Contemporary Mathematics
http://ojs.wiserpub.com/index.php/CM/

UNIVERSAL WISER PUBLISHER

Research Article

# Optimization of Fuzzy Mathematical Model of Regular Octagon-Shaped Parking Space

**Arun Prasath GM[1*], Saad Salman Ahmed[1], Bushra Hibras Al Sulaimi[1], Kamal Kumar M[1], Madhusudhana Rao B[1], Alia Khalfan Mohamed Salim Al Rahbi[2], Sara Ahmed Said Hilal Al Barwani[3], Abdulmalik Said Harib Hassan Al Hadhrami[3], Khadija Bilal Yousuf Al Bahrani[3]**

[1]Department of Information Technology, College of Computing and Information Sciences, University of Technology and Applied Sciences, Muscat 133, Oman
[2]Department of Applied Sciences, College of Applied Sciences and Pharmacy, University of Technology and Applied Sciences, Muscat 133, Oman
[3]Department of Engineering, College of Applied Sciences and Pharmacy, University of Technology and Applied Sciences, Muscat 133, Oman
E-mail: arun.prasath@utas.edu.om

**Received:** 2 August 2024; **Revised:** 23 September 2024; **Accepted:** 8 October 2024

**Abstract:** In the vigorous development of the city population, there is a need to set up clear dimensions for the parking space. Car parking plays a significant part in the residential apartments and commercial buildings. Roadside parking spaces will create a huge traffic problem if the parking lots are not properly designed. Sometimes, it leads to the accident. Hence, keeping all these causes in mind, the flexible parking space is the necessary of the time. In the proposed research work, the parking space considered in the problem is of a regular octagon shape, and the mathematical model is developed under a fuzzy environment. LINGO Optimization Software is used to solve the mathematical model and MATLAB (Matrix Laboratory) is used to define the fuzzy variable. At the outset, the results will reveal the importance of making the length of the parking lot under fuzzy environment.

***Keywords*:** linear programming problem, LINGO, parking space, optimization, triangular fuzzy number, MATLAB

**MSC:** 9004, 90C05, 90B06, 90B20, 90C70, 90C90

## Abbreviation

| | |
|---|---|
| IR | Interior Rectangle |
| ER | Exterior Rectangles |
| RT | Right-angled Triangles |
| DM | Dimension |


DOI: https://doi.org/10.37256/cm.5420245435

# 1. Introduction

We acknowledge that the submitted paper is the extended version of the article “Optimization of regular octagon-shaped parking space” authored by Saad Salman Ahmed and Arun Prasath GM in 2021 [1]. Reducing the length of the parking lot will consequently increase the number of parking lots in the allotted parking space. So, in this paper, the length of the parking lot will be considered a variable, and its lower and upper values are defined as 5.0 m and 5.5 m, respectively.

The design of the parking lots is strategically orientated towards maximizing the number of parking lots and efficient utilization of the allotted parking space. Key determinants for optimizing parking space include dimensions, layout, angles, and accommodation for a range of vehicle sizes and types. Various vehicles necessitate tailored parking spaces and configurations. This study focuses specifically on different types and sizes of cars.

The parking angles under consideration will be denoted as and as illustrated in Figure 1. These angles represent widely utilized configurations in parking lot scenarios. The selection of parking angles holds a significant sway in the optimization of parking space.

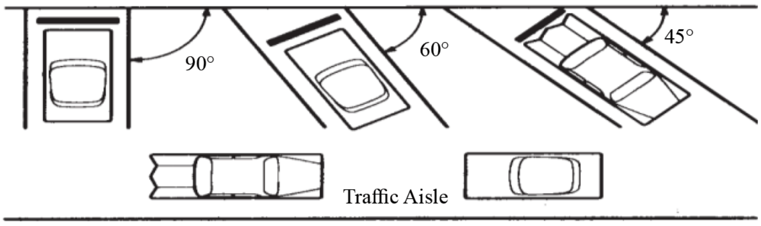


**Figure 1.** Parking angles

Different types of cars with different body types are given in Figure 2 and are available in the market. Most of the cars have the same breadth and different lengths. In Muscat, the parking lot size is fixed at 2.5 $m \times 5.5\ m$. Dubai municipality fixed the parking lot dimension as 2.5 $m \times 5.0\ m$ [2], and Oman fixed the parking lot dimension for the Duqm city urban planning and development project as 2.5 $m \times 5.5\ m$ [1].

Here the discussion arises about the length of the parking lot. Different countries follow parking lot dimensions for their own convenience. After the observation, it is understood that the countries fixed the breadth of the parking lot and compromised its length. With the use of sonar sensors, the car decides the turning angle, and with the use of fuzzy logic, the car can automatically decide the motion direction [3].

A cloud-based parking lot reservation system was created, and it will monitor the available parking lots for the cars. The time consumption and fuel consumption will be reduced by reserving the parking lot [4]. The availability of the parking lot and the weight of the car are considered to decide the suitable parking lot to maintain the equilibrium of the parking space [5]. Most of the research papers focus on parking the cars in the suitable parking lot. A novel approach is taken in this paper to consider the parking lot length as a variable under a fuzzy environment to optimize the number of parking lots in the available parking space.

In this paper, the mathematical model for octagon-shaped parking spaces is taken under a fuzzy environment. The length of the parking lot is taken as a fuzzy variable. Also, three different parking lot dimensions are taken into consideration for a numerical example. It is denoted as follows: Dimension 1 (DM1: 2.5 $m \times 5.0\ m$), Dimension 2 (DM2: 2.5 $m \times 5.25\ m$), and Dimension 3 (DM3: 2.5 $m \times 5.5\ m$).

In the allotted parking space, vehicles can be parked for long term or short term. Transportation infrastructure includes parking lots as the essential one. Always to utilize the parking area effectively, the parking lots will be planned to accommodate the maximum number of vehicles. The influential factors in optimizing the parking area will be the length and breadth of the parking lot, parking area design, parking angles, and variety of vehicles with different sizes. Different types and sizes of vehicles will require different sizes of parking lots and parking modes. In this paper, different types and sizes of cars are only considered. Common angles 0°, 45°, 60° and 90° considered in parking lots given in Figure 1 are used in the developed parking lot problems. Parking angles are playing an influential role in optimizing the parking space.

