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Research Article

On a General Subclass of q-Starlike and q-Convex Analytic Functions

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Abstract: We introduce a certain subclass of analytic functions in the open unit disk $\mathbb{U}$ involving the q-derivative operator. Some convolution results and Fekete-Szegö inequalities for the analytic functions belonging to this class are derived. We have also provided some results as corollaries of our theorems.

***Keywords*:** univalent function, convolution, q-starlike, q-convex, fekete-szegö problem

MSC: 30C45

1. Introduction

Let $\mathscr{A}$ denote the class of analytic functions of the form:

$$\psi(\xi) = \xi + \sum_{k=2}^{\infty} \rho_k \xi^k \tag{1}$$

in the open unit disk $\mathbb{U} = \{\xi \in \mathbb{C} : |\xi| < 1\}$. If $\psi(\xi)$ and $\phi(\xi)$ are analytic in $\mathbb{U}$, we say that $\psi(\xi)$ is subordinate to $\phi(\xi)$, written $\psi(\xi) \prec \phi(\xi)$ if there exists a Schwarz function ω, which (by definition) is analytic in $\mathbb{U}$ with $\omega(0) = 0$ and $|\omega(\xi)| < 1$ for all $\xi \in \mathbb{U}$, such that $\psi(\xi) = \phi(\omega(\xi))$, $\xi \in \mathbb{U}$(see [1] and [2]). For functions $\psi(\xi)$ given by (1) and $\phi(\xi)$ given by

$$\phi(\xi) = \xi + \sum_{k=2}^{\infty} \sigma_k \xi^k, \tag{2}$$

the Hadamard product or convolution of $\psi(\xi)$ and $\phi(\xi)$ is defined by

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$$(\psi * \phi)(\xi) = \xi + \sum_{k=2}^{\infty} \rho_k \, \sigma_k \xi^k = (\phi * \psi)(\xi). \tag{3}$$

For $-1 \leq B < A \leq 1$, we introduce the subclasses $\mathscr{S}[A,\, B]$ and $\mathscr{C}[A,\, B]$ of $\mathscr{A}$ as follows:

$$\mathscr{S}[A,\, B] = \left\{ \psi \in \mathscr{A} : \frac{\xi \psi'(\xi)}{\psi(\xi)} \prec \frac{1 + A\xi}{1 + B\xi} \right\}, \tag{4}$$

and

$$\mathscr{C}[A,\, B] = \left\{ \psi \in \mathscr{A} : \frac{(\xi \psi'(\xi))'}{\psi'(\xi)} \prec \frac{1 + A\xi}{1 + B\xi} \right\}, \tag{5}$$

where $\mathscr{S}[A,\, B]$ and $\mathscr{C}[A,\, B]$ are defined by Janowski [3, 4] (see also [5–11]). We note that $\mathscr{S}[1-2\alpha,\, -1] = \mathscr{S}(\alpha)$ and $\mathscr{C}[1-2\alpha,\, -1] = \mathscr{C}(\alpha)$ $(0 \leq \alpha < 1)$, where $\mathscr{S}(\alpha)$ and $\mathscr{C}(\alpha)$ denote the subclasses of $\mathscr{A}$ that consists, respectively, of starlike of order α and convex of order α in $\mathbb{U}$ (see [12, 13]).

Recently, the q-derivative and q-integral defined by Jackson [14, 15] has played a crucial role in the theory of univalent analytic functions especially in defining operators and classes of analytic functions, and studying interesting properties which are related to the Geometric Function Theory (see [16–31]). For $0 < q < 1$, the q-derivative of a function ψ is defined as

$$\mathscr{D}_q \psi(\xi) = \frac{\psi(\xi) - \psi(q\xi)}{(1-q)\,\xi} \quad (\xi \neq 0;\ 0 < q < 1), \tag{6}$$

and $\mathscr{D}_q \psi(0) = \psi'(0)$ provided that $\psi(\xi)$ is differentiable at 0. From (1) and (6), we deduce that

$$\mathscr{D}_q \psi(\xi) = 1 + \sum_{k=2}^{\infty} [k]_q \; \rho_k \xi^{k-1}, \tag{7}$$

where $[k]_q$ is the q-number given by

$$[k]_q = \frac{1-q^k}{1-q} = 1 + q + ... + q^{k-2} + q^{k-1}. \tag{8}$$

We note that, if $\psi,\ \phi \in \mathscr{A}$, then

(i) $\mathscr{D}_q [\gamma_1 \; \psi(\xi) \pm \gamma_2 \; \phi(\xi)] = \gamma_1 \; \mathscr{D}_q \psi(\xi) \pm \gamma_2 \; \mathscr{D}_q \phi(\xi)$, where $\gamma_1,\ \gamma_2$ are constants;

(ii) $\mathscr{D}_q [\psi(\xi) \phi(\xi)] = \phi(\xi) \; \mathscr{D}_q \psi(\xi) + \psi(q\xi) \; \mathscr{D}_q \phi(\xi)$;

(ii) $\mathscr{D}_q \left[\dfrac{\psi(\xi)}{\phi(\xi)} \right] = \dfrac{\phi(\xi) \mathscr{D}_q \psi(\xi) - \psi(\xi) \mathscr{D}_q \phi(\xi)}{\phi(\xi) \phi(q\xi)}$.

Also, the q-integral of the function ψ on $[0,\ \xi]$ is defined as

$$\int_0^{\xi} \psi(t)\ d_q t = (1-q)\,\xi \sum_{k=0}^{\infty} q^k \psi\left(q^k \xi\right) \quad (0 < q < 1)\,. \tag{9}$$

In particular, the q-integral of the function $\psi \in \mathscr{A}$ defined by (1) is given by

$$\int_0^{\xi} \psi(t)\ d_q t = \frac{\xi^2}{[2]_q} + \sum_{k=2}^{\infty} \rho_k \frac{\xi^{k+1}}{[k+1]_q}. \tag{10}$$

Again, since $[k+1]_q \to k+1$ as $q \to 1^-$, therefore for $q \to 1^-$, we have

$$\int_0^{\xi} \psi(t)\ d_q t \to \int_0^{\xi} \psi(t)\ dt,$$

which is the ordinary integral of the function $\psi(\xi)$ on $[0,\ \xi]$.

