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Research Article

Information Asymmetric Cooperative Games with Agreements Self-Implemented

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Abstract: This paper introduces asymmetric information into the analysis of cooperative games with agreements self-implemented, establishes theoretical models of a one-shot information asymmetric cooperative game with agreements self-implemented, aims to provide analytical tools for the study of political science, economics, sociology, and other humanities disciplines. In an information asymmetric cooperative game with agreements self-implemented, the players make their decisions through their virtual games on the basis of their own information sets. By introducing the virtual games of the players, this paper defines the coalition equilibrium of an information asymmetric cooperative game with agreements self-implemented and examines the condition for its existence. This paper demonstrates that in an information asymmetric cooperative game with agreements self-implemented, the condition for the existence of its coalition equilibrium is that information is symmetric after the game is completed. This paper defines the distribution equilibrium of the cooperative payoff of a coalition in the coalition equilibrium (if it does exist) as the Nash equilibrium of the bargaining game between the core members of the coalition. When the core members are unallied in the bargaining game, the distribution of a coalition member is the sum of his cooperative payoff distribution when the estimations of the optimal strategic combination choice of all the core members are all correct and the distribution he gets in the "misjudgment" cooperation; when the core members are allied in the bargaining game, the distribution of a cooperative team is the sum of its cooperative payoff distribution when the estimations of the optimal strategic combination choice of all teams are all correct and the distribution it gets in the "misjudgment" cooperation.

Keywords: information asymmetry, information asymmetric cooperative game with agreements self-implemented, virtual game, coalition equilibrium, bargaining game, core coalition, extensive coalition

MSC: 91A12, 91A44

1. Introduction

An important hypothesis in classical cooperative game theory is that there is no asymmetric information among the players, and that all the players have complete information is common knowledge of all players. However, even a person who knows game theory just a little will feel doubtful about the complete information hypothesis. So, it's not surprising that Kadan et al. [1] asked whether game models according to the above belief really make sense. Obviously,

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this hypothesis neglects the brass tacks that things the players believe rely on the situations they know. However, in the real cooperative social games, players are usually information asymmetric rather than information symmetric.

The classical cooperative game theory usually assumes that cooperation agreements are implemented by third parties at no cost, while ignoring the fact that in reality, there are a large number of cooperation agreements that are self-implemented by the coalition members. The reasons why cooperation agreements are self-implemented by coalition members are first due to the lack of a third party to implemented the agreements in many cases; secondly, in some cases, the information required for a third party to implement the agreements is lacking or the cost of obtaining such information is very high, thus, the third party is unable to implement the cooperation agreements; thirdly, some cooperation agreements implemented by a third party are inherently self-implemented. Obviously, the implementation mode of cooperation agreements will have a significant impact on the coalition formation and the distribution of the cooperative payoff of a coalition in cooperative games.

In the classical literature on cooperative games of complete information, the three main areas that researchers focus on include:

(1) the coalition formation in the game;

(2) the strategic combination choices of the coalitions in the game;

(3) the distribution of the cooperative payoff of a coalition.

Obviously, the game among coalitions is typically non-cooperative, the cooperative Nash equilibrium of the cooperative game is just the non-cooperative Nash equilibrium of the non-cooperative game among coalitions, therefore, the classical literature focuses only on the coalition formation in a cooperative game and the distribution rule of the cooperative payoff of a coalition which can bring a Pareto improvement to all its members, especially on the latter.

In cooperative game theory, researchers often assumed that there is only one coalition in a cooperative game, if there is no dummy in the game, all the players will join the only coalition; and if there are dummies in the game, all players except the dummies will join this coalition. Such an assumption can only accord with a special situation, in a general cooperative game, the situation may be different from the assumption above. In many cases, there may be not only one coalition, but a series of coalitions. If there are "dummies" for a coalition in a cooperative game, there may be synergies among the "dummies". Therefore, they may form one or more coalitions to benefit from cooperation.

Based on the equilibrium definition introduced by Konishi et al. [2] and the extended by Hyndman et al. [3], and also the solution concept used in Gomes et al. [4] and Gomes [5], Aumann [6] discussed the coalition formation in a repeated cooperative game, and defined the equilibrium process of coalition formation (EPCF), which is a process of coalition formation with the property that at every history, every active coalition, faced with a given set of potential partners, makes a profitable and maximal move. An implicit assumption of Aumann's dynamic model is that the information between players is asymmetric in the repeated cooperative game in study.

By the blocking approach, a lot of researches discussed the possible ranges of the distribution scheme of the cooperative payoff of a coalition. Beginning with the monumental work of von Neumann et al. [7], this kind of literature includes notions such as the stable set, the core, and the bargaining set (Aumann et al. [8]; Gillies [9]; Shapley [10]; von Neumann et al. [7]). Extensions of these ideas to incorporate notions of farsighted behavior were introduced by Harsanyi [11], and later by Aumann et al. [12]. The farsightedness notion was further developed by Chwe [13], Ray et al. [14], Diamantoudi et al. [15], and others. Of course, this type of research does not end up with a distribution rule of the cooperative payoff of a coalition.

Some researchers have attempted to obtain a one-point solution to a fully cooperative game, that is, the only distribution scheme of the cooperative payoff of a coalition, such as Shapley value (Shapley [16]) and Nucleolus (Schmeidler [17]). These models are often based on "collectivism", in these models, coalition members are assumed to pursue a collective goal in the distribution process. In fact, the distribution process is the social interaction among the coalition members for their own welfares, in which a rational individual would not replace his welfare maximization objective with a collective one. And in the distribution process, individual rationality and collective rationality are often in conflict (this conflict is often shown in the so-called prisoner's dilemma). Nash [18] realized that the distribution of the cooperative payoff of a coalition is the result of the bargaining among the members of the coalition, after establishing some axioms, Nash proved that there exists only one bargaining process that can satisfy the axioms which should be satisfied, this only bargaining process is called the Nash negotiation solution. Unfortunately, due to his unreasonable axiom assumptions, Nash did not find a satisfactory negotiation solution.

Another type of literature on the distribution of the cooperative payoff is called non-fully cooperative game theory, in which the participation levels of players in the cooperation are introduced. Aubin [19] introduced the solution to a cooperative game with fuzzy coalitions, Sakawa et al. [20], Molina et al. [21] proposed the lexicographical solution to a cooperative game with fuzzy coalitions.

So far, classical cooperative game theory has not provided satisfactory solutions to the two basic problems of coalition formation and cooperative payoff distribution in a one-shot, finite cooperative game of complete information with agreements implemented by a third party.

In recent years the asymmetric information coordination has been studied in networked system theory. Shang [22] studied a simple three-body consensus model, which favorably incorporates higher-order network interactions, higher-order dimensional states, group reinforcement effect as well as homophily principle, proposed a system model of three-body interactions in complex networks. Shang [23] introduced a novel multiplex network presentation for directed graphs and its associated connectivity concepts including the pseudo-strongly connectivity and graph robustness, which provide a resilience characterization in the presence of malicious nodes. Qi et al. [24] investigated the linear quadratic (LQ) control problem for a stochastic system (https://www.sciencedirect.com/topics/mathematics/stochastic-system) with different intermittent observations. However, the goal of such literature of networked system theory is clearly not to establish a general information asymmetry cooperative game theory.

However, these conclusions drawn from the analysis of evolutionary games on the basis of different assumptions about the behavior patterns of players obviously cannot explain the cooperation of players in a one-shot information asymmetric cooperative game with agreements self-implemented.

Recently, Chen [25] examined the coalition formation in an information symmetric cooperative game with agreements implemented by a third party, provided the existence proof and an algorithm of the coalition equilibrium; moreover, Chen analyzed the equilibrium of the bargaining game on the distribution of the cooperative payoff of a coalition under the coalition equilibrium, and examined the distribution equilibrium of cooperative payoff of a coalition. Chen [26] examined an information asymmetric cooperative game with agreements implemented by a third party, defined the virtual cooperative games of the players and demonstrated the equilibrium of the virtual cooperative game of a player; proposed the condition for the existence of the coalition equilibrium in an information asymmetric cooperative game with agreements implemented by a third party, defined and provided the existence proof of this coalition equilibrium when it does exist; defined the public choice game of a coalition on the strategic combination choice in a coalition situation, provided the existence proof of the equilibrium of this game; examined the condition for the existence of the bargaining game on the distribution of the cooperative payoff of a coalition, and provided the existence proof of the bargaining game, when the coalition members are allied or unallied in the bargaining game.

Chen [27] provided an analytical framework for a cooperative games with agreements self-implemented in three scenarios: (1) the possible opportunistic behaviors in the distribution process are ignored; (2) coalitions centralize all the payoffs their members get in the game to inhibit the possible opportunistic behaviors in the distribution process; (3) coalitions distribute their cooperative payoffs before the game begins to inhibit the possible opportunistic behaviors in the distribution process. In each scenario, Chen examined the formation of the coalitions and the distribution process of the cooperative payoff of a coalition, defined and provided the existence proof of the coalition equilibrium of a cooperative game with agreements self-implemented, provided the existence proof of the equilibrium in the bargaining game of a coalition on the distribution of its cooperative payoff, when its members cooperate in the game or not.

Chen [27] proposed the basic methodology for the analysis of an cooperative game with agreements self-implemented: the formation of the coalition equilibrium is the result of the choices of the players who pursue the maximization of their individual welfares, and the cooperative payoff of a coalition can always be decomposed into the common payoffs of different member sets, the equilibrium of the bargaining game of a coalition on the cooperative payoff distribution can easily be obtained by applying the distribution rule of common payoffs. Meanwhile, Chen [26] also provided a basic idea for introducing asymmetric information into the analysis of an cooperative game with agreements implemented by a third party: players make decisions through their virtual games on the basis of their own information sets, the criteria for each player to choose his coalition is the maximization of his expected cooperative payoff distribution or the minimization of his expected escape payoff deriving from deviation; the distribution equilibrium of the cooperative payoff of a coalition is also the result of the negotiations between the coalition members in the bargaining game on the distribution of the cooperative payoff of the coalition. This paper aims to introduce asymmetric information into the analysis of a cooperative game with agreements self-implemented, using the basic methodologies mentioned above. Based on Chen [26, 27], this paper tries to analyze the coalition formation in an information asymmetric cooperative game with agreements self-implemented, defines the coalition equilibrium and examines the condition for its existence, and investigates the Nash equilibrium of the bargaining game on the distribution of the cooperative payoff of a coalition in the coalition equilibrium (if it does exist).

The significance of the society lies in cooperation, and many social interactions can be compared to cooperative games. Among them, a large number take self-implemented agreements, and there is information asymmetry between the players. This paper aims to provide an analytical tool for a one-shot information asymmetric cooperative game with agreements self-implemented. By analyzing the virtual games of the players, this paper defines the coalition equilibrium of the coalition equilibrium of an information asymmetric cooperative game with agreements self-implemented, and investigates the condition for its existence. Meanwhile, this paper defines the distribution equilibrium of the cooperative payoff of a coalition under the coalition equilibrium, when the coalition members are allied in the bargaining game or not. The methodology and conclusions presented in this paper can be widely applied to the study of political science, economics, sociology, and other humanities disciplines.

First, in Sections 2-4, we ignore the opportunistic behaviors of coalition members in the cooperative payoff distribution process, or we assume that the opportunistic behaviors of coalition members in the distribution process of cooperative payoff are negligible. In Section 5, we assume that each coalition concentrates the payoffs that its members get in the game to prevent coalition members from engaging in opportunistic behaviors in the distribution process of cooperative payoff.

2. The virtual game of a player: ignoring the opportunistic behaviors in distribution process

In this section, we will analyze information asymmetric cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented and assume that the coalitions are publicly-owned and that the members of a coalition are not allied in the bargaining game on the distribution of its cooperative payoff, where N denotes the player set, $N = \{1, 2, \cdots, n\}$; S_i denotes the strategy set of any player i, $S_i = \{s_{i1}, s_{i2}, \cdots, s_{im_i}\}$, $i = 1, 2, \cdots, n$; and u_i denotes the payoff function of any player i. First, we do not consider the opportunistic behaviors of coalition members in the distribution process of cooperative payoff, or, we assume that the opportunistic behaviors of coalition members in the distribution process of cooperative payoff are negligible.

Due to information asymmetry, in information asymmetric cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented, each player virtualizes his own game on the basis of his information set and makes his decisions according to his virtual game. The reason why a player retains private information is that the private information he retains is beneficial for expanding the expected cooperative payoff distribution he can receive from the coalition he belongs to. A player can exaggerate his marginal contribution to the coalition and his escape payoff deriving from deviation by publishing false information to the core coalition he belongs to (from which he will not escape) in order to obtain higher cooperative payoff distribution; he can also influence the decisions of other core coalitions by publishing false information to them (including claiming to join some coalition but ultimately not keeping his promise) to increase the expected cooperative payoff of the core coalition he belongs to, thus increase the expected cooperative payoff distribution he receives from his core coalition.

Obviously, when cooperation agreements are self-implemented, a player's promise to join some coalition cannot be implemented by force. Therefore, our discussion starts with a player's false promise and his escape path. Of course, not every player's false promise is trustworthy, it must gain the trust of the deceived coalition.

In this section, we will examine the escape paths of a player in a coalition situation and the conditions for him to be trusted by the coalition he joins. In each coalition situation c, a player decides his best escape strategy on the basis of his estimation of the escape strategy of any other player. However, his escape strategy itself is also a kind of information release which will change the information sets of other players. By analyzing a player's n-level virtual games, we can obtain the virtual game of this player with the information sets of all players stable.

Next, we'll examine the information sets-stable virtual game of this player level by level until a stable solution to his optimization problem appears, we can obtain the coalition equilibrium of the virtual game of this player. Of course, due to different information sets, the coalition equilibria in the virtual games of different players are different. However, the information transmission, communication, and negotiation between the players can ultimately lead to the convergence of the coalition equilibria of the virtual games of all players.

2.1 *Escape path and trust condition*

An important difference between a cooperative game with agreements self-implemented and a cooperative game with agreements implemented by a third party is that in a cooperative game with agreements self-implemented, not only each member needs to have synergy with the coalition he belongs to, but he should be trusted by other members of the coalition too, that is, the members of a coalition are considered not to have the motivation to escape through deviation.

In an information asymmetric cooperative game with agreements implemented by a third party, as long as a player believes that he has the greatest synergy with a coalition, he can apply to become a member of the coalition. Of course, the cooperative payoff distribution he obtains is not necessarily the cooperative payoff distribution in his virtual game.

However, in an information asymmetric cooperative game with agreements self-implemented, even if a player in his virtual game believes that he has the greatest expected synergy with some coalition, and considers that he does not have the motivation to escape through deviation, he may not be accepted by the coalition, because he may be considered to have a motive to escape from the coalition through deviation. Due to information asymmetry, in other members' virtual games, the judgments of the trustworthy members who really have synergies with the coalition, as potential partners, are not consistent.

In an information asymmetric cooperative game with agreements self-implemented, the conditions that some player is trusted by other members of a coalition and is accepted by this coalition are:

(1) the marginal contribution of this player to the coalition is considered to be higher than his escape-payoff deriving from deviation;

(2) the marginal contribution of any member set which this player belongs to is considered to be higher than the sum of the escape-payoffs deriving from deviation of the members in the member set;

(3) each member of the coalition believes that the cooperative payoff of the coalition is higher than the sum of the escape-payoffs of all the coalition members.

2.1.1 *Possible escape paths*

First, we examine the escape paths of member k_2 of some coalition C_k in the virtual game of player k_1, in coalition situation c of information asymmetric cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented.

In information symmetric cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented, in coalition situation c, the escape paths of any member of some coalition C_k are clear: as long as when he escapes from coalition C_k through deviation, he can obtain an escape-payoff deriving from deviation that is greater than his marginal contribution to coalition (this is the maximum cooperative payoff distribution he can obtain from the target coalition when he escapes from coalition C_k through deviation), his escape is out of question, not only the player himself can recognize this, but other players can also do; obviously, he will escape through deviation to a coalition that enables him to obtain the maximum cooperative payoff distribution after his escaping, that is to say, he will escape through deviation to the coalition to which his marginal contribution is the greatest when he escapes from coalition C_k through deviation. In this escape process, the escape path includes the initial node (the original coalition that this player belongs to) and the terminal node (the target coalition he chooses).

In coalition situation c of information asymmetric cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented, an escape path of some member k_2 of coalition C_k not only includes the initial node (the original coalition that this player in name belongs to) and the terminal node (the target coalition he chooses when he escapes through deviation), but also the intermediate nodes: member k_2 of coalition C_k can choose some coalition $C_{k'}$, promise that he would join the coalition through deviation from coalition C_k though his actual escape target is not coalition $C_{k'}$. That is to say, the purpose of player k_2's commitment is not to obtain the escape-payoff deriving from deviation when he pretends to escape from coalition C_k to coalition $C_{k'}$, but to provide false information to coalition $C_{k'}$, so as to facilitate the increase of the cooperative payoff of his real escape target (a coalition other than coalition $C_{k'}$, or possibly coalition C_k which he currently belongs to). At the initial node and the intermediate node of the escape path, his commitments to the non-terminal nodes are all false.

Player k_2 may make false promises to more than one intermediate node coalitions, and the terminal node coalition as his escape target may be coalition which he currently belongs to. Herein, it is assumed that in the escape path of player k_2, the initial node coalition and the terminal node coalition can be the same, but the initial node coalition and an intermediate coalition cannot be the same, and the terminal node coalition and an intermediate node coalition cannot be the same too; in addition, the same coalition cannot become the intermediate node twice, nor can two coalitions become initial node coalitions or terminal node coalitions at the same time.

In some coalition situation c, when each player chooses one of his escape paths, an escape situation is formed.

2.1.2 *Possible coalition situations and trust conditions of a coalition*

Obviously, not each coalition situation c is feasible: the basic condition for a coalition situation to be feasible is that any member of the coalition in this coalition situation is trusted by the coalition that he belongs to. A self-implemented agreement must make the members of the coalition be considered to have no opportunistic deviations from the coalition, because the deviation of a member means a loss of the coalition, and may even a significant reduction in its cooperative payoff. Therefore, in coalition situation c, if player k_1 is a core member of coalition C_k (that is to say, player k_1 as a member of coalition C_k will not escape through deviation), one of the conditions for some player k_2 to be trusted by core member k_1 of the coalition is that in the virtual game of core member k_1, the marginal contribution of player k_2 to core coalition $C_k^{c(k_1)}$ that core member k_1 belongs to is considered to be higher than his escape-payoff deriving from deviation. [There are two ways for player k_2 to apply for membership of coalition C_k in coalition situation c. The first one is that player k_2 at the beginning is a nominal member of coalition C_k; the second is that although player k_2 is nominal member of another coalition, he hopes to escape through deviation and join coalition C_k. In these two cases, the marginal contributions of player k_2 to coalition C_k and his escape-payoffs deriving from deviation when he escapes from coalition C_k through deviation are different].

$$Mv_{k_2}^{(k_1)}(C_k^{c(k_1)}) > W_{k_2}^{-C_k^{c(k_1)}(k_1)},\ k_1 \neq k_2,$$

where $Mv_{k_2}^{(k_1)}(C_k^{c(k_1)})$ represents the marginal contribution of *player* k_2 to coalition $C_k^{c(k_1)}$ in player k_1's virtual game, $W_{k_2}^{-C_k^{c(k_1)}(k_1)}$ represents the escape-payoff deriving from deviation of *player* k_2 when he escapes from coalition $C_k^{c(k_1)}$ via deviation in player k_1's virtual game.

If in the distribution process of cooperative payoff of coalition C_k, all the members of coalition C_k are bound to be responsible for their own misjudgments of the cooperative game, player k_1, as a core member of coalition C_k, can get the maximum expected cooperative payoff distribution according to his own "correct" judgment, including his judgment of the choices of his partners. Obviously, for core member k_1 of coalition C_k, the condition that a potential coalition member can be accepted by coalition C_k is that in his virtual game, the potential coalition member's contribution to core coalition $C_k^{c(k_1)}$ is greater than his escape-payoff deriving from deviation when he escape from core coalition $C_k^{c(k_1)}$ through deviation.

However, if player k_1 is a nominal member or an extensive member of coalition C_k rather than a core one, that is to say, whether player k_1 is a nominal member of coalition C_k in coalition situation c, or he escapes from other coalitions through deviation, as long as player k_1 chooses to escape from coalition C_k through deviation, he certainly does not aim at the maximum expected cooperative payoff of coalition C_k when applying for the membership of coalition C_k, but should aim at the maximum expected cooperative payoff of the target coalition when he escapes from coalition C_k through deviation. But, in order not to reveal his intention to escape from coalition C_k through deviation, player k_1 will judge whether the joining condition of potential coalition member k_2 is satisfied according to the false signals $i_{k_1}^{*(k_1^F)}$ he issued to coalition C_k. That is, according to the false signals issued by player k_1 to coalition C_k, potential coalition member k_2's contribution to "core" coalition $C_k^{c(k_1^F)}$ which player k_1 claims is greater than the escape-payoff deriving from deviation of player k_2 when he escapes from "core" coalition $C_k^{c(k_1^F)}$ through deviation:

$$Mv_{k_2}^{(k_1^F)}(C_k^{c(k_1^F)}) > W_{k_2}^{-C_k^{c(k_1^F)}(k_1^F)},\ k_1 \neq k_2.$$

In coalition situation c, one of the conditions that player k_2 as a coalition member is trusted by coalition C_k is that, for any other member i of coalition C_k,

$$Mv_{k_2}^{(i)}(C_k^{c(i)}) > W_{k_2}^{-C_k^{c(i)}(i)},\ i \in C_k^c,\ i \neq k_2;$$

$$Mv_{k_2}^{(i^F)}(C_k^{c(i^F)}) > W_{k_2}^{-C_k^{c(i^F)}(i^F)},\ i \in C_k^+,\ i \notin C_k^c,\ i \neq k_2.$$

The above condition is not easy to be satisfied. In fact, it is too harsh for potential member k_2 who asks to join the coalition to satisfy the trust conditions of all other members.

Under information asymmetry, when coalition C_k is deciding whether to accept potential member k_2 or not, if the coalition members draw different conclusions according to their own virtual games, does coalition C_k accept the application of potential member k_2? What is the condition for coalition C_k accepting the application of potential member k_2?

Theorem 1 In information asymmetric cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented, given the coalition situation and the escape situation of the game, one of the conditions that potential member k_2 is accepted by coalition C_k and is trusted by coalition C_k (that is to say, he can sign a cooperation agreement with coalition C_k) is that the contribution of player k_2 to coalition C_k can satisfy the condition shown as follows:

$$\sum_{i\in C_k^c}\left[Mv_{k_2}^{(i)}(C_k^{c(i)}) - W_{k_2}^{-C_k^{c(i)}(i)}\right] + \sum_{j\in C_k^+,\ j\notin C_k^c}\left[Mv_{k_2}^{(j^F)}(C_k^{c(j^F)}) - W_{k_2}^{-C_k^{c(j^F)}(j^F)}\right] > 0,\ i,\ j \neq k_2.$$

Proof. Build the public choice game in which all the members of coalition C_k except player k_2 decide whether to accept player k_2. The strategies of all the players in the public choice game include "acceptance" and "rejection"; the payoff function of any player is his expected cooperative payoff distribution in each strategic situation of the game.

Obviously, the goal of the maximum expected cooperative payoff distribution of any member (a player in the public choice game) is consistent with the goal of the coalition's maximum expected cooperative payoff. Therefore, in the public choice game mentioned above, the payoff function of each member in the form of his expected cooperative payoff distribution is equivalent to the payoff function in the form of his expected cooperative payoff of the coalition.

In this public choice game, all the players will form a unique coalition, and the coalition's equilibrium public choice is the public strategy which maximizes the sum of the expected cooperative payoffs of all the players. Thus, the objective of the above public choice game is the solution to the following optimization problem:

$$\max\left\{\sum_{i\in C_k^c} V_{C_k^{c(i)}}^{(i)}+\sum_{j\in C_k^+,\ j\notin C_k^c} V_{C_k^{c(j^F)}}^{(j^F)}\right\},\ i,\ j\neq k_2,$$

where $V_{C_k^{c(i)}}^{(i)}$ represents the cooperative payoff of coalition $C_k^{c(i)}$ in player i's virtual game, $V_{C_k^{c(j^F)}}^{(j^F)}$ represents the cooperative payoff of coalition $C_k^{c(j^F)}$ in the virtual game of player i as a non-core member of the coalition who will eventually escape from the coalition.

Therefore, when $\sum_{i\in C_k^c}\left[Mv_{k_2}^{(i)}(C_k^{c(i)})-W_{k_2}^{-C_k^{c(i)}(i)}\right]+\sum_{j\in C_k^+,\ j\notin C_k^c}\left[Mv_{k_2}^{(j^F)}(C_k^{c(j^F)})-W_{k_2}^{-C_k^{c(j^F)}(j^F)}\right]>0,\ (i,\ j\neq k_2)$, coalition C_k will accept player k_2, when $\sum_{i\in C_k^c}\left[Mv_{k_2}^{(i)}(C_k^{c(i)})-W_{k_2}^{-C_k^{c(i)}(i)}\right]+\sum_{j\in C_k^+,\ j\notin C_k^c}\left[Mv_{k_2}^{(j^F)}(C_k^{c(j^F)})-W_{k_2}^{-C_k^{c(j^F)}(j^F)}\right]$ $\leq 0,\ (i,\ j\neq k_2)$, coalition C_k will reject player k_2. □

The marginal contribution of a coalition member to the coalition is considered to be higher than his escape-payoff deriving from deviation does not mean that the coalition member must be trustworthy. According to an analysis similar to the proof of Theorem 1, we can also draw the conclusion in Theorem 2.

Theorem 2 In information asymmetric cooperative game $\Gamma(N,\{S_i\},\{u_i\})$ with agreements self-implemented, given the coalition situation and the escape situation of the game, the second condition that potential member k_2 is accepted by coalition C_k and is trusted by coalition C_k (that is to say, he can sign a cooperation agreement with coalition C_k) is that the contribution of each member set containing player k_2 to coalition C_k can satisfy:

$$\sum_{i\in C_k^c}\left[Mv_{T_h}^{(i)}(C_k^{c(i)})-\sum_{t\in T_h} W_t^{-C_k^{c(i)}(i)}\right]+\sum_{j\in C_k^+,\ j\notin C_k^c}\left[Mv_{T_h}^{(j^F)}(C_k^{c(j^F)})-\sum_{t\in T_h} W_t^{-C_k^{c(j^F)}(j^F)}\right]>0;$$

$$i,\ j\neq k_2,\ k_2\in T_h\subset C_k.$$

The third condition that potential member k_2 is accepted by coalition C_k and is trusted by coalition C_k (that is to say, he can sign a cooperation agreement with coalition C_k) is that each member of coalition C_k consider the sum of the contributions of all the members to coalition C_k is higher than the cooperative payoff of coalition C_k in his virtual game:

$$V_{C_k^c}^{(i)}-\sum_{t\in C_k^c} W_t^{-C_k^{c(i)}(i)}>0,\ i\in C_k^c;$$

$$V_{C_k^{c(j^F)}}^{(j^F)}-\sum_{t\in C_k^{c(j^F)}} W_t^{-C_k^{c(j^F)}(j^F)}>0,\ j\in C_k^+,\ j\notin C_k^c.$$

Proof. Build the public choice game in which all the members of coalition C_k except team T_h decide whether to accept player T_h. According to Theorem 1, the following condition should be met:

$$\sum_{i\in C_k^c}\left[Mv_{T_h}^{(i)}(C_k^{c(i)})-W_{T_h}^{-C_k^{c(i)}(i)}\right]+\sum_{j\in C_k^+,\ j\notin C_k^c}\left[Mv_{T_h}^{(j^F)}(C_k^{c(j^F)})-W_{T_h}^{-C_k^{c(j^F)}(j^F)}\right]>0.$$

Obviously,

$$\sum_{t\in T_h}W_t^{-C_k^{c(i)}(i)}\geq W_{T_h}^{-C_k^{c(i)}(i)},\ \sum_{t\in T_h}W_t^{-C_k^{c(j^F)}(j^F)}\geq W_{T_h}^{-C_k^{c(j^F)}(j^F)},$$

therefore, $\sum_{i\in C_k^c}\left[Mv_{T_h}^{(i)}(C_k^{c(i)})-\sum_{t\in T_h}W_t^{-C_k^{c(i)}(i)}\right]+\sum_{j\in C_k^+,\ j\notin C_k^c}\left[Mv_{T_h}^{(j^F)}(C_k^{c\left(j^F\right)})-\sum_{t\in T_h}W_t^{-C_k^{c\left(j^F\right)}\left(j^F\right)}\right]>0$ should be met.

On the other hand, as a member of coalition C_k, if $V_{C_k^c}^{(i)}-\sum_{t\in C_k^c}W_t^{-C_k^{c(i)}(i)}<0(i\in C_k^c)$ or $V_{C_k^{c(i^F)}}^{(i^F)}-\sum_{t\in C_k^{c}\left(i^F\right)}W_t^{-C_k^{c\left(i^F\right)}\left(i^F\right)}$ $<0\left(i\in C_k^+,\ i\notin C_k^c\right)$, player i would not join coalition C_k. □

In Theorem 2, condition (2) can guarantee that each member set of the coalition is trusted, and condition (3) can guarantee that the cooperation agreement of the coalition is feasible from the perspective of cooperative payoff distribution.

For convenience, we will simply denote trust condition (1),

$$\sum_{i\in C_k^c}\left[Mv_{k_2}^{(i)}(C_k^{c(i)})-W_{k_2}^{-C_k^{c(i)}(i)}\right]+\sum_{j\in C_k^+,\ j\notin C_k^c}\left[Mv_{k_2}^{(j^F)}(C_k^{c(j^F)})-W_{k_2}^{-C_k^{c(j^F)}(j^F)}\right]>0,\ i,\ j\neq k_2,$$

as:

$$\sum_{i\in C_k}\left[Mv_{k_2}^{(i)}(C_k)-W_{k_2}^{-C_k(i)}\right]>0,\ i\neq k_2.$$

Similarly, we denote trust condition (2),

$$\sum_{i\in C_k^c}\left[Mv_{T_h}^{(i)}(C_k^{c(i)})-\sum_{t\in T_h}W_t^{-C_k^{c(i)}(i)}\right]+\sum_{j\in C_k^+,\ j\notin C_k^c}\left[Mv_{T_h}^{(j^F)}(C_k^{c(j^F)})-\sum_{t\in T_h}W_t^{-C_k^{c(j^F)}(j^F)}\right]>0,\ i,\ j\neq k_2,\ k_2\in T_h\subset C_k,$$

as:

$$\sum_{i\in C_k}\left[Mv_{T_h}^{(i)}(C_k)-\sum_{t\in T_h}W_t^{-C_k(i)}\right]>0,\ i\neq k_2,\ k_2\in T_h\subset C_k;$$

and denote trust condition (3)

$$V_{C_k^c}^{(i)} - \sum_{t \in C_k^c} W_t^{-C_k^{c(i)}(i)} > 0,\ i \in C_k^c;\ V_{C_k^{c(j^F)}}^{(j^F)} - \sum_{t \in C_k^{c(j^F)}} W_t^{-C_k^{c(j^F)}(j^F)} > 0,\ j \in C_k^+,\ j \notin C_k^c,$$

as:

$$V_{C_k}^{(i)} - \sum_{t \in C_k} W_t^{-C_k(i)} > 0,\ i \in C_k.$$

In an information asymmetric cooperative game with agreements self-implemented, to guarantee a coalition member who meets the above conditions no longer has the motive to escape through deviation, in the distribution scheme of expected cooperative payoff, the expected cooperative payoff distribution that this member can get should be no less than his expected escape-payoff deriving from deviation. That is to say, in the distribution process of expected cooperative payoff, this member's expected escape-payoff deriving from deviation in his virtual game is his reservation cooperative payoff distribution.

2.1.3 *Feasible escape paths and trust conditions*

Similarly, not all possible escape paths are feasible. In coalition situation c, when player k_2 choose to escape from coalition C_k through deviation and join coalition C_z (whether the coalition is an intermediate node coalition or a terminal node coalition), the first condition for player k_2 to be trusted by other coalition members and reach a cooperation agreement with coalition C_z is shown as follows:

$$\sum_{i \in C_z^c} \left[Mv_{k_2}^{(i)}(C_z^{c(i)}) - W_{k_2}^{-C_z^{c(i)}(i)} \right] + \sum_{j \in C_z^+,\ j \notin C_z^c} \left[Mv_{k_2}^{(j^F)}(C_z^{c(j^F)}) - W_{k_2}^{-C_z^{c(j^F)}(j^F)} \right] > 0,\ i,\ j \neq k_2. \tag{1}$$

The second condition is shown as follows:

$$\sum_{i \in C_z^c} \left[Mv_{T_h}^{(i)}(C_z^{c(i)}) - \sum_{t \in T_h} W_t^{-C_z^{c(i)}(i)} \right] + \sum_{j \in C_z^+,\ j \notin C_z^c} \left[Mv_{T_h}^{(j^F)}(C_z^{c(j^F)}) - \sum_{t \in T_h} W_t^{-C_z^{c(j^F)}(j^F)} \right] > 0,$$
$$i,\ j \neq k_2,\ k_2 \in T_h \subset C_z. \tag{2}$$

The third condition is shown as follows:

$$V_{C_z^c}^{(i)} - \sum_{t \in C_z^c} W_t^{-C_z^{c(i)}(i)} > 0,\ i \in C_z^c;\ V_{C_z^{c(j^F)}}^{(j^F)} - \sum_{t \in C_z^{c(j^F)}} W_t^{-C_z^{c(j^F)}(j^F)} > 0,\ j \in C_z^+,\ j \notin C_z^c. \tag{3}$$

In the above analysis, we did not give the definition of some member k_2's marginal contribution to coalition C_z (member k_2 as a nominal member of coalition C_z or as a member who escapes from another coalition through deviation), and his escape-payoff deriving from deviation when he escapes from coalition C_z through deviation. These two important concepts will be discussed in the following.

2.2 *Escape situations in coalition situation c and the signal releases of players*

Next, we will analyze the escape situations in coalition situation c of the information asymmetric cooperative game with agreements self-implemented.

Denote e as an escape situation in coalition situation c. In this escape situation, the escape strategy of each player i is denoted as $e_i = (e_1^i, \ldots, e_j^i, \ldots, e_n^i)$. Escape strategy variable $e_j^i = T$ represents that player i will eventually join coalition C_j, whether C_j is the coalition which contains player i in coalition situation c or not; escape strategy variable $e_j^i = F$ represents that player i promises to join coalition C_j but will eventually escape through deviation; escape strategy variable $e_j^i = 0$ represents that player i does not commit to join and finally does not join coalition C_j.

In an information asymmetric cooperative game with agreements self-implemented, some player who belongs to some core coalition may have motivation to retain private information to the coalition. Moreover, in an information asymmetric cooperative game with agreements self-implemented, players may also commit to join but do not actually join a core coalition. At this time, such a player's goal of committing to join this coalition is to release false information to guide the strategic combination choice of this coalition and make it beneficial for the improvement of the cooperative payoff of the core coalition that he will actually join.

Therefore, in each player's virtual game of the information asymmetric cooperative game with agreements self-implemented, in addition to estimating the actual strategy sets and the payoff functions of other players, he must also estimate the false signals that other players release in the coalitions they falsely join.

2.3 *Escape situations and virtual game of player k_1: his virtual game when information sets are unstable*

Under information symmetry, an important difference between a cooperative game with agreements implemented by a third party and a cooperative game with agreements self-implemented is that in a cooperative game with agreements self-implemented, since the members of any coalition may escape through deviation, the condition that a player joins his coalition is more harsh: in an information asymmetric cooperative game with agreements self-implemented, for other members of the coalition, the condition for a player being accepted as a member of the coalition is that his marginal contribution to the coalition must be at least no less than his escape-payoff deriving from deviation, and at the same time, the marginal contribution of any member set that he belongs to must be no less than the sum of the escape-payoffs deriving from deviation of all the members in the member set. And in a cooperative game with agreements implemented by a third party, once the cooperation agreement is signed, it is impossible for a member to escape through deviation.

Under information asymmetry, even if all other members of the coalition consider that the marginal contribution of some member to the coalition is greater than his escape-payoff deriving from deviation, and the marginal contribution of any member set this member belongs to is greater than the sum of escape-payoffs deriving from deviation of all the members in the member set, this member may still escape through deviation. The reason is that this member may retain private information and the information sets of other members are incomplete.

In an information asymmetric cooperative game with agreements self-implemented, the escape through deviation of any member of a coalition itself is a process of information release. The different escape strategic choices of some player mean that they may change the information sets of all the players. In the virtual game of player i_1, when he decides and plays his escape strategy on the basis of his estimation of the information sets of other members, his escape strategy as information release will change the information set and the strategic choice of any other member i_2 $(i_2 \neq i_1)$. But this is not the end of the matter. In fact, in the virtual game of player i_1, the change of the escape strategy choice of i_2 as information release will change the information set and thus the strategic choice of any other member i_3 $(i_3 \neq i_2)$ too, ..., and so on, until player i_1 has estimated any other member $i_2's$ $(i_2 \neq i_1)$ estimation of any other member $i_3's$ $(i_3 \neq i_2)$ estimation of ... any other member $i_n's$ $(i_n \neq i_{n-1})$ change in his information set and strategic choice. At this point, in the virtual game of player i_1, the influences of the optimal escape strategy choices of all players on the information sets and strategic choices of all players are taken into account, and the information sets of all players become stable.

Herein, our methodology is finding all kinds of possible information sets, then analyzing the equilibrium of a player's virtual game on the basis of stable information sets.

Given the feasible coalition situation c of the game, in the virtual game of player k_1, the feasibility of coalition situation c means that in coalition situation c, all the players are trusted by other members of the coalitions which they belong to. For any player $j \in C^+$, the conditions that he is trusted by other coalition members are shown as follows:

(1) $\sum_{i\in C_k^c}\left[Mv_{k_2}^{(i)}(C_k^{c(i)}) - W_{k_2}^{-C_k^{c(i)}(i)}\right] + \sum_{j\in C_k^+,\ j\notin C_k^c}\left[Mv_{k_2}^{(j^F)}(C_k^{c(j^F)}) - W_{k_2}^{-C_k^{c(j^F)}(j^F)}\right] > 0,\ i,\ j \neq k_2;$

(2) $\sum_{i\in C_k^c}\left[Mv_{T_h}^{(i)}(C_k^{c(i)}) - \sum_{t\in T_h} W_t^{-C_k^{c(i)}(i)}\right] + \sum_{j\in C_k^+,\ j\notin C_k^c}\left[Mv_{T_h}^{(j^F)}(C_k^{c(j^F)}) - \sum_{t\in T_h} W_t^{-C_k^{c(j^F)}(j^F)}\right] > 0,\ i,\ j \neq k_2,\ k_2 \in T_h \subset C_k;$

(3) $V_{C_k^c}^{(i)} - \sum_{t\in C_k^c} W_t^{-C_k^{c(i)}(i)} > 0,\ i \in C_k^c;\ V_{C_k^{c(j^F)}}^{(j^F)} - \sum_{t\in C_k^{c(j^F)}} W_t^{-C_k^{c(j^F)}(j^F)} > 0,\ j \in C_k^+,\ j \notin C_k^c.$

The above conditions can be simply denoted as:

$$\sum_{i\in C_k}\left[Mv_{k_2}^{(i)}(C_k) - W_{k_2}^{-C_k(i)}\right] > 0,\ i \neq k_2;$$

$$\sum_{i\in C_k}\left[Mv_{T_h}^{(i)}(C_k) - \sum_{t\in T_h} W_t^{-C_k(i)}\right] > 0,\ i \neq k_2,\ k_2 \in T_h \subset C_k;$$

$$V_{C_k}^{(i)} - \sum_{t\in C_k} W_t^{-C_k(i)} > 0,\ i \in C_k.$$

Herein, that some member is trusted by coalition C refers to that he is trusted by the extensive coalition C^+. Extensive coalition C^+ includes all its nominal members, as well as those who are considered to escape from other coalitions through deviation and join coalition C.

