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Research Article

Entrepreneurial Strategies in Livestock Inventory Management with Trade Credit

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Abstract: Amelioration and deterioration rates play a vital role in inventory management. Some animals lose value due to illness, death, or other incidents as they gain weight when kept on farms in good health. Organizations and entrepreneurs can maximize the operational efficiency and health of their animals through a well-organized and efficient inventory system. The proposed inventory model provides a more realistic assumption of livestock with a Weibull amelioration rate and constant deterioration rate. In the inventory domain, trade credit plays a vital role in minimizing costs, in maintaining an adequate livestock supply, and in maximizing the credit period. The main objective of this article is not only to improve financial performance but also to enable livestock operations to satisfy market demands, to manage resources, and to successfully navigate the complex world of credit-based transactions. This inventory system is mathematically modelled by differential equations, and the total cost is determined. The optimality of the solved nonlinear minimization problem is verified by using pseudo concavity. Partial Swam Optimization (PSO) and Genetic Algorithm (GA) are designed to determine the optimal cycle length, the optimal credit period and the order quantity. Numerical examples are presented to illustrate the solution procedure for various cases of the proposed inventory model. Sensitivity analysis serves as a validation tool, providing a discerning assessment of the strength and flexibility of the model in response to variations in key parameters.

***Keywords*:** ameliorating, cashflow, credit period, livestock, metaheuristic

MSC: 90B05

Highlights:

• Introduces a groundbreaking fusion of trade credit principles with livestock inventory management, unlocking new dimensions for optimizing cash flow dynamics within the industry and enhancing financial flexibility.

• Innovatively addresses the complex challenge of managing both amelioration and deterioration rates in livestock inventories, providing a comprehensive and adaptive framework for dynamic inventory control that ensures optimal stock levels.

• Utilizes the Weibull distribution to accurately model the dynamic nature of amelioration rates, significantly enhancing the precision and realism of inventory forecasting and planning, leading to more informed decision-making.

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• Employs new strategic optimization techniques to fine-tune order quantities and replenishment cycles, minimizing costs and maximizing operational efficiency, thereby ensuring a highly effective inventory management strategy tailored specifically for the poultry farming industry.

Abbreviation

PSO	Partial Swam Optimization
GA	Genetic Algorithm
EOQ	Economic Order Quantity
EPQ	Economic Production Quantity
CA	Conventional Algorithm
FIFO	First In, First Out
LIFO	Last In, First Out

1. Introduction

Inventory management is a cornerstone of any successful entrepreneur's business endeavour, ensuring the timely availability of products in the correct quantity at optimal costs. Beyond mere logistics, it encompasses a strategic understanding of how items within inventory evolve over time, either growing or declining in value. In the realm of poultry farming, for instance, livestock steadily improve in value as they mature, a phenomenon known as amelioration. This dynamic amelioration is particularly evident in sectors such as agriculture, where livestock, e.g., ducks, pigs, broilers, and fish, undergo natural processes of growth and transformation. Conversely, the passage of time also increases the risk of deterioration through illness, mortality, or other unforeseen events. This dual trajectory underscores the complexity of inventory management, where decision-makers must navigate the interplay of appreciating and depreciating assets. Recognizing the simultaneous forces of amelioration and deterioration is paramount in effective inventory modelling. As the inventory fluctuates in response to these opposing trends, informed decision-making becomes beneficial and imperative. Whether for businesses, entrepreneurs, governmental bodies, or individuals, a nuanced understanding of these dynamics empowers stakeholders to optimize their inventory strategies, ensuring resilience and efficiency in supply chain operations.

In 1997, Hwang [1] first developed an inventory model for fast-growing items. Based on the Weibull amelioration rate, the economic order quantity (EOQ) and the partial selling quantity (PSQ) are generated for ameliorating items. Later, Hwang [2] proposed an inventory model for ameliorating and for deteriorating items with issuing policies-first in, first out (FIFO) and last in, first out (LIFO). Law and Wee [3] proposed inventory for both ameliorating and deteriorating inventory models with time discounts and partial backlogging. Optimization techniques are incorporated with discount cash flow to obtain an optimal solution.

Sana [4] analysed a multi-item inventory model for ameliorating and deteriorating items, where the Euler-Lagrange method was used to maximize the total profit by determining the optimal order quantity. Mondal et al. [5] proposed price-dependent ameliorating items with the Weibull distribution. Mahata and De [6] proposed an EOQ inventory model for ameliorating items under a retailer's partial trade credit. An algorithm was derived to determine the optimal pricing and inventory policies for retailers. Vandana and Sana [7] established a two-level inventory model for ameliorating items. A Weibull distribution amelioration rate with two parameters and a content deterioration rate was considered, where the deterioration rate was less than the amelioration rate. In recent years, a profuse inventory model was designed by Hatibaruah and Saha [8], Moon et al. [9] and Nodust et al. [10] for deteriorating and ameliorating items.

Deteriorating inventory models are subject to various influencing factors, such as price, stock, time, or combinations of these factors. Among these factors, price-sensitive demand appears to be a critical principle, where demand for stocks rises as prices rise and falls as prices fall. In the realm of inventory management, an inadequately determined order quantity can result in either shortages or waste, underscoring the critical importance of establishing the economic order quantity (EOQ). A plethora of methodologies have been crafted to pinpoint the most systematic order quantity in inventory modelling, spanning production models, integration models, and echelon models. However, the pivotal role of sales considerations is often overlooked within these methodologies, despite its undeniable importance. Forward-thinking entrepreneurs employ a diverse array of marketing tactics, including trade credit, discounts, and targeted advertising campaigns, to productively mitigate inventory deterioration and to maximize profitability. Trade credit serves as a predominant strategy for satisfying customer demands while concurrently optimizing inventory management practices. By intertwining these tactics, entrepreneurs can effectively navigate the complexities of deteriorating inventory models, enhancing both customer satisfaction and financial performance.

The conventional inventory system obligates retailers to settle their accounts promptly upon order receipt. However, with trade credit, suppliers extend a grace period during which retailers can settle their accounts without incurring interest charges. This arrangement allows retailers to defer payment for a specified duration, known as the credit period. Trade-credit financing offers dual advantages to suppliers. First, it entices new customers by presenting the deferred payment option as a price reduction incentive. Second, it encourages loyalty among existing customers, as they are inclined to settle their accounts promptly to capitalize on the opportunity for more frequent delays in payment. An optimal order strategy for the lot size model was designed by Haley and Higgins [11] by using an inventory policy and trade credit financing. Goyal [12] used trade credit to extend a traditional EOQ model in which the order quantity and replenishment period increase dramatically as the total cost decreases. Udayakumar et al. [13] proposed a non-instantaneous deterioration model with inflation and a time discount under permissible delay in payment. By using a computational algorithm, an optimal replenishment policy was derived to minimize the total inventory cost. Mahato et al. [14] developed an echelon inventory model for growing items with price and stock-dependent demand under a trade credit. The inventory cycle duration was influenced more by the growth patterns of livestock items than by the cost patterns. Notable studies include Mittal and Sharma [15], who examined ordering policies for growing items like poultry under trade-credit financing, offering a robust framework for managing amelioration in livestock contexts.

