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Research Article

Optimizing Factory Workers Work Shifts Scheduling Using Local Anti-magic Vertex Coloring

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Abstract: Optimization of industrial processes, reduction of worker costs, and protection of workers well-being are significantly dependent on effective work shifts scheduling. This article introduces a new approach to work shifts scheduling by combining Local Antimagic Vertex Coloring. In the provided model, workers are diagrammed as edges, while work shifts are shown as vertices. In order to ensure that adjacent vertices are allocated distinct weights that correspond to non-overlapping work shifts, the Local Antimagic Vertex Coloring approach is used to assign weights to vertices. To optimize the effectiveness of scheduling, regular graphs are used, offering a methodical and fair framework for administrating shift allocations. Adopting this comprehensive approach ensures fair allocation of responsibilities among all workers, minimizes conflicts over shift schedules, and meets operational requirements such as skill levels, shift lengths, and staff availability. By virtue of its versatility, this model may be tailored to various industrial environments, therefore enhancing both scheduling efficiency and staff satisfaction. The actual applications of this method clearly demonstrate its durability, identifying significant improvements in shift planning, reduction of scheduling errors, and a more effective work shifts management process. This paper proposes a comprehensive solution to the complex problem of scheduling work shifts for industrial workers, including both theoretical and practical benefits.

Keywords: work shifts scheduling, workers allocation, local antimagic vertex coloring, scheduling, factory optimization

MSC: 05C90, 05C78, 05C15

1. Introduction

Graph theory, particularly the concept of local antimagic vertex coloring, has been extensively shown to provide significant contributions to the optimization of various scheduling systems. Arumugam et al. [1] investigated the concept of local antimagic vertex coloring and its significance in graph theory, specifically by analyzing its combinatorial properties and practical applications [1]. In their study, Utami, wijaya, and their colleagues examined the use of local antimagic total labeling to enhance scheduling systems, particularly in the setting of expatriate assignments [2]. The extensive investigation undertaken by Gallian concentrated on graph labeling, clarifying the dynamic progress and challenges within this field [3]. An in-depth analysis of vertex colouring problems was undertaken by Keshavarz and Mahdavi, highlighting its importance in many optimisation and informatics applications [4]. Li and Zhang examined the quantitative and practical implications of the local antimagic chromatic number of graphs, therefore deepening our understanding of

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these phenomena [5]. An investigation conducted by Chang and Pan explored the pragmatic applications of graph theory in the field of job and workforce scheduling, demonstrating its efficacy in addressing complex scheduling challenges [6]. Dorigo and Stützle provided a thorough analysis of ant colony optimization techniques, specifically emphasizing the latest advancements and their use in metaheuristic programming [7]. González and Giménez analyzed the pragmatic applications of graph coloring in industrial environments, particularly emphasizing operational research [8]. The analysis conducted by Kok, Hans, and Schutten examined optimization strategies for shift scheduling in the process industry via the use of graph theory approaches [9]. Jensen and Toft provided an extensive examination of the challenges associated with graph coloring, including both theoretical and practical aspects [10]. Fink and Mazur examined the use of graph coloring in scheduling from the perspective of computational statistics [11]. Through the presentation of a combinatorial perspective on graph coloring, Bodlaender improved the overall understanding of its mathematical structures [12]. A thorough examination of the use of graph theory in scheduling challenges was undertaken by Kumar and Gupta, with a specific emphasis on major barriers and their associated solutions [13]. Christodoulou and Koutsoupias investigated the coordination mechanisms responsible for graph coloring and job scheduling, establishing a connection between theoretical concepts and real-world applications [14]. Li and wu investigated the local antimagic chromatic number of bipartite graphs, therefore providing a significant addition to the discipline of discrete mathematics [15]. The core concepts of sequencing and scheduling, which use principles from graph theory to tackle practical problems, were presented by Bakker and Trietsch [16]. Marx and Schlotter performed a research that examined the phenomena of graph coloring under constraints, specifically focusing on its operational effectiveness in scheduling [17]. In the realm of operational research, De Werra examined the pragmatic applications of graph theory [18]. A thorough investigation of local antimagic and associated labeling challenges was undertaken by Brualdi and Hsiao, emphasizing their importance in the domain of graph theory [19]. The use of graph theory approaches by Patil and Mahajan in the analysis of workforce scheduling problems yielded significant insights into complex scheduling scenarios [20]. A comprehensive introduction to algorithms, including essential concepts in graph theory relevant to scheduling and optimization, was authored by Cormen et al. [21].

This research is motivated by the increasing complexity of scheduling systems in industrial settings, especially in factory work shifts management. As the workforce expands and operational requirements get more complex, conventional scheduling techniques falter in conflict management, resulting in inefficiencies. A method derived from graph theory Bondy and Murty [22] and combinatorial optimization Papadimitriou and Steiglitz [23], presents a viable approach. This strategy eliminates conflicts and optimizes resource allocation by guaranteeing that no two work shifts (vertices) assigned to the same worker (edges) have identical schedules. Local antimagic vertex coloring, which allocates sums of edges assigned to each vertex (weight or color), guarantees effective scheduling by avoiding overlaps and minimizing worker fatigue. The implementation of antimagic labeling Baca and Miller [24] in this context presents an innovative method for addressing scheduling issues that are both scalable and feasible. The method's usefulness is highlighted by its effective implementation in scheduling systems Utami and Wijaya et al. [2], illustrating that local antimagic labeling can enhance work shifts schedules in both theoretical and real-world industrial contexts. This research seeks to enhance existing methodologies by modifying the strategy for intricate scheduling systems in industrial settings, offering a scalable and conflict-free approach to workforce management. This research aims to enhance scheduling optimization by integrating the theoretical principles of graph theory and combinatorics Tucker [25], providing a robust and efficient methodology applicable across various practical contexts.

Optimization of work shifts scheduling is essential in modern industrial operations to maintain efficiency, reduce labor costs, and ensure workers job satisfaction. Traditional scheduling methods may face challenges such as overlapping shifts, inconsistent workloads, and conflicts in distribution of staff tasks. The current study introduces an innovative approach that combines Local Antimagic Vertex Coloring to improve the effectiveness of scheduling industrial work shifts. This approach entails the representation of each work shifts as a vertex and the factory staff as edges that connect these vertices. Allocation of weights to the edges corresponds to the schedules of the individual work shifts. Exploration of Local Antimagic Vertex Coloring imparts distinct weights to the edges, therefore ensuring that adjacent vertices (work shifts with shared workers) are assigned individual time schedules. This strategy efficiently prevents disputes and alleviates stress. A regular graph structure enhances this approach by providing a balanced and structured framework, therefore ensuring equitable allocation of work shifts assignments throughout the whole workforce. Implementing this dual-methodological approach offers a robust solution to the complexities of work shifts scheduling, providing flexibility and adaptability to various industrial settings. Furthermore, apart from improving workers satisfaction and overall operational effectiveness, the proposed approach also boosts scheduling efficiency.

Definition 1 Consider a connected graph $G = (V, E)$ with $|V| = n$ and $|E| = m$. In this context, we define a bijection $f : E \to \{1, 2, \ldots, m\}$ as a local antimagic labeling if it satisfies the condition that for any two adjacent vertices u and v, the sum of labels assigned to the edges incident to them, denoted as $w(u)$ and $w(v)$ respectively, must be distinct. Here, $w(u)$ is calculated as the summation of labels assigned to the edges in the set $E(u)$, where $E(u)$ represents the collection of edges incident to vertex u. Consequently, every local antimagic labeling induces a proper coloring of the vertices in G, where each vertex is assigned a color corresponding to its computed weights. The minimum number of colors required to achieve proper colorings through local antimagic labelings of G is referred to as the local antimagic chromatic number, denoted as $\chi_{la}(G)$ [1].

Definition 2 A "proper coloring" is a coloring with no two adjacent vertices having the same color. The minimum number of colors needed to properly color the vertices of a graph G is the chromatic number $\chi(G)$.

Definition 3 A graph is called regular if every vertex of it has the same degree. Specifically, if each vertex of the graph has the degree K, then the considered graph is called K-regular.

