac{8}{3g-4}\right)^{\kappa} \quad (7)$$

□

Corollary 13 The first hyper-temperature index of convex polytope R_η is

$$\frac{4g}{(g-1)^2}+\left(\frac{64g^3-160g^2+100g}{(3g^2-8g+5)^2}\right)+\frac{100g}{(3g-5)^2}+\left(\frac{1458g^3-4320g^2+3200g}{(9g^2-27g+20)^2}\right)+\frac{64g}{(3g-4)^2}.$$

Corollary 14 The sum-connectivity temperature index of convex polytope R_η is

$$\frac{g}{\sqrt{2}}\sqrt{g-1}+g\sqrt{\frac{3g^2-8g+5}{8g-10}}+\frac{g}{\sqrt{10}}\sqrt{3g-5}+2g\sqrt{\frac{9g^2-27g+20}{27g-40}}+\frac{g}{\sqrt{8}}\sqrt{3g-4}.$$

Proof. Put $\kappa=2$ & $-\frac{1}{2}$ in Equation 7, then we get the desired result respectively. □

Theorem 10 The convex polytope R_η has general second temperature index

$$g\left(\frac{1}{g-1}\right)^{2\kappa}+g\left(\frac{5}{3g^2-8g+5}\right)^{\kappa}+g\left(\frac{5}{3g-5}\right)^{2\kappa}+2g\left(\frac{20}{9g^2-27g+20}\right)^{\kappa}+g\left(\frac{4}{3g-4}\right)^{2\kappa}.$$

Proof. Let R_η be a convex polytope. By definition, we have

$$T_2^\kappa(R_\eta)=\sum_{edges}(T_rT_s)^\kappa.$$

then by using edge partitioning in the Table 5, we deduce

$$T_2^\kappa(R_\eta)=|E_1|\left(\frac{3}{3g-3}\times\frac{3}{3g-3}\right)^\kappa+|E_2|\left(\frac{3}{3g-3}\times\frac{5}{3g-5}\right)^\kappa+|E_3|\left(\frac{5}{3g-5}\times\frac{5}{3g-5}\right)^\kappa+$$
$$|E_4|\left(\frac{5}{3g-5}\times\frac{4}{3g-4}\right)^\kappa+|E_5|\left(\frac{4}{3g-4}\times\frac{4}{3g-4}\right)^\kappa$$
$$T_2^\kappa(R_\eta)=g\left(\frac{3}{3g-3}\right)^{2\kappa}+g\left(\frac{15}{9g^2-24g+15}\right)^{\kappa}+g\left(\frac{5}{3g-5}\right)^{2\kappa}+$$
$$2g\left(\frac{20}{9g^2-27g+20}\right)^{\kappa}+g\left(\frac{4}{3g-4}\right)^{2\kappa}$$
$$T_2^\kappa(R_\eta)=g\left(\frac{1}{g-1}\right)^{2\kappa}+g\left(\frac{5}{3g^2-8g+5}\right)^{\kappa}+g\left(\frac{5}{3g-5}\right)^{2\kappa}+$$
$$2g\left(\frac{20}{9g^2-27g+20}\right)^{\kappa}+g\left(\frac{4}{3g-4}\right)^{2\kappa} \tag{8}$$

□

Corollary 15 The second hyper-temperature index of convex polytope R_η is

$$\frac{g}{(g-1)^4}+\frac{25g}{(3g^2-8g+5)^2}+\frac{625g}{(3g-5)^4}+\frac{800g}{(9g^2-27g+20)^2}+\frac{256g}{(3g-4)^4}.$$

Corollary 16 The product-connectivity temperature index of convex polytope R_η is

$$\frac{47}{20}g^2-3g+\frac{g}{\sqrt{5}}\left(\sqrt{3g^2-8g+5}+\sqrt{9g^2-27g+20}\right).$$

Corollary 17 The reciprocal product-connectivity temperature index of convex polytope R_η is

$$\frac{g}{g-1}+\frac{5g}{3g-5}+g\sqrt{5}\left(\frac{1}{\sqrt{3g^2-8g+5}}+\frac{4}{\sqrt{9g^2-27g+20}}\right)+\frac{4g}{3g-4}.$$

Proof. Put $\kappa=2,-\frac{1}{2}$ & $\frac{1}{2}$ in Equation 8, then we get the desired result respectively. □

Theorem 11 The arithmetic-geometric temperature index of convex polytope R_η is

$$3g+\left(\frac{4g^2-5g}{\sqrt{15g^2-40g+25}}\right)+\left(\frac{27g^2-40g}{2\sqrt{45g^2-135g+100}}\right).$$

Proof. Let R_η be a convex polytope. By definition, we have

$$AGT(R_\eta)=\sum_{edges}\left(\frac{T_r+T_s}{2\sqrt{T_rT_s}}\right).$$

then by using edge partitioning in the Table 5, we deduce

$$AGT(R_\eta)=|E_1|\left(\frac{\frac{3}{3g-3}+\frac{3}{3g-3}}{2\sqrt{\left(\frac{3}{3g-3}\right)\left(\frac{3}{3g-3}\right)}}\right)+|E_2|\left(\frac{\frac{3}{3g-3}+\frac{5}{3g-5}}{2\sqrt{\left(\frac{3}{3g-3}\right)\left(\frac{5}{3g-5}\right)}}\right)+|E_3|\left(\frac{\frac{5}{3g-5}+\frac{5}{3g-5}}{2\sqrt{\left(\frac{5}{3g-5}\right)\left(\frac{5}{3g-5}\right)}}\right)+$$

$$|E_4|\left(\frac{\frac{5}{3g-5}+\frac{4}{3g-4}}{2\sqrt{\left(\frac{5}{3g-5}\right)\left(\frac{4}{3g-4}\right)}}\right)+|E_5|\left(\frac{\frac{4}{3g-4}+\frac{4}{3g-4}}{2\sqrt{\left(\frac{4}{3g-4}\right)\left(\frac{4}{3g-4}\right)}}\right)$$

$$AGT(R_\eta)=g\left(\frac{\frac{6}{3g-3}}{2\sqrt{\left(\frac{3}{3g-3}\right)^2}}\right)+g\left(\frac{\frac{24g-30}{9g^2-24g+15}}{2\sqrt{\frac{15}{9g^2-24g+15}}}\right)+g\left(\frac{\frac{10}{3g-5}}{2\sqrt{\left(\frac{5}{3g-5}\right)^2}}\right)+$$

$$2g\left(\frac{\frac{27g-40}{9g^2-27g+20}}{2\sqrt{\frac{20}{9g^2-27g+20}}}\right)+g\left(\frac{\frac{8}{3g-4}}{2\sqrt{\left(\frac{4}{3g-4}\right)^2}}\right)$$

$$AGT(R_\eta)=3g+\left(\frac{4g^2-5g}{\sqrt{15g^2-40g+25}}\right)+\left(\frac{27g^2-40g}{2\sqrt{45g^2-135g+100}}\right).$$

