Contemporary Mathematics
http://ojs.wiserpub.com/index.php/CM/

UNIVERSAL WISER PUBLISHER

Research Article

Fermatean Neutrosophic Soft Structure and its Ideals in Hyper BCK-Algebras

M. Kaviyarasu[1], Mohammed Alqahtani[2*], M. Rajeshwari[3], Kholood Alnefaie[4]

[1] Department of Mathematics, Vel Tech Rangarajan Dr Sagunthala R and D Institute of Science and Technology, 60002, Chennai, India
[2] Department of Basic Sciences, College of Science and Theoretical Studies, Saudi Electronic University, P.O. Box 93499, Riyadh 11673, Saudi Arabia
[3] Department of Mathematics, School of Engineering, Presidency University, Bangalure, India
[4] Department of Mathematics, College of Science, Taibah University, Madinah 42353, Saudi Arabia
E-mail: m.alqahtani@seu.edu.sa

Received: 28 September 2024; **Revised:** 29 October 2024; **Accepted:** 8 November 2024

Abstract: The goal of this paper is to investigate the use of fermatean neutrosophic soft sets (FNSSs) in the context of hyper BCK-algebras. While examining their characteristics and connections, it presents several ideas, including fermatean neutrosophic soft hyper BCK-ideals (FNSH BCK-ideals), fermatean neutrosophic soft weak hyper BCK-ideals (FNSWH BCK-ideals), fermatean neutrosophic soft s-weak hyper BCK-ideals (FNSs-WH BCK-ideals), and fermatean neutrosophic soft strong hyper BCK-ideals (FNSSH BCK-ideals). The criteria under which an FNSWH BCK-ideal can be categorized as an FNSs-WH BCK-ideal are also investigated, along with the classification of FNSWH BCK-ideals. Furthermore, standards are given for determining if an FNSS is an FNSSH BCK-ideal.

***Keywords*:** fermatean neutrosophic soft sets, hyper BCK-algebras, fermatean neutrosophic soft hyper BCK-ideals, fermatean neutrosophic soft weak hyper BCK-ideals, fermatean neutrosophic soft s-weak hyper BCK-ideals, fermatean neutrosophic soft strong hyper BCK-ideals, fuzzy logic

MSC: 06F35, 08A72

Abbreviation

FNSSs	Fermatean neutrosophic soft sets
FNSH BCK-ideals	Fermatean neutrosophic soft hyper BCK-ideals
FNSWH BCK-ideals	Fermatean neutrosophic soft weak hyper BCK-ideals
FNSs-WH BCK-ideals	Fermatean neutrosophic soft s-weak hyper BCK-ideals
FNSSH BCK-ideals	Fermatean neutrosophic soft strong hyper BCK-ideals

1. Introduction

Many fields, including economics, engineering, environmental research, medical science, and social science, face the difficulty of managing uncertainty. Due to their intrinsic constraints, traditional approaches frequently find it difficult to

DOI: https://doi.org/10.37256/cm.6220255814

address these uncertainties effectively. In response, Molodtsov [1] proposed a unique method for representing uncertainty that he called soft set theory. Building on this foundation, Jun [2] used soft sets to study BCK/BCI-algebra theory, while Jun et al. [3] used soft set theory to study BCK/BCI-algebra ideal theory. By building upon this paradigm, Maji et al. [4] increased the range of uncertainty modeling by investigating fuzzy soft sets in addition to soft sets. To demonstrate their flexibility in managing uncertainties, they presented the idea of fuzzy soft sets, which are broader versions of normal soft sets. They also showed how fuzzy soft sets may be used in a decision-making setting. By combining the advantages of soft sets with Atanassov's neutrosophic sets, Maji et al. [5] added NSSs to the theoretical framework. Fuzzy soft sets are useful in algebraic structures; Jun et al. [6] built upon these advances by applying them to the domain of BCK/BCI-algebras. For more studies on soft sets, see [7, 8].

Marty introduced hypergroups in 1934, which sparked the study of their characteristics and their application to relational algebraic functions and groups [9]. This marked the beginning of hyperstructure theory. Classical algebraic structures, in which combining two elements yields a set rather than merely another element, naturally extend to algebraic hyperstructures. This area has been thoroughly investigated in a large number of books and publications, indicating its importance in many different computer science and mathematics disciplines. Ameri in 2003 [10] dived into the categories of hypergroups and hypermodules, investigating their structures and features. In [11], Ameri collaborated with Rosenberg to research multialgebra congruences, concentrating on their characteristics within the context of multivalued logic and soft computing. Ameri and Zahedi [12] examined hyperalgebraic systems' theoretical basis and practical consequences in mathematical analysis and applications. Corsini [13] work "Prolegomena of Hypergroup Theory" presented an introductory survey of hypergroup theory, shedding light on its fundamental notions and theoretical foundations. Corsini and Leoreanu [14] investigated the applications of hyperstructure theory in their collaborative work, emphasizing its relevance and significance in a variety of mathematical situations. In [15], Leoreanu-Fotea and Davvaz investigated join n-spaces and lattices, looking at their characteristics and interactions within the contexts of multivalued logic and soft computing, they also published a paper on [16], n-hypergroups and binary relations, which advances our knowledge of hyperstructures and their applications in combinatorial mathematics, notably in the setting of binary relations and algebraic representations. Pelea [17] investigated the direct product of multialgebras, looking at its features and structure of algebraic structures. Pickett [18] focused on homomorphisms and subalgebras of multialgebras, shedding light on the links between various algebraic structures and their maps. Schweigert [19] focused on multialgebra congruence relations, namely the equivalence relations that maintain algebraic operations and attributes inside these mathematical frameworks. Serafimidis, et al. [20] presented the L-fuzzy Corsini join hyperoperation, which adds to the research of fuzzy hyperoperations and its applications in mathematical analysis and soft computing. Vougioukli [21] investigated hyperstructures' theoretical basis, characteristics, and applications in mathematics and computer science, providing thorough coverage and excellent explanations.

Several articles supported the concepts of fuzzy and neutrosophic in algebraic and applied scenarios in recent years. Kousar et al. [22] surveyed congruencies on generalized fuzzy G-acts in another paper in which they expanded the notion of group theory into fuzzy systems. Complex interval-valued Q-neutrosophic subbisemirings were developed by Syed Ahmad et al. in [23], giving rise to a new structure. In robotic sensors, Murugan et al. [24] used Q-rung complex diophantine neutrosophic sets to enhance precision and decision-making. These works show how fuzzy and neutrosophic structures can be used in abstract algebra and in applications to robotics and engineering. To improve decision-making accuracy under uncertainty, Mahapatra et al. [25] proposed a new correlation measure of neutrosophic sets in developing an efficient health insurance provider selection applying TOPSIS. Their approach also shows the usefulness of neutrosophic logic in the assessment of the available health insurance. In the same way, Kumar Rath et al. [26] studied requisite and core-optimal redundant fuzzy components, which enriched the theoretical research of fuzzy logic. They improve computational efficiency in algebraic structures and foundational concepts in algebraic structures as their research.

In [27], Jun et al. introduced the idea of hyper BCK-algebras as an extension of BCK-algebras and extended hyperstructures to BCK-algebras. Later, in [3, 28, 29], Jun et al. built upon this work by investigating other concepts and outcomes. Furthermore, in [30] and [31], several fuzzy forms of hyper BCK-algebras have been studied. More recently, in [32], Davvaz et al. gave a thorough summary of the state of the field's research on fuzzy hyperstructures. Zail et al.