In coalition situation c, the n nominal coalitions have been formed, what each player k_1 should decide includes not only his escape strategy through deviation, but also the strategic choice he will adopt in the terminal node (core) coalition which he will join (his false strategic choice in an intermediate node coalition is just a means of information publishing, not a real strategic choice).

In fact, the strategic choice of some player k_1 in his escape target coalition must be subject to the goal of the target coalition's maximum expected cooperative payoff, therefore, in feasible coalition situation c, what player k_1 needs to decide includes his escape strategy $e_{k_1}^{(k_1)}$, and the strategic combination choice that "should" be adopted by the core coalition which he belongs to. Herein, the core coalition is his terminal node coalition in his virtual game, consisting of the nominal members who are considered not to escape from the coalition through deviation, and those who claim to escape from other coalition through deviation and are considered to take the core coalition as his terminal node coalition. A member of the core coalition is considered to honor his promise and take the core coalition as his terminal node coalition in player k_1's virtual game.

Escape strategy $e_{k_1}^{(k_1)}$ of player k_1 should be considered to be feasible too, that is to say, if $e_k^{k_1(k_1)} \neq 0$, then,

$$\sum_{i\in C_k^{c(k_1)}}\left[Mv_{k_1}^{(k_1,\, i)}(C_k^{c(k_1,\, i)})-W_{k_1}^{-C_k^{c(k_1,\, i)}(k_1,\, i)}\right]+\sum_{j\in C_k^{+(k_1)},\; j\notin C_k^{c(k_1)}}\left[Mv_{k_1}^{\left(k_1,\, j^F\right)}(C_k^{c\left(k_1,\, j^F\right)})-W_{k_1}^{-C_k^{c\left(k_1,\, j^F\right)}\left(k_1,\, j^F\right)}\right]>0,$$

$$i,\; j\neq k_1;$$

$$\sum_{i\in C_k^{c(k_1)}}\left[Mv_{T_h}^{(k_1,\, i)}(C_k^{c(k_1,\, i)})-\sum_{t\in T_h}W_t^{-C_k^{c(k_1,\, i)}(k_1,\, i)}\right]+\sum_{j\in C_k^{+},\; j\notin C_k^{c(k_1)}}\left[Mv_{T_h}^{\left(k_1,\, j^F\right)}(C_k^{c\left(k_1,\, j^F\right)})-\sum_{t\in T_h}W_t^{-C_k^{c\left(k_1,\, j^F\right)}\left(k_1,\, j^F\right)}\right]>0,$$

$$i,\; j\neq k_1,\; k_1\in T_h\subset C_k;$$

$$V_{C_k^{c(k_1,\, i)}}^{(k_1,\, i)}-\sum_{t\in C_k^{c(k_1,\, i)}}W_t^{-C_k^{c(k_1,\, i)}(k_1,\, i)}>0,\; i\in C_k^{c(k_1,\, i)};$$

$$V_{C_k^{c(k_1,\, j^F)}}^{(k_1,\, j^F)}-\sum_{t\in C_k^{c(k_1,\, j^F)}}W_t^{-C_k^{c(k_1,\, j^F)}(k_1,\, j^F)}>0,\; j\in C_k^{c(k_1,\, j^F)};$$

we can denote the above conditions as:

$$\sum_{i\in C_k}\left[Mv_{k_1}^{(k_1,\, i)}(C_k)-W_{k_1}^{-C_k(k_1,\, i)}\right]>0,\; i\neq k_1;$$

$$\sum_{i\in C_k}\left[Mv_{T_h}^{(k_1,\, i)}(C_k)-\sum_{t\in T_h}W_t^{-C_k(k_1,\, i)}\right]>0,\; i\neq k_1,\; k_1\in T_h\subset C_k;$$

$$V_{C_k}^{(k_1,\, i)}-\sum_{t\in C_k}W_t^{-C_k(k_1,\, i)}>0,\; i\in C_k.$$

That is, whether he honors his commitment to escape through deviation or not, player k_1 must be trusted by his escape target coalition.

Obviously, in coalition situation c, in the virtual game of player k_1, his optimal escape strategy $e_{k_1}^{*(k_1)}$ and the optimal strategic combination choice $s_{C_T}^{*(k_1)}$ of the core coalition (as his terminal node coalition) are the optimal response to his estimations of the optimal escape strategies $e_{-k_1}^{*(k_1)}$ of other players and other core coalitions' optimal strategic combination choices $s_{-C_T}^{*(k_1)}$, the goal of his decision-making is to maximize his expected cooperative payoff distribution (from his escape target core coalition):

$$(e_{k_1}^{*(k_1)},\, s_{C_T^c}^{*(k_1)}) = \underset{\left(e_{k_1}^{(k_1)},\, s_{C_T^c}^{(k_1)}\right)}{argmax}\ \widetilde{x}_{k_1}^{(k_1)}(C_T^c)\left[\left(e_{k_1}^{(k_1)},\, s_{C_T^c}^{(k_1)}\right);\ \left(e_{-k_1}^{*(k_1)},\, s_{-C_T^c}^{*(k_1)}\right)\right],$$

where C_T^c is the escape target coalition of player k_1 in escape situation $(e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)})$, $e_T^{k_1(k_1)} = T$; $\widetilde{x}_{k_1}^{(k_1)}(C_T^c)$ is player k_1's estimation of his expected cooperative payoff distribution from core coalition C_T^c he belongs to.

Or, because player k_1's goal of the maximum expected cooperative payoff distribution is consistent with his goal of the minimum expected escape-payoff deriving from deviation, the goal of player k_1's decision-making is also his minimum expected escape-payoff deriving from deviation:

$$(e_{k_1}^{*(k_1)},\, s_{C_T^c}^{*(k_1)}) = \underset{(e_{k_1}^{(k_1)},\, s_{C_T^c}^{(k_1)})}{argmin}\ W_{k_1}^{-C_T^c(k_1)}\left[(e_{k_1}^{(k_1)},\, s_{C_T^c}^{(k_1)});\ (e_{-k_1}^{*(k_1)},\, s_{-C_T^c}^{*(k_1)})\right],$$

where $W_{k_1}^{-C_T^c(k_1)}$ represents the expected escape-payoff deriving from deviation of player k_1 when he escapes from core coalition C_T^c through deviation.

The *n* levels virtual game of player k_1.

In coalition situation c, assume that player k_1 is a member of coalition C, $k_1 \in C$, given player k_1's estimation of the optimal strategic combination choices $e_{-k_1}^{*(k_1)}$ of other coalitions, when player k_1 decides his optimal escape strategy and the optimal strategic combination choice that the core coalition he belongs to "should" adopt, the goal of his decision-making is his maximum expected cooperative payoff distribution (from his escape target core coalition C_T^c when he escapes from C through deviation). As mentioned above, in escape situation $(e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)})$, the goal of player k_1's maximum expected cooperative payoff distribution from the escape target core coalition C_T^c is consistent with the goal of maximum expected cooperative payoff of core coalition C_T^c.

In the first level virtual game of player k_1, in coalition situation c, given the escape situation $\left\{e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1},\, e_{-k_1}^{*(k_1)}\right)\right]\right\}$ (where $e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\right)\right]$ are the equilibrium escape strategic choices of other players when his escape strategy is $e_{k_1}^{(k_1)}$, and the information sets of all the players are $I^{(k_1)}\left[e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\right]$). At this time, player k_1 can make up three different member sets of coalition C_i:

(1) Nominal member set $M_i^{(k_1)}$, that is, the member set of coalition C_i in coalition situation c when there is no escape behavior of the members to be considered; nominal coalition C_i is formed by all the nominal members;

(2) Extensive member set $M_i^{+(k_1)}\left\{e_{k_1}^{(k_1)}, e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)}, e_{-k_1}^{*(k_1)}\right)\right]\right\}$, that is, the member set of coalition C_i formed after all its members escape from other coalitions through deviation (including false escape through deviation). In this member set, all the members in the nominal member set are included, as well as the players in other coalitions who claim to join coalition C_i through deviation; the extensive members form extensive coalition $C_i^{+(k_1)}$;

(3) Core member set $M_i^{c(k_1)}\left\{e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\right)\right]\right\}$, that is, in extensive member set $M_i^{+(k_1)}\left\{e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\right)\right]\right\}$, the member subset formed by those who are considered to eventually join coalition C_i and decide not to escape from coalition C_i through deviation (that is to say, the extensive members who are considered to take coalition C_i as their escape target coalition); the core members form core coalition $C_i^{c(k_1)}$.

At the same time, in player k_1's first level virtual game, when some player plays different escape strategies, player k_1's estimation of the information sets of other players are different too, because the implementation process of escape strategy itself is a process of information release. When a player plays different escape strategies, the information he releases will be different. Therefore, player k_1 has different estimations of the optimal escape strategies of other players in different escape situations. Denote player k_1's estimation of the optimal escape strategic choice of any player when player k_1 plays escape strategy $e_{k_1}^{(k_1)}$ which he considers feasible as $e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\right)\right]$, in coalition situation c of the virtual game of player k_1, when player k_1 plays feasible escape strategy $e_{k_1}^{(k_1)}$, his estimation of the escape situation will be $\left\{e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\right)\right]\right\}$.

When player k_1 chooses escape strategy $e_{k_1}^{(k_1)}$ that is considered feasible, first of all, player k_1 needs to estimate the information sets $I^{(k_1)}(e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)})$ of all the players and the corresponding escape strategic choices $e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\right)\right]$ of other players, here, the condition that his escape strategy $e_{k_1}^{(k_1)}$ is considered feasible is that, if $e_k^{k_1(k_1)} \neq 0$ (that is, player k_1 chooses coalition C_k as his escape target coalition ($e_k^{k_1(k_1)} = T$) or his false escape target coalition ($e_k^{k_1(k_1)} = F$), then,

$$\sum_{i\in C_k}\left[Mv_{k_1}^{(k_1,\ i)}(C_k) - W_{k_1}^{-C_k(k_1,\ i)}\right] > 0,\ i \neq k_1;$$

$$\sum_{i\in C_k}\left[Mv_{T_h}^{(k_1,\ i)}(C_k) - \sum_{t\in T_h} W_t^{-C_k(k_1,\ i)}\right] > 0,\ i \neq k_1,\ k_1 \in T_h \subset C_k;$$

$$V_{C_k}^{(k_1,\ i)} - \sum_{t\in C_k} W_t^{-C_k(k_1,\ i)} > 0,\ i \in C_k.$$

When the information sets of all the players are $I^{(k_1)}(e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)})$, assume that player k_1's estimation of the strategic combination choice of any other core coalition $C_h^{c(k_1)}\left\{e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\right)\right]\right\}$ $\left(C_h^{c(k_1)}\left\{e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\right)\right]\right\} \neq C_T^{c(k_1)}\left\{e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\right)\right]\right\}\right)$ is $s^{**(k_1)}_{C_h^c\left\{e_{k_1}^{(k_1)},\ e_{-k_1}^{(k_1)*}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\right)\right]\right\}}$, then the optimal strategic combination that the escape target coalition of player k_1 should adopt (that is, in player k_1's first level virtual game, the optimal strategic combination which maximizes the cooperative payoff of core coalition $C_T^{c(k_1)}\left\{e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\right)\right]\right\}$ is considered to be:

$$
\begin{aligned}
& s^{\circ(k_1)}_{C_T^{c(k_1)}\left\{e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\right)\right]\right\}} \\
&= \underset{s^{(k_1)}_{C_T^{c(k_1)}\left\{e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\right)\right]\right\}}}{argmax}\; V^{(k_1)}_{C_T^{c(k_1)}(\cdot)}\left(s^{(k_1)}_{C_T^{c(k_1)}\left\{e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\right)\right]\right\}},\right. \\
&\qquad \left. s^{**(k_1)}_{-C_T^{c(k_1)}\left\{e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\right)\right]\right\}}\right) \\
&= \underset{s^{(k_1)}_{C_T^{c(k_1)}\left\{e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\right)\right]\right\}}}{argmax}\; \sum_{i\in C_T^{c(k_1)}\left\{e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\right)\right]\right\}} u_i^{(k_1)}\left(s^{(k_1)}_{C_T^{c(k_1)}\left\{e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\right)\right]\right\}},\right. \\
&\qquad \left. s^{**(k_1)}_{-C_T^{c(k_1)}\left\{e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\right)\right]\right\}}\right).
\end{aligned}
$$

This maximization model above determines the optimal strategic combination choice of core coalition $C_T^{c(k_1)}\left\{e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\right)\right]\right\}$ in escape situation $\left\{e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\right)\right]\right\}$ (in player k_1's first level virtual game); of course, it also determines player k_1's optimal strategy in escape situation $\left\{e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\right)\right]\right\}$ (in player k_1's first level virtual game):

$$
s_{k_1}^{\circ(k_1)}\left(e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\right) \in s^{\circ(k_1)}_{C_T^{c(k_1)}\left\{e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\right)\right]\right\}}.
$$

Strategic combination $s^{\circ(k_1)}_{C_T^{c(k_1)}\left\{e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\right)\right]\right\}}$ is the one that player k_1's escape target core coalition $C_T^{c(k_1)}\left\{e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\right)\right]\right\}$ should adopt when player k_1 plays escape strategy $e_{k_1}^{(k_1)}$ in player k_1's first level virtual game, this strategic combination is considered to maximize the cooperative payoff of core coalition $C_T^{c(k_1)}\left\{e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\right)\right]\right\}$ when player k_1 plays escape strategy $e_{k_1}^{(k_1)}$ and at the same time maximize the cooperative payoff distribution of player k_1 himself. Denote the cooperative payoff distribution that player k_1 gets from core coalition $C_T^{c(k_1)}\left\{e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\right)\right]\right\}$ as $\widetilde{x}_{k_1}^{\circ(k_1)}\left\{e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\right)\right]\right\}$, and his escape-payoff deriving from deviation as $W_{k_1}^{\circ-C_T^{c(k_1)}\left\{e_{k_1}^{(k_1)}, e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)}, e_{-k_1}^{*(k_1)}\right)\right]\right\}(k_1)}$ when he escapes from core coalition $C_T^{c(k_1)}\left\{e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\right)\right]\right\}$ through deviation (we will examine the cooperative payoff distribution $\widetilde{x}_{k_1}^{\circ(k_1)}\left\{e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\right)\right]\right\}$ that player k_1 gets from core coalition $C_T^{c(k_1)}\left\{e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\right)\right]\right\}$ and his escape-payoff deriving from deviation $C_T^{c(k_1)}$ in the following).

Under the escape strategy $e_{k_1}^{(k_1)}$, the reason why player k_1 considers that core coalition $C_T^{c(k_1)}\left\{e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\right)\right]\right\}$ "should" adopt strategic combination $s^{\circ(k_1)}_{C_T^{c(k_1)}\left\{e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\right)\right]\right\}}$ which can maximize the cooperative payoff of core coalition $C_T^{c(k_1)}\left\{e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\right)\right]\right\}$ is that core coalition $C_T^{c(k_1)}\left\{e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\right)\right]\right\}$'s maximum cooperative payoff means the maximum cooperative payoff distributions of all the core members of core coalition $C_T^{c(k_1)}\left\{e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\right)\right]\right\}$, and also the maximum cooperative payoff distribution $\widetilde{x}_{k_1}^{\circ(k_1)}\left\{e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\right)\right]\right\}$ of player k_1. Therefore, player k_1's escape strategic choice is aimed at his maximum cooperative payoff $\widetilde{x}_{k_1}^{\circ(k_1)}\left\{e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\right)\right]\right\}$ distribution from his target core coalition $C_T^{c(k_1)}\left\{e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\right)\right]\right\}$:

$$e_{k_1}^{*(k_1)} = \underset{e_{k_1}^{(k_1)}}{argmax}\widetilde{x}_{k_1}^{\circ(k_1)}\left(C_T^{c(k_1)}\left\{e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\right)\right]\right\}\right).$$

Or, the goal of his escape strategic choice is his minimum expected escape-payoff deriving from deviation:

$$e_{k_1}^{*(k_1)} = \underset{e_{k_1}^{(k_1)}}{argmin}W_{k_1}^{\circ - C_T^{c(k_1)}\left\{e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\right)\right]\right\}(k_1)}.$$

The escape target core coalition of player k_1 is $C_T^{c(k_1)}\left\{e_{k_1}^{*(k_1)},\ e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{*(k_1)},\ e_{-k_1}^{*(k_1)}\right)\right]\right\}$, the strategic combination that this core coalition "should" adopt is:

$$s^{*(k_1)}_{C_T^{c(k_1)}\left\{e_{k_1}^{*(k_1)},\ e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{*(k_1)},\ e_{-k_1}^{*(k_1)}\right)\right]\right\}} = s^{\circ(k_1)}_{C_T^{c(k_1)}\left\{e_{k_1}^{*(k_1)},\ e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{*(k_1)},\ e_{-k_1}^{*(k_1)}\right)\right]\right\}}.$$

The cooperative payoff that core coalition $C_T^{c(k_1)}\left\{e_{k_1}^{*(k_1)},\ e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{*(k_1)},\ e_{-k_1}^{*(k_1)}\right)\right]\right\}$ "should" get is:

$$V^{(k_1)}_{C_T^{c(k_1)}(\cdot)}\Big(s^{*(k_1)}_{C_T^{c(k_1)}\left\{e_{k_1}^{*(k_1)},\ e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{*(k_1)},\ e_{-k_1}^{*(k_1)}\right)\right]\right\}},\ s^{**(k_1)}_{-C_T^{c(k_1)}\left\{e_{k_1}^{*(k_1)},\ e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{*(k_1)},\ e_{-k_1}^{*(k_1)}\right)\right]\right\}}\Big)$$

$$= \sum_{i\in C_T^{c(k_1)}\left\{e_{k_1}^{*(k_1)},\ e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{*(k_1)},\ e_{-k_1}^{*(k_1)}\right)\right]\right\}} u_i^{(k_1)}\Big(s^{*(k_1)}_{C_T^{c(k_1)}\left\{e_{k_1}^{*(k_1)},\ e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{*(k_1)},\ e_{-k_1}^{*(k_1)}\right)\right]\right\}},$$

$$s^{**(k_1)}_{-C_T^{c(k_1)}\left\{e_{k_1}^{*(k_1)},\ e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{*(k_1)},\ e_{-k_1}^{*(k_1)}\right)\right]\right\}}\Big).$$

The expected cooperative payoff distribution $\tilde{x}_{k_1}^{o(k_1)}\left(C_T^{c(k_1)}\left\{e_{k_1}^{*(k_1)},\ e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{*(k_1)},\ e_{-k_1}^{*(k_1)}\right)\right]\right\}\right)$ of player k_1 is the maximum expected cooperative payoff distribution that player k_1 can get in coalition situation c:

$$\widetilde{x}_{k_1}^{*(k_1)} = \widetilde{x}_{k_1}^{\circ(k_1)}\left(C_T^{c(k_1)}\left\{e_{k_1}^{*(k_1)},\ e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{*(k_1)},\ e_{-k_1}^{*(k_1)}\right)\right]\right\}\right).$$

In player k_1's first level virtual game, when player k_1 decides his own optimal escape strategy and the optimal strategic combination choices of his core coalition, he must estimate the optimal escape strategy $e_{k_2}^{*(k_1)}$ of any other player $k_2(k_2 \neq k_1)$ and the strategic combination choices $s^{**(k_1)}_{-C_T^{c(k_1)}\left\{e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\right)\right]\right\}}$ of the core coalitions other than his own. This estimation relies on player k_1's estimation of any other player k_2's estimation of his own equilibrium choices based on his own information set in player k_1's virtual game. Player k_1's estimation of the escape strategic choice $e_{k_2}^{*(k_1)}$ of any other player k_2 depends on player k_2's choice based on his own information set in player k_1's virtual game:

$$e_{k_2}^{*(k_1)} = e_{k_2}^{*(k_1,\ k_2)}.$$

The strategic combination choice $s^{**(k_1)}_{C_h^{c(k_1)}\left\{e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\right)\right]\right\}}$ of any other core coalition $C_h^{c(k_1)}\left\{e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\right)\right]\right\}$ $\left(C_h^{c(k_1)}\left\{e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\right)\right]\right\} \neq C_T^{c(k_1)}\left\{e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\right)\right]\right\}\right)$ depends on the equilibrium public choice of extensive coalition $C_h^{+(k_1)}\left\{e_{k_1}^{*(k_1)},\ e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{*(k_1)},\ e_{-k_1}^{*(k_1)}\right)\right]\right\}$:

$$s^{**(k_1)}_{C_h^{c(k_1)}\left\{e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\right)\right]\right\}} \subseteq s^{**\left(k_1,\ C_h^{+(k_1)}\right)}_{C_h^{+(k_1)}\left\{e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\right)\right]\right\}};$$

$$s^{**(k_1)}_{C_h^{c(k_1)}\left\{e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\ e_{-k_1}^{*(k_1)}\right)\right]\right\}} = argmax\left\{\sum_{i\in C_h^{c(k_1)}(\cdot)} V^{(k_1,\ i)}_{C_T^{c(k_1,\ i)}(\cdot)}\left(s^{(k_1,\ i)}_{C_T^{c(k_1,\ i)}(\cdot)},\ s^{**(k_1,\ i)}_{-C_T^{c(k_1,\ i)}(\cdot)}\right)\right.$$

$$\left. + \sum_{\substack{j\in C_h^{+(k_1)}(\cdot)\\ j\notin C_h^{c(k_1)}(\cdot)}} V^{(k_1,\ j^F)}_{C_h^{c(k_1,\ j)}(\cdot)}\left(s^{(k_1,\ j^F)}_{C_h^{c(k_1,\ j)}(\cdot)},\ s^{**(k_1,\ j^F)}_{C_T^{c(k_1,\ j)}(\cdot)},\ s^{**(k_1,\ j^F)}_{-C_h^{c(k_1,\ j)}(\cdot),\ -C_T^{c(k_1,\ j)}(\cdot)}\right)\right\};$$

$$V^{\left(k_1,\ j^F\right)}_{C_h^{c(k_1,\ j)}(\cdot)}\left(s^{\left(k_1,\ j^F\right)}_{C_h^{c(k_1,\ j)}(\cdot)},\ s^{**\left(k_1,\ j^F\right)}_{C_T^{c(k_1,\ j)}(\cdot)},\ s^{**\left(k_1,\ j^F\right)}_{-C_h^{c(k_1,\ j)}(\cdot),\ -C_T^{c(k_1,\ j)}(\cdot)}\right) = V^{\left(k_1,\ j^F\right)}_{C_h^{c(k_1,\ j)}(\cdot)}\left(s^{\left(k_1,\ j^F\right)}_{C_h^{c(k_1,\ j)}(\cdot)},\ i_j^{\left(k_1,\ j^F\right)}\right)\left(i_j^{\left(k_1,\ j^F\right)} \in I_j^{\left(k_1,\ j^F\right)}\right);$$

where $i \in C_h^{c(k_1)}(\cdot)$. Therefore, the escape target core coalition $C_T^{c(k_1,\ i)}(\cdot)$ of player i is core coalition $C_h^{c(k_1)}(\cdot)$, $j \in C_h^{+(k_1)}(\cdot)$, $j \notin C_h^{c(k_1)}(\cdot)$, core coalition $C_h^{c(k_1,\ i)}(\cdot)$ and core coalition $C_h^{c(k_1)}(\cdot)$ are the same, the escape target coalition $C_T^{c(k_1,\ j)}(\cdot)$of player j when he escapes through deviation is not core coalition $C_h^{c(k_1)}(\cdot)$; $V^{(k_1,\ j^F)}_{C_T^{c(k_1,\ j)}(\cdot)}\left(s^{(k_1,\ j^F)}_{C_h^{c(k_1,\ j)}(\cdot)},\ s^{**(k_1,\ j^F)}_{C_T^{c(k_1,\ j)}(\cdot)},\ s^{**(k_1,\ j^F)}_{-C_h^{c(k_1,\ j)}(\cdot),\ -C_T^{c(k_1,\ j)}(\cdot)}\right)$ is player k_1's estimation of player j's (false) estimation of the cooperative payoff of core coalition $C_T^{c(k_1,\ j)}(\cdot)$ according to the false signal released by player j, $i_j^{(k_1,\ j^F)}$ is player k_1's estimation of the false signal of player j, $s^{**(k_1,\ j^F)}_{C_T^{c(k_1,\ j)}(\cdot)}$, $s^{**(k_1,\ j^F)}_{-C_h^{c(k_1,\ j)}(\cdot),\ -C_T^{c(k_1,\ j)}(\cdot)}$ is player k_1's estimation of player j's (false) estimation of the strategic combination choices of core coalitions (including core coalition $C_T^{c(k_1,\, j)}(\cdot)$ which player j finally joins) other than player j's false escape target core coalition $C_h^{c(k_1,\, j)}(\cdot)$ accor-ding to the false signal released by player j, $i_j^{(k_1,\, j^F)}$.

Therefore, in player k_1's virtual game, player j's $\left[j \in C_h^{+(k_1)}(\cdot),\ j \notin C_h^{c(k_1)}(\cdot)\right]$ estimation of the strategic combination choice $s^{**(k_1,\, j)}_{C_h^{c(k_1,\, j)}(\cdot)}$ of core coalition $C_h^{c(k_1,\, j)}(\cdot)$ is a function of his false signal $i_j^{(k_1,\, j^F)}$:

$$s^{**(k_1,\, j)}_{C_h^{c(k_1,\, j)}(\cdot)} = s^{**(k_1,\, j)}_{C_h^{c(k_1,\, j)}(\cdot)}\left(i_j^{(k_1,\, j^F)}\right)$$

$$= argmax \left\{ \sum_{i \in C_h^{c(k_1,\, j)}(\cdot)} V^{(k_1,\, j,\, i)}_{C_T^{c(k_1,\, j,\, i)}(\cdot)} \left(s^{(k_1,\, j,\, i)}_{C_T^{c(k_1,\, j,\, i)}(\cdot)},\ s^{**(k_1,\, j,\, i)}_{-C_T^{c(k_1,\, j,\, i)}(\cdot)} \right) + V^{(k_1,\, j^F)}_{C_h^{c(k_1,\, j)}(\cdot)} \left(s^{(k_1,\, j^F)}_{C_h^{c(k_1,\, j)}(\cdot)},\ i_j^{(k_1,\, j^F)} \right) \right.$$

$$\left. + \sum_{\substack{k \in C_h^{+(k_1, j)}(\cdot) \\ k \notin C_h^{c(k_1, j)}(\cdot) \\ k \neq j}} V^{(k_1,\, j,\, k^F)}_{C_h^{c(k_1,\, j,\, k)}(\cdot)} \left(s^{(k_1,\, j,\, k^F)}_{C_h^{c(k_1,\, j,\, k)}(\cdot)},\ i_j^{*(k_1,\, j,\, k^F)} \right) \right\},$$

where $i \in C_h^{c(k_1,\, j)}(\cdot)$, therefore the escape target core coalition $C_T^{c(k_1,\, j,\, i)}(\cdot)$ of player i is core coalition $C_h^{c(k_1,\, j)}(\cdot)$, $k \in C_h^{+(k_1,\, j)}(\cdot)$, $k \notin C_h^{c(k_1,\, j)}(\cdot)$, core coalition $C_h^{c(k_1,\, j,\, k)}(\cdot)$ and core coalition $C_h^{c(k_1,\, j)}(\cdot)$ are the same, the escape target core coalition $C_T^{c(k_1,\, j,\, k)}(\cdot)$ of player k when he escapes through deviation is not core coalition $C_h^{c(k_1,\, j)}(\cdot)$; $i_j^{*(k_1,\, j,\, k^F)}$ is player j's estimation of the optimal false signal of any extensive member k $\left[k \in C_h^{+(k_1,\, j)}(\cdot),\ k \notin C_h^{c(k_1,\, j)}(\cdot),\ k \neq j\right]$ in player j's virtual game.

Player j's purpose of releasing false signal to his intermediate node coalition $C_h^{c(k_1,\, j)}(\cdot)$ is to influence the public choice of the strategic combination choice of core coalition $C_h^{c(k_1,\, j)}(\cdot)$, thereby maximize the expected cooperative payoff of player j's terminal node core coalition $C_T^{c(k_1,\, j)}(\cdot)$:

$$i_j^{*(k_1,\, j^F)} = \underset{i_j^{(k_1,\, j^F)} \in I_j^{(k_1,\, j^F)}}{argmax}\ V^{(k_1,\, j)}_{C_T^{c(k_1,\, j)}(\cdot)} \left(s^{**(k_1,\, j)}_{C_h^{c(k_1,\, j)}(\cdot)}\left(i_j^{(k_1,\, j^F)}\right),\ s^{*(k_1,\, j)}_{C_T^{c(k_1,\, j)}(\cdot)},\ s^{**(k_1,\, j)}_{-C_h^{c(k_1,\, j)}(\cdot),\, -C_T^{c(k_1,\, j)}(\cdot)} \right).$$

Obviously, to estimate the strategic combination choice $s^{**(k_1)}_{C_h^{c(k_1)}\left\{e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\left[I^{(k_1)}\left(e_{k_1}^{(k_1)},\, e_{-k_1}^{*(k_1)}\right)\right]\right\}}$ of any other core coalition $C_h^{c(k_1)}(\cdot)$, player k_1 needs to estimate any core member i's [who belongs to core coalition $C_h^{c(k_1)}(\cdot)$] estimation of the strategic combination choices $s^{**(k_1,\, i)}_{-C_T^{c(k_1,\, i)}(\cdot)}$ of other core coalitions $-C_h^{c(k_1)}(\cdot)$, and any extensive member j's [who belongs to core coalition $C_h^{c(k_1)}(\cdot)$] estimation of the strategic combination choice that core coalition $C_h^{c(k_1)}(\cdot)$ "should" adopt, extensive member j's estimation of the strategic combination choices $s^{**(k_1,\, j)}_{-C_h^{c(k_1,\, j)}(\cdot),\, -C_T^{c(k_1,\, j)}(\cdot)}$ of the core coalitions other than core coalition $C_h^{c(k_1,\, j)}(\cdot)$ and his terminal node core coalition $C_T^{c(k_1,\, j)}(\cdot)$, as well as extensive member j's estimation of any extensive member k's optimal signal $i_j^{*(k_1,\, j,\, k^F)}$.

Thus, the virtual game of player k_1 enters the second level. In the second level virtual game, in order to get his optimal escape strategy in coalition situation c, player k_1 needs to estimate the escape strategy $e_{k_2}^{*(k_1,\, k_2)}\left(e_{k_1}^{(k_1)}\right)$ of any other player $k_2 (k_2 \neq k_1)$ when he plays any feasible escape strategy $e_{k_1}^{(k_1)}$. And in order to get his optimal escape strategy, player

$k_2(k_2 \neq k_1)$ needs to estimate the escape strategy $e_{k_3}^{*(k_1,\, k_2,\, k_3)}(e_{k_1}^{(k_1)},\, e_{k_2}^{(k_1,\, k_2)})$ of any other player $k_3(k_3 \neq k_2)$ when he plays any feasible escape strategy $e_{k_2}^{(k_1,\, k_2)}$. At this point, player k_1's virtual game enters the third level, …

Examine player k_1's virtual game level by level up to the n-th level, we get the virtual game of player k_1 with the information sets of other players stable.

The model of the t-th $(1 \leq t \leq n)$ level virtual game of player k_1 is shown as follows.

The *t*-th level virtual game of player k_1

In player k_1's t-th level virtual game, player k_1 needs to estimate any player k_2's estimation of … any player k_t's estimation of the escape strategy $e_{k_{t+1}}^{*(k_1,\, \cdots,\, k_{t+1})}$ of any other player $k_{t+1}(k_{t+1} \neq k_t)$ and the strategic combination choices $s^{**(k_1,\, k_2,\, \cdots,\, k_t)}_{C_h^{c(k_1,\, k_2,\, \cdots,\, k_t)}\left\{e_{k_t}^{(k_1,\, k_2,\, \cdots,\, k_t)},\, e_{-k_t}^{*(k_1,\, k_2,\, \cdots,\, k_t)}\left[I^{(k_1,\, \cdots,\, k_t)}\left(e_{k_1}^{(k_1)},\, \cdots;\, e_{k_t}^{(k_1,\, k_2,\, \cdots,\, k_t)},\, e_{-k_1,\, -k_2,\, \cdots,\, -k_t}^{*(k_1,\, k_2,\, \cdots,\, k_t)}\right)\right]\right\}}$ of core coalitions $C_h^{c(k_1,\, k_2,\, \cdots,\, k_t)}$ $\left\{e_{k_t}^{(k_1,\, k_2,\, \cdots,\, k_t)},\, e_{-k_t}^{*(k_1,\, k_2,\, \cdots,\, k_t)}\left[I^{(k_1,\, \cdots,\, k_t)}\left(e_{k_1}^{(k_1)},\, \cdots;\, e_{k_t}^{(k_1,\, k_2,\, \cdots,\, k_t)},\, e_{-k_1,\, -k_2,\, \cdots,\, -k_t}^{*(k_1,\, k_2,\, \cdots,\, k_t)}\right)\right]\right\}$ other than the escape target core coalition of player k_t, when the feasible escape strategies that player k_1, player k_2, …, player k_{t-1} play are respectively $e_{k_1}^{(k_1)}, e_{k_2}^{(k_1,\, k_2)},\, e_{k_3}^{(k_1,\, k_2,\, k_3)},\, \ldots,\, e_{k_{t-1}}^{(k_1,\, k_2,\, \cdots,\, k_{t-1})}$.

In player k_1's t-th level virtual game, when the feasible escape strategies that player k_1, player k_2, …, player k_{t-1} play are respectively $e_{k_1}^{(k_1)}, e_{k_2}^{(k_1,\, k_2)}, e_{k_3}^{(k_1,\, k_2,\, k_3)},\, \ldots,\, e_{k_{t-1}}^{(k_1,\, k_2,\, \cdots,\, k_{t-1})}$, first player k_t needs to estimate the information sets $I^{(k_1,\, \cdots,\, k_t)}(e_{k_1}^{(k_1)},\, \cdots,\, e_{k_t}^{(k_1,\, \cdots,\, k_t)},\, e_{-k_1,\, -k_2,\, \cdots,\, -k_t}^{*(k_1,\, k_2,\, \cdots,\, k_t)})$ of all the players, his optimal escape strategic $e_{k_t}^{*(k_1,\, \cdots,\, k_t)}\Big[I^{(k_1,\, \cdots,\, k_t)}$ $\left(e_{k_1}^{(k_1)},\, \cdots,\, e_{k_t}^{(k_1,\, k_2,\, \cdots,\, k_t)},\, e_{-k_1,\, -k_2,\, \cdots,\, -k_t}^{*(k_1,\, k_2,\, \cdots,\, k_t)}\right)\Big]$, choice and the strategic combination choice $s_{C_T^{c(k_1,\, k_2,\, \cdots,\, k_t)}\left\{e_{k_t}^{(k_1,\, k_2,\, \cdots,\, k_t)},\, e_{-k_t}^{*(k_1,\, k_2,\, \cdots,\, k_t)}\left[I^{(k_1,\, \cdots,\, k_t)}\left(e_{k_1}^{(k_1)},\, \cdots;\, e_{k_t}^{(k_1,\, k_2,\, \cdots,\, k_t)},\, e_{-k_1,\, -k_2,\, \cdots,\, -k_t}^{*(k_1,\, k_2,\, \cdots,\, k_t)}\right)\right]\right\}}^{*(k_1,\, k_2,\, \cdots,\, k_t)}$ of his escape target core coalition, where the condition that player k_1 considers player k_2 considers … player k_t considers some escape strategy $e_{k_t}^{(k_1,\, k_2,\, \cdots,\, k_t)}$ is feasible is that if $e_k^{k_t(k_1,\, k_2,\, \cdots,\, k_t)} \neq 0$ ($e_k^{k_t(k_1,\, k_2,\, \cdots,\, k_t)} = T$, or, $e_k^{k_t(k_1,\, k_2,\, \cdots,\, k_t)} = F$), then,

$$\sum_{i \in C_k}\left[Mv_{k_1}^{(k_1,\, k_2,\, \cdots,\, k_t,\, i)}(C_k) - W_{k_1}^{-C_k(k_1,\, k_2,\, \cdots,\, k_t,\, i)}\right] > 0,\ i \neq k_t;$$

$$\sum_{i \in C_k}\left[Mv_{T_h}^{(k_1,\, k_2,\, \cdots,\, k_t,\, i)}(C_k) - \sum_{t \in T_h} W_t^{-C_k(k_1,\, k_2,\, \cdots,\, k_t,\, i)}\right] > 0,\ i \neq k_t,\ k_t \in T_h \subset C_k;$$

$$V_{C_k}^{(k_1,\, k_2,\, \cdots,\, k_t,\, i)} - \sum_{t \in C_k} W_t^{-C_k(k_1,\, k_2,\, \cdots,\, k_t,\, i)} > 0,\ i \in C_k.$$