Making use of the q-derivative $\mathscr{D}_q \psi(\xi)$ given by (6) and the definition of the subordination, we introduce the subclass $\mathscr{SC}_q[\alpha,\ \beta;\ A,\ B]$ of $\mathscr{A}$ for as follows:

Definition 1 A function $\psi \in \mathscr{A}$ is said to be in $\mathscr{SC}_q[\alpha,\ \beta;\ A,\ B]$ if it satisfies the following subordination condition:

$$\frac{\alpha \xi \mathscr{D}_q \psi(\xi) + \beta \xi \mathscr{D}_q(\xi \mathscr{D}_q \psi(\xi))}{\alpha \psi(\xi) + \beta \xi \mathscr{D}_q \psi(\xi)} \prec \frac{1 + A\xi}{1 + B\xi} \tag{11}$$

$$(\xi \in \mathbb{U};\ 0 < q < 1;\ \alpha,\ \beta \geq 0;\ -1 \leq B < A \leq 1)\,.$$

The class $\mathscr{SC}_q[\alpha,\ \beta;\ A,\ B]$ is not empty since the function $\psi(\xi) = z + \dfrac{\alpha + \beta}{2\left(\alpha + [2]_q \beta\right)} z^2$ belongs to $\mathscr{SC}_q[\alpha,\ \beta;\ A,\ B]$. We note that

1. $\mathscr{SC}_q[\alpha,\ 0;\ A,\ B] = \mathscr{S}_q[A,\ B]$ (see [32])

$$\mathscr{S}_q[A,\ B] = \left\{ \psi \in \mathscr{A} : \frac{\xi \mathscr{D}_q \psi(\xi)}{\psi(\xi)} \prec \frac{1 + A\xi}{1 + B\xi} \right\}.$$

2. $\mathscr{SC}_q[0,\ \beta;\ A,\ B] = \mathscr{C}_q[A,\ B]$ (see [32])

$$\mathscr{C}_q[A,\ B] = \left\{ \psi \in \mathscr{A} : \frac{\mathscr{D}_q(\xi \mathscr{D}_q \psi(\xi))}{\mathscr{D}_q \psi(\xi)} \prec \frac{1 + A\xi}{1 + B\xi} \right\}.$$

3. $\lim_{q \to 1^-} \mathscr{SC}_q[\alpha,\ \beta;\ A,\ B] = \mathscr{SC}[\alpha,\ \beta;\ A,\ B]$

$$\mathscr{SC}[\alpha,\ \beta;\ A,\ B] = \left\{ \psi \in \mathscr{A} : \frac{\alpha \xi \psi'(\xi) + \beta \xi (\xi \psi'(\xi))'}{\alpha \psi(\xi) + \beta \xi \psi'(\xi)} \prec \frac{1 + A\xi}{1 + B\xi} \right\},$$

$\mathscr{SC}\left[\alpha,\ 0;\ A,\ B\right]=\mathscr{S}\left[A,\ B\right]$ and $\mathscr{SC}\left[0,\ \beta;\ A,\ B\right]=\mathscr{C}\left[A,\ B\right]$(see [3, 4]).

4. $\mathscr{SC}_q\left[\alpha,\ \beta;\ 1-2\gamma,\ -1\right]=\mathscr{SC}_q\left(\alpha,\ \beta;\ \gamma\right)\left(0\leq\gamma<1\right)$

$$\mathscr{SC}_q\left(\alpha,\ \beta;\ \gamma\right)=\left\{\psi\in\mathscr{A}:\operatorname{Re}\left(\frac{\alpha\xi\mathscr{D}_q\psi\left(\xi\right)+\beta\xi\mathscr{D}_q\left(\xi\mathscr{D}_q\psi\left(\xi\right)\right)}{\alpha\psi\left(\xi\right)+\beta\xi\mathscr{D}_q\psi\left(\xi\right)}\right)>\gamma\right\},$$

and $\lim_{q\to1^-}\mathscr{SC}_q\left(\alpha,\ \beta;\ \gamma\right)=\mathscr{SC}\left(\alpha,\ \beta;\ \gamma\right)\left(0\leq\gamma<1\right)$

$$\mathscr{SC}_q\left(\alpha,\ \beta;\ \gamma\right)=\left\{\psi\in\mathscr{A}:\operatorname{Re}\left(\frac{\alpha\xi\psi'\left(\xi\right)+\beta\xi\left(\xi\psi'\left(\xi\right)\right)'}{\alpha\psi\left(\xi\right)+\beta\xi\psi'\left(\xi\right)}\right)>\gamma\right\}.$$

5. For $0\leq\gamma<1,\ \mathscr{SC}_q\left(\alpha,\ 0;\ \gamma\right)=\mathscr{S}_q\left(\gamma\right)$ (see [33])

$$\mathscr{S}_q\left(\gamma\right)=\left\{\psi\in\mathscr{A}:\operatorname{Re}\left(\frac{\xi\mathscr{D}_q\psi\left(\xi\right)}{\psi\left(\xi\right)}\right)>\gamma\right\},$$

and $\lim_{q\to1^-}\mathscr{S}_q\left(\gamma\right)=\mathscr{S}\left(\gamma\right)$ (see [12]).

6. For $0\leq\gamma<1,\ \mathscr{SC}_q\left(0,\ \beta;\ \gamma\right)=\mathscr{C}_q\left(\gamma\right)$ (see [33])

$$\mathscr{C}_q\left(\gamma\right)=\left\{\psi\in\mathscr{A}:\operatorname{Re}\left(\frac{\mathscr{D}_q\left(\xi\mathscr{D}_q\psi\left(\xi\right)\right)}{\mathscr{D}_q\psi\left(\xi\right)}\right)>\gamma\right\},$$

and $\lim_{q\to1^-}\mathscr{C}_q\left(\gamma\right)=\mathscr{C}\left(\gamma\right)$ (see [12]).