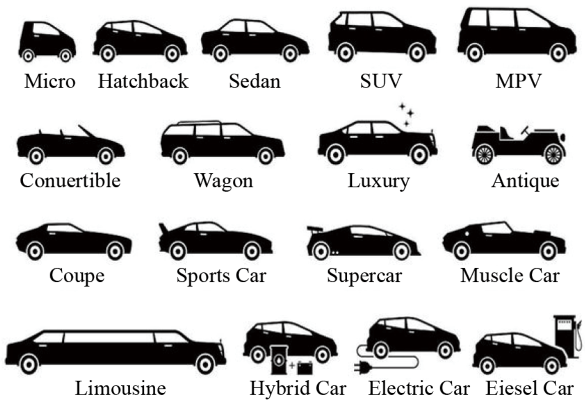


**Figure 2.** Different types of cars

## 2. Literature review

To attain the optimum number of parking lots with various designs and dimensions, the mathematical model was developed [2]. Different algorithms are used to obtain the optimum results of the RPCDV mathematical model of the parking lot network graph of the evaluation index and suitable path search efficiency.

There is a need to allot parking lots in maximum numbers and utilize parking space effectively. The optimization of car parking space is done by using the Evolutionary Algorithm with its customized fitness function. Parking designs are separated as perpendicular, diagonal, and parallel [6]. Automated vehicle's impact on the parking facilities is analyzed by using the mixed integer nonlinear mathematical model and the results are compared with the standard industry requirements of the manual driving vehicles [7].

The inadequate public parking spaces in busy urban areas need to be analyzed by using a mathematical model. New public parking sites need to be created to minimize the environmental and economic costs. The fuzzy MCDM, AHP, and fuzzy AHP-TOPSIS are used to address the selection criteria for site selection and the challenges and uncertainties in the decision-making process of finding the new public parking lot locations [8]. The vehicle door, size of the vehicle, and free space are the effective measures in the parking lots. There will be more unutilized spaces in the parking space of triangular shape. In the procedure of consuming the maximum area of the allotted parking space and obtaining the parking lots in maximum numbers, the mathematical model of different types of triangles likely equilateral and isosceles is developed and solved using the integer linear programming method.

In recent times, most of the buildings are old with less number of parking stalls. Due to the low number of parking lots, some cars are parked randomly near the parking lots making unnecessary traffic near to the parking lots. To try to increase the number of parking lots, a formation of cutting-stock is used for redesigning the available stalls on the university campus and to increase the number of parking stalls [9]. The network graph connecting parking lots was developed and the optimal path was identified by using algorithm [10].

In the supermarkets with a huge number of customers and a big company with a huge number of workers will face the spatial efficiency in the parking lot shape. A mathematical model was developed with a single steering manoeuvre to improve the spatial efficiency and the parking angles were taken as variables. The angled car parking lot is more than 10 percent more efficient than the rectangular car parking lot [11].

The parking spare area of parallelogram shape consists rectangle and two right triangles to optimize the parking lot numbers [12]. The parallelogram shape land discussed in two different cases for motorbikes and cars namely parallelogram direct shape and parallelogram separated into two right triangles and one rectangle. After the discussion, it is noticed that the parallelogram separation is more efficient than parallelogram direct shape [13]. The mathematical model of triangular-shaped parking space was developed [14].

## 3. Parking design

### 3.1 *Notations*

Following notations are used in the mathematical model:

$a$-length of the regular octagon

$b$-Breadth of the outer rectangle

$d$-Length of the inner rectangle

$A_1$-Bay width

$A_2$-Bay width, Parallel to aisle

$B_1$-Bay length

$B_2$-Length of line between bays

$C_1$-Bay depth to wall

$C_2$-Bay depth to curb

$C_3$-Bay depth to interlock

$D$-Aisle width between bay lines

$E$-Bumper overhang (typical)

$F_1$-Module, wall to interlock

$F_2$-Module, curb to interlock

$F_3$-Module interlock to interlock

$F_4$-Interior width

$LIR$-Width of outer and inner bay lines of IR

$LER$-Width of outer and inner bay lines of ER

$L_1$-Length of the first bay line

$L_2$-Length of the second bay line

$L_3$-Length of the third bay line

$L_4$-Length of the fourth bay line

### 3.2 *Parking space design*

The pictorial representation of the proposed parking area with a regular octagon shape is given in Figure 3. The parking area consists of three parts as given in Figure 3.

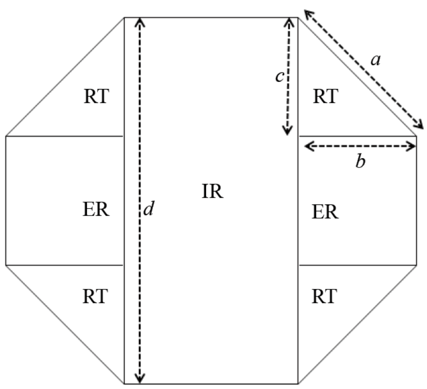


**Figure 3.** Pictorial representation of regular octagon shaped parking space

For the proposed parking design, the pictorial representation of rectangular parking spaces is presented in Figure 4. For angles 0°, 45°, 60° and 90°, the following notations are used:

NORIR$\theta$ = total number of rows outside of the interior rectangle with as the angle $\theta$

NIRIR$\theta$ = total number of rows inside of the interior rectangle with as the angle $\theta$

NORER$\theta$ = total number of rows outside of the exterior rectangle with the angle $\theta$

NIRER$\theta$ = total number of rows inside of the exterior rectangle with as the angle $\theta$

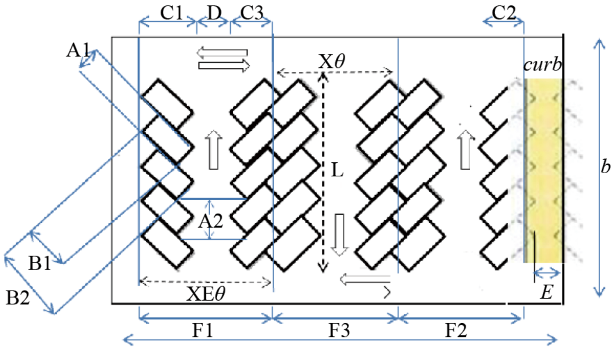


**Figure 4.** Pictorial representation of rectangular shaped parking area

The pictorial representation of a right triangle parking space is given in Figure 5. The variable value for F4 is taken for the discussion. In addition, the height or length of the first, second, third, and fourth parking rows in the interior of the rectangle of a right triangle is given as $L_1$, $L_2$, $L_3$, $L_4$. For the right triangle parking space design, the following are the notations used for different angles 0°, 45°, 60° and 90°.