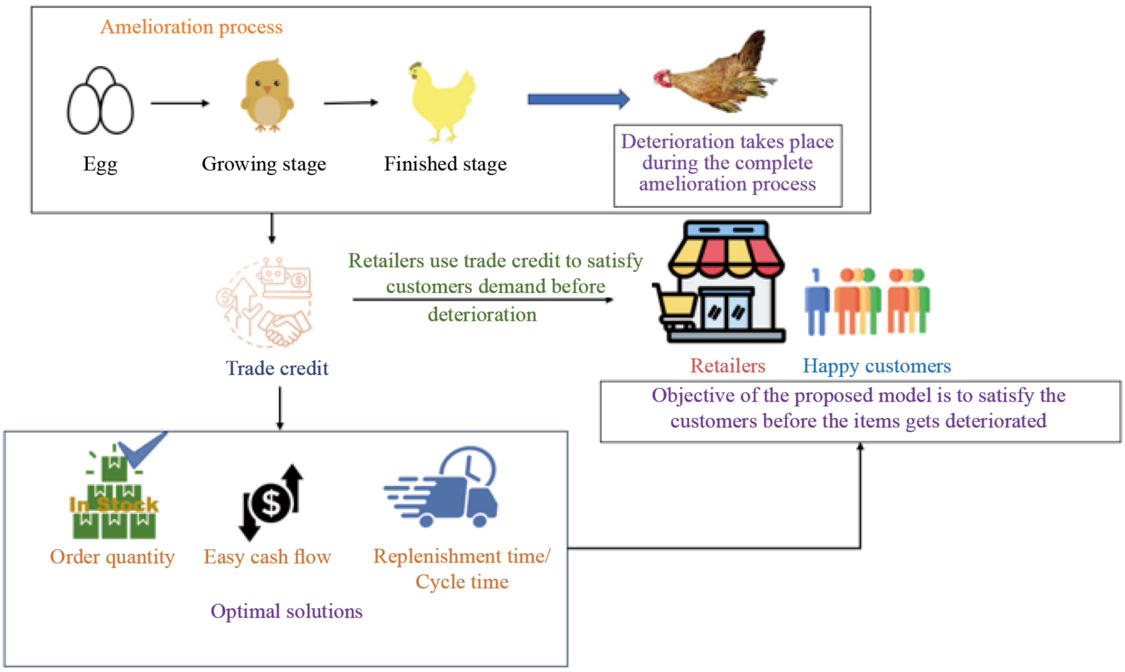


Figure 1. Schematic representation of the proposed model

The collection of articles listed in Table 1 underscores a significant lack of focus on livestock inventory model concerning the financial and economic losses attributed to the deterioration of items. Inventory management is imperative for livestock inventories since concurrent processes of amelioration and deterioration emphasize the need for proactive management, which is an elucidative influence on entrepreneurs in delineating optimal order quantities, particularly in the context of introducing trade credit to retailers. This strategic approach is pivotal for averting inventory loss and for ensuring the fulfilment of customer demands. The visual depiction of the proposed model is presented in Figure 1, offering a clear and comprehensive illustration of its key components and dynamics. In this study, it is emphasized that the ramifications of inventory loss stemming from deterioration extend beyond mere financial considerations, encompassing broader economic implications. strategic approach is pivotal for averting inventory loss and for ensuring the fulfilment of customer demands.

Table 1. Key characteristics of the present study with a review of the existing inventory model on ameliorating and deteriorating items

Source	EOQ/ EPQ	Demand	Deterioration	Amelioration	Trade credit	Framework
Mahata and De [6]	EOQ	Price sensitive	Constant	Constant	Partial trade credit	Efficient algorithm
Vandana and Sana [7]	EOQ	Constant	Constant	Weibull	-	Single-vendor multi-buyer
Hatibaruah and Saha [8]	EOQ	Stock & price	Constant	Weibull	-	Preservation investment
Moon et al. [9]	EOQ	Time-varying	Constant	Constant	-	Inflation and time discount
Nodoust et al. [10]	EPQ	Time	Constant	Constant	-	Evidential reasoning approach
Mittal and Sharma [15]	EOQ	Constant	Constant	Constant	Trade credit	Ordering policy
Shaikh et al. [16]	EOQ	Display shelf space, stock-dependent, expiration rate	Constant	-	Two-level trade credit	Non-zero inventory with partial backlogging
Jayashri and Umamaheswari [17]	EOQ	Stock	Weibull	-	Complete trade credit	Shortage with complete backlogging
Hatibaruah and Saha [18]	EPQ	Price, time & advertisement	Weibull	Weibull	-	Inflation
Mandal [19]	EOQ	Cubic	Constant	Constant	Permissible delay in payment	Salvage cost
Palanivel and Vetriselvi [20]	EOQ	Polynomial	Non-instantaneous	-	-	Warehouse model with advance payment
Padiyar [21]	EPQ	Dynamic	Instantaneous	-	-	Inflation and value of money
Singh and Rana [22]	EPQ	Linear	Time	Constant	1 level trade credit	Partial backlogging
Rai [23]	EOQ	Constant	Weibull	Weibull	2 Level trade credit	Suppliers, manufacturers and retailers relationship
Rana et al. [24]	EOQ	Time	Linear	Time varying	Permissible delay in payment	Carbon emission
Khan et al. [25]	EOQ	Constant	Constant	Constant	Advance payment	Discounts and carbon policy
Proposed work	EOQ	Price	Constant	Weibull	Complete trade credit	Metaheuristic algorithm

Research gap: Livestock inventory models are underexplored regarding integrating trade credit concepts for facilitating easy cash flow. Trade credit in livestock operations has not been adequately evaluated in the current literature, leading to a research gap. To address these challenges and to optimize resource utilization in livestock inventories, the aim of this article is to determine the optimal order quantity and replenishment cycle time. The focus is specifically on incorporating the trade credit model to manage cash flow effectively while ensuring the maintenance of the right inventory levels.