7. $\mathscr{SC}_q\left[\alpha,\ \beta;\ \left(1-2\gamma\right)\delta,\ -\delta\right]=\mathscr{SC}_q\left(\alpha,\ \beta;\ \gamma,\ \delta\right)\left(0\leq\gamma<1,\ 0<\delta\leq1\right)$

$$\mathscr{SC}_q\left(\alpha,\ \beta;\ \gamma,\ \delta\right)=\left\{\psi\in\mathscr{A}:\left|\frac{\dfrac{\alpha\xi\mathscr{D}_q\psi\left(\xi\right)+\beta\xi\mathscr{D}_q\left(\xi\mathscr{D}_q\psi\left(\xi\right)\right)}{\alpha\psi\left(\xi\right)+\beta\xi\mathscr{D}_q\psi\left(\xi\right)}-1}{\dfrac{\alpha\xi\mathscr{D}_q\psi\left(\xi\right)+\beta\xi\mathscr{D}_q\left(\xi\mathscr{D}_q\psi\left(\xi\right)\right)}{\alpha\psi\left(\xi\right)+\beta\xi\mathscr{D}_q\psi\left(\xi\right)}+1-2\gamma}\right|<\delta\right\}$$

and $\lim_{q\to1^-}\mathscr{SC}_q\left(\alpha,\ \beta;\ \gamma,\ \delta\right)=\mathscr{SC}\left(\alpha,\ \beta;\ \gamma,\ \delta\right)\left(0\leq\gamma<1,\ 0<\delta\leq1\right)$

$$\mathscr{SC}\left(\alpha,\ \beta;\ \gamma,\ \delta\right)=\left\{\psi\in\mathscr{A}:\left|\frac{\dfrac{\alpha\xi\psi'\left(\xi\right)+\beta\xi\left(\xi\psi'\left(\xi\right)\right)'}{\alpha\psi\left(\xi\right)+\beta\xi\psi'\left(\xi\right)}-1}{\dfrac{\alpha\xi\psi'\left(\xi\right)+\beta\xi\left(\xi\psi'\left(\xi\right)\right)'}{\alpha\psi\left(\xi\right)+\beta\xi\psi'\left(\xi\right)}+1-2\gamma}\right|<\delta\right\}.$$

8. $\mathscr{SC}_q\left(\alpha,\ 0;\ \gamma,\ \delta\right)=\mathscr{S}_q\left(\gamma,\ \delta\right)\left(0\leq\gamma<1,\ 0<\delta\leq1\right)$

$$\mathscr{S}_q(\gamma,\ \delta) = \left\{ \psi \in \mathscr{A} : \left| \frac{\dfrac{\xi \mathscr{D}_q \psi(\xi)}{\psi(\xi)} - 1}{\dfrac{\xi \mathscr{D}_q \psi(\xi)}{\psi(\xi)} + 1 - 2\gamma} \right| < \delta \right\};$$

and $\lim_{q \to 1^-} \mathscr{S}_q(\gamma,\ \delta) = \mathscr{S}(\gamma,\ \delta)$ (see [34]).

9. $\mathscr{SC}_q(0,\ \beta;\ \gamma,\ \delta) = \mathscr{C}_q(\gamma,\ \delta)\,(0 \le \gamma < 1,\ 0 < \delta \le 1)$

$$\mathscr{C}_q(\gamma,\ \delta) = \left\{ \psi \in \mathscr{A} : \left| \frac{\dfrac{\mathscr{D}_q(\xi \mathscr{D}_q \psi(\xi))}{\mathscr{D}_q \psi(\xi)} - 1}{\dfrac{\mathscr{D}_q(\xi \mathscr{D}_q \psi(\xi))}{\mathscr{D}_q \psi(\xi)} + 1 - 2\gamma} \right| < \delta \right\},$$

and $\lim_{q \to 1^-} \mathscr{C}_q(\gamma,\ \delta) = \mathscr{C}(\gamma,\ \delta)$ (see [34]).

The aim of the present investigation is to define a general subclass $\mathscr{SC}_q[\alpha,\ \beta;\ A,\ B]$ of q-starlike and q-convex analytic functions by using the q-derivative operator. We then investigate some convolution properties and coefficient estimates for functions belonging to this subclass. Furthermore, Fekete-Szegö problems and several inequalities are studied. Various corollaries and consequences of most of our results are connected with earlier works related to the field of investigation here.

2. Convolution properties and coefficient estimates

Unless otherwise mentioned, we assume throughout this paper that $\theta \in [0,\ 2\pi)$, $\alpha,\ \beta \ge 0$, $-1 \le B < A \le 1$, $0 < q < 1$, $\xi \in \mathbb{U}$ and $\psi \in \mathscr{A}$ given by (1).

Theorem 1 $\psi \in \mathscr{SC}_q[\alpha,\ \beta;\ A,\ B]$ if and only if

$$\frac{1}{\xi}\left[\psi(\xi) * \frac{\xi - \left(\dfrac{\alpha q - \beta}{\alpha + \beta} + \dfrac{\alpha + \beta(1+q)}{\alpha + \beta}\Omega\right) q\xi^2 + \dfrac{\alpha}{\alpha + \beta}\Omega q^3 \xi^3}{(1-\xi)(1-q\xi)(1-q^2\xi)}\right] \ne 0, \tag{12}$$

where Ω is given by

$$\Omega = \Omega(\theta,\ A,\ B) = \frac{\mathrm{e}^{-i\theta} + A}{A - B}. \tag{13}$$

Proof. If $\psi \in \mathscr{SC}_q[\alpha,\ \beta;\ A,\ B]$, then there is a Schwarz function $\omega(\xi)$ in $\mathbb{U}$ such that

$$\frac{\alpha \xi \mathscr{D}_q \psi(\xi) + \beta \xi \mathscr{D}_q(\xi \mathscr{D}_q \psi(\xi))}{\alpha \psi(\xi) + \beta \xi \mathscr{D}_q \psi(\xi)} = \frac{1 + A\omega(\xi)}{1 + B\omega(\xi)}, \tag{14}$$

hence

$$\frac{\alpha\xi\mathscr{D}_q\psi(\xi)+\beta\xi\mathscr{D}_q(\xi\mathscr{D}_q\psi(\xi))}{\alpha\psi(\xi)+\beta\xi\mathscr{D}_q\psi(\xi)} \neq \frac{1+A\mathrm{e}^{i\theta}}{1+B\mathrm{e}^{i\theta}} \quad (0\leq\theta<2\pi),$$

which is equivalent to

$$\frac{1}{\xi}\left[\left(1+B\mathrm{e}^{i\theta}\right)\left\{\alpha\xi\mathscr{D}_q\psi(\xi)+\beta\xi\mathscr{D}_q(\xi\mathscr{D}_q\psi(\xi))\right\}-\left(1+A\mathrm{e}^{i\theta}\right)\left\{\alpha\psi(\xi)+\beta\mathscr{D}_q\psi(\xi)\right\}\right]\neq 0. \tag{15}$$