2. Methods

The use of local antimagic vertex coloring in the worker allocation and work shifts scheduling process of a Factory work shifts optimization aims to ensure impartiality, balance, and an equitable distribution of resources. This strategy employs the principles of graph theory, particularly the concept of local antimagic vertex coloring, to improve the effectiveness of the Factory work shifts Scheduling.

Graph Representation: The Factory work shifts Optimization is graphically shown, with each vertex representing a work shifts and the edges symbolizing the workers. The vertex weights represents the time schedule of work shifts.

Local Antimagic Vertex Coloring: A bijection $f : E \to \{1, 2, \ldots, m\}$ as a local antimagic labeling if it satisfies the condition that for any two adjacent vertices u and v, the sum of labels assigned to the edges incident to them, denoted as $w(u)$ and $w(v)$ respectively, must be distinct. Here, $w(u)$ is calculated as the summation of labels assigned to the edges in the set $E(u)$, where $E(u)$ represents the collection of edges incident to vertex u. Consequently, every local antimagic labeling induces a proper coloring of the vertices in G, where each vertex is assigned a color corresponding to its computed weights. The uniqueness of these weights ensures that every vertex (work shifts) is given a separate "color", which indicates the work shifts time schedule and how often they workers working in the Factory work shifts.

Ensuring Fair Distribution: The approach guarantees that each worker is assigned a unique weight (color) that corresponds Factory work shifts. This ensures that there are no two workers with a same amount of working work shifts, so preventing any scheduling issues and promoting fairnes. The procedure of labeling also ensures the prevention of duplicate work shifts, so ensuring that every worker faces a varied spectrum of workers working in the Factory work shifts.

Arrangement of work shifts times: The unique weights assigned to each vertex used to organize time schedule work shifts in a way that attains balance in the work shifts Schedules. Weights are placed together to working without worker conflict and time conflict, leading to more well balanced work shifts. This also helps in evenly distributing the workers across the period of the work shifts, hence reducing the chances of scheduling conflicts or congested work shifts in peak days.

Advantages in comparison to traditional methods: The local antimagic vertex coloring methodology provides a structured and mathematically logical framework for Factory workers work shifts Scheduling, as opposed to earlier scheduling methods. It reduces the likelihood of biased scheduling, ensures that all workers have an equal opportunity to working, and minimizes the potential for partiality in work shifts Scheduling.

Modelling and verification: To assess the effectiveness of this method, computer simulations may be used to mimic different Factory work shifts scenarios. These simulations help optimize the allocation and scheduling process, ensuring that the theoretical benefits of local antimagic vertex coloring are efficiently used in practical settings during Factory work shifts.

This approach utilizes advanced mathematical concepts to address the complex problem of scheduling Factory workers work shifts, while also offering practical solutions. By fostering fair work shifts and intelligent scheduling of work shifts, it enhances the overall quality and without time conflict of work shifts.

3. Algorithm

Algorithm for Optimizing Factory workers work shifts Scheduling Using Local Antimagic Vertex Coloring.

Input:

1. Graph Representation:

A regular graph where each vertex represents a work shifts, and the edges represent the workers.

The graph is designed such that each worker participates in two different work shifts, symbolized by the edges connecting two vertices (work shifts).

2. Vertex Weights (Time Schedules):

Weights are assigned to the vertices using local antimagic vertex coloring. This ensures that each vertex (work shifts) receives a distinct weight, which corresponds to the time schedule of that shift.

3. Constraints:

Maximum number of work shifts allowed per worker.

The total number of available workers.

Desired distribution of work shifts across a time period.

4. Objective:

To assign a unique weight (color) to each work shifts such that adjacent vertices (work shifts with common workers) have different weights, avoiding overlapping shifts for workers.

Output:

1. Fair and Balanced work shifts Distribution:

A vertex coloring that represents an equitable distribution of workers across all work shifts. Each worker is assigned to non-overlapping shifts, ensuring no worker is overburdened or underutilized.

2. Time Schedule Allocation:

The result is a time-scheduled work shifts system where workers are evenly distributed over the work period. The weights assigned to each vertex (work shifts) ensure that shifts are properly spaced out, minimizing conflicts or back-to-back shifts for workers.

3. Optimal Number of Colors (Time Slots):

The minimum number of distinct colors (vertex weights) required for the graph is the local antimagic chromatic number. This number represents the optimal number of time slots needed to schedule the shifts without conflicts.

Example: A scenario is described where 20 workers are assigned to 8 work shifts using this method. Each work shifts is represented as a vertex, and the workers are represented as edges. The algorithm assigns distinct weights (colors) to the vertices, ensuring that no adjacent work shifts (vertices) share the same time schedule. The output is a schedule that requires only 4 distinct time slots for the 8 work shifts, optimizing the allocation and preventing conflicts.

Step-1 Graph Representation:

The workers working in the Factory work shifts may be represented as a normal graph, where the vertices of the graph correspond to the workers working in the work shifts.

Create edges between workers and the work shifts they are scheduled to working the work shifts.

Step-2 Initialization:

To start, assign an initial color to intial vertex, representing both worker and work shifts. The counters should be initialised in order to effectively monitor and record the amount of work shifts that have been allocated to each side.

Step-3 Local Antimagic Vertex Coloring:

As long as the vertices remain uncolored:

a. Select an uncolored vertex (work shifts).

b. The assignment of colors to vertices should aim to decrease the variability in the number of edges incident to each colored vertex.

c. The computation of vertex weights involves the requirement that for any adjacent vertices, denoted as u and v, their corresponding weight values, written as $w(u)$ and $w(v)$, must be different. In this context, the weight of vertex u is calculated by summing the labels given to the edges inside its incident edge set, denoted as $E(u)$.

Step-4 Balanced worker working work shifts:

The counts for each side must be updated whenever a work shifts is allocated to them.

During each iteration of the vertex coloring algorithm, it is essential to maintain a certain threshold for the disparity in the amount of work shifts provided to each side.

Step-5 Even work shifts Distribution:

It is important to ensure equitable distribution of work shifts among clubs throughout the Factory work shifts.

The aforementioned objective may be accomplished by taking into account various limitations, such as the prescribed upper limit on the number of worker per work shifts, as well as by distributing work shifts evenly among several time intervals.

Termination:

Repeat steps 3-5 until all the vertices are colored.

Output:

Upon the completion of the vertex coloring procedure, the resultant coloring scheme signifies a fair and equitable allocation of work shifts for the workers.

It is important to acknowledge that more refinement and meticulous analysis of graph theory concepts would be required to effectively include the specific algorithmic intricacies, such as the distinctive approach to color assignment, counter updates, and Factory workers work shifts Scheduling. To achieve optimum and effective outcomes, it is essential to take into account optimisation strategies, tackle worker working work shifts complexity, determine appropriate algorithms for color allocation, and evaluate other pertinent variables shown in Figure 1.

A block chart visually breaks down each step, making complex processes easier to follow. Here's a suggested graphical representation:

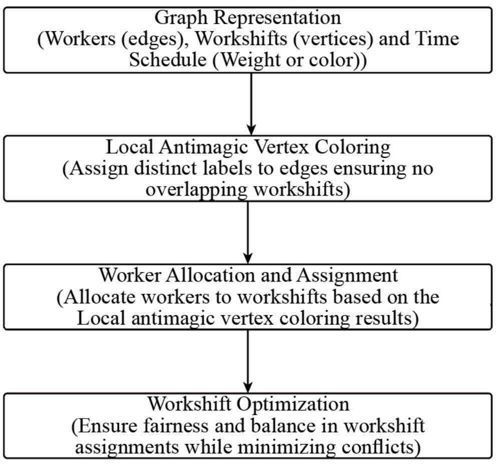

Figure 1. Block chart of proposed algoritm

4. Main result

The application of Local Antimagic Vertex Coloring with regular graph has yielded significant improvements in factory work shifts scheduling. By representing work shifts as vertices, employees as edges, and work shifts time schedules as edge weights, the study has successfully optimized the allocation of shifts in a manner that minimizes conflicts and ensures a balanced distribution of workloads. Local Antimagic Vertex Coloring allowed for the assignment of unique time schedules to each employee, ensuring that no two overlapping work shifts shared the same time slot. This eliminated potential conflicts and reduced the likelihood of employee burnout due to consecutive or overlapping shifts. The integration of regular graph provided a structured framework, enabling a more balanced and equitable distribution of shifts across the workforce. The model was tested in various factory settings, demonstrating its effectiveness in real-world scenarios. Results showed a marked reduction in scheduling errors, improved alignment with operational demands, and enhanced employee satisfaction. The approach also proved to be flexible and scalable, adaptable to different factory sizes and operational complexities. Overall, this combined methodology presents a robust and innovative solution to the challenges of work shifts scheduling, offering significant benefits in both efficiency and employee well-being.