Research problem: In this research article, an ameliorating inventory model is developed for livestock items under a trade credit. More importance is given to livestock as amelioration and deterioration occur at the same time. By incorporating trade credit terms, the aim is to strike a balance among maintaining adequate livestock supply, minimizing costs, and capitalizing on favourable credit terms. Regarding livestock, the Weibull distribution is considered to indicate that the product amelioration rate is faster in the beginning and declines in the end. This approach not only improves financial performance but also ensures that livestock operations can effectively satisfy market demand, manage resources, and manage the complexities of credit-based transactions. In the proposed system, the total profit is calculated by solving differential equations via a PSO and to determine the optimal order quantity for inventory to clear stock in the warehouse before deterioration. The methodology for designing the proposed model, highlighting the factors, solution approaches, and optimization techniques utilized, is illustrated in Figure 2. This model is particularly suitable for entrepreneurs in the poultry farming industry to maximize their profits, which leads to improved financial performance and operational efficiency.

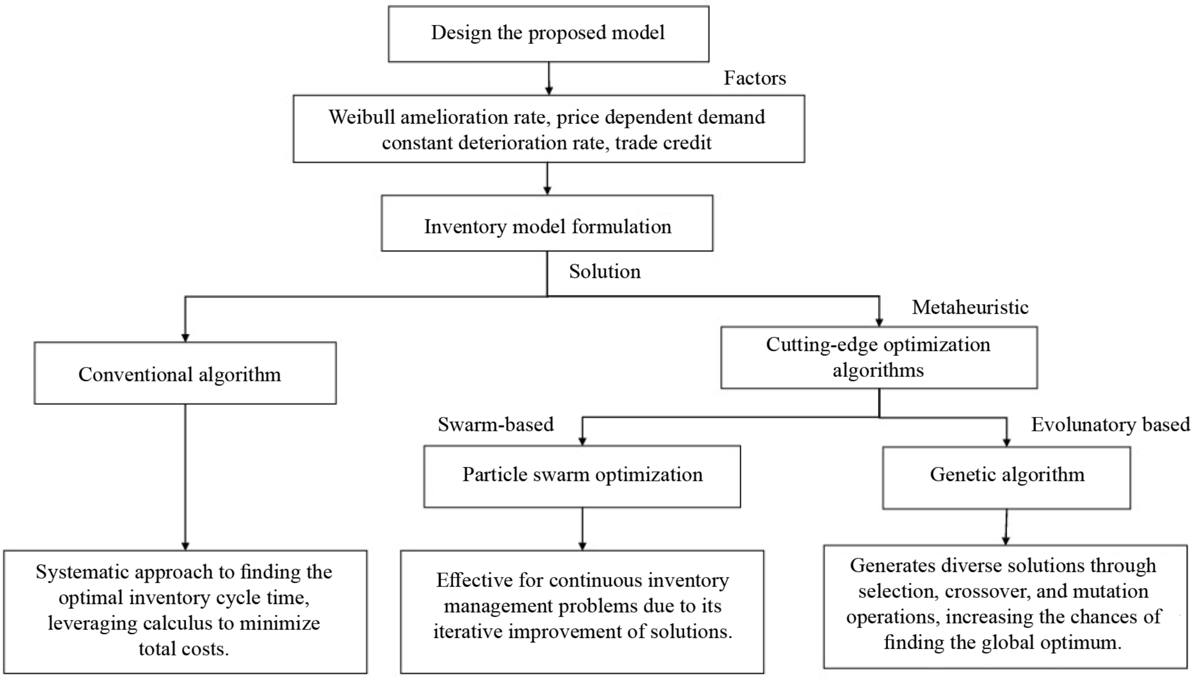


Figure 2. Research layout of the proposed model

Section 2 introduces the key assumptions and notations, providing the foundation for the subsequent analysis. In Section 3, the mathematical formulation of the problem is presented. Section 4 describes the CA, which serves as a benchmark for performance evaluation. Section 5 delves the cutting-edge optimization algorithms, namely GA and PSO, examining their effectiveness in addressing the problem. Section 6 presents the numerical analysis, showcasing experimental results that compare the performance of the algorithms. In Section 7, a sensitivity analysis is conducted to assess the impact of variations in input parameters on the algorithm's behaviour and results. Finally, Section 8 offers the conclusion, summarizing the key findings and proposing directions for future research.

2. Assumptions and notations

A sophisticated mathematical model was meticulously developed based on the provided assumptions and notation.

• Demand is a price-dependent function, where a represents the initial demand rate and b represents the initial change in demand parameters with price p, $D = a - bp$, where $a \geq 0$ and $b \geq 0$ [18].

• The amelioration rate 'A' can be represented as a Weibull distribution with two parameters based on scale parameter 'u' and shape parameters 'v', $A = uvt^{(v-1)}$, u, $v \geq 0$ [8].

• Amelioration and deterioration occur when the items are effectively in stock.

• Lead time is negligible.

• With an infinite time horizon, the replenishment rate is instantaneously infinite, while the size is finite.

• Neither shortage nor backlogging is considered.

• No replacement policy for deterioration.

Table 2 provides a comprehensive and structured representation of the variables, parameters, and notations used in the study, detailing the nomenclature of the proposed model.

Table 2. Nomenclature

Notations	Description
c	Unit cost of an item
a_c	Amelioration cost per unit item
d_c	Deterioration cost per unit item
h_c	Holding cost per unit item
O_c	Ordering cost per unit per order
$Q(t)$	Inventory level at any time t
I_p	Interest paid per year
I_e	Interest earned per year
R	Replenishment at time T
M	Credit period offered by the suppliers to their retailer's (time unit)
TC	Total cost per unit of time
θ	Deterioration rate, where $0 \leq \theta \leq 1$

3. Mathematical formulation

The differential equation describing the inventory at any given moment is $[0, T]$

$$\frac{dQ(t)}{dt} + \left(\theta - uvt^{v-1}\right) Q(t) = -(a - bp) \tag{1}$$

With the initial boundary conditions, $Q(0) = R$ and $Q(T) = 0$.

By solving the above differential equation, we obtain

$$Q(t) = (a - bp)\{1 - \theta t + ut^v\}\left[\left(T + \frac{\theta T^2}{2} - \frac{uT^{v+1}}{v+1}\right) - \left(t + \frac{\theta t^2}{2} - \frac{ut^{v+1}}{v+1}\right)\right] \tag{2}$$

Retailers achieve effective ameliorating inventory management by scrutinizing the rate of change of deterioration (D_T) and amelioration (A_T) and the inventory quantity (I_T) over a period $[0, T]$.