It is easy to verify that

$$\psi(\xi)*\frac{\xi}{1-\xi}=\psi(\xi), \tag{16}$$

$$\psi(\xi)*\frac{\xi}{(1-\xi)(1-q\xi)}=\xi\mathscr{D}_q\psi(\xi), \tag{17}$$

and

$$\psi(\xi)*\frac{\xi(1+q\xi)}{(1-\xi)(1-q\xi)(1-q^2\xi)}=\xi\mathscr{D}_q(\xi\mathscr{D}_q\psi(\xi)). \tag{18}$$

Using (16), (17) and (18), Eq. (15) may be written as

$$\frac{1}{\xi}\left[\left(1+B\mathrm{e}^{i\theta}\right)\left\{\psi(\xi)*\frac{\alpha\xi}{(1-\xi)(1-q\xi)}+\psi(\xi)*\frac{\beta\xi(1+q\xi)}{(1-\xi)(1-q\xi)(1-q^2\xi)}\right\}\right.$$

$$\left.-\left(1+A\mathrm{e}^{i\theta}\right)\left\{\psi(\xi)*\frac{\alpha\xi}{1-\xi}+\psi(\xi)*\frac{\beta\xi}{(1-\xi)(1-q\xi)}\right\}\right]$$

$$=\frac{(\alpha+\beta)(B-A)\mathrm{e}^{i\theta}}{\xi}\left[\psi(\xi)*\frac{\xi-\left(\frac{-\beta}{\alpha+\beta}+\frac{\alpha+\beta(1+q)}{\alpha+\beta}\frac{\mathrm{e}^{-i\theta}+A}{A-B}\right)q\xi^2+\frac{\alpha}{\alpha+\beta}\frac{\mathrm{e}^{-i\theta}+A}{A-B}q^3\xi^3}{(1-\xi)(1-q\xi)(1-q^2\xi)}\right]$$

$$=\frac{(\alpha+\beta)(B-A)\mathrm{e}^{i\theta}}{\xi}\left[\psi(\xi)*\frac{\xi-\left(\frac{\alpha q-\beta}{\alpha+\beta}+\frac{\alpha+\beta(1+q)}{\alpha+\beta}\Omega\right)q\xi^2+\frac{\alpha}{\alpha+\beta}\Omega q^3\xi^3}{(1-\xi)(1-q\xi)(1-q^2\xi)}\right]\neq 0$$

which leads to (12), which proves the necessary condition of Theorem 1.

Reversely, since, it was shown in the first part of the proof that the assumption (15) is equivalent to (12), we obtain that

$$\frac{\alpha\xi\mathscr{D}_q\psi(\xi)+\beta\xi\mathscr{D}_q(\xi\mathscr{D}_q\psi(\xi))}{\alpha\psi(\xi)+\beta\xi\mathscr{D}_q\psi(\xi)}\neq\frac{1+Ae^{i\theta}}{1+Be^{i\theta}}\quad(0\leq\theta<2\pi), \tag{19}$$

if we denote

$$\varphi(\xi)=\frac{\alpha\xi\mathscr{D}_q\psi(\xi)+\beta\xi\mathscr{D}_q(\xi\mathscr{D}_q\psi(\xi))}{\alpha\psi(\xi)+\beta\xi\mathscr{D}_q\psi(\xi)}$$

and

$$\psi(\xi)=\frac{1+A\xi}{1+B\xi},$$

the relation (19) shows that $\varphi(\mathbb{U})\cap\psi(\partial\mathbb{U})=\emptyset$. Thus, the simply-connected domain $\varphi(\mathbb{U})$ is included in a connected component of $\mathbb{C}\backslash\psi(\partial\mathbb{U})$. From here, using the fact that $\varphi(0)=\psi(0)$ together with the univalence of the function ψ, it follows that $\varphi(\xi)\prec\psi(\xi)$, which represents in fact the subordination (13), i.e. $\psi\in\mathscr{SC}_q[\alpha,\ \beta;\ A,\ B]$. This complete the proof of Theorem 1. □

Letting $q\to 1^-$ in Theorem 1, we obtain

Corollary 1 $\psi\in\mathscr{SC}[\alpha,\ \beta;\ A,\ B]$ if and only if

$$\frac{1}{\xi}\left[\psi(\xi)*\frac{\xi-\left(\frac{\alpha-\beta}{\alpha+\beta}+\frac{\alpha+2\beta}{\alpha+\beta}\Omega\right)\xi^2+\frac{\alpha}{\alpha+\beta}\Omega\xi^3}{(1-\xi)^3}\right]\neq 0,$$

where Ω is given by (13).

Putting $\beta=0$ in Theorem 1, we obtain

Corollary 2 [32, Theorem 1] $\psi\in\mathscr{S}_q[A,\ B]$ if and only if

$$\frac{1}{\xi}\left[\psi(\xi)*\frac{\xi-\Omega q\xi^2}{(1-\xi)(1-q\xi)}\right]\neq 0,$$

where Ω is given by (13).

Remark 1 Letting $q\to 1^-$ in Corollary 2, we derive the convolution result of the subclass $\mathscr{S}[A,\ B]$ which improves the result in [11, Theorem 1] (see also [10, Theorem 7]).

Putting $\alpha=0$ in Theorem 1, we obtain

Corollary 3 [32, Theorem 5] $\psi\in\mathscr{C}_q[A,\ B]$ if and only if

$$\frac{1}{\xi}\left[\psi(\xi)*\frac{\xi-[\Omega(1+q)-1]q\xi^2}{(1-\xi)(1-q\xi)(1-q^2\xi)}\right]\neq 0,$$

where Ω is given by (13).

Remark 2 Letting $q \to 1^-$ in Corollary 3, we obtain the convolution result of the subclass $\mathscr{C}[A, B]$ which improves the result in [11, Theorem 2].