IT$\theta$ = total number of rows in the interior area of the triangle with $\theta$ as the angle

XET$\theta$ = total number of rows in the external area of the triangle with $\theta$ as the angle

XT$\theta$ = quantity of rows filled in the interior area of the triangle with $\theta$ as the angle

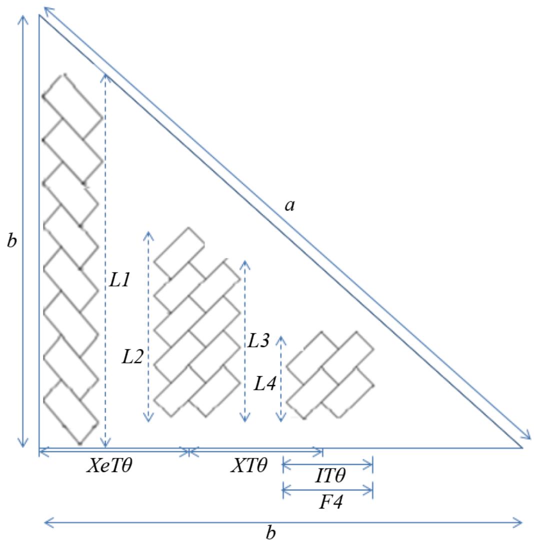


**Figure 5.** Pictorial representation of right triangle parking space

## 4. Triangular fuzzy number

Triangular Fuzzy Number [15] is a number Ã=(a,b,c) if its membership function is given by

$$\mu_A(x) = \begin{cases} 0, & x < a \\ \dfrac{x-a}{b-a}, & a \le x \le b \\ \dfrac{c-x}{c-b}, & b \le x \le c \\ 0, & x > c \end{cases} \tag{1}$$

## 5. Mathematical model for the proposed parking design

Mathematical model for the proposed regular octagon shaped parking space is developed by using the following assumptions:

### 5.1 *Assumptions*

1. Exterior lines and the wall are not adjacent
2. One-way circulation is considered across the aisles
3. Bay length is taken under fuzzy environment
4. Two exits/entries will be available
5. Two-way circulation is considered in perpendicular to the rows
6. At the adjacent to the wall, no exterior rows are allowed

Assumption (5) can be taken as one-way traffic perpendicular to the rows, and assumption (6) can be taken as the exterior rows are adjacent to the wall according to the parking lot design. The objective function and constraints of the proposed mathematical model are given below. The length of the parking lot is considered a fuzzy environment. The proposed mathematical model includes the following decision variables for different angles 0°,45°,60°, and 90°.

### 5.2 *Notations*

NET$\theta$ = total cars in the exterior surface with $\theta$ as angle
NT$\theta$ = total cars in the interior surface with $\theta$ as angle
NIT$\theta$ = total cars in the interior surface with $\theta$ as angle
NIPER$\theta$ = parking area available inside in the rectangle in exterior with $\theta$ as angle
NOPER$\theta$ = parking area available outside in the rectangle in exterior with $\theta$ as angle
NIPIR$\theta$ = parking area available inside in the inner rectangle with $\theta$ as angle
NOPIR$\theta$ = parking area available outside in the inner rectangle with $\theta$ as angle

### 5.3 *Objective function*

The objective of the proposed mathematical model of the regular octagon-shaped parking space is to maximize the number of cars in the allotted parking space. With the constraints like space, parking lot dimensions, and parking angles. The objective function is defined in equation (2).

$$Max\ \tilde{z} = \sum_{\theta=0^\circ,\ 45^\circ,\ 60^\circ,\ 90^\circ} (NIPIR\theta + NOPIR\theta + 2NIPER\theta + 2NOPER\theta + 4NET\theta + 4NT\theta + 4NIT\theta) \tag{2}$$

## 5.4 *Constraints*

In the regular octagon-shaped parking space, one interior rectangle, two exterior rectangles, and four right-angled triangles are available. Equations (3), (4), (5), (6), (10)-(13) are related to the interior and exterior rectangular-shaped parking space. Equation (5), (8), (9), (14)-(16) are related to the right-angled triangle shaped parking space. Equation (3) and (4) discussed that the sum of the lengths of the number of cars in the interior row and the exterior row should not be more than the length of the parking space. Equation (6) and (7) discussed that the total number of outside rows should be less than or equal to two.

$$\sum_{\theta=0^\circ,\ 45^\circ,\ 60^\circ,\ 90^\circ} (\tilde{F}_3 NIRIR\theta + (D + \tilde{C}_1) NORIR\theta) \le d \tag{3}$$

$$\sum_{\theta=0^\circ,\ 45^\circ,\ 60^\circ,\ 90^\circ} (\tilde{F}_3 NIRER\theta + (D + \tilde{C}_1) NORER\theta) \le a \tag{4}$$

$$\sum_{\theta=0^\circ,\ 45^\circ,\ 60^\circ,\ 90^\circ} (\tilde{F}_1 XET\theta + \tilde{F}_3 XT\theta + \tilde{F}_4 IT\theta) \le b \tag{5}$$

$$\sum_{\theta=0^\circ,\ 45^\circ,\ 60^\circ,\ 90^\circ} NORIR\theta \le 2 \tag{6}$$

$$\sum_{\theta=0^\circ,45^\circ,60^\circ,90^\circ} NORER\theta \le 2 \tag{7}$$

$$\sum_{\theta=0^\circ,\ 45^\circ,\ 60^\circ,\ 90^\circ} XET\theta \le 1 \tag{8}$$

$$\sum_{\theta=0^\circ,\ 45^\circ,\ 60^\circ,\ 90^\circ} IT\theta \le 1 \tag{9}$$

For 0°, 45°, 60° and 90°.