[33] explored neutrosophic BCK-algebras and Ω-BCK-algebras, including their features and applications in neutrosophic algebra. Santhakumar et al. [34] provided a fresh way to comprehend the algebraic structure of neutrosophic SuperHyper algebra, which may introduce new theoretical frameworks or computational tools for studying such structures. Hamidi [35] focused on extended BCK-ideal theory by including single-valued neutrosophic hyper BCK-ideals, presumably to investigate the extension of standard BCK-ideal conceptions in neutrosophic algebra. Borzooei et al. [36] examined MBJ-neutrosophic subalgebras and filters in BE-algebras, perhaps looking into particular substructures or features linked to neutrosophic algebra within the context of BE-algebras. Xin et al. [37] investigated the concept of intuitionistic fuzzy soft hyper BCK-algebras, examining the integration of intuitionistic fuzzy logic and soft set theory into the framework of hyper BCK-algebras, potentially introducing new theoretical constructs or providing real-world applications.

This article presents new ideas of FNSH BCK-ideals, FNSWH BCK-ideals, FNSs-WH BCK-ideals, and FNSSH BCK-ideals are some of the new ideas in the field of hyper BCK-algebras. It goes into an advanced level of examination of these newly defined absolutes and the complete system of their mutual connections, offering theoretical insight toward their study. The work also analyzes the relations between Fermatean neutrosophic hyper BCK-ideal and Fermatean neutrosophic level cut sets for the improvement of the theory of hyper BCK-algebras. In this sense, the article enriches algebraic hyperstructure theory and Fermatean neutrosophic algebraic systems with Fermatean neutrosophic structures on hyper BCK-algebras.

1.1 *Novelty*

As no studies have been reported so far to generalize the above-mentioned concepts, the aim of this article is as follows:

• In this context, the idea of Fermatean neutrosophic structure will be employed to construct a hyper BCK-algebra.

• First, new concepts of Fermatean neutrosophic (weak, *s*-weak, strong) hyper BCK-ideal of hyper BCK-algebra are enshrined and explored.

• To investigate Fermatean neutrosophic (weak, strong) hyper BCK-ideals for the Fermatean neutrosophic level cut sets.

1.2 *Motivation*

• In the further study, Fermatean neutrosophic structures are expanded to hyper BCK-algebras due to their superior capability of dealing with uncertainty with more significant membership, indeterminacy, and non-membership levels.

• It proposes the use of hyper BCK-ideals with Fermatean neutrosophic structure, and being weak, *s*-weak, strong: it enriches the theory of hyper BCK-algebras.

• In the present study, an attempt is made to analyze the structural characteristics of Fermatean neutrosophic hyper BCK-ideals and Fermatean neutrosophic level cut sets.

• In this way, riching the given subject and filling in the existing gap, the study offers new valuable information about the use of Fermatean neutrosophic structures in algebraic systems.

To proceed, the rest of the article is organized as follows: In Section 2, we provide a brief overview of basic notions that are essential to the work. In Section 3, we define theFermatean neutrosophic soft ideals in hyper BCK-algebra. In Section 4, we provide the application in medical decision-making. In Section 5, we provide the Comparison Analysis. Lastly, the conclusion and direction for future work of the study are provided in Section 6.

2. Preliminaries

Let P be a nonempty set with a hyper operation $\circ$, where $\circ$ is a function from $P \times P$ to $\Im^*(P)$. For two subsets R and S of P, write $R \circ S$ as $\cup\{r \circ s | r \in R,\ s \in S\}$. Instead of using $\{\lambda\} \circ \{\mu\}$, $\lambda \circ \{\mu\}$ or $\{\lambda\} \circ\ \mu$, we will use $\lambda\ \circ\ \mu$.

A nonempty set P fitted with a hyper operations $\circ$ and a constant 0 that stratifies the following axioms is called a hyper BCK-algebra.

(B_1) $(\lambda \circ \gamma) \circ (\mu \circ \gamma) \ll (\lambda \circ \mu)$,
(B_2) $(\lambda \circ \mu) \circ \gamma = (\lambda \circ \gamma) \circ \mu$,
(B_3) $\lambda \circ P \ll \{\lambda\}$,
(B_4) $\lambda \ll \mu$ *and* $\mu \ll \lambda \Rightarrow \lambda = \mu$,

for all $\lambda,\ \mu,\ \gamma \in P$, where $\lambda \ll \mu$ is define by $0 \in (\lambda \circ \mu)$ and for any $R,\ S \subseteq P$, $R \ll S$ is defined by for all $r \in R$, $\exists\, s \in S$ such that $r \ll s$.

The condition (B_3) in hyper BCK-algebra P may be written equivalently as follows: $(\lambda \circ \mu) \ll \{\lambda\}\ \forall\ \lambda,\ \mu \in P$. The following holds in any types of BCK-algebra P.

$$\lambda \circ 0 \ll \{\lambda\},\ 0 \circ \lambda \ll \{0\},\ 0 \circ 0 \ll \{\ 0\ \} \tag{1}$$

$$(R \circ S) \circ Q = (R \circ Q) \circ S,\ R \circ S \ll R,\ 0 \circ R \ll \{0\}, \tag{2}$$

$$0 \ll \lambda,\ \ \lambda \ll \lambda,\ R \ll R \tag{3}$$

$$R \subseteq S \Rightarrow R \ll S \tag{4}$$

$$0 \circ \lambda = \{0\},\ 0 \circ R = \{0\} \tag{5}$$

$$R \ll \{0\} \Rightarrow R = \{0\} \tag{6}$$

$$\lambda \in \lambda \circ 0 \tag{7}$$

for every $\lambda,\ \mu,\ \gamma \in P$ as well has every nonempty subset of P, that is $R,\ S$ and Q.

- In a hyper BCK-algebra P, define a subset R as a hyper BCK-ideal if it meets specific requirements:

$$0 \in R, \tag{8}$$

$$(\forall \lambda,\ \mu \in P)\,(\lambda \circ \mu \ll R,\ \mu \in R \implies \lambda \in R). \tag{9}$$

- A subset R of a hyper BCK-algebra P qualities as a strong hyper BCK-ideal if it meets condition (2.8) and

$$(\forall \lambda,\ \mu \in P)\,((\lambda \circ \mu) \cap R \neq \phi,\ \mu \in R \implies \lambda \in R). \tag{10}$$

- A subset R of a hyper BCK-algebra P is considered as a weak hyper BCK-ideal if it meets condition (2.8) and

$$(\forall \lambda,\ \mu \in P)\,((\lambda \circ \mu) \subseteq R,\ \mu \in R \implies \lambda \in R). \tag{11}$$

It is importance to note that each strong hyper BCK-ideal is essentially a hyper BCK-ideal as demonstrated in [29]. Molodtsov [1] defined a soft set as follows: Consider the original universe set U and its parameter J. Let $P(U)$ be the power set of U and R be a subset of J. A soft set over U is defined as a pair $(\eta,\ R)$, where η is a specified mapping $\eta: R \to P(U)$. A soft set over U may be viewed as a collection of subsets of the universe defined by a set R. $\eta(\psi)$ represents the set of ψ-approximate elements in the soft set $(\eta,\ R)$, for each elements $\psi \in R$. Consider an initial universe set U and a set of parameters J. Denote $Z(u)$ as the set that includes all fuzzy sets in U. The fuzzy set $(Z,\ R)$ over U [4] is defined as a mapping $Z: R \to Z(u)$, where R is a subset of J.

For each parameter $u \in R$, $Z(u)$ represents a fuzzy set in U known as the parameter fuzzy value set. If $Z(u)$ is a crisp subset of U for every $u \in R$, then the pair $(Z,\ R)$ is reduced to the usual soft set.

Thus, based on this definition, it is clear that a fuzzy soft set expands the idea of a conventional soft set.

3. Fermatean neutrosophic soft ideals in hyper BCK-algebra

In this context, treat P as a hyper BCK-algebra and J as a collection of parameters.