□

Theorem 12 The general temperature index of convex polytope R_η is

$$3g\left(\frac{1}{g-1}\right)^\kappa+5g\left(\frac{5}{3g-5}\right)^\kappa+4g\left(\frac{4}{3g-4}\right)^\kappa.$$

Proof. Let R_η be a convex polytope. By definition, we have

$$T_\kappa(R_\eta)=\sum_{edges}(T_r^\kappa+T_s^\kappa).$$

then by using edge partitioning in the Table 5, we deduce

$$\begin{aligned}T_\kappa(R_\eta)=&|E_1|\left(\left(\frac{3}{3g-3}\right)^\kappa+\left(\frac{3}{3g-3}\right)^\kappa\right)+|E_2|\left(\left(\frac{3}{3g-3}\right)^\kappa+\left(\frac{5}{3g-5}\right)^\kappa\right)+\\&|E_3|\left(\left(\frac{5}{3g-5}\right)^\kappa+\left(\frac{5}{3g-5}\right)^\kappa\right)+|E_4|\left(\left(\frac{5}{3g-5}\right)^\kappa+\left(\frac{4}{3g-4}\right)^\kappa\right)+\\&|E_5|\left(\left(\frac{4}{3g-4}\right)^\kappa+\left(\frac{4}{3g-4}\right)^\kappa\right)\end{aligned}$$

$$\begin{aligned}T_\kappa(R_\eta)=&g\left(2\left(\frac{3}{3g-3}\right)^\kappa\right)+g\left(\left(\frac{3}{3g-3}\right)^\kappa+\left(\frac{5}{3g-5}\right)^\kappa\right)+\\&g\left(2\left(\frac{5}{3g-5}\right)^\kappa\right)+2g\left(\left(\frac{5}{3g-5}\right)^\kappa+\left(\frac{4}{3g-4}\right)^\kappa\right)+\\&g\left(2\left(\frac{4}{3g-4}\right)^\kappa\right)\end{aligned}$$

$$T_{\kappa}(R_{\eta}) = 3g\left(\frac{1}{g-1}\right)^{\kappa} + 5g\left(\frac{5}{3g-5}\right)^{\kappa} + 4g\left(\frac{4}{3g-4}\right)^{\kappa} \tag{9}$$

□

Corollary 18 The F-temperature index of convex polytope R_{η} is

$$\frac{3g}{(g-1)^2} + \frac{125g}{(3g-5)^2} + \frac{64g}{(3g-4)^2}.$$

Proof. Put $\kappa = 2$ in Equation 9, then we get the result. □

4.4 *Computational results of temperature indices on η-dimensional convex polytope T_{η}*

The temperature-based topological indices of the convex polytope T_{η} with dimension η are calculated in this section. Firstly we introduce the graph of convex polytope T_{η} in Figure 6. For any arbitrary vertices r and s of T_{η} we denote T_r for temperature index of vertex r and T_s is for temperature index of vertex s then the edge partitioning of the given graph is represented by (T_r, T_s), and it is further explained in the Table 6 that follows.

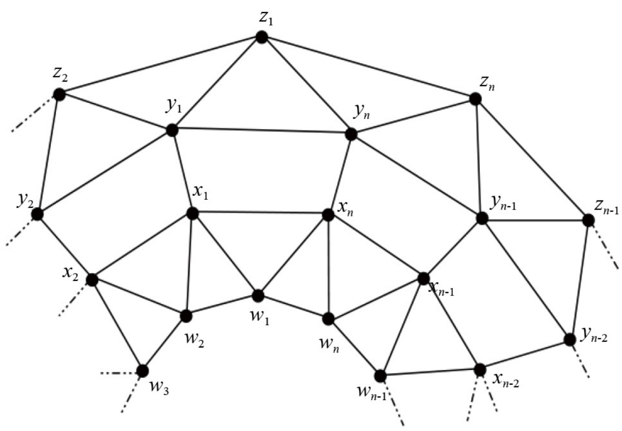

Figure 6. The convex polytope T_{η} with η dimension

Table 6. Edge Partition of convex polytope T_{η}

$(T_r, T_s) \setminus rs \in E(G)$	Number of edges
$\left(\frac{4}{4g-4}, \frac{4}{4g-4}\right)$	$2g$
$\left(\frac{4}{4g-4}, \frac{5}{4g-5}\right)$	$4g$
$\left(\frac{5}{4g-5}, \frac{5}{4g-5}\right)$	$3g$

We calculate temperature indices in the following theorems based on this edge partitioning of T_{η}:

Theorem 13 The convex polytope T_η has the general first temperature index

$$2g\left(\frac{2}{g-1}\right)^{\kappa}+4g\left(\frac{9g-10}{4g^2-9g+5}\right)^{\kappa}+3g\left(\frac{10}{4g-5}\right)^{\kappa}.$$

Proof. Let T_η be a convex polytope. By definition, we have

$$T_1^{\kappa}(T_\eta)=\sum_{edges}(T_r+T_s)^{\kappa}.$$

then by using edge partitioning in the Table 6, we deduce

$$\begin{aligned}T_1^{\kappa}(T_\eta)=&|E_1|\left(\frac{4}{4g-4}+\frac{4}{4g-4}\right)^{\kappa}+|E_2|\left(\frac{4}{4g-4}+\frac{5}{4g-5}\right)^{\kappa}+\\&|E_3|\left(\frac{5}{4g-5}+\frac{5}{4g-5}\right)^{\kappa}\\T_1^{\kappa}(T_\eta)=&2g\left(\frac{2}{g-1}\right)^{\kappa}+4g\left(\frac{9g-10}{4g^2-9g+5}\right)^{\kappa}+3g\left(\frac{10}{4g-5}\right)^{\kappa}\end{aligned}\tag{10}$$

□

Corollary 19 The first hyper-temperature index of convex polytope T_η is

$$\frac{8g}{(g-1)^2}+\frac{324g^3-720g^2+400g}{(4g^2-9g+5)^2}+\frac{300g}{(4g-5)^2}.$$

Corollary 20 The sum-connectivity temperature index of convex polytope T_η is

$$g\sqrt{2g-2}+4g\sqrt{\frac{4g^2-9g+5}{9g-10}}+\frac{3g}{\sqrt{10}}\sqrt{4g-5}.$$

Proof. Put $\kappa=2$ & $-\frac{1}{2}$ in Equation 10, then we get the desired result respectively. □

Theorem 14 The convex polytope T_η has general the second temperature index

$$2g\left(\frac{1}{g-1}\right)^{2\kappa}+4g\left(\frac{5}{4g^2-9g+5}\right)^{\kappa}+3g\left(\frac{5}{4g-5}\right)^{2\kappa}.$$