$$\tilde{A}_2 NIPIR\theta - 2(\tilde{L}_{ir}) NIRIR\theta \le 0 \tag{10}$$

$$\tilde{A}_2 NOPIR\theta - \tilde{L}_{ir} NORIR\theta \le 0 \tag{11}$$

$$\tilde{A}_2 NIPER\theta - 2(\tilde{L}_{er}) NIRER\theta \le 0 \tag{12}$$

$$\tilde{A}_2 NOPER\theta - \tilde{L}_{er} NORER\theta \le 0 \tag{13}$$

$$\tilde{A}_2 NET\theta - (\tilde{L}_1 + \tilde{L}_2) XET\theta \leq 0 \tag{14}$$

$$\tilde{A}_2 NT\theta - (\tilde{L}_3 + \tilde{L}_4) XT\theta \leq 0 \tag{15}$$

$$\tilde{A}_2 NIT\theta - \tilde{L}_4 IT\theta \leq 0 \tag{16}$$

$$NIPER\theta,\ NIRER\theta,\ NOPER\theta,\ NORER\theta \geq 0$$

$$NET\theta,\ NT\theta,\ NIT\theta,\ XET\theta,\ XT\theta,\ IT\theta \geq 0$$

## 6. Numerical example

To illustrate the proposed mathematical model, the numerical example is taken into consideration. The numerical example was solved by using LINGO software and MATLAB is used to define the the fuzzy variable. Here the length of the parking lot is taken as a fuzzy variable.

### 6.1 *Membership function*

The length of the parking lot is considered under a fuzzy environment. The membership function for the triangular fuzzy number $\tilde{B}_1 = (5.0,\ 5.25,\ 5.5)$ is given below Equation (17) and Figure 6

$$\mu_{B_1}(x) = \begin{cases} 0, & x < 5 \\ \dfrac{x-5}{5.25-5}, & 5 \leq x \leq 5.25 \\ \dfrac{5.5-x}{5.5-5.25}, & 5.25 \leq x \leq 5.5 \\ 0, & x > 5.5 \end{cases} \tag{17}$$

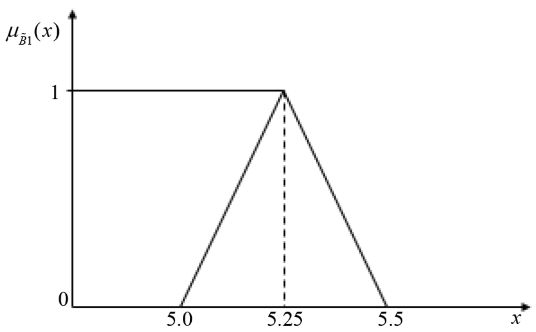


**Figure 6.** Membership function graph

## 6.2 *Input values*

To solve the proposed model for the different dimensions DM1, DM2, and DM3, the dimensions of the parking lots [1] are defined as given in Table 1.

# 7. Results and discussion

The LINGO software is used to solve the numerical example and the outputs are presented in the below Table 2.

Using LINGO software the numerical example solved for $2.5 \times 5.0\ m$, $2.5 \times 5.25\ m$, and $2.5 \times 5.5\ m$, and the following global optimum results obtained 502 parking lots, 488 parking lots, and 481 parking lots respectively. From Table 2, it is identified that the parking lot length and number of cars are inversely proportional because of the adjustment in the parking lot length. The output values are given in Figure 7 only for the length of the parking lot as 5.0, 5.25, and 5.5.

To take different values of parking lot length from 5 to 5.5 is inevitable to acquire the productive outcomes of the proposed mathematical model. A forecast graph and its equation can be drawn by using the available output values. Figure 8 illustrates the forecast of the output values and its equation for further process.

**Table 1.** Parking lot dimensions

| Dimensions | Degrees | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $2.5 \times 5.0\ m$ | | | | $2.5 \times 5.25\ m$ | | | | $2.5 \times 5.5\ m$ | | | |
| | 0 | 45 | 60 | 90 | 0 | 45 | 60 | 90 | 0 | 45 | 60 | 90 |
| $a$ | 50.00 | 50.00 | 50.00 | 50.00 | 50.00 | 50.00 | 50.00 | 50.00 | 50.00 | 50.00 | 50.00 | 50.00 |
| $b$ | 35.36 | 35.36 | 35.36 | 35.36 | 35.36 | 35.36 | 35.36 | 35.36 | 35.36 | 35.36 | 35.36 | 35.36 |
| $d$ | 120.72 | 120.72 | 120.72 | 120.72 | 120.72 | 120.72 | 120.72 | 120.72 | 120.72 | 120.72 | 120.72 | 120.72 |
| $A_1$ | 6.50 | 2.50 | 2.50 | 2.50 | 6.50 | 2.50 | 2.50 | 2.50 | 6.50 | 2.50 | 2.50 | 2.50 |
| $A_2$ | 6.50 | 3.54 | 2.89 | 2.50 | 6.50 | 3.57 | 2.90 | 2.50 | 6.50 | 3.60 | 2.91 | 2.50 |
| $B_1$ | 2.50 | 5.00 | 5.00 | 5.00 | 2.50 | 5.25 | 5.25 | 5.25 | 2.50 | 5.50 | 5.50 | 5.50 |
| $B_2$ | 6.50 | 6.80 | 6.25 | 5.00 | 6.50 | 6.95 | 6.43 | 5.25 | 6.50 | 7.10 | 6.60 | 5.50 |
| $C_1$ | 2.50 | 5.30 | 5.60 | 5.00 | 2.50 | 5.45 | 5.80 | 5.25 | 2.50 | 5.60 | 6.00 | 5.50 |
| $C_2$ | 2.50 | 4.70 | 4.90 | 4.25 | 2.50 | 4.85 | 5.10 | 4.50 | 2.50 | 5.00 | 5.30 | 4.75 |
| $C_3$ | 2.50 | 4.40 | 5.05 | 5.00 | 2.50 | 4.60 | 5.30 | 5.25 | 2.50 | 4.70 | 5.50 | 5.50 |
| $D$ | 3.00 | 3.75 | 4.50 | 7.00 | 3.00 | 3.75 | 4.50 | 7.00 | 3.00 | 3.75 | 4.50 | 7.00 |
| $E$ | 0.00 | 0.60 | 0.70 | 0.75 | 0.00 | 0.60 | 0.70 | 0.75 | 0.00 | 0.60 | 0.70 | 0.75 |
| $F_1$ | 8.00 | 13.45 | 15.15 | 17.00 | 8.00 | 13.80 | 15.60 | 17.50 | 8.00 | 14.05 | 16.00 | 18.00 |
| $F_2$ | 8.00 | 12.85 | 14.45 | 16.25 | 8.00 | 13.20 | 14.90 | 16.75 | 8.00 | 13.45 | 15.30 | 17.25 |
| $F_3$ | 8.00 | 12.55 | 14.60 | 17.00 | 8.00 | 12.95 | 15.10 | 17.50 | 8.00 | 13.15 | 15.50 | 18.00 |
| $F_4$ | 5.00 | 8.80 | 10.10 | 10.00 | 5.00 | 9.20 | 10.60 | 10.50 | 5.00 | 9.40 | 11.00 | 11.00 |
| $LIR$ | 36.00 | 36.00 | 36.00 | 36.00 | 36.00 | 36.00 | 36.00 | 36.00 | 36.00 | 36.00 | 36.00 | 36.00 |
| $LER$ | 35.36 | 35.36 | 35.36 | 35.36 | 35.36 | 35.36 | 35.36 | 35.36 | 35.36 | 35.36 | 35.36 | 35.36 |
| $L_1$ | 32.36 | 31.61 | 30.86 | 28.36 | 32.36 | 31.61 | 30.86 | 28.36 | 32.36 | 31.61 | 30.86 | 28.36 |
| $L_2$ | 29.36 | 27.86 | 26.36 | 21.36 | 29.36 | 27.86 | 26.36 | 21.36 | 29.36 | 27.86 | 26.36 | 21.36 |
| $L_3$ | 26.36 | 24.11 | 21.86 | 14.36 | 26.36 | 24.11 | 21.86 | 14.36 | 26.36 | 24.11 | 21.86 | 14.36 |
| $L_4$ | 23.36 | 20.36 | 17.36 | 7.36 | 23.36 | 20.36 | 17.36 | 7.36 | 23.36 | 20.36 | 17.36 | 7.36 |