Theorem 2 $\psi \in \mathscr{S}\mathscr{C}_q[\alpha, \beta; A, B]$ if and only if

$$1-\sum_{k=2}^{\infty}\left(\frac{\beta [k]_q+\alpha}{\alpha+\beta}\right)\frac{\left([k]_q-1\right)\left(\mathrm{e}^{-i\theta}+B\right)-A+B}{(A-B)}\rho_k\xi^{k-1}\neq 0. \tag{20}$$

Proof. From Theorem 1, we have $\psi \in \mathscr{S}\mathscr{C}_q[\alpha, \beta; A, B]$ if and only if

$$\frac{1}{\xi}\left[\psi(\xi)*\frac{\xi-\left[\frac{\alpha q-\beta}{\alpha+\beta}+\frac{\alpha+\beta(1+q)}{\alpha+\beta}\Omega\right]q\xi^2+\frac{\alpha}{\alpha+\beta}\Omega q^3\xi^3}{(1-\xi)(1-q\xi)(1-q^2\xi)}\right]\neq 0 \tag{21}$$

for all Ω given by (13). The left hand side of (21) can be written as

$$\frac{1}{\xi}\left[\psi(\xi)*\left(\frac{\alpha\Omega}{\alpha+\beta}\frac{\xi}{1-\xi}+\frac{\alpha-(\alpha-\beta)\Omega}{\alpha+\beta}\frac{\xi}{(1-\xi)(1-q\xi)}+\frac{\beta(1-\Omega)}{\alpha+\beta}\frac{\xi(1+q\xi)}{(1-\xi)(1-q\xi)(1-q^2\xi)}\right)\right]$$

$$=\frac{1}{\xi}\left[\frac{\alpha\Omega}{\alpha+\beta}\psi(\xi)+\frac{\alpha-(\alpha-\beta)\Omega}{\alpha+\beta}\xi\mathscr{D}_q\psi(\xi)+\frac{\beta(1-\Omega)}{\alpha+\beta}\xi\mathscr{D}_q(\xi\mathscr{D}_q\psi(\xi))\right]$$

$$=1-\sum_{k=2}^{\infty}\left(\frac{\beta(\Omega-1)}{\alpha+\beta}\left([k]_q\right)^2+\frac{(\alpha-\beta)\Omega-\alpha}{\alpha+\beta}[k]_q-\frac{\alpha\Omega}{\alpha+\beta}\right)\rho_k\xi^{k-1}$$

$$=1-\sum_{k=2}^{\infty}\left(\frac{\beta [k]_q+\alpha}{\alpha+\beta}\right)\frac{\left([k]_q-1\right)\left(\mathrm{e}^{-i\theta}+B\right)-A+B}{A-B}\rho_k\xi^{k-1}.$$

Thus, the proof of Theorem 2 is completed.

Letting $q \to 1^-$ in Theorem 2, we obtain

Corollary 4 $\psi \in \mathscr{S}\mathscr{C}[\alpha, \beta; A, B]$ if and only if

$$1-\sum_{k=2}^{\infty}\left(\frac{\alpha+\beta k}{\alpha+\beta}\right)\frac{(k-1)\left(\mathrm{e}^{-i\theta}+B\right)-A+B}{A-B}\rho_k\xi^{k-1}\neq 0.$$

Taking $\beta=0$ in Theorem 2, we get

Corollary 5 [32, Theorem 9] $\psi \in \mathscr{S}_q[A, B]$ if and only if

$$1-\sum_{k=2}^{\infty}\frac{\left([k]_q-1\right)\left(\mathrm{e}^{-i\theta}+B\right)-A+B}{A-B}\rho_k\xi^{k-1}\neq 0.$$

Taking $\alpha = 0$ in Theorem 2, we get

Corollary 6 [32, Theorem 13] $\psi \in \mathscr{C}_q[A, B]$ if and only if

$$1 - \sum_{k=2}^{\infty} [k]_q \frac{\left([k]_q - 1\right)\left(e^{-i\theta} + B\right) - A + B}{A - B} \rho_k \xi^{k-1} \neq 0.$$

Theorem 3 If $\psi \in \mathscr{A}$ satisfy the following inequality

$$\sum_{k=2}^{\infty} \left(\frac{\beta [k]_q + \alpha}{\alpha + \beta}\right) \left\{\left([k]_q - 1\right)(1 - B) + A - B\right\} |\rho_k| \leq A - B,$$

then $\psi \in \mathscr{S}\mathscr{C}_q[\alpha, \beta; A, B]$.

Proof. Since

$$\left|1 - \sum_{k=2}^{\infty} \left(\frac{\beta [k]_q + \alpha}{\alpha + \beta}\right) \frac{\left([k]_q - 1\right)\left(e^{-i\theta} + B\right) - A + B}{A - B} \rho_k \xi^{k-1}\right|$$

$$\geq 1 - \left|\sum_{k=2}^{\infty} \left(\frac{\beta [k]_q + \alpha}{\alpha + \beta}\right) \frac{\left([k]_q - 1\right)\left(e^{-i\theta} + B\right) - A + B}{A - B} \rho_k \xi^{k-1}\right|$$

$$\geq 1 - \sum_{k=2}^{\infty} \left(\frac{\beta [k]_q + \alpha}{\alpha + \beta}\right) \frac{\left([k]_q - 1\right)(1 - B) + A - B}{A - B} |\rho_k| > 0.$$

Thus, the result follows from Theorem 2. □

Letting $q \to 1^-$ in Theorem 3, we obtain

Corollary 7 If $\psi \in \mathscr{A}$ satisfy the following inequality

$$\sum_{k=2}^{\infty} \left(\frac{\alpha + k\beta}{\alpha + \beta}\right) \{(k - 1)(1 - B) + A - B\} |\rho_k| \leq A - B,$$

then $\psi \in \mathscr{S}\mathscr{C}[\alpha, \beta; A, B]$.

Remark 3

(i) Taking $\beta = 0$ in Theorem 3, we get the coefficient estimates for the subclass $\mathscr{S}_q[A, B]$ (see [32, Theorem 17]);

(ii) Taking $\alpha = 0$ in Theorem 3, we get the coefficient estimates for the subclass $\mathscr{C}_q[A, B]$ (see [32, Theorem 21]);

3. Fekete-Szegö problems

In this section, we study the Fekete-Szegö problems for the subclass $\mathscr{S}\mathscr{C}_q[\alpha, \beta; A, B]$. In order to establish our results, we need the following lemmas.