Assume that under information sets $I^{(k_1,\, \cdots,\, k_t)}(e_{k_1}^{(k_1)},\, \cdots,\, e_{k_t}^{(k_1,\, \cdots,\, k_t)},\, e_{-k_1,\, -k_2,\, \cdots,\, -k_t}^{*(k_1,\, k_2,\, \cdots,\, k_t)})$, player k_1's estimation of player k_1's estimation of … player k_t's estimation of the strategic combination choice of any core coalition $C_h^{c(k_1,\, k_2,\, \cdots,\, k_t)}\left\{e_{k_t}^{(k_1,\, k_2,\, \cdots,\, k_t)},\, e_{-k_t}^{*(k_1,\, k_2,\, \cdots,\, k_t)}\left[I^{(k_1,\, \cdots,\, k_t)}\left(e_{k_1}^{(k_1)},\, \cdots;\, e_{k_t}^{(k_1,\, k_2,\, \cdots,\, k_t)},\, e_{-k_1,\, -k_2,\, \cdots,\, -k_t}^{*(k_1,\, k_2,\, \cdots,\, k_t)}\right)\right]\right\}\Big(\neq C_T^{c(k_1,\, k_2,\, \cdots,\, k_t)}$ $\left\{e_{k_t}^{(k_1,\, k_2,\, \cdots,\, k_t)},\, e_{-k_t}^{*(k_1,\, k_2,\, \cdots,\, k_t)}\left[I^{(k_1,\, \cdots,\, k_t)}\left(e_{k_1}^{(k_1)},\, \cdots;\, e_{k_t}^{(k_1,\, k_2,\, \cdots,\, k_t)},\, e_{-k_1,\, -k_2,\, \cdots,\, -k_t}^{*(k_1,\, k_2,\, \cdots,\, k_t)}\right)\right]\right\}\Big)$ is $s_{C_h^{c(k_1,\, k_2,\, \cdots,\, k_t)}\left\{e_{k_t}^{(k_1,\, k_2,\, \cdots,\, k_t)},\, e_{-k_t}^{*(k_1,\, k_2,\, \cdots,\, k_t)}\left[I^{(k_1,\, \cdots,\, k_t)}\left(e_{k_1}^{(k_1)},\, \cdots;\, e_{k_t}^{(k_1,\, k_2,\, \cdots,\, k_t)},\, e_{-k_1,\, -k_2,\, \cdots,\, -k_t}^{*(k_1,\, k_2,\, \cdots,\, k_t)}\right)\right]\right\}}^{**(k_1,\, k_2,\, \cdots,\, k_t)}$, the strategic combination choice that core coalition $C_T^{c(k_1,\, k_2,\, \cdots,\, k_t)}\left\{e_{k_t}^{(k_1,\, k_2,\, \cdots,\, k_t)},\, e_{-k_t}^{*(k_1,\, k_2,\, \cdots,\, k_t)}\left[I^{(k_1,\, \cdots,\, k_t)}\left(e_{k_1}^{(k_1)},\, \cdots;\, e_{k_t}^{(k_1,\, k_2,\, \cdots,\, k_t)},\, e_{-k_1,\, -k_2,\, \cdots,\, -k_t}^{*(k_1,\, k_2,\, \cdots,\, k_t)}\right)\right]\right\}$ which is player k_t's escape target "should" adopt (that is, the strategic combination choice that is considered to maximize the cooperative payoff of core coalition $C_T^{c(k_1,\, k_2,\, \cdots,\, k_t)}\left\{e_{k_t}^{(k_1,\, k_2,\, \cdots,\, k_t)},\, e_{-k_t}^{*(k_1,\, k_2,\, \cdots,\, k_t)}\left[I^{(k_1,\, \cdots,\, k_t)}\left(e_{k_1}^{(k_1)},\, \cdots;\, e_{k_t}^{(k_1,\, k_2,\, \cdots,\, k_t)},\, e_{-k_1,\, -k_2,\, \cdots,\, -k_t}^{*(k_1,\, k_2,\, \cdots,\, k_t)}\right)\right]\right\}$ is:

$$s^{\circ(k_1, k_2, \cdots, k_t)}_{C_T^{c(k_1, k_2, \cdots, k_t)}\left\{e_{k_t}^{(k_1, k_2, \cdots, k_t)}, e_{-k_t}^{*(k_1, k_2, \cdots, k_t)}\left[I^{(k_1, \cdots, k_t)}\left(e_{k_1}^{(k_1)}, \cdots; e_{k_t}^{(k_1, k_2, \cdots, k_t)}, e^{*(k_1, k_2, \cdots, k_t)}_{-k_1, -k_2, \cdots, -k_t}\right)\right]\right\}}$$

$$= \underset{s^{(k_1, k_2, \cdots, k_t)}_{C_T^{c(k_1, k_2, \cdots, k_t)}\left\{e_{k_t}^{(k_1, k_2, \cdots, k_t)}, e_{-k_t}^{*(k_1, k_2, \cdots, k_t)}\left[I^{(k_1, \cdots, k_t)}\left(e_{k_1}^{(k_1)}, \cdots; e_{k_t}^{(k_1, k_2, \cdots, k_t)}, e^{*(k_1, k_2, \cdots, k_t)}_{-k_1, -k_2, \cdots, -k_t}\right)\right]\right\}}}{argmax}$$

$$V^{(k_1, k_2, \cdots, k_t)}_{C_T^{c(k_1, k_2, \cdots, k_t)}}(\cdot)\left(s^{(k_1, k_2, \cdots, k_t)}_{C_T^{c(k_1, k_2, \cdots, k_t)}\left\{e_{k_t}^{(k_1, k_2, \cdots, k_t)}, e_{-k_t}^{*(k_1, k_2, \cdots, k_t)}\left[I^{(k_1, \cdots, k_t)}\left(e_{k_1}^{(k_1)}, \cdots; e_{k_t}^{(k_1, k_2, \cdots, k_t)}, e^{*(k_1, k_2, \cdots, k_t)}_{-k_1, -k_2, \cdots, -k_t}\right)\right]\right\}},\right.$$

$$\left. s^{**(k_1, k_2, \cdots, k_t)}_{-C_T^{c(k_1, k_2, \cdots, k_t)}\left\{e_{k_t}^{(k_1, k_2, \cdots, k_t)}, e_{-k_t}^{*(k_1, k_2, \cdots, k_t)}\left[I^{(k_1, \cdots, k_t)}\left(e_{k_1}^{(k_1)}, \cdots; e_{k_t}^{(k_1, k_2, \cdots, k_t)}, e^{*(k_1, k_2, \cdots, k_t)}_{-k_1, -k_2, \cdots, -k_t}\right)\right]\right\}}\right)$$

$$= \underset{s^{(k_1, k_2, \cdots, k_t)}_{C_T^{c(k_1, k_2, \cdots, k_t)}\left(s^{(k_1, k_2, \cdots, k_t)}_{C_T^{c(k_1, k_2, \cdots, k_t)}\left\{e_{k_t}^{(k_1, k_2, \cdots, k_t)}, e_{-k_t}^{*(k_1, k_2, \cdots, k_t)}\left[I^{(k_1, \cdots, k_t)}\left(e_{k_1}^{(k_1)}, \cdots; e_{k_t}^{(k_1, k_2, \cdots, k_t)}, e^{*(k_1, k_2, \cdots, k_t)}_{-k_1, -k_2, \cdots, -k_t}\right)\right]\right\}}\right.}}{argmax}$$

$$\sum_{i \in C_T^{c(k_1, k_2, \cdots, k_t)}\left(s^{(k_1, k_2, \cdots, k_t)}_{C_T^{c(k_1, k_2, \cdots, k_t)}\left\{e_{k_t}^{(k_1, k_2, \cdots, k_t)}, e_{-k_t}^{*(k_1, k_2, \cdots, k_t)}\left[I^{(k_1, \cdots, k_t)}\left(e_{k_1}^{(k_1)}, \cdots; e_{k_t}^{(k_1, k_2, \cdots, k_t)}, e^{*(k_1, k_2, \cdots, k_t)}_{-k_1, -k_2, \cdots, -k_t}\right)\right]\right\}}\right.}$$

$$u_i^{(k_1, k_2, \cdots, k_t)}\left(s^{(k_1, k_2, \cdots, k_t)}_{C_T^{c(k_1, k_2, \cdots, k_t)}\left(s^{(k_1, k_2, \cdots, k_t)}_{C_T^{c(k_1, k_2, \cdots, k_t)}\left\{e_{k_t}^{(k_1, k_2, \cdots, k_t)}, e_{-k_t}^{*(k_1, k_2, \cdots, k_t)}\left[I^{(k_1, \cdots, k_t)}\left(e_{k_1}^{(k_1)}, \cdots; e_{k_t}^{(k_1, k_2, \cdots, k_t)}, e^{*(k_1, k_2, \cdots, k_t)}_{-k_1, -k_2, \cdots, -k_t}\right)\right]\right\}}\right.},\right.$$

$$\left. s^{**(k_1, k_2, \cdots, k_t)}_{-C_T^{c(k_1, k_2, \cdots, k_t)}\left(s^{(k_1, k_2, \cdots, k_t)}_{C_T^{c(k_1, k_2, \cdots, k_t)}\left\{e_{k_t}^{(k_1, k_2, \cdots, k_t)}, e_{-k_t}^{*(k_1, k_2, \cdots, k_t)}\left[I^{(k_1, \cdots, k_t)}\left(e_{k_1}^{(k_1)}, \cdots; e_{k_t}^{(k_1, k_2, \cdots, k_t)}, e^{*(k_1, k_2, \cdots, k_t)}_{-k_1, -k_2, \cdots, -k_t}\right)\right]\right\}}\right.}\right).$$

This maximization model also determines (player k_1's estimation of player k_2's estimation of ...) player k_t's estimation of his optimal strategic choice in the escape situation $\left\{e_{k_t}^{(k_1, k_2, \cdots, k_t)}, e_{-k_t}^{*(k_1, k_2, \cdots, k_t)}\left[I^{(k_1, \cdots, k_t)}\left(e_{k_1}^{(k_1)}, \cdots; e_{k_t}^{(k_1, k_2, \cdots, k_t)}, e^{*(k_1, k_2, \cdots, k_t)}_{-k_1, -k_2, \cdots, -k_t}\right)\right]\right\}$:

$$s_{k_t}^{\circ(k_1, k_2, \cdots, k_t)}\left(e_{k_t}^{(k_1, \cdots, k_t)}, e_{k_{-t}}^{*(k_1, \cdots, k_{t+1})}\right)$$

$$\in s^{\circ(k_1, k_2, \cdots, k_t)}_{C_T^{c(k_1, k_2, \cdots, k_t)}\left\{e_{k_t}^{(k_1, k_2, \cdots, k_t)}, e_{-k_t}^{*(k_1, k_2, \cdots, k_t)}\left[I^{(k_1, \cdots, k_t)}\left(e_{k_1}^{(k_1)}, \cdots; e_{k_t}^{(k_1, k_2, \cdots, k_t)}, e^{*(k_1, k_2, \cdots, k_t)}_{-k_1, -k_2, \cdots, -k_t}\right)\right]\right\}}.$$

$s^{\circ(k_1,\,k_2,\,\cdots,\,k_t)}_{C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}\left\{e_{k_t}^{(k_1,\,k_2,\,\cdots,\,k_t)},\,e_{-k_t}^{*(k_1,\,k_2,\,\cdots,\,k_t)}\left[I^{(k_1,\,\cdots,\,k_t)}\left(e_{k_1}^{(k_1)},\,\cdots;\,e_{k_t}^{(k_1,\,k_2,\,\cdots,\,k_t)},\,e_{-k_1,\,-k_2,\,\cdots,\,-k_t}^{*(k_1,\,k_2,\,\cdots,\,k_t)}\right)\right]\right\}}$ is (player k_1's estimation of player k_2's estimation of ...) player k_t's estimation of the strategic combination that his escape target core coalition "should" adopt in the escape situation $\left\{e_{k_t}^{(k_1,\,k_2,\cdots,k_t)},\,e_{-k_t}^{*(k_1,\,k_2,\cdots,\,k_t)}\left[I^{(k_1,\cdots,\,k_t)}\left(e_{k_1}^{(k_1)},\,\cdots;\,e_{k_t}^{(k_1,\,k_2,\cdots,\,k_t)},\,e_{-k_1,-k_2,\cdots,-k_t}^{*(k_1,k_2,\cdots,k_t)}\right)\right]\right\}$ when player k_t plays escape strategy $e_{k_t}^{(k_1,\,\cdots,\,k_t)}$, this strategic combination will maximize the cooperative payoff of player k_t's escape target core coalition $C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)$ when he plays escape strategy $e_{k_t}^{(k_1,\,\cdots,\,k_t)}$, and it also means that player k_t gets the maximum expected cooperative payoff distribution from the core coalition. Denote player k_t's expected cooperative payoff distribution from core coalition $C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)$ as $\widetilde{x}_{k_2}^{\circ(k_1,\,k_2)}\left(C_T^{c(k_1,\,k_2)}\left\{e_{k_t}^{(k_1,\,k_2,\,\cdots,\,k_t)},\,e_{-k_t}^{*(k_1,\,k_2,\,\cdots,\,k_t)}\left[I^{(k_1,\,\cdots,\,k_t)}\left(e_{k_1}^{(k_1)},\,\cdots;\,e_{k_t}^{(k_1,\,k_2,\,\cdots,\,k_t)},\,e_{-k_1,\,-k_2,\,\cdots,\,-k_t}^{*(k_1,\,k_2,\,\cdots,\,k_t)}\right)\right]\right\}\right)$, and player k_t's estimation of his escape-payoff deriving from deviation when he escapes from core coalition $C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)$ through deviation as $W_{k_t}^{\circ-C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)(k_1,\,k_2,\,\cdots,\,k_t)}$.

Under escape strategy $e_{k_t}^{(k_1,\,\cdots,\,k_t)}$, the reason why in player k_1's t-th level virtual game player k_t considers the strategic combination that his escape target core coalition $C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)$ "should" adopt is $s_{C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}\left\{e_{k_t}^{(k_1,\,k_2,\,\cdots,\,k_t)},\,e_{-k_t}^{*(k_1,\,k_2,\,\cdots,\,k_t)}\left[I^{(k_1,\,\cdots,\,k_t)}\left(e_{k_1}^{(k_1)},\,\cdots;\,e_{k_t}^{(k_1,\,k_2,\,\cdots,\,k_t)},\,e_{-k_1,\,-k_2,\,\cdots,\,-k_t}^{*(k_1,\,k_2,\,\cdots,\,k_t)}\right)\right]\right\}}^{\circ(k_1,\,k_2,\,\cdots,\,k_t)}$, which can maximize the expected cooperative payoff $V_{C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)}^{(k_1,\,k_2,\,\cdots,\,k_t)}\left(s_{C_T^{c(k_1,k_2,\,\cdots,\,k_t)}(\cdot)}^{(k_1,\,k_2,\,\cdots,\,k_t)},\,s_{-C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)}^{**(k_1,\,k_2,\,\cdots,\,k_t)}\right)$ of core coalition $C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)$, is that the maximum cooperative payoff of his escape target core coalition $C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)$ under his escape strategy $e_{k_t}^{(k_1,\,\cdots,\,k_t)}$ means his expected cooperative payoff distribution $\widetilde{x}_{k_t}^{\circ(k_1,\,k_2,\,\cdots,\,k_t)}\left[C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)\right]$ maximized at the same time. Therefore, the goal of player k_t is his maximum expected cooperative payoff distribution $\widetilde{x}_{k_t}^{\circ(k_1,\,k_2,\,\cdots,\,k_t)}\left[C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)\right]$ when he chooses escape strategy $e_{k_t}^{(k_1,\,\cdots,\,k_t)}$:

$$e_{k_t}^{*(k_1,\,\cdots,\,k_t)}(e_{k_1}^{(k_1)},\,\cdots,\,e_{k_{t-1}}^{(k_1,\,\cdots,\,k_t)}) = \underset{e_{k_t}^{(k_1,\,\cdots,\,k_t)}}{argmax}\,\widetilde{x}_{k_t}^{\circ(k_1,\,k_2,\,\cdots,\,k_t)}\left[C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)\right].$$

Or, the goal of player k_t's escape strategic choice is his minimum expected escape-payoff deriving from deviation:

$$e_{k_t}^{*(k_1,\,\cdots,\,k_t)}(e_{k_1}^{(k_1)},\,\cdots,\,e_{k_{t-1}}^{(k_1,\,\cdots,\,k_t)}) = \underset{e_{k_t}^{(k_t,\,\cdots,\,k_t)}}{argmin}\,W_{k_t}^{\circ-C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)(k_1,\,k_2,\,\cdots,\,k_t)}.$$

The escape target core coalition of player k_t is

$$C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}\left\{e_{k_t}^{*(k_1,\,k_2,\,\cdots,\,k_t)}\left(e_{k_1}^{(k_1)},\,\cdots,\,e_{k_{t-1}}^{(k_1,\,\cdots,\,k_{t-1})}\right),\,e_{-k_t}^{*(k_1,\,k_2,\,\cdots,\,k_t)}\left[I^{(k_1,\,\cdots,\,k_t)}(\cdot)\right]\right\},$$

$$I^{(k_1,\,\cdots,\,k_t)}(\cdot) = I^{(k_1,\,\cdots,\,k_t)}\left(e_{k_1}^{(k_1)},\,e_{k_2}^{(k_1,\,k_2)},\,\cdots,\,e_{k_{t-1}}^{(k_1,\,\cdots,\,k_{t-1})},\,e_{-k_1,\,-k_2,\,\cdots,-k_t}^{*(k_1,\,k_2,\,\cdots,\,k_t)}\right).$$

The strategic combination that core coalition $C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)$ "should" adopt is considered to be:

$$s^{*(k_1,\, k_2,\, \cdots,\, k_t)}_{C_T^{c(k_1,\, k_2,\, \cdots,\, k_t)}\left\{e_{k_t}^{*(k_1,\, k_2,\, \cdots,\, k_t)}\left(e_{k_1}^{(k_1)},\, \cdots,\, e_{k_{t-1}}^{(k_1,\, \cdots,\, k_{t-1})}\right),\, e_{-k_t}^{*(k_1,\, k_2,\, \cdots,\, k_t)}\left[I^{(k_1,\, \cdots,\, k_t)}(\cdot)\right]\right\}}$$

$$=s^{\circ(k_1,\, k_2,\, \cdots,\, k_t)}_{C_T^{c(k_1,\, k_2,\, \cdots,\, k_t)}\left\{e_{k_t}^{*(k_1,\, k_2,\, \cdots,\, k_t)}\left(e_{k_1}^{(k_1)},\, \cdots,\, e_{k_{t-1}}^{(k_1,\, \cdots,\, k_{t-1})}\right),\, e_{-k_t}^{*(k_1,\, k_2,\, \cdots,\, k_t)}\left[I^{(k_1,\, \cdots,\, k_t)}(\cdot)\right]\right\}}.$$

The cooperative payoff that core coalition $C_T^{c(k_1,\, k_2,\, \cdots,\, k_t)}(\cdot)$ "should" get is considered to be:

$$V^{(k_1,\, k_2,\, \cdots,\, k_t)}_{C_T^{c(k_1,\, k_2,\, \cdots,\, k_t)}(\cdot)}\Big(s^{\circ(k_1,\, k_2,\, \cdots,\, k_t)}_{C_T^{c(k_1,\, k_2,\, \cdots,\, k_t)}\left\{e_{k_t}^{*(k_1,\, k_2,\, \cdots,\, k_t)}\left(e_{k_1}^{(k_1)},\, \cdots,\, e_{k_{t-1}}^{(k_1,\, \cdots,\, k_{t-1})}\right),\, e_{-k_t}^{*(k_1,\, k_2,\, \cdots,\, k_t)}\left[I^{(k_1,\, \cdots,\, k_t)}(\cdot)\right]\right\}},$$

$$s^{**(k_1,\, k_2,\, \cdots,\, k_t)}_{-C_T^{c(k_1,\, k_2,\, \cdots,\, k_t)}\left\{e_{k_t}^{*(k_1,\, k_2,\, \cdots,\, k_t)}\left(e_{k_1}^{(k_1)},\, \cdots,\, e_{k_{t-1}}^{(k_1,\, \cdots,\, k_{t-1})}\right),\, e_{-k_t}^{*(k_1,\, k_2,\, \cdots,\, k_t)}\left[I^{(k_1,\, \cdots,\, k_t)}(\cdot)\right]\right\}}\Big)$$

$$=\sum_{i\in C_T^{c(k_1,\, k_2,\, \cdots,\, k_t)}\left\{e_{k_t}^{*(k_1,\, k_2,\, \cdots,\, k_t)}\left(e_{k_1}^{(k_1)},\, \cdots,\, e_{k_{t-1}}^{(k_1,\, \cdots,\, k_{t-1})}\right),\, e_{-k_t}^{*(k_1,\, k_2,\, \cdots,\, k_t)}\left[I^{(k_1,\, \cdots,\, k_t)}(\cdot)\right]\right\}}$$

$$u_i^{(k_1,\, k_2,\, \cdots,\, k_t)}\Big(s^{\circ(k_1,\, k_2,\, \cdots,\, k_t)}_{C_T^{c(k_1,\, k_2,\, \cdots,\, k_t)}\left\{e_{k_t}^{*(k_1,\, k_2,\, \cdots,\, k_t)}\left(e_{k_1}^{(k_1)},\, \cdots,\, e_{k_{t-1}}^{(k_1,\, \cdots,\, k_{t-1})}\right),\, e_{-k_t}^{*(k_1,\, k_2,\, \cdots,\, k_t)}\left[I^{(k_1,\, \cdots,\, k_t)}(\cdot)\right]\right\}},$$

$$s^{**(k_1,\, k_2,\, \cdots,\, k_t)}_{-C_T^{c(k_1,\, k_2,\cdots,\, k_t)}\left\{e_{k_t}^{*(k_1,\, k_2,\, \cdots,\, k_t)}\left(e_{k_1}^{(k_1)},\, \cdots,\, e_{k_{t-1}}^{(k_1,\, \cdots,\, k_{t-1})}\right),\, e_{-k_t}^{*(k_1,\, k_2,\, \cdots,\, k_t)}\left[I^{(k_1,\, \cdots,\, k_t)}(\cdot)\right]\right\}}\Big),$$

$$I^{(k_1,\, \cdots,\, k_t)}(\cdot)=I^{(k_1,\, \cdots,\, k_t)}\left[e_{k_1}^{(k_1)},\, e_{k_2}^{(k_1,\, k_2)},\, \cdots,\, e_{k_t}^{*(k_1,\, k_2,\, \cdots, k_t)}(e_{k_1}^{(k_1)},\, \cdots,\, e_{k_{t-1}}^{(k_1,\, \cdots,\, k_{t-1})}),\, e^{*(k_1,\, k_2,\, \cdots,\, k_t)}_{-k_1, -k_2,\, \cdots,\, -k_t}\right].$$

Player k_t's estimation of his maximum expected cooperative payoff distribution is considered to be:

$$\widetilde{x}_{k_t}^{*(k_1,\, \cdots,\, k_t)}(e_{k_1}^{(k_1)},\, \cdots,\, e_{k_{t-1}}^{(k_1,\, \cdots,\, k_{t-1})})$$

$$=\widetilde{x}_{k_t}^{\circ(k_1,\, \cdots,\, k_t)}(e_{k_1}^{(k_1)},\, \cdots,\, e_{k_{t-1}}^{(k_1,\, \cdots,\, k_{t-1})})(C_T^{c(k_1,\, k_2,\, \cdots,\, k_t)}\left\{e_{k_t}^{*(k_1,\, k_2,\, \cdots,\, k_t)}\left(e_{k_1}^{(k_1)},\, \cdots,\, e_{k_{t-1}}^{(k_1,\, \cdots,\, k_{t-1})}\right),\right.$$

$$\left. e_{-k_t}^{*(k_1,\, k_2,\, \cdots,\, k_t)}\left[I^{(k_1,\, \cdots,\, k_t)}(\cdot)\right]\right\});$$

$$I^{(k_1,\, \cdots,\, k_t)}(\cdot)=I^{(k_1,\, \cdots,\, k_t)}\left[e_{k_1}^{(k_1)},\, e_{k_2}^{(k_1,\, k_2)},\, \cdots,\, e_{k_t}^{*(k_1,\, k_2,\, \cdots,\, k_t)}(e_{k_1}^{(k_1)},\, \cdots,\, e_{k_{t-1}}^{(k_1,\, \cdots,\, k_{t-1})}),\, e^{*(k_1,\, k_2,\, \cdots,\, k_t)}_{-k_1,\, -k_2,\, \cdots,\, -k_t}\right].$$

In player k_1's t-th level virtual game, player k_t must estimate the escape strategic choice $e_{-k_t}^{*(k_1,\,k_2,\,\cdots,\,k_t)}$ of any other player $-k_t$ on the basis of player $-k_t$'s own information set and the strategic combination choices $s^{**(k_1,\,k_2,\,\cdots,\,k_t)}_{-C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}\left[e_{k_t}^{(k_1,\,\cdots,\,k_t)},\,e_{-k_t}^{*(k_1,\,k_2,\,\cdots,\,k_t)}(I^{(k_1,\,\cdots,\,k_t)}(\cdot)\right]}$ of the core coalitions other than player k_t's escape target core coalition when he decides his optimal escape strategy and the optimal strategic combination choice of his escape target core coalition. Player k_t's decision-making relies on his estimation of the equilibrium escape strategic choice of any player $-k_t$ in player $-k_t$'s virtual game. Player k_t's estimation of any other player $-k_t$'s estimation of his own escape strategic choice $e_{-k_t}^{*(k_1,\,k_2,\,\cdots,\,k_t)}$ depends on player $-k_t$'s choice based on his own information set:

$$e_{-k_t}^{*(k_1,\,k_2,\,\cdots,\,k_t)} = e_{-k_t}^{*(k_1,\,k_2,\,\cdots,\,k_t,\,-k_t)}.$$

In player k_1's t-th level virtual game, strategic combination choice $s^{**(k_1,k_2,\cdots,k_t)}_{C_h^{c(k_1,k_2,\cdots,k_t)}\left\{e_{k_t}^{(k_1,\cdots,k_t)},e_{-k_t}^{*(k_1,k_2,\cdots,k_t)}\left[I^{(k_1,\cdots,k_t)}(\cdot)\right]\right\}}$ of any core coalition $C_h^{c(k_1,\,k_2,\,\cdots,\,k_t)}\left\{e_{k_t}^{(k_1,\,\cdots,\,k_t)},\,e_{-k_t}^{*(k_1,\,k_2,\,\cdots,\,k_t)}\left[I^{(k_1,\,\cdots,\,k_t)}(\cdot)\right]\right\}$ $(\neq C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}\left\{e_{k_t}^{(k_1,\,\cdots,\,k_t)},\,e_{-k_t}^{*(k_1,\,k_2,\,\cdots,\,k_t)}\left[I^{(k_1,\,\cdots,\,k_t)}(\cdot)\right]\right\})$ depends on the public choice game of extensive coalition $C_h^{+(k_1,\,k_2,\,\cdots,\,k_t)}\left\{e_{k_t}^{(k_1,\,\cdots,\,k_t)},\,e_{-k_t}^{*(k_1,\,k_2,\,\cdots,\,k_t)}\left[I^{(k_1,\,\cdots,\,k_t)}(\cdot)\right]\right\}$ among its extensive members:

$$s^{*(k_1,\,k_2,\,\cdots,\,k_t)}_{C_h^{c(k_1,\,k_2,\,\cdots,\,k_t)}\left\{e_{k_t}^{(k_1,\,\cdots,\,k_t)},\,e_{-k_t}^{*(k_1,\,k_2,\,\cdots,\,k_t)}\left[I^{(k_1,\,\cdots,\,k_t)}\left(e_{k_1}^{(k_1)},\,\cdots,\,e_{k_t}^{*(k_1,\,k_2,\,\cdots,\,k_t)}\left(e_{k_1}^{(k_1)},\,\cdots,\,e_{k_{t-1}}^{(k_1,\,\cdots,\,k_{t-1})}\right),\,e_{-k_1,\,-k_2,\,\cdots,\,-k_t}^{*(k_1,\,k_2,\,\cdots,\,k_t)}\right)\right]\right\}}$$

$$\subseteq s^{*(k_1,\,k_2,\,\cdots,\,k_t,\,C_h^{+(k_1,\,k_2,\,\cdots,\,k_t)})}_{C_h^{+(k_1,\,k_2,\,\cdots,\,k_t)}\left\{e_{k_t}^{(k_1,\,\cdots,\,k_t)},\,e_{-k_t}^{*(k_1,\,k_2,\,\cdots,\,k_t)}\left[I^{(k_1,\,\cdots,\,k_t)}\left(e_{k_1}^{(k_1)},\,\cdots,\,e_{k_t}^{*(k_1,\,k_2,\,\cdots,\,k_t)}\left(e_{k_1}^{(k_1)},\,\cdots,\,e_{k_{t-1}}^{(k_1,\,\cdots,\,k_{t-1})}\right),\,e_{-k_1,\,-k_2,\,\cdots,\,-k_t}^{*(k_1,\,k_2,\,\cdots,\,k_t)}\right)\right]\right\}};$$

$$s^{*(k_1,\,k_2,\,\cdots,\,k_t)}_{C_h^{c(k_1,\,k_2,\,\cdots,\,k_t)}\left\{e_{k_t}^{(k_1,\,\cdots,\,k_t)},\,e_{-k_t}^{*(k_1,\,k_2,\,\cdots,k_t)}\left[I^{(k_1,\,\cdots,\,k_t)}\left(e_{k_1}^{(k_1)},\,\cdots,\,e_{k_t}^{*(k_1,\,k_2,\,\cdots,\,k_t)}\left(e_{k_1}^{(k_1)},\,\cdots,\,e_{k_{t-1}}^{(k_1,\,\cdots,\,k_{t-1})}\right),\,e_{-k_1,\,-k_2,\,\cdots,\,-k_t}^{*(k_1,\,k_2,\,\cdots,\,k_t)}\right)\right]\right\}}$$

$$= \underset{s^{(k_1,\,k_2,\,\cdots,\,k_t)}_{C_h^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)}}{argmax} \left\{ \sum_{i\in C_h^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)} V^{(k_1,\,\cdots,\,k_t,\,i)}_{C_T^{c(k_1,\,k_2,\,\cdots,\,k_t,\,i)}(\cdot)}(s^{(k_1,\,\cdots,\,k_t,\,i)}_{C_T^{c(k_1,\,k_2,\,\cdots,\,k_t,\,i)}(\cdot)},\,s^{**(k_1,\,\cdots,\,k_t,\,i)}_{-C_T^{c(k_1,\,k_2,\,\cdots,\,k_t,\,i)}(\cdot)}) \right.$$

$$\left. + \sum_{\substack{j\in C_h^{+(k_1,k_2,\cdots,k_t)}(\cdot)\\ j\notin C_h^{c(k_1,k_2,\cdots,k_t)}(\cdot)}} V^{(k_1,\,k_2,\,\cdots,\,k_t,\,j^F)}_{C_h^{c(k_1,\,k_2,\,\cdots,\,k_t,\,j)}(\cdot)}(s^{(k_1,\,k_2,\,\cdots,\,k_t,\,j^F)}_{C_h^{c(k_1,\,k_2,\,\cdots,\,k_t,\,j)}(\cdot)},\,s^{**(k_1,\,k_2,\,\cdots,\,k_t,\,j^F)}_{C_T^{c(k_1,\,k_2,\,\cdots,\,k_t,\,j)}(\cdot)},\,s^{**(k_1,\,k_2,\,\cdots,\,k_t,\,j^F)}_{-C_h^{c(k_1,\,k_2,\,\cdots,\,k_t,\,j)}(\cdot),\,-C_T^{c(k_1,\,k_2,\,\cdots,k_t,\,j)}(\cdot)}) \right\};$$

$$V^{(k_1,\,k_2,\,\cdots,\,k_t,\,j^F)}_{C_h^{c(k_1,\,k_2,\,\cdots,\,k_t,\,j)}(\cdot)}(s^{(k_1,\,k_2,\,\cdots,\,k_t,\,j^F)}_{C_h^{c(k_1,\,k_2,\,\cdots,\,k_t,\,j)}(\cdot)},\,s^{**(k_1,\,k_2,\,\cdots,\,k_t,\,j^F)}_{C_T^{c(k_1,\,k_2,\,\cdots,\,k_t,\,j)}(\cdot)},\,s^{**(k_1,\,k_2,\,\cdots,\,k_t,\,j^F)}_{-C_h^{c(k_1,\,k_2,\,\cdots,\,k_t,\,j)}(\cdot),\,-C_T^{c(k_1,\,k_2,\,\cdots,\,k_t,\,j)}(\cdot)})$$

$$= V^{(k_1,\,k_2,\,\cdots,\,k_t,\,j^F)}_{C_h^{c(k_1,\,k_2,\,\cdots,\,k_t,\,j)}(\cdot)}(s^{(k_1,\,k_2,\,\cdots,\,k_t,\,j^F)}_{C_h^{c(k_1,\,k_2,\,\cdots,\,k_t,\,j)}(\cdot)},\,i_j^{(k_1,\,k_2,\,\cdots,\,k_t,\,j^F)}),\ i_j^{(k_1,\,k_2,\,\cdots,\,k_t,\,j^F)} \in I_j^{(k_1,\,k_2,\,\cdots,\,k_t,\,j^F)};$$

where $i \in C_h^{c(k_1, k_2, \cdots, k_t)}(\cdot)$, therefore the escape target core coalition $C_T^{c(k_1, k_2, \cdots, k_t, i)}(\cdot)$ of player i is core coalition $C_h^{c(k_1, k_2, \cdots, k_t)}(\cdot)$, $j \in C_h^{+(k_1, k_2, \cdots, k_t)}(\cdot)$, $j \notin C_h^{c(k_1, k_2, \cdots, k_t)}(\cdot)$, core coalition $C_h^{c(k_1, k_2, \cdots, k_t)}(\cdot)$ and core coalition $C_h^{c(k_1, k_2)}(\cdot)$ are the same, player j's escape target core coalition $C_T^{c(k_1, k_2, \cdots, k_t, j)}(\cdot)$ is not core coalition $C_h^{c(k_1, k_2, \cdots, k_t)}(\cdot)$, $V^{(k_1, k_2, \cdots, k_t, j^F)}_{C_h^{c(k_1, k_2, \cdots, k_t, j)}(\cdot)}\left(s^{(k_1, k_2, \cdots, k_t, j^F)}_{C_h^{c(k_1, k_2, \cdots, k_t, j)}(\cdot)}, s^{**(k_1, k_2, \cdots, k_t, j^F)}_{C_T^{c(k_1, k_2, \cdots, k_t, j)}(\cdot)}, s^{**(k_1, k_2, \cdots, k_t, j^F)}_{-C_h^{c(k_1, k_2, \cdots, k_t, j)}(\cdot), -C_T^{c(k_1, k_2, \cdots, k_t, j)}(\cdot)}\right)$ represents player k_t's estimation of the cooperative payoff of core coalition $C_T^{c(k_1, k_2, \cdots, k_t, j)}(\cdot)$ according to the false signal released by player j, $i_j^{(k_1, k_2, \cdots, k_t, j^F)}$ is player k_t's estimation of the false signal released by player j, $I_j^{(k_1, k_2, \cdots, k_t, j^F)}$ is player k_t's estimation of player j's feasible false signal set, $s^{**(k_1, k_2, \cdots, k_t, j^F)}_{C_T^{c(k_1, k_2, \cdots, k_t, j)}(\cdot)}$, $s^{**(k_1, k_2, \cdots, k_t, j^F)}_{-C_h^{c(k_1, k_2, \cdots, k_t, j)}(\cdot), -C_T^{c(k_1, k_2, \cdots, k_t, j)}(\cdot)}$ is player k_t's estimation of player j's estimation of the strategic combination choices of core coalitions [including player j's escape target core coalition $C_T^{c(k_1, k_2, \cdots, k_t, j)}(\cdot)$] other than player j's intermediate node core coalition $C_h^{c(k_1, k_2, \cdots, k_t, j)}(\cdot)$.

Therefore, in player k_1's t-th level virtual game, player j's $\left[j \in C_h^{+(k_1, k_2, \cdots, k_t)}(\cdot),\ j \notin C_h^{c(k_1, k_2, \cdots, k_t)}(\cdot)\right]$ estimation of the strategic combination choice $s^{**(k_1, k_2, \cdots, k_t, j)}_{C_h^{c(k_1, k_2, \cdots, k_t, j)}(\cdot)}$ of core coalition $C_h^{c(k_1, k_2, \cdots, k_t, j)}(\cdot)$ is a function of the false signal $i_j^{(k_1, k_2, \cdots, k_t, j^F)}$ released by him:

$$
\begin{aligned}
& s^{**(k_1, k_2, \cdots, k_t, j)}_{C_h^{c(k_1, k_2, \cdots, k_t, j)}(\cdot)} = s^{**(k_1, k_2, \cdots, k_t, j)}_{C_h^{c(k_1, k_2, \cdots, k_t, j)}(\cdot)}\left(i_j^{(k_1, k_2, \cdots, k_t, j^F)}\right) \\
=\,& \mathit{argmax}\left\{\sum_{i \in C_h^{c(k_1, k_2, \cdots, k_t, j)}(\cdot)} V^{(k_1, k_2, \cdots, k_t, j, i)}_{C_T^{c(k_1, k_2, \cdots, k_t, j, i)}(\cdot)}\left(s^{(k_1, k_2, \cdots, k_t, j, i)}_{C_T^{c(k_1, k_2, \cdots, k_t, j, i)}(\cdot)}, s^{**(k_1, k_2, \cdots, k_t, j, i)}_{-C_T^{c(k_1, k_2, \cdots, k_t, j, i)}(\cdot)}\right)\right. \\
& + V^{(k_1, k_2, \cdots, k_t, j^F)}_{C_h^{c(k_1, k_2, \cdots, k_t, j)}(\cdot)}\left(s^{(k_1, k_2, \cdots, k_t, j^F)}_{C_h^{c(k_1, k_2, \cdots, k_t, j)}(\cdot)}, i_j^{(k_1, k_2, \cdots, k_t, j^F)}\right) \\
& \left. + \sum_{\substack{k \in C_h^{+(k_1, k_2, \cdots, k_t, j)}(\cdot) \\ k \notin C_h^{c(k_1, k_2, \cdots, k_t, j)}(\cdot) \\ k \neq j}} V^{(k_1, k_2, \cdots, k_t, j, k^F)}_{C_h^{c(k_1, k_2, \cdots, k_t, j, k)}(\cdot)}\left(s^{(k_1, k_2, \cdots, k_t, j, k^F)}_{C_h^{c(k_1, k_2, \cdots, k_t, j, k)}(\cdot)}, i_j^{*(k_1, k_2, \cdots, k_t, j, k^F)}\right)\right\},
\end{aligned}
$$

where $i \in C_h^{c(k_1, k_2, \cdots, k_t, j)}(\cdot)$, the escape target core coalition $C_T^{c(k_1, k_2, \cdots, k_t, j, i)}(\cdot)$ of player i is core coalition $C_h^{c(k_1, k_2, \cdots, k_t, j)}(\cdot)$, $k \in C_h^{+(k_1, k_2, \cdots, k_t, j)}(\cdot)$, $k \notin C_h^{c(k_1, k_2, \cdots, k_t, j)}(\cdot)$, core coalition $C_h^{c(k_1, k_2, \cdots, k_t, j, k)}(\cdot)$ and core coalition $C_h^{c(k_1, k_2, \cdots, k_t, j)}(\cdot)$ are the same, the escape target core coalition $C_T^{c(k_1, k_2, \cdots, k_t, j, k)}(\cdot)$ of player k is not core coalition $C_h^{c(k_1, k_2, \cdots, k_t, j)}(\cdot)$; $i_j^{*(k_1, k_2, \cdots, k_t, j, k^F)}$ is player j's estimation of the optimal false signal of any extensive member $k\left[k \in C_h^{+(k_1, k_2, \cdots, k_t, j)}(\cdot),\ k \notin C_h^{c(k_1, k_2, \cdots, k_t, j)}(\cdot),\ k \neq j\right]$ in his virtual game.

Player j's purpose of releasing false signals to his intermediate node core coalition $C_h^{c(k_1, k_2, \cdots, k_t, j)}(\cdot)$ is to influence the public choice $s^{**(k_1, k_2, \cdots, k_t)}_{C_h^{c(k_1, k_2, \cdots, k_t, j)}(\cdot)}$ of strategic combination of core coalition $C_h^{c(k_1, k_2, \cdots, k_t, j)}(\cdot)$, thereby maximize the cooperative payoff of his escape target core coalition $C_T^{c(k_1, k_2, \cdots, k_t, j)}(\cdot)$:

$$i_j^{*(k_1,\, k_2,\, \cdots,\, k_t,\, j^F)} = \underset{i_j^{(k_1,\, k_2,\, \cdots,\, k_t,\, j^F)} \in I_j^{(k_1,\, k_2,\, \cdots,\, k_t,\, j^F)}}{argmax} V^{(k_1,\, k_2,\, \cdots,\, k_t,\, j)}_{C_T^{c(k_1,\, k_2,\, \cdots,\, k_t,\, j)}(\cdot)} \left[s^{**(k_1,\, k_2,\, \cdots,\, k_t,\, j)}_{C_h^{c(k_1,\, k_2,\, \cdots,\, k_t,\, j)}(\cdot)} \left(i_j^{(k_1,\, k_2,\, \cdots,\, k_t,\, j^F)} \right), \right.$$

$$\left. s^{*(k_1,\, k_2,\, \cdots,\, k_t,\, j)}_{C_T^{c(k_1,\, k_2,\, \cdots,\, k_t,\, j)}(\cdot)},\ s^{**(k_1,\, k_2,\, \cdots,\, k_t,\, j)}_{-C_h^{c(k_1,\, k_2,\, \cdots,\, k_t,\, j)}(\cdot),\, -C_T^{c(k_1,\, k_2,\, \cdots,\, k_t,\, j)}(\cdot)} \right].$$

Obviously, in order to estimate the strategic combination choice $s^{**(k_1,\, k_2,\, \cdots,\, k_t)}_{C_h^{c(k_1,\, k_2,\, \cdots,\, k_t)}(\cdot)}$ of any other core coalition $C_h^{c(k_1,\, k_2,\, \cdots,\, k_t)}(\cdot)$, player k_t needs to estimate any core member i's [who belongs to core coalition $C_h^{c(k_1,\, k_2,\, \cdots,\, k_t)}(\cdot)$] estimation of the strategic combination choices $s^{**(k_1,\, k_2,\, \cdots,\, k_t,\, i)}_{-C_T^{c(k_1,\, k_2,\, \cdots,\, k_t,\, i)}(\cdot)}$ of other core coalitions $-C_h^{c(k_1,\, k_2,\, \cdots,\, k_t)}(\cdot)$, any extensive member j's [who nominally belongs to core coalition $C_h^{c(k_1,\, k_2,\, \cdots,\, k_t)}(\cdot)$] estimation of the strategic combination $s^{*(k_1,\, k_2,\, \cdots,\, k_t,\, j)}_{C_T^{c(k_1,\, k_2,\, \cdots,\, k_t,\, j)}(\cdot)}$ that this core coalition "should" adopt, member j's estimation of the strategic combinations $s^{**(k_1,\, k_2,\, \cdots,\, k_t,\, j)}_{-C_h^{c(k_1,\, k_2,\, \cdots,\, k_t,\, j)}(\cdot),\, -C_T^{c(k_1,\, k_2,\, \cdots,\, k_t,\, j)}(\cdot)}$ of the core coalitions other than core coalition $C_h^{c(k_1,\, k_2,\, \cdots,\, k_t)}(\cdot)$ and his escape target core coalition, and member j's estimation of the optimal signal $i_j^{*(k_1,\, k_2,\, \cdots,\, k_t,\, j,\, j^F)}$ of any other extensive member k.