**Table 2.** Output table

| 2.5 × 5.0 $m$ | | | 2.5 × 5.25 $m$ | | | 2.5 × 5.5 $m$ | | |
|---|---|---|---|---|---|---|---|---|
| Variable | Value | Reduced Value | Variable | Value | Reduced Value | Variable | Value | Reduced Value |
| NIPIR0 | 0 | -1 | NIPIR0 | 0 | -1 | NIPIR0 | 0 | -1 |
| NOPIR0 | 0 | -1 | NOPIR0 | 0 | -1 | NOPIR0 | 0 | -1 |
| NIPIR45 | 0 | -1 | NIPIR45 | 0 | -1 | NIPIR45 | 0 | -1 |
| NOPIR45 | 0 | -1 | NOPIR45 | 0 | -1 | NOPIR45 | 0 | -1 |
| NIPIR60 | 174 | -1 | NIPIR60 | 24 | -1 | NIPIR60 | 74 | -1 |
| NOPIR60 | 0 | -1 | NOPIR60 | 0 | -1 | NOPIR60 | 0 | -1 |
| NIPIR90 | 28 | -1 | NIPIR90 | 172 | -1 | NIPIR90 | 115 | -1 |
| NOPIR90 | 0 | -1 | NOPIR90 | 0 | -1 | NOPIR90 | 0 | -1 |
| NIPER0 | 0 | -2 | NIPER0 | 0 | -2 | NIPER0 | 0 | -2 |
| NOPER0 | 0 | -2 | NOPER0 | 0 | -2 | NOPER0 | 0 | -2 |
| NIPER45 | 0 | -2 | NIPER45 | 0 | -2 | NIPER45 | 0 | -2 |
| NOPER45 | 0 | -2 | NOPER45 | 0 | -2 | NOPER45 | 0 | -2 |
| NIPER60 | 24 | -2 | NIPER60 | 48 | -2 | NIPER60 | 48 | -2 |
| NOPER60 | 0 | -2 | NOPER60 | 0 | -2 | NOPER60 | 0 | -2 |
| NIPER90 | 56 | -2 | NIPER90 | 28 | -2 | NIPER90 | 28 | -2 |
| NOPER90 | 0 | -2 | NOPER90 | 0 | -2 | NOPER90 | 0 | -2 |
| NET0 | 0 | -4 | NET0 | 0 | -4 | NET0 | 0 | -4 |
| NT0 | 7 | -4 | NT0 | 7 | -4 | NT0 | 7 | -4 |
| NIT0 | 0 | -4 | NIT0 | 0 | -4 | NIT0 | 0 | -4 |
| NET45 | 16 | -4 | NET45 | 16 | -4 | NET45 | 16 | -4 |
| NT45 | 12 | -4 | NT45 | 12 | -4 | NT45 | 12 | -4 |
| NIT45 | 0 | -4 | NIT45 | 0 | -4 | NIT45 | 0 | -4 |
| NET60 | 0 | -4 | NET60 | 0 | -4 | NET60 | 0 | -4 |
| NT60 | 0 | -4 | NT60 | 0 | -4 | NT60 | 0 | -4 |
| NIT60 | 0 | -4 | NIT60 | 0 | -4 | NIT60 | 0 | -4 |
| NET90 | 0 | -4 | NET90 | 0 | -4 | NET90 | 0 | -4 |
| NT90 | 0 | -4 | NT90 | 0 | -4 | NT90 | 0 | -4 |
| NIT90 | 0 | -4 | NIT90 | 0 | -4 | NIT90 | 0 | -4 |
| NIRIR0 | 0 | 0 | NIRIR0 | 0 | 0 | NIRIR0 | 0 | 0 |
| NIRIR45 | 0 | 0 | NIRIR45 | 0 | 0 | NIRIR45 | 0 | 0 |
| NIRIR60 | 7 | 0 | NIRIR60 | 1 | 0 | NIRIR60 | 3 | 0 |
| NIRIR90 | 1 | 0 | NIRIR90 | 6 | 0 | NIRIR90 | 4 | 0 |
| NORIR0 | 0 | 0 | NORIR0 | 0 | 0 | NORIR0 | 0 | 0 |
| NORIR45 | 0 | 0 | NORIR45 | 0 | 0 | NORIR45 | 0 | 0 |
| NORIR60 | 0 | 0 | NORIR60 | 0 | 0 | NORIR60 | 0 | 0 |
| NORIR90 | 0 | 0 | NORIR90 | 0 | 0 | NORIR90 | 0 | 0 |
| NIRER0 | 0 | 0 | NIRER0 | 0 | 0 | NIRER0 | 0 | 0 |
| NIRER45 | 0 | 0 | NIRER45 | 0 | 0 | NIRER45 | 0 | 0 |
| NIRER60 | 1 | 0 | NIRER60 | 2 | 0 | NIRER60 | 2 | 0 |
| NIRER90 | 2 | 0 | NIRER90 | 1 | 0 | NIRER90 | 1 | 0 |
| NORER0 | 0 | 0 | NORER0 | 0 | 0 | NORER0 | 0 | 0 |
| NORER45 | 0 | 0 | NORER45 | 0 | 0 | NORER45 | 0 | 0 |
| NORER60 | 0 | 0 | NORER60 | 0 | 0 | NORER60 | 0 | 0 |
| NORER90 | 0 | 0 | NORER90 | 0 | 0 | NORER90 | 0 | 0 |
| XET0 | 0 | 0 | XET0 | 0 | 0 | XET0 | 0 | 0 |
| XT0 | 1 | 0 | XT0 | 1 | 0 | XT0 | 1 | 0 |
| IT0 | 0 | 0 | IT0 | 0 | 0 | IT0 | 0 | 0 |
| XET45 | 1 | 0 | XET45 | 1 | 0 | XET45 | 1 | 0 |
| XT45 | 1 | 0 | XT45 | 1 | 0 | XT45 | 1 | 0 |
| IT45 | 0 | 0 | IT45 | 0 | 0 | IT45 | 0 | 0 |
| XET60 | 0 | 0 | XET60 | 0 | 0 | XET60 | 0 | 0 |
| XT60 | 0 | 0 | XT60 | 0 | 0 | XT60 | 0 | 0 |
| IT60 | 0 | 0 | IT60 | 0 | 0 | IT60 | 0 | 0 |
| XET90 | 0 | 0 | XET90 | 0 | 0 | XET90 | 0 | 0 |
| XT90 | 0 | 0 | XT90 | 0 | 0 | XT90 | 0 | 0 |
| IT90 | 0 | 0 | IT90 | 0 | 0 | IT90 | 0 | 0 |