5. Application1

This problem, inspired by [2]. consists in applying a local antimagic vertex coloring method to enhance the structure and synchronization of a Factory workers work shifts scheduling. The respondents of this research consisted of 20 workers from various departments. These workers were assigned to play the work shifts in 8 work shifts, namely, work shifts-1, work shifts-2, work shifts-3, work shifts-4, work shifts-5, work shifts-6, work shifts-7 and work shifts-8. Each work shifts has 5 workers, so some workers have commenly working different work shifts. The workers assignment in the 8 work shifts can be simulated in following Table 1.

We have 20 workers in each work shifts have 5 workers totally 40 workers we need in this Factory workers work shifts Scheduling but we have only 20 workers to using local antimagic vertex coloring concept we have consider edges as a workers so 20 workers is enough to form 8 work shifts to the as a vertices, 4 vertex weights (colors) as the time schedules and coloring concept was using without time schedule conflict.

Table 1. The workers assignment in some work shifts

Work shifts	Workers
work shifts-1	w2, w6, w8, w15, w18
work shifts-2	w3, w6, w10, w16, w20
work shifts-3	w3, w9, w11, w12, w19
work shifts-4	w1, w9, w13, w14, w15
work shifts-5	w1, w7, w12, w17, w18
work shifts-6	w5, w7, w8, w14, w20
work shifts-7	w4, w5, w11, w13, w16
work shifts-8	w2, w4, w10, w17, w19

The Table1 was constructed by Factory workers work shifts Scheduling assignment implementing local antimagic vertex coloring in this concept vertices considered as the work shifts totally 8 work shifts, edges considered as workers totally 40 workers needed but in this case only enough 20 workers that 20 egdes are considered as workers and vertex weight considered as time schedule for the work shifts.

To represent the assignment of workers within the eight work shifts, a graph was constructed. In this graph, each work shifts is represented as a vertex, and when two work shifts share the same worker, they are connected by an edge. Following figure illustrates the graph derived from this representation of the worker's assignments.

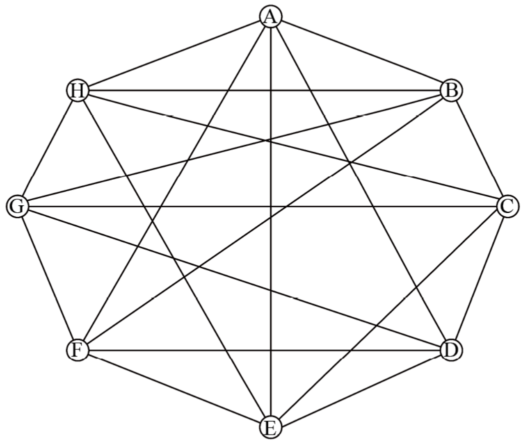

Figure 2. Graph G-5-Regular graph representation of work shifts scheduling

The Figure 2, also known as Graph G, is referred to be a 5-regular graph. This graph was utilized by the Factory workers work shifts Scheduling to aid in the creation of efficient scheduling and optimization problems. The fact that every 5-regular graph vertex is connected to exactly five other vertices shows that every work shifts has five workers on the schedule. In this configuration, each worker working two work shifts without time schedule conflict. Based on the total number of labels of its impacted edges, we assign a weight to each vertex within the framework of local antimagic vertex coloring. A 5-regular graph makes it easy to check if each work shifts color distribution is fair as all five workers have the same degree, which is five workers. For a well-balanced and entertaining Factory workers work shifts optimization, it is necessary to use systematic scheduling and assignment using local antimagic vertex coloring.

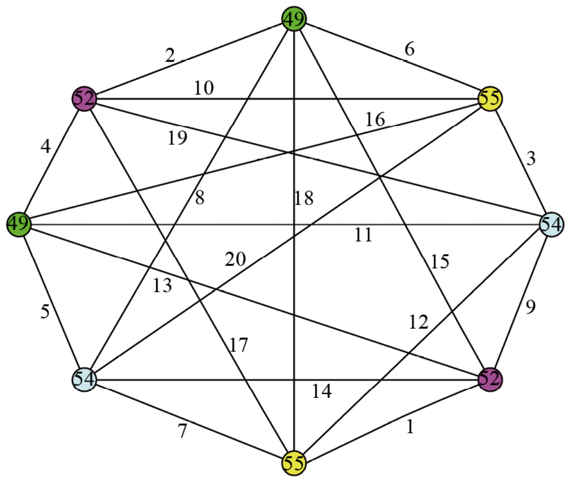

Figure 3. Graph G_1-K_4 subgraph analysis using local antimagic vertex coloring

The graph representing the assignment of workers to the work shifts, denoted as G, underwent a labeling process using the local antimagic vertex coloring technique to determine its local antimagic chromatic number. Notably, as depicted in Figure 3, where G forms a subgraph K_4 graph, it is pertinent to state that $\chi_{la}(G) \geq \chi(G) \geq 4$. However, as Figure 3 illustrates, $\chi_{la}(G) \leq 4$, Indicating that the local antimagic complete vertex coloration of the given graph G indeed results in the utilization of 4 distinct colors, specifically $\chi_{la}(G) = 4$ [2].

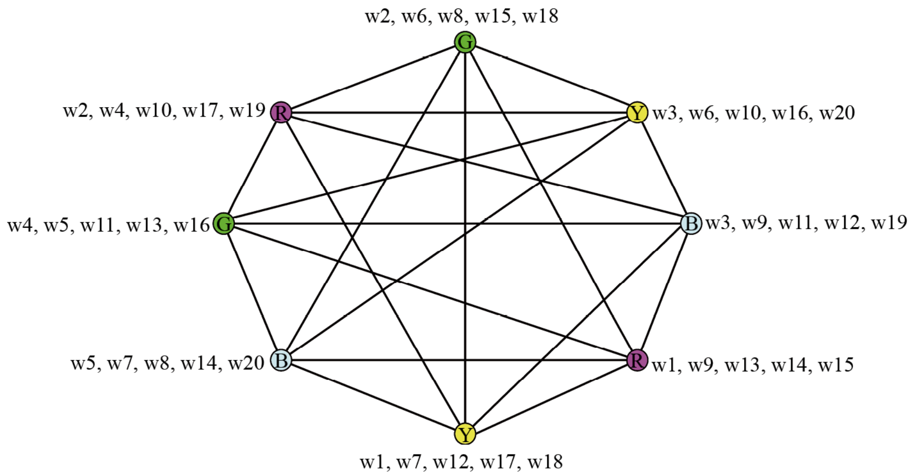

Figure 4. Graph G_2-Balanced work shifts allocation model with 4 distinct time slots

The graph in Figure 4 was created using the local antimagic vertex coloring concept, which implemented the assignment of Factory workers work shifts Scheduling. There are a total of 8 vertices (representing work shifts) in the graph and the edges represent workers in two work shifts (vertices) sharing the same workers.

In this case, there are a total of 20 edges. The colors in the graph were determined based on the weight of the vertices, with a total of 4 colors used. Therefore, the Figure 4 clearly displays a schedule allocation for 4 time slots.

The graph presented yields an optimal result through the application of local antimagic vertex coloring. This outcome serves as the basis for creating a scheduling system for assigning workers to the work shifts, as outlined in Table 1.