Order quantity: The order quantity determines the total inventory to be replenished during a cycle, considering both amelioration and deterioration rates. This metric is critical for balancing supply and demand while maintaining optimal stock levels, especially for items with growth-dependent dynamics, such as livestock.

$$R = Q(0) = (a - bp)\left[T + \frac{\theta T^2}{2} - \frac{uT^{v+1}}{v+1}\right] \tag{3}$$

Inventory units over the cycle [0, T]: This refers to the inventory levels at any given time during the replenishment cycle. It incorporates the interplay of amelioration (growth) and deterioration, providing a dynamic perspective on stock availability throughout the cycle.

$$\begin{aligned} I_T &= (a - bp)\int_0^T Q(t)dt \\ &= \frac{(a-bp)}{2(v+1)}\left[\frac{u^2T^{2v+2}}{(v+1)(2v+2)} - \frac{\theta uT^{v+3}}{2(v+3)} - \frac{uvT^{v+2}}{(v+1)(v+2)} + \frac{\theta^2}{8}T^4 + \frac{\theta}{6}T^3 - \frac{2uT^{v+2}}{v+1} + \frac{(\theta T^2 + 2T)}{2}T \right. \\ &\left. - \frac{uT^{2v+2}}{v+1} + \frac{(uT^2\theta + 2Tu)}{2(v+1)}T^{v+1} - \frac{(T\theta + 1)}{2}T^2 + \frac{(2u\theta T^{v+1} - T^2\theta^2(uv+1))}{4(v+1)}T^2\right] \end{aligned} \tag{4}$$

Deteriorating units in the inventory cycle: Deteriorating units represent the inventory losses incurred due to factors like spoilage, aging, or value depreciation during the cycle. Accurately assessing these losses allows businesses to implement strategies to minimize waste and associated costs.

$$\begin{aligned} D_T &= (a - bp)\int_0^T \theta Q(t)dt \\ &= \theta\frac{(a-bp)}{2(v+1)}\left[\frac{u^2T^{2v+2}}{(v+1)(2v+2)} - \frac{\theta uT^{v+3}}{2(v+3)} - \frac{uvT^{v+2}}{(v+1)(v+2)} + \frac{\theta^2}{8}T^4 + \frac{\theta}{6}T^3 - \frac{2uT^{v+2}}{v+1} \right. \\ &\left. + \frac{(\theta T^2 + 2T)}{2}T - \frac{uT^{2v+2}}{v+1} + \frac{(uT^2\theta + 2Tu)}{2(v+1)}T^{v+1} - \frac{(T\theta + 1)}{2}T^2 + \frac{(2u\theta T^{v+1} - T^2\theta^2(uv+1))}{4(v+1)}T^2\right] \end{aligned} \tag{5}$$

Ameliorating units in the inventory cycle: Amelioration units refer to the portion of inventory that grows or improves in quality or value during the replenishment cycle. This metric is vital for inventory systems with growth dynamics, ensuring that ameliorated items are factored into supply planning.

$$A_T = (a-bp)\int_0^T uvt^{(v-1)}Q(t)dt$$

$$= uv(a-bp)\left[\frac{\theta uT^{2v+3}}{2(2v+3)} + \frac{uT^{2v+2}}{2v+1} + \frac{\theta^2 T^{v+3}}{2(v+3)} - \frac{\theta^2 T^{v+4} + \theta T^{v+2}}{2(v+2)} - \frac{(2T\theta - T^2\theta)}{2(v+1)}T^{v+1} + \frac{T^{v+1}}{v}\right.$$

$$\left. + \frac{u^2 T^{3v+2}}{(v+1)(3v+2)} - \frac{\theta uT^{2v+2}}{(v+1)(2v+2)} + \frac{u(1-uT^{v+1})T^{2v+1}}{(v+1)(2v+1)} + \frac{\theta uT^{2v+2}}{(v+1)^2} - \frac{uT^{2v+1}}{v(v+1)}\right] \tag{6}$$

Below are the detailed calculations for the cost components include ordering cost, deterioration cost, carrying cost, ameliorating cost, holding cost, interest paid, and interest earned. These cost factors emphasize the imperative need for thorough consideration.

• **Ordering cost:** Ordering costs denote the expenses incurred by retailers when placing orders to restock items. These costs typically encompass administrative expenses, handling fees, and other charges directly associated with procuring inventory from suppliers or manufacturers.

$$OC = O_c$$

• **Ameliorating cost:** Amelioration costs are the expenses associated with improving or enhancing stock over time. These costs may include specialized storage, monitoring, or processing fees required to increase the quality, value, or utility of inventory, such as maturation or refinement processes.

$$AC = a_c * A_T$$

$$= a_c uv(a-bp)\left[\frac{\theta uT^{2v+3}}{2(2v+3)} + \frac{uT^{2v+2}}{2v+1} + \frac{\theta^2 T^{v+3}}{2(v+3)} - \frac{\theta^2 T^{v+4} + \theta T^{v+2}}{2(v+2)} - \frac{(2T\theta - T^2\theta)}{2(v+1)}T^{v+1}\right.$$

$$\left. + \frac{T^{v+1}}{v} + \frac{u^2 T^{3v+2}}{(v+1)(3v+2)} - \frac{\theta uT^{2v+2}}{(v+1)(2v+2)} + \frac{u(1-uT^{v+1})T^{2v+1}}{(v+1)(2v+1)} + \frac{\theta uT^{2v+2}}{(v+1)^2} - \frac{uT^{2v+1}}{v(v+1)}\right] \tag{7}$$

• **Deteriorating cost:** Deterioration costs represent the losses incurred when stock quality declines due to spoilage, obsolescence, or natural wear and tear. These costs often encompass the value of unusable inventory, as well as any expenses associated with the disposal or replacement of degraded items.

$DC = d_c * D_T$

$$d_c\theta\frac{(a-bp)}{2(v+1)}\left[\frac{u^2T^{2v+2}}{(v+1)(2v+2)}-\frac{\theta uT^{v+3}}{2(v+3)}-\frac{uvT^{v+2}}{(v+1)(v+2)}+\frac{\theta^2}{8}T^4+\frac{\theta}{6}T^3-\frac{2uT^{v+2}}{v+1}\right.$$

$$\left.+\frac{(\theta T^2+2T)}{2}T-\frac{uT^{2v+2}}{v+1}+\frac{(uT^2\theta+2Tu)}{2(v+1)}T^{v+1}-\frac{(T\theta+1)}{2}T^2+\frac{(2u\theta T^{v+1}-T^2\theta^2(uv+1))}{4(v+1)}T^2\right] \tag{8}$$

• **Holding cost:** Holding costs refer to the expenses incurred for storing and maintaining stock over time. These costs include warehousing fees, insurance, depreciation, and the opportunity cost of capital tied up in the inventory, as well as expenses related to utilities and security for storage facilities.