Lemma 1 [35] If

$$\varphi(\xi)=1+\varkappa_1\xi+\varkappa_2\xi^2+.....$$

is an analytic function with positive real part in $\mathbb{U}$ and v is a complex number, then

$$\left|\varkappa_2-v\varkappa_1^2\right|\leq 2\max\{1;\ |2v-1|\}.$$

The result is sharp for

$$\varphi(\xi)=\frac{1+\xi^2}{1-\xi^2}\ \text{ and }\ \varphi(\xi)=\frac{1+\xi}{1-\xi}.$$

Lemma 2 [35] If

$$\varphi(\xi)=1+\varkappa_1\xi+\varkappa_2\xi^2+...$$

is an analytic function with a positive real part in $\mathbb{U}$, then

$$\left|\varkappa_2-\kappa\varkappa_1^2\right|\leq\begin{cases}-4\kappa+2 & (\kappa\leq 0),\\ 2 & (0\leq\kappa\leq 1),\\ 4\kappa-2 & (\kappa\geq 1),\end{cases}$$

when $\kappa<0$ or $\kappa>1$, the equality holds if and only if $\varphi(\xi)=\frac{1+\xi}{1-\xi}$ or one of its rotations. If $0<\kappa<1$, then the equality holds if and only if $\varphi(\xi)=\frac{1+\xi^2}{1-\xi^2}$ or one of its rotations. If $\kappa=0$, the equality holds if and only if

$$\varphi(\xi)=\left(\frac{1+\lambda}{2}\right)\frac{1+\xi}{1-\xi}+\left(\frac{1-\lambda}{2}\right)\frac{1-\xi}{1+\xi}\,(0\leq\lambda\leq 1)$$

or one of its rotations. If $\kappa=1$, the equality holds if and only if $\varphi(\xi)$ is the reciprocal of one of the functions such that equality holds in the case of $\kappa=0$.

Also the above upper bound is sharp, and it can be improved as follows when $0<\kappa<1$:

$$\left|\varkappa_2-\kappa\varkappa_1^2\right|+\kappa|\varkappa_1|^2\leq 2\ \left(0\leq\kappa\leq\frac{1}{2}\right)$$

and

$$\left|\varkappa_2-\kappa\varkappa_1^2\right|+(1-\kappa)|\varkappa_1|^2\leq 2\ \left(\frac{1}{2}\leq\kappa\leq 1\right).$$

Theorem 4 If $\psi \in \mathscr{SC}_q[\alpha,\ \beta;\ A,\ B]$, then

$$\left|\rho_3 - \mu\rho_2^2\right| \leq \frac{(A-B)(\alpha+\beta)}{q(1+q)[\alpha+(1+q+q^2)\beta]}$$

$$\max\left\{1;\ \left|B - \frac{A-B}{q}\left(1 - \frac{q(1+q)(\alpha+\beta)\left[\alpha+\left(1+q+q^2\right)\beta\right]}{q[\alpha+(1+q)\beta]^2}\mu\right)\right|\right\}. \tag{22}$$

Proof. If $\psi \in \mathscr{SC}_q[\alpha,\ \beta;\ A,\ B]$, then there is a Schwarz function $\omega(\xi)$ in $\mathbb{U}$ such that

$$\frac{\alpha\xi\mathscr{D}_q\psi(\xi) + \beta\xi\mathscr{D}_q(\xi\mathscr{D}_q\psi(\xi))}{\alpha\psi(\xi) + \beta\xi\mathscr{D}_q\psi(\xi)} = \frac{1+A\omega(\xi)}{1+B\omega(\xi)}. \tag{23}$$

Define $\varphi(\xi)$ by

$$\varphi(\xi) = \frac{1+\omega(\xi)}{1-\omega(\xi)} = 1 + \varkappa_1\xi + \varkappa_2\xi^2 + \dots. \tag{24}$$

Since $\omega(\xi)$ is a Schwarz function, we see that $\Re\{\varphi(\xi)\} > 0$ and $\varphi(0) = 1$. Therefore,

$$\frac{1+A\omega(\xi)}{1+B\omega(\xi)} = \frac{1-A+(1+A)\varphi(\xi)}{1-B+(1+B)\varphi(\xi)}$$

$$= 1 + \frac{(A-B)}{2}\varkappa_1\xi + \frac{(A-B)}{2}\left[\varkappa_2 - \frac{(1+B)}{2}\varkappa_1^2\right]\xi^2 + \dots. \tag{25}$$

Substituting (25) in (23), we have

$$\frac{\alpha\xi\mathscr{D}_q\psi(\xi) + \beta\xi\mathscr{D}_q(\xi\mathscr{D}_q\psi(\xi))}{\alpha\psi(\xi) + \beta\xi\mathscr{D}_q\psi(\xi)} = 1 + \frac{(A-B)}{2}\varkappa_1\xi + \frac{(A-B)}{2}\left[\varkappa_2 - \frac{(1+B)}{2}\varkappa_1^2\right]\xi^2 + \dots. \tag{26}$$

From (26), we obtain

$$\frac{\alpha+(1+q)\beta}{\alpha+\beta}\rho_2 = \frac{(A-B)}{2}\varkappa_1$$

and

$$\frac{q(1+q)\left[\alpha+\left(1+q+q^2\right)\beta\right]}{\alpha+\beta}\rho_3 - \frac{q[\alpha+(1+q)\beta]^2}{(\alpha+\beta)^2}\rho_2^2 = \frac{(A-B)}{2}\left[\varkappa_2 - \frac{(1+B)}{2}\varkappa_1^2\right],$$

or,

$$\rho_2 = \frac{(A-B)(\alpha+\beta)}{2q[\alpha+(1+q)\beta]}\varkappa_1$$

and

$$\rho_3 = \frac{(A-B)(\alpha+\beta)}{2q(1+q)[\alpha+(1+q+q^2)\beta]}\left[\varkappa_2 - \frac{1}{2}\left(1+B-\frac{A-B}{q}\right)\varkappa_1^2\right].$$

Hence, we have

$$\rho_3 - \mu\rho_2^2 = \frac{(A-B)(\alpha+\beta)}{2q(1+q)[\alpha+(1+q+q^2)\beta]}\left\{\varkappa_2 - \kappa\varkappa_1^2\right\}, \tag{27}$$

where

$$\kappa = \frac{1}{2}\left[1+B-\frac{A-B}{q}\left(1-\frac{q(1+q)(\alpha+\beta)\left[\alpha+\left(1+q+q^2\right)\beta\right]}{q[\alpha+(1+q)\beta]^2}\mu\right)\right]. \tag{28}$$

Our result (22) now follows from Lemma 1. This completes the proof of Theorem 1. □