Definition 1 Let R is a subset of J and $Z_1(P)$ is the collection of all fernatean neutrosophic sets in P. Then a pair $(\bar{y},\ R)$ is termed an FNSS over P in which $\bar{y}$ is a function that can be described as

$$\bar{y}: R \to Z_1(P). \tag{12}$$

The fermatean neutrosophic value set of parameter $j \in R$, denoted as $\bar{y}(j)$, has the following format:

$$\bar{y}(j) = \left\{ \left\langle \lambda,\ T_{\bar{y}_{(j)}}(\lambda),\ I_{\bar{y}_{(j)}}(\lambda),\ F_{\bar{y}_{(j)}}(\lambda) \right\rangle | \lambda \in P \right\}, \tag{13}$$

where $T_{\bar{y}_{(j)}}(\lambda)$ is the degree of membership, $I_{\bar{y}_{(j)}}(\lambda)$ is the degree of indeterminacy membership, $F_{\bar{y}_{(j)}}(\lambda)$is the degree of non-membership and $\bar{y}_{(j)}$ is a fermatean neutrosophic set in P.

Definition 2 The FNSS $(\bar{y},\ R)$ over P is defined as an FNSH BCK-ideal based on parameter $j \in R$ if the fermatean neutrosophic value set $\bar{y}(j)$ of j meets the following conditions:

$$(\forall \lambda,\ \mu \in P) \left((\lambda \ll \mu) \implies \begin{array}{c} T_{\bar{y}_{(j)}}(\lambda) \leq T_{\bar{y}_{(j)}}(\mu), \\ I_{\bar{y}_{(j)}}(\lambda) \geq I_{\bar{y}_{(j)}}(\mu), \\ F_{\bar{y}_{(j)}}(\lambda) \geq F_{\bar{y}_{(j)}}(\mu) \end{array} \right), \tag{14}$$

$$(\forall,\ \lambda,\ \mu \in P) \left(\begin{array}{c} T_{\bar{y}_{(j)}}(\lambda) \leq \wedge \left\{ inf_{r \in (\lambda \circ \mu)}\ T_{\bar{y}_{(j)}}(r),\ T_{\bar{y}_{(j)}}(\mu) \right\} \\ I_{\bar{y}_{(j)}}(\lambda) \geq \vee \left\{ sup_{r \in (\lambda \circ \mu)}\ I_{\bar{y}_{(j)}}(r),\ I_{\bar{y}_{(j)}}(\mu) \right\} \\ F_{\bar{y}_{(j)}}(\lambda) \geq \vee \left\{ sup_{r \in (\lambda \circ \mu)}\ F_{\bar{y}_{(j)}}(r),\ F_{\bar{y}_{(j)}}(\mu) \right\} \end{array} \right). \tag{15}$$

If $(\bar{y},\ R)$ is a j-FNSH BCK-ideal for any $j \in R$, then it is called an FNSH BCK-ideal of P.

Example 1 Let $P = 0,\ r,\ s$ be the hyper BCK-algebra. The hyper operation "$\circ$" on P is shown in Table 1.

Table 1. Representation of the hyper operation $\circ$

$\circ$	0	r	s
0	$\{0\}$	$\{0\}$	$\{0\}$
r	$\{r\}$	$\{0, r\}$	$\{0, r\}$
s	$\{s\}$	$\{r, s\}$	$\{0, r, s\}$

As shown in Table 2, we have an FNSS $(\bar{y}, R)$ defined on the set P.

Table 2. Representation of $(\bar{y}, R)$

$\bar{y}$	λ	μ
0	(0.2, 0.7, 0.12)	(0.5, 0.6, 0.12)
r	(0.4, 0.4, 0.34)	(0.4, 0.3, 0.03)
s	(0.8, 0.3, 0.43)	(0.9, 0.4, 0.24)

Hence, it can be concluded that $\bar{y}(\lambda)$ satisfies criteria (3.3) and (3.4), proving that $(\bar{y}, R)$ is an FNSH BCK-ideal based on λ over P. However, $\bar{y}(\mu)$ does not meet condition (3.3) due to $r \ll s$ and $T_{\bar{y}_{(j)}}(r) \leq T_{\bar{y}_{(j)}}(s)$, $I_{\bar{y}_{(j)}}(r) \geq I_{\bar{y}_{(j)}}(s)$, *or* $F_{\bar{y}_{(j)}}(r) \geq F_{\bar{y}_{(j)}}(s)$. Thus, it is not an FNSH BCK-ideal based on μ over P.

Proposition 1 For every parameter $j \in R$, the following statements are valid in every circumstance, where $(\bar{y}, R)$ denotes an FNSH BCK-ideal of P.

1). $(\bar{y}, R)$ meets the requirement:

$$(\forall \lambda \in P) \begin{pmatrix} T_{\bar{y}_{(j)}}(0) \leq T_{\bar{y}_{(j)}}(\lambda) \\ I_{\bar{y}_{(j)}}(0) \geq I_{\bar{y}_{(j)}}(\lambda) \\ F_{\bar{y}_{(j)}}(0) \geq F_{\bar{y}_{(j)}}(\lambda) \end{pmatrix}. \tag{16}$$

2). If $(\bar{y}, R)$ meets the requirement

$$(\forall K,\ L \in 2^P)(\exists (\lambda_0,\ \mu_0) \in K \times L) \begin{pmatrix} T_{\bar{y}_{(j)}}(\lambda_0) = inf_{r \in K}\ T_{\bar{y}_{(j)}}(r) \\ I_{\bar{y}_{(j)}}(\lambda_0) = sup_{r \in K}\ I_{\bar{y}_{(j)}}(s) \\ F_{\bar{y}_{(j)}}(\lambda_0) = sup_{s \in L}\ F_{\bar{y}_{(j)}}(s) \end{pmatrix}. \tag{17}$$

then the following claim is true:

$$(\forall \lambda,\ \mu \in P)\ (\exists\ r,\ s \in \lambda \circ \mu) \begin{pmatrix} T_{\bar{y}_{(j)}}(\lambda) \leq \wedge \left\{ T_{\bar{y}_{(j)}}(r),\ T_{\bar{y}_{(j)}}(\mu) \right\} \\ I_{\bar{y}_{(j)}}(\lambda) \geq \vee \left\{ I_{\bar{y}_{(j)}}(s),\ I_{\bar{y}_{(j)}}(\mu) \right\} \\ F_{\bar{y}_{(j)}}(\lambda) \geq \vee \left\{ F_{\bar{y}_{(j)}}(s),\ F_{\bar{y}_{(j)}}(\mu) \right\} \end{pmatrix}. \tag{18}$$

Proof. Assume that $0 \ll \lambda$ for all $\lambda \in P$. Then, we have

$$T_{\bar{y}_{(j)}}(0) \leq T_{\bar{y}_{(j)}}(\lambda),\ I_{\bar{y}_{(j)}}(0) \geq I_{\bar{y}_{(j)}}(\lambda),\ \text{and}\ F_{\bar{y}_{(j)}}(0) \geq F_{\bar{y}_{(j)}}(\lambda)$$

according to the condition (3.3). For any $\lambda,\ \mu \in P$, there exist $\lambda_0,\ \mu_0 \in \lambda \circ \mu$ such that

$$T_{\bar{y}_{(j)}}(\lambda_0) = inf_{r\in\lambda\circ\mu}\ T_{\bar{y}_{(j)}}(r)$$

$$I_{\bar{y}_{(j)}}(\mu_0) = sup_{r\in\lambda\circ\mu}\ I_{\bar{y}_{(j)}}(s)$$

$$F_{\bar{y}_{(j)}}(\mu_0) = sup_{s\in\lambda\circ\mu}\ F_{\bar{y}_{(j)}}(s)$$

as derived from equation (3.6). Condition (3.4) allows us to conclude the following

$$T_{\bar{y}_{(j)}}(\lambda) = \wedge\left\{inf_{r\in\lambda\circ\mu}\ T_{\bar{y}_{(j)}}(r),\ T_{\bar{y}_{(j)}}(\mu)\right\}$$

$$= \wedge\left\{T_{\bar{y}_{(j)}}(\lambda_0),\ T_{\bar{y}_{(j)}}(\mu)\right\},$$

$$I_{\bar{y}_{(j)}}(\lambda) = \vee\left\{sup_{r\in\lambda\circ\mu}\ I_{\bar{y}_{(j)}}(s),\ I_{\bar{y}_{(j)}}(\mu)\right\}$$

$$= \vee\left\{I_{\bar{y}_{(j)}}(\mu_0),\ I_{\bar{y}_{(j)}}(\mu)\right\},$$

and

$$F_{\bar{y}_{(j)}}(\lambda) = \vee\left\{sup_{s\in\lambda\circ\mu}\ F_{\bar{y}_{(j)}}(s),\ F_{\bar{y}_{(j)}}(\mu)\right\}$$

$$= \vee\left\{F_{\bar{y}_{(j)}}(\mu_0),\ F_{\bar{y}_{(j)}}(\mu)\right\}.$$

This is the intended outcome.