Thus, the virtual game of player k_1 enters the $(t+1)$-th level.

…

In the n-th level virtual games of player k_1, the information sets of player k_n are stable. In this information asymmetric cooperative game on the basis of stable information sets, how does player k_n virtualize his game to determine his own escape strategy and the strategic combination choice that his escape target core coalition "should" choose?

2.4 *Escape strategy and player k_1's virtual game: the virtual game under stable information sets*

In order to get player k_1's estimations of the expected escape-payoffs deriving from deviation of players $k_1, \cdots, k_n$,

$$W_{k_1}^{*-C_T^{c(k_1)}(k_1)}(c),\ W_{k_2}^{*-C_T^{c(k_1,\, k_2)}(k_1,\, k_2)}(c),\ \ldots,\ \ W_{k_n}^{*-C_T^{c(k_1,\, k_n)}(k_1,\, k_n)}(c),$$

in his virtual game in coalition situation c, in his n-th level virtual game player k_1 must estimate any player k_n's virtual game based on the stable information sets $I^{(k_1,\, \cdots,\, k_n)}(e_{k_1}^{(k_1)}, \cdots, e_{k_n}^{(k_1,\, \cdots,\, k_n)})$.

In player k_1's n-th level virtual game, when the escape strategies of player k_1, player $k_2 (k_2 \neq k_1)$, player $k_3 (k_3 \neq k_2)$, …, and player $k_n (k_n \neq k_{n-1})$ are respectively $e_{k_1}^{(k_1)}, e_{k_2}^{(k_1,\, k_2)}, \cdots, e_{k_n}^{(k_1,\, \cdots,\, k_n)}$, all the players get the stable information sets $I^{(k_1,\, \cdots,\, k_n)}(e_{k_1}^{(k_1)}, \cdots, e_{k_n}^{(k_1,\, \cdots,\, k_n)})$, thus player k_1 can get virtual game $\Gamma_{k_n}^{(k_1,\, \cdots,\, k_n)}(e_{k_1}^{(k_1)}, \cdots, e_{k_n}^{(k_1,\, \cdots,\, k_n)})$ in which any player k_n's estimation of any other player's optimal escape strategic choices are $e_{-k_n}^{*(k_1,\, \cdots,\, k_n)}\left[I^{(k_1,\, \cdots,\, k_n)}(e_{k_1}^{(k_1)}, \cdots, e_{k_n}^{(k_1,\, \cdots,\, k_n)})\right]$, and the optimal strategic combination choices of the core coalitions other than his escape target core coalition are $s^{**(k_1,\, k_2,\, \cdots,\, k_n)}_{-C_T^{c(k_1,\, k_2,\, \cdots,\, k_n)}}\left(e_{k_n}^{(k_1,\, \cdots,\, k_n)}, e_{-k_n}^{*(k_1,\, k_2,\, \cdots,\, k_n)}\right)$.

In fact, virtual game $\Gamma_{k_n}^{(k_1,\, \cdots,\, k_n)}(e_{k_1}^{(k_1)}, \cdots, e_{k_n}^{(k_1,\, \cdots,\, k_n)})$ is player k_n's estimation model of the escape strategy of any other player and the strategic combinations of other core coalition when the escape strategies of player k_1, player $k_2 (k_2 \neq k_1)$, player $k_3 (k_3 \neq k_2)$, …, and player $k_n (k_n \neq k_{n-1})$ are respectively $e_{k_1}^{(k_1)}, e_{k_2}^{(k_1,\, k_2)}, \cdots, e_{k_n}^{(k_1,\, \cdots,\, k_n)}$. On the basis of virtual game $\Gamma_{k_n}^{(k_1,\, \cdots,\, k_n)}(e_{k_1}^{(k_1)}, \cdots, e_{k_n}^{(k_1,\, \cdots,\, k_n)})$, player k_n can estimate the strategic combination that his escape target core coalition "should" adopt.

The virtual game $\Gamma_{k_n}^{(k_1,\cdots,k_n)}(e_{k_1}^{(k_1)},\cdots,e_{k_n}^{(k_1,\cdots,k_n)})$.

In game $\Gamma_{k_n}^{(k_1,\cdots,k_n)}(e_{k_1}^{(k_1)},\cdots,e_{k_n}^{(k_1,\cdots,k_n)})$, under information sets $I_{k_n}^{(k_1,\cdots,k_n)}(e_{k_1}^{(k_1)},\cdots,e_{k_n}^{(k_1,\cdots,k_n)})$, given player k_n's feasible escape strategy $e_{k_n}^{(k_1,\cdots,k_n)}$, the strategic combination that player k_n's escape target core coalition "should" adopt depends on his estimation of the escape strategies of other players and the strategic combination choices of the core coalitions other than his escape target core coalition.

In coalition situation c, in the first level of virtual game $\Gamma_{k_n}^{(k_1,\cdots,k_n)}(e_{k_1}^{(k_1)},\cdots,e_{k_n}^{(k_1,\cdots,k_n)})$, given the escape situation $(e_{k_1}^{(k_1)},\cdots,e_{k_n}^{(k_1,\cdots,k_n)})$, assume that the strategic combination of any core coalition $C_h^{c(k_1,\ldots,k_n)}\left(e_{k_n}^{(k_1,\ldots,k_n)},e_{-k_n}^{*(k_1,\ldots,k_n)}\right)$ $\left[C_h^{c(k_1,\ldots,k_n)}\left(e_{k_n}^{(k_1,\ldots,k_n)},e_{-k_n}^{*(k_1,\ldots,k_n)}\right)\neq C_T^{c(k_1,\ldots,k_n)}\left(e_{k_n}^{(k_1,\ldots,k_n)},e_{-k_n}^{*(k_1,\ldots,k_n)}\right)\right]$ other than player k_n's escape target core coalition $C_T^{c(k_1,k_2,\cdots,k_n)}\left(e_{k_n}^{(k_1,\cdots,k_n)},e_{-k_n}^{*(k_1,k_2,\cdots,k_n)}\right)$ is player k_n's escape target core coalition in escape situation $\left(e_{k_n}^{(k_1,\cdots,k_n)},e_{-k_n}^{*(k_1,k_2,\cdots,k_n)}\right)$ is $s^{**(k_1,k_2,\cdots,k_n)}_{C_h^{c(k_1,\ldots,k_n)}(e_{k_n}^{(k_1,\ldots,k_n)},e_{-k_n}^{*(k_1,\ldots,k_n)})}$, the strategic combination that player k_n considers his escape target core coalition $C_T^{c(k_1,k_2,\cdots,k_n)}\left(e_{k_n}^{(k_1,\cdots,k_n)},e_{-k_n}^{*(k_1,k_2,\cdots,k_n)}\right)$ "should" adopt [that is to say, the strategic combination which maximizes the expected cooperative payoff of core coalition $C_T^{c(k_1,k_2,\cdots,k_n)}\left(e_{k_n}^{(k_1,\cdots,k_n)},e_{-k_n}^{*(k_1,k_2,\cdots,k_n)}\right)$ is:

$$
\begin{aligned}
& s^{*(k_1,k_2,\cdots,k_n)}_{C_T^{c(k_1,k_2,\cdots,k_n)}(e_{k_n}^{(k_1,\cdots,k_n)},e_{-k_n}^{*(k_1,k_2,\cdots,k_n)})} \\
= & \underset{s^{(k_1,k_2,\cdots,k_n)}_{C_T^{c(k_1,k_2,\cdots,k_n)}(e_{k_n}^{(k_1,\cdots,k_n)},e_{-k_n}^{*(k_1,k_2,\cdots,k_n)})}}{argmax} V^{(k_1,k_2,\cdots,k_n)}_{C_T^{c(k_1,k_2,\cdots,k_n)}(\cdot)}(s^{(k_1,k_2,\cdots,k_n)}_{C_T^{c(k_1,k_2,\cdots,k_n)}(e_{k_n}^{(k_1,\cdots,k_n)},e_{-k_n}^{*(k_1,k_2,\cdots,k_n)})}, \\
& s^{**(k_1,k_2,\cdots,k_n)}_{-C_T^{c(k_1,k_2,\cdots,k_n)}(e_{k_n}^{(k_1,\cdots,k_n)},e_{-k_n}^{*(k_1,k_2,\cdots,k_n)})}) \\
= & \underset{s^{(k_1,k_2,\cdots,k_n)}_{C_T^{c(k_1,k_2,\cdots,k_n)}(e_{k_n}^{(k_1,\cdots,k_n)},e_{-k_n}^{*(k_1,k_2,\cdots,k_n)})}}{argmax} \sum_{i\in C_T^{c(k_1,k_2,\cdots,k_n)}(e_{k_n}^{(k_1,\cdots,k_n)},e_{-k_n}^{*(k_1,k_2,\cdots,k_n)})} \\
& u_i^{(k_1,k_2,\cdots,k_n)}(s^{(k_1,k_2,\cdots,k_n)}_{C_T^{c(k_1,k_2,\cdots,k_n)}(e_{k_n}^{(k_1,\cdots,k_n)},e_{-k_n}^{*(k_1,k_2,\cdots,k_n)})}, s^{**(k_1,k_2,\cdots,k_n)}_{-C_T^{c(k_1,k_2,\cdots,k_n)}(e_{k_n}^{(k_1,\cdots,k_n)},e_{-k_n}^{*(k_1,k_2,\cdots,k_n)})}).
\end{aligned}
$$

In the first level of virtual game $\Gamma_{k_n}^{(k_1,\cdots,k_n)}(e_{k_1}^{(k_1)},\cdots,e_{k_n}^{(k_1,\cdots,k_n)})$, in order to decide his own optimal escape strategy and the strategic combination that his escape target core coalition "should" adopt, player k_n needs to estimate the escape strategic choice $e_{-k_n}^{*(k_1,k_2,\cdots,k_n)}$ of any other player $-k_n$ and the strategic combination choices $s^{**(k_1,k_2,\cdots,k_n)}_{-C_T^{c(k_1,k_2,\cdots,k_n)}(e_{k_n}^{(k_1,\cdots,k_n)},e_{-k_n}^{*(k_1,k_2,\cdots,k_n)})}$ of the core coalitions other than his escape target core coalition, which relies on his estimation of the equilibrium escape strategic choice of any other player $-k_n$ in the virtual game based on player $-k_n$'s own information set. Player k_n's estimation of the escape strategic choice $e_{-k_n}^{*(k_1,k_2,\cdots,k_n)}$ of any other player $-k_n$ depends on player $-k_n$'s choice based on player $-k_n$'s own information set:

$$
e_{-k_n}^{*(k_1,k_2,\cdots,k_n)}=e_{-k_n}^{*(k_1,k_2,\cdots,k_n,-k_n)}.
$$

The strategic combination choice $s^{**(k_1, k_2, \cdots, k_n)}_{C_h^{c(k_1, k_2, \cdots, k_n)}(e_{k_n}^{(k_1, \cdots, k_n)}, e_{-k_n}^{*(k_1, k_2, \cdots, k_n)})}$ of any other core coalition $C_h^{c(k_1, k_2, \cdots, k_n)}$ $(e_{k_n}^{(k_1, \cdots, k_n)}, e_{-k_n}^{*(k_1, k_2, \cdots, k_n)})$ depends on the public choice of strategic combination of extensive coalition $C_h^{+(k_1, k_2, \cdots, k_n)}$ $(e_{k_n}^{(k_1, \cdots, k_n)}, e_{-k_n}^{*(k_1, k_2, \cdots, k_n)})$:

$$s^{**(k_1, k_2, \cdots, k_n)}_{C_h^{c(k_1, k_2, \cdots, k_n)}\left(e_{k_n}^{(k_1, \cdots, k_n)}, e_{-k_n}^{*(k_1, k_2, \cdots, k_n)}\right)} \subseteq s^{**(k_1, k_2, \cdots, k_n)}_{C_h^{+(k_1, k_2, \cdots, k_n)}\left(e_{k_n}^{(k_1, \cdots, k_n)}, e_{-k_n}^{*(k_1, k_2, \cdots, k_n)}\right)};$$

$$s^{**(k_1, k_2, \cdots, k_n)}_{C_h^{c(k_1, k_2, \cdots, k_n)}\left(e_{k_n}^{(k_1, \cdots, k_n)}, e_{-k_n}^{*(k_1, k_2, \cdots, k_n)}\right)}$$

$$= \underset{s^{(k_1, k_2, \cdots, k_n)}_{C_h^{c(k_1, k_2, \cdots, k_n)}(\cdot)}}{argmax} \left\{ \sum_{i \in C_h^{c(k_1, k_2, \cdots, k_n)}(\cdot)} V^{(k_1, k_2, \cdots, k_n, i)}_{C_T^{c(k_1, k_2, \cdots, k_n, i)}(\cdot)} (s^{(k_1, k_2, \cdots, k_n, i)}_{C_T^{c(k_1, k_2, \cdots, k_n, i)}(\cdot)}, s^{**(k_1, k_2, \cdots, k_n, i)}_{-C_T^{c(k_1, k_2, \cdots, k_n, i)}(\cdot)}) \right.$$

$$+ \sum_{\substack{j \in C_h^{+(k_1, k_2, \cdots, k_n)}(\cdot) \\ j \notin C_h^{c(k_1, k_2, \cdots, k_n)}(\cdot)}} V^{\left(k_1, k_2, \cdots, k_n, j^F\right)}_{C_h^{c(k_1, k_2, \cdots, k_n, j)}(\cdot)} (s^{\left(k_1, k_2, \cdots, k_n, j^F\right)}_{C_h^{c(k_1, k_2, \cdots, k_n, j)}(\cdot)},$$

$$\left. s^{**\left(k_1, k_2, \cdots, k_n, j^F\right)}_{C_T^{c(k_1, k_2, \cdots, k_n, j)}(\cdot)}, s^{**\left(k_1, k_2, \cdots, k_n, j^F\right)}_{-C_h^{c(k_1, k_2, \cdots, k_n, j)}(\cdot), -C_T^{c(k_1, k_2, \cdots, k_n, j)}(\cdot)}) \right\};$$

$$V^{\left(k_1, k_2, \cdots, k_n, j^F\right)}_{C_h^{c(k_1, k_2, \cdots, k_n, j)}(\cdot)} \left(s^{\left(k_1, k_2, \cdots, k_n, j^F\right)}_{C_h^{c(k_1, k_2, \cdots, k_n, j)}(\cdot)}, s^{**\left(k_1, k_2, \cdots, k_n, j^F\right)}_{C_T^{c(k_1, k_2, \cdots, k_n, j)}(\cdot)}, s^{**\left(k_1, k_2, \cdots, k_n, j^F\right)}_{-C_h^{c(k_1, k_2, \cdots, k_n, j)}(\cdot), -C_T^{c(k_1, k_2, \cdots, k_n, j)}(\cdot)} \right)$$

$$= V^{\left(k_1, k_2, \cdots, k_n, j^F\right)}_{C_h^{c(k_1, k_2, \cdots, k_n, j)}(\cdot)} \left(s^{\left(k_1, k_2, \cdots, k_n, j^F\right)}_{C_h^{c(k_1, k_2, \cdots, k_n, j)}(\cdot)}, i_j^{\left(k_1, k_2, \cdots, k_n, j^F\right)} \right), i_j^{\left(k_1, k_2, \cdots, k_n, j^F\right)} \in I_j^{\left(k_1, k_2, \cdots, k_n, j^F\right)};$$

where $i \in C_h^{c(k_1, k_2, \cdots, k_n)}(\cdot)$, therefore the escape target core coalition $C_T^{c(k_1, k_2, \cdots, k_n, i)}(\cdot)$ of player i is core coalition $C_h^{c(k_1, k_2, \cdots, k_n)}(\cdot)$, $j \in C_h^{+(k_1, k_2, \cdots, k_n)}(\cdot)$, $j \notin C_h^{c(k_1, k_2, \cdots, k_n)}(\cdot)$, core coalition $C_h^{c(k_1, k_2, \cdots, k_n, i)}(\cdot)$ and core coalition $C_h^{c(k_1, k_2, \cdots, k_n)}(\cdot)$ are the same, player j's escape target core coalition $C_T^{c(k_1, k_2, \cdots, k_n, j)}(\cdot)$ is not core coalition $C_h^{c(k_1, k_2, \cdots, k_n)}$ $(\cdot)$; $V^{\left(k_1, k_2, \cdots, k_n, j^F\right)}_{C_h^{c(k_1, k_2, \cdots, k_n, j)}(\cdot)} \left(s^{\left(k_1, k_2, \cdots, k_n, j^F\right)}_{C_h^{c(k_1, k_2, \cdots, k_n, j)}(\cdot)}, s^{**\left(k_1, k_2, \cdots, k_n, j^F\right)}_{C_T^{c(k_1, k_2, \cdots, k_n, j)}(\cdot)}, s^{**\left(k_1, k_2, \cdots, k_n, j^F\right)}_{-C_h^{c(k_1, k_2, \cdots, k_n, j)}(\cdot), -C_T^{c(k_1, k_2, \cdots, k_n, j)}(\cdot)} \right)$ represents player k_n's estimation of the cooperative payoff of core coalition $C_T^{c(k_1, k_2, \cdots, k_n, j)}(\cdot)$ according to the false signal released by player j, $i_j^{\left(k_1, k_2, \cdots, k_n, j^F\right)}$ is player k_n's estimation of the false signal released by player j, $I_j^{\left(k_1, k_2, \cdots, k_n, j^F\right)}$ is player k_n's estimation of player j's feasible false signal set, $s^{**\left(k_1, k_2, \cdots, k_n, j^F\right)}_{C_T^{c(k_1, k_2, \cdots, k_n, j)}(\cdot)}, s^{**\left(k_1, k_2, \cdots, k_n, j^F\right)}_{-C_h^{c(k_1, k_2, \cdots, k_n, j)}(\cdot), -C_T^{c(k_1, k_2, \cdots, k_n, j)}(\cdot)}$ is player k_n's estimation of player j's estimation of the strategic combination choices of the core coalitions [including player j's escape target core coalition $C_T^{c(k_1, k_2, \cdots, k_n, j)}(\cdot)$] other than player j's intermediate node core coalition $C_h^{c(k_1, k_2, \cdots, k_n, j)}(\cdot)$.

Therefore, player j's $\left[j \in C_h^{+(k_1, k_2, \cdots, k_n)}(\cdot),\ j \notin C_h^{c(k_1, k_2, \cdots, k_n)}(\cdot)\right]$ estimation of the strategic combination choice $s^{**(k_1, k_2, \cdots, k_n, j)}_{C_h^{c(k_1, k_2, \cdots, k_n, j)}(\cdot)}$ of core coalition $C_h^{c(k_1, k_2, \cdots, k_n, j)}(\cdot)$ is a function of the false signal $i_j^{\left(k_1, k_2, \cdots, k_n, j^F\right)}$ released by him:

$$
\begin{aligned}
s^{**(k_1, k_2, \cdots, k_n, j)}_{C_h^{c(k_1, k_2, \cdots, k_n, j)}(\cdot)} =& s^{**(k_1, k_2, \cdots, k_n, j)}_{C_h^{c(k_1, k_2, \cdots, k_n, j)}(\cdot)}\left(i_j^{\left(k_1, k_2, \cdots, k_n, j^F\right)}\right) \\
&= argmax\left\{\sum_{i \in C_h^{c(k_1, k_2, \cdots, k_n, j)}(\cdot)} V^{(k_1, k_2, \cdots, k_n, j, i)}_{C_T^{c(k_1, k_2, \cdots, k_n, j, i)}(\cdot)}\left(s^{(k_1, k_2, \cdots, k_n, j, i)}_{C_T^{c(k_1, k_2, \cdots, k_n, j, i)}(\cdot)},\ s^{**(k_1, k_2, \cdots, k_n, j, i)}_{-C_T^{c(k_1, k_2, \cdots, k_n, j, i)}(\cdot)}\right)\right. \\
&\quad + V^{\left(k_1, k_2, \cdots, k_n, j^F\right)}_{C_h^{c(k_1, k_2, \cdots, k_n, j)}(\cdot)}\left(s^{\left(k_1, k_2, \cdots, k_n, j^F\right)}_{C_h^{c(k_1, k_2, \cdots, k_n, j)}(\cdot)},\ i_j^{\left(k_1, k_2, \cdots, k_n, j^F\right)}\right) \\
&\quad \left. + \sum_{\substack{k \in C_h^{+(k_1, k_2, \cdots, k_n, j)}(\cdot) \\ k \notin C_h^{c(k_1, k_2, \cdots, k_n, j)}(\cdot) \\ k \neq j}} V^{\left(k_1, k_2, \cdots, k_n, j, k^F\right)}_{C_h^{c(k_1, k_2, \cdots, k_n, j, k)}(\cdot)}\left(s^{\left(k_1, k_2, \cdots, k_n, j, k^F\right)}_{C_h^{c(k_1, k_2, \cdots, k_n, j, k)}(\cdot)},\ i_j^{*\left(k_1, k_2, \cdots, k_n, j, k^F\right)}\right)\right\}
\end{aligned}
$$

where $i \in C_h^{c(k_1, k_2, \cdots, k_n, j)}(\cdot)$, the escape target core coalition $C_T^{c(k_1, k_2, \cdots, k_n, j, i)}(\cdot)$ of player i is core coalition $C_h^{c(k_1, k_2, \cdots, k_n, j)}(\cdot)$, $k \in C_h^{+(k_1, k_2, \cdots, k_n, j)}(\cdot)$, $k \notin C_h^{c(k_1, k_2, \cdots, k_n, j)}(\cdot)$, core coalition $C_h^{c(k_1, k_2, \cdots, k_n, j, k)}(\cdot)$ and core coalition $C_h^{c(k_1, k_2, \cdots, k_n, j)}(\cdot)$ are the same, the escape target core coalition $C_T^{c(k_1, k_2, \cdots, k_n, j, k)}(\cdot)$ of player k is not core coalition $C_h^{c(k_1, k_2, \cdots, k_n, j)}(\cdot)$; $i_j^{*\left(k_1, k_2, \cdots, k_n, j, k^F\right)}$ is player j's estimation of the optimal false signal of any extensive member $k\left[k \in C_h^{+(k_1, k_2, \cdots, k_n, j)}(\cdot),\ k \notin C_h^{c(k_1, k_2, \cdots, k_n, j)}(\cdot),\ k \neq j\right]$ in his virtual game.

Player j's purpose of releasing false signals to his intermediate node core coalition $C_h^{c(k_1, k_2, \cdots, k_n, j)}(\cdot)$ is to influence the public choice $s^{**(k_1, k_2, \cdots, k_n)}_{C_h^{c(k_1, k_2, \cdots, k_n, j)}(\cdot)}$ of strategic combination of core coalition $C_h^{c(k_1, k_2, \cdots, k_n, j)}(\cdot)$, thereby maximize the cooperative payoff of his escape target core coalition $C_T^{c(k_1, k_2, \cdots, k_n, j)}(\cdot)$:

$$
\begin{aligned}
i_j^{*\left(k_1, k_2, \cdots, k_t, j^F\right)} = & \underset{i_j^{\left(k_1, k_2, \cdots, k_t, j^F\right)} \in I_j^{\left(k_1, k_2, \cdots, k_t, j^F\right)}}{argmax} V^{(k_1, k_2, \cdots, k_t, j)}_{C_T^{c(k_1, k_2, \cdots, k_t, j)}(\cdot)}\left[s^{**(k_1, k_2, \cdots, k_t, j)}_{C_h^{c(k_1, k_2, \cdots, k_t, j)}(\cdot)}\left(i_j^{\left(k_1, k_2, \cdots, k_t, j^F\right)}\right),\right. \\
& \left. s^{*(k_1, k_2, \cdots, k_t, j)}_{C_T^{c(k_1, k_2, \cdots, k_t, j)}(\cdot)},\ s^{**(k_1, k_2, \cdots, k_t, j)}_{-C_h^{c(k_1, k_2, \cdots, k_t, j)}(\cdot),\ -C_T^{c(k_1, k_2, \cdots, k_t, j)}(\cdot)}\right].
\end{aligned}
$$

Obviously, in order to estimate the strategic combination choice $s^{**(k_1, k_2, \cdots, k_n)}_{C_h^{c(k_1, k_2, \cdots, k_n)}(\cdot)}$ of any other core coalition $C_h^{c(k_1, k_2, \cdots, k_n)}(\cdot)$, player k_n needs to estimate any core member i's [who belongs to core coalition $C_h^{c(k_1, k_2, \cdots, k_n)}(\cdot)$] estimation of the strategic combination choices $s^{**(k_1, k_2, \cdots, k_n, i)}_{-C_T^{c(k_1, k_2, \cdots, k_n, i)}(\cdot)}$ of other core coalitions $-C_h^{c(k_1, k_2, \cdots, k_n)}(\cdot)$, and any extensive member j's [who nominally belongs to core coalition $C_h^{c(k_1, k_2, \cdots, k_n)}(\cdot)$] estimation of the strategic combination $s^{*(k_1,\, k_2,\, \cdots,\, k_n,\, j)}_{C_T^{c(k_1,\, k_2,\, \cdots,\, k_n,\, j)}(\cdot)}$ that this core coalition "should" adopt, member j's estimation of the strategic combinations $s^{**(k_1,\, k_2,\, \cdots,\, k_n,\, j)}_{-C_h^{c(k_1,\, k_2,\, \cdots,\, k_n,\, j)}(\cdot),\, -C_T^{c(k_1,\, k_2,\, \cdots,\, k_n,\, j)}(\cdot)}$ of the core coalitions other than core coalition $C_h^{c(k_1,\, k_2,\, \cdots,\, k_n)}(\cdot)$ and his escape target core coalition, and member j's estimation of the optimal signal $i_j^{*\left(k_1,\, k_2,\, \cdots,\, k_n,\, j,\, k^F\right)}$ of any other extensive member k.

Thus, the virtual game $\Gamma_{k_n}^{(k_1,\, \cdots,\, k_n)}(e_{k_1}^{(k_1)},\, \cdots,\, e_{k_n}^{(k_1,\, \cdots,\, k_n)})$ enters the second, the third, … level up to the one in which the equilibrium solution to game $\Gamma_{k_n}^{(k_1,\, \cdots,\, k_n)}(e_{k_1}^{(k_1)},\, \cdots,\, e_{k_n}^{(k_1,\, \cdots,\, k_n)})$ can be achieved.

The model of t-th level of virtual game $\Gamma_{k_n}^{(k_1,\, \cdots,\, k_n)}(e_{k_1}^{(k_1)},\, \cdots,\, e_{k_n}^{(k_1,\, \cdots,\, k_n)})$ is shown as follows.

The *t*-th level of virtual game $\Gamma_{k_n}^{(k_1,\, \cdots,\, k_n)}(e_{k_1}^{(k_1)},\, \cdots,\, e_{k_n}^{(k_1,\, \cdots,\, k_n)})$.

In the t-th level of virtual game $\Gamma_{k_n}^{(k_1,\, \cdots,\, k_n)}(e_{k_1}^{(k_1)},\, \cdots,\, e_{k_n}^{(k_1,\, \cdots,\, k_n)})$, player i_{t-1} needs to estimate the optimal escape strategy $e_{i_t}^{(k_1,\, \cdots,\, k_n,\, i_1,\, \cdots,\, i_t)}$ of any other player $i_t(i_t \neq i_{t-1})$, and the optimal strategic combination choices of the core coalitions other than player i_{t-1}'s escape target core coalition.

In coalition situation c, in the t-th level of virtual game $\Gamma_{k_n}^{(k_1,\, \cdots,\, k_n)}(e_{k_1}^{(k_1)},\, \cdots,\, e_{k_n}^{(k_1,\, \cdots,\, k_n)})$, assume that the strategic combination of any core coalition $C_h^{c(k_1,\, ...,\, k_n,\, i_1,\, \cdots,\, i_{t-1})}\left(e_{i_{t-1}}^{(k_1,\, ...,\, k_n,\, i_1,\, \cdots,\, i_{t-1})},\, e_{-i_{t-1}}^{*(k_1,\, ...,\, k_n,\, i_1,\, \cdots,\, i_{t-1})}\right)$ $\left[C_h^{c(k_1,\, ...,\, k_n,\, i_1,\, \cdots,\, i_{t-1})}\left(e_{i_{t-1}}^{(k_1,\, ...,\, k_n,\, i_1,\, \cdots,\, i_{t-1})},\, e_{-i_{t-1}}^{*(k_1,\, ...,\, k_n,\, i_1,\, \cdots,\, i_{t-1})}\right) \neq C_T^{c(k_1,\, ...,\, k_n,\, i_1,\, \cdots,\, i_{t-1})}\left(e_{i_{t-1}}^{(k_1,\, ...,\, k_n,\, i_1,\, \cdots,\, i_{t-1})},\, e_{-i_{t-1}}^{*(k_1,\, ...,\, k_n,\, i_1,\, \cdots,\, i_{t-1})}\right)\right]$ other than player i_{t-1}'s escape target core coalition $\left[C_T^{c(k_1,\, ...,\, k_n,\, i_1,\, \cdots,\, i_{t-1})}(e_{i_{t-1}}^{(k_1,\, ...,\, k_n,\, i_1,\, \cdots,\, i_{t-1})},\, e_{-i_{t-1}}^{*(k_1,\, ...,\, k_n,\, i_1,\, \cdots,\, i_{t-1})})\right.$ is player i_{t-1}'s escape target core coalition in escape situation $\left.(e_{i_{t-1}}^{(k_1,\, ...,\, k_n,\, i_1,\, \cdots,\, i_{t-1})},\, e_{-i_{t-1}}^{*(k_1,\, ...,\, k_n,\, i_1,\, \cdots,\, i_{t-1})})\right]$ is $s^{**(k_1,\, ...,\, k_n,\, i_1,\, \cdots,\, i_{t-1})}_{C_h^{c(k_1,\, ...,\, k_n,\, i_1,\, \cdots,\, i_{t-1})}\left(e_{i_{t-1}}^{(k_1,\, ...,\, k_n,\, i_1,\, \cdots,\, i_{t-1})},\, e_{-i_{t-1}}^{*(k_1,\, ...,\, k_n,\, i_1,\, \cdots,\, i_{t-1})}\right)}$, the strategic combination that player i_{t-1} considers that his escape target core coalition $C_T^{c(k_1,\, ...,\, k_n,\, i_1,\, \cdots,\, i_{t-1})}(e_{i_{t-1}}^{(k_1,\, ...,\, k_n,\, i_1,\, \cdots,\, i_{t-1})},\, e_{-i_{t-1}}^{*(k_1,\, ...,\, k_n,\, i_1,\, \cdots,\, i_{t-1})})$ "should" adopt [that is to say, the strategic combination which maximizes the expected cooperative payoff of core coalition $C_T^{c(k_1,\, ...,\, k_n,\, i_1,\, \cdots,\, i_{t-1})}(e_{i_{t-1}}^{(k_1,\, ...,\, k_n,\, i_1,\, \cdots,\, i_{t-1})},\, e_{-i_{t-1}}^{*(k_1,\, ...,\, k_n,\, i_1,\, \cdots,\, i_{t-1})})$] is:

$$s^{\circ(k_1,\, ...,\, k_n,\, i_1,\, \cdots,\, i_{t-1})}_{C_T^{c(k_1,\, ...,\, k_n,\, i_1,\, \cdots,\, i_{t-1})}(e_{i_{t-1}}^{(k_1,\, ...,\, k_n,\, i_1,\, \cdots,\, i_{t-1})},\, e_{-i_{t-1}}^{*(k_1,\, ...,\, k_n,\, i_1,\, \cdots,\, i_{t-1})})}$$

$$= \underset{s^{(k_1,\, ...,\, k_n,\, i_1,\, \cdots,\, i_{t-1})}_{C_T^{c(k_1,\, ...,\, k_n,\, i_1,\, \cdots,\, i_{t-1})}\left(e_{i_{t-1}}^{(k_1,\, ...,\, k_n,\, i_1,\, \cdots,\, i_{t-1})},\, e_{-i_{t-1}}^{*(k_1,\, ...,\, k_n,\, i_1,\, \cdots,\, i_{t-1})}\right)}}{argmax}$$

$$V^{(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}_{C_T^{c(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}(\cdot)}\left(s^{(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}_{C_T^{c(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}\left(e_{i_{t-1}}^{(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})},\ e_{-i_{t-1}}^{*(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}\right)},\right.$$

$$\left.s^{**(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}_{-C_T^{c(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}\left(e_{i_{t-1}}^{(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})},\ e_{-i_{t-1}}^{*(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}\right)}\right)$$

$$= \underset{s^{(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}_{C_T^{c(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}\left(e_{i_{t-1}}^{(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})},\ e_{-i_{t-1}}^{*(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}\right)}}{argmax}$$

$$\sum_{i\in C_T^{c(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}\left(e_{i_{t-1}}^{(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})},\ e_{-i_{t-1}}^{*(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}\right)} u_i^{(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}$$

$$\left(s^{(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}_{C_T^{c(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}\left(e_{i_{t-1}}^{(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})},\ e_{-i_{t-1}}^{*(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}\right)},\right.$$

$$\left.s^{**(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}_{-C_T^{c(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}\left(e_{i_{t-1}}^{(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})},\ e_{-i_{t-1}}^{*(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}\right)}\right).$$

Denote player i_{t-1}'s estimation of his escape-payoff deriving from deviation when he escapes from core coalition $C_T^{c(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}\left(e_{i_{t-1}}^{(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})},\ e_{-i_{t-1}}^{*(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}\right)$ deriving from deviation as $W_{i_{t-1}}^{\circ -C_T^{c(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}\left(e_{i_{t-1}}^{(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})},\ e_{-i_{t-1}}^{*(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}\right)(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}$, obviously, the goal of player i_{t-1}'s escape strategic choice is his minimum expected escape-payoff deriving from deviation:

$$e_{i_{t-1}}^{*(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}$$

$$= \underset{e_{i_{t-1}}^{(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}}{argmin}\ W_{i_{t-1}}^{\circ -C_T^{c(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}\left(e_{i_{t-1}}^{(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})},\ e_{-i_{t-1}}^{*(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}\right)(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}.$$

The strategic combination that core coalition $C_T^{c(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}\left(e_{i_{t-1}}^{*(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})},\ e_{-i_{t-1}}^{*(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}\right)$ "should" adopt is considered to be:

$$s^{*(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}_{C_T^{c(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}\left(e_{i_{t-1}}^{*(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})},\ e_{-i_{t-1}}^{*(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}\right)}$$

$$= \underset{s^{(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}_{C_T^{c(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}\left(e_{i_{t-1}}^{*(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})},\ e_{-i_{t-1}}^{*(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}\right)}}{argmax}$$

$$V^{(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}_{C_T^{c(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}(\cdot)}\left(s^{(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}_{C_T^{c(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}\left(e_{i_{t-1}}^{*(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})},\ e_{-i_{t-1}}^{*(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}\right)},\right.$$

$$\left. s^{**(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}_{-C_T^{c(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}\left(e_{i_{t-1}}^{*(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})},\ e_{-i_{t-1}}^{*(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}\right)}\right).$$

In the t-th level of virtual game $\Gamma_{k_n}^{(k_1,\ \cdots,\ k_n)}(e_{k_1}^{(k_1)},\ \cdots,\ e_{k_n}^{(k_1,\ \cdots,\ k_n)})$, player i_{t-1} must estimate the escape strategic choice $e_{-i_{t-1}}^{*(k_1,\ ...,\ k_n,\ i_1,\ \cdots,i_{t-1})}$ of any other player $-i_{t-1}$ on the basis of player $-i_{t-1}$'s own information set and the strategic combination choices $s^{**(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}_{-C_T^{c(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}\left(e_{i_{t-1}}^{*(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})},\ e_{-i_{t-1}}^{*(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}\right)}$ of the core coalitions other than player i_{t-1}'s escape target core coalition when he decides his optimal escape strategy and the optimal strategic combination choice of his escape target core coalition. Player i_{t-1}'s decision-making relies on his estimation of the equilibrium escape strategic choice of any player $-i_{t-1}$ in player $-i_{t-1}$'s virtual game. Player i_{t-1}'s estimation of any other player $-i_{t-1}$'s estimation of his own escape strategic choice $e_{-i_{t-1}}^{*(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}$ depends on player $-k_t$'s choice based on his own information set:

$$e_{-i_{t-1}}^{*(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})} = e_{-i_{t-1}}^{*(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1},\ -i_{t-1})}.$$

The strategic combination choice $s^{**(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}_{C_h^{c(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}\left(e_{i_{t-1}}^{(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})},\ e_{-i_{t-1}}^{*(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}\right)}$ of any core coalition $C_h^{c(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}\left(e_{i_{t-1}}^{(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})},\ e_{-i_{t-1}}^{*(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}\right)$ depends on the public choice game of extensive coalition $C_h^{+(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}\left(e_{i_{t-1}}^{(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})},\ e_{-i_{t-1}}^{*(k_1,\ ...,\ k_n,\ i_1,\ \cdots,\ i_{t-1})}\right)$ among its extensive members:

$$s^{**(k_1, ..., k_n, i_1, \cdots, i_{t-1})}_{C_h^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1})}(e_{i_{t-1}}^{(k_1, ..., k_n, i_1, \cdots, i_{t-1})}, e_{-i_{t-1}}^{*(k_1, ..., k_n, i_1, \cdots, i_{t-1})})}$$

$$\subseteq s^{**(k_1, ..., k_n, i_1, \cdots, i_{t-1})}_{C_h^{+(k_1, ..., k_n, i_1, \cdots, i_{t-1})}(e_{i_{t-1}}^{(k_1, ..., k_n, i_1, \cdots, i_{t-1})}, e_{-i_{t-1}}^{*(k_1, ..., k_n, i_1, \cdots, i_{t-1})})};$$

$$s^{**(k_1, ..., k_n, i_1, \cdots, i_{t-1})}_{C_h^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1})}(e_{i_{t-1}}^{(k_1, ..., k_n, i_1, \cdots, i_{t-1})}, e_{-i_{t-1}}^{*(k_1, ..., k_n, i_1, \cdots, i_{t-1})})}$$

$$= \underset{s^{(k_1, ..., k_n, i_1, \cdots, i_{t-1})}_{C_h^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1})}(\cdot)}}{argmax} \left\{ \sum_{i \in C_h^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1})}(\cdot)} V^{(k_1, ..., k_n, i_1, \cdots, i_{t-1}, i)}_{C_T^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1}, i)}(\cdot)} (s^{(k_1, ..., k_n, i_1, \cdots, i_{t-1}, i)}_{C_T^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1}, i)}(\cdot)}, \right.$$

$$s^{**(k_1, ..., k_n, i_1, \cdots, i_{t-1}, i)}_{-C_T^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1}, i)}(\cdot)})$$

$$+ \sum_{\substack{j \in C_h^{+(k_1, ..., k_n, i_1, \cdots, i_{t-1})}(\cdot) \\ j \notin C_h^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1})}(\cdot)}} V^{(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j^F)}_{C_h^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j)}(\cdot)} (s^{(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j^F)}_{C_h^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j)}(\cdot)},$$

$$s^{**(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j^F)}_{C_T^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j)}(\cdot)}, s^{**(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j^F)}_{-C_h^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j)}(\cdot), -C_T^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j)}(\cdot)});$$

$$V^{(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j^F)}_{C_h^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j)}(\cdot)} (s^{(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j^F)}_{C_h^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j)}(\cdot)},$$

$$s^{**(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j^F)}_{C_T^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j)}(\cdot)}, s^{**(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j^F)}_{-C_h^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j)}(\cdot), -C_T^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j)}(\cdot)})$$

$$= V^{(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j^F)}_{C_h^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j)}(\cdot)} (s^{(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j^F)}_{C_h^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j)}(\cdot)},$$

$$i_j^{(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j^F)}), \; i_j^{(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j^F)} \in I_j^{(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j^F)};$$

where $i \in C_h^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1})}(\cdot)$, therefore the escape target core coalition $C_T^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1}, i)}(\cdot)$ of player i is core coalition $C_h^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1})}(\cdot)$, $j \in C_h^{+(k_1, ..., k_n, i_1, \cdots, i_{t-1})}(\cdot)$, $j \notin C_h^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1})}(\cdot)$, core coalition $C_h^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1}, i)}(\cdot)$ and core coalition $C_h^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1})}(\cdot)$ are the same, player j's escape target core coalition $C_T^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j)}(\cdot)$ is not core coalition $C_h^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1})}(\cdot)$; and $(s^{(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j^F)}_{C_h^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j)}(\cdot)}$,

$s^{**\left(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j^F\right)}_{C_T^{c(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j)}(\cdot)}$, $s^{**\left(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j^F\right)}_{-C_h^{c(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j)}(\cdot),\,-C_T^{c(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j)}(\cdot)}$) represents player i_{t-1}'s estimation of the cooperative payoff of core coalition $C_T^{c(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j)}(\cdot)$ according to the false signal released by player j, $i_j^{\left(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j^F\right)}$ is player i_{t-1}'s estimation of the false signal released by player j, $I_j^{\left(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j^F\right)}$ is player i_{t-1}'s estimation of player j's feasible false signal set, $s^{**\left(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j^F\right)}_{-C_T^{c(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j)}(\cdot)}$, $s^{**\left(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},j^F\right)}_{-C_h^{c(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j)}(\cdot),\,-C_T^{c(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j)}(\cdot)}$ is player i_{t-1}'s estimation of player j's estimation of the strategic combination choices of the core coalitions [including player j's escape target core coalition $C_T^{c(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j)}(\cdot)$] other than player j's intermediate node core coalition $C_h^{c(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j)}(\cdot)$.