Letting $q \to 1^-$ in Theorem 4, we obtain

Corollary 8 If $\psi \in \mathscr{SC}[\alpha,\ \beta;\ A,\ B]$, then

$$\left|\rho_3 - \mu\rho_2^2\right| \le \frac{(A-B)(\alpha+\beta)}{2(\alpha+3\beta)}\max\left\{1;\ \left|B-(A-B)\left(1-\frac{2(\alpha+\beta)(\alpha+3\beta)}{(\alpha+2\beta)^2}\mu\right)\right|\right\}.$$

Putting $\beta = 0$ in Theorem 4, we obtain

Corollary 9 If $\psi \in \mathscr{S}_q[A,\ B]$, then

$$\left|\rho_3 - \mu\rho_2^2\right| \le \frac{A-B}{q(1+q)}\max\left\{1;\ \left|B-\frac{A-B}{q}(1-(1+q)\mu)\right|\right\}.$$

Putting $\alpha = 0$ in Theorem 4, we obtain

Corollary 10 If $\psi \in \mathscr{C}_q[A,\ B]$, then

$$\left|\rho_3 - \mu\rho_2^2\right| \le \frac{A-B}{q(1+q)(1+q+q^2)}\max\left\{1;\ \left|B-\frac{A-B}{q}\left(1-\frac{1+q+q^2}{1+q}\mu\right)\right|\right\}. \tag{29}$$

Theorem 5 Let

$$\chi_1 = \frac{[\alpha+(1+q)\beta]^2[A-(1+q)B-q]}{(1+q)(\alpha+\beta)[\alpha+(1+q+q^2)\beta](A-B)},$$

$$\chi_2 = \frac{[\alpha+(1+q)\beta]^2[A-(1+q)B+q]}{(1+q)(\alpha+\beta)[\alpha+(1+q+q^2)\beta](A-B)},$$

$$\chi_3 = \frac{[\alpha+(1+q)\beta]^2[A-(1+q)B]}{(1+q)(\alpha+\beta)[\alpha+(1+q+q^2)\beta](A-B)}.$$

If $\psi \in \mathscr{SC}_q[\alpha,\ \beta;\ A,\ B]$, then

$$\left|\rho_3-\mu\rho_2^2\right| \le \begin{cases} -\dfrac{(A-B)(\alpha+\beta)}{q(1+q)[\alpha+(1+q+q^2)\beta]} \\ \left[B-\dfrac{A-B}{q}\left(1-\dfrac{(1+q)(\alpha+\beta)\left[\alpha+\left(1+q+q^2\right)\beta\right]\mu}{[\alpha+q(1+q)\beta]^2}\right)\right] & (\mu \le \chi_1), \\ \dfrac{(A-B)(\alpha+\beta)}{q(1+q)[\alpha+(1+q+q^2)\beta]} & (\chi_1 \le \mu \le \chi_2), \\ \dfrac{(A-B)(\alpha+\beta)}{q(1+q)[\alpha+(1+q+q^2)\beta]} \\ \left[B-\dfrac{A-B}{q}\left(1-\dfrac{(1+q)(\alpha+\beta)\left[\alpha+\left(1+q+q^2\right)\beta\right]\mu}{[\alpha+q(1+q)\beta]^2}\right)\right] & (\mu \ge \chi_2). \end{cases}$$

Further, if $\chi_1 \le \mu \le \chi_3$, then

$$\left|\rho_3-\mu\rho_2^2\right| + \frac{q[\alpha+(1+q)\beta]^2}{(1+q)(\alpha+\beta)[\alpha+(1+q+q^2)\beta]}$$

$$\left[\frac{1+B}{A-B}-\frac{1}{q}\left(1-\frac{(1+q)(\alpha+\beta)[\alpha+(1+q)\beta]\mu}{[\alpha+q(1+q)\beta]^2}\right)\right]|\rho_2|^2 \le \frac{(A-B)(\alpha+\beta)}{q(1+q)[\alpha+q(1+q)\beta]}.$$

If $\chi_3 \le \mu \le \chi_2$, then

$$\left|\rho_3-\mu\rho_2^2\right| + \frac{q[\alpha+(1+q)\beta]^2}{(1+q)(\alpha+\beta)[\alpha+(1+q+q^2)\beta]}$$

$$\left[\frac{1-B}{A-B}+\frac{1}{q}\left(1-\frac{(1+q)(\alpha+\beta)[\alpha+(1+q)\beta]\mu}{[\alpha+q(1+q)\beta]^2}\right)\right]|\rho_2|^2 \le \frac{(A-B)(\alpha+\beta)}{q(1+q)[\alpha+q(1+q)\beta]}.$$

Proof. Applying Lemma 2 to (27) and (28), we can obtain our results asserted by Theorem 5. □

Letting $q \to 1^-$ in Theorem 5, we obtain
Corollary 11 Let

$$\chi_4 = \frac{(\alpha+2\beta)^2\{A-2B-1\}}{2(\alpha+\beta)(\alpha+3\beta)(A-B)},$$

$$\chi_5 = \frac{(\alpha+2\beta)^2\{A-2B+1\}}{2(\alpha+\beta)(\alpha+3\beta)(A-B)},$$

$$\chi_6 = \frac{(\alpha+2\beta)^2(A-2B)}{2(\alpha+\beta)(\alpha+3\beta)(A-B)}.$$

If $\psi \in \mathscr{SC}[\alpha,\ \beta;\ A,\ B]$, then

$$\left|\rho_3-\mu\rho_2^2\right| \leq \begin{cases} -\dfrac{(A-B)(\alpha+\beta)}{2(\alpha+3\beta)} \\ \left[B-(A-B)\left(1-\dfrac{2(\alpha+\beta)(\alpha+3\beta)}{(\alpha+2\beta)^2}\mu\right)\right] & (\mu \leq \chi_4), \\ \dfrac{(A-B)(\alpha+\beta)}{2(\alpha+3\beta)} & (\chi_4 \leq \mu \leq \chi_5), \\ \dfrac{(A-B)(\alpha+\beta)}{2(\alpha+3\beta)} \\ \left[B-(A-B)\left(1-\dfrac{2(\alpha+\beta)(\alpha+3\beta)}{(\alpha+2\beta)^2}\mu\right)\right] & (\mu \geq \chi_5). \end{cases}$$