Proof. Let T_η be a convex polytope. By definition, we have

$$T_2^{\kappa}(T_\eta) = \sum_{edges} (T_r T_s)^{\kappa}.$$

then by using edge partitioning in the Table 6, we deduce

$$T_2^{\kappa}(T_\eta) = |E_1|\left(\frac{4}{4g-4}\times\frac{4}{4g-4}\right)^{\kappa} + |E_2|\left(\frac{4}{4g-4}\times\frac{5}{4g-5}\right)^{\kappa} + |E_3|\left(\frac{5}{4g-5}\times\frac{5}{4g-5}\right)^{\kappa}$$

$$T_2^{\kappa}(T_\eta) = 2g\left(\frac{4}{4g-4}\right)^{2\kappa} + 4g\left(\frac{20}{16g^2-36g+20}\right)^{\kappa} + 3g\left(\frac{5}{4g-5}\right)^{2\kappa}$$

$$T_2^{\kappa}(T_\eta) = 2g\left(\frac{1}{g-1}\right)^{2\kappa} + 4g\left(\frac{5}{4g^2-9g+5}\right)^{\kappa} + 3g\left(\frac{5}{4g-5}\right)^{2\kappa} \tag{11}$$

□

Corollary 21 The second hyper-temperature index of convex polytope T_η is

$$\frac{2g}{(g-1)^4} + \frac{100g}{(4g^2-9g+5)^2} + \frac{1875g}{(4g-5)^4}.$$

Corollary 22 The product-connectivity temperature index of convex polytope T_η is

$$\frac{22}{5}g^2 - 5g + \frac{4g}{\sqrt{5}}\sqrt{4g^2-9g+5}.$$

Corollary 23 The reciprocal Product-connectivity temperature index of convex polytope T_η is

$$\frac{2g}{g-1} + \frac{4\sqrt{5}g}{\sqrt{4g^2-9g+5}} + \frac{15g}{4g-5}.$$

Proof. Put $\kappa = 2, -\frac{1}{2}$ & $\frac{1}{2}$ in Equation 11, then we get the required respectively. □

Theorem 15 The arithmetic-geometric temperature index of convex polytope T_η is

$$5g + \frac{18g^2-20g}{\sqrt{20g^2-45g+25}}.$$

Proof. Let T_η be a convex polytope. By definition, we have

$$AGT(T_\eta) = \sum_{edges}\left(\frac{T_r+T_s}{2\sqrt{T_rT_s}}\right).$$

then by using edge partitioning in the Table 6, we deduce

$$AGT(T_\eta) = |E_1|\left(\frac{\frac{4}{4g-4}+\frac{4}{4g-4}}{2\sqrt{\left(\frac{4}{4g-4}\right)\left(\frac{4}{4g-4}\right)}}\right)+|E_2|\left(\frac{\frac{4}{4g-4}+\frac{5}{4g-5}}{2\sqrt{\left(\frac{4}{4g-4}\right)\left(\frac{5}{4g-5}\right)}}\right)+|E_3|\left(\frac{\frac{5}{4g-5}+\frac{5}{4g-5}}{2\sqrt{\left(\frac{5}{4g-5}\right)\left(\frac{5}{4g-5}\right)}}\right)$$

$$AGT(T_\eta) = 2g\left(\frac{\frac{8}{4g-4}}{2\sqrt{\left(\frac{4}{4g-4}\right)^2}}\right)+4g\left(\frac{\frac{36g-40}{16g^2-36g+20}}{2\sqrt{\frac{20}{(16g^2-36g+20)}}}\right)+3g\left(\frac{\frac{10}{4g-5}}{2\sqrt{\left(\frac{5}{4g-5}\right)^2}}\right)$$

$$AGT(T_\eta) = 5g+\frac{18g^2-20g}{\sqrt{20g^2-45g+25}}.$$

□

Theorem 16 The general temperature index of convex polytope T_η is

$$8g\left(\frac{1}{g-1}\right)^\kappa+10g\left(\frac{5}{4g-5}\right)^\kappa.$$

Proof. Let T_η be a convex polytope. By definition, we have

$$T_\kappa(T_\eta) = \sum_{edges}(T_r^\kappa+T_s^\kappa).$$

then by using edge partitioning in Table 6, we deduce

$$T_\kappa(T_\eta) = |E_1|\left(\left(\frac{4}{4g-4}\right)^\kappa+\left(\frac{4}{4g-4}\right)^\kappa\right)+|E_2|\left(\left(\frac{4}{4g-4}\right)^\kappa+\left(\frac{5}{4g-5}\right)^\kappa\right)+$$

$$|E_3|\left(\left(\frac{5}{4g-5}\right)^\kappa+\left(\frac{5}{4g-5}\right)^\kappa\right)$$

$$T_\kappa(T_\eta) = 2g\left(2\left(\frac{4}{4g-4}\right)^\kappa\right)+4g\left(\left(\frac{4}{4g-4}\right)^\kappa+\left(\frac{5}{4g-5}\right)^\kappa\right)+3g\left(2\left(\frac{5}{4g-5}\right)^\kappa\right)$$

$$T_\kappa(T_\eta) = 8g\left(\frac{1}{g-1}\right)^\kappa+10g\left(\frac{5}{4g-5}\right)^\kappa \tag{12}$$

□

Corollary 24 The F-temperature index of convex polytope T_η is

$$\frac{8g}{(g-1)^2} + \frac{250g}{(4g-5)^2}.$$

Proof. Put $\kappa = 2$ in Equation 12, then we get the result. □

4.5 *Computational results of temperature indices on η-dimensional convex polytope C_η*

The temperature-based topological indices of the convex polytope C_η with dimension η are calculated in this section. Firstly we introduce the graph of convex polytope C_η in Figure 7.