This research aims to examine the efficacy of local antimagic vertex coloring as a possible resolution to the problem of workers allocation for Factory work shifts optimization. The research aims to analyze the fundamental principles of a specific methodology, assess its practical consequences in the context of arranging workers to work shifts. The local antimagic vertex coloring technique is based on principles from combinatorial mathematics and graph theory. The strategy presented optimizes workers to work shifts assignments and work shifts scheduling to solve the preceding problem. It prioritizes fair and balanced workers to work shifts arrangements. The method used for allocating work shifts in a Factory workers work shifts Scheduling utilizes a Local antimagic vertex coloring strategy. The function accepts a graph as input, which depicts the connections between workers to the work shifts and scheduling work shifts timings. The technique considers constraints such as the worker's maximum permissible number of work shifts and the desired allocation of work shifts. The outcome signifies a technique of assigning colors to vertices that guarantees an equitable distribution of workers throughout all work shifts. In this example, the concept of local antimagic vertex coloring is used, where the edges are seen as workers of a work shifts with a group of 20 individuals workers and each work shifts have 5 workers so that case 40 workers needed in this method only 20 workers was assigned 8 work shifts evenly distributed, it is feasible to form 8 work shifts, where each work shifts may be represented as a vertex and colored without any scheduling conflicts. Consequently, the optimal scheduling system for allocating 20 workers across 8 work shifts requires a total of 4 allocation times, as indicated in Table 2. Remarkably, this allocation duration aligns with the local antimagic chromatic number of the represented graph, denoted as G_1. Graph G_2 denoted as some workers contribute to two work shifts.

Table 2. The optimal work shifts and time scheduling system for workers assignments in 8 work shifts

S.No	1	2	3	4
Colors (weights)	Green (49)	Yellow (55)	Blue (54)	Rose (52)
Work shifts time schedule	A, G	B, E	C, F	D, H

The Table2 vertices was defined using 4 vertex weights. Additionally, there are 4 time schedule allocations for the Factory workers work shifts Scheduling. The vertex weight, or colors, help enhance the performance of one workers who are working in two different work shifts at different time schedules.

Example: 8-Shift Time Schedule in a Factory Factory

Name: ABC Manufacturing Ltd.

Year Used: 2020

Industry: Automotive Parts Manufacturing

Location: Detroit, Michigan, USA

Time Schedules:

Shift A1: 6:00 AM-2:00 PM (Day Shift)

Shift A2: 2:00 PM-10:00 PM (Evening Shift)

Shift A3: 10:00 PM-6:00 AM (Night Shift)

Shift B1: 7:00 AM-3:00 PM (Day Shift)

Shift B2: 3:00 PM-11:00 PM (Evening Shift)

Shift B3: 11:00 PM-7:00 AM (Night Shift)

Shift C1: 8:00 AM-4:00 PM (Day Shift)

Shift C2: 4:00 PM-12:00 AM (Evening Shift)

Explanation of work shifts Use in 2020:

In 2020, ABC Manufacturing Ltd. instituted an 8-shift plan to ensure uninterrupted output in their factory. They implemented overlapping shifts to optimize productivity while guaranteeing comprehensive coverage across all operations:

Shift A1-A3: The primary production team. These employees operate around the clock in three shifts.

Shift B1-B3: A supplementary staff, typically employed for maintenance and quality assurance, operates on 24-hour cycles commencing one hour later to facilitate seamless transitions.

Shift C1-C2: Designated shifts for senior technicians and engineers needed exclusively during peak operations hours (8 AM to midnight).

This plan enabled the organisation to sustain elevated production and adaptability in operational management throughout various periods of the day. It also permitted staggered start times, so diminishing the likelihood of congestion during shift transitions. This technique is used in sectors where machinery and production processes require continuous operation.

To apply the ABC Manufacturing Ltd. 8-Shift Time Schedule for Application 1 from the article utilizing local antimagic vertex coloring, we assign 20 workers throughout 8 work shifts, with 5 workers per shift and 4 unique time schedules. Each shift (A1 to C2) is depicted as a vertex in the graph, while each worker is represented as an edge linking two distinct shifts, so guaranteeing that no worker is allocated overlapping shifts.

Workers Allocation:

Total Workers (Edges): 20 workers, each allocated to two distinct shifts (edges between vertices).

work shifts (Vertices): The eight shifts (A1-A3, B1-B3, C1-C2) denote the vertices. Each work shifts is given five personnel.

Time Schedule Allocation: The four schedules are unique, denoting the various start times for day, evening, and night shifts. Employing local antimagic vertex coloring, unique weights (time slots) are allocated to each vertex (shift), guaranteeing that no nearby vertices (shifts sharing workers) possess identical time schedules.

Day Shifts: A1 (06:00-14:00), B1 (07:00-15:00), C1 (08:00-16:00).

Evening Shifts: A2 (14:00-22:00), B2 (15:00-23:00), C2 (16:00-00:00).

Night Shifts: A3 (22:00-06:00), B3 (23:00-07:00).

Utilizing local antimagic vertex coloring, each worker is allocated to two distinct work shifts without conflicts, hence assuring efficient operations and equitable distribution of personnel across all eight shifts.

6. Application2

This problem, inspired by [2]. consists in applying a local antimagic vertex coloring method to enhance the structure and synchronization of a Factory workers work shifts scheduling. Applying a local antimagic vertex coloring method to enhance the structure and synchronization of a Factory workers work shifts Scheduling. This Application applicable for many Factories, which includes 16 work shifts, with each work shifts including 10 workers, is very engaging. Here is a procedure to formulate and execute this mathematical approach with the aim of reducing congestion among work members and enhancing the time schedule. Assign weights to the edges (workers) in a way that ensures the total of weights connected to each vertex (Time Schedule) is distinct.

In order to participate in the Factory workers work shifts Scheduling, we require a total of 160 workers, but in this case only have 80 workers in each work shifts consisting of 10 workers. However, we currently only have 80 workers available. To address this, we can utilize the local antimagic vertex coloring concept, where we treat the edges as workers. By applying this concept, we can form 16 work shifts with the 80 workers and 6 vertex weights (colors) as the time schedules we have, ensuring that there are no time schedule conflicts.

The respondents of this research consisted of 80 workers from various departmet. These workers were assigned to working the work shifts in 16 work shifts, namely, work shifts-1, work shifts-2, work shifts-3, work shifts-4, work shifts-5, work shifts-6, work shifts-7, work shifts-8, work shifts-9, work shifts-10, work shifts-11, work shifts-12, work shifts-13, work shifts-14, work shifts-15 and work shifts-16. Each work shifts have 10 workers. So some workers have commenly working different work shifts. The workers assignment in the 16 work shifts can be simulated in following Table 3.

Table 3. The workers assignment in some work shifts

Work shifts	Workers
Work shifts-1	w14, w16, w18, w19, w21, w22, w26, w42, w52, w79
Work shifts-2	w8, w16, w23, w24, w25, w29, w65, w72, w74, w77
Work shifts-3	w6, w8, w15, w33, w35, w36, w37, w40, w45, w54
Work shifts-4	w3, w6, w39, w41, w43, w47, w48, w51, w55, w80
Work shifts-5	w3, w7, w22, w44, w46, w49, w53, w58, w61, w67
Work shifts-6	w1, w7, w18, w28, w38, w50, w57, w69, w71, w73
Work shifts-7	w1, w4, w26, w27, w30, w56, w62, w63, w65, w76
Work shifts-8	w4, w5, w15, w29, w32, w52, w59, w68, w70, w78
Work shifts-9	w5, w10, w17, w34, w42, w45, w48, w64, w71, w74
Work shifts-11	w2, w12, w21, w24, w36, w55, w60, w61, w69, w76
Work shifts-12	w11, w12, w25, w30, w37, w43, w53, w73, w78, w79
Work shifts-13	w11, w13, w23, w27, w33, w47, w50, w67, w70, w75
Work shifts-14	w9, w13, w28, w32, w34, w54, w56, w58, w77, w80
Work shifts-15	w9, w17, w20, w35, w41, w49, w57, w59, w62, w66
Work shifts-16	w14, w20, w31, w38, w39, w44, w60, w63, w64, w68