Therefore, in the t-th level of virtual game $\Gamma_{k_n}^{(k_1,\,\cdots,\,k_n)}(e_{k_1}^{(k_1)},\,\cdots,\,e_{k_n}^{(k_1,\,\cdots,\,k_n)})$, player j's $\left[j \in C_h^{+(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1})}(\cdot),\ j \notin C_h^{c(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1})}(\cdot)\right]$ estimation of the strategic combination choice $s^{**(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j)}_{C_h^{c(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j)}(\cdot)}$ of core coalition $C_h^{c(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j)}(\cdot)$ is a function of the false signal $i_j^{\left(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j^F\right)}$ released by him:

$$
\begin{aligned}
& s^{**(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j)}_{C_h^{c(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j)}(\cdot)} \\
&= s^{**(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j)}_{C_h^{c(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j)}(\cdot)}\left(i_j^{\left(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j^F\right)}\right) \\
&= \operatorname{argmax}\left\{ \sum_{i \in C_h^{c(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j)}(\cdot)} V^{(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j,\,i)}_{C_T^{c(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j,\,i)}(\cdot)}\left(s^{(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j,\,i)}_{C_T^{c(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j,\,i)}(\cdot)},\, s^{**(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j,\,i)}_{-C_T^{c(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j,\,i)}(\cdot)}\right) \right. \\
&\quad + V^{\left(k_1,\,...,\,k_n,\,i_1,\,\cdots,i_{t-1},\,j^F\right)}_{C_h^{c(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j)}(\cdot)}\left(s^{\left(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j^F\right)}_{C_h^{c(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j)}(\cdot)},\, i_j^{\left(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j^F\right)}\right) \\
&\quad \left. + \sum_{\substack{k \in C_h^{+(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j)}(\cdot) \\ k \notin C_h^{c(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j)}(\cdot) \\ k \neq j}} V^{\left(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j,\,k^F\right)}_{C_h^{c(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j,\,k)}(\cdot)}\left(s^{\left(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j,\,k^F\right)}_{C_h^{c(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j,\,k)}(\cdot)},\, i_j^{*\left(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j,\,k^F\right)}\right) \right\},
\end{aligned}
$$

where $i \in C_h^{c(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j)}(\cdot)$, the escape target core coalition $C_T^{c(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j,\,i)}(\cdot)$ of player i is core coalition $C_h^{c(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j)}(\cdot)$, $k \in C_h^{+(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j)}(\cdot)$, $k \notin C_h^{c(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j)}(\cdot)$, core coalition $C_h^{c(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j,\,k)}(\cdot)$ and core coalition $C_h^{c(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j)}(\cdot)$ are the same, the escape target core coalition $C_T^{c(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j,\,k)}(\cdot)$ of player k is not core coalition $C_T^{c(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j)}(\cdot)$; $i_j^{*\left(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j,\,k^F\right)}$ is player j's estimation of the optimal false signal of any extensive member k $\left[k \in C_h^{+(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j)}(\cdot),\ k \notin C_h^{c(k_1,\,...,\,k_n,\,i_1,\,\cdots,\,i_{t-1},\,j)}(\cdot),\ k \neq j\right]$ in his virtual game.

Player j's purpose of releasing false signal to his intermediate node core coalition $C_h^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j)}(\cdot)$ is to influence the public choice $s^{**(k_1, ..., k_n, i_1, \cdots, i_{t-1})}_{C_h^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j)}(\cdot)}$ of strategic combination of core coalition $C_h^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j)}(\cdot)$, thereby maximize the cooperative payoff of his escape target core coalition $C_T^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j)}(\cdot)$:

$$i_j^{*\left(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j^F\right)}$$

$$= \underset{i_j^{\left(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j^F\right)} \in I_j^{\left(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j^F\right)}}{argmax} V^{(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j)}_{C_T^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j)}(\cdot)}$$

$$\left(s^{**(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j)}_{C_h^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j)}(\cdot)} \left(i_j^{\left(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j^F\right)} \right), \right.$$

$$\left. s^{*(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j)}_{C_T^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j)}(\cdot)}, s^{**(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j)}_{-C_h^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j)}(\cdot), -C_T^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j)}(\cdot)} \right).$$

Obviously, in order to estimate the strategic combination choice $s^{**(k_1, ..., k_n, i_1, \cdots, i_{t-1})}_{C_h^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1})}(\cdot)}$ of any other core coalition $C_h^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1})}(\cdot)$, player i_{t-1} needs to estimate any core member i's [who belongs to core coalition $C_h^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1})}(\cdot)$] estimation of the strategic combination choices $s^{**(k_1, ..., k_n, i_1, \cdots, i_{t-1}, i)}_{-C_T^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1}, i)}(\cdot)}$ of other core coalitions $-C_T^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1})}(\cdot)$, any extensive member j's [who nominally belongs to core coalition $C_h^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1})}(\cdot)$] estimation of the strategic combination $s^{*(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j)}_{C_T^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j)}(\cdot)}$ which this core coalition "should" adopt, member j's estimation of the strategic combinations $s^{**(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j)}_{-C_h^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j)}(\cdot), -C_T^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j)}(\cdot)}$ of the core coalitions other than core coalition $C_h^{c(k_1, ..., k_n, i_1, \cdots, i_{t-1})}(\cdot)$ and his escape target core coalition, and member j's estimation of the optimal signal $i_j^{*\left(k_1, ..., k_n, i_1, \cdots, i_{t-1}, j, k^F\right)}$ of any other extensive member k.

Thus, the virtual game of $\Gamma_{k_n}^{(k_1, \cdots, k_n)}(e_{k_1}^{(k_1)}, \cdots, e_{k_n}^{(k_1, \cdots, k_n)})$ enters the (t+1)-th level.

$\cdots$

Solution to virtual game $\Gamma_{k_n}^{(k_1, \cdots, k_n)}(e_{k_1}^{(k_1)}, \cdots, e_{k_n}^{(k_1, \cdots, k_n)})$.

If for the p-th level of virtual game $\Gamma_{k_n}^{(k_1, \cdots, k_n)}(e_{k_1}^{(k_1)}, \cdots, e_{k_n}^{(k_1, \cdots, k_n)})$, conditions

$$s_i^{\left(k_1, ..., k_n, i_1, \cdots, i_{p-1}\right)} = s_i^{\left(k_1, ..., k_n, i_1, \cdots, i_p\right)}, u_i^{\left(k_1, ..., k_n, i_1, \cdots, i_{p-1}\right)} = u_i^{\left(k_1, ..., k_n, i_1, \cdots, i_p\right)}$$

always hold (that is to say, in the p-th level of virtual game $\Gamma_{k_n}^{(k_1, \cdots, k_n)}(e_{k_1}^{(k_1)}, \cdots, e_{k_n}^{(k_1, \cdots, k_n)})$, any player i_{p-1}'s estimation of the virtual game of any other player is the same as the one of player i_{p-1}, in which the strategy sets and the payoff functions of all the players are the same), virtual game $\Gamma_{k_n}^{(k_1, \cdots, k_n)}(e_{k_1}^{(k_1)}, \cdots, e_{k_n}^{(k_1, \cdots, k_n)})$ is called a p-order game. At this time, the equilibrium solution $e_{i_{p-1}}^{*(k_1, ..., k_n, i_1, \cdots, i_{p-1})}$ to the p-level of virtual game $\Gamma_{k_n}^{(k_1, \cdots, k_n)}(e_{k_1}^{(k_1)}, \cdots, e_{k_n}^{(k_1, \cdots, k_n)})$ is the Nash equilibrium of the coalition-choosing game under the criterion of minimum escape-payoff deriving from deviation under information symmetry, that is, the coalition equilibrium of information symmetric cooperative game

$\Gamma_{k_n}^{(k_1,\ldots,k_n,i_1,\cdots,i_{p-1})}\left(N,\ \left\{S_i^{(k_1,\ldots,k_n,i_1,\cdots,i_{p-1})}\right\},\ \left\{u_i^{(k_1,\ldots,k_n,i_1,\cdots,i_{p-1})}\right\}\right)$ with agreements self-implemented. By backward induction, from the p-th level of virtual game $\Gamma_{k_n}^{(k_1,\cdots,k_n)}(e_{k_1}^{(k_1)},\cdots,e_{k_n}^{(k_1,\cdots,k_n)})$ to the first level of virtual game $\Gamma_{k_n}^{(k_1,\cdots,k_n)}(e_{k_1}^{(k_1)},\cdots,e_{k_n}^{(k_1,\cdots,k_n)})$, we can finally get the equilibrium solution to game $\Gamma_{k_n}^{(k_1,\cdots,k_n)}(e_{k_1}^{(k_1)},\cdots,e_{k_n}^{(k_1,\cdots,k_n)})$.

2.5 *Solution to the virtual game of player* k_1

According to the analysis above, under the criterion of maximum expected cooperative payoff distribution, player k_1 can get the expected cooperative payoff distribution of all the players, $\widetilde{x}_{k_1}^{*(k_1)}(c)$, $\widetilde{x}_{k_2}^{*(k_1,k_2)}(c)$, $\cdots$, $\widetilde{x}_{k_n}^{*(k_1,k_n)}(c)$, in coalition situation c in his virtual game, that is to say, when the coalition-choosing strategy vector of all the players is $c=(c_1,\cdots,c_n)$, the cooperative payoff distribution vector of all the players is

$$\widetilde{x}=\left[\widetilde{x}_{k_1}^{*(k_1)}(c),\ \widetilde{x}_{k_2}^{*(k_1,k_2)}(c),\ \cdots,\ \widetilde{x}_{k_n}^{*(k_1,k_n)}(c)\right].$$

At this point, we get coalition-choosing game $\Gamma_{\Gamma}^{(k_1)}(N,\ \{C_i\},\ \left\{\widetilde{x}_{k_i}^{*(k_1,i)}(c)\right\})$ of playerk_1's virtual game, the equilibrium of this non-cooperative game is the solution to player k_1's virtual game. The equilibrium of the coalition-choosing game $\Gamma_{\Gamma}^{(k_1)}(N,\{C_i\},\left\{\widetilde{x}_{k_i}^{*(k_1,i)}(c)\right\})$ of player k_1's virtual game is the coalition equilibrium of player k_1's virtual game under the criterion of maximum expected cooperative payoff distribution (when the members of each coalition trust each other).

Under the criterion of minimum expected escape-payoff deriving from deviation (when the members of each coalition trust each other), with the expected escape-payoffs deriving from deviation of the players when they escape from the core coalitions they belong to through deviation, $W_{k_1}^{*-C_T^{c(k_1)}(k_1)}(c)$, $W_{k_2}^{*-C_T^{c(k_1,k_2)}(k_1,k_2)}(c)$, $\cdots$, $W_{k_n}^{*-C_T^{c(k_1,k_n)}(k_1,k_n)}(c)$, in coalition situation c in the virtual game of player k_1, we can get the coalition-choosing game $\Gamma_{\Gamma}^{(k_1)}(N,\ \{C_i\},\ \left\{-W_i^{*-C_T^{c(k_1,i)}(k_1,i)}(c)\right\}\right)$ under the criterion of minimum expected escape-payoff deriving from deviation (when the members of each coalition trust each other) in which the estimation vector of the payoff functions of all the players is

$$W=\left[-W_{k_1}^{*-C_T^{c(k_1)}(k_1)}(c),\ -W_{k_2}^{*-C_T^{c(k_1,k_2)}(k_1,k_2)}(c),\ \cdots,\ -W_{k_n}^{*-C_T^{c(k_1,k_n)}(k_1,k_n)}(c)\right].$$

Since the minimization of the expected escape-payoff deriving from deviation (when the members of each coalition trust each other) of any player and the maximization of his expected cooperative payoff distribution (when the members of each coalition trust each other) are mutually necessary and sufficient conditions, the coalition-choosing game $\Gamma_{\Gamma}^{(k_1)}(N,\{C_i\},\ \left\{\widetilde{x}_{k_i}^{*(k_1,i)}(c)\right\})$ under the criterion of maximum expected cooperative payoff distribution (when the members of each coalition trust each other) in player k_1's virtual game and the coalition-choosing game $\Gamma_{\Gamma}^{(k_1)}(N,\{C_i\},\ \left\{-W_i^{*-C_T^{c(k_1,i)}(k_1,i)}(c)\right\}\right)$ under the criterion of minimum expected escape-payoff deriving from deviation (when the members of each coalition trust each other) have the same Nash equilibrium.

Of course, the Nash equilibrium $c^{*(k_1)}$ of coalition-choosing game $\Gamma_{\Gamma}^{(k_1)}(N,\ \{C_i\},\ \left\{-W_i^{*-C_T^{c(k_1,i)}(k_1,i)}(c)\right\})$ under the criterion of maximum expected cooperative payoff distribution (when the members of each coalition trust each other) in player k_1's virtual game is not the actual Nash equilibrium of the coalition-choosing game of the information asymmetric cooperative game with agreements self-implemented, because the players have different information sets. When all the players form their virtual games according to their own information sets, they often get different coalition equilibria, that is, usually,

$$c^{*(k_1)} \neq c^{*(-k_1)}.$$

If the coalition equilibrium in any player k_1's virtual game is different from the actual possible coalition equilibrium, player k_1 has the incentive to continue collecting information in order to understand the possible coalition equilibrium, because the misjudgment of other players' information sets will cause him to suffer loss. Therefore, before the coalition equilibrium of information asymmetric cooperative game is formed, player k_1 should release and transmit information through communication and negotiation and receive information released by other players. The changes in the players' information sets will ultimately make:

$$c'^{*(k_1)} = c'^{*(-k_1)} = c'^{*}.$$

At this point, the information asymmetric cooperative game with agreements self-implemented finally reaches its coalition equilibrium. Obviously, if the information set of some player we refer to in the above analysis is the final information set before the coalition equilibrium of the information asymmetric cooperative game with agreements self-implemented is reached, then:

$$c^{*(k_1)} = c^{*(-k_1)} = c^{*}.$$

Before the formation of the coalition equilibrium, the players need to release and receive information, and finally form a consistent judgment of the coalition equilibrium. Therefore, the information set on the basis of which player k_1 forms his virtual game we mentioned above refers to the final information set of player k_1 before the coalition equilibrium is reached.

Secondly, what needs to be mentioned here is that even if we have mentioned in the above model, only the escape strategy that is considered feasible will be discussed, however, before the end of the backstepping process, it is actually impossible to accurately determine which escape strategies of the players are considered feasible. Therefore, an alternative approach is to discuss the virtual game model in which all the escape strategies of the players are considered at first, then the escape strategies which are considered infeasible are gradually eliminated from the players' escape strategy sets.

In the above discussion about the coalition equilibrium of player k_1's virtual game, we still miss two important issues that have not been solved. One is the measurement of the expected cooperative payoff distribution that a player gets from the core coalition he belongs to, and the other is the measurement of the expected escape-payoff deriving from deviation of a player when he escapes from the coalition he belongs to through deviation, although we have shown that the goal of the maximum expected cooperative payoff of a core coalition is consistent with the goal of the maximum expected cooperative payoff distribution of some core member of this core coalition.

3. Coalition equilibrium of the game and the distribution of the cooperative payoff: ignoring the opportunistic behaviors in the distribution process

In this section, we will examine the coalition equilibrium of the game and the distribution process of the actual cooperative payoff of a core coalition when members are unallied in the bargaining game, ignoring the opportunistic behaviors in the distribution process.

The basic methodology proposed by Chen [27] can be well applied to the analysis of an information asymmetric cooperative game with agreements self-implemented: the formation of the coalition equilibrium is the result of the choices of players who pursue the maximization of their expected cooperative payoff distributions, and the equilibrium of the bargaining game of a coalition on the cooperative payoff distribution can be obtained by applying the distribution rule of common payoff. In this section, we'll examine the condition for the existence of the coalition equilibrium of the game and define the coalition equilibrium when it does exist. Meanwhile, we'll divide the actual cooperative payoff of a coalition into two parts: the cooperative payoff of the coalition when all core members' judgments of the strategic combination of their coalition are all correct, and the cooperative payoff of the coalition caused by the misjudgment "cooperation" between the coalition members. When members are unallied in the bargaining game, the actual cooperative payoff distribution obtained by a coalition member is the sum of the distributions he obtains in the distribution process of these two cooperative payoffs mentioned above. In Section 4, we will discuss the situation when coalition members are allied in the bargaining games.

3.1 *Unallied bargaining game and the distribution of the cooperative payoff*

If information is still asymmetric after the cooperative game with agreements self-implemented is completed, the Nash equilibrium of the unallied bargaining game of a coalition does not exist, thus the coalition equilibrium of the information asymmetric cooperative game with agreements self-implemented does not exist. Therefore, in this section we only discuss the distribution of the cooperative payoff of a coalition when information is symmetric after the cooperative game with agreements self-implemented is completed. Here, we assume that members of each coalition are unallied in the bargaining game of the coalition.

Assume that in the coalition equilibrium of player k_1's virtual game, the equilibrium strategic combination that his escape target core coalition "should" adopt is $s^{*(k_1)}_{C_T^{c(k_1)}}$. Of course, this is not to say that core coalition $C_T^{c(k_1)}$ will take this strategic combination as its choice. In fact, because the extensive members of the coalition have different estimations of the "correct" or "should be adopted" equilibrium strategic combination of the core coalition, the extensive members will decide the strategic combination of the coalition through a public choice game. The public choice of strategic combination of the extensive coalition is the result of the compromise of the extensive members of the coalition. In fact, in coalition equilibrium $c^{*(k_1)}$ of the virtual game of player k_1, according to his information set, the public choice of strategic combination of core coalition $C_T^{c(k_1)}$ should be:

$$s^{**(k_1)}_{C_T^{c(k_1)}} \subseteq s^{**(k_1)}_{C_T^{+(k_1)}};$$

$$s^{**(k_1)}_{C^{c(k_1)}_T} = argmax \sum_{i \in C^{c(k_1)}_T} V^{(k_1,\, i)}_{C^{c(k_1,\, i)}_T} \left(s^{(k_1,\, i)}_{C^{c(k_1,\, i)}_T},\, s^{*(k_1, i)}_{-C^{c(k_1,\, i)}_T}\right)$$

$$+ \sum_{\substack{j \in C^{+(k_1)}_T \\ j \notin C^{c(k_1)}_T}} V^{\left(k_1,\, j^F\right)}_{C^{c(k_1,\, j)}_h(\cdot)} \left(s^{\left(k_1,\, j^F\right)}_{C^{c(k_1,\, j)}_h(\cdot)},\, s^{**\left(k_1,\, j^F\right)}_{C^{c(k_1,\, j)}_T(\cdot)},\, s^{**\left(k_1,\, j^F\right)}_{-C^{c(k_1,\, j)}_h(\cdot),\, -C^{c(k_1,\, j)}_T(\cdot)}\right)$$

$$= argmax \left\{ \sum_{i \in C^{c(k_1)}_T} V^{(k_1,\, i)}_{C^{c(k_1,\, i)}_T} \left(s^{(k_1,\, i)}_{C^{c(k_1,\, i)}_T},\, s^{*(k_1,\, i)}_{-C^{c(k_1,\, i)}_T}\right)\right.$$

$$\left. + \sum_{\substack{j \in C^{+(k_1)}_T \\ j \notin C^{c(k_1)}_T}} V^{\left(k_1,\, j^F\right)}_{C^{c(k_1,\, j)}_h(\cdot)} \left(s^{\left(k_1,\, j^F\right)}_{C^{c(k_1,\, j)}_h(\cdot)},\, i^{\left(k_1,\, j^F\right)}_j\right) \right\},\ i^{\left(k_1,\, j^F\right)}_j \in I^{\left(k_1,\, j^F\right)}_j;$$

where $C^{c(k_1,\, j)}_h = C^{c(k_1)}_T \neq C^{c(k_1,\, j)}_T$.

According to player k_1's information set, core coalition $C^{c(k_1)}_T$ "should" adopt strategic combination $s^{*(k_1)}_{C^{c(k_1)}_T}$. Under this strategic combination, the expected cooperative payoff that core coalition $C^{c(k_1)}_T$ "should" obtain is $V^{*(k_1)}_{C^{c(k_1)}_T}\left(s^{*(k_1)}_{C^{c(k_1)}_T},\, s^{**(k_1)}_{-C^{c(k_1)}_T}\right)$. According to player k_1's information set, how should this expected cooperative payoff be distributed?

Assume that all the members of each coalition are responsible for their own misjudgments, and that the above assumption is common knowledge of all the players, according to player k_1's information set, the cooperative payoff $V^{*(k_1)}_{C^{c(k_1)}_T}\left(s^{*(k_1)}_{C^{c(k_1)}_T},\, s^{**(k_1)}_{-C^{c(k_1)}_T}\right)$ of the core coalition "should" be distributed in accordance with the distribution rule of cooperative payoff of the coalition when its members are unallied in the bargaining game (Chen [25]).

Assume that there are m members in the member set of core coalition $C^{c(k_1)}_T$, member set $M_{q_1,\, q_2,\, \cdots,\, q_k}$ composed of members $q_1,\ q_2,\ \cdots,\ q_k$ is a subset of the coalition member set M of core coalition $C^{c(k_1)}_T$, $M_{q_1,\, q_2,\, \cdots,\, q_k} \subseteq M (k \leq m)$, $\theta^{(k_1)}(M_{q_1,\, q_2,\, \cdots,\, q_k})$ is called the common payoff of member set $M_{q_1,\, q_2,\, \cdots,\, q_k}$ in the virtual game of player k_1:

$$\theta^{(k_1)}(M_{q_1,\, q_2,\, \cdots,\, q_k}) = V^{(k_1)}_{M_{q_1,\, q_2,\, \cdots,\, q_k}} - \sum_{i=1}^{k} W^{-C^{c(k_1)}_T(k_1)}_{q_i} - \sum \theta^{(k_1)}_{(2)}(M_{q_1,\, q_2,\, \cdots,\, q_k}) - \cdots - \sum \theta^{(k_1)}_{(k-1)}(M_{q_1,\, q_2,\, \cdots,\, q_k}),$$

where $V^{(k_1)}_{M_{q_1,\, q_2,\, \cdots,\, q_k}}$ is player k_1's estimation of the cooperative payoff of coalition $C^{c(k_1)}_T$ when all the members except those in member set $M_{q_1,\, q_2,\, \cdots,\, q_k}$ escape from the coalition and join the same coalition as a whole to maximize their escape-payoff, while members of other coalitions keep their coalition-choosing strategies unchanged, $\sum \theta^{(k_1)}_{(j)}(M_{q_1,\, q_2,\, \cdots,\, q_k})$ is the sum of the common payoffs of all the j-member subsets of member set $M_{q_1,\, q_2,\, \cdots,\, q_k}$, $\sum_{i=1}^{k} W^{-C^{c(k_1)}_T(k_1)}_{q_i}$ is the sum of the escape-payoffs deriving from deviation of all the members in member set $M_{q_1,\, q_2,\, \cdots,\, q_k}$.

In the virtual game of player k_1, in coalition equilibrium $C^{*(k_1)}$, if the strategic combination that core coalition $C_T^{c(k_1)}$ "should" adopt is $s^{*(k_1)}_{C_T^{c(k_1)}}$ and the strategic combination choices of other core coalitions are $s^{**(k_1)}_{-C_T^{c(k_1)}}$, the expected cooperative payoff distribution of any member q_i of core coalition $C_T^{c(k_1)}$ is:

$$\widetilde{x}_{q_i}^{(k_1)}\left(s^{*(k_1)}_{C_T^{c(k_1)}},\ s^{**(k_1)}_{-C_T^{c(k_1)}}\right) = W_{q_i}^{-C_T^c(k_1)}$$

$$+\frac{1}{2}\sum_{\substack{q_j=1\\ q_j\neq q_i}}^{m}\theta^{(k_1)}(M_{q_i,\, q_j})+\frac{1}{3}\sum_{\substack{q_j=1\\ q_j\neq q_i}}^{m}\sum_{\substack{q_k=1\\ q_k\neq q_i}}^{q_j-1}\theta^{(k_1)}(M_{q_i,\, q_j,\, q_k})+\cdots+\frac{1}{m}\theta^{(k_1)}(M_{1,\, 2,\, \cdots,\, m}).$$

According to the virtual game of player k_1, in coalition equilibrium $C^{*(k_1)}$, when the cooperative payoff $V^{*(k_1)}_{C_T^{c(k_1)}}\left(s^{*(k_1)}_{C_T^{c(k_1)}},\ s^{**(k_1)}_{-C_T^{c(k_1)}}\right)$ of core coalition $C_T^{c(k_1)}$ is distributed according to the rule mentioned above, the expected cooperative payoff distribution that player k_1 gets is:

$$\widetilde{x}_{k_1}^{(k_1)}\left(s^{*(k_1)}_{C_T^{c(k_1)}},\ s^{**(k_1)}_{-C_T^{c(k_1)}}\right) = W_{k_1}^{-C_T^c(k_1)}$$

$$+\frac{1}{2}\sum_{\substack{q_j=1\\ q_j\neq k_1}}^{m}\theta^{(k_1)}(M_{k_1,\, q_j})+\frac{1}{3}\sum_{\substack{q_j=1\\ q_j\neq k_1}}^{m}\sum_{\substack{q_k=1\\ q_k\neq k_1}}^{q_j-1}\theta^{(k_1)}(M_{k_1,\, q_j,\, q_k})+\cdots+\frac{1}{m}\theta^{(k_1)}(M_{1,\, 2,\, \cdots,\, m}).$$

Next, we will examine the distribution of the actual cooperative payoff of core coalition C_T^c. Assume that the strategic combination adopted by core coalition C_T^c is $s^{**}_{C_T^c}\left(s^{**}_{C_T^c}\subseteq s^{**}_{C_T^+}\right)$ is determined by the public choice game among the extensive members of the coalition, and that the actual strategic combination choices of other core coalitions are $s^{**}_{-C_T^c}$, the actual cooperative payoff that core coalition C_T^c gets is:

$$V_{C_T^c}\left(s^{**}_{C_T^c},\ s^{**}_{-C_T^c}\right)=\sum_{i=1}^{m^*}u_i\left(s^{**}_{C_T^c},\ s^{**}_{-C_T^c}\right),$$

where m^* is the actual number of the members of core coalition C_T^c. Cooperative payoff $V_{C_T^c}\left(s^{**}_{C_T^c},\ s^{**}_{-C_T^c}\right)=\sum_{i=1}^{m^*}u_i\left(s^{**}_{C_T^c},\ s^{**}_{-C_T^c}\right)$ is core coalition C_T^c's actual cooperative payoff that can ultimately be distributed. It does not necessarily equal the sum of the cooperative payoff distribution expected by the members of the core coalition. Obviously, due to information asymmetry, the public choice of strategic combination of core coalition C_T^c is not the optimal response to the strategic combination choices $s^{**}_{-C_T^c}$ of other core coalitions.

If the estimations of all the core members of core coalition C_T^c of the optimal strategic combination choice of their core coalition are correct, the maximum cooperative payoff that this core coalition can obtain is:

$$maxV_{C_T^c}\left(s_{C_T^c},\ s^{**}_{-C_T^c}\right) = max\sum_{i=1}^{m^*} u_i\left(s_{C_T^c},\ s^{**}_{-C_T^c}\right),$$

$$s^*_{C_T^c} = ^{argmax}_{s_{C_T^c}}\ V_{C_T^c}\left(s_{C_T^c},\ s^{**}_{-C_T^c}\right) = ^{argmax}_{s_{C_T^c}}\ \sum_{i=1}^{m^*} u_i\left(s_{C_T^c},\ s^{**}_{-C_T^c}\right).$$

The cooperative payoff surplus,

$$Gap_{C_T^c} = V_{C_T^c}\left(s^{**}_{C_T^c},\ s^{**}_{-C_T^c}\right) - V_{C_T^c}\left(s^*_{C_T^c},\ s^{**}_{-C_T^c}\right) \leq 0,$$

is caused by the inappropriate public choice of strategic combination of the core coalition. And the inappropriate public choice of strategic combination of the core coalition is caused by the coalition members' inappropriate public choice of the strategic combination of the core coalition.

If the estimations of all the core members of core coalition C_T^c of the optimal strategic combination choice of their core coalition are correct, in coalition equilibrium c^*, when the public choice of strategic combination of core coalition C_T^c is actually optimal response to the actual strategic combination choices of other core coalitions, that is, $s^{**}_{C_T^c} = s^*_{C_T^c}$, the cooperative payoff of core coalition C_T^c satisfies

$$V_{C_T^c}\left(s^{**}_{C_T^c},\ s^{**}_{-C_T^c}\right) = V_{C_T^c}\left(s^*_{C_T^c},\ s^{**}_{-C_T^c}\right).$$

Assume that after the game is completed, there is no longer information asymmetry among the players, and that the above assumption is common knowledge of all the players, if the estimations of all the core members of core coalition C_T^c of the optimal strategic combination choice of their core coalition are correct, the distribution of the cooperative payoff of core coalition C_T^c will be carried out according to the distribution rule in the unallied bargaining game of this core coalition (Chen [26]):

$$\widetilde{x}^{**}_{k_1}(s^*_{C_T^c},\ s^{**}_{-C_T^c}) = W_{k_1}^{**-C_T^c} + \frac{1}{2}\sum_{\substack{q_j=1\\ q_j\neq k_1}}^{m^*} \theta^{**}(M_{k_1,\ q_j}) + \frac{1}{3}\sum_{\substack{q_j=1\\ q_j\neq k_1}}^{m^*}\ \sum_{\substack{q_k=1\\ q_k\neq k_1}}^{q_j-1} \theta^{**}(M_{k_1,\ q_j,\ q_k}) + \cdots + \frac{1}{m^*}\theta^{**}(M_{1,\ 2,\ \cdots,\ m^*}).$$

Core coalition C_T^c's cooperative payoff $V_{C_T^c}\left(s^{**}_{C_T^c},\ s^{**}_{-C_T^c}\right)$ can be regarded as the "cooperation" outcome starting from strategic combination $s^*_{C_T^c}$ and through the misjudgments of the public choices of strategic combination of other coalitions. The distribution of the cooperative payoff $V_{C_T^c}\left(s^{**}_{C_T^c},\ s^{**}_{-C_T^c}\right)$ should be based on the distribution of the cooperative payoff $V_{C_T^c}\left(s^*_{C_T^c},\ s^{**}_{-C_T^c}\right)$, with an additional distribution of the "cooperative" payoff,

$$Gap_{C_T^c} = V_{C_T^c}\left(s^{**}_{C_T^c},\ s^{**}_{-C_T^c}\right) - V_{C_T^c}\left(s^*_{C_T^c},\ s^{**}_{-C_T^c}\right),$$

which is caused by the misjudgment cooperation among the core members of core coalition C_T^c.

For any member k_1 of core coalition C_T^c, when he escapes from "cooperation" state $s_{C_T^c}^{**}$ to state $s_{C_T^c}^{*}$, that is, if member k_1 misjudges but the other members still keep their judgments correct, the loss of the cooperative payoff of core coalition C_T^c shall obviously be the responsibility of member k_1, which is member k_1's escape-payoff. When member k_1 escapes, core coalition C_T^c's strategic combination choice is:

$$s_{C_T^c}^{*}(EM_{k_1}) = argmax\left\{\sum_{\substack{q_1=1\\ q_1\neq k_1}}^{m^*} V_{C_T^c}\left(s_{C_T^c},\ s_{-C_T^c}^{**}\right) + V_{C_T^c}\left(s_{C_T^c},\ s_{-C_T^c}^{**(k_1)}\right)\right\}$$

$$= argmax\left\{(m^*-1)V_{C_T^c}\left(s_{C_T^c},\ s_{-C_T^c}^{**}\right) + V_{C_T^c}\left(s_{C_T^c},\ s_{-C_T^c}^{**(k_1)}\right)\right\},$$

where $s_{C_T^c}^{*}(EM_{k_1})$ stands for core coalition C_T^c's strategic combination choice after member k_1 escapes. The escape-payoff of member k_1 is:

$$W_{k_1}^{*} = V_{C_T^c}\left[s_{C_T^c}^{*}\left(EM_{k_1}\right),\ s_{-C_T^c}^{**}\right] - V_{C_T^c}\left(s_{C_T^c}^{*},\ s_{-C_T^c}^{**(k_1)}\right).$$

If members k_1 and k_2 escape at the same time, core coalition C_T^c's strategic combination choice is:

$$s_{C_T^c}^{*}(EM_{k_1,\,k_2}) = argmax\left\{\sum_{\substack{q_1=1\\ q_1\neq k_1,\,k_2}}^{m^*} V_{C_T^c}\left(s_{C_T^c},\ s_{-C_T^c}^{**}\right) + V_{C_T^c}\left(s_{C_T^c},\ s_{-C_T^c}^{**(k_1)}\right) + V_{C_T^c}\left(s_{C_T^c},\ s_{-C_T^c}^{**(k_2)}\right)\right\}$$

$$= argmax\left\{(m^*-2)V_{C_T^c}\left(s_{C_T^c},\ s_{-C_T^c}^{**}\right) + V_{C_T^c}\left(s_{C_T^c},\ s_{-C_T^c}^{**(k_1)}\right) + V_{C_T^c}\left(s_{C_T^c},\ s_{-C_T^c}^{**(k_2)}\right)\right\}.$$

The common payoff of members k_1 and k_2 is:

$$\delta^{*}\left(M_{k_1,\,k_2}\right) = V_{C_T^c}\left[s_{C_T^c}^{*}\left(EM_{k_1,\,k_2}\right),\ s_{-C_T^c}^{**}\right] - V_{C_T^c}\left(s_{C_T^c}^{*},\ s_{-C_T^c}^{**}\right) - W_{k_1}^{*} - W_{k_2}^{*}.$$

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When all the members in member set $M_{k_1,\,k_2,\,\cdots,\,k_i}$ escape, core coalition C_T^c's strategic combination choice is:

$$s_{C_T^c}^{*}(EM_{k_1,\,k_2,\,\cdots,\,k_i}) = argmax\left\{\sum_{\substack{q_1=1\\ q_1\neq k_1,\,k_2,\,\cdots,\,k_i}}^{m^*} V_{C_T^c}\left(s_{C_T^c},\ s_{-C_T^c}^{**}\right) + \sum_{r_1\neq k_1}^{k_i} V_{C_T^c}\left(s_{C_T^c},\ s_{-C_T^c}^{**(r_1)}\right)\right\}$$

$$= argmax\left\{(m^*-i)V_{C_T^c}\left(s_{C_T^c},\ s_{-C_T^c}^{**}\right) + \sum_{r_1\neq k_1}^{k_i} V_{C_T^c}\left(s_{C_T^c},\ s_{-C_T^c}^{**(r_1)}\right)\right\}.$$

The common payoff of member set $M_{k_1,\, k_2,\, \cdots,\, k_i}$ is:

$$\delta^*\left(M_{k_1,\, k_2,\, \cdots,\, k_i}\right) = V_{C_T^c}\left[s^*_{C_T^c}\left(EM_{k_1,\, k_2,\, \cdots,\, k_i}\right),\, s^{**}_{-C_T^c}\right] - V_{C_T^c}(s^*_{C_T^c},\, s^{**}_{-C_T^c})$$

$$-\sum_{j=1}^{i} W^*_{k_j} - \sum \delta_{(2)}(M_{k_1,\, k_2,\, \cdots,\, k_i}) - \cdots - \sum \delta_{(k-1)}(M_{k_1,\, k_2,\, \cdots,\, k_i}),$$

where $\sum_{j=1}^{i} W^*_{k_j}$ is the sum of the escape-payoffs of all the members in member set $M_{k_1,\, k_2,\, \cdots,\, k_i}$, $\sum \delta_{(j)}(M_{k_1,\, k_2,\, \cdots,\, k_i})$ is the sum of the common payoffs of all the j-member subsets of member set $M_{k_1,\, k_2,\, \cdots,\, k_i}$.

The cooperative payoff distribution that member k_1 gets from the misjudgment "cooperation" is:

$$\widetilde{x}^*_{k_1} = W^*_{k_1} + \frac{1}{2}\sum_{\substack{q_j=1 \\ q_j \neq k_1}}^{m^*} \delta^*(M_{k_1,\, q_j}) + \frac{1}{3}\sum_{\substack{q_j=1 \\ q_j \neq k_1}}^{m^*} \sum_{\substack{q_k=1 \\ q_k \neq k_1}}^{q_j-1} \delta^*(M_{k_1,\, q_j,\, q_k}) + \cdots + \frac{1}{m^*}\delta^*(M_{1,\, 2,\, \cdots,\, m^*}).$$

Herein, in the misjudgment "cooperation", the members of core coalition C_T^c cannot escape through deviation. Therefore, in the above-mentioned distribution process, player k_1's reservation distribution is his escape payoff $W^*_{k_i}$ in the misjudgment "cooperation".

And the actual total cooperative payoff distribution that member k_1 gets is:

$$x_{k_1} = \widetilde{x}^*_{k_1} + \widetilde{x}^{**}_{k_1}.$$

If in the bargaining game on the distribution of the cooperative payoff $V_{C_T^c}(s^{**}_{C_T^c},\, s^{**}_{-C_T^c})$, information is still asymmetric among the coalition members, the Nash equilibrium of the bargaining game on the distribution of the cooperative payoff does not exist, because the Nash equilibria of the virtual bargaining games of the members are different.