$HC = h_c * I_T$

$$=\frac{h_c(a-bp)}{2(v+1)}\left[\frac{u^2T^{2v+2}}{(v+1)(2v+2)}-\frac{\theta uT^{v+3}}{2(v+3)}-\frac{uvT^{v+2}}{(v+1)(v+2)}+\frac{\theta^2}{8}T^4+\frac{\theta}{6}T^3-\frac{2uT^{v+2}}{v+1}\right.$$

$$\left.+\frac{(\theta T^2+2T)}{2}T-\frac{uT^{2v+2}}{v+1}+\frac{(uT^2\theta+2Tu)}{2(v+1)}T^{v+1}-\frac{(T\theta+1)}{2}T^2+\frac{(2u\theta T^{v+1}-T^2\theta^2(uv+1))}{4(v+1)}T^2\right] \tag{9}$$

• The interest earned and the interest paid are determined according to the credit periods and time intervals as $M < T$ and $M \geq T$.

Case 1: $M < T$

When the total inventory cycle is greater than the credit period, the supplier charges interest for the unsold inventory. The interest earned is calculated as follows. Figure 3 represents the retailer's revenue within this framework.

$$\text{Interest Earned (IE)} = (a-bp)pI_e\left[\int_0^T t\,dt\right]$$

$$= pI_e\left[(a-bp)T^2/2\right] \tag{10}$$

The interest paid is calculated as follows.

$$\text{Interest Paid (IP)} = cI_P \int_M^T Q(t)dt$$

$$= \frac{pI_P}{2(v+1)} \Bigg[\Bigg(\frac{u^2 M^{2v+2}}{(v+1)(2v+2)} - \frac{\theta u M^{v+3}}{2(v+3)} - \frac{uvM^{v+2}}{(v+1)(v+2)} + \frac{\theta^2}{8} M^4 + \frac{\theta}{6} M^3$$

$$- \frac{2uM^{v+2}}{v+1} + \frac{(\theta M^2 + 2M)}{2} M - \frac{uM^{2v+2}}{v+1} + \frac{(uM^2\theta + 2Mu)}{2(v+1)} M^{v+1}$$

$$- \frac{(M\theta + 1)}{2} M^2 + \frac{(2u\theta M^{v+1} - M^2\theta^2(uv+1))}{4(v+1)} M^2 \Bigg)$$

$$- \Bigg(\frac{u^2 T^{2v+2}}{(v+1)(2v+2)} - \frac{\theta u T^{v+3}}{2(v+3)} - \frac{uvT^{v+2}}{(v+1)(v+2)} + \frac{\theta^2}{8} T^4$$

$$+ \frac{\theta}{6} T^3 - \frac{2uT^{v+2}}{v+1} + \frac{(\theta T^2 + 2T)}{2} T$$

$$- \frac{uT^{2v+2}}{v+1} + \frac{(uT^2\theta + 2Tu)}{2(v+1)} T^{v+1} - \frac{(T\theta + 1)}{2} T^2 + \frac{(2u\theta T^{v+1} - T^2\theta^2(uv+1))}{4(v+1)} T^2 \Bigg) \Bigg] \tag{11}$$

The total inventory cost per unit time is the sum of the ordering cost, deterioration cost, carrying cost, ameliorating cost, holding cost and interest paid minus interest earned.

$$TC = \frac{1}{T} \left[OC + DC + AC + HC - IE + IP \right]$$

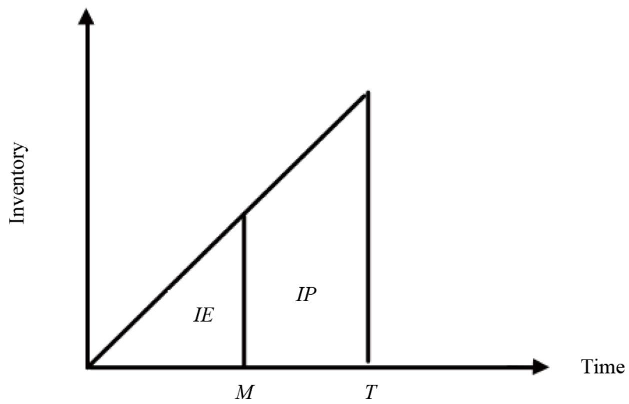


Figure 3. Interest earned and interest paid for the case $M \leq T$

Case 2: $M \geq T$

When the total inventory cycle is less than the credit period, there is no paid interest, and the interest earned is calculated as follows. Figure 4 illustrates the retailer's revenue in this context of this scenario.

$$\text{Interest Earned (IE)} = (a - bp)\, pI_e \left[\int_0^T t\, dt + T(M - T) \right]$$

$$= (a - bp) pI_e \left[\frac{T^2}{2} + T(M - T) \right] \tag{12}$$

The total inventory cost per unit time is the sum of the ordering cost, deterioration cost, carrying cost, ameliorating cost, holding cost and interest paid minus interest earned.

$$TC = \frac{1}{T} \left[OC + DC + AC + HC - IE \right]$$

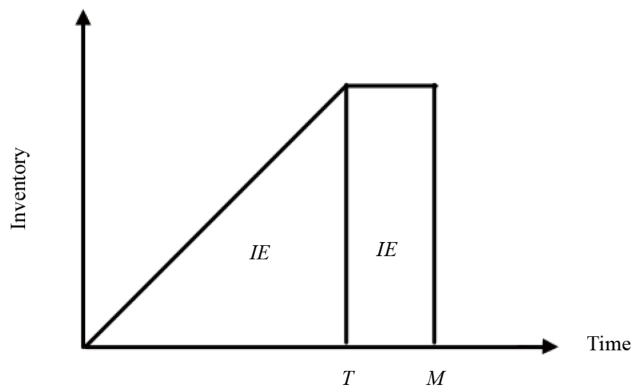


Figure 4. Interest earned and interest paid for the case $M > T$

4. Conventional algorithm (CA)

The conventional algorithm (CA) is a systematic optimization method grounded in classical calculus-based principles. It aims to find optimal solutions-minima or maxima-of an objective function by identifying and analyzing critical points. The CA is not limited to single-variable problems; it can also be applied to multi-variable functions by extending its framework to include partial derivatives and second-order conditions for multi-dimensional spaces. The CA is versatile and applicable to problems with one or multiple decision variables. Its precision makes it ideal for smooth and differentiable functions commonly encountered in fields like inventory management, engineering optimization, and financial modeling. The CA provides exact solutions for well-behaved, continuous, and differentiable functions but not suitable for non-smooth, non-convex, or highly complex objective functions. In cases with multiple local optima, it may fail to find the global optimum, necessitating the use of heuristic or metaheuristic methods for more complex problems. This description and analysis are synthesized from insights discussed in the articles by Law and Wee [3], Arunadevi and Umamaheswari [26] and Mahata et al. [6].