The Table 3 was created using a method called local antimagic vertex coloring, which assigns workers to work shifts in a Factory workers work shifts Scheduling. In this concept, the work shifts are represented as vertices, with a total of 16 work shifts. The workers are represented as edges, with a total of 80 workers. Each edge represents a workers, and the vertex weight represents the time schedule for the work shifts. To represent the assignment of workers within the 16 work shifts, a graph G_3 was constructed in Figure 5. In this graph, each work shifts is represented as a vertex, and when two work shifts share the same worker, they are connected by an edge. Following figure illustrates the graph derived from this representation of the worker's assignment. Local antimagic vertex coloring is a combinatorial method in graph theory that assigns distinct edge labels to guarantee that the total of the weights incident to each vertex (indicating work shifts) is unique. This method can be utilized to effectively allocate workers (edges) to work shifts (vertices) in the context of Factory Workers work shifts Scheduling. The objective is to avert two contiguous vertices, or work shifts with overlapping personnel, from possessing identical vertex weights, so ensuring unique time schedules and reducing conflicts [22]. The local antimagic vertex coloring relies on antimagic characteristics, wherein the sums of edge weights at each vertex are distinct, rendering it an effective method for addressing intricate scheduling challenges [24]. In the graph G_3, vertices denote work shifts, whereas edges signify the workers assigned to various shifts. This method assigns unique labels to edges according to the total of incident edge calculated as vertex weights, guaranteeing that adjacent work shifts do not have overlapping workers, hence preventing scheduling conflicts [23]. This method facilitates the development of a conflict-free schedule, ensuring that no two consecutive work shifts have the same worker assignments, hence offering an ideal resolution to the scheduling dilemma [25].

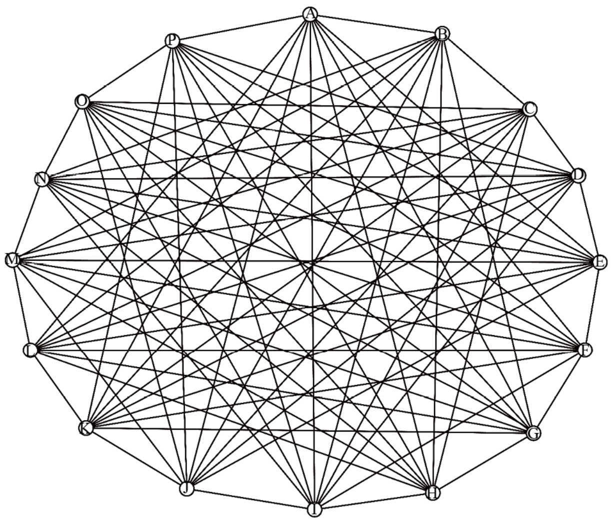

Figure 5. Graph G_3-10-Regular graph representation of work shifts scheduling

The Figure 5, also known as Graph G_3, is referred to be a 10-regular graph. This graph was utilized by the Factory workers work shifts Scheduling to aid in the creation of efficient scheduling and optimization problems.

$$A = \begin{pmatrix}
0&1&0&0&1&1&1&1&1&1&1&1&0&0&0&1\\
1&0&1&0&0&0&1&1&1&1&1&1&1&1&0&0\\
0&1&0&1&0&0&0&1&1&1&1&1&1&1&1&0\\
0&0&1&0&1&0&0&0&1&1&1&1&1&1&1&1\\
1&0&0&1&0&1&0&0&0&1&1&1&1&1&1&1\\
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1&1&1&1&1&0&0&0&1&0&1&0&1&0&1&1\\
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1&1&1&1&1&1&1&1&0&0&1&0&1&0&0&0\\
0&1&1&1&1&1&1&1&0&1&0&1&0&1&0&0\\
0&1&1&1&1&1&1&1&1&0&0&0&1&0&1&0\\
0&0&1&1&1&1&1&1&1&1&0&0&0&1&0&1\\
1&0&0&1&1&1&1&1&1&1&1&0&0&0&1&0
\end{pmatrix}$$

Figure 6. Graph G_4-Adjacency matrix for work shifts interrelations in scheduling optimization

The adjacency matrix of graph G_4, shown in Figure 6, demonstrates the relationships or connections between the different vertices (work shifts) in the Factory work shifts optimizations assignment. Each element in the matrix represents two work shifts share the same worker, they are connected by an edge. The rows and columns of the adjacency matrix corresponding to the workers doing the work in different time schedule of work shifts.

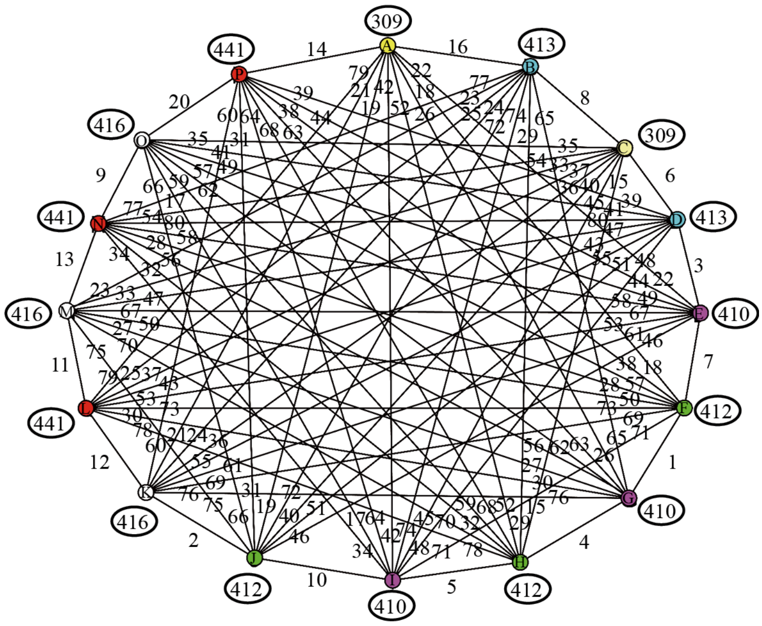

Figure 7. Graph G_5-6 Color-based workshift scheduling with local antimagic vertex coloring

The graph representing the assignment of workers allocating the work shifts, denoted as G_3, underwent a labeling process using the local antimagic vertex coloring technique to determine its local antimagic chromatic number. Notably, as depicted in Figure 7, it is pertinent to state that $\chi_{la}(G) \geq \chi(G) \geq 6$. However, as Figure 7 illustrates, $\chi_{la}(G) \leq 6$, Indicating that the local antimagic complete vertex coloration of the given graph G_3 indeed results in the utilization of 6 distinct colors, specifically $\chi_{la}(G) = 6$ [2].

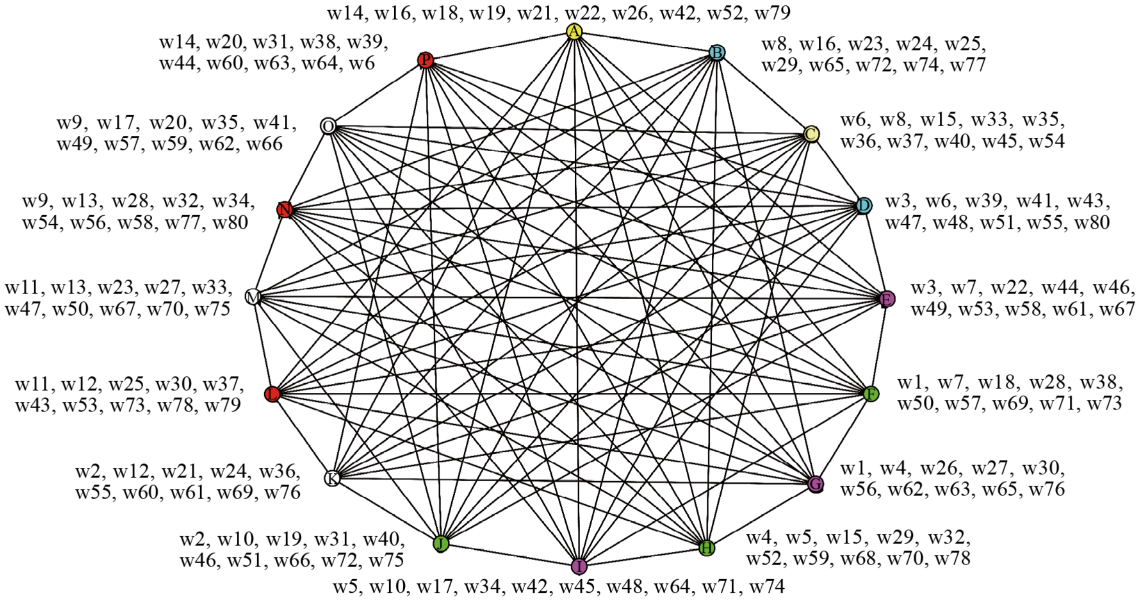

Figure 8. Graph G_6-Enhanced scheduling for dual-work shifts workers using local antimagic vertex coloring

The graph presented yields an optimal result through the application of local antimagic vertex coloring. This outcome serves as the basis for creating a scheduling system for assigning workers in work shifts, as outlined in Table 4.