Similarly, in the bargaining game on the distribution of the "cooperative" payoff,

$$Gap_{C_T^c} = V_{C_T^c}\left(s^{**}_{C_T^c},\, s^{**}_{-C_T^c}\right) - V_{C_T^c}(s^*_{C_T^c},\, s^{**}_{-C_T^c}),$$

caused by the coalition members' misjudgment "cooperation", if information is still asymmetric after the game is completed, the core members of core coalition C_T^c cannot reach the Nash equilibrium of the bargaining game on the distribution of the "cooperative" payoff

$$Gap_{C_T^c} = V_{C_T^c}\left(s^{**}_{C_T^c},\, s^{**}_{-C_T^c}\right) - V_{C_T^c}(s^*_{C_T^c},\, s^{**}_{-C_T^c}).$$

3.2 *Unallied bargaining game and the coalition equilibrium of the information asymmetric cooperative game with agreements self-implemented*

After investigating the virtual games of the players, now we can define and give the existence proof of the coalition equilibrium of an information asymmetric cooperative game with agreements self-implemented, when information is symmetric after the game is completed. It is easy to reach the conclusions in Theorems 3 and 4.

Theorem 3 Ignoring the opportunistic behaviors of coalition members in the distribution process of cooperative payoff, in information asymmetric cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented, assume that information is still asymmetric after the game is completed, and that the above assumption is common knowledge of all the players, there exists no Nash equilibrium in the unallied bargaining game of any coalition C. At the same time, there exists no coalition equilibrium in information asymmetric cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented under the criterion of maximum expected cooperative payoff distribution (when the members of each coalition trust each other).

Proof. The equilibrium of information asymmetric cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented includes two interrelated aspects: the coalition equilibrium of the game and the distribution equilibrium of the cooperative payoff of each coalition.

If information is still asymmetric after the game is completed, in the bargaining game on the distribution of the cooperative payoff of a coalition, members will present their requirements for cooperative payoff distribution on the basis of their virtual games, and the sum of their requirements for cooperative payoff distribution do not necessarily equal the actual cooperative payoff received by the coalition. That is to say, the distribution equilibrium of the cooperative payoff of the coalition cannot be achieved; on the other hand, if the distribution equilibrium of cooperative payoff between the coalition members cannot be achieved, the coalition equilibrium of the game cannot be reached either. □

Herein, there exists no coalition equilibrium under the criterion of maximum expected cooperative payoff distribution (when the members of each coalition trust each other) in information asymmetric cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented, or, there exists no coalition equilibrium under the criterion of minimum expected escape-payoff deriving from deviation (when the members of each coalition trust each other) in the information asymmetric cooperative game with agreements self-implemented does not mean that there is no cooperative coalition in the game. Some players with a high degree of information symmetry (after the completion of the cooperative game) may still establish cooperative coalitions which aim at exploiting the synergies among them, and reach cooperative payoff distribution agreements with some kinds of compensation mechanisms. In addition, even if the degree of information asymmetry among the players is still high after the completion of the cooperative game, those who agree with each other on the synergy expectations and do not need distribution compensations (perhaps they can set up some compensation mechanism to benefit from their cooperation) may also reach some forms of distribution agreements and establish cooperative coalitions designed to take advantage of the synergy expectations among them.

When information among the players is still asymmetric after the completion of the cooperative game, in information asymmetric cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented, there is at least a coalition situation shown as follows which is feasible.

Theorem 4 Ignoring the opportunistic behaviors of coalition members in the distribution process of cooperative payoff, in information asymmetric cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented, assume that information is still asymmetric after the game is completed, and that the above assumption is common knowledge of all the players, if in the cooperative game there exists no compensation mechanism (or, the distribution of any member of a coalition is just the payoff that he gets in the game), the following coalition situation under the criterion of maximum expected payoff (when the members of each coalition trust each other) is feasible:

$$c_i^* = \begin{cases} i, \ \text{if for any } c_i \neq i, \ u_i^{(i)}(i,\ c_{-i}^*) \geq u_i^{(i)}(c_i,\ c_{-i}^*), \ \text{or}, \ u_i^{(j)}(c_i,\ c_{-i}^*) - W_i^{-C_{c_i}(j)} \leq 0, \\ \text{or}, \ u_j^{(i)}(c_i,\ c_{-i}^*) \leq W_j^{-C_{c_i}(i)} (j \in C_{c_i},\ j \neq i); \\ \arg\max_{c_i} u_i^{(i)}(c_i,\ c_{-i}^*), \ \text{if at least for a certain } c_i \neq i, \ u_i^{(i)}(i,\ c_{-i}^*) < u_i^{(i)}(c_i,\ c_{-i}^*), \ u_i^{(j)}(c_i,\ c_{-i}^*) - W_i^{-C_{c_i}(j)} > 0, \\ \text{and } u_j^{(i)}(c_i,\ c_{-i}^*) > W_j^{-C_{c_i}(i)} (j \in C_{c_i},\ j \neq i). \end{cases}$$

The proof of Theorem 4 is omitted.

Assume that information is symmetric after the game is completed, in information asymmetric cooperative game $\Gamma(N,\ \{S_i\},\ \{u_i\})$ with agreements self-implemented, there exists the mixed strategic coalition equilibrium under the criterion of maximum expected cooperative payoff distribution.

Theorem 5 Ignoring the opportunistic behaviors of coalition members in the distribution process of cooperative payoff, in information asymmetric cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented, assume that information is symmetric after the game is completed, and that the above assumption is common knowledge of all the players, there exists the mixed strategic coalition equilibrium under the criterion of maximum expected cooperative payoff distribution (when the members of each coalition trust each other):

$$c_i^* = \begin{cases} i, \ \text{if for any } c_i \neq i, \ \widetilde{x}_i^{(i)}(i,\ c_{-i}^*) \geq \widetilde{x}_i^{(i)}(c_i,\ c_{-i}^*), \ \text{or}, \ \sum\limits_{\substack{j \in C_{c_i} \\ j \neq i}} \left[Mv_i^{(j)}(C_{c_i}) - W_i^{-C_{c_i}(j)} \right] \leq 0, \\ \text{or}, \ \sum\limits_{\substack{j \in C_{c_i} \\ j \neq i}} \left[Mv_{T_h}^{(j)}(C_{c_i}) - \sum\limits_{k \in T_h} W_k^{-C_{c_i}(j)} \right] \leq 0 (i \in T_h \subseteq C_{c_i}); \\ \arg\max_{c_i} \widetilde{x}_i^{(i)}(c_i,\ c_{-i}^*), \ \text{if at least for } a \text{ certain } c_i \neq i, \ \widetilde{x}_i^{(i)}(i,\ c_{-i}^*) < \widetilde{x}_i^{(i)}(c_i,\ c_{-i}^*), \\ \sum\limits_{\substack{j \in C_{c_i} \\ j \neq i}} \left[Mv_i^{(j)}(C_{c_i}) - W_i^{-C_{c_i}(j)} \right] > 0, \ \text{and} \ \sum\limits_{\substack{j \in C_{c_i} \\ j \neq i}} \left[Mv_{T_h}^{(j)}(C_{c_i}) - \sum\limits_{k \in T_h} W_k^{-C_{c_i}(j)} \right] > 0 (i \in T_h \subseteq C_{c_i}). \end{cases}$$

Proof. If in coalition situation c, any player i has synergy with the coalition he belongs to, player i and each member set he belongs to are trusted by his coalition, then coalition situation c is feasible; if in coalition situation c, some player i has no synergy with the coalition he belongs to, or, he or some member set he belongs to is not trusted by his coalition, coalition situation c is infeasible.

Assume that C is the feasible situation set. First and foremost, C must not be an empty set because at least the coalition situation in which each player chooses to create a 1-person coalition and solely participates in the game is feasible.

In information asymmetric cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented, when the opportunistic behaviors in the distribution process are ignored, all the players must make consistent decisions by negotiations on choosing coalition situations. Therefore, in the virtual game of any player i, the probability with which each player chooses some coalition situation in the mixed-strategic coalition equilibrium must be the same. Assume that all the players select coalition situations in vector $(C_1, C_2, \cdots, C_M)$ (where M is the number of feasible coalition situations) with the same probability vector $p = (p_1, p_2, \cdots, p_M)$, the decision-making problem of player i is $\Psi(N, p)$:

$$\widetilde{x}_i^{(i)}(p^*) = \max_{p \in P} \widetilde{x}_i^{(i)}(p), \ \forall i \in N.$$

It can be further expressed as

$$\max_p \widetilde{x}_i^{(i)}(p) = \max_p \sum_{I=1}^{M} p_I \widetilde{x}_i^{(i)}(c_I).$$

$$s.t. \ \max_p \widetilde{x}_{-i}^{(-i)}(p) = \max_p \sum_{I=1}^{M} p_I \widetilde{x}_{-i}^{(-i)}(c_I);$$

$$p = (p_1, p_2, \cdots, p_M) \in P, \quad (p_1, p_2, \cdots, p_M) | p_I(c_I) \geq 0, I = 1, 2, \cdots, M;$$

$$\sum_{I=1}^{M} p_I = 1.$$

If the decision-making problem above of any player i has an optimal solution p^*, p^* is the mixed-strategic coalition situation in information asymmetric cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented when the opportunistic behaviors in the distribution process are ignored.

According to Kakutani Fixed Point Theorem, it's easy to prove that the decision-making problem above has an optimal solution, but p^* may not be one and only. □

We can also draw the conclusion in Theorem 6.

Theorem 6 Ignoring the opportunistic behaviors of coalition members in the distribution process of cooperative payoff, in information asymmetric cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented, assume that information is symmetric after the game is completed, and that the above assumption is common knowledge of all the players, there exists the mixed strategic coalition equilibrium under the criterion of minimum expected escape-payoff deriving from deviation (when the members of each coalition trust each other):

$$c_i^* = \begin{cases} i, \ if \ for \ any \ c_i \neq i, \ W_i^{-C_i(i)}(i, \ c_{-i}^*) \leq W_i^{-C_{c_i}(i)}(c_i, \ c_{-i}^*), \ or, \ \sum\limits_{\substack{j \in C_{c_i} \\ j \neq i}} \left[Mv_i^{(j)}(C_{c_i}) - W_i^{-C_{c_i}(j)} \right] \leq 0, \\ or, \ \sum\limits_{\substack{j \in C_{c_i} \\ j \neq i}} \left[Mv_{T_h}^{(j)}(C_{c_i}) - \sum\limits_{k \in T_h} W_k^{-C_{c_i}(j)} \right] \leq 0 (i \in T_h \subseteq C_{c_i}); \\ \arg\min\limits_{c_i} W_i^{-C_{c_i}(i)}(c_i, \ c_{-i}^*), \ \ if \ at \ least \ for \ a \ certain \ c_i \neq i, \ \ W_i^{-C_i(i)}(i, \ c_{-i}^*) > W_i^{-C_{c_i}(i)}(c_i, \ c_{-i}^*), \\ \sum\limits_{\substack{j \in C_{c_i} \\ j \neq i}} \left[Mv_i^{(j)}(C_{c_i}) - W_i^{-C_{c_i}(j)} \right] > 0, \ \text{and} \ \sum\limits_{\substack{j \in C_{c_i} \\ j \neq i}} \left[Mv_{T_h}^{(j)}(C_{c_i}) - \sum\limits_{k \in T_h} W_k^{-C_{c_i}(j)} \right] > 0 (i \in T_h \subseteq C_{c_i}). \end{cases}$$

If the distribution scheme of each coalition meets the competitive distribution condition, that is, the distribution of any member is no less than his escape payoff deriving from deviation and no more than his contribution to the coalition, the coalition equilibrium under the criterion of minimum expected escape-payoff deriving from deviation (when the members of each coalition trust each other) is equivalent to the one under the criterion of maximum expected cooperative payoff distribution (when the members of each coalition trust each other).

Proof. According to an analysis similar to the proof of Theorem 5, it is easy to prove that ignoring the opportunistic behaviors of coalition members in the distribution process of cooperative payoff, in information asymmetric cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented, assume that information is symmetric after the game is completed, and that the above assumption is common knowledge of all the players, there exists the mixed strategic coalition equilibrium under the criterion of minimum expected escape-payoff deriving from deviation (when the members of each coalition trust each other). Next, we will prove that the above coalition equilibrium is equivalent to the one under the criterion of minimum expected escape-payoff deriving from deviation (when the members of each coalition trust each other).

Ignoring the opportunistic behaviors of coalition members in the distribution process of cooperative payoff, when competitive distribution condition is satisfied, obviously,

$$c_i^* = i, \text{ if for any } c_i \neq i, \ \widetilde{x}_i^{(i)}(i,\ c_{-i}^*) \geq \widetilde{x}_i^{(i)}(c_i,\ c_{-i}^*), \text{ or,}$$

$$\sum_{\substack{j\in C_{c_i}\\ j\neq i}} \left[Mv_i^{(j)}(C_{c_i}) - W_i^{-C_{c_i}(j)}\right] \leq 0, \text{ or, } \sum_{\substack{j\in C_{c_i}\\ j\neq i}} \left[Mv_{T_h}^{(j)}(C_{c_i}) - \sum_{k\in T_h} W_k^{-C_{c_i}(j)}\right] \leq 0 (i \in T_h \subseteq C_{c_i}).$$

$$\Leftrightarrow c_i^* = i, \text{ if for any } c_i \neq i, \ W_i^{-C_i(i)}(i,\ c_{-i}^*) \leq W_i^{-C_{c_i}(i)}(c_i,\ c_{-i}^*), \text{ or, } \sum_{\substack{j\in C_{c_i}\\ j\neq i}} \left[Mv_i^{(j)}(C_{c_i}) - W_i^{-C_{c_i}(j)}\right] \leq 0, \text{ or,}$$

$$\sum_{\substack{j\in C_{c_i}\\ j\neq i}} \left[Mv_{T_h}^{(j)}(C_{c_i}) - \sum_{k\in T_h} W_k^{-C_{c_i}(j)}\right] \leq 0 (i \in T_h \subseteq C_{c_i}).$$

To prove Theorem 6, what we need to do is to prove that when the agreements are self-implemented,

$$c_i^* = \arg\min W_i^{-C_{c_i}(i)}(c_i,\ c_{-i}^*) \Leftrightarrow c_i^* = \arg\max \widetilde{x}_i^{(i)}(c_i,\ c_{-i}^*).$$

That is, when some player i minimizes his escape-payoff deriving from deviation in the coalition equilibrium under the principle of minimum expected escape-payoff deriving from deviation, he can get the highest expected cooperative payoff distribution. Meanwhile, when some player i maximizes his expected cooperative payoff distribution in the coalition equilibrium under the principle of expected maximum distribution (when the members of each coalition trust each other), he can get the lowest expected escape-payoff deriving from deviation.

(1) If $c_i^* = \arg\max\ \widetilde{x}_i^{(i)}(c_i,\ c_{-i}^*)$, coalition situation $(c_i^*,\ c_{-i}^*)$ is the coalition equilibrium under the principle of maximum expected distribution (when the members of each coalition trust each other). In this coalition equilibrium of the virtual game of player i, the distribution $\widetilde{x}_i^{(i)}\left(c_i^*,\ c_{-i}^*\right)$ satisfies the competitive distribution condition:

$$Mv_i^{(i)}(C_{c_i^*})(c_i^*,\ c_{-i}^*) \geq \widetilde{x}_i^{(i)}(c_i^*,\ c_{-i}^*) \geq W_i^{-C_{c_i^*}(i)}(c_i^*,\ c_{-i}^*).$$

Assume that the coalition-choosing strategy of player i is $c_i \neq c_i^*$, according to the definition of coalition equilibrium under the principle of maximum expected distribution (when the members of each coalition trust each other), $\widetilde{x}_i^{(i)}\left(c_i,\ c_{-i}^*\right) \leq \widetilde{x}_i^{(i)}\left(c_i^*,\ c_{-i}^*\right)$. Therefore, player i has the motivation to betray coalition C_{c_i} and join the corresponding coalition $C_{c_i^*}$ in situation $\left(c_i,\ c_{-i}^*\right)$ through deviation. When player i withdraws from coalition C_{c_i} through deviation and join coalition $C_{c_i^*}$, his expected escape-payoff deriving from deviation satisfies:

$$Mv_i^{(i)}(C_{c_i^*}) \leq W_i^{-C_{c_i}(i)}(c_i,\ c_{-i}^*).$$

Coalition $C_{c_i^*}$ wouldn't give a distribution more than $Mv_i^{(i)}(C_{c_i^*})$ to player i, because $Mv_i^{(i)}(C_{c_i^*})$ is the marginal contribution of player i to coalition $C_{c_i^*}$ through his deviation. According to the competitive distribution condition, we have:

$$\widetilde{x}_i^{(i)}(c_i^*,\ c_{-i}^*) \le Mv_i^{(i)}(C_{c_i^*}).$$

Therefore,

$$W_i^{-C_{c_i^*}(i)}(c_i^*,\ c_{-i}^*) \le \widetilde{x}_i^{(i)}(c_i^*,\ c_{-i}^*) \le W_i^{-C_{c_i}(i)}(c_i,\ c_{-i}^*).$$

So,

$$W_i^{-C_{c_i^*}(i)}(c_i^*,\ c_{-i}^*) \le W_i^{-C_{c_i}(i)}(c_i,\ c_{-i}^*),$$

$$c_i^* = argmax\widetilde{x}_i^{(i)}(c_i,\ c_{-i}^*) \Rightarrow argminW_i^{-C_{c_i}(i)}(c_i,\ c_{-i}^*).$$

(2) If $c_i^* = argminW_i^{-C_{c_i}(i)}(c_i,\ c_{-i}^*)$, coalition situation $(c_i^*,\ c_{-i}^*)$ is the coalition equilibrium under the principle of minimum expected escape-payoff deriving from deviation (when the members of each coalition trust each other). In this coalition equilibrium, the expected escape-payoff deriving from deviation of player i, $W_i^{-C_{c_i^*}(i)}(c_i^*,\ c_{-i}^*)$, satisfies the competitive distribution condition:

$$\widetilde{x}_i^{(i)}(c_i^*,\ c_{-i}^*) \ge W_i^{-C_{c_i^*}(i)}(c_i^*,\ c_{-i}^*).$$

Assume that the coalition-choosing strategy of player i is $c_i \neq c_i^*$, according to the definition of coalition equilibrium under the principle of minimum expected escape-payoff deriving from deviation (when the members of each coalition trust each other), $W_i^{-C_{c_i}(i)}(c_i,\ c_{-i}^*) \ge W_i^{-C_{c_i^*}(i)}(c_i^*,\ c_{-i}^*)$. In coalition situation $(c_i^*,\ c_{-i}^*)$ when other players keep their coalition-choosing strategies unchanged, and player i plays coalition-choosing strategy c_i, $Mv_i^{(i)}(C_{c_i}) > W_i^{-C_{c_i}(i)}$, the expected distribution that player i gets from the corresponding coalition C_{c_i} which he joins satisfies:

$$\widetilde{x}_i^{(i)}(c_i,\ c_{-i}^*) \le W_i^{-C_{c_i^*}(i)}(c_i^*,\ c_{-i}^*),$$

because $W_i^{-C_{c_i^*}(i)}(c_i^*, c_{-i}^*)$ is the expected marginal contribution of player i to coalition C_{c_i} through deviation, we have:

$$\widetilde{x}_i^{(i)}(c_i^*,\ c_{-i}^*) \ge \widetilde{x}_i^{(i)}(c_i,\ c_{-i}^*).$$

So,

$$c_i^* = \arg\min W_i^{-C_{c_i}(i)}(c_i,\ c_{-i}^*) \Rightarrow c_i^* = \arg\max \widetilde{x}_i^{(i)}(c_i,\ c_{-i}^*).$$

That is to say,

$$c_i^* = \arg\min_{c_i} W_i^{-C_{c_i}(i)}(c_i,\ c_{-i}^*),\ \ \text{if at least for } a \text{ certain } c_i \neq i,\ \ W_i^{-C_i(i)}(i,\ c_{-i}^*) > W_i^{-C_{c_i}(i)}(c_i,\ c_{-i}^*),$$

$$\sum_{\substack{j\in C_{c_i}\\ j\neq i}} \left[Mv_i^{(j)}(C_{c_i}) - W_i^{-C_{c_i}(j)} \right] > 0, \quad \text{and} \sum_{\substack{j\in C_{c_i}\\ j\neq i}} \left[Mv_{T_h}^{(j)}(C_{c_i}) - \sum_{k\in T_h} W_k^{-C_{c_i}(j)} \right] > 0 (i \in T_h \subseteq C_{c_i}).$$

$$\Leftrightarrow c_i^* = \arg\max_{c_i} \widetilde{x}_i^{(i)}(c_i,\ c_{-i}^*),\ \ \text{if at least for } a \text{ certain } c_i \neq i,\ \ \widetilde{x}_i^{(i)}(i,\ c_{-i}^*) < \widetilde{x}_i^{(i)}(c_i,\ c_{-i}^*),$$

$$\sum_{\substack{j\in C_{c_i}\\ j\neq i}} \left[Mv_i^{(j)}(C_{c_i}) - W_i^{-C_{c_i}(j)} \right] > 0, \ \text{and} \sum_{\substack{j\in C_{c_i}\\ j\neq i}} \left[Mv_{T_h}^{(j)}(C_{c_i}) - \sum_{k\in T_h} W_k^{-C_{c_i}(j)} \right] > 0 (i \in T_h \subseteq C_{c_i}).$$

□

4. Allied bargaining game of a coalition: ignoring the opportunistic behaviors in the distribution process

When coalition members are allied in the bargaining games, in information asymmetric cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented, each player's escape payoff deriving from deviation remains unchanged, just the same one when coalition members are unallied in the bargaining game. Therefore, when coalition members are allied in the bargaining games, the coalition equilibrium of the game remains the same as the one under the criterion of minimum expected escape-payoff deriving from deviation (when the members of each coalition trust each other) when coalition members are unallied in the bargaining games. Therefore, in this section, we won't examine the coalition equilibrium of the game anymore.

In this section, we will examine the distribution process of the actual cooperative payoff of a coalition when core members are allied in the bargaining game, ignoring the opportunistic behaviors in the distribution process. First, we'll examine the coalition equilibrium of the bargaining game of a coalition. Secondly, with the methodology used in analyzing the distribution equilibrium when coalition members are unallied in the bargaining game, we'll examine the distribution of the cooperative payoff of a coalition between the cooperative teams, presents the actual cooperative payoff distribution obtained by a cooperative team of the coalition in the coalition equilibrium.

4.1 *Coalition equilibrium of the bargaining game*

In information asymmetric cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented, assume that information is symmetric after the game is completed, and that the above assumption is common knowledge of all the players, there exists the Nash equilibrium in the allied bargaining game of any coalition, and there exists the mixed strategic coalition equilibrium in the information asymmetric cooperative game with agreements self-implemented when coalition members are allied in the bargaining games. When coalition members are allied in the bargaining games, the coalition equilibrium in information asymmetric cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented is the same as the one in the information asymmetric cooperative game with agreements self-implemented when coalition members are unallied in the bargaining games, the public choice of strategic combination of a coalition when members of the coalition are allied in the bargaining game is the same as the one when members of the coalition are unallied in the bargaining game.

If information is still asymmetric after the game is completed, there is no Nash equilibria in the allied bargaining games of coalitions, therefore, when members of each coalition are allied in the bargaining game on the distribution of its cooperative payoff, there exists no coalition equilibrium in information asymmetric cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented under the criterion of maximum expected cooperative payoff distribution(when the members of each coalition trust each other), or, there exists no coalition equilibrium under the criterion of minimum expected escape-payoff deriving from deviation (when the members of each coalition trust each other) in the information asymmetric cooperative game with agreements self-implemented.

In this section we only examine the coalition equilibrium of the allied bargaining game of a coalition and the distribution of the cooperative payoff of the coalition in the case of information symmetry after the completion of the game.

In the virtual game of player k_1, when the core members of core coalition $C_T^{c(k_1)}$ set up the cooperative teams in some coalition situation, the competition among the m core members (m is the number of the core members of core coalition $C_T^{c(k_1)}$) of core coalition $C_T^{c(k_1)}$ in the unallied bargaining game is replaced by the competition among the m cooperative teams in the allied bargaining game. It is easy to prove that whether the coalition members are allied in the bargaining game or not, the coalition equilibria of information asymmetric cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented are the same.

In the virtual game of player k_1, in the allied bargaining game $\Gamma^{(k_1)}(M,\ \{T_i\},\ \left\{\tilde{x}_i^{(k_1)}\right\})$ of core coalition $C_T^{c(k_1)}$, let $M_{m_1,\ m_2,\ \ldots,\ m_k}$ denote a subset consisting of cooperative teams $m_1, m_2, \ldots, m_k$ of team set M of core coalition $C_T^{c(k_1)}$ in coalition situation t, $M_{m_1,\ m_2,\ \ldots,\ m_k} \subseteq M (k \leq m)$, the common payoff $\theta^{(k_1)}(M_{m_1,\ m_2,\ \cdots,\ m_k})$of team set $M_{m_1,\ m_2,\ \ldots,\ m_k}$ is defined as following:

$$\theta^{(k_1)}(M_{m_1,\ m_2,\ \cdots,\ m_k}) = V^{(k_1)}_{M_{m_1,\ m_2,\ \cdots,\ m_k}} - \sum_{i=1}^{k} W_{m_i}^{-C_T^{c(k_1)}(k_1)} - \sum \theta^{(k_1)}_{(2)}(M_{m_1,\ m_2,\ \cdots,\ m_k}) - \cdots - \sum \theta^{(k_1)}_{(k-1)}(M_{m_1,\ m_2,\ \cdots,\ m_k}),$$

where $V^{(k_1)}_{M_{m_1,\ m_2,\ \cdots,\ m_k}}$ is the cooperative payoff of core coalition $C_T^{c(k_1)}$ when all the members except those of the teams in set $M_{m_1,\ m_2,\ \ldots,\ m_k}$ have escaped from the coalition and join the same coalition as a whole, while other coalitions keep unchanged in the virtual allied bargaining game of player k_1; $\sum \theta^{(k_1)}_{(j)}(M_{m_1,\ m_2,\ \cdots,\ m_k})$ is the sum of the common payoffs of all the j-team subsets of $M_{m_1,\ m_2,\ \cdots,\ m_k}$ in the virtual allied bargaining game of player k_1; $\sum_{i=1}^{k} W_{m_i}^{-C_T^{c(k_1)}(k_1)}$ is the sum of escape-payoffs deriving from deviation of all the core members of the k teams of $M_{m_1,\ m_2,\ \cdots,\ m_k}$ in the virtual allied bargaining game of player k_1.

According to Chen [25], in coalition situation t of the bargaining game in the virtual game of player k_1, the Nash equilibrium in the bargaining game among teams $M_{m_1,\ m_2,\ \cdots,\ m_k}$ about the distribution of the common payoff $\theta^{(k_1)}(M_{m_1,\ m_2,\ \cdots,\ m_k})$ is:

$$y_{m_1}^{*(k_1)} = \frac{1}{k}\theta^{(k_1)}\left(M_{m_1,\ m_2,\ \cdots,\ m_k}\right),\ i = 1,\ 2,\ \cdots,\ k.$$

That's to say, the teams that belong to set $M_{m_1,\ m_2,\ \cdots,\ m_k}$ will get the same common payoff distribution.

So, in the virtual game of player k_1, in some coalition situation t of the allied bargaining game $\Gamma^{(k_1)}(M,\ \{T_i\},\ \left\{\tilde{x}_i^{(k_1)}\right\})$ of core coalition $C_T^{c(k_1)}$ on the distribution of the cooperative payoff surplus, the cooperative payoff surplus distribution that some cooperative team m_i can get from core coalition $C_T^{c(k_1)}$ is:

$$\widetilde{y}_{m_i}^{(k_1)} = \frac{1}{2} \sum_{\substack{m_j=1 \\ m_j \neq m_i}}^{m} \theta^{(k_1)}(M_{m_i,\, m_j}) + \frac{1}{3} \sum_{\substack{m_j=1 \\ m_j \neq m_i}}^{m} \sum_{\substack{m_k=1 \\ m_k \neq m_i}}^{m_j-1} \theta^{(k_1)}(M_{m_i,\, m_j,\, m_k}) + \cdots + \frac{1}{m}\theta^{(k_1)}(M_{1,\, 2,\, \cdots,\, m}).$$

The total distribution that team m_i can get is:

$$\widetilde{x}_{m_i}^{(k_1)} = W_{m_i}^{-C_T^c(k_1)} + \widetilde{y}_{m_i}^{(k_1)} = W_{m_i}^{-C_T^c(k_1)}$$

$$+ \frac{1}{2} \sum_{\substack{m_j=1 \\ m_j \neq m_i}}^{m} \theta^{(k_1)}(M_{m_i,\, m_j}) + \frac{1}{3} \sum_{\substack{m_j=1 \\ m_j \neq m_i}}^{m} \sum_{\substack{m_k=1 \\ m_k \neq m_i}}^{m_j-1} \theta^{(k_1)}(M_{m_i,\, m_j,\, m_k}) + \cdots + \frac{1}{m}\theta^{(k_1)}(M_{1,\, 2,\, \cdots,\, m}).$$

Thus, we get the Nash equilibrium of the allied bargaining game $\Gamma^{(k_1)}(M,\ \{T_i\},\ \left\{\widetilde{x}_i^{(k_1)}\right\})$ of core coalition $C_T^{c(k_1)}$ on the distribution of the cooperative payoff surplus among the cooperative teams formed in any coalition situation t.

After discussing the equilibrium of the non-cooperative game among cooperative teams of core coalition $C_T^{c(k_1)}$ in coalition situation t of the bargaining game in the virtual game of player k_1, we will continue to analyze the coalition equilibrium of the bargaining game $\Gamma^{(k_1)}(M,\ \{T_i\},\ \left\{\widetilde{x}_i^{(k_1)}\right\})$ of core coalition $C_T^{c(k_1)}$ in the virtual game of player k_1.

Ignoring the opportunistic behaviors of coalition members in the distribution process of cooperative payoff, assume that information is symmetric after the game is completed, and that the above assumption is common knowledge of all the players, in the virtual game of player k_1, if the coalition situation $t^{*(k_1)} = (t_1^{*(k_1)},\ t_2^{*(k_1)},\ \cdots,\ t_n^{*(k_1)})$ of virtual bargaining game $\Gamma^{(k_1)}(M,\ \{T_i\},\ \left\{\widetilde{x}_i^{(k_1)}\right\})$ of player k_1 is feasible, and the team-choosing strategy of each coalition member is the best response to the collective actions of other coalition members, coalition situation $t^{*(k_1)} = (t_1^{*(k_1)},\ t_2^{*(k_1)},\ \cdots,\ t_n^{*(k_1)})$ is called the coalition equilibrium under the criterion of maximum expected cooperative payoff distribution.

Theorem 7 Ignoring the opportunistic behaviors of coalition members in the distribution process of cooperative payoff, in information asymmetric cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented, assume that information is symmetric after the game is completed, and that the above assumption is common knowledge of all the players, in the virtual game of player k_1, in the allied bargaining game $\Gamma^{(k_1)}(M,\ \{T_i\},\ \left\{\widetilde{x}_i^{(k_1)}\right\})$ of core coalition $C_T^{c(k_1)}$, there exists the (mixed strategic) coalition equilibrium under the criterion of maximum expected cooperative payoff distribution:

$$\forall i = 1,\ 2,\ \cdots,\ m,$$

$$t_i^{*(k_1)} = \begin{cases} i, & \text{if for any } t_i,\ \widetilde{x}_i^{(k_1)}(i,\ t_{-i}^{*(k_1)}) \geq \widetilde{x}_i^{(k_1)}(t_i,\ t_{-i}^{*(k_1)}); \\ argmax\widetilde{x}_i^{(k_1)}(t_i,\ t_{-i}^{*(k_1)}), & \text{if at least for a certain } t_i \neq i,\ \widetilde{x}_i^{(k_1)}(i,\ t_{-i}^{*(k_1)}) < \widetilde{x}_i^{(k_1)}(t_i,\ t_{-i}^{*(k_1)}). \end{cases}$$

At the same time, in the allied bargaining game $\Gamma^{(k_1)}(M,\ \{T_i\},\ \left\{\widetilde{x}_i^{(k_1)}\right\})$ of core coalition $C_T^{c(k_1)}$, there exists the (mixed strategic) coalition equilibrium under the criterion of minimum expected escape-payoff:

$\forall i = 1,\ 2,\ \cdots,\ m,$

$$t_i^{*(k_1)} = \begin{cases} i, \ \text{if for any } t_i,\ w_i^{(k_1)}(i,\ t_{-i}^{*(k_1)}) \leq w_i^{(k_1)}(t_i,\ t_{-i}^{*(k_1)}); \\ \\ argminw_i^{(k_1)}(t_i,\ t_{-i}^{*(k_1)}), \ \text{if at least for a certain } t_i \neq i,\ w_i^{(k_1)}(i,\ t_{-i}^{*(k_1)}) > w_i^{(k_1)}(t_i,\ t_{-i}^{*(k_1)}). \end{cases}$$

If the distribution scheme of each team satisfies the competitive distribution condition, the coalition equilibrium under the criterion of maximum expected cooperative payoff distribution in allied bargaining game $\Gamma^{(k_1)}(M,\ \{T_i\},\ \left\{\widetilde{x}_i^{(k_1)}\right\})$ of core coalition $C_T^{c(k_1)}$ is equivalent to the one under the criterion of minimum expected escape-payoff in game $\Gamma^{(k_1)}(M,\ \{T_i\},\ \left\{\widetilde{x}_i^{(k_1)}\right\})$.

The proof of Theorem 7 is omitted.

In the virtual game of player k_1, after the formation of coalition equilibrium $t^{*(k_1)}$ of the allied bargaining game $\Gamma^{(k_1)}(M,\ \{T_i\},\ \left\{\widetilde{x}_i^{(k_1)}\right\})$ of core coalition $C_T^{c(k_1)}$, the competition among the m members (m is the number of the members of core coalition $C_T^{c(k_1)}$) of core coalition $C_T^{c(k_1)}$ in the unallied bargaining game is replaced by the competition among the m cooperative teams in the allied bargaining game. In coalition equilibrium $t^{*(k_1)}$ of allied bargaining game $\Gamma^{(k_1)}(M,\ \{T_i\},\ \left\{\widetilde{x}_i^{(k_1)}\right\})$ of core coalition $C_T^{c(k_1)}$, assume that the team-choosing strategies of the m member of core coalition $C_T^{c(k_1)}$ are respectively $t_1^{*(k_1)},\ t_2^{*(k_1)},\ \cdots,\ t_m^{*(k_1)}$, obviously, due to the different information sets, for some members $k_1,\ -k_1(-k_1 \neq k_1)$, usually, we have:

$$(t_1^{*(k_1)},\ t_2^{*(k_1)},\ \cdots,\ t_m^{*(k_1)}) \neq (t_1^{*(-k_1)},\ t_2^{*(-k_1)},\ \cdots,\ t_m^{*(-k_1)}).$$

Obviously, if player k_1 finds that his estimation of the coalition equilibrium of the bargaining game is different from the actual possible coalition equilibrium, his information set is obviously not complete enough. Therefore, player k_1 is motivated to further collect information, which is helpful for increasing his expected cooperative payoff distribution. Thus, before the cooperative teams sign the cooperation agreements, all the coalition members will further collect information, such that

$$\left(t_1^{'*(k_1)},\ t_2^{'*(k_1)},\ \cdots,\ t_m^{'*(k_1)}\right) = \left(t_1^{'*(-k_1)},\ t_2^{'*(-k_1)},\ \cdots,\ t_m^{'*(-k_1)}\right) = (t_1^*,\ t_2^*,\ \cdots,\ t_m^*).$$

Of course, if the information set that we refer to of a coalition member is his final information set before the cooperation agreements are signed by cooperative teams, then

$$\left(t_1^{*(k_1)},\ t_2^{*(k_1)},\ \cdots,\ t_m^{*(k_1)}\right) = \left(t_1^{*(-k_1)},\ t_2^{*(-k_1)},\ \cdots,\ t_m^{*(-k_1)}\right) = (t_1^*,\ t_2^*,\ \cdots,\ t_m^*).$$

That is,

$$t^{*(-k_1)} = t^{*(-k_1)} = t^*.$$

4.2 *Coalition equilibrium of the bargaining game and the distribution of the cooperative payoff*

After the formation of coalition equilibrium t^*, in coalition equilibrium t^* of the bargaining game of core coalition $C_T^{c(k_1)}$, if team set $M_{m_1^*,\, m_2^*,\, \cdots,\, m_k^*}$ is a subset of team set M^* of core coalition $C_T^{c(k_1)}$, which consists of cooperative teams $m_1^*, m_2^*, \cdots, m_k^*$, $M_{m_1^*,\, m_2^*,\, \cdots,\, m_k^*} \subseteq M^* (k \le m)$, common payoff $\theta^{(k_1)}(M_{m_1^*,\, m_2^*,\, \cdots,\, m_k^*})$ of team set $M_{m_1^*,\, m_2^*,\, \cdots,\, m_k^*}$ is defined as following:

$$\theta^{(k_1)}(M_{m_1^*,\, m_2^*,\, \cdots,\, m_k^*}) = V^{(k_1)}_{M_{m_1^*,\, m_2^*,\, \cdots,\, m_k^*}} - \sum_{i=1}^{k} W_{m_i^*}^{-C_T^{c(k_1)}(k_1)} - \sum \theta_{(2)}^{(k_1)}(M_{m_1^*,\, m_2^*,\, \cdots,\, m_k^*}) - \cdots - \sum \theta_{(k-1)}^{(k_1)}(M_{m_1^*,\, m_2^*,\, \cdots,\, m_k^*}),$$

where $V^{(k_1)}_{M_{m_1^*,\, m_2^*,\, \cdots,\, m_k^*}}$ is the cooperative payoff of core coalition $C_T^{c(k_1)}$ when all the members of other teams except those of the teams in set $M_{m_1^*,\, m_2^*,\, \cdots,\, m_k^*}$ have escaped from the core coalition and join the same coalition as a whole to maximize their escape-payoff, while members of other coalitions keep their coalition-choosing strategies unchanged in player k_1's virtual game; $\sum \theta_{(j)}^{(k_1)}\left(M_{m_1^*,\, m_2^*,\, \cdots,\, m_k^*}\right)$ is the sum of the common payoffs of all the j-team subsets of set $M_{m_1^*,\, m_2^*,\, \cdots,\, m_k^*}$; $\sum_{i=1}^{k} W_{m_i^*}^{-C_T^{c(k_1)}(k_1)}$ is the sum of the escape-payoffs deriving from deviation of all the members of the k teams in set $M_{m_1^*,\, m_2^*,\, \cdots,\, m_k^*}$.