Considering a FNSS $(\bar{y},\ R)$ over P and we define the following sets:

$$\left.\begin{aligned} \mathfrak{X}_\rho &= \left\{\lambda \in P \mid T_{\bar{y}_{(j)}} \leq \rho\right\} \\ \mathfrak{Y}_\sigma &= \left\{\lambda \in P \mid I_{\bar{y}_{(j)}} \geq \sigma\right\} \\ \mathfrak{Z}_\omega &= \left\{\lambda \in P \mid F_{\bar{y}_{(j)}} \geq \omega\right\} \end{aligned}\right\} \tag{19}$$

where $(\rho,\ \sigma,\ \omega) \in [0,\ 1] \times [0,\ 1] \times [0,\ 1]$ and j is a parameter in R.

Lemma 1 Let T be a hyper BCK ideal of P such that $R \ll T$ and R be a subset of hyper BCK algebra P. Thus, it follows that R is contained completely inside T.

Theorem 1 The nonempty set $\mathfrak{X}_\rho$, $\mathfrak{Y}_\sigma$ and $\mathfrak{Z}_\omega$ are hyper BCK-ideals of P for all $(\rho,\ \sigma,\ \omega) \in [0,\ 1] \times [0,\ 1] \times [0,\ 1]$ if and only if the $FNSS$ $(\bar{y},\ R)$ over P constitutes an FNSH BCK-ideal of P.

Proof. Assume that $(\bar{y},\ R)$ is an FNSH BCK-ideal of P and $\mathfrak{X}_\rho,\ \mathfrak{Y}_\sigma$ and $\mathfrak{Z}_\omega$ are nonempty for all $(\rho,\ \sigma,\ \omega) \in [0,\ 1]$. Subsequently, r belongs to $\mathfrak{X}_\rho$ and s belongs to $\mathfrak{Y}_\sigma$ and $\mathfrak{Z}_\omega$, so that $T_{\bar{y}_{(j)}}(r) \leq \rho,\ \ I_{\bar{y}_{(j)}}(s) \geq \sigma,$ and $F_{\bar{y}_{(j)}}(s) \geq \omega$. Consequently, it may be inferred from (3.5) that

$$T_{\bar{y}_{(j)}}(0) \leq T_{\bar{y}_{(j)}}(r) \leq \rho,$$

$$I_{\bar{y}_{(j)}}(0) \geq I_{\bar{y}_{(j)}}(s) \geq \sigma,$$

and

$$F_{\bar{y}_{(j)}}(0) \geq F_{\bar{y}_{(j)}}(s) \geq \omega.$$

Thus, $0 \in \mathfrak{X}_\rho \cap \mathfrak{Y}_\sigma \cap \mathfrak{Z}_\omega$. Consider $\lambda,\ \mu \in P$ such that $(\lambda \circ \mu) \ll \mathfrak{X}_\rho$ and $\mu \in \mathfrak{X}_\rho$. Then there exists $r_0 \in \mathfrak{X}_\rho,$ such that $r \ll r_0$ for any $r \in (\lambda \circ \mu)$. Therefore, $T_{\bar{y}_{(j)}}(r) \leq T_{\bar{y}_{(j)}}(r_0) \leq \rho$ by (3.3), it may be shown from (3.4) that

$$T_{\bar{y}_{(j)}}(\lambda) \leq \wedge \left\{ inf_{r \in (\lambda \circ \mu)}\ T_{\bar{y}_{(j)}}(r),\ T_{\bar{y}_{(j)}}(\mu) \right\}$$

$$\leq \wedge \left\{ \rho,\ T_{\bar{y}_{(j)}}(\mu) \right\}$$

$$\leq \rho.$$

As a result, $\lambda \in \mathfrak{X}_\rho$ and so $\mathfrak{X}_\rho$ is a hyper BCK ideal of P.

Similarly if $(r \circ s) \ll \mathfrak{Y}_\sigma$ and $s \in \mathfrak{Y}_\sigma$, then there exists $\lambda_0 \in \mathfrak{Y}_\sigma$ such that $\lambda \ll \lambda_0$ for every $\lambda \in r \circ s$. Therefore, $I_{\bar{y}_{(j)}}(\lambda) \geq I_{\bar{y}_{(j)}}(\lambda_0) \geq \sigma$ by (3.3), it may be shown from (3.4) that

$$I_{\bar{y}_{(j)}}(\lambda) \geq \wedge \left\{ sup_{r \in (\lambda \circ \mu)}\ I_{\bar{y}_{(j)}}(\lambda),\ I_{\bar{y}_{(j)}}(s) \right\}$$

$$\geq \wedge \left\{ \sigma,\ I_{\bar{y}_{(j)}}(s) \right\}$$

$$\geq \sigma.$$

Therefore, $\lambda \in \mathfrak{Y}_\sigma$ and so $\mathfrak{Y}_\sigma$ is a hyper BCK ideal of P. Now, assume that $r \circ s \ll \mathfrak{Z}_\sigma$ and $s \in \mathfrak{Z}_\omega$ for every $r,\ s \in P$. Then, there exists $\lambda_0 \in \mathfrak{Z}_\omega$ such that $\lambda \ll \lambda_0$ for every $\lambda \in r \circ s$.

Thus, $F_{\bar{y}_{(j)}}(\lambda) \geq F_{\bar{y}_{(j)}}(\lambda_0) \geq \omega$ by (3.3), it may be shown from (3.4) that

$$F_{\bar{y}_{(j)}}(r) \geq \vee \left\{ sup_{\lambda \in (r \circ s)} F_{\bar{y}_{(j)}}(\lambda),\ F_{\bar{y}_{(j)}}(s) \right\}$$

$$\geq \wedge \left\{ \omega,\ F_{\bar{y}_{(j)}}(s) \right\}$$

$$\geq \omega.$$

Therefore, $r \in \mathfrak{Z}_\omega$ and so $\mathfrak{Z}_\omega$ is a hyper BCK-ideal of P. Conversely assume that the non empty sets $\mathfrak{X}_\rho,\ \mathfrak{Y}_\sigma$ and $\mathfrak{Z}_\omega$ are hyper BCK-ideals of P for every $(\rho,\ \sigma,\ \omega) \in [0,\ 1] \times [0,\ 1] \times [0,\ 1]$. Assume that $\lambda,\ \mu,\ \varepsilon,\ \delta \in P$ such that $\lambda \ll \mu,\ T_{\bar{y}_{(j)}}(\mu) = \rho$, $\varepsilon \ll \delta\ I_{\bar{y}_{(j)}}(\delta) = \sigma$, and $F_{\bar{y}_{(j)}}(\delta) = \omega$. Then, μ belongs to $\mathfrak{X}_P$ and δ belongs to $\mathfrak{Y}_\sigma$ and $\mathfrak{Z}_\omega$, we have $\lambda \ll \mathfrak{X}_\rho,\ \varepsilon \ll \mathfrak{Y}_\sigma$ and $\varepsilon \ll \mathfrak{Z}_\omega$. Thus, Lemma 1 indicates that λ belongs $\mathfrak{X}_\rho$ and ε belongs to $\mathfrak{Y}_\sigma$ and $\mathfrak{Z}_\omega$. Thus,