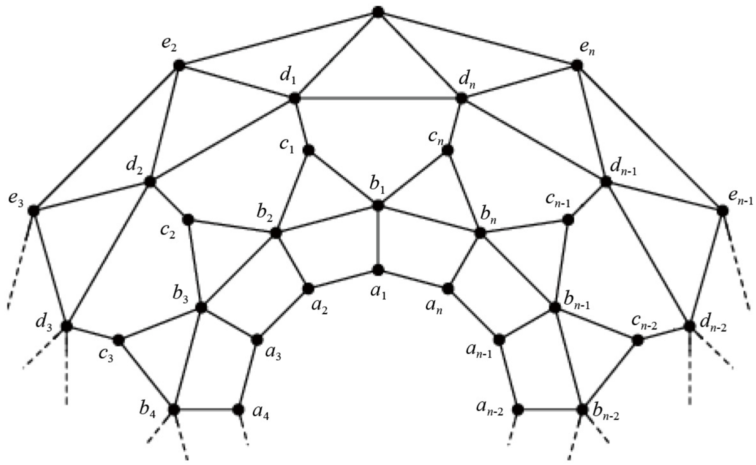

Figure 7. The convex polytope C_η in η dimensions

For any arbitrary vertices r and s of C_η, we denote T_r for temperature index of vertex r and T_s is for temperature index of vertex s then the edge partitioning of the given graph is represented by (T_r, T_s), and it is further explained in Table 7 that follows. We calculate temperature indices in the following theorems based on this edge partitioning of C_η:

Table 7. Edge Partition of convex polytope C_η

$(T_r, T_s) \setminus rs \in E(G)$	Number of edges
$\left(\frac{3}{5g-3}, \frac{3}{5g-3}\right)$	g
$\left(\frac{3}{5g-3}, \frac{5}{5g-5}\right)$	$4g$
$\left(\frac{5}{5g-5}, \frac{5}{5g-5}\right)$	$2g$
$\left(\frac{5}{5g-5}, \frac{4}{5g-4}\right)$	$2g$
$\left(\frac{4}{(5g-4}, \frac{4}{5g-4}\right)$	g

Theorem 17 The convex polytope C_η has the general first temperature index

$$g\left(\frac{6}{5g-4}\right)^{\kappa}+4g\left(\frac{8g-6}{5g^2-8g+3}\right)^{\kappa}+2g\left(\frac{2}{g-1}\right)^{\kappa}+2g\left(\frac{9g-8}{5g^2-9g+4}\right)^{\kappa}+g\left(\frac{8}{5g-4}\right)^{\kappa}.$$

Proof. Let C_η be a convex polytope. By definition, we have

$$T_1^{\kappa}(C_\eta)=\sum_{edges}(T_r+T_s)^{\kappa}.$$

then by using edge partitioning in the Table 7, we deduce

$$T_1^{\kappa}(C_\eta)=|E_1|\left(\frac{3}{5g-3}+\frac{3}{5g-3}\right)^{\kappa}+|E_2|\left(\frac{3}{5g-3}+\frac{5}{5g-5}\right)^{\kappa}+|E_3|\left(\frac{5}{5g-5}+\frac{5}{5g-5}\right)^{\kappa}$$

$$+|E_4|\left(\frac{5}{5g-5}+\frac{4}{5g-4}\right)^{\kappa}+|E_5|\left(\frac{4}{5g-4}+\frac{4}{5g-4}\right)^{\kappa}$$

$$T_1^{\kappa}(C_\eta)=g\left(\frac{6}{5g-4}\right)^{\kappa}+4g\left(\frac{40g-30}{25g^2-40g+15}\right)^{\kappa}+2g\left(\frac{10}{5g-5}\right)^{\kappa}+$$

$$2g\left(\frac{45g-40}{25g^2-45g+20}\right)^{\kappa}+g\left(\frac{8}{5g-4}\right)^{\kappa}$$

$$T_1^{\kappa}(C_\eta)=g\left(\frac{6}{5g-4}\right)^{\kappa}+4g\left(\frac{8g-6}{5g^2-8g+3}\right)^{\kappa}+2g\left(\frac{2}{g-1}\right)^{\kappa}+$$

$$2g\left(\frac{9g-8}{5g^2-9g+4}\right)^{\kappa}+g\left(\frac{8}{5g-4}\right)^{\kappa} \tag{13}$$

□

Corollary 25 The first hyper-temperature index of convex polytope C_η is

$$\frac{36g}{(5g-4)^2}+\left(\frac{256g^3-384g^2+144g}{(5g^2-8g+3)^2}\right)+\frac{8g}{(g-1)^2}+\left(\frac{162g^3-288g^2+128g}{(5g^2-9g+4)^2}\right)+\frac{64g}{(5g-4)^2}.$$

Corollary 26 The sum-connectivity temperature index of convex polytope C_η is

$$\frac{g}{\sqrt{6}}\sqrt{5g-4}+4g\sqrt{\frac{5g^2-8g+3}{8g-6}}+g\sqrt{2g-2}+2g\sqrt{\frac{5g^2-9g+4}{9g-8}}+\frac{g}{\sqrt{8}}\sqrt{5g-4}.$$

Proof. Put $\kappa=2$ & $\kappa=-\frac{1}{2}$ in Equation 13, then we get the required result respectively. □

Theorem 18 The convex polytope C_η has the general second temperature index

$$g\left(\frac{3}{5g-3}\right)^{2\kappa}+4g\left(\frac{3}{5g^2-8g+3}\right)^{\kappa}+2g\left(\frac{1}{g-1}\right)^{2\kappa}+2g\left(\frac{4}{5g^2-9g+4}\right)^{\kappa}+g\left(\frac{4}{5g-4}\right)^{2\kappa}.$$

Proof. Let C_η be a convex polytope. By definition, we have

$$T_2^\kappa(C_\eta)=\sum_{edges}(T_rT_s)^\kappa.$$

then by using edge partitioning in the Table 7, we deduce

$$T_2^\kappa(C_\eta)=|E_1|\left(\frac{3}{5g-3}\times\frac{3}{5g-3}\right)^\kappa+|E_2|\left(\frac{3}{5g-3}\times\frac{5}{5g-5}\right)^\kappa+|E_3|\left(\frac{5}{5g-5}\times\frac{5}{5g-5}\right)^\kappa+$$

$$|E_4|\left(\frac{5}{5g-5}\times\frac{4}{5g-4}\right)^\kappa+|E_5|\left(\frac{4}{5g-4}\times\frac{4}{5g-4}\right)^\kappa$$

$$T_2^\kappa(C_\eta)=g\left(\frac{3}{5g-3}\right)^{2\kappa}+4g\left(\frac{15}{25g^2-40g+15}\right)^{\kappa}+2g\left(\frac{5}{5g-5}\right)^{2\kappa}+$$

$$2g\left(\frac{20}{25g^2-45g+20}\right)^{\kappa}+g\left(\frac{4}{5g-4}\right)^{2\kappa}$$

$$T_2^\kappa(C_\eta)=g\left(\frac{3}{5g-3}\right)^{2\kappa}+4g\left(\frac{3}{5g^2-8g+3}\right)^{\kappa}+2g\left(\frac{1}{g-1}\right)^{2\kappa}+$$

$$2g\left(\frac{4}{5g^2-9g+4}\right)^{\kappa}+g\left(\frac{4}{5g-4}\right)^{2\kappa} \tag{14}$$

□

Corollary 27 The second hyper-temperature index of convex polytope C_η is

$$\frac{81g}{(5g-3)^4}+\frac{36g}{(5g^2-8g+3)^2}+\frac{2g}{(g-1)^4}+\frac{32g}{(5g^2-9g+4)^2}+\frac{256g}{(5g-4)^4}.$$