According to the distribution rule of common payoff, in player k_1's virtual bargaining game of core coalition $C_T^{c(k_1)}$, the common payoff distribution that any cooperative team m_i^* in team set $M_{m_1^*,\, m_2^*,\, \cdots,\, m_k^*}$ can get is:

$$\widetilde{y}_{m_i^*}^{*(k_1)} = \frac{1}{k}\theta^{(k_1)}\left(M_{m_1^*,\, m_2^*,\, \cdots,\, m_k^*}\right),\ i = 1,\ 2,\ \cdots,\ k.$$

Therefore, in player k_1's virtual bargaining game $\Gamma^{(k_1)}(M,\ \{T_i\},\ \left\{\widetilde{x}_i^{(k_1)}\right\})$ of core coalition $C_T^{c(k_1)}$, the cooperative payoff surplus distribution that some cooperative team m_i^* of core coalition $C_T^{c(k_1)}$ can get is:

$$\widetilde{y}_{m_i^*}^{(k_1)} = \frac{1}{2}\sum_{\substack{m_j^*=1 \\ m_j^* \ne m_i^*}}^{m} \theta^{(k_1)}(M_{m_i^*,\, m_j^*}) + \frac{1}{3}\sum_{\substack{m_j^*=1 \\ m_j^* \ne m_i^*}}^{m} \sum_{\substack{m_k^*=1 \\ m_k^* \ne m_i^*}}^{m_j^*-1} \theta^{(k_1)}(M_{m_i^*,\, m_j^*,\, m_k^*}) + \cdots + \frac{1}{m}\theta^{(k_1)}(M_{1,\, 2,\, \cdots,\, m}).$$

The total cooperative payoff distribution that cooperative team m_i^* of core coalition $C_T^{c(k_1)}$ can get is:

$$\widetilde{x}_{m_i^*}^{(k_1)} = \widetilde{y}_{m_i^*}^{(k_1)} + W_{m_i^*}^{-C_T^c(k_1)} = W_{m_i^*}^{-C_T^c(k_1)}$$

$$+ \frac{1}{2}\sum_{\substack{m_j^*=1 \\ m_j^* \ne m_i^*}}^{m} \theta^{(k_1)}(M_{m_i^*,\, m_j^*}) + \frac{1}{3}\sum_{\substack{m_j^*=1 \\ m_j^* \ne m_i^*}}^{m} \sum_{\substack{m_k^*=1 \\ m_k^* \ne m_i^*}}^{m_j^*-1} \theta^{(k_1)}(M_{m_i^*,\, m_j^*,\, m_k^*}) + \cdots + \frac{1}{m}\theta^{(k_1)}(M_{1,\, 2,\, \cdots,\, m}).$$

Assume that information is symmetric after the game is completed, and that the above assumption is common knowledge of all the players, now we can get the Nash equilibrium of core coalition $C_T^{c(k_1)}$'s allied bargaining game

$\Gamma^{(k_1)}(M, \{T_i\}, \{\tilde{x}_i^{(k_1)}\})$ on the distribution of the cooperative payoff, in the coalition equilibrium t^* of the game in player k_1's virtual game.

If the number of the members of a cooperative team is 3 or more than 3, after examining the distribution process of cooperative payoff in the coalition level (that is, the first level) bargaining game, we need to extend the above-mentioned distribution process-that is, we need to examine the second level, third level, forth level, ... bargaining games, …, step by step, finally we'll get the distribution vector of a core coalition with limited members.

Assume that after the game is completed, the information among all the players, including all the members of core coalition $C_T^{c(k_1)}$, is symmetric, and that all the members or teams of each core coalition must be responsible for their own misjudgments. In coalition equilibrium c^* of the information asymmetric cooperative game with agreements self-implemented, assume that the public choice of strategic combination of core coalition $C_T^{c(k_1)}$ is $s^{**}_{C_T^{c(k_1)}}$, and that the actual public choices of strategic combination of other core coalitions are $s^{**T}_{-C_T^{c(k_1)}}$, the cooperative payoff actually obtained by core coalition $C_T^{c(k_1)}$ after the game is completed is $V_{C_T^{c(k_1)}}(s^{**}_{C_T^{c(k_1)}}, s^{**T}_{-C_T^{c(k_1)}}) = \sum_{i=1}^{m} u_i(s^{**}_{C_T^{c(k_1)}}, s^{**T}_{-C_T^{c(k_1)}})$, this cooperative payoff is actually available to be distributed by core coalition $C_T^{c(k_1)}$. If the information is symmetric among all the players after the game is completed, if the estimations of all the members of core coalition $C_T^{c(k_1)}$ of the strategic combination choice of the core coalition are correct, that is:

$$s^{*(i)}_{C_T^{c(k_1)}} = s^{*}_{C_T^{c(k_1)}},\ i \in C_T^{c(k_1)},$$

and accordingly,

$$s^{**}_{C_T^{c(k_1)}} = s^{*}_{C_T^{c(k_1)}},$$

where $s^{*(i)}_{C_T^{c(k_1)}}$ is the best response of core coalition $C_T^{c(k_1)}$ to the strategic combination choices $s^{**T}_{-C_T^{c(k_1)}}$ of other core coalitions.

At this point, the distribution of the cooperative payoff $V_{C_T^{c(k_1)}}(s^{*}_{C_T^{c(k_1)}}, s^{**T}_{-C_T^{c(k_1)}})$ of core coalition $C_T^{c(k_1)}$ will be carried out according to the distribution rule in the information symmetric allied bargaining game of core coalition $C_T^{c(k_1)}$ (Chen [25]):

$$\tilde{x}^{**}_{m_i^*} = W_{m_i^*}^{-c_T^{c(k_1)}} + \frac{1}{2}\sum_{\substack{m_j^*=1 \\ m_j^* \neq m_i^*}}^{m} \delta^{**}\left(M_{m_i^*,\, m_j^*}\right) + \frac{1}{3}\sum_{\substack{m_j^*=1 \\ m_j^* \neq m_i^*}}^{m} \sum_{\substack{m_k^*=1 \\ m_k^* \neq m_i^*}} \delta^{**}\left(M_{m_i^*,\, m_j^*,\, m_k^*}\right) + \cdots + \frac{1}{m}\delta^{**}\left(M_{1,\,2,\,\cdots,\,m}\right),$$

where $w_{m_i}^{-C_T^{c(k_1)}}$ is the sum of the escape-payoffs deriving through deviation of all the members of cooperative team m_i^*, $\delta^*\left(M_{m_i^*,\, m_j^*}\right)$, $\delta^*\left(M_{m_i^*,\, m_j^*,\, m_k^*}\right)$, $\cdots$, $\delta^*\left(M_{1,\,2,\,\cdots,\,m}\right)$ are respectively the common payoffs of cooperative teams $M_{m_i^*,\, m_j^*}$, $M_{m_i^*,\, m_j^*,\, m_k^*}$, $\cdots$, $M_{1,\,2,\,\cdots,\,m}$. By examining the first level, second level, ... bargaining games of core coalition $C_T^{c(k_1)}$, we can finally get the cooperative payoff distribution $\tilde{x}^{**}_{m_i}$ that any member k_1 of core coalition $C_T^{c(k_1)}$ can get.

If not all judgments of the members of core coalition $C_T^{c(k_1)}$ of the public choices of strategic combination of other core coalitions are correct, then $s^{**}_{C_T^{c(k_1)}} \neq s^{*}_{C_T^{c(k_1)}}$ (correspondingly, $V_{C_T^{c(k_1)}}(s^{**}_{C_T^{c(k_1)}}, s^{**T}_{-C_T^{c(k_1)}}) \neq V_{C_T^{c(k_1)}}(s^{*}_{C_T^{c(k_1)}}, s^{**T}_{-C_T^{c(k_1)}})$), core coalition $C_T^{c(k_1)}$'s cooperative payoff $V_{C_T^{c(k_1)}}(s^{**}_{C_T^{c(k_1)}}, s^{**T}_{-C_T^{c(k_1)}})$ can be regarded as the "cooperation" outcome of core coalition $C_T^{c(k_1)}$ starting from strategic combination $s^*_{C_T^{c(k_1)}}$ and through the misjudgments of core coalition $C_T^{c(k_1)}$'s public choice of strategic combination and the public choices of strategic combinations of other core coalitions. The distribution of the cooperative payoff $V_{C_T^{c(k_1)}}(s^{**}_{C_T^{c(k_1)}}, s^{**T}_{-C_T^{c(k_1)}})$ should be based on the distribution of the cooperative payoff $V_{C_T^{c(k_1)}}(s^*_{C_T^{c(k_1)}}, s^{**T}_{-C_T^{c(k_1)}})$, with an additional distribution of the "cooperative" payoff $V_{C_T^{c(k_1)}}(s^{**}_{C_T^{c(k_1)}}, s^{**T}_{-C_T^{c(k_1)}}) - V_{C_T^{c(k_1)}}(s^*_{C_T^{c(k_1)}}, s^{**T}_{-C_T^{c(k_1)}})$ which is caused by the misjudgment cooperation among the members of core coalition $C_T^{c(k_1)}$.

The cooperative payoff distribution of any member k_1 of core coalition $C_T^{c(k_1)}$ in the misjudgment "cooperation" is:

$$\widetilde{x}^*_{k_1} = W^*_{k_1} + \frac{1}{2}\sum_{\substack{q_j=1\\ q_j\neq k_1}}^{m^*} \delta^*(M_{k_1,\, q_j}) + \frac{1}{3}\sum_{\substack{q_j=1\\ q_j\neq k_1}}^{m^*}\sum_{\substack{q_k=1\\ q_k\neq k_1}}^{q_j-1} \delta^*(M_{k_1,\, q_j,\, q_k}) + \cdots + \frac{1}{m^*}\delta^*(M_{1,\, 2,\, \cdots,\, m^*}).$$

Therefore, the total cooperative payoff distribution that cooperative team m_i^* of core coalition $C_T^{c(k_1)}$ obtains is:

$$\widetilde{x}_{m_i^*} = \widetilde{x}^{**}_{m_i^*} + \sum_{j\in m_i^*} \widetilde{x}^*_j.$$

The cooperative payoff distribution that any member k_1 of core coalition $C_T^{c(k_1)}$ can obtain is:

$$\widetilde{x}_{k_1} = \widetilde{x}^*_{k_1} + \widetilde{x}^{**}_{k_1}.$$

5. Coalitions centralizes all the payoffs that their members get in the game to prevent opportunistic behaviors in the distribution process

Next, we will relax the assumption that coalition members carry no opportunistic behaviors in the distribution process of cooperative payoff. If coalition members may carry opportunistic behaviors in the distribution process of cooperative payoff, a coalition can inhibit the opportunistic behaviors of members by centralizing all the payoffs its members get in the game, the distribution scheme of cooperative payoff can get implemented.

When information is asymmetric, a coalition cannot ensure that its members carry no opportunistic behavior of refusing the distribution scheme through the prior distribution of its cooperative payoff. Therefore, we do not consider the situation when the coalitions distribute their cooperative payoffs before the game begins.

However, if information among the players in the game is asymmetric, before the game is completed, the coalition members cannot accurately estimate the cooperative payoff of the coalition, and it is actually difficult for the coalition to perform cooperative payoff distribution before the game begins. Therefore, here we only discuss the situation in which coalitions concentrate the payoffs that all their members get in the game to prevent the members from carrying opportunistic behaviors in the distribution process of cooperative payoff.

In this section, assuming that coalitions centralize all the payoffs that their members get in the game, we'll examine the coalition equilibrium of the game, investigate the condition for its existence, examine the distribution process of the cooperative payoff of a coalition in the coalition equilibrium if it does exist, when the coalition members are allied or not in the bargaining game. The methodology used in this section is just the same as that used in analyzing the situation when the opportunistic behaviors in the distribution process are ignored.

When coalitions centralize all the payoffs that their members get in the game to prevent opportunistic behaviors in the distribution process, the coalition equilibrium of the game and the condition for its existence, as well as the distribution equilibrium of the cooperative payoff of a coalition under the coalition equilibrium are similar to those mentioned in the previous two sections when opportunistic behaviors in the distribution process are ignored. The only difference is that, because the coalitions concentrate all members' payoffs gotten in the game, when a member escapes through deviation from the coalition he belongs to, his escape payoff deriving from deviation, the cooperative payoff of his target coalition and his marginal contribution to his target coalition will change.

5.1 *Coalition equilibrium and the unallied bargaining games of coalitions*

First, assume that members of each coalition are unallied in the bargaining game on the distribution of its cooperative payoff, we will examine an information asymmetric cooperative game with agreements self-implemented, when the coalitions centralize all the payoffs that their members get in the game to inhibit the opportunistic behaviors in the distribution process.

5.1.1 *Escape-payoff deriving from deviation*

When the coalitions centralize all the payoffs that their members get in the game to inhibit the opportunistic behaviors in the distribution process, between the virtual game of any player k_1 and the one when the opportunistic behaviors of coalition members in the distribution process of cooperative payoff are negligible, the fundamental difference is that the escape-payoff deriving from deviation of a coalition member has changed: when the coalitions centralize all the payoffs that their members get in the game to inhibit the opportunistic behaviors in the distribution process, whatever escape strategy is played by the deviating member, the payoff that he gets in the game is attributed to his nominal coalition and cannot be attributed to his escape target core coalition. That is,

$$\widehat{V}_{C_i^c}(\cdot) = V_{C_i}(\cdot) = \sum_{j \in C_i} u_i(\cdot).$$

If the payoffs that the members get in the game are attributed to their nominal coalitions, in some escape situation e in coalition situation c, the cooperative payoff of a core coalition must be the cooperative payoff of the corresponding nominal coalition.

When the coalitions centralize all the payoffs that their members get in the game to inhibit the opportunistic behaviors in the distribution process, the conditions that player k_1 is trusted by other members of nominal coalition C_k^+ are shown as follows:

(1) $\sum_{i \in C_k^c} \left[Mv_{k_2}^{(i)}(C_k^{c(i)}) - \widehat{W}_{k_2}^{-C_k^{c(i)}(i)} \right] + \sum_{j \in C_k^+,\ j \notin C_k^c} \left[Mv_{k_2}^{(j^F)}(C_k^{c(j^F)}) - \widehat{W}_{k_2}^{-C_k^{c(j^F)}(j^F)} \right] > 0,\ \ i,\ j \neq k_2;$

(2) $\sum_{i \in C_k^c} \left[Mv_{T_h}^{(i)}(C_k^{c(i)}) - \sum_{t \in T_h} \widehat{W}_t^{-C_k^{c(i)}(i)} \right] + \sum_{j \in C_k^+,\ j \notin C_k^c} \left[Mv_{T_h}^{(j^F)}(C_k^{c(j^F)}) - \sum_{t \in T_h} \widehat{W}_t^{-C_k^{c(j^F)}(j^F)} \right] > 0,\ i,\ j \neq k_2,\ k_2 \in T_h \subset C_k;$

(3) $\mathrm{V}_{C_k^c}^{(i)} - \sum_{t \in C_k^c} \widehat{W}_t^{-C_k^{c(i)}(i)} > 0,\ i \in C_k^c;\ V_{C_k^{c(j^F)}}^{(j^F)} - \sum_{t \in C_k^{c(j^F)}} \widehat{W}_t^{-C_k^{c(j^F)}(j^F)} > 0,\ j \in C_k^+,\ j \notin C_k^c.$

The above conditions can be simply denoted as:

$$\sum_{i\in C_k}\left[Mv_{k_2}^{(i)}(C_k)-\widehat{W}_{k_2}^{-C_k(i)}\right]>0,\ i\neq k_2;$$

$$\sum_{i\in C_k}\left[Mv_{T_h}^{(i)}(C_k)-\sum_{t\in T_h}\widehat{W}_t^{-C_k(i)}\right]>0,\ i\neq k_2,\ k_2\in T_h\subset C_k;$$

$$V_{C_k}^{(i)}-\sum_{t\in C_k}\widehat{W}_t^{-C_k(i)}>0,\ i\in C_k.$$

In player k_1's t-th level virtual game, in coalition situation c, assume that (player k_1's estimation of player k_2's estimation of ...) the feasible escape strategy that player k_t plays is $e_{k_t}^{(k_1,\,\cdots,\,k_t)}$, his escape target core coalition is $C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}(e_{k_t}^{(k_1,\,k_2,\,\cdots,\,k_t)},\ e_{-k_t}^{*(k_1,\,k_2,\,\cdots,\,k_t)})$, his estimation of the strategic combination choices of the core coalitions other than core coalition $C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}(e_{k_t}^{(k_1,\,k_2,\,\cdots,\,k_t)},\ e_{-k_t}^{*(k_1,\,k_2,\,\cdots,\,k_t)})$ are $s_{-C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)}^{**(k_1,\,k_2,\,\cdots,\,k_t)}$, and that his estimation of the escape strategic choices of other players are $e_{-k_t}^{*(k_1,\,k_2,\,\cdots,\,k_t)}$. Assume that under his estimation of the information sets $I^{(k_1,\,\cdots,\,k_t)}(e_{k_1}^{(k_1)},\ \cdots,\ e_{k_t}^{(k_1,\,\cdots,\,k_t)},\ e_{-k_1,\,-k_2,\,\cdots,\,-k_t}^{*(k_1,\,k_2,\,\cdots,\,k_t)})$, player k_t considers that the strategic combination that his escape target core coalition $C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}(e_{k_t}^{(k_1,\,\cdots,\,k_t)},\ e_{-k_t}^{*(k_1,\,k_2,\,\cdots,\,k_t)})$ "should" adopt is $s_{C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)}^{*(k_1,\,k_2,\,\cdots,\,k_t)}$, at this point, the cooperative payoff that core coalition $C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)$ "should" get is:

$$\widehat{V}_{C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}}^{(k_1,\,k_2,\,\cdots,\,k_t)}(s_{C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)}^{*(k_1,\,k_2,\,\cdots,\,k_t)},\ s_{-C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)}^{**(k_1,\,k_2,\,\cdots,\,k_t)})=\sum_{i\in C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}}u_i^{(k_1,\,k_2,\,\cdots,\,k_t)}(s_{C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)}^{*(k_1,\,k_2,\,\cdots,\,k_t)},\ s_{-C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)}^{**(k_1,\,k_2,\,\cdots,\,k_t)}).$$

If player k_t escapes from core coalition $C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)$ through deviation, that is to say, he chooses a feasible escape strategy $e_{k_t}^{\prime(k_1,\,k_2,\,\cdots,\,k_t)}$ which is different from escape strategy $e_{k_t}^{(k_1,\,k_2,\,\cdots,\,k_t)}$, at this point, player$k_t$'s escape target core coalition changes to $C_{T'}^{c(k_1,\,k_2,\,\cdots,\,k_t)}\left(e_{k_t}^{\prime(k_1,\,k_2,\,\cdots,\,k_t)},\ e_{-k_t}^{\prime*(k_1,\,k_2,\,\cdots,\,k_t)}\right)$ $\left[C_{T'}^{c(k_1,\,k_2,\,\cdots,\,k_t)}\left(e_{k_t}^{\prime(k_1,\,k_2,\,\cdots,\,k_t)},\ e_{-k_t}^{\prime*(k_1,\,k_2,\,\cdots,\,k_t)}\right)\right.$ $\left.\neq C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}\left(e_{k_t}^{(k_1,\,k_2,\,\cdots,\,k_t)},\ e_{-k_t}^{*(k_1,\,k_2,\,\cdots,\,k_t)}\right)\right]$, his estimation of the escape strategic choices of other players changes to $e_{-k_t}^{\prime*(k_1,\,k_2,\,\cdots,\,k_t)}$, and his estimation of the strategic combination choices of the core coalitions other than core coalition $C_{T'}^{c(k_1,\,k_2,\,\cdots,\,k_t)}\left(e_{k_t}^{\prime(k_1,\,k_2,\,\cdots,\,k_t)},\ e_{-k_t}^{\prime*(k_1,\,k_2,\,\cdots,\,k_t)}\right)$ changes to $s_{-C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}\left(e_{k_t}^{\prime(k_1,\,k_2,\,\cdots,\,k_t)},\ e_{-k_t}^{\prime*(k_1,\,k_2,\,\cdots,\,k_t)}\right)}^{\prime**(k_1,\,k_2,\,\cdots,\,k_t)}$.

Under his new estimation of information sets $I^{(k_1,\,\cdots,\,k_t)}(e_{k_1}^{(k_1)},\ e_{k_2}^{(k_1,\,k_2)},\ \cdots,\ e_{k_{t-1}}^{\prime(k_1,\,k_2,\,\cdots,\,k_{t-1})},\ e_{-k_1,\,-k_2,\,\cdots,\,-k_t}^{\prime*(k_1,\,k_2,\,\cdots,\,k_t)})$ of all the players, player k_t considers that the strategic combination that his escape target core coalition "should" adopt is $s_{C_{T'}^{c(k_1,\,k_2,\,\cdots,\,k_t)}\left(e_{k_t}^{\prime(k_1,\,k_2,\,\cdots,\,k_t)},\ e_{-k_t}^{\prime*(k_1,\,k_2,\,\cdots,\,k_t)}\right)}^{\prime*(k_1,\,k_2,\,\cdots,\,k_t)}$, at this point, the cooperative payoff that core coalition $C_{T'}^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)$ "should" get is:

$$\widehat{V}_{C_{T'}^{c(k_1,\,k_2,\,\cdots,\,k_t)}}^{(k_1,\,k_2,\,\cdots,\,k_t)}(s_{C_{T'}^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)}^{\prime*(k_1,\,k_2,\,\cdots,\,k_t)},\ s_{-C_{T'}^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)}^{\prime**(k_1,\,k_2,\,\cdots,\,k_t)})=\sum_{i\in C_{T'}^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)}u_i^{(k_1,\,k_2,\,\cdots,\,k_t)}(s_{C_{T'}^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)}^{\prime*(k_1,\,k_2,\,\cdots,\,k_t)},\ s_{-C_{T'}^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)}^{\prime**(k_1,\,k_2,\,\cdots,\,k_t)}).$$

Before player k_t escapes from core coalition $C_T^{c(k_1, k_2, \cdots, k_t)}(\cdot)$ through deviation, the cooperative payoff that core coalition $C_{T'}^{c(k_1, k_2, \cdots, k_t)}(\cdot)$ "should" get is

$$\widehat{V}^{(k_1, k_2, \cdots, k_t)}_{C_{T'}^{c(k_1, k_2, \cdots, k_t)}}(s^{*(k_1, k_2, \cdots, k_t)}_{C_{T'}^{c(k_1, k_2, \cdots, k_t)}(\cdot)}, s^{**(k_1, k_2, \cdots, k_t)}_{-C_{T'}^{c(k_1, k_2, \cdots, k_t)}(\cdot)}) = \sum_{i \in C_{T'}^{c(k_1, k_2, \cdots, k_t)}(\cdot)} u_i^{(k_1, k_2, \cdots, k_t)}(s^{*(k_1, k_2, \cdots, k_t)}_{C_{T'}^{c(k_1, k_2, \cdots, k_t)}(\cdot)}, s^{**(k_1, k_2, \cdots, k_t)}_{-C_{T'}^{c(k_1, k_2, \cdots, k_t)}(\cdot)}),$$

$s^{*(k_1, k_2, \cdots, k_t)}_{C_{T'}^{c(k_1, k_2, \cdots, k_t)}(\cdot)}$, $s^{**(k_1, k_2, \cdots, k_t)}_{-C_{T'}^{c(k_1, k_2, \cdots, k_t)}(\cdot)}$ are respectively the strategic combination choice that core coalition $C_{T'}^{c(k_1, k_2, \cdots, k_t)}(\cdot)$ "should" adopt and the strategic combination choices of the core coalitions other than core coalition $C_{T'}^{c(k_1, k_2, \cdots, k_t)}(\cdot)$ before player k_t escapes. Therefore, the marginal contribution of player k_t to core coalition $C_{T'}^{c(k_1, k_2, \cdots, k_t)}(\cdot)$ when he escapes through deviation from core coalition $C_T^{c(k_1, k_2, \cdots, k_t)}(\cdot)$ to core coalition $C_{T'}^{c(k_1, k_2, \cdots, k_t)}(\cdot)$ is:

$$\begin{aligned} Mv_{k_t}^{(k_1, k_2, \cdots, k_t)}\left[C_{T'}^{c(k_1, k_2, \cdots, k_t)}(\cdot)\right] =& \widehat{V}^{(k_1, k_2, \cdots, k_t)}_{C_{T'}^{c(k_1, k_2, \cdots, k_t)}}\left(s'^{*(k_1, k_2, \cdots, k_t)}_{C_{T'}^{c(k_1, k_2, \cdots, k_t)}(\cdot)}, s'^{**(k_1, k_2, \cdots, k_t)}_{-C_{T'}^{c(k_1, k_2, \cdots, k_t)}(\cdot)}\right) \\ &- \widehat{V}^{(k_1, k_2, \cdots, k_t)}_{C_{T'}^{c(k_1, k_2, \cdots, k_t)}}\left(s^{*(k_1, k_2, \cdots, k_t)}_{C_{T'}^{c(k_1, k_2, \cdots, k_t)}(\cdot)}, s^{**(k_1, k_2, \cdots, k_t)}_{-C_{T'}^{c(k_1, k_2, \cdots, k_t)}(\cdot)}\right), \end{aligned}$$

where $s'^{*(k_1, k_2, \cdots, k_t)}_{C_{T'}^{c(k_1, k_2, \cdots, k_t)}(\cdot)}$, $s'^{**(k_1, k_2, \cdots, k_t)}_{-C_{T'}^{c(k_1, k_2, \cdots, k_t)}(\cdot)}$ are respectively the strategic combination choice that core coalition $C_{T'}^{c(k_1, k_2, \cdots, k_t)}(\cdot)$ "should" adopt and the strategic combination choices of the core coalitions other than core coalition $C_{T'}^{c(k_1, k_2, \cdots, k_t)}(\cdot)$ after player k_t escapes.

Define the marginal contribution of player k_t to his escape target core coalition $C_{T'}^{c(k_1, k_2, \cdots, k_t)}(\cdot)$ $\left[C_{T'}^{c(k_1, k_2, \cdots, k_t)}(\cdot) \neq C_T^{c(k_1, k_2, \cdots, k_t)}(\cdot)\right]$ as his expected escape-payoff deriving from deviation when he escapes to core coalition $C_{T'}^{c(k_1, k_2, \cdots, k_t)}(\cdot)$ through deviation:

$$\widehat{W}_{k_t}^{(k_1, k_2, \cdots, k_t)}\left[C_T^{c(k_1, k_2, \cdots, k_t)}(\cdot) \longrightarrow C_{T'}^{c(k_1, k_2, \cdots, k_t)}(\cdot)\right] = Mv_{k_t}^{(k_1, k_2, \cdots, k_t)}\left[C_{T'}^{c(k_1, k_2, \cdots, k_t)}(\cdot)\right]$$

$$= \widehat{V}^{(k_1, k_2, \cdots, k_t)}_{C_{T'}^{c(k_1, k_2, \cdots, k_t)}}\left(s'^{*(k_1, k_2, \cdots, k_t)}_{C_{T'}^{c(k_1, k_2, \cdots, k_t)}(\cdot)}, s'^{**(k_1, k_2, \cdots, k_t)}_{-C_{T'}^{c(k_1, k_2, \cdots, k_t)}(\cdot)}\right) - \widehat{V}^{(k_1, k_2, \cdots, k_t)}_{C_{T'}^{c(k_1, k_2, \cdots, k_t)}}\left(s^{*(k_1, k_2, \cdots, k_t)}_{C_{T'}^{c(k_1, k_2, \cdots, k_t)}(\cdot)}, s^{**(k_1, k_2, \cdots, k_t)}_{-C_{T'}^{c(k_1, k_2, \cdots, k_t)}(\cdot)}\right).$$

Obviously, if player k_t escapes from core coalition $C_T^{c(k_1, k_2, \cdots, k_t)}(\cdot)$ through deviation, he will choose an escape strategy which can maximize his expected escape-payoff deriving from deviation, therefore, the expected escape-payoff deriving from deviation of player k_t when he escapes from core coalition $C_T^{c(k_1, k_2, \cdots, k_t)}(\cdot)$ through deviation is:

$$\widehat{W}_{k_t}^{-C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)(k_1,\,k_2,\,\cdots,\,k_t)} = \sum_{\substack{e_{k_t}^{\prime(k_1,\,k_2,\,\cdots,\,k_t)} \neq e_{k_t}^{(k_1,\,k_2,\,\cdots,\,k_t)Max} \\ C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)} \neq C_{T'}^{c(k_1,\,k_2,\,\cdots,\,k_t)}}} Mv_{k_t}^{(k_1,\,k_2,\,\cdots,\,k_t)}\left[C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot) \longrightarrow C_{T'}^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)\right]$$

$$= \sum_{\substack{e_{k_t}^{\prime(k_1,\,k_2,\,\cdots,\,k_t)} \neq e_{k_t}^{(k_1,\,k_2,\,\cdots,\,k_t)Max} \\ C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)} \neq C_{T'}^{c(k_1,\,k_2,\,\cdots,\,k_t)}}} \left\{\widehat{V}_{C_{T'}^{c(k_1,\,k_2,\,\cdots,\,k_t)}}^{(k_1,\,k_2,\,\cdots,\,k_t)}\left(s_{C_{T'}^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)}^{\prime*(k_1,\,k_2,\,\cdots,\,k_t)},\, s_{-C_{T'}^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)}^{\prime**(k_1,\,k_2,\,\cdots,\,k_t)}\right) - \widehat{V}_{C_{T'}^{c(k_1,\,k_2,\,\cdots,\,k_t)}}^{(k_1,\,k_2,\,\cdots,\,k_t)}\left(s_{C_{T'}^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)}^{*(k_1,\,k_2,\,\cdots,\,k_t)},\, s_{-C_{T'}^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)}^{**(k_1,\,k_2,\,\cdots,\,k_t)}\right)\right\}.$$

Assume that the escape strategic choice of player k_t is $e_{k_t}^{\prime(k_1,\,k_2,\,\cdots,\,k_t)}$, and that his escape target core coalition is $C_{T'*}^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)$ (when player k_t escapes from core coalition $C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)$ to core coalition $C_{T'*}^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)$ through deviation, his marginal contribution to his escape target core coalition $C_{T'*}^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)$ reaches the maximum value), his estimation of the strategic combination choice that his escape target core coalition "should" adopt and the strategic combination choices of the core coalitions other than his escape target core coalition are respectively $s_{C_{T'*}^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)}^{\prime\prime*(k_1,\,k_2,\,\cdots,\,k_t)}$ and $s_{-C_{T'*}^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)}^{\prime\prime*(k_1,\,k_2,\,\cdots,\,k_t)}$, after player k_t escapes from core coalition $C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)$ through deviation the cooperative payoff that core coalition $C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)$ can get is $\widehat{V}_{C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}}^{(k_1,\,k_2,\,\cdots,\,k_t)}\left(s_{C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)}^{\prime\prime*(k_1,\,k_2,\,\cdots,\,k_t)},\, s_{-C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)}^{\prime\prime*(k_1,\,k_2,\,\cdots,\,k_t)}\right)$, therefore, the expected marginal contribution of player k_t to core coalition $C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)$ is:

$$Mv_{k_t}^{(k_1,\,k_2,\,\cdots,\,k_t)}\left(C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)\right) = \widehat{V}_{C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}}^{(k_1,\,k_2,\,\cdots,\,k_t)}\left(s_{C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)}^{*(k_1,\,k_2,\,\cdots,\,k_t)},\, s_{-C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)}^{*(k_1,\,k_2,\,\cdots,\,k_t)}\right) - \widehat{V}_{C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}}^{(k_1,\,k_2,\,\cdots,\,k_t)}\left(s_{C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)}^{\prime\prime*(k_1,\,k_2,\,\cdots,\,k_t)},\, s_{-C_T^{c(k_1,\,k_2,\,\cdots,\,k_t)}(\cdot)}^{\prime\prime*(k_1,\,k_2,\,\cdots,\,k_t)}\right).$$

5.1.2 *Coalition equilibrium and unallied bargaining games*

In this section, for the sake of simplicity of analysis, the analysis of the virtual game of any player k_1 is omitted. When the coalitions centralize all the payoffs their members get in the game, the virtual game of player k_1 is just similar to the one when the opportunistic behaviors of the coalition members in the distribution process are negligible.

When the coalitions centralize all the payoffs their members get in the game, the public choice of strategic combination of an extensive coalition is the outcome of the compromise of the extensive members of the coalition. In fact, in coalition situation c in the virtual game of the player k_1, according to the information set of player k_1, the public choice of strategic combination of core coalition $C_T^{c(k_1)}$ should be:

$$s^{**(k_1)}_{C_T^{c(k_1)}} \subseteq s^{**(k_1)}_{C_T^{+(k_1)}};$$

$$s^{**(k_1)}_{C_T^{c(k_1)}} = argmax\left\{\sum_{i\in C_T^{c(k_1)}} \widehat{V}^{(k_1,\, i)}_{C_T^{c(k_1,\, i)}}\left(s^{(k_1,\, i)}_{C_T^{c(k_1,\, i)}},\, s^{*(k_1,\, i)}_{-C_T^{c(k_1,\, i)}}\right)\right.$$

$$\left. + \sum_{\substack{k\in C_h^{+(k_1)}(\cdot)\\ k\notin C_h^{c(k_1)}(\cdot)}} \widehat{V}^{(k_1,\, j^F)}_{C_h^{c(k_1,\, j)}(\cdot)}\left(s^{(k_1,\, j^F)}_{C_h^{c(k_1,\, j)}(\cdot)},\, s^{**(k_1,\, j^F)}_{C_T^{c(k_1,\, j)}(\cdot)},\, s^{**(k_1,\, j^F)}_{-C_h^{c(k_1,\, j)}(\cdot),\, -C_T^{c(k_1,\, j)}(\cdot)}\right)\right\}$$

$$= argmax\left\{\sum_{i\in C_T^{c(k_1)}} \widehat{V}^{(k_1,\, i)}_{C_T^{c(k_1,\, i)}}\left(s^{(k_1,\, i)}_{C_T^{c(k_1,\, i)}},\, s^{*(k_1,\, i)}_{-C_T^{c(k_1,\, i)}}\right) + \sum_{\substack{k\in C_h^{+(k_1)}(\cdot)\\ k\notin C_h^{c(k_1)}(\cdot)}} \widehat{V}^{\left(k_1,\, j^F\right)}_{C_h^{c(k_1,\, j)}(\cdot)}\left(s^{\left(k_1,\, j^F\right)}_{C_h^{c(k_1,\, j)}(\cdot)},\, i_j^{\left(k_1,\, j^F\right)}\right)\right\}$$

$$\left[i_j^{\left(k_1,\, j^F\right)} \in I_j^{\left(k_1,\, j^F\right)}\right],$$

where $C_h^{c(k_1,\, j)} = C_T^{c(k_1)} \neq C_T^{c(k_1,\, j)}$.

Assume that all the members of each coalition are responsible for their own misjudgments, and that the above assumption are common knowledge of all the players, according to player k_1's information set, the cooperative payoff $\widehat{V}^{(k_1)}_{C_T^{c(k_1)}}\left(s^{*(k_1)}_{C_T^{c(k_1)}},\, s^{**(k_1)}_{-C_T^{c(k_1)}}\right)$ of the core coalition "should" be distributed in accordance with the distribution rule of cooperative payoff of the coalition when its members are unallied in the bargaining game (Chen [25]).

Assume that there are m members in the member set M of core coalition $C_T^{c(k_1)}$, member set $M_{q_1,\, q_2,\, \cdots,\, q_k}$ composed of members $q_1,\ q_2,\ \cdots,\ q_k$ is a subset of coalition member set M of core coalition $C_T^{c(k_1)}$, $M_{q_1,\, q_2,\, \cdots,\, q_k} \subseteq M(k \leq m)$, $\widehat{\theta}^{(k_1)}(M_{q_1,\, q_2,\, \cdots,\, q_k})$ is called the common payoff of member set $M_{q_1,\, q_2,\, \cdots,\, q_k}$ in the virtual game of player k_1:

$$\widehat{\theta}^{(k_1)}(M_{q_1,\, q_2,\, \cdots,\, q_k}) = \widehat{V}^{(k_1)}_{M_{q_1,\, q_2,\, \cdots,\, q_k}} - \sum_{i=1}^{k} \widehat{W}_{q_i}^{-C_T^{c(k_1)}(k_1)} - \sum \widehat{\theta}^{(k_1)}_{(2)}(M_{q_1,\, q_2,\, \cdots,\, q_k}) - \cdots - \sum \widehat{\theta}^{(k_1)}_{(k-1)}(M_{q_1,\, q_2,\, \cdots,\, q_k}),$$

where $\widehat{V}^{(k_1)}_{M_{q_1,\, q_2,\, \cdots,\, q_k}}$ is player k_1's estimation of the cooperative payoff of coalition $C_T^{c(k_1)}$ when all the members except those in member set $M_{q_1,\, q_2,\, \cdots,\, q_k}$ escape from the coalition and join the same coalition as a whole to maximize their escape-payoff, while members of other coalitions keep their coalition-choosing strategies unchanged, $\sum \widehat{\theta}^{(k_1)}_{(j)}(M_{q_1,\, q_2,\, \cdots,\, q_k})$ is the sum of the common payoffs of all the j-member subsets of member set $M_{q_1,\, q_2,\, \cdots,\, q_k}$, $\sum_{i=1}^{k} \widehat{W}_{q_i}^{-C_T^{c(k_1)}(k_1)}$ is the sum of the escape-payoffs deriving from deviation of all the members in member set $M_{q_1,\, q_2,\, \cdots,\, q_k}$.

In the virtual game of player k_1, in coalition equilibrium $c^{*(k_1)}$, if the strategic combination that core coalition $C_T^{c(k_1)}$ "should" adopt is $s^{*(k_1)}_{C_T^{c(k_1)}}$ and the strategic combination choices of other core coalitions are $s^{**(k_1)}_{-C_T^{c(k_1)}}$, the expected cooperative payoff distribution of some member q_i of core coalition $C_T^{c(k_1)}$ is:

$$\widehat{x}_{q_i}^{(k_1)}(s^{*(k_1)}_{C_T^{c(k_1)}},\ s^{**(k_1)}_{-C_T^{c(k_1)}}) = \widehat{W}_{q_i}^{-C_T^c(k_1)} + \frac{1}{2}\sum_{\substack{q_j=1\\ q_j\neq q_i}}^{m} \widehat{\theta}^{(k_1)}(M_{q_i,\, q_j})$$

$$+\frac{1}{3}\sum_{\substack{q_j=1\\ q_j\neq q_i}}^{m}\sum_{\substack{q_k=1\\ q_k\neq q_i}}^{q_j-1} \widehat{\theta}^{(k_1)}(M_{q_i,\, q_j,\, q_k}) + \cdots + \frac{1}{m}\widehat{\theta}^{(k_1)}(M_{1,\, 2,\, \cdots,\, m}).$$

According to the virtual game of player k_1, in coalition equilibrium $c^{*(k_1)}$, when cooperative payoff $\widehat{V}^{*(k_1)}_{C_T^{c(k_1)}}(s^{*(k_1)}_{C_T^{c(k_1)}},\ s^{**(k_1)}_{-C_T^{c(k_1)}})$ of core coalition $C_T^{c(k_1)}$ is distributed according to the rule mentioned above, the expected cooperative payoff distribution that player k_1 gets is:

$$\widehat{x}_{k_1}^{(k_1)}(s^{*(k_1)}_{C_T^{c(k_1)}},\ s^{**(k_1)}_{-C_T^{c(k_1)}}) = \widehat{W}_{k_1}^{-C_T^c(k_1)}$$

$$+\frac{1}{2}\sum_{\substack{q_j=1\\ q_j\neq k_1}}^{m} \widehat{\theta}^{(k_1)}(M_{k_1,\, q_j}) + \frac{1}{3}\sum_{\substack{q_j=1\\ q_j\neq k_1}}^{m}\sum_{\substack{q_k=1\\ q_k\neq k_1}}^{q_j-1} \widehat{\theta}^{(k_1)}(M_{k_1,\, q_j,\, q_k}) + \cdots + \frac{1}{m}\widehat{\theta}^{(k_1)}(M_{1,\, 2,\, \cdots,\, m}).$$

Next, we will examine the distribution of the actual cooperative payoff of core coalition C_T^c. Assume that the strategic combination adopted by core coalition C_T^c is $s^{**}_{C_T^c}\left(s^{**}_{C_T^c} \subseteq s^{**}_{C_T^+}\right)$, which is determined by the public choice game among the extensive members of the coalition, assume that the actual strategic combination choices of other core coalitions are $s^{**}_{-C_T^c}$, the actual cooperative payoff that core coalition C_T^c gets is:

$$\widehat{V}_{C_T^c}\left(s^{**}_{C_T^c},\ s^{**}_{-C_T^c}\right) = \sum_{i=1}^{m^*} u_i\left(s^{**}_{C_T^c},\ s^{**}_{-C_T^c}\right),\ \ i\in C_T^c,$$

where m^* is the actual number of the members of core coalition C_T^c. The cooperative payoff $\widehat{V}_{C_T^c}\left(s^{**}_{C_T^c},\ s^{**}_{-C_T^c}\right) = \sum_{i=1}^{m^*} u_i\left(s^{**}_{C_T^c},\ s^{**}_{-C_T^c}\right)$ is the actual cooperative payoff of core coalition C_T^c that can ultimately be distributed. The cooperative payoff surplus $Gap_{C_T^c} = \widehat{V}_{C_T^c}\left(s^{**}_{C_T^c},\ s^{**}_{-C_T^c}\right) - \widehat{V}_{C_T^c}\left(s^{*}_{C_T^c},\ s^{**}_{-C_T^c}\right)$ is caused by the inappropriate choice of the strategic combination of the core coalition. And the inappropriate choice of the strategic combination of the core coalition is caused by the coalition members' inappropriate choices of the strategic combination of the core coalition.