The research aims to examine the efficacy of local antimagic vertex coloring as a possible resolution to the problem of workers allocation for Factory work shifts optimizationing. The research aims to analyze the fundamental principles of a specific methodology, assess its practical consequences in the context of arranging work shifts. The local antimagic vertex coloring technique is based on principles from combinatorial mathematics and graph theory. The strategy presented optimizes workers assignments and work shifts time allocation to solve the preceding problem. It prioritizes fair and balanced workers arrangements. The method used for allocating workers in a Factory workers work shifts Scheduling utilizes a regional antimagic vertex coloring strategy. The function accepts a graph as input, which depicts the connections between workers and scheduling work shifts. The technique considers constraints such as the worker's maximum permissible number of work shifts and the desired allocation of work shifts. The outcome signifies a technique of assigning colors to vertices that guarantees an equitable distribution of workers throughout all work shifts. In this example, the concept of local antimagic vertex coloring is used, where the edges are seen as workers of a work shifts. with a group of 80 individuals worker and each work shifts have 10 workers so that case 160 workers needed in this method only 80 workers was assigned 16 work shifts evenly distributed, it is feasible to form 16 work shifts, where each work shifts may be represented as a vertex and colored without any scheduling conflicts. The local antimagic vertex coloring idea may be used to create 16 work shifts without any time schedule conflicts, given that there are 80 available workers. The working in this study were 80 individuals who were workers of different work shifts. The Factory workers working by a total of 16 work shifts, specifically referred to as work shifts 1 to work shifts 16.

Consequently, the optimal scheduling system for allocating 80 workers across 16 work shifts requires a total of 6 allocation times, as indicated in Table 4. Remarkably, this allocation duration aligns with the local antimagic chromatic number of the represented graph in Figure 7, denoted as G_5. Graph G_6 denoted as some workers contribute to two work shifts shown in Figure 8.

The graph in Figure 8 was created using the local antimagic vertex coloring, which implemented the assignment of Factory workers work shifts Scheduling. There are a total of 16 vertices (representing work shifts) in the graph, and the edges represent workers sharing the same work shifts. In this case, there are a total of 80 edges. The colors in the graph were determined based on the weight of the vertices, with a total of 6 colors used. Therefore, the Figure 7 clearly displays a work shifts schedule optimization for 6 time slots.

Table 4. The optimal workers and time scheduling system for workers assignments in 16 work shifts

S.No	1	2	3	4	5	6
Colors (weights) time schedule	Yellow (309)	Blue (413)	Magenta (410)	Green (412)	white (416)	Red (441)
work shifts	A, C	B, D	E, G, I	F, H, J	K, M, O	L, N, P

The Table 4 vertices was defined using 6 vertex weights. Additionally, there are 6 time schedule optimization for the Factory workers work shifts Scheduling assignment. The vertex weight, or colors, help enhance the performance of two workers who are working in work shifts for separate workers at different time schedules.

Example: 16-Shift time schedule in a factory factory

Name: Pharma tech solutions

Year used: 2018

Industry: Pharmaceutical manufacturing

Location: Basel, switzerland

Time Schedules: The factory operated with 16 distinct shifts over a 4-day cycle, split between morning, evening, and night shifts.

Shift 1: 6:00 AM-2:00 PM (Day Shift, Group A)

Shift 2: 6:30 AM-2:30 PM (Day Shift, Group B)

Shift 3: 7:00 AM-3:00 PM (Day Shift, Group C)

Shift 4: 7:30 AM-3:30 PM (Day Shift, Group D)

Shift 5: 2:00 PM-10:00 PM (Evening Shift, Group A)

Shift 6: 2:30 PM-10:30 PM (Evening Shift, Group B)

Shift 7: 3:00 PM-11:00 PM (Evening Shift, Group C)

Shift 8: 3:30 PM-11:30 PM (Evening Shift, Group D)

Shift 9: 10:00 PM-6:00 AM (Night Shift, Group A)

Shift 10: 10:30 PM-6:30 AM (Night Shift, Group B)

Shift 11: 11:00 PM-7:00 AM (Night Shift, Group C)

Shift 12: 11:30 PM-7:30 AM (Night Shift, Group D)

Shift 13: 8:00 AM-4:00 PM (Specialized RD Shift)

Shift 14: 9:00 AM-5:00 PM (Management and Operations)

Shift 15: 8:30 PM-4:30 AM (Emergency Maintenance)

Shift 16: 10:00 PM-6:00 AM (Critical Quality Control)

Explanation of work shifts Use in 2018:

In 2018, Pharma Tech Solutions instituted a 16-shift schedule to facilitate seamless and uninterrupted manufacturing, especially for time-critical drug manufacture. The schedule was organized as follows:

Groups A to D: These groups alternated among day, evening, and night shifts within a 4-day cycle. This method facilitated overlap and seamless transitions between shifts, reducing downtime and guaranteeing continuous operations. Shifts 1 to 12 were structured to encompass essential production with staggered commencement and conclusion periods to prevent bottlenecks.

Specialized Shifts (13-16):

Shift 13 was established for the RD team, which operated during conventional office hours while providing support for production through testing and innovation.

Shift 14 was designated for management and operational supervision, facilitating critical decision-making during business hours.

Shift 15 was designated for the emergency maintenance personnel to address machine repairs and emergencies during the night.

Shift 16 was essential for quality control, guaranteeing that overnight-produced pharmaceutical batches complied with regulatory standards.

The 16-shift approach enabled Pharma Tech Solutions to optimize production while preserving the adaptability to support various teams, including dedicated shifts for RD, management, and essential services. This method was implemented to minimize human error, enhance product quality, and guarantee continuous operations without overburdening any single group of employees. The technology was particularly critical for the pharmaceutical sector, as downtime can impact both production efficiency and product safety.

The Pharma Tech Solutions 16-Shift Time Schedule is applied to Application 2 by modeling the 16 work shifts as vertices in a graph, with the 80 workers represented as connections connecting these vertices with local antimagic vertex coloring. Each work shifts necessitates 10 workers, with each worker engaged in two shifts, thereby providing seamless operations without overlaps or schedule issues.

Worker Allocation:

Total Workers (Edges): 80 employees are distributed across the 16 work shifts.

work shifts (Vertices): The 16 shifts (1 to 16) denote the vertices, with 10 workers allocated to each shift.

Time Schedule Allocation: The 6 separate time schedules denote various commencement times for day, evening, and night shifts:

Day Shifts: Shifts 1 through 4, 13 and 14.

Evening Shifts: Shifts 5 through 8 and Shift 15.

Night Shifts: Shifts 9 to 12 and Shift 16.

Employing local antimagic vertex coloring, each shift (vertex) is allocated a unique time slot (color), guaranteeing that no nearby vertices (shifts with shared workers) possess identical time schedules. Groups A to D alternate among day, evening, and night hours, while specialty shifts (1316) are allocated specific periods for research and development, management, maintenance, and quality control.

This allocation approach equilibrates worker assignments, guarantees uninterrupted output, and mitigates the danger of overlap, allowing Pharma Tech to sustain continuous operations while fulfilling regulatory and operational requirements.