$$T_{\bar{y}_{(j)}}(\lambda) \leq \rho = T_{\bar{y}_{(j)}}(\mu),$$

$$I_{\bar{y}_{(j)}}(\varepsilon) \geq \sigma = I_{\bar{y}_{(j)}}(\delta),$$

and

$$F_{\bar{y}_{(j)}}(\varepsilon) \geq \omega = F_{\bar{y}_{(j)}}(\delta).$$

Now, for any $\lambda,\ \mu,\ \varepsilon,\ \delta \in P$, let

$$\rho = \wedge \left\{ inf_{r \in (\lambda \circ \mu)}\ T_{\bar{y}_{(j)}}(r),\ T_{\bar{y}_{(j)}}(\mu) \right\},$$

$$\sigma = \vee \left\{ sup_{r \in (\lambda \circ \mu)}\ I_{\bar{y}_{(j)}}(s),\ I_{\bar{y}_{(j)}}(\delta) \right\},$$

and

$$\omega = \vee \left\{ sup_{s \in (\varepsilon \circ \delta)}\ F_{\bar{y}_{(j)}}(s),\ F_{\bar{y}_{(j)}}(\delta) \right\}.$$

Then, for each $r \in (\lambda \circ \mu)$ and $s \in (\varepsilon \circ \delta)$, we have

$$T_{\bar{y}_{(j)}}(r) \leq inf_{r\in(\lambda\circ\mu)} T_{\bar{y}_{(j)}}(r)$$

$$\leq \wedge \left\{ inf_{r\in(\lambda\circ\mu)}\ T_{\bar{y}_{(j)}}(r),\ T_{\bar{y}_{(j)}}(\mu) \right\}$$

$$= \rho,$$

$$I_{\bar{y}_{(j)}}(r) \geq sup_{r\in(\lambda\circ\mu)} I_{\bar{y}_{(j)}}(s)$$

$$\geq \vee \left\{ sup_{r\in(\lambda\circ\mu)}\ I_{\bar{y}_{(j)}}(s),\ I_{\bar{y}_{(j)}}(\delta) \right\}$$

$$= \sigma,$$

and

$$F_{\bar{y}_{(j)}}(s) \geq sup_{s\in(\varepsilon\circ\delta)} F_{\bar{y}_{(j)}}(s)$$

$$\geq \vee \left\{ sup_{s\in(\varepsilon\circ\delta)}\ F_{\bar{y}_{(j)}}(s),\ F_{\bar{y}_{(j)}}(\delta) \right\}$$

$$= \omega.$$

As a result, $r \in \mathfrak{X}_\rho,\ s \in \mathfrak{Y}_\sigma$ and $s \in \mathfrak{Z}_\omega$. According by (2.4) $(\lambda \circ \mu) \ll \mathfrak{X}_\rho$, $(\lambda \circ \mu) \ll \mathfrak{Y}_\sigma$ and $(\varepsilon \circ \delta) \ll \mathfrak{Z}_\omega$. Since μ belongs to $\mathfrak{X}_\sigma$, δ belongs to $\mathfrak{Y}_\delta$ and $\mathfrak{Z}_\omega$, $\mathfrak{X}_\sigma$, $\mathfrak{Y}_\delta$ and $\mathfrak{Z}_\omega$ are hyper BCK-ideals of P, we can deduce that λ belongs to $\mathfrak{X}_\rho$ and ε belongs to $\mathfrak{Y}_\sigma$ and $\mathfrak{Z}_\omega$. Thus,

$$T_{\bar{y}_{(j)}}(\lambda) \leq \rho = \wedge \left\{ inf_{r\in(\lambda\circ\mu)}\ T_{\bar{y}_{(j)}}(r),\ T_{\bar{y}_{(j)}}(\mu) \right\},$$

$$I_{\bar{y}_{(j)}}(\lambda) \geq \sigma = \vee \left\{ sup_{r\in(\lambda\circ\mu)}\ I_{\bar{y}_{(j)}}(s),\ I_{\bar{y}_{(j)}}(\delta) \right\},$$

and

$$F_{\bar{y}_{(j)}}(\varepsilon) \geq \omega = \vee \left\{ sup_{s\in(\varepsilon\circ\delta)}\ F_{\bar{y}_{(j)}}(s),\ F_{\bar{y}_{(j)}}(\delta) \right\}.$$

Hence, $(\bar{y},\ R)$ is an FNSH BCK-ideal of P. □

Definition 3 Then an FNSS $(\bar{y},\ R)$ over P is called a j-FNSWH BCK-ideal of P if the following claims are true.

$$T_{\bar{y}_{(j)}}(0) \leq T_{\bar{y}_{(j)}}(\lambda) \leq \wedge \left\{ sup_{r\in(\lambda\circ\mu)}\, T_{\bar{y}_{(j)}}(r),\ T_{\bar{y}_{(j)}}(\mu) \right\},$$

$$I_{\bar{y}_{(j)}}(0) \geq I_{\bar{y}_{(j)}}(\lambda) \geq \vee \left\{ inf_{r\in(\lambda\circ\mu)}\, I_{\bar{y}_{(j)}}(s),\ I_{\bar{y}_{(j)}}(\delta) \right\}$$

and

$$F_{\bar{y}_{(j)}}(0) \geq F_{\bar{y}_{(j)}}(\varepsilon) \geq \vee \left\{ inf_{s\in(\varepsilon\circ\delta)}\, F_{\bar{y}_{(j)}}(s),\ F_{\bar{y}_{(j)}}(\delta) \right\}.$$

Definition 4 If the fermatean neutrosophic value set $\bar{y}_{(j)}$ meets the conditions (3.5) and (3.7), then an FNSS $(\bar{y},\ R)$ over P is called a j-FNSs-WH BCK-ideal of P.

Example 2 An FNSWH BCK-ideal of P is the FNSS $(\bar{y},\ R)$ as shown in Example 1. Every FNSH BCK-ideal may be classified as a weak hyper BCK ideal. On the other hand, the converse is not true. In particular, the FNSWH BCK-ideal of P shown in Example 2 does not satisfy the requirements to be identified as an FNSH BCK-ideal of H, since it does not display the characteristics of an FNSH BCK-ideal according to the parameters μ over P.

Theorem 2 The nonempty set $\mathfrak{X}_\rho$, $\mathfrak{Y}_\sigma$ and $\mathfrak{Z}_\omega$ are hyper BCK-ideal of P for all $(\rho,\ \sigma,\ \omega) \in [0,\ 1] \times [0,\ 1] \times [0,\ 1]$ if and only if an FNSS $(\bar{y},\ R)$ over P constitutes an FNSs-WH BCK-ideal of P. □

Proof. The proof is similar to the proof of Theorem 1.

Theorem 3 Any FNSs-WH BCK-ideal implies an FNSWH BCK-ideal.

Proof. Let $(\bar{y},\ R)$ be the FNSs-WH BCK-ideal of P and let $\lambda,\ \mu \in P$. Then, $\exists s,\ r \in \lambda \circ \mu$ such that

$$T_{\bar{y}_{(j)}}(\lambda) \leq \wedge \left\{ T_{\bar{y}_{(j)}}(r),\ T_{\bar{y}_{(j)}}(\mu) \right\} \leq \wedge \left\{ inf_{q\in(\lambda\circ\mu)}\, T_{\bar{y}_{(j)}}(q),\ T_{\bar{y}_{(j)}}(\mu) \right\},$$

$$I_{\bar{y}_{(j)}}(\lambda) \geq \vee \left\{ I_{\bar{y}_{(j)}}(s),\ I_{\bar{y}_{(j)}}(\mu) \right\} \geq \vee \left\{ sup_{c\in(\lambda\circ\mu)}\, I_{\bar{y}_{(j)}}(c),\ I_{\bar{y}_{(j)}}(\mu) \right\},$$

and

$$F_{\bar{y}_{(j)}}(\lambda) \geq \vee \left\{ F_{\bar{y}_{(j)}}(s),\ F_{\bar{y}_{(j)}}(\mu) \right\} \geq \vee \left\{ sup_{c\in(\varepsilon\circ\delta)}\, F_{\bar{y}_{(j)}}(c),\ F_{\bar{y}_{(j)}}(\mu) \right\}.$$

As a result $(\bar{y},\ R)$ is an FNSWH BCK-ideal of P.