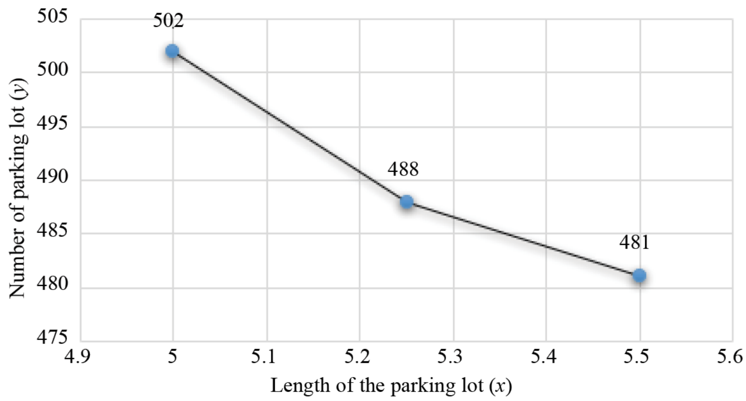


**Figure 7.** Output graph

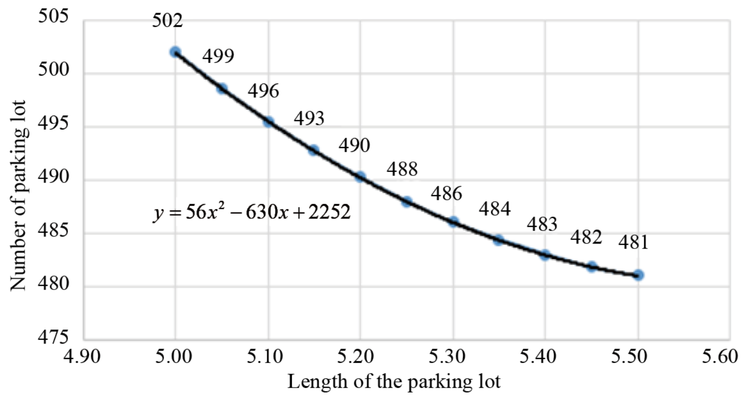


**Figure 8.** Forecast results graph and its equation

## 7.1 *Fuzzy logic controller*

MATLAB is used to define the Fuzzy Logic Controller. The parking lot length will be considered as a triangular fuzzy number and its minimum and maximum values are 5.0 *m* and 5.5 *m* respectively. One input, one output, and Mamdani rules-based fuzzy logic controller are developed and presented in Figure 9. Here the parking lot lengths are input values and the number of parking lots will be the output values. Membership functions for the triangular fuzzy number of the proposed problem are given below:

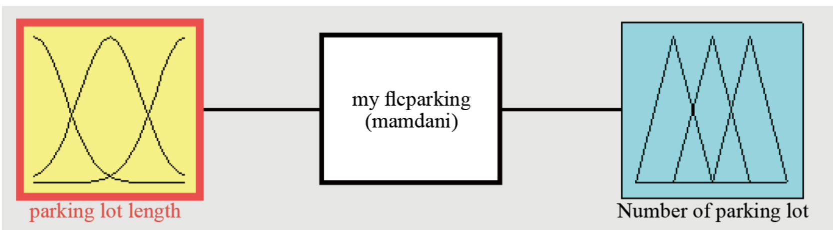


**Figure 9.** One input, one output, and Mamdani rule-based fuzzy number

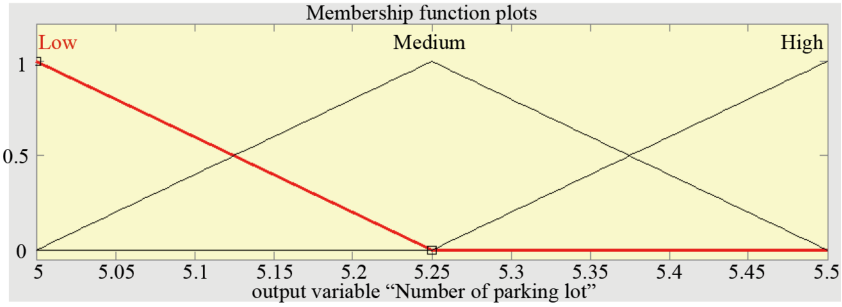


**Figure 10.** Membership function for input variable parking lot length

The triangular Membership Function is taken in the problem. The lower limit is 5.0 *m* and the upper limit is 5.5 *m* and it is presented in Figure 10.