Total cost optimization with the conventional algorithm

Step 1: Start

Step 2: Assign initial values for all required parameters, including demand rate, amelioration and deterioration rates, costs, and credit period.

Step 3: Derive the total cost function $TC(T)$ for the given model.

Step 4: Solve $\dfrac{\partial TC}{\partial T} = 0$ to find all critical points Tc in the feasible domain of T.

Step 5: For each critical point Tc, compute $\dfrac{\partial^2 TC}{\partial T^2}$. If $\dfrac{\partial^2 TC}{\partial T^2} > 0$ for all Tc, Confirm Tc is a local minimum then proceed to Step 6. Otherwise, explore alternative methods to solve $TC(T)$.

Step 6: Verify whether $\dfrac{\partial^2 TC}{\partial T^2} > 0$ for all T in the domain to confirm the global convexity of $TC(T)$. If confirmed, declare the solution as a global minimum.

Step 7: Assign $T^* = Tc$, where $TC(T)$ is minimized.

Step 8: Evaluate the minimum total cost $TC^* = TC(T^*)$.

Step 9: Return T^* and TC^* as the optimal replenishment cycle time and total cost, respectively.

Step 10: End.

5. Cutting-edge optimization algorithms

In recent years, metaheuristic approaches have proven effective in addressing optimization challenges across various domains in science and engineering. Notable algorithms such as genetic algorithms (GA), particle swarm optimization (PSO), ant colony optimization, and differential evolution have demonstrated success. In the realm of inventory optimization, GAs and PSO algorithms consistently outperform alternative algorithms, offering the best solutions. Noteworthy researchers, including Sadeghi et al. [27] and Bhunia et al. [28], have successfully utilized GA and PSO to solve inventory models, highlighting the effectiveness of these algorithms in achieving superior solutions for complex inventory management challenges.

5.1 *Particle swarm optimization*

Particle swarm optimization is a nature-inspired optimization algorithm based on the social behaviour of birds and fish. PSO follows the current optimum solution through the search space. This approach was developed in 1995 by Kennedy and Eberhart to solve optimization and search problems, and it is influenced by the movement and intelligence of swarms. Many researchers, such as Shaikh et al. [29], Ruidas et al. [30], and Das et al. [31], have proposed various PSO algorithms for inventory modelling.

The PSO algorithm iteratively refines the particle positions and velocities in the search space, leveraging both individual and swarm knowledge to converge towards an optimal solution. The balance between exploration and exploitation allows the algorithm to efficiently explore the solution space and to discover optimal solutions for complex optimization problems. Adjustments to parameters influence the behaviour of the algorithm. The PSO was formulated and implemented using Python (3.10.2), leveraging its capabilities to efficiently handle complex optimization problems.

Total cost minimization using PSO

Step 1: Initialization

- Define the optimization problem:
- Fitness Function: Minimize $total\_cost\ (T)$
- Variable: T
- Bounds: $T\_bounds = (0.1, 35.0)$
- Import necessary modules for particle swarm optimization (PSO)

Step 2: Define the Total Cost Function

- Create a function '$total\_cost\ (T)$':

- Calculate the total cost based on T
- Return the calculated cost

Step 3: Generate a Range of Values for Evaluation
- Generate T_values as a range of evenly spaced values between 0.1 and 35.0

Step 4: Evaluate Total Costs Across the Range
- Loop through T_values to compute $total\_cost$ for each T
- Store the results in a list '$total\_costs$'

Step 5: Set the algorithm parameters
- Num_particles = 25
- max_iterations = 1,000

Step 6: Run particle swarm optimization (PSO)
- Use PSO to find the optimal value of T that minimizes $total\_cost$
- Inputs: $total\_cost$ function, bounds (0.1, 35.0)
- Outputs: $optimal\_T$ (optimal T value), $optimal\_cost$ (minimum total cost)

Step 7: Retrieve Optimization Results
- Print the optimal value of T and the corresponding minimum total cost

Step 8: Plot the Results
- Plot T_values against $total\_costs$ to visualize the curve
- Mark the optimal point ($optimal\_T$, $optimal\_cost$) on the graph
- Add labels, title, and legend to the plot
- Display the plot.

5.2 *Genetic algorithm*

Genetic algorithms are search-based optimization techniques based on evolutionary processes that occur in natural systems. Holland [32] was the first to propose a GA to address difficult decision-making in the field of science and technology. GA uses crossover, mutation, and selection to achieve the best optimal solution. The general structure of the GA for optimization problems was proposed by Gen and Cheng [33]. Later, many researchers considered the GA to solve large optimization problems. The GA computations and optimizations were performed in Python (3.10.2), utilizing its specialized functions for evolutionary problem-solving.

Algorithm: GA for total cost minimization

Step 1: Initialization
Create the optimization problem: FitnessMin, Individual (T), $T\_bounds = (0.1, 100.0)$
Toolbox: $attr\_float$, individual, population

Step 2: Define the Total Cost Function
$total\_cost\ (T)$:
Return cost

Step 3: Define the evaluation function
evaluate (individual):
T = individual[0]
$total\_cost\ (T)$

Step 4: Register GA Operators
mate: Blend crossover $(alpha = 0.5)$, mutate: Gaussian mutation $(mu = 0,\ sigma = 1,\ indpb = 0.2)$
select: Tournament selection $(tournsize = 3)$, evaluate

Step 5: Set the algorithm parameters
$population_size = 100,\ generations = 100$

Step 6: Create the Initial Population
population = toolbox.population ($n = population\_size$)

Step 7: Run the GA

algorithms.eaMuPlusLambda (population, toolbox, $mu = population\_size$, $lambda_{=}population\_size$, $cxpb = 0.7$, $mutpb = 0.2$, $ngen = generations$)