7. Comparative study of two scheduling approaches utilizing local antimagic labeling techniques

The two papers, Optimizing Factory Workers work shifts Scheduling Using Local Antimagic Vertex Coloring and Application of the Local Antimagic Total Labeling of Graphs to Optimize Scheduling System for an Expatriate Assignment [2], both tackle the problem of scheduling using graph theory, but in very different contexts. One deals with long-term industrial work shifts management, while the other focuses on short-term expatriate task assignments across divisions in multinational companies. In this study, we will compare their approaches and prove that Optimizing Factory Workers work shifts Scheduling Using Local Antimagic Vertex Coloring is a more efficient method than the expatriate scheduling approach using local antimagic total labeling.

Comparison of scenarios:

1. Scheduling of expatriate assignments

The purpose of the expatriate assignment scheduling paper is to maximize the task allocation for expatriates employed in various divisions of a corporation. These expatriates are engaged in short-term missions, and the challenge is to guarantee that no two divisions utilizing the same expatriates have overlapping responsibilities. The issue is represented as a graph in which divisions serve as vertices and tasks shared within divisions function as edges. Local antimagic total labeling is employed, which entails labeling both vertices (divisions) and edges (tasks), guaranteeing that the weights of two neighboring vertices are unique. This mitigates scheduling issues for expatriates operating across many divisions.

This method guarantees efficient work distribution; however, the intricacy stems from the necessity to name both vertices and edges. As the quantity of expatriates and divisions rises, the labeling process becomes increasingly computationally intensive. The extra responsibility of guaranteeing distinct labels for both vertices and edges diminishes the method's scalability, particularly when managing several shared activities across various divisions.

2. Scheduling of work shifts for factory workers

The paper on factory work shifts scheduling is on maximizing the long-term, continuous scheduling of plant personnel. In this context, work shifts are depicted as vertices, while workers are illustrated as edges linking the shifts. The method employs local antimagic vertex coloring, ensuring that adjacent vertices (work shifts) do not share identical weights, hence averting problems in worker allocations across shifts. This ensures that employees are assigned to non-overlapping shifts, facilitating an equitable and balanced distribution of labor.

This method's simplicity is attributed to its exclusive emphasis on vertex labeling. The absence of separate labels for the edges (workers) decreases computing complexity. The conventional graph structure and the straightforward nature of vertex-only labeling enable this method to scale effectively, even with an increase in the number of work shifts and workers.

Essential comparative metrics 1. Scalability

Expatriate assignment: The local antimagic total labeling employed in this methodology encompasses the labeling of both vertices (divisions) and edges (tasks). As the quantity of divisions and tasks escalates, the intricacy of sustaining distinct labels for both amplifies. This method has poor scalability for larger organizations with numerous interdivisional shared tasks, as each new edge adds complexity to the labeling process.

Factory workers work shifts scheduling: The local antimagic vertex coloring exclusively pertains to the designation of vertices (work shifts), omitting the necessity for edge labeling (workers). This enhances scalability, allowing for a substantial increase in the number of work shifts with no computing expense. Emphasizing straightforward vertex coloring facilitates the management of extensive processes.

2. Implementation

Expatriate assignment: The necessity of labeling both vertices and edges introduces an additional layer of complexity to the implementation. With each additional task or division, both the vertices and edges must be modified with distinct weights, complicating maintenance and implementation as the system expands.

Factory workers work shifts scheduling: The ease of merely labeling the vertices renders implementation uncomplicated. Allocating unique weights to the vertices (work shifts) facilitates management and maintenance throughout time, particularly in prolonged activities characterized by repetitive shifts.

3. Fairness and resource allocation

Expatriate assignment: The system prevents the double-booking of expatriates across divisions; however, it does not assure an equitable allocation of jobs. Certain expatriates may encounter inconsistent workloads based on the number of divisions to which they are allocated. Consequently, it emphasizes conflict prevention but does not inherently guarantee equitable resource distribution.

Factory workers work shifts scheduling: The conventional graph structure and vertex-exclusive labeling provide equitable distribution of workers across shifts, averting both overload and underutilization. Every employee possesses a balanced workload, which is essential for sustaining job happiness and preventing burnout. This renders the strategy more appropriate for prolonged, recurrent scheduling where equilibrium is essential.

4. Computational complexity

Expatriate assignment: The temporal complexity for local antimagic complete labeling is $(V+E)$, as both vertices and edges require labeling. As the quantity of jobs (edges) escalates, the labeling procedure gets increasingly more intricate, rendering the system less efficient for extensive scheduling.

Factory workers work shifts scheduling: Conversely, local antimagic vertex coloring has a temporal complexity of V, since only the vertices require labeling. This leads to a markedly reduced computational load, particularly when the number of shifts escalates. The lack of edge labeling streamlines the algorithm and decreases the time needed to produce the schedule.

Proof of efficiency: The Superiority of Factory work shifts Scheduling

The efficacy of a scheduling method is characterized by its computational simplicity, scalability, and capacity for equitable resource distribution. The Factory Workers work shifts Scheduling approach demonstrates greater efficiency than the expatriate assignment method for various reasons:

1. Lower computational complexity:

The expatriate technique exhibits increased computational complexity owing to the requirement for comprehensive labeling, necessitating the labeling of both vertices and edges. Conversely, the factory work shifts approach necessitates simply vertex labeling, hence diminishing the overall complexity to V, resulting in enhanced speed and efficiency as the system expands.

The computational burden of the expatriate method increases substantially with the quantity of shared tasks, rendering it inappropriate for extensive scheduling challenges. The factory work shifts method, utilizing a simplified vertex-only coloring approach, manages scaling significantly more efficiently.

2. Equity in resource distribution:

The factory technique ensures an equitable allocation of tasks among shifts owing to its consistent graph configuration. Workers are uniformly assigned to shifts, promoting equitable distribution of task and reducing disagreements. Conversely, although the expatriate technique prevents task redundancy, it does not guarantee an equitable distribution of responsibilities, resulting in potential disparities in burden among expats.

The consistency of the manufacturing work shifts model guarantees that workers are neither excessively burdened nor underutilized, which is essential for sustained operations.

3. Simplicity and scalability:

The factory method's emphasis on vertex-only labeling simplifies implementation and enhances maintainability, even with an increasing number of shifts. This scalability is essential in extensive factories with multiple employees and shifts, where repeated and effective scheduling is crucial.

The expatriate approach, characterized by comprehensive labeling, becomes intricate and cumbersome as the quantity of divisions and jobs escalates. Its inefficiency in scaling within intricate task-sharing contexts renders it impractical for large enterprises.

In this case although both approaches effectively enhance scheduling through local antimagic labeling techniques, Factory Workers work shifts Scheduling is the more efficient method. Its reduced computational complexity, enhanced scalability, and capacity to guarantee fairness in resource allocation render it superior, particularly in extensive, long-term scheduling contexts. The expatriate assignment approach is efficient for short-term, task-specific scheduling but becomes increasingly impractical as the number of tasks and divisions expands, particularly in larger, more complicated organizations.

Some scenarios for other applications

1. Education: University course scheduling:

Scenario: Within higher education institutions, instructors are responsible for teaching many courses, each scheduled at certain time intervals.

Example: In the above scenario, a professor may provide a morning session on Monday and a subsequent session in the afternoon on wednesday. In this model, the professor, shown by the edge, is linked to two separate time slots, described by the vertices, each of which corresponds to a different class schedule.

2. Transportation: Airline crew scheduling problem

Scenario: Crew members often manage many flights, each linked to unique departure and arrival timetables.

Example: A pilot may embark on a trip from New York to London during one work period and subsequently from London to Paris during another. Considering that both flights occur at separate durations, the pilot (edge) is associated with two flight schedules (vertices).

3. Healthcare: Nurse shift allocation

Scenario: Nurses in hospitals often work across different shifts to provide continuous patient care.

Example: Consider a situation where a nurse is assigned to the morning shift in the emergency room on one day and the night shift in the critical care unit on another. The nurse (edge) is associated with two distinct time slots (vertices), each having specific work schedules.