Theorem 4 If $(\bar{y},\ R)$ of P is an FNSWH BCK-ideal and it satisfies condition (3.6), then $(\bar{y},\ R)$ is also an FNSs-WH BCK-ideal of P.

Proof. As stated by condition (3.6) for $\lambda,\ \mu \in P$, $\exists\ \lambda_0,\ \mu_0 \in (\lambda \circ \mu)$ such that

$$T_{\bar{y}_{(j)}}(\lambda_0) \leq inf_{r\in(\lambda\circ\mu)}\, T_{\bar{y}_{(j)}}(r),$$

$$I_{\bar{y}_{(j)}}(\lambda_0) \geq sup_{s\in(\lambda\circ\mu)}\, I_{\bar{y}_{(j)}}(s),$$

and

$$F_{\bar{y}_{(j)}}(\lambda_0) \geq sup_{s\in(\lambda\circ\mu)}\, F_{\bar{y}_{(j)}}(s).$$

This is also supported by (3.4), which suggests that

$$T_{\bar{y}_{(j)}}(\lambda) \leq \wedge\left\{inf_{r\in(\lambda\circ\mu)}\, T_{\bar{y}_{(j)}}(r),\, T_{\bar{y}_{(j)}}(\mu)\right\}$$

$$= \wedge\left\{T_{\bar{y}_{(j)}}(\lambda_0),\, T_{\bar{y}_{(j)}}(\mu)\right\},$$

$$I_{\bar{y}_{(j)}}(\lambda) \geq \vee\left\{sup_{r\in(\lambda\circ\mu)}\, I_{\bar{y}_{(j)}}(r),\, I_{\bar{y}_{(j)}}(\mu)\right\}$$

$$= \vee\left\{I_{\bar{y}_{(j)}}(\mu_0),\, I_{\bar{y}_{(j)}}(\mu)\right\},$$

and

$$F_{\bar{y}_{(j)}}(\lambda) \geq \vee\left\{sup_{r\in(\lambda\circ\mu)}\, F_{\bar{y}_{(j)}}(r),\, F_{\bar{y}_{(j)}}(\mu)\right\}$$

$$= \vee\left\{F_{\bar{y}_{(j)}}(\mu_0),\, F_{\bar{y}_{(j)}}(\mu)\right\}.$$

Hence, $(\bar{y}_{(j)},\, R)$ is a j-FNSs-WH BCK-ideal of P. □

Remark 1 Every FNS in a finite hyper BCK-algebra meets condition (3.6). Thus, in a finite hyper BCK-algebra, the notions of FNSs-WH BCK-ideal and FNSWH BCK-ideal coincide.

Definition 5 A FNSS $(\bar{y},\, R)$ in P is called a j-NSSH BCK-ideal over P if it satisfies:

$$(\forall\lambda,\, \mu\in P)\begin{pmatrix} T_{\bar{y}_{(j)}}(\lambda) \leq \wedge\left\{sup_{r\in(\lambda\circ\mu)}\, T_{\bar{y}_{(j)}}(r),\, T_{\bar{y}_{(j)}}(\mu)\right\} \\ I_{\bar{y}_{(j)}}(r) \geq \vee\left\{inf_{r\in(\lambda\circ\mu)}\, I_{\bar{y}_{(j)}}(r),\, I_{\bar{y}_{(j)}}(\mu)\right\} \\ F_{\bar{y}_{(j)}}(r) \geq \vee\left\{inf_{r\in(\lambda\circ\mu)}\, F_{\bar{y}_{(j)}}(r),\, F_{\bar{y}_{(j)}}(\mu)\right\} \end{pmatrix}. \tag{20}$$

and

$$(\forall\lambda\in P)\begin{pmatrix} inf_{r\in(\lambda\circ\lambda)} T_{\bar{y}_{(j)}}(r) \leq T_{\bar{y}_{(j)}}(\lambda) \\ sup_{r\in(\lambda\circ\lambda)} I_{\bar{y}_{(j)}}(r) \geq I_{\bar{y}_{(j)}}(\lambda) \\ sup_{r\in(\lambda\circ\lambda)} F_{\bar{y}_{(j)}}(r) \geq F_{\bar{y}_{(j)}}(\lambda) \end{pmatrix}. \tag{21}$$

Then, $(\bar{y},\, R)$ is an FNSSH BCK-ideal of P.

Proposition 2 Any FNSSH BCK-ideal $(\bar{y},\, R)$ of P meets the following claims:

(1) $(\bar{y},\, R)$ meets the condition (3.5) for every $j\in R$.

(2)

$$(\forall\, \lambda,\ \mu \in P)(\forall j \in R)\left(\lambda \ll \mu \implies \begin{array}{c} T_{\bar{y}_{(j)}}(\lambda) \leq T_{\bar{y}_{(j)}}(\mu) \\ I_{\bar{y}_{(j)}}(\lambda) \geq I_{\bar{y}_{(j)}}(\mu) \\ F_{\bar{y}_{(j)}}(\lambda) \geq F_{\bar{y}_{(j)}}(\mu) \end{array}\right).$$

(3)

$$(\forall\, r,\ \lambda,\ \mu \in P)(\forall j \in R)\left(r \in \lambda \circ \mu \implies \begin{array}{c} T_{\bar{y}_{(j)}}(\lambda) \leq \wedge\left\{T_{\bar{y}_{(j)}}(r),\ T_{\bar{y}_{(j)}}(\mu)\right\} \\ I_{\bar{y}_{(j)}}(\lambda) \geq \vee\left\{I_{\bar{y}_{(j)}}(r),\ I_{\bar{y}_{(j)}}(\mu)\right\} \\ F_{\bar{y}_{(j)}}(\lambda) \geq \vee\left\{F_{\bar{y}_{(j)}}(r),\ F_{\bar{y}_{(j)}}(\mu)\right\} \end{array}\right).$$

Proof. (1) Since $0 \in (\lambda \circ \lambda)$ for all $\lambda \in R$, we have

$$T_{\bar{y}_{(j)}}(0) \leq inf_{r\in(\lambda\circ\lambda)}\ T_{\bar{y}_{(j)}}(r) \leq T_{\bar{y}_{(j)}}(\lambda),$$

$$I_{\bar{y}_{(j)}}(0) \geq sup_{r\in(\lambda\circ\lambda)}\ I_{\bar{y}_{(j)}}(r) \geq I_{\bar{y}_{(j)}}(\lambda),$$

and

$$F_{\bar{y}_{(j)}}(0) \geq sup_{r\in(\lambda\circ\lambda)}\ F_{\bar{y}_{(j)}}(r) \geq F_{\bar{y}_{(j)}}(\lambda)\ for\ all\ \lambda\ \in P\ by\ (3.10).$$