Corollary 28 The product-connectivity temperature index of convex polytope C_η is

$$\frac{59}{12}g^2-4g+\frac{4g}{\sqrt{3}}\sqrt{5g^2-8g+3}+g\sqrt{5g^2-9g+4}.$$

Corollary 29 The reciprocal product-connectivity temperature index of convex polytope C_η is

$$\frac{3g}{(5g-3)}+\frac{4\sqrt{3}g}{\sqrt{5g^2-8g+3}}+\frac{2g}{(g-1)}+\frac{4g}{\sqrt{5g^2-9g+4}}+\frac{4g}{(5g-4)}.$$

Proof. Put $\kappa = 2, -\frac{1}{2}$ & $\frac{1}{2}$ in Equation 14, then we get the desired result respectively. □

Theorem 19 The arithmetic-geometric temperature index of convex polytope C_η is

$$4g+\left(\frac{16g^2-12g}{\sqrt{15g^2-24g+9}}\right)+\left(\frac{9g^2-8g}{2\sqrt{5g^2-9g+4}}\right).$$

Proof. Let C_η be a convex polytope. By definition, we have

$$AGT(C_\eta)=\sum_{edges}\left(\frac{T_r+T_s}{2\sqrt{T_rT_s}}\right).$$

then by using edge partitioning in the Table 7, we deduce

$$AGT(C_\eta)=|E_1|\left(\frac{\frac{3}{5g-3}+\frac{3}{5g-3}}{2\sqrt{\left(\frac{3}{5g-3}\right)\left(\frac{3}{5g-3}\right)}}\right)+|E_2|\left(\frac{\frac{3}{5g-3}+\frac{5}{5g-5}}{2\sqrt{\left(\frac{3}{5g-3}\right)\left(\frac{5}{5g-5}\right)}}\right)+|E_3|\left(\frac{\frac{5}{5g-5}+\frac{5}{5g-5}}{2\sqrt{\left(\frac{5}{5g-5}\right)\left(\frac{5}{5g-5}\right)}}\right)+$$

$$|E_4|\left(\frac{\frac{5}{5g-5}+\frac{4}{5g-4}}{2\sqrt{\left(\frac{5}{5g-5}\right)\left(\frac{4}{5g-4}\right)}}\right)+|E_5|\left(\frac{\frac{4}{5g-4}+\frac{4}{5g-4}}{2\sqrt{\left(\frac{4}{5g-4}\right)\left(\frac{4}{5g-4}\right)}}\right)$$

$$AGT(C_\eta)=g\left(\frac{\frac{6}{5g-3}}{2\sqrt{\left(\frac{3}{5g-3}\right)^2}}\right)+4g\left(\frac{\frac{40g-30}{(25g^2-40g+15)}}{2\sqrt{\frac{15}{(25g^2-40g+15)}}}\right)+2g\left(\frac{\frac{10}{5g-5}}{2\sqrt{\left(\frac{5}{5g-5}\right)^2}}\right)+$$

$$2g\left(\frac{\frac{45g-40}{(25g^2-45g+20)}}{2\sqrt{\frac{20}{(25g^2-45g+20)}}}\right)+g\left(\frac{\frac{8}{5g-4}}{2\sqrt{\left(\frac{4}{5g-4}\right)^2}}\right)$$

$$AGT(C_\eta)=4g+\left(\frac{16g^2-12g}{\sqrt{15g^2-24g+9}}\right)+\left(\frac{9g^2-8g}{2\sqrt{5g^2-9g+4}}\right).$$

□

Theorem 20 The general F-temperature index of convex polytope C_η is

$$6g\left(\frac{3}{5g-3}\right)^{\kappa}+10g\left(\frac{1}{g-1}\right)^{\kappa}+4g\left(\frac{4}{5g-4}\right)^{\kappa}.$$

Proof. Let C_η be a convex polytope. By definition, we have

$$T_\kappa(C_\eta)=\sum_{edges}(T_r^\kappa+T_s^\kappa).$$

then by using edge partitioning in the Table 7, we deduce

$$\begin{aligned}
T_\kappa(C_\eta)=&|E_1|\left(\left(\frac{3}{5g-3}\right)^{\kappa}+\left(\frac{3}{5g-3}\right)^{\kappa}\right)+|E_2|\left(\left(\frac{3}{5g-3}\right)^{\kappa}+\left(\frac{5}{5g-5}\right)^{\kappa}\right)+\\
&|E_3|\left(\left(\frac{5}{5g-5}\right)^{\kappa}+\left(\frac{5}{5g-5}\right)^{\kappa}\right)+|E_4|\left(\left(\frac{5}{5g-5}\right)^{\kappa}+\left(\frac{4}{5g-4}\right)^{\kappa}\right)+\\
&|E_5|\left(\left(\frac{4}{5g-4}\right)^{\kappa}+\left(\frac{4}{5g-4}\right)^{\kappa}\right)\\
T_\kappa(C_\eta)=&g\left(2\left(\frac{3}{5g-3}\right)^{\kappa}\right)+4g\left(\left(\frac{3}{5g-3}\right)^{\kappa}+\left(\frac{5}{5g-5}\right)^{\kappa}\right)+\\
&2g\left(2\left(\frac{5}{5g-5}\right)^{\kappa}\right)+2g\left(\left(\frac{5}{5g-5}\right)^{\kappa}+\left(\frac{4}{5g-4}\right)^{\kappa}\right)+\\
&g\left(2\left(\frac{4}{5g-4}\right)^{\kappa}\right)\\
T_\kappa(C_\eta)=&6g\left(\frac{3}{5g-3}\right)^{\kappa}+10g\left(\frac{1}{g-1}\right)^{\kappa}+4g\left(\frac{4}{5g-4}\right)^{\kappa} \qquad (15)
\end{aligned}$$

□

Corollary 30 The F-temperature index of convex polytope C_η is

$$\frac{54g}{(5g-3)^2}+\frac{10g}{(g-1)^2}+\frac{64g}{(5g-4)^2}.$$

Proof. Put $\kappa=2$ in Equation 15, then we obtain the result. □

4.6 *Computational results of temperature indices on η-dimensional convex polytope D_η*

The temperature-based topological indices of the convex polytope D_η with dimension η are calculated in this section. Firstly we introduce the graph of convex polytope D_η in Figure 8.