4. Event management: Conference speaker scheduling

Scenario: Within the framework of prolonged conferences, speakers have the opportunity to offer oral presentations or participate in panel discussions at different times and locations.

Example: A speaker may deliver a keynote address in the morning and then participate in a panel discussion in the afternoon. The speaker, shown by the edge, is linked to two separate time intervals, or vertices, which indicate their designated duties.

5. Sports: Athlete training and competition scheduling

Scenario: Within the domain of sports, it is customary for athletes to schedule their training sessions and competing events at distinct time intervals.

Example: A swimmer may engage in a morning training session, followed by an evening session of competition. Two separate time periods (vertices) are used to represent the athlete's training and competition schedules (edge).

6. Public services: Firefighter shift scheduling

Scenario: In the context of public services is the operation of allocating firemen to rotating shifts to provide continuous coverage for all hours of the day.

Example: By alternating between working a day shift on one day and a night shift on another, a firefighter may ensure a consistent personnel level at the fire station. In this diagram, the firefighter, shown as the edge, is connected to two distinct shifts, indicated as vertices, each with their own time schedules.

7. Retail: Employee shift management in stores

Scenario: In the retail sector, employee shift management in shops is the systematic arrangement of retail staff for different shifts to provide comprehensive coverage of shop hours.

Example: A retail employee may be allocated to do an initial shift on one day and a final duty on another, especially during peak buying periods like holidays. An individual worker is linked to two shifts, which align with their respective work schedules.

8. Logistics: Delivery driver routing

Scenario: Delivery drivers may be allocated certain routes at different times of the day.

Example: Here is a hypothetical situation in which a delivery driver is allocated to a morning route in one city and an afternoon route in another. Each individual driver (edge) is associated with two distinct time slots (vertices), each of which corresponds to a distinct delivery schedule.

9. Broadcast media: Television production scheduling

Scenario: In Broadcast Media Television production crew members may be allocated to many shows or parts, each with individually defined recording timetables.

Example: A camera operator may capture a morning news telecast and then participate in an evening discussion program. The operator (edge) connects two separate time slots (vertices) that correspond to their individual work assignments.

10. Military: Troop deployment scheduling

Scenario: In the military domain, troop deployment scheduling pertains to the assignment of military personnel to various planned operations or patrols scheduled at distinct time intervals.

Example: A soldier may be assigned to patrol duty during daylight hours and participate in a training simulation at nighttime. Each edge soldier is connected to two vertices that represent distinct time slots, each of which maintains a unique operational schedule.

Notes: The graph model enables the optimization of resource scheduling and allocation in many areas, therefore ensuring efficiency and avoiding conflicts. An edge, which represents a single participant, participating in several vertices representing distinct time slots, is a widely utilized and fundamental concept for maintaining the uninterrupted flow of activities in many scenarios.

8. Conclusion

This study introduces a new scheduling technique that uses local antimagic vertex coloring. This technique helps to distribute work shifts evenly among workers and prevents overlapping shifts. By using a limited color scheme to represent specific work shift hours, this approach ensures fairness and efficiency in scheduling. The study focuses on determining the local antimagic chromatic number, a key factor in creating a highly accurate scheduling system. However, it's important to note that this model may not account for all the complexities of real-world scheduling, such as varying resource availability and facility limitations. Future research will compare this new technique with existing methods to evaluate its strengths and weaknesses. Furthermore, it will assess its potential for expansion into larger industrial settings and its efficacy in various scenarios. Ultimately, this study aims to establish a foundation for future advancements in local antimagic vertex coloring for industrial work shifts scheduling, leading to more equitable and efficient work systems.

9. Open problem

The local antimagic vertex coloring method presents an effective approach to optimize industrial work shifts scheduling. However, several challenges persist. A key challenge lies in determining the minimum number of colors required to achieve a valid local antimagic vertex coloring for a given work shifts scheduling graph. Additionally, developing efficient algorithms to handle large-scale and dynamic work shifts scheduling problems, particularly in real-time scenarios, remains a significant hurdle. Furthermore, incorporating real-world constraints such as worker preferences, skill requirements, and equipment availability into the local antimagic vertex coloring framework requires further investigation. Rigorous evaluation of the performance of local antimagic vertex coloring-based scheduling algorithms against traditional methods in various industrial settings is crucial. Future research should focus on addressing these challenges to fully realize the potential of this technique in practical applications.

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Conflict of interest

The authors affirm that they do not own any identifiable conflicting financial interests or personal ties that might have potentially influenced the findings presented in this study.

References

[1] Arumugam S, Premalatha K, Bača M, Semaničová-Feňovčíková A. Local antimagic vertex coloring of a graph. *Graphs and Combinatorics*. 2017; 33: 275-285.

[2] Utami W, Wijaya K. Application of the local antimagic total labeling of graphs to optimize scheduling system for an expatriate assignment. *Journal of Physics: Conference Series*. 2020; 1538: 13-20.

[3] Gallian JA. A dynamic survey of graph labeling. *Electronic Journal of Combinatorics*. 2022; 6(25): 4-623.

[4] Keshavarz P, Mahdavi. A survey on vertex coloring problems and their applications. *International Journal of Combinatorial Optimization Problems and Informatics*. 2017; 8(2): 15-30.
[5] Li X, Zhang w. On the local antimagic chromatic number of a graph. *Discrete Applied Mathematics*. 2012; 160(15): 2214-2218.
[6] Chang F, Pan Z. Applications of graph theory in the scheduling of jobs and workforce. *Journal of Scheduling*. 2020; 23(4): 405-418.
[7] Dorigo M, Stützle T. Ant colony optimization: overview and recent advances. In Handbook of Metaheuristics. 2019.
[8] González SC, Giménez MM. Applications of graph coloring in industrial settings. *Annals of Operations Research*. 2019; 276(1-2): 181-204.
[9] Kok AL, Hans EW, Schutten JM. Optimizing shift scheduling in the process industry. *OR Spectrum*. 2012; 34(4): 783-818.
[10] Jensen TR, Toft B. *Graph Coloring Problems*. John Wiley and Sons; 2011.
[11] Fink J, Mazur P. Graph coloring with applications to scheduling. *Wiley Interdisciplinary Reviews: Computational Statistics*. 2015; 7(1): 29-49.
[12] Bodlaender HL. *Graph Coloring: A Combinatorial Perspective*. Cambridge University Press; 2019.
[13] Kumar R, Gupta S. Graph theory applications in scheduling problems. *International Journal of Operations Research*. 2016; 13(1): 67-75.
[14] Christodoulou G, Koutsoupias E. Graph coloring and job scheduling: A study in coordination mechanisms. *Theoretical Computer Science*. 2013; 497: 13-24.
[15] Li H, wu X. Local antimagic chromatic number of bipartite graphs. *Discrete Mathematics*. 2019; 342(2): 519-529.
[16] Baker KR, Trietsch D. *Principles of Sequencing and Scheduling*. John Wiley and Sons; 2017.
[17] Marx D, Schlotter I. Graph coloring with constraints: The role of graph theory in scheduling. *Journal of Scheduling*. 2012; 15(1): 111-129.
[18] De Werra D. Graph theory and scheduling applications in operational research. *Discrete Applied Mathematics*. 2019; 274: 62-72.
[19] Brualdi RA, Hsiao HJ. A survey of local antimagic and other labeling problems on graphs. *Utilitas Mathematica*. 2015; 98: 241-264.
[20] Patil BH, Mahajan RK. Graph theory approaches in workforce scheduling problems. *Journal of Scheduling*. 2018; 21(6): 635-653.
[21] Cormen TH, Leiserson CE, Rivest RL, Stein C. *Introduction to Algorithms*. MIT Press; 2009.
[22] Bondy JA, Murty USR. *Graph Theory with Applications*. Springer; 2008.
[23] Papadimitriou CH, Steiglitz K. *Combinatorial Optimization: Algorithms and Complexity*. Dover Publications; 1998.
[24] Baca M, Miller M. *Antimagic Labeling of Graphs*. CRC Press; 2020.
[25] Roberts FS, Tesman B. *Applied Combinatorics*. CRC Press; 2024.