Step 8: Retrieve the best individual

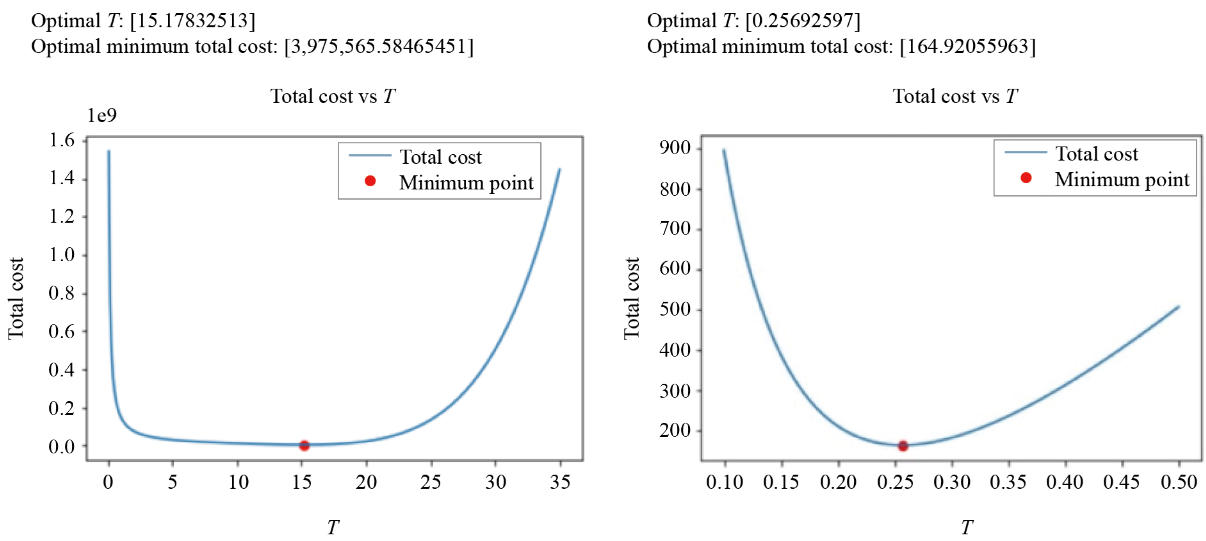
$best\_individual = tools.selBest(population,\ k = 1)[0]\ optimal\_T,\ optimal\_TC = best\_individual[0],\ evaluate(best\_individual)[0]$.

6. Numerical analysis

The following numerical examples are provided to illustrate the proposed model. To obtain the retailer's total inventory cost and cycle time, the initial parameters are $O_c = 200$ order, $h_c = 1$ per unit year, $D = 180$ units order, where $D = (a - bp)$ $I_e = 0.13$ years, $I_p = 0.15$ years, $a_c = 6$ per unit, $d_c = 9$ per unit, $u = 0.9$, $v = 0.9$, $a = 200$, $b = 0.1$, $p = 200$, $\theta = 0.8$ and $c = 150$ in appropriate units.

Table 3. Optimization results for various credit periods

Cases	Algorithm	D (Units)	I_e	I_p	M	T_*	TC_*
	CA					15.197	3,975,730
$M < T$	PSO	180	0.13	0.15	14	15.178*	3,975,565*
	GA					15.178*	3,975,565*
	CA					0.257	164.925
$M \geq T$	PSO	180	0.13	0	0.31	0.256*	164.921*
	GA					0.256*	164.921*



Case 1: $M < T$ Case 2: $M \geq T$

Figure 5. Graphical representation of the numerical analysis

Inference:

• From Table 3

- When the credit period is short, retailers prefer to order less, which leads to less interest as the retailers can pay on time.

- When the credit period is long, retailers can earn a large interest as the order quantity increases gradually.

• From Figure 5

- The concavity of the total cost is strictly concave with the dependent variable T.

- This implies that the credit period is a great support for enterprises to earn interest.

The optimal solution of the proposed inventory model presented in Table 3 is graphically represented in Figure 5.

The optimal total cost ($TC*$) is ₹ 164.92 when the credit period is greater than or equal to the cycle length $T* = 0.25$ weeks and $M = 0.31$ weeks. If the credit period is less than the cycle length, the optimal cycle length is 15 weeks, and $TC*$ is ₹ 3975730.

7. Sensitivity analysis

Sensitivity analysis is performed by altering the values of the key parameters associated with this model is illustrated in Table 4 and Figure 6 to determine the optimal solution.

The following observations can be drawn from Table 4:

• The total cost (TC) increases as the ameliorating parameter (u), the demand parameter (b), the price, and the deterioration rate increase, which indicates the minimization of inventory costs from the retailer's perspective.

• The total cost (TC) decreases the total profit as ameliorating parameter v, demand parameter a and cost parameter c increase.

• The total cost depends on the demand parameters a and b and the cost parameter c, while other variables are given relatively low importance.

• The replenishment cycle time T is influenced by demand parameter a and unit cost parameter c, with comparatively less emphasis on other variables.

Significant of the model: The proposed model represents a significant advancement in livestock inventory management, as it seamlessly integrates trade credit concepts to optimize operational efficiency and financial performance. By considering both amelioration and deterioration rates simultaneously, utilizing the Weibull distribution for nuanced modelling, and optimizing order quantity and replenishment cycle time, the model offers a holistic approach to inventory control. The significance of this model is further underscored by its applicability to the poultry farming industry, where it provides practical solutions for maximizing profits and meeting market demands. Through rigorous mathematical analysis and advanced validation techniques, including cutting-edge optimization algorithms such as swarm-based and evolutionary-based metaheuristic algorithms, the model proves its effectiveness and reliability in real-world scenarios. Moreover, insights from the conclusion emphasize the model's responsiveness to market dynamics, highlighting its potential to enhance operational efficiency, reduce costs, and ensure a consistent supply of livestock resources. By empowering entrepreneurs with strategic tools for inventory management and trade credit utilization, the model not only addresses critical gaps in the literature but also sets a new benchmark for best practices in livestock inventory management, ultimately driving sustainable growth and success in the industry.

Table 4. Optimization results for various credit periods

Key parameters	Values	T	TC	% change in TC value
u	0.7	0.33	98.32	-40.385
	0.8	0.33	98.47	-40.2941
	1	0.33	98.75	-40.1243
	1.1	0.33	98.88	-40.0455
v	0.7	0.33	98.87	-40.0515
	0.8	0.33	98.74	-40.1304
	1	0.33	98.47	-40.2941
	1.1	0.33	98.34	-40.3729
a	180	0.35	151.81	-7.9521
	190	0.34	125.7	-23.7835
	210	0.32	70.61	-57.1866
	220	0.31	41.75	-74.6855
b	0.2	0.35	151.81	-7.9521
	0.3	0.38	200.64	21.6553
	0.4	0.41	244.13	48.02486
	0.5	0.45	280.85	70.28953
p	180	0.33	93.08	-43.5622
	190	0.33	95.86	-41.8766
	210	0.33	101.35	-38.5478
	220	0.33	104.1	-36.8804
θ	0.6	0.33	98.62	-40.2031
	0.7	0.33	98.61	-40.2092
	0.9	0.33	98.61	-40.2092
	1	0.33	98.61	-40.2092
c	130	0.36	161.95	-1.80385
	140	0.35	131.02	-20.5578
	160	0.32	64.88	-60.6609
	170	0.31	29.98	-81.822