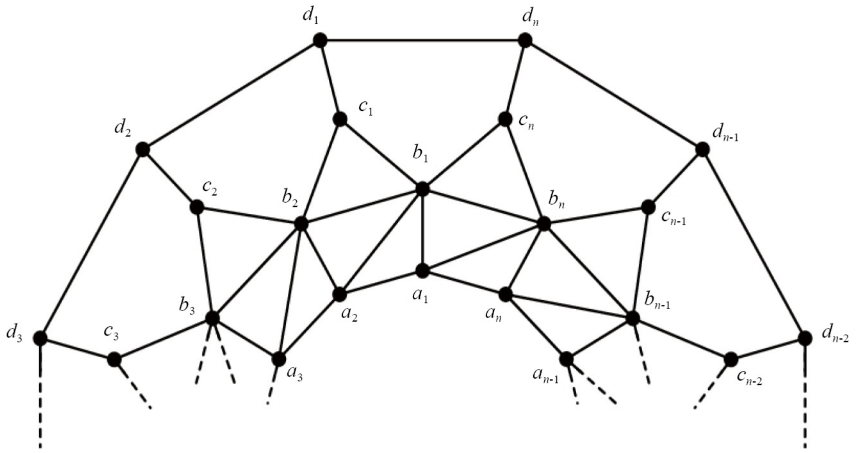

Figure 8. The convex polytope D_η in η dimensions

For any arbitrary vertices r and s of D_η we denote T_r for the temperature index of vertex r and T_s is for the temperature index of vertex s then the edge partitioning of the given graph is represented by (T_r, T_s), and it is further explained in Table 8 that follows. We calculate temperature indices in the following theorems based on this edge partitioning of D_η:

Table 8. Edge Partition of convex polytope D_η

$(T_r, T_s) \setminus rs \in E(G)$	Number of edges
$\left(\frac{4}{4g-4}, \frac{4}{4g-4}\right)$	g
$\left(\frac{4}{4g-4}, \frac{6}{4g-6}\right)$	$2g$
$\left(\frac{6}{4g-6}, \frac{6}{4g-6}\right)$	g
$\left(\frac{6}{4g-6}, \frac{3}{4g-3}\right)$	$2g$
$\left(\frac{3}{4g-3}, \frac{3}{4g-3}\right)$	$2g$

Theorem 21 The convex polytope D_η has general first temperature index

$$g\left(\frac{2}{g-1}\right)^{\kappa}+2g\left(\frac{5g-6}{2g^2-5g+3}\right)^{\kappa}+g\left(\frac{6}{2g-3}\right)^{\kappa}+2g\left(\frac{18(g-1)}{8g^2-18g+9}\right)^{\kappa}+2g\left(\frac{6}{4g-3}\right)^{\kappa}.$$

Proof. Let D_η be a convex polytope. By definition, we have

$$T_1^{\kappa}(D_\eta) = \sum_{edges} (T_r + T_s)^{\kappa}.$$

then by using edge partitioning in the Table 8, we deduce

$$T_1^{\kappa}(D_\eta) = |E_1|\left(\frac{4}{4g-4}+\frac{4}{4g-4}\right)^{\kappa} + |E_2|\left(\frac{4}{4g-4}+\frac{6}{4g-6}\right)^{\kappa} + |E_3|\left(\frac{6}{4g-6}+\frac{6}{4g-6}\right)^{\kappa} +$$

$$|E_4|\left(\frac{6}{4g-6}+\frac{3}{4g-3}\right)^{\kappa} + |E_5|\left(\frac{3}{4g-3}+\frac{3}{4g-3}\right)^{\kappa}$$

$$T_1^{\kappa}(D_\eta) = g\left(\frac{8}{4g-4}\right)^{\kappa} + 2g\left(\frac{40g-48}{16g^2-40g+24}\right)^{\kappa} + g\left(\frac{12}{4g-6}\right)^{\kappa} +$$

$$2g\left(\frac{36g-36}{16g^2-36g+18}\right)^{\kappa} + 2g\left(\frac{6}{4g-3}\right)^{\kappa}$$

$$T_1^{\kappa}(D_\eta) = g\left(\frac{2}{g-1}\right)^{\kappa} + 2g\left(\frac{5g-6}{2g^2-5g+3}\right)^{\kappa} + g\left(\frac{6}{2g-3}\right)^{\kappa} +$$

$$2g\left(\frac{18(g-1)}{8g^2-18g+9}\right)^{\kappa} + 2g\left(\frac{6}{4g-3}\right)^{\kappa} \tag{16}$$

□

Corollary 31 The first hyper-temperature index of convex polytope D_η is

$$\frac{4g}{(g-1)^2} + \frac{50g^3-120g^2+72g}{(2g^2-5g+3)^2} + \frac{36g}{(2g-3)^2} + \frac{648g^3-1296g^2+648g}{(8g^2-18g+9)^2} + \frac{72g}{(4g-3)^2}.$$

Corollary 32 The sum-connectivity temperature index of convex polytope D_η is

$$\frac{g}{\sqrt{2}}\sqrt{g-1} + 2g\sqrt{\frac{2g^2-5g+3}{5g-6}} + \frac{g}{\sqrt{6}}\sqrt{2g-3} + \frac{g}{3}\sqrt{\frac{16g^2-36g+18}{g-1}} + \frac{g}{\sqrt{3}}\sqrt{8g-6}.$$

Proof. Put $\kappa = 2$ & $\kappa = -\frac{1}{2}$ in Equation 16, then we get the desired result respectively. □

Theorem 22 The convex polytope D_η has the general second temperature index

$$g\left(\frac{1}{g-1}\right)^{2\kappa} + 2g\left(\frac{3}{2g^2-5g+3}\right)^{\kappa} + g\left(\frac{3}{2g-3}\right)^{2\kappa} + 2g\left(\frac{9}{8g^2-18g+9}\right)^{\kappa} + 2g\left(\frac{3}{4g-3}\right)^{2\kappa}.$$

Proof. Let D_η be a convex polytope. By definition, we have

$$T_2^\kappa(D_\eta) = \sum_{edges}(T_rT_s)^\kappa.$$

then by using edge partitioning in the Table 8, we deduce

$$\begin{aligned}
T_2^\kappa(D_\eta) =&|E_1|\left(\frac{4}{4g-4}\times\frac{4}{4g-4}\right)^\kappa+|E_2|\left(\frac{4}{4g-4}\times\frac{6}{4g-6}\right)^\kappa+|E_3|\left(\frac{6}{4g-6}\times\frac{6}{4g-6}\right)^\kappa+\\
&|E_4|\left(\frac{6}{4g-6}\times\frac{3}{4g-3}\right)^\kappa+|E_5|\left(\frac{3}{4g-3}\times\frac{3}{4g-3}\right)^\kappa\\
T_2^\kappa(D_\eta) =&g\left(\frac{4}{4g-4}\right)^{2\kappa}+2g\left(\frac{24}{16g^2-40g+24}\right)^\kappa+g\left(\frac{6}{4g-6}\right)^{2\kappa}+\\
&2g\left(\frac{18}{16g^2-36g+18}\right)^\kappa+2g\left(\frac{3}{4g-3}\right)^{2\kappa}\\
T_2^\kappa(D_\eta) =&g\left(\frac{1}{g-1}\right)^{2\kappa}+2g\left(\frac{3}{2g^2-5g+3}\right)^\kappa+g\left(\frac{3}{2g-3}\right)^{2\kappa}+\\
&2g\left(\frac{9}{8g^2-18g+9}\right)^\kappa+2g\left(\frac{3}{4g-3}\right)^{2\kappa}
\end{aligned}\tag{17}$$