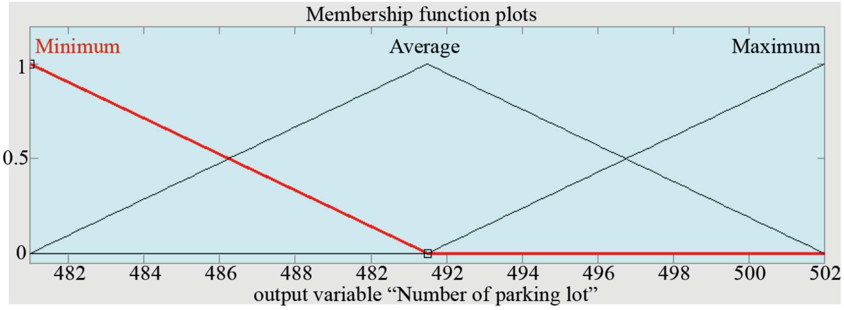


**Figure 11.** Membership function for output variable number of parking lot

Membership function plots for the output variable number of parking lots with a minimum of 481 parking lots and a maximum of 502 parking lots are given in Figure 11. Fuzzy logic controller rules are defined and given below:

If (Parking lot length is low) then (Number of parking lot is maximum)

If (Parking lot length is medium) then (Number of parking lot is average)

If (Parking lot length is high) then (Number of parking lot is minimum)

The fuzzy logic controller was executed by using MATLAB for input value 5.0 *m*. The result is given in the Figure 12.

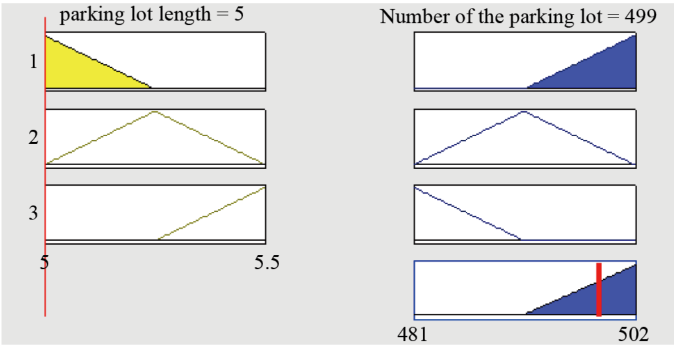


**Figure 12.** Output value for the input value parking lot length (5 *m*)

The fuzzy logic controller was executed by using MATLAB for an input value of 5.25 *m*. The result is given in Figure 13.

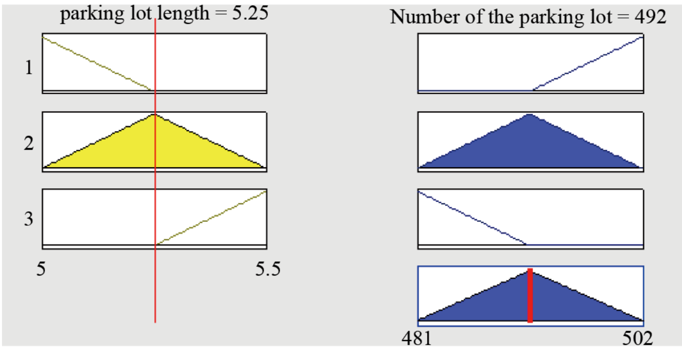


**Figure 13.** Output value for the input value parking lot length (5.25 *m*)

The fuzzy logic controller was executed by using MATLAB for input value 5.5 *m*. The result is shown in Figure 14.

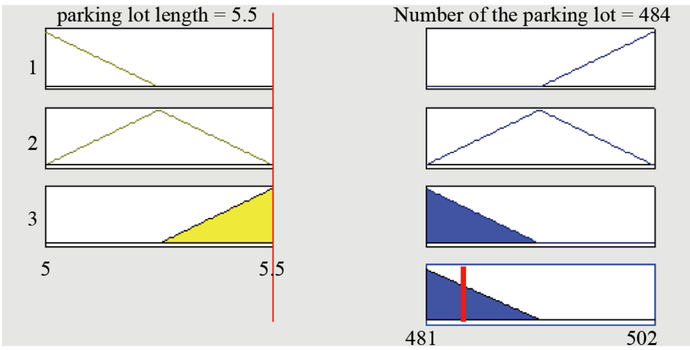


**Figure 14.** Output value for the input value parking lot length (5.5 *m*)

A surface view of the fuzzy results of the proposed fuzzy logic controller is given in Figure 15.

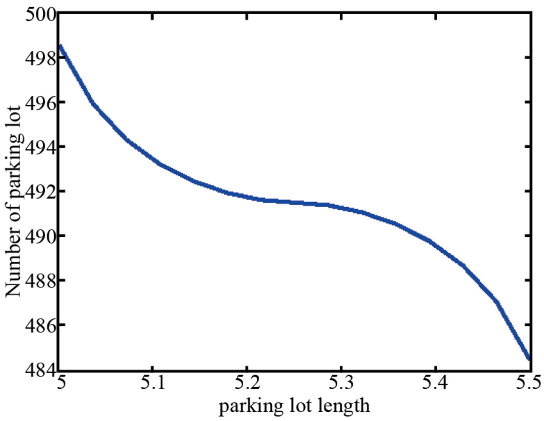


**Figure 15.** Surface view of the fuzzy results

The results of the LINGO, MATLAB, and Forecast graphs are presented in Table 3 for the comparison and its graph is presented in Figure 16.

**Table 3.** Results of the mathematical model

| Length of the Parking lot | MATLAB results | Forecast graph results | LINGO results |
|---|---|---|---|
| 5.00 | 499 | 502 | 502 |
| 5.05 | 495 | 499 | - |
| 5.10 | 493 | 496 | - |
| 5.15 | 492 | 493 | - |
| 5.20 | 492 | 490 | - |
| 5.25 | 492 | 488 | 488 |
| 5.30 | 491 | 486 | - |
| 5.35 | 491 | 484 | - |
| 5.40 | 490 | 483 | - |
| 5.45 | 488 | 482 | - |
| 5.50 | 484 | 481 | 481 |

## 7.2 *Sensitivity analysis*

The results of the mathematical model using LINGO software are compared with the forecast graph and MATLAB results in Table 3, and they are presented in Figure 16. It shows the sensitivity of the results obtained from MATLAB, Excel solver, and LINGO. The forecast graph is developed from the LINGO results. A Fuzzy Inference System (FIS) is created from the LINGO results, and the results are obtained by executing the FIS.