If the estimations of all the core members of core coalition C_T^c of the optimal strategic combination choice of the core coalition are correct, in coalition equilibrium c^*, when the public choice of strategic combination of core coalition C_T^c is actually optimal response to the actual strategic combination choices of other core coalitions, that is, $s^{**}_{C_T^c} = s^{*}_{C_T^c}$, the cooperative payoff of core coalition C_T^c satisfies $\widehat{V}_{C_T^c}\left(s^{**}_{C_T^c},\ s^{**}_{-C_T^c}\right) = \widehat{V}_{C_T^c}\left(s^{*}_{C_T^c},\ s^{**}_{-C_T^c}\right)$. If the estimations of all the core members of core coalition C_T^c of the optimal strategic combination choice of the core coalition are correct, the distribution of the cooperative payoff of core coalition C_T^c will be carried out according to the distribution rule in the unallied bargaining game of this core coalition (Chen [25]):

$$\widehat{x}_{k_1}^{**}(s_{C_T^c}^{*},\ s_{-C_T^c}^{**}) = \widehat{W}_{k_1}^{**-C_T^c} + \frac{1}{2}\sum_{\substack{q_j=1\\ q_j\neq k_1}}^{m^*} \widehat{\theta}^{**}(M_{k_1,\ q_j}) + \frac{1}{3}\sum_{\substack{q_j=1\\ q_j\neq k_1}}^{m^*}\sum_{\substack{q_k=1\\ q_k\neq k_1}}^{q_j-1} \widehat{\theta}^{**}(M_{k_1,\ q_j,\ q_k}) + \cdots + \frac{1}{m^*}\widehat{\theta}^{**}(M_{1,\ 2,\ \cdots,\ m^*}).$$

Core coalition C_T^c's cooperative payoff $\widehat{V}_{C_T^c}\left(s_{C_T^c}^{**},\ s_{-C_T^c}^{**}\right)$ can be regarded as the "cooperation" outcome of core coalition C_T^c starting from strategic combination $s_{C_T^c}^{*}$ and through the misjudgments of core coalition C_T^c's public choice of strategic combination and the public choices of strategic combinations of other coalitions. The distribution of the cooperative payoff $\widehat{V}_{C_T^c}\left(s_{C_T^c}^{**},\ s_{-C_T^c}^{**}\right)$ should be based on the distribution of the cooperative payoff $\widehat{V}_{C_T^c}\left(s_{C_T^c}^{*},\ s_{-C_T^c}^{**}\right)$, with an additional distribution of the "cooperative" payoff $Gap_{C_T^c} = \widehat{V}_{C_T^c}\left(s_{C_T^c}^{**},\ s_{-C_T^c}^{**}\right) - \widehat{V}_{C_T^c}\left(s_{C_T^c}^{*},\ s_{-C_T^c}^{**}\right)$ which is caused by the misjudgment cooperation among the core members of core coalition C_T^c.

According to an analysis similar to the one in the previous sections, member k_1's cooperative payoff distribution from the misjudgment "cooperation" is:

$$\widehat{x}_{k_1}^{*} = \widehat{W}_{k_1}^{*} + \frac{1}{2}\sum_{\substack{q_j=1\\ q_j\neq k_1}}^{m^*} \widehat{\delta}^{*}(M_{k_1,\ q_j}) + \frac{1}{3}\sum_{\substack{q_j=1\\ q_j\neq k_1}}^{m^*}\sum_{\substack{q_k=1\\ q_k\neq k_1}}^{q_j-1} \widehat{\delta}^{*}(M_{k_1,\ q_j,\ q_k}) + \cdots + \frac{1}{m^*}\widehat{\delta}^{*}(M_{1,\ 2,\ \cdots,\ m^*}).$$

And the actual total cooperative payoff distribution that member k_1 gets is:

$$\widehat{x}_{k_1} = \widehat{x}_{k_1}^{*} + \widehat{x}_{k_1}^{**}.$$

If in the unallied bargaining game on the distribution of the cooperative payoff $\widehat{V}_{C_T^c}\left(s_{C_T^c}^{*},\ s_{-C_T^c}^{**}\right)$ of core coalition C_T^c, information is still asymmetric among coalition members, the coalition equilibrium of the information asymmetric cooperative game with agreements self-implemented does not exist.

Similarly, if information is still asymmetric after the information asymmetric cooperative game with agreements self-implemented is completed, there exists no coalition equilibrium in the bargaining game on the distribution of the cooperative payoff surplus,

$$Gap_{C_T^c} = \widehat{V}_{C_T^c}\left(s_{C_T^c}^{**},\ s_{-C_T^c}^{**}\right) - \widehat{V}_{C_T^c}\left(s_{C_T^c}^{*},\ s_{-C_T^c}^{**}\right),$$

which is caused by the misjudgment "cooperation".

According to an analysis similar to the one in the previous sections, we can get Theorems 8 and 10.

Theorem 8 If the coalitions centralize all the payoffs that their members get in the game to inhibit the opportunistic behaviors in the distribution process, in information asymmetric cooperative game $\Gamma(N,\ \{S_i\},\ \{u_i\})$ with agreements self-implemented, assume that information is still asymmetric after the game is completed, and assume that the above assumption is common knowledge of all the players, there exists no coalition equilibrium under the criterion of maximum expected cooperative payoff distribution (when the members of each coalition trust each other) and no coalition equilibrium under the criterion of minimum expected escape-payoff deriving from deviation (when the members of each coalition trust each other).

Proof. The equilibrium of information asymmetric cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented includes two interrelated aspects: the coalition equilibrium of the game and the distribution equilibrium of the cooperative payoff of each coalition.

If information is still asymmetric after the game is completed, in the bargaining game on the distribution of the cooperative payoff of some coalition, each member will present his requirement for cooperative payoff distribution on the basis of his virtual game, and the sum of the core members' requirements for cooperative payoff distribution do not necessarily equal the actual cooperative payoff of the coalition. That is, the distribution equilibrium of the cooperative payoff of the coalition cannot be achieved; on the other hand, if the distribution equilibria of the cooperative payoffs of the coalitions cannot be achieved, the coalition equilibrium cannot be reached either. □

Herein, that there exists no coalition equilibrium under the criterion of maximum expected cooperative payoff distribution (when the members of each coalition trust each other) in information asymmetric cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented, or, there exists no coalition equilibrium under the criterion of minimum expected escape-payoff deriving from deviation (when the members of each coalition trust each other) in the information asymmetric cooperative game with agreements self-implemented does not mean that there is no cooperative coalition in the game. Some players with a high degree of information symmetry (after the completion of the cooperative game) may still establish cooperative coalitions which aim at exploiting the synergies among them, and reach cooperative payoff distribution agreements with some kinds of compensation mechanisms. In addition, even if the degree of information asymmetry among the players is still high after the completion of the cooperative game, those who agree with each other on the synergy expectations and do not need distribution compensations (perhaps they can set up some compensation mechanisms to benefit from their cooperation) may also reach some distribution agreements and establish cooperative coalitions designed to take advantage of the synergy expectations among them.

When information among the players is still high asymmetric after the completion of the cooperative game, in information asymmetric cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented, there is at least a coalition situation shown as follows which is feasible.

Theorem 9 If the coalitions centralize all the payoffs that their members get in the game to inhibit the opportunistic behaviors in the distribution process, in information asymmetric cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented, assume that information is still asymmetric after the game is completed, and the above assumption is common knowledge of all the players, and that in the cooperative game there exists no compensation mechanism (or, the distribution of any member of a coalition is just the payoff that he gets in the game), the following coalition situation under the criterion of maximum expected payoff (when the members of each coalition trust each other) is feasible:

$$c_i^* = \begin{cases} i, \ \text{if for any } c_i \neq i, \ u_i^{(i)}(i, c_{-i}^*) \geq u_i^{(i)}(c_i, c_{-i}^*), \ \text{or}, \ u_i^{(j)}(c_i, c_{-i}^*) - \widehat{W}_i^{-C_{c_i}(j)} \leq 0, \\ \\ \text{or}, \ u_j^{(i)}(c_i, c_{-i}^*) \leq \widehat{W}_j^{-C_{c_i}(i)} (j \in C_{c_i}, \ j \neq i); \\ \\ \arg\max\limits_{c_i} u_i^{(i)}(c_i, c_{-i}^*), \ \text{if at least for } a \text{ certain } c_i \neq i, \ u_i^{(i)}(i, c_{-i}^*) < u_i^{(i)}(c_i, c_{-i}^*), \ u_i^{(j)}(c_i, c_{-i}^*) - \widehat{W}_i^{-C_{c_i}(j)} > 0, \\ \\ \text{and } u_j^{(i)}(c_i, c_{-i}^*) > \widehat{W}_j^{-C_{c_i}(i)} (j \in C_{c_i}, \ j \neq i). \end{cases}$$

Where $\widehat{W}_i^{-C_{c_i}}$ is the escape-payoff deriving from deviation that member i can get when he escapes from coalition C_{c_i} through deviation.

The proof of Theorem 9 is omitted.

Assume that information is symmetric after the game is completed, in information asymmetric cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented, there exists the mixed strategic coalition equilibrium under the criterion of maximum expected cooperative payoff distribution.

Theorem 10 If the coalitions centralize all the payoffs that their members get in the game to inhibit the opportunistic behaviors in the distribution process, in information asymmetric cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented, assume that information is symmetric after the game is completed, and that the above assumption is common knowledge of all the players, there exists the mixed strategic coalition equilibrium under the criterion of maximum expected cooperative payoff distribution (when the members of each coalition trust each other):

$$c_i^* = \begin{cases} i, \text{ if for any } c_i \neq i, \widehat{x}_i^{(i)}(i, c_{-i}^*) \geq \widehat{x}_i^{(i)}(c_i, c_{-i}^*), \text{ or, } \sum\limits_{\substack{j \in C_{c_i} \\ j \neq i}} [Mv_i^{(j)}(C_{c_i}) - \widehat{W}_i^{-C_{c_i}(j)}] \leq 0, \\ \text{or, } \sum\limits_{\substack{j \in C_{c_i} \\ j \neq i}} [Mv_{T_h}^{(j)}(C_{c_i}) - \sum\limits_{k \in T_h} \widehat{W}_k^{-C_{c_i}(j)}] \leq 0 (i \in T_h \subseteq C_{c_i}); \\ \arg\max\limits_{c_i} \widehat{x}_i^{(i)}(c_i, c_{-i}^*), \text{ if for at least for } a \text{ certain } c_i \neq i, \ \widehat{x}_i^{(i)}(i, c_{-i}^*) < \widehat{x}_i^{(i)}(c_i, c_{-i}^*), \\ \sum\limits_{\substack{j \in C_{c_i} \\ j \neq i}} [Mv_i^{(j)}(C_{c_i}) - \widehat{W}_i^{-C_{c_i}(j)}] > 0, \text{ and } \sum\limits_{\substack{j \in C_{c_i} \\ j \neq i}} [Mv_{T_h}^{(j)}(C_{c_i}) - \sum\limits_{k \in T_h} \widehat{W}_k^{-C_{c_i}(j)}] > 0 (i \in T_h \subseteq C_{c_i}). \end{cases}$$

$\forall i = 1, 2, \cdots, n.$

The proof of Theorem 10 is omitted.

Similarly, we can also draw the conclusion in Theorem 11.

Theorem 11 If the coalitions centralize all the payoffs that their members get in the game to inhibit the opportunistic behaviors in the distribution process, in information asymmetric cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented, assume that information is symmetric after the game is completed, and that the above assumption is common knowledge of all the players, there exists the mixed strategic coalition equilibrium under the criterion of minimum expected escape-payoff deriving from deviation (when the members of each coalition trust each other):

$$c_i^* = \begin{cases} i, \text{ if for any } c_i \neq i, \widehat{W}_i^{(i)}(i, c_{-i}^*) \leq \widehat{W}_i^{(i)}(c_i, c_{-i}^*), \text{or}, \sum_{\substack{j \in C_{c_i} \\ j \neq i}} [Mv_i^{(j)}(C_{c_i}) - \widehat{W}_i^{-C_{c_i}(j)}] \leq 0, \\ \text{or}, \sum_{\substack{j \in C_{c_i} \\ j \neq i}} [Mv_{T_h}^{(j)}(C_{c_i}) - \sum_{k \in T_h} \widehat{W}_k^{-C_{c_i}(j)}] \leq 0 (i \in T_h \subseteq C_{c_i}); \\ \arg\max_{c_i} \widehat{x}_i^{(i)}(c_i, c_{-i}^*), \text{ if at least for } a \text{ certain } c_i \neq i, \widehat{W}_i^{(i)}(i, c_{-i}^*) > \widehat{W}_i^{(i)}(c_i, c_{-i}^*), \\ \sum_{\substack{j \in C_{c_i} \\ j \neq i}} [Mv_i^{(j)}(C_{c_i}) - \widehat{W}_i^{-C_{c_i}(j)}] > 0, \text{ and } \sum_{\substack{j \in C_{c_i} \\ j \neq i}} [Mv_{T_h}^{(j)}(C_{c_i}) - \sum_{k \in T_h} \widehat{W}_k^{-C_{c_i}(j)}] > 0 (i \in T_h \subseteq C_{c_i}). \end{cases}$$

$$\forall i = 1, 2, \cdots, n.$$

If the distribution scheme of each coalition meets the competitive distribution condition, the coalition equilibrium under the criterion of maximum expected cooperative payoff distribution (when the members of each coalition trust each other) is equivalent to the one under the criterion of minimum expected escape-payoff deriving from deviation (when the members of each coalition trust each other).

The proof of Theorem 11 is omitted.

5.2 *Allied bargaining game and the distribution of the cooperative payoff*

If the coalitions centralize all the payoffs that their members get in the game to inhibit the opportunistic behaviors in the distribution process, in information asymmetric cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented, assume that information is symmetric after the game is completed, and that the above assumption is common knowledge of all the players, there exists the coalition equilibrium in the allied bargaining game of each coalition, and there exists the mixed strategic coalition equilibrium in the information asymmetric cooperative game with agreements self-implemented when coalition members are allied in the bargaining games. It is easy to prove that when coalition members are allied in the bargaining games, the coalition equilibrium in information asymmetric cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented is just the same as the one in the information asymmetric cooperative game with agreements self-implemented when coalition members are unallied in the bargaining games, the public choice of strategic combination of each coalition when members of the coalition are allied in the bargaining game is the same as the one when members of the coalition are unallied in the bargaining game.

If information is still asymmetric after the game is completed, there is no Nash equilibrium in the unallied bargaining game of any coalition, therefore, there exists no coalition equilibrium under the criterion of maximum expected cooperative payoff distribution (when the members of each coalition trust each other) in information asymmetric cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented, or, there exists no coalition equilibrium under the criterion of minimum expected escape-payoff deriving from deviation (when the members of each coalition trust each other) in the information asymmetric cooperative game with agreements self-implemented.

Assume that information is symmetric after the game is completed, and that the above assumption is common knowledge of all the players, in the virtual game of player k_1, when the members of core coalition $C_T^{c(k_1)}$ set up the cooperative teams in some coalition situation, the competition among the m core members (m is the number of the members of core coalition $C_T^{c(k_1)}$) of core coalition $C_T^{c(k_1)}$ in the unallied bargaining game is replaced by the competition among the m cooperative teams in the allied bargaining game.

In the virtual game of player k_1, in the allied bargaining game $\Gamma^{(k_1)}(M,\ \{T_i\},\ \{\widehat{x}_i^{(k_1)}\})$ of core coalition $C_T^{c(k_1)}$, let $M_{m_1,\ m_2,\ \cdots,\ m_k}$ denote a subset consisting of cooperative teams $m_1,\ m_2,\ \cdots,\ m_k$ of team set M of core coalition $C_T^{c(k_1)}$ in coalition situation t, $M_{m_1,\ m_2,\ \ldots,\ m_k} \subseteq M(k \le m)$, the common payoff $\widehat{\theta}^{(k_1)}(M_{m_1,\ m_2,\ \cdots,\ m_k})$ of team set $M_{m_1,\ m_2,\ \ldots,\ m_k}$ is defined as following:

$$\widehat{\theta}^{(k_1)}(M_{m_1,\ m_2,\ \cdots,\ m_k}) = \widehat{V}^{(k_1)}_{M_{m_1,\ m_2,\ \cdots,\ m_k}} - \sum_{i=1}^{k} \widehat{W}_{m_i}^{-C_T^{c(k_1)}(k_1)} - \sum \widehat{\theta}^{(k_1)}_{(2)}(M_{m_1,\ m_2,\ \cdots,\ m_k}) - \cdots - \sum \widehat{\theta}^{(k_1)}_{(k-1)}(M_{m_1,\ m_2,\ \cdots,\ m_k}),$$

where $\widehat{V}^{(k_1)}_{M_{m_1,\ m_2,\ \cdots,\ m_k}}$ is the cooperative payoff of core coalition $C_T^{c(k_1)}$ when all the members except those of the teams in set $M_{m_1,\ m_2,\ \cdots,\ m_k}$ have escaped from the coalition and join the same coalition as a whole to maximize their escape-payoff, while members of other coalitions keep their coalition-choosing strategies unchanged in the virtual allied bargaining game of player k_1; $\sum \widehat{\theta}^{(k_1)}_{(j)}(M_{m_1,\ m_2,\ \cdots,\ m_k})$ is the sum of the common payoffs of all the j-team subsets of set $M_{m_1,\ m_2,\ \cdots,\ m_k}$ in the virtual allied bargaining game of player k_1; $\sum_{i=1}^{k} \widehat{W}_{m_i}^{-C_T^{c(k_1)}(k_1)}$ is the sum of the escape-payoffs deriving from deviation of all the core members of the k teams in set $M_{m_1,\ m_2,\ \cdots,\ m_k}$ in the virtual allied bargaining game of player k_1.

In the coalition situation t of the bargaining game in the virtual game of player k_1, the Nash equilibrium in the bargaining game among teams $m_1,\ m_2,\ \cdots,\ m_k$ about the distribution of the common payoff $\widehat{\theta}^{(k_1)}(M_{m_1,\ m_2,\ \cdots,\ m_k})$ is:

$$\widehat{y}_{m_1}^{*(k_1)} = \frac{1}{k}\widehat{\theta}^{(k_1)}\left(M_{m_1,\ m_2,\ \cdots,\ m_k}\right),\ i = 1,\ 2,\ \cdots,\ k.$$

That's to say, the teams that belong to set $M_{m_1,\ m_2,\ \cdots,\ m_k}$ will get the same common payoff distribution.

So, in the virtual game of player k_1, in some coalition situation t of allied bargaining game $\Gamma^{(k_1)}(M,\ \{T_i\},\ \left\{\widehat{x}_i^{(k_1)}\right\})$ of core coalition $C_T^{c(k_1)}$ on the distribution of the cooperative payoff surplus, the cooperative payoff surplus distribution that some cooperative team m_i can get from core coalition $C_T^{c(k_1)}$ is:

$$\widehat{y}_{m_i}^{(k_1)} = \frac{1}{2}\sum_{\substack{m_j=1\\ m_j\neq m_i}}^{m} \widehat{\theta}^{(k_1)}(M_{m_i,\ m_j}) + \frac{1}{3}\sum_{\substack{m_j=1\\ m_j\neq m_i}}^{m}\ \sum_{\substack{m_k=1\\ m_k\neq m_i}}^{m_j-1} \widehat{\theta}^{(k_1)}(M_{m_i,\ m_j,\ m_k}) + \cdots + \frac{1}{m}\widehat{\theta}^{(k_1)}(M_{1,\ 2,\ \cdots,\ m}).$$

The total distribution that team m_i can get is:

$$\begin{aligned}\widehat{x}_{m_i}^{(k_1)} &= \widehat{W}_{m_i}^{-C_T^{c}(k_1)} + \widehat{y}_{m_i}^{(k_1)}\\ &= \widehat{W}_{m_i}^{-C_T^{c}(k_1)} + \frac{1}{2}\sum_{\substack{m_j=1\\ m_j\neq m_i}}^{m} \widehat{\theta}^{(k_1)}(M_{m_i,\ m_j}) + \frac{1}{3}\sum_{\substack{m_j=1\\ m_j\neq m_i}}^{m}\ \sum_{\substack{m_k=1\\ m_k\neq m_i}}^{m_j-1} \widehat{\theta}^{(k_1)}(M_{m_i,\ m_j,\ m_k}) + \cdots + \frac{1}{m}\widehat{\theta}^{(k_1)}(M_{1,\ 2,\ \cdots,\ m}).\end{aligned}$$

If the coalitions centralize all the payoffs that their members get in the game to inhibit the opportunistic behaviors in the distribution process, assume that information is symmetric after the game is completed, and that the above assumption is common knowledge of all the players, in the virtual game of player k_1, if the coalition situation

$t^{*(k_1)} = (t_1^{*(k_1)},\ t_2^{*(k_1)},\ \cdots,\ t_n^{*(k_1)})$ of the virtual bargaining game $\Gamma^{(k_1)}(M,\ \{T_i\},\ \left\{\widehat{x}_i^{(k_1)}\right\})$ of player k_1 is feasible, and the team-choosing strategy of each coalition member is the best response to the collective actions of other coalition members, coalition situation $t^{*(k_1)} = (t_1^{*(k_1)},\ t_2^{*(k_1)},\ \cdots,\ t_n^{*(k_1)})$ is called the coalition equilibrium under the criterion of maximum expected cooperative payoff distribution.

Theorem 12 If the coalitions centralize all the payoffs that their members get in the game to inhibit the opportunistic behaviors in the distribution process, in information asymmetric cooperative game $\Gamma(N,\ \{S_i\},\ \{u_i\})$ with agreements self-implemented, assume that information is symmetric after the game is completed, and that the above assumption is common knowledge of all the players, in the virtual game of player k_1, in allied bargaining game $\Gamma^{(k_1)}(M,\ \{T_i\},\ \left\{\widehat{x}_i^{(k_1)}\right\})$ of core coalition $C_T^{c(k_1)}$ there exists the (mixed strategic) coalition equilibrium under the criterion of maximum expected cooperative payoff distribution:

$$t_i^{*(k_1)} = \begin{cases} i, \text{ if for any } t_i,\ \widehat{x}_i^{(k_1)}\left(i,\ t_{-i}^{*(k_1)}\right) \geq \widehat{x}_i^{(k_1)}\left(t_i,\ t_{-i}^{*(k_1)}\right); \\ \\ \text{argmax } \widehat{x}_i^{(k_1)}\left(t_i,\ t_{-i}^{*(k_1)}\right), \text{ if at least for a certain } t_i \neq i,\ \widehat{x}_i^{(k_1)}\left(i,\ t_{-i}^{*(k_1)}\right) < \widehat{x}_i^{(k_1)}\left(t_i,\ t_{-i}^{*(k_1)}\right). \end{cases}$$

At the same time, in the allied bargaining game $\Gamma^{(k_1)}(M,\ \{T_i\},\ \left\{\widehat{x}_i^{(k_1)}\right\})$ of core coalition $C_T^{c(k_1)}$ there exists the (mixed strategic) coalition equilibrium under the criterion of minimum expected escape-payoff too:

$$t_i^{*(k_1)} = \begin{cases} i, \text{ if for any } t_i,\ w_i^{(k_1)}\left(i,\ t_{-i}^{*(k_1)}\right) \leq w_i^{(k_1)}\left(t_i,\ t_{-i}^{*(k_1)}\right); \\ \\ \text{argmin } \mathrm{w}_i^{(k_1)}\left(t_i,\ t_{-i}^{*(k_1)}\right), \text{ if at least for a certain } t_i \neq i,\ w_i^{(k_1)}\left(i,\ t_{-i}^{*(k_1)}\right) > w_i^{(k_1)}\left(t_i,\ t_{-i}^{*(k_1)}\right). \end{cases}$$

If the distribution schemes of all teams satisfy the competitive distribution condition, the coalition equilibrium under the criterion of maximum expected cooperative payoff distribution is equivalent to the one under the criterion of minimum expected escape-payoff.

The proof of Theorem 12 is omitted.

After the formation of coalition equilibrium t^*, in coalition equilibrium t^* of the bargaining game of core coalition $C_T^{c(k_1)}$, let team set $M_{m_1^*,\ m_2^*,\ \cdots,\ m_k^*}$ denote a subset of team set M^* of core coalition $C_T^{c(k_1)}$, which consists of cooperative teams $m_1^*,\ m_2^*,\ \cdots,\ m_k^*$, $M_{m_1^*,\ m_2^*,\ \cdots,\ m_k^*} \subseteq M^*(k \leq m)$, the common payoff $\widehat{\theta}^{(k_1)}(M_{m_1^*,\ m_2^*,\ \cdots,\ m_k^*})$ of team set $M_{m_1^*,\ m_2^*,\ \cdots,\ m_k^*}$ is defined as following:

$$\widehat{\theta}^{(k_1)}(M_{m_1^*,\ m_2^*,\ \cdots,\ m_k^*}) = \widehat{V}^{(k_1)}_{M_{m_1^*,\ m_2^*,\ \cdots,\ m_k^*}} - \sum_{i=1}^{k} \widehat{W}_{m_i^*}^{-C_T^{c(k_1)}(k_1)} - \sum \widehat{\theta}^{(k_1)}_{(2)}(M_{m_1^*,\ m_2^*,\ \cdots,\ m_k^*}) - \cdots - \sum \widehat{\theta}^{(k_1)}_{(k-1)}(M_{m_1^*,\ m_2^*,\ \cdots,\ m_k^*}),$$

where $\widehat{V}^{(k_1)}_{M_{m_1^*,\ m_2^*,\ \cdots,\ m_k^*}}$ is the cooperative payoff of core coalition $C_T^{c(k_1)}$ when all the members except those of the teams in set $M_{m_1^*,\ m_2^*,\ \cdots,\ m_k^*}$ have escaped from the core coalition and join the same coalition as a whole to maximize their escape-payoff, while members of other coalitions keep their coalition-choosing strategies unchanged in player k_1's virtual game; $\sum \widehat{\theta}^{(k_1)}_{(j)}(M_{m_1^*,\ m_2^*,\ \cdots,\ m_k^*})$ is the sum of the common payoffs of all the j-team subsets of set $M_{m_1^*,\ m_2^*,\ \cdots,\ m_k^*}$; $\sum_{i=1}^{k} \widehat{W}_{m_i^*}^{-C_T^{c(k_1)}(k_1)}$ is the sum of the escape-payoffs deriving from deviation of all the members of the k teams in set $M_{m_1^*,\ m_2^*,\ \cdots,\ m_k^*}$.

According to the distribution rule of common payoff (Chen [25]), in player k_1's virtual bargaining game of core coalition $C_T^{c(k_1)}$, the common payoff distribution that any cooperative team m_i^* in team set $M_{m_1^*, m_2^*, \cdots, m_k^*}$ can get is:

$$\widehat{y}_{m_i^*}^{*(k_1)} = \frac{1}{k}\widehat{\theta}^{(k_1)}\left(M_{m_1^*, m_2^*, \cdots, m_k^*}\right), \ i = 1, 2, \cdots, k.$$

Therefore, in player k_1's virtual bargaining game $\Gamma^{(k_1)}(M, \{T_i\}, \{\widehat{x}_i^{(k_1)}\})$ of core coalition $C_T^{c(k_1)}$, the cooperative payoff surplus distribution that some cooperative team m_i^* of core coalition $C_T^{c(k_1)}$ can get is:

$$\widehat{y}_{m_i^*}^{(k_1)} = \frac{1}{2}\sum_{\substack{m_j^*=1\\ m_j^*\neq m_i^*}}^{m}\widehat{\theta}^{(k_1)}(M_{m_i^*, m_j^*}) + \frac{1}{3}\sum_{\substack{m_j^*=1\\ m_j^*\neq m_i^*}}^{m}\sum_{\substack{m_k^*=1\\ m_k^*\neq m_i^*}}^{m_j^*-1}\widehat{\theta}^{(k_1)}(M_{m_i^*, m_j^*, m_k^*}) + \cdots + \frac{1}{m}\widehat{\theta}^{(k_1)}(M_{1, 2, \cdots, m}).$$

The total cooperative payoff distribution that cooperative team m_i^* of core coalition $C_T^{c(k_1)}$ can get is:

$$\begin{aligned}\widehat{x}_{m_i^*}^{(k_1)} &= \widehat{y}_{m_i^*}^{(k_1)} + \widehat{W}_{m_i^*}^{-C_T^c(k_1)}\\ &= \widehat{W}_{m_i^*}^{-C_T^c(k_1)} + \frac{1}{2}\sum_{\substack{m_j^*=1\\ m_j^*\neq m_i^*}}^{m}\widehat{\theta}^{(k_1)}(M_{m_i^*, m_j^*}) + \frac{1}{3}\sum_{\substack{m_j^*=1\\ m_j^*\neq m_i^*}}^{m}\sum_{\substack{m_k^*=1\\ m_k^*\neq m_i^*}}^{m_j^*-1}\widehat{\theta}^{(k_1)}(M_{m_i^*, m_j^*, m_k^*}) + \cdots + \frac{1}{m}\widehat{\theta}^{(k_1)}(M_{1, 2, \cdots, m}).\end{aligned}$$

Assume that information is symmetric after the game is completed, and that the above assumption is common knowledge of all the players, now we can get the Nash equilibrium of core coalition $C_T^{c(k_1)}$'s allied bargaining game $\Gamma^{(k_1)}(M, \{T_i\}, \{\widehat{x}_i^{(k_1)}\})$ on the distribution of the cooperative payoff, in the coalition equilibrium t^* of the game in player k_1's virtual game.

Assume that in coalition equilibrium c^* of the information asymmetric cooperative game with agreements self-implemented, the public choice of strategic combination of core coalition $C_T^{c(k_1)}$ is $s^{**}_{C_T^{c(k_1)}}$, and the actual public choices of strategic combination of other core coalitions are denoted as $s^{**T}_{-C_T^{c(k_1)}}$, the cooperative payoff actually obtained by core coalition $C_T^{c(k_1)}$ after the game is completed is $\widehat{V}_{C_T^{c(k_1)}}(s^{**}_{C_T^{c(k_1)}}, s^{**T}_{-C_T^{c(k_1)}})$, this cooperative payoff is actually available to be distributed by core coalition $C_T^{c(k_1)}$. If information is symmetric among all the players after the game is completed, and the estimations of all the members of core coalition $C_T^{c(k_1)}$ of the strategic combination choice of the core coalition are correct, that is:

$$s^{*(i)}_{C_T^{c(k_1)}} = s^{*}_{C_T^{c(k_1)}}, \ i \in C_T^{c(k_1)},$$

and accordingly,

$$s^{**}_{C_T^{c(k_1)}} = s^{*}_{C_T^{c(k_1)}},$$

where $s^{*}_{C_T^{c(k_1)}}$ is the best response of core coalition $C_T^{c(k_1)}$ to the strategic combination choices $s^{**T}_{-C_T^{c(k_1)}}$ of other core coalitions.

At this point, the distribution of the cooperative payoff $\widehat{V}_{C_T^{c(k_1)}}(s^{*}_{C_T^{c(k_1)}}, s^{**T}_{-C_T^{c(k_1)}})$ of core coalition $C_T^{c(k_1)}$ will be distributed according to the distribution rule in the information symmetric allied bargaining game of core coalition $C_T^{c(k_1)}$ (Chen [25]):

$$\widehat{x}^{**}_{m_i^*} = \widehat{W}^{-C_T^{c(k_1)}(k_1)}_{m_i^*} + \frac{1}{2}\sum_{\substack{m_j^*=1\\ m_j^*\neq m_i^*}}^{m} \widehat{\delta}^{**}(M_{m_i^*,\, m_j^*}) + \frac{1}{3}\sum_{\substack{m_j^*=1\\ m_j^*\neq m_i^*}}^{m}\sum_{\substack{m_k^*=1\\ m_k^*\neq m_i^*}}^{m_j^*-1} \widehat{\delta}^{**}(M_{m_i^*,\, m_j^*,\, m_k^*}) + \cdots + \frac{1}{m}\widehat{\delta}^{**}(M_{1,\, 2,\, \cdots,\, m}),$$

where $\widehat{W}^{-C_T^{c(k_1)}}_{m_i^*}$ is the sum of escape-payoffs of all the members of cooperative team m_i^*, $\{\widehat{\delta}^{**}(M_{m_i^*,\, m_j^*}),\ \widehat{\delta}^{**}(M_{m_i^*,\, m_j^*,\, m_k^*}),\ \cdots,\ \widehat{\delta}^{**}(M_{1,\, 2,\, \cdots,\, m})$ are respectively the common payoffs of cooperative teams $M_{m_i^*,\, m_j^*}$, $M_{m_i^*,\, m_j^*,\, m_k^*}$, $\cdots$, $M_{1,\, 2,\, \cdots,\, m}$. By analyzing the first level, second level, ... bargaining games of core coalition $C_T^{c(k_1)}$, we can finally get the cooperative payoff distribution $\widehat{x}^{**}_{k_1}$ that any member k_1 of core coalition $C_T^{c(k_1)}$ can get.

If not all judgments of the members of core coalition $C_T^{c(k_1)}$ of the public choices of strategic combination of other core coalitions are correct, then $s^{**}_{C_T^{c(k_1)}} \neq s^{*}_{C_T^{c(k_1)}}$, core coalition $C_T^{c(k_1)}$'s cooperative payoff $\widehat{V}_{C_T^{c(k_1)}}(s^{**}_{C_T^{c(k_1)}}, s^{**T}_{-C_T^{c(k_1)}})$ can be regarded as the "cooperation" outcome of core coalition $C_T^{c(k_1)}$ starting from strategic combination $s^{*}_{C_T^{c(k_1)}}$ and through the misjudgments of core coalition $C_T^{c(k_1)}$'s public choice of strategic combination and the public choices of strategic combinations of other core coalitions. The distribution of the cooperative payoff $\widehat{V}_{C_T^{c(k_1)}}(s^{**}_{C_T^{c(k_1)}}, s^{**T}_{-C_T^{c(k_1)}})$ should be based on the distribution of the cooperative payoff $\widehat{V}_{C_T^{c(k_1)}}(s^{*}_{C_T^{c(k_1)}}, s^{**T}_{-C_T^{c(k_1)}})$, with an additional distribution of the "cooperative" payoff $\widehat{V}_{C_T^{c(k_1)}}\left(s^{**}_{C_T^{c(k_1)}}, s^{**T}_{-C_T^{c(k_1)}}\right) - \widehat{V}_{C_T^{c(k_1)}}(s^{*}_{C_T^{c(k_1)}}, s^{**T}_{-C_T^{c(k_1)}})$ which is caused by the misjudgment cooperation among the members of core coalition $C_T^{c(k_1)}$.

The cooperative payoff distribution of any member k_1 of core coalition $C_T^{c(k_1)}$ in the misjudgment "cooperation" is:

$$\widehat{x}^{*}_{k_1} = \widehat{W}^{*}_{k_1} + \frac{1}{2}\sum_{\substack{q_j=1\\ q_j\neq k_1}}^{m} \widehat{\delta}^{*}(M_{k_1,\, q_j}) + \frac{1}{3}\sum_{\substack{q_j=1\\ q_j\neq k_1}}^{m}\sum_{\substack{q_k=1\\ q_k\neq k_1}}^{q_j-1} \widehat{\delta}^{*}(M_{k_1,\, q_j,\, q_k}) + \cdots + \frac{1}{m}\widehat{\delta}^{*}(M_{1,\, 2,\, \cdots,\, m}).$$

The total cooperative payoff distribution that cooperative team m_i^* of core coalition $C_T^{c(k_1)}$ obtains is:

$$\widehat{x}_{m_i^*} = \widehat{x}^{**}_{m_i^*} + \sum_{j\in m_i^*} \widehat{x}^{*}_{j}.$$

6. Conclusions

In this paper we have examined a one-shot information asymmetric cooperative game with agreements self-implemented, investigated the virtual game of a player, the coalition formation and the bargaining game on the distribution of the cooperative payoff of a coalition.

In the virtual game of a player, in each coalition situation c, he decides his optimal escape strategy under the criterion of maximum expected cooperative payoff distribution on the basis of his estimation of the escape strategies of others. However, his escape strategy itself is also a kind of information release. By analyzing a player's n-level virtual games, we can get his virtual game with information sets stable. Then, analyzing the virtual game of this player as the information sets keeps stable level by level until a stable solution appears, we get the coalition equilibrium of the virtual game of this player. Of course, due to different information sets, the coalition equilibria of the virtual games of different players are different. However, the information transmission, communication, and negotiation between the players can ultimately lead to the convergence of the coalition equilibria of the virtual games of all players.

Ignoring the opportunistic behaviors in the distribution process, assume that information is still asymmetric after the game is completed, whether the coalition members are allied in the bargaining games or not, there exists no distribution equilibrium in the bargaining game of a coalition. At the same time, there exists no coalition equilibrium in the information asymmetric cooperative game with agreements self-implemented. Of course, this does not mean that there is no form of cooperation in the game.

Ignoring the opportunistic behaviors in the distribution process, in an information asymmetric cooperative game with agreements self-implemented, assume that information is symmetric after the game is completed, whether the coalition members are allied in the bargaining games or not, there exists the distribution equilibrium in the bargaining game of each coalition, and there exists the coalition equilibrium under the criterion of maximum expected cooperative payoff distribution (when the members of each coalition trust each other) and also the coalition equilibrium under the criterion of minimum expected escape-payoff deriving from deviation (when the members of each coalition trust each other) in the game. If the distribution rule of each coalition meets the competitive distribution condition, the above two coalition equilibria are equivalent.

Ignoring the opportunistic behaviors in the distribution process, when members are unallied in the bargaining game, in the coalition equilibrium of the game (if it does exist), the distribution of a core coalition's actual cooperative payoff would be based on the distribution of the maximum cooperative payoff at the optimal strategic combination of this core coalition when all its members judge the strategic combination of their coalition correctly, with an additional distribution of the "cooperative" payoff caused by the misjudgment "cooperation" between its core members. When the core members are allied in the bargaining games, there exists the coalition equilibrium under the criterion of minimum expected escape-payoff, or the coalition equilibrium under the criterion of minimum expected escape-payoff which is equivalent to the former. In the bargaining game among the allied teams of a core coalition, the distribution of the coalition's actual cooperative payoff should similarly be based on the distribution of the maximum cooperative payoff at the optimal strategic combination of the coalition when all the teams judge the strategic combination of their coalition correctly, with an additional distribution of the "cooperative" payoff caused by the misjudgment "cooperation" between all the teams.

When coalitions centralize all the payoffs that their members get in the game to prevent opportunistic behaviors in the distribution process, the escape payoff deriving from deviation of a player, the cooperative payoff of his target coalition, and his marginal contribution to the target coalition will change when he escapes through deviation from the coalition he belongs to. However, by similar analysis, the coalition equilibrium of the game and the condition for its existence, as well as the distribution equilibrium of the cooperative payoff of a coalition under the coalition equilibrium are similar to those when opportunistic behaviors in the distribution process are ignored.

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Conflict of interest

There is no conflict of interest in this study.

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