□

Corollary 33 The second hyper-temperature index of convex polytope D_η is

$$\frac{g}{(g-1)^4}+\frac{18g}{(2g^2-5g+3)^2}+\frac{81g}{(2g-3)^4}+\frac{162g}{(8g^2-18g+9)^2}+\frac{162g}{(4g-3)^4}.$$

Corollary 34 The product-connectivity temperature index of convex polytope D_η is

$$\frac{13}{3}g^2-4g+\frac{2g}{\sqrt{3}}\sqrt{2g^2-5g+3}+\frac{2g}{3}\sqrt{8g^2-18g+9}.$$

Corollary 35 The reciprocal product-connectivity temperature index of convex polytope D_η is

$$\frac{g}{g-1}+\frac{2\sqrt{3}g}{\sqrt{2g^2-5g+3}}+\frac{3g}{2g-3}+\frac{6g}{\sqrt{8g^2-18g+9}}+\frac{6g}{4g-3}.$$

Proof. Put $\kappa = 2$, $\frac{-1}{2}$ & $\frac{1}{2}$ in Equation 17, then we get the desired result respectively. $\square$

Theorem 23 The arithmetic-geometric temperature index of convex polytope D_η is

$$4g + \left(\frac{5g^2 - 6g}{\sqrt{6g^2 - 15g + 9}}\right) + \left(\frac{6g^2 - 6g}{\sqrt{8g^2 - 18g + 9}}\right).$$

Proof. Let D_η be a convex polytope. By definition, we have

$$AGT(D_\eta) = \sum_{edges} \left(\frac{T_r + T_s}{2\sqrt{T_r T_s}}\right).$$

then by using edge partitioning in Table 8, we deduce

$$AGT(D_\eta) = |E_1| \left(\frac{\frac{4}{4g-4} + \frac{4}{4g-4}}{2\sqrt{\left(\frac{4}{4g-4}\right)\left(\frac{4}{4g-4}\right)}}\right) + |E_2| \left(\frac{\frac{4}{4g-4} + \frac{6}{4g-6}}{2\sqrt{\left(\frac{4}{4g-4}\right)\left(\frac{6}{4g-6}\right)}}\right) + |E_3| \left(\frac{\frac{6}{4g-6} + \frac{6}{4g-6}}{2\sqrt{\left(\frac{6}{4g-6}\right)\left(\frac{6}{4g-6}\right)}}\right) +$$

$$|E_4| \left(\frac{\frac{6}{4g-6} + \frac{3}{4g-3}}{2\sqrt{\left(\frac{6}{4g-6}\right)\left(\frac{3}{4g-3}\right)}}\right) + |E_5| \left(\frac{\frac{3}{4g-3} + \frac{3}{4g-3}}{2\sqrt{\left(\frac{3}{4g-3}\right)\left(\frac{3}{4g-3}\right)}}\right)$$

$$AGT(D_\eta) = g \left(\frac{\frac{8}{4g-4}}{2\sqrt{\left(\frac{4}{4g-4}\right)^2}}\right) + 2g \left(\frac{\frac{40g-48}{16g^2-40g+24}}{2\sqrt{\frac{24}{16g^2-40g+24}}}\right) + g \left(\frac{\frac{12}{4g-6}}{2\sqrt{\left(\frac{6}{4g-6}\right)^2}}\right) +$$

$$2g \left(\frac{\frac{36g-36}{(16g^2-36g+18)}}{2\sqrt{\frac{18}{(16g^2-36g+18)}}}\right) + 2g \left(\frac{\frac{6}{4g-3}}{2\sqrt{\left(\frac{3}{4g-3}\right)^2}}\right)$$

$$AGT(D_\eta) = 4g + \left(\frac{5g^2 - 6g}{\sqrt{6g^2 - 15g + 9}}\right) + \left(\frac{6g^2 - 6g}{\sqrt{8g^2 - 18g + 9}}\right).$$

$\square$

Theorem 24 The general temperature index of convex polytope D_η is

$$4g\left(\frac{1}{g-1}\right)^\kappa + 6g\left(\frac{6}{4g-6}\right)^\kappa + 6g\left(\frac{3}{4g-3}\right)^\kappa.$$

Proof. Let D_η be a convex polytope. By definition, we have

$$T_\kappa(D_\eta) = \sum_{edges} (T_r^\kappa + T_s^\kappa).$$

then by using edge partitioning in Table 8, we deduce

$$\begin{aligned} T_\kappa(D_\eta) =& |E_1|\left(\left(\frac{4}{4g-4}\right)^\kappa + \left(\frac{4}{4g-4}\right)^\kappa\right) + |E_2|\left(\left(\frac{4}{4g-4}\right)^\kappa + \left(\frac{6}{4g-6}\right)^\kappa\right) + \\ & |E_3|\left(\left(\frac{6}{4g-6}\right)^\kappa + \left(\frac{6}{4g-6}\right)^\kappa\right) + |E_4|\left(\left(\frac{6}{4g-6}\right)^\kappa + \left(\frac{3}{4g-3}\right)^\kappa\right) + \\ & |E_5|\left(\left(\frac{3}{4g-3}\right)^\kappa + \left(\frac{3}{4g-3}\right)^\kappa\right) \\ T_\kappa(D_\eta) =& g\left(2\left(\frac{4}{4g-4}\right)^\kappa\right) + 2g\left(\left(\frac{4}{4g-4}\right)^\kappa + \left(\frac{6}{4g-6}\right)^\kappa\right) + \\ & g\left(2\left(\frac{6}{4g-6}\right)^\kappa\right) + 2g\left(\left(\frac{6}{4g-6}\right)^\kappa + \left(\frac{3}{4g-3}\right)^\kappa\right) + 2g\left(2\left(\frac{3}{4g-3}\right)^\kappa\right) \\ T_\kappa(D_\eta) =& 4g\left(\frac{1}{g-1}\right)^\kappa + 6g\left(\frac{6}{4g-6}\right)^\kappa + 6g\left(\frac{3}{4g-3}\right)^\kappa \end{aligned} \tag{18}$$

□

Corollary 36 The F-temperature index of convex polytope D_η is

$$\frac{4g}{(g-1)^2} + \frac{216g}{(4g-6)^2} + \frac{54g}{(4g-3)^2}.$$

Proof. Put $\kappa = 2$ in Equation 18, then we obtain the result. □

5. Conclusions

In this study, a special family of geometrical graphs i.e., convex polytopes and their temperature indices are discussed. We emphasized different convex polytopes, also computed the general first temperature index, the general second temperature index, the first hyper-temperature index, the second hyper-temperature index, the sum-connectivity temperature index, the product-connectivity temperature index, the reciprocal product-connectivity temperature index, the arithmetic-geometric temperature index and the F-temperature index of all these convex polytopes.