(2) Let $\lambda,\ \mu \in P$ such that $\lambda \ll \mu$. Then, $0 \in \lambda \circ \mu$ and

$$T_{\bar{y}_{(j)}}(0) \leq sup_{r\in(\lambda\circ\mu)}\ T_{\bar{y}_{(j)}}(r),$$

$$I_{\bar{y}_{(j)}}(0) \geq inf_{r\in(\lambda\circ\mu)}\ I_{\bar{y}_{(j)}}(r),$$

and

$$F_{\bar{y}_{(j)}}(0) \geq inf_{r\in(\lambda\circ\mu)}\ F_{\bar{y}_{(j)}}(r).$$

According to the Definition (5) and (1), we get

$$T_{\bar{y}_{(j)}}(\lambda) \leq \wedge \left\{ sup_{r \in (\lambda \circ \mu)} \; T_{\bar{y}_{(j)}}(r),\; T_{\bar{y}_{(j)}}(\mu) \right\}$$

$$\leq \wedge \left\{ T_{\bar{y}_{(j)}}(0),\; T_{\bar{y}_{(j)}}(\mu) \right\}$$

$$= T_{\bar{y}_{(j)}}(\mu),$$

$$I_{\bar{y}_{(j)}}(\lambda) \geq \vee \left\{ inf_{r \in (\lambda \circ \mu)} \; I_{\bar{y}_{(j)}}(r),\; I_{\bar{y}_{(j)}}(\mu) \right\}$$

$$\geq \vee \left\{ I_{\bar{y}_{(j)}}(0),\; I_{\bar{y}_{(j)}}(\mu) \right\}$$

$$= I_{\bar{y}_{(j)}}(\mu),$$

and

$$F_{\bar{y}_{(j)}}(\lambda) \geq \vee \left\{ inf_{r \in (\lambda \circ \mu)} \; F_{\bar{y}_{(j)}}(r),\; F_{\bar{y}_{(j)}}(\mu) \right\}$$

$$\geq \vee \left\{ F_{\bar{y}_{(j)}}(0),\; F_{\bar{y}_{(j)}}(\mu) \right\}$$

$$= F_{\bar{y}_{(j)}}(\mu).$$

(3) Let $r,\ \lambda,\ \mu \in P$, such that $r \in \lambda \circ \mu$. Then,

$$sup_{s \in (\lambda \circ \mu)} T_{\bar{y}_{(j)}}(s) \geq T_{\bar{y}_{(j)}}(r),$$

$$inf_{q \in (\lambda \circ \mu)} I_{\bar{y}_{(j)}}(q) \leq I_{\bar{y}_{(j)}}(r),$$

and

$$inf_{q \in (\lambda \circ \mu)} F_{\bar{y}_{(j)}}(q) \leq F_{\bar{y}_{(j)}}(r).$$

This implies from (3.9) that

$$T_{\bar{y}_{(j)}}(\lambda) \geq \wedge \left\{ sup_{s \in (\lambda \circ \mu)} T_{\bar{y}_{(j)}}(s),\ T_{\bar{y}_{(j)}}(\mu) \right\}$$

$$\geq \wedge \left\{ T_{\bar{y}_{(j)}}(r),\ T_{\bar{y}_{(j)}}(\mu) \right\},$$

$$I_{\bar{y}_{(j)}}(\lambda) \leq \vee \left\{ inf_{q \in (\lambda \circ \mu)} I_{\bar{y}_{(j)}}(q),\ I_{\bar{y}_{(j)}}(\mu) \right\}$$

$$\leq \vee \left\{ I_{\bar{y}_{(j)}}(r),\ I_{\bar{y}_{(j)}}(\mu) \right\},$$

and

$$F_{\bar{y}_{(j)}}(\lambda) \leq \vee \left\{ inf_{q \in (\lambda \circ \mu)} F_{\bar{y}_{(j)}}(q),\ F_{\bar{y}_{(j)}}(\mu) \right\}$$

$$\leq \vee \left\{ F_{\bar{y}_{(j)}}(r),\ F_{\bar{y}_{(j)}}(\mu) \right\}.$$

This demonstrates (3). □

Corollary 1 Any FNSH BCK-ideal $(\bar{y},\ R)$ of P fulfills the subsequent requirement:

$$(\forall j \in R)(\forall \lambda,\ \mu \in P) \begin{pmatrix} T_{\bar{y}_{(j)}}(\lambda) \leq \wedge \left\{ inf_{r \in \lambda \circ \mu}\ T_{\bar{y}_{(j)}}(r),\ T_{\bar{y}_{(j)}}(\mu) \right\} \\ I_{\bar{y}_{(j)}}(\lambda) \geq \vee \left\{ sup_{r \in \lambda \circ \mu}\ I_{\bar{y}_{(j)}}(r),\ I_{\bar{y}_{(j)}}(\mu) \right\} \\ F_{\bar{y}_{(j)}}(\lambda) \geq \vee \left\{ sup_{r \in \lambda \circ \mu}\ F_{\bar{y}_{(j)}}(r),\ F_{\bar{y}_{(j)}}(\mu) \right\} \end{pmatrix}. \tag{22}$$

Corollary 2 Any FNSSH BCK-ideal is also an FNSs-WH BCK-ideal and an FNSH BCK-ideal.

Proof. Straightforward. □

Example 3 Let us take the hyper BCK-algebra $P = \{0,\ r,\ s\}$ in Example 1. Consider a collection of parameters $R = \{d,\ e,\ f\}$. Let $(\bar{y},\ R)$ be a representation of an FNSS over P, defined by Table 3.

Table 3. FNSWH BCK- ideal

$\bar{y}$	0	r	s
d	(0.9, 0.6, 0.1)	(0.8, 0.65, 0.25)	(0.7, 0.7, 0.45)
e	(0.6, 0.4, 0.25)	(0.5, 0.5, 0.65)	(0.3, 0.5, 0.65)
f	(0.8, 0.7, 0.01)	(0.3, 0.76, 0.25)	(0.1, 0.77, 0.75)

Subsequently, $(\bar{y},\ R)$ represents an FNSWH BCK-ideal of P. However, it is not an FNSSH BCK-ideal of P because

$$T_{\bar{y}_{(e)}}(s) = 0.3 < 0.5 = \wedge\left\{sup_{t\in(s\circ r)}T_{\bar{y}_{(e)}}(t),\ T_{\bar{y}_{(e)}}(r)\right\},$$

$$I_{\bar{y}_{(e)}}(s) = 0.5 = 0.5 = \vee\left\{inf_{t\in(s\circ r)}I_{\bar{y}_{(e)}}(t),\ I_{\bar{y}_{(e)}}(r)\right\},$$

and

$$F_{\bar{y}_{(e)}}(s) = 0.65 = 0.65 = \vee\left\{inf_{t\in(s\circ r)}F_{\bar{y}_{(e)}}(t),\ F_{\bar{y}_{(e)}}(r)\right\}.$$

Theorem 5 If $(\bar{y},\ R)$ constitutes an FNSSH BCK-ideal of P, then for every $(\rho,\ \sigma,\ \omega) \in [0,\ 1]\times[0,\ 1]\times[0,\ 1]$, such that $0 \le \rho+\sigma+\omega \le 3$, the nonempty sets $\mathfrak{X}_\rho$, $\mathfrak{Y}_\sigma$ and $\mathfrak{Z}_\omega$, given in Lemma 1, emerge as strong hyper BCK ideals of P.