Further, if $\chi_4 \leq \mu \leq \chi_6$, then

$$\left|\rho_3-\mu\rho_2^2\right| + \frac{(\alpha+2\beta)^2}{2(\alpha+3\beta)(\alpha+\beta)}$$

$$\left[\frac{1+B}{A-B} - \left(1-\frac{2(\alpha+\beta)(\alpha+3\beta)}{(\alpha+2\beta)^2}\mu\right)\right]|\rho_2|^2 \leq \frac{(A-B)(\alpha+\beta)}{2(\alpha+3\beta)}.$$

If $\chi_6 \leq \mu \leq \chi_5$, then

$$\left|\rho_3-\mu\rho_2^2\right| + \frac{(\alpha+2\beta)^2}{2(\alpha+3\beta)(\alpha+\beta)}$$

$$\left[\frac{1-B}{A-B} + 1 - \frac{2(\alpha+\beta)(\alpha+3\beta)}{(\alpha+2\beta)^2}\mu\right]|\rho_2|^2 \leq \frac{(A-B)(\alpha+\beta)}{2(\alpha+3\beta)}.$$

Putting $\beta = 0$ in Theorem 5, we obtain
Corollary 12 Let

$$\chi_7 = \frac{A-(1+q)B-q}{(1+q)(A-B)},$$

$$\chi_8 = \frac{A-(1+q)B+q}{(1+q)(A-B)},$$

$$\chi_9 = \frac{A-(1+q)B}{(1+q)(A-B)}.$$

If $\psi \in \mathscr{S}_q[A,\ B]$, then

$$\left|\rho_3 - \mu\rho_2^2\right| \leq \begin{cases} -\dfrac{A-B}{q(1+q)}\left[B - \dfrac{A-B}{q}(1-(1+q)\mu)\right] & (\mu \leq \chi_7), \\ \dfrac{A-B}{q(1+q)} & (\chi_7 \leq \mu \leq \chi_8), \\ \dfrac{A-B}{q(1+q)}\left[B - \dfrac{A-B}{q}(1-(1+q)\mu)\right] & (\mu \geq \chi_8). \end{cases}$$

Further, if $\chi_7 \leq \mu \leq \chi_9$, then

$$\left|\rho_3 - \mu\rho_2^2\right| + \frac{q}{1+q}\left[\frac{1+B}{A-B} - \frac{1}{q}(1-(1+q)\mu)\right]|\rho_2|^2 \leq \frac{A-B}{q(1+q)}.$$

If $\chi_9 \leq \mu \leq \chi_8$, then

$$\left|\rho_3 - \mu\rho_2^2\right| + \frac{q}{1+q}\left[\frac{1-B}{A-B} + \frac{1}{q}(1-(1+q)\mu)\right]|\rho_2|^2 \leq \frac{A-B}{q(1+q)}.$$

Putting $\alpha = 0$ in Theorem 5, we obtain
Corollary 13 Let

$$\chi_{10} = \frac{(1+q)[A-(1+q)B-q]}{A-B},$$

$$\chi_{11} = \frac{(1+q)[A-(1+q)B+q]}{A-B},$$

$$\chi_{12} = \frac{(1+q)[A-(1+q)B]}{A-B}.$$

If $\psi \in \mathscr{C}_q[A, B]$, then

$$\left|\rho_3-\mu\rho_2^2\right| \leq \begin{cases} -\dfrac{A-B}{q(1+q)(1+q+q^2)}\left[B-\dfrac{A-B}{q}\left(1-\dfrac{1+q+q^2}{1+q}\mu\right)\right] & (\mu \leq \chi_{10}), \\ \dfrac{A-B}{q(1+q)(1+q+q^2)} & (\chi_{10} \leq \mu \leq \chi_{11}), \\ \dfrac{A-B}{q(1+q)(1+q+q^2)}\left[B-\dfrac{A-B}{q}\left(1-\dfrac{1+q+q^2}{1+q}\mu\right)\right] & (\mu \geq \chi_{11}). \end{cases}$$

Further, if $\chi_{10} \leq \mu \leq \chi_{12}$, then

$$\left|\rho_3-\mu\rho_2^2\right|+\frac{q(1+q)}{1+q+q^2}\left[\frac{1+B}{A-B}-\frac{1}{q}\left(1-\frac{1+q+q^2}{1+q}\mu\right)\right]|\rho_2|^2 \leq \frac{A-B}{q(1+q)(1+q+q^2)}.$$

If $\chi_{12} \leq \mu \leq \chi_{11}$, then

$$\left|\rho_3-\mu\rho_2^2\right|+\frac{q(1+q)}{1+q+q^2}\left[\frac{1-B}{A-B}+\frac{1}{q}\left(1-\frac{1+q+q^2}{1+q}\mu\right)\right]|\rho_2|^2 \leq \frac{A-B}{q(1+q)(1+q+q^2)}.$$

Remark 4 For different choices of the parameters α, β, A, B, and q in the above theorems, we can get the corresponding results for each of the following subclasses $\mathscr{S}[A, B]$, $\mathscr{C}[A, B]$, $\mathscr{SC}_q(\alpha, \beta; \gamma)$, $\mathscr{SC}(\alpha, \beta; \gamma)$, $\mathscr{S}_q(\gamma)$, $\mathscr{S}(\gamma)$, $\mathscr{C}_q(\gamma)$, $\mathscr{C}(\gamma)$, $\mathscr{SC}_q(\alpha, \beta; \gamma, \delta)$, $\mathscr{SC}(\alpha, \beta; \gamma, \delta)$, $\mathscr{S}_q(\gamma, \delta)$, $\mathscr{S}(\gamma, \delta)$, $\mathscr{C}_q(\gamma, \delta)$ and $\mathscr{C}(\gamma, \delta)$ which are defined in Introduction section.

4. Conclusions

In our present investigation, we have defined a general subclass $\mathscr{SC}_q[\alpha, \beta; A, B]$ of normalized analytic functions associated with q-derivative operator. For functions belonging to this subclass, we have derived some interesting results such as convolution properties, coefficient estimates and the Fekete-Szegö problems the estimates. Furthermore, interesting corollaries and particular cases are shown for each of those results for particular choices of parameters found in the definition of this subclass. Our results are connected with those in several earlier works, which are related to the Geometric Function Theory.

For future studies, we can define the same subclass in the case of multivalent analytic functions by using q-derivative operator and study the same properties that we studied in this paper.

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Conflict of interest

The authors declare that they have no competing interests.

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