Future work may include the extension of this study:

• Find sharp upper and lower bounds on the arithmetic-geometric temperature index of graphs with a given number of vertices, the number of edges, the vertex connectivity, and the edge-connectivity.

• Do the computational results in this paper on temperature indices of convex polytopes deliver some topological insights to better characterize their geometrical structure?

Acknowledgement

The authors are grateful to all the reviewers for their remarks and constructive criticism which significantly improved the submitted version of the paper.

Disclosure

The authors declare that no financial or other disclosures are to be declared.

Conflict of interest

The authors declare no competing financial interest.

References

[1] Basak SC, Balaban AT, Grunwald GD, Gute BD. Topological indices: Their nature and mutual relatedness. *Journal of Chemical Information and Computer Sciences*. 2000: 40(4): 891-898.
[2] Asadpour J. Computing some topological indices of nano structures of bridge graph. *Digest Journal of Nanomaterials and Biostructures*. 2012; 7(1): 19-22.
[3] Kulli VR. On K Banhatti indices and K hyper-Banhatti indices of V-Phenylenic nanotubes and nanotorus. *Journal of Computer and Mathematical Sciences*. 2016; 7(6): 302-307.
[4] Kulli VR. Some multiplicative temperature indices of $HC_5C_7[p, q]$ nanotubes. *International Journal of Fuzzy Mathematical Archive*. 2019; 17(2): 91-98.
[5] Kulli VR. Computing some multiplicative temperature indices of certain networks. *Journal of Mathematics and Informatics*. 2020; 18; 139-143.
[6] Kulli VR. Inverse sum temperature index and multiplicative inverse temperature index of certain nanotubes. *International Journal of Recent Scientific Research*. 2021; 12: 40935-40939.
[7] Kulli VR. The (a, b)-temperature index of H-Naphtalenic nanotubes. *Annals of Pure and Applied Mathematics*. 2019; 20(2): 85-90.
[8] Kulli VR. Multiplicative (a, b)-KA temperature indices of certain nanostructure. *International Journal of Mathematics Trends and Technology*. 2020; 66(5): 137-142.
[9] Kulli VR. (a, b)-KA temperature indices of tetrameric 1,3-adamantane. *International Journal of Recent Scientific Research*. 2021; 12: 40929-40933.
[10] Jahanbani A, Khoeilar R, Cancan M. On the temperature indices of molecular structures of some networks. *Journal of Mathematics*. 2023; 2023: 815268.
[11] Zhang Y, Khalid A, Siddiqui MK, Rehman H, Ishtiaq M, Cancan M. On analysis of temperature based topological indices of some Covid-19 drugs. *Polycyclic Aromatic Compounds*. 2023; 43(4): 3810-3826.
[12] Kulli VR. Temperature Sombor and temperature Nirmala indices. *International Journal of Mathematics and Computer Research*. 2022; 10(9): 2910-2915.
[13] Zhang YF, Ghani MU, Sultan F, Inc M, Cancan M. Connecting SiO_4 in silicate and silicate chain networks to compute Kulli temperature indices. *Molecules*. 2022; 27(21): 7533.
[14] Kulli VR. Some new temperature indices of oxide and honeycomb networks. *Annals of Pure and Applied Mathematics*. 2020; 21(2): 129-133.

[15] Hayat S, Khan A, Khan S, Liu J-B. Hamilton connectivity of convex polytopes with applications to their detour index. *Complexity*. 2021; 2021: 684784.

[16] Turaci T. On combinatorial properties of two convex polytopes via eccentricity based topological indices. *Journal of Modern Technology and Engineering*. 2022; 7(3): 216-225.

[17] Yoong KK, Hasni R, Lau GC, Asim MA, Ahmad A. Reflexive edge strength of convex polytopes and corona product of cycle with path. *AIMS Mathematics*. 2022; 7(7): 11784-11800.

[18] Imran M, Siddiqui HMA. Computing the metric dimension of convex polytopes generated by wheel related graphs. *Hungarian Mathematical Journal*. 2016; 149(1): 10-30.

[19] Hayat S, Malik MYH, Ahmad A, Khan S, Yousafzai F, Hasni R. On hamilton-connectivity and detour index of certain families of convex polytopes. *Mathematical Problems in Engineering*. 2021; 2021(1): 5553216.

[20] Bača M. On magic labellings of convex polytopes. *Annals of Discrete Mathematics*. 1992; 51: 13-16.

[21] Bača M. Labelings of two classes of convex polytopes. *Mathematical Utility*. 1988; 34: 24-31.

[22] Imran M, Bokhary SAUH, Baig AQ. On the metric dimension of rotationally-symmetric convex polytopes. *Journal of Algebra Combinatorics Discrete Structures and Applications*. 2015; 3(2): 45-49.

[23] Gutman I, Tošović J. Testing the quality of molecular structure descriptors. Vertex-degree-based topological indices. *Journal of the Serbian Chemical Society*. 2013; 78(6): 805-810.

[24] NIST Standard Reference Database. *NIST Chemistry WebBook, SRD 69*. National Institute of Standards and Technology, U.S. Department of Commerce; 2023. Available from: https://doi.org/10.18434/T4D303 [Accessed 27th October 2023].

[25] Hayat S, Khan A, Ali K, Liu J-B. Structure-property modeling for thermodynamic properties of benzenoid hydrocarbons by temperature-based topological indices. *Ain Shams Engineering Journal*. 2024; 15(3): 102586.

[26] Hayat S, Liu J-B. Comparative analysis of temperature-based graphical indices for correlating the total π-electron energy of benzenoid hydrocarbons. *International Journal of Modern Physics B*. 2024; 2550047. Available from: https://doi.org/10.1142/S021797922550047X.

[27] Diudea MV, Gutman I, Lorentz J. *Molecular topology*. Nova Science Publishers; 1997.

[28] Buck G, Litherland RA. The geometry of molecular models. *Mathematical Intelligencer*. 1997; 19(3): 30-37.

[29] Zhou H, Du G. Computing symmetry and stability of molecules with convex polytopes. *Journal of Molecular Modeling*. 2008; 14(8): 753-763.

[30] Balaban AT. Applications of graph theory in chemistry. *Journal of Chemical Information and Computer Sciences*. 1983; 23(5): 268-275.