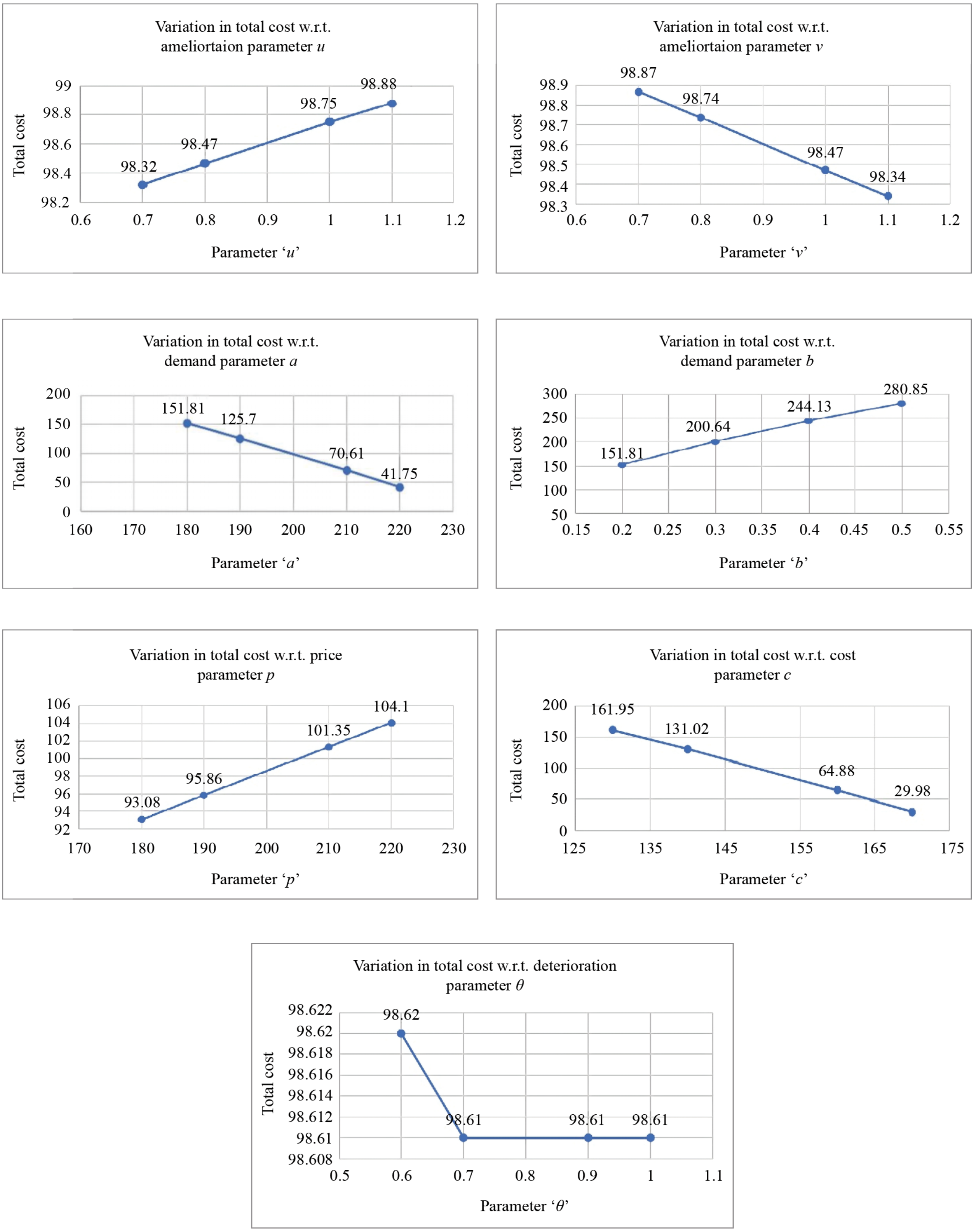


Figure 6. Graphical representation of the sensitivity analysis

8. Conclusion

An Ameliorating Inventory model is incorporated within the framework of trade credit in this seminal study of economic order quantity (EOQ). The amelioration rates decrease with the Weibull distribution over time, as the growth of livestock is faster at the beginning and slower at the end. The analysis involves the development of an inventory model with trade credit for livestock with an amelioration rate in the Weibull distribution and constant deterioration. In this proposed model, the amelioration rate is greater than the deterioration rate. The objective of the proposed model is to determine the optimal replenishment time and the credit period to minimize the total inventory costs. This model, crafted through the utilization of trade credit, is tailored for retailers to enhance profits and to mitigate the deterioration rate in their livestock inventory. Trade credit participation emerges as a distinctive and unparalleled strategy, delivering exceptional results, especially in the nuanced management of livestock inventories. Trade credit provides a pathway to sustainable living, empowering individuals to triumph over financial challenges and achieve success. This model is intended to enhance operational efficiency, reduce costs, and ensure a constant supply of livestock resources. By combining inventory management with trade credit opportunities, a balanced approach is achieved to meet market demand while managing resources efficiently.

In this article, both conventional and cutting-edge optimization algorithms are used to determine the optimal inventory solution. In comparison to the conventional model, cutting-edge optimization has the best optimal solution. In the pragmatic landscape of business, substantial changes can also have a large influence. The increase in deterioration is directly linked to a rise in total costs, highlighting the significance of opting for fewer orders. Significantly, the crucial elements that determine total costs are fluctuations in demand and unit prices, with their impact surpassing that of other variables. This model can be of great help to the blooming entrepreneur to sustain in this business world with the help of trade credit. The model is limited to ameliorating items such as young, fast-growing items (ducks, pigs, broilers, etc.), perishable items (flowers, fruits, green vegetables, etc.), and age-value products (cheese, wine, spirits, etc.). The future scope of this model can also be extended to the dual channel inventory model under inflation and the time value of money. In the context of deterioration and amelioration rates, one can assume a three-level Weibull distribution. The research is limited by its focus on specific inventory types such as fast-growing livestock and perishable goods, which restricts its broader applicability. The ameliorating inventory model with trade credit is constrained by its emphasis on specific product categories, and its assumption that the amelioration rate consistently exceeds the deterioration rate, which may not be applicable to all inventory types. Additionally, the model does not account for external factors such as market fluctuations or changes in consumer demand, which could impact inventory levels and trade credit decisions.

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Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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