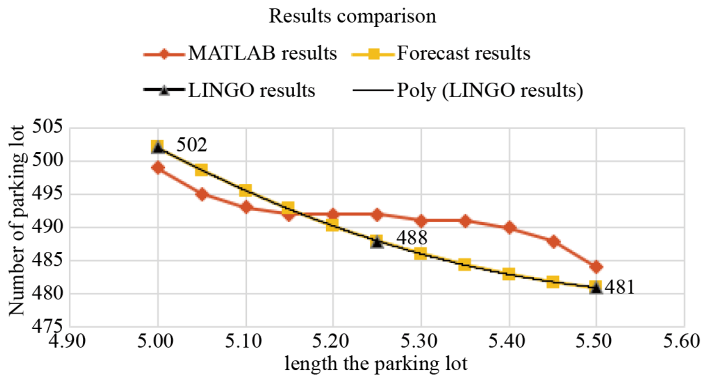


**Figure 16.** Results comparison

MATLAB results, forecast graphs, and LINGO results are independent of each other. So, the pictorial sensitivity analysis to compare the results with respect to the different parking lot lengths was done in Figure 16. In Figure 16, there is not much difference between the results. Hence, it is recommended that using MATLAB we can predict the results without solving the problem by using LINGO software for all values of parking lot length. Also, using MATLAB software will reduce the working labor time, and it will be easy to choose the value for parking lot length.

From Table 3, it is understood that the different parking lot length values give different numbers of parking lots. If parking lot length increases, then the number of parking lots will decrease relatively. Also, it is perceived that there is not much difference between the results achieved from MATLAB, LINGO, and Forecast graph.

## 8. Conclusion

The mathematical model of a regular octagon-shaped parking space is defined and developed under a fuzzy environment. Parking lot length is taken under a fuzzy environment. As per the defined parking lot dimensions, the proposed model is solved by using LINGO software. The forecast graph is developed by using the LINGO results. MATLAB is used to define the fuzzy logic control for the output results of LINGO software. From the results, it is perceived that the number of parking lots increases when the parking lot length is reduced. As we are defining the parking lot dimensions manually, it will occupy more labor work and time. To reduce the labor time to find the results it is suggested to take parking lot length as a fuzzy variable and proceed. From the results, it is observed that the parking lot dimensions that were defined for Oman are now user-friendly. Hence, based on the available area the number of parking lots can be defined by using the proposed mathematical model. In the future, different values can be determined for the different parking lot lengths to achieve more accurate and exact results.


## Acknowledgement

This work was supported by the "The Research Council (TRC)" of the Sultanate of Oman, under Block Funding Program Agreement Number: MoHERI/BFP/UTAS/2023, Project ID: BFP/URG/EBR/23/006 Project Title: "Optimization of mathematical model of regular octagon-shaped parking space in Fuzzy Environment". I thank all the authors involved in this work and acknowledge the valuable suggestions for improvement of the article from the peer reviewers.


## Conflict of interest

The authors declare no competing financial interest.

## References


[1] Arun Prasath GM, Ahmed SS. Optimization of regular octagon-shaped parking space. *Journal Information and Optimization Sciences*. 2021; 42(6): 1295-1306.

[2] Abdelfatah AS, Taha MA. Parking capacity optimization using linear programming. *Journal of Traffic and Logistics Engineering*. 2014; 2(3): 176-181.

[3] Demirli K, Khoshnejad M. Autonomous parallel parking of a car-like mobile robot by a neuro-fuzzy sensor-based controller. *Fuzzy Sets and Systems*. 2009; 160(19): 2876-2891.

[4] Tejal L, Minla C, Ankita M, Madhusmita O, Anjali Y. Smart parking application. *International Journal of Computer Applications*. 2016; 149(9): 32-37.

[5] Udo EU, Agwu EO, Udeani HU. Mathematical modelling of a fuzzy logic based car parking system. *Journal of Engineering Research Innovation and Scientific Development*. 2023; 4(1): 19-28.

[6] Ibrahim S, Syahir A, Aziz A. Customized fitness function of evolutionary algorithm for optimization of car park space. *International Journal for Research in Applied Science Engineering Technology*. 2017; 5(6): 500-505.

[7] Kong Y, Le Vine S, Liu X. Capacity impacts and optimal geometry of automated cars surface parking facilities. *Journal of Advanced Transportation*. 2018; 2018(1): 6908717.

[8] Kazazi Darani S, Akbari Eslami A, Jabbari M, Asefi H. Parking lot site selection using a fuzzy AHP-topsis framework in tuyserkan, iran. *Journal of Urban Planning and Development*. 2018; 144(3): 1-10.

[9] Yıldırım G, Yıldırım B, Satır B. Design of a rectangular parking lot using a cutting-stock formulation. *Cankaya University Journal of Science and Engineering*. 2019; 16(1): 1-16.

[10] Wang Y, Wu J, Wang Y. An improved hierarchical A* algorithm in the optimization of parking lots. *AIP Conference Proceedings*. 2017; 1864(1): 020031.

[11] Boroiu A, Neagu E, Boroiu AA, Vieru I. Mathematical modelling of the spatial efficiency of car parks. *IOP Conference Series: Materials Science and Engineering*. 2019; 564(1): 012124.

[12] Hasbiyati I, Putri W, Adnan A. Parking lot optimization in parallelogram using the concept area of rectangular and right triangle. *Pure and Applied Mathematics Journal*. 2019; 8(4): 77-82.

[13] Putri W, Hasbiyati I, Gamal MDH. Optimization of parking lot in the forms of parallelogram and right triangle for cars and motorbikes. *Mathematical Modelling and Applications*. 2019; 4(4): 64-71.

[14] Syahrini I, Sundari T, Iskandar T, Halfiani V, Munzir S, Ramli M. Mathematical model of Parking Space unit for triangular parking area. *IOP Conference Series: Materials Sciences and Engineering*. 2018; 300(1): 012012.

[15] Anand MCJ, Bharatraj J. Theory of Triangular Fuzzy Number. In: *Proceedings of NCATM*. National Conference on Advanced Trends in Mathematics (NCATM-2017); 2017. p.80-83.