Proof. Consider $(\bar{y},\ R)$ as an FNSSH BCK-ideal P. Let $(\rho,\ \sigma,\ \delta) \in [0,\ 1]\times[0,\ 1]\times[0,\ 1]$ such that $0 \le \rho+\sigma+\omega \le 3$. Then, there exist $r \in \mathfrak{X}_\rho,\ i \in \mathfrak{Y}_\sigma$ and $s \in \mathfrak{Z}_\omega$, and so $T_{\bar{y}_{(j)}}(r) \le \rho,\ I_{\bar{y}_{(j)}}(s) \ge \sigma$ and $F_{\bar{y}_{(j)}}(s) \ge \omega$. By Proposition 2 (1):

$$T_{\bar{y}_{(j)}}(0) \le T_{\bar{y}_{(j)}}(r) \le \rho$$

$$I_{\bar{y}_{(j)}}(0) \ge I_{\bar{y}_{(j)}}(s) \ge \sigma$$

$$F_{\bar{y}_{(j)}}(0) \ge F_{\bar{y}_{(j)}}(s) \ge \omega, \text{ and hence } 0 \in \mathfrak{X}_\rho \cup \mathfrak{Y}_\sigma \cap \mathfrak{Z}_\omega.$$

Let $\lambda,\ \mu \in P$ be such that $(\lambda\circ\mu)\cap\mathfrak{X}_\rho \neq \emptyset$ and $\mu \in \mathfrak{X}_\rho$. Then, $T_{\bar{y}_{(j)}}(\mu) \le \rho$ and there exists $r_0 \in (\lambda\circ\mu)\cap\mathfrak{X}_\rho$. Equation (3.9) indicates that

$$\begin{aligned} T_{\bar{y}_{(j)}}(\lambda) &\le \wedge\left\{sup_{t\in(\lambda\circ\mu)}\ T_{\bar{y}_{(j)}}(t),\ T_{\bar{y}_{(j)}}(\mu)\right\} \\ &\le \wedge\left\{T_{\bar{y}_{(j)}}(r_0),\ T_{\bar{y}_{(j)}}(\mu)\right\} \\ &\le \rho. \end{aligned}$$

Thus, $\lambda \in \mathfrak{X}_\rho$ and so $\mathfrak{X}_\rho$ is a strong hyper BCK-ideal of P. Now, let $(\lambda\circ\mu)\cap\mathfrak{Y}_\sigma \neq \emptyset$ and $\mu \in \mathfrak{Y}_\sigma$. Then, there exists $s_0 \in (\lambda\circ\mu)\cap\mathfrak{Y}_\sigma$, from (3.9), we get

$$I_{\bar{y}_{(j)}}(\lambda) \geq \vee\left\{inf_{t\in(\lambda\circ\mu)}\, I_{\bar{y}_{(j)}}(t),\; I_{\bar{y}_{(j)}}(\mu)\right\}$$

$$\geq \vee\left\{I_{\bar{y}_{(j)}} s_0),\; I_{\bar{y}_{(j)}}(\mu)\right\}$$

$$\geq \sigma.$$

Thus, $\lambda \in \mathfrak{Y}_\sigma$ and $\mathfrak{Y}_\sigma$ is a strong hyper BCK-ideal of P.

Let $(\lambda\circ\mu)\cap\mathfrak{Z}_\omega \neq \emptyset$ and $\mu \in \mathfrak{Z}_\omega$ for all $\lambda,\ \mu \in P$. Then, there exists $s_0 \in (\lambda\circ\mu)\cap\mathfrak{Z}_\omega$ and $F_{\bar{y}_{(j)}}(\mu) \leq \omega$. From (3.9), we get

$$F_{\bar{y}_{(j)}}(\lambda) \geq \vee\left\{inf_{t\in(\lambda\circ\mu)}\, F_{\bar{y}_{(j)}}(t),\; F_{\bar{y}_{(j)}}(\mu)\right\}$$

$$\geq \vee\left\{F_{\bar{y}_{(j)}}(s_0),\; F_{\bar{y}_{(j)}}(\mu)\right\}$$

$$\geq \omega.$$

Thus, $\lambda \in \mathfrak{Z}_\omega$ and so $\mathfrak{Z}_\omega$ is a strong hyper BCK-ideal of P. □

Theorem 6 Let $(\bar{y},\ R)$ be an FNSS over P with

$$(\forall I \subset P)(\exists \lambda_0,\ \mu_0 \in I)\begin{pmatrix} T_{\bar{y}_{(j)}}(\lambda_0) = sup_{r\in I}\, T_{\bar{y}_{(j)}}(r) \\ I_{\bar{y}_{(j)}}(\lambda_0) = inf_{s\in I}\, I_{\bar{y}_{(j)}}(s) \\ F_{\bar{y}_{(j)}}(\mu_0) = inf_{s\in I}\, F_{\bar{y}_{(j)}}(s) \end{pmatrix}. \tag{23}$$

Then, $(\bar{y},\ R)$ is an FNSSH BCK-ideal of P if the sets $\mathfrak{X}_\rho$, $\mathfrak{Y}_\sigma$ and $\mathfrak{Z}_\omega$ in Equation (3.8) are nonempty strong hyper BCK-ideals of P for all $(\rho,\ \sigma,\ \omega) \in [0,\ 1]\times[0,\ 1]\times[0,\ 1]$.

Proof. Let $\lambda \in P$ and $j \in R$. Take $T_{\bar{y}_{(j)}}(\lambda) = \rho,\ I_{\bar{y}_{(j)}}(\lambda) = \sigma$ and $F_{\bar{y}_{(j)}}(\lambda) = \omega$. Then, λ belonges to $\mathfrak{X}_\rho$, $\mathfrak{Y}_\sigma$ and $\mathfrak{Z}_\omega$.

Lemma 1 implies that $\lambda\circ\lambda \subseteq \mathfrak{X}_\rho$. As a results, we obtain $T_{\bar{y}_{(j)}}(r) \leq \rho,\ \ I_{\bar{y}_{(j)}}(r)\ \geq \sigma$ and $F_{\bar{y}_{(j)}}(r) \geq \omega$ for every $r \in \lambda\circ\lambda,$ and so

$$inf_{r\in(\lambda\circ\lambda)}\, T_{\bar{y}_{(j)}}(r) \leq \rho = T_{\bar{y}_{(j)}}(\lambda),$$

$$sup_{r\in(\lambda\circ\lambda)}\, I_{\bar{y}_{(j)}}(r) \geq \sigma = I_{\bar{y}_{(j)}}(\lambda),$$

$$sup_{r\in(\lambda\circ\lambda)}\, F_{\bar{y}_{(j)}}(r) \geq \omega = F_{\bar{y}_{(j)}}(\lambda).$$

Let

$$v = \wedge \left\{ sup_{r \in (\lambda \circ \mu)} \ T_{\bar{y}_{(j)}}(r),\ T_{\bar{y}_{(j)}}(\mu) \right\},$$

$$m = \vee \left\{ inf_{r \in (\lambda \circ \mu)} \ I_{\bar{y}_{(j)}}(r),\ I_{\bar{y}_{(j)}}(\mu) \right\},$$

and

$$n = \vee \left\{ inf_{r \in (\lambda \circ \mu)} \ F_{\bar{y}_{(j)}}(r),\ F_{\bar{y}_{(j)}}(\mu) \right\}.$$

Then, $\mathfrak{X}_v$, $\mathfrak{Y}_m$ and $\mathfrak{Z}_n$ are nonempty strong hyper BCK-ideals of P. Applying condition (3.12), we get

$$T_{\bar{y}_{(j)}}(r_0) = sup_{r \in (\lambda \circ \mu)} \ T_{\bar{y}_{(j)}}(r),$$

$$I_{\bar{y}_{(j)}}(s_0) = inf_{r \in (\lambda \circ \mu)} \ I_{\bar{y}_{(j)}}(r),$$

and

$$F_{\bar{y}_{(j)}}(s_0) = inf_{r \in (\lambda \circ \mu)} \ F_{\bar{y}_{(j)}}(r),$$

for some $r_0,\ s_0 \in \lambda \circ \mu$. Then,

$$\begin{aligned} T_{\bar{y}_{(j)}}(r_0) &= sup_{r \in (\lambda \circ \mu)} \ T_{\bar{y}_{(j)}}(r) \\ &\leq \wedge \left\{ sup_{r \in (\lambda \circ \mu)} \ T_{\bar{y}_{(j)}}(r),\ T_{\bar{y}_{(j)}}(\mu) \right\} \\ &= v, \\ I_{\bar{y}_{(j)}}(r_0) &= inf_{r \in (\lambda \circ \mu)} \ I_{\bar{y}_{(j)}}(r) \\ &\geq \vee \left\{ inf_{r \in (\lambda \circ \mu)} \ I_{\bar{y}_{(j)}}(r),\ I_{\bar{y}_{(j)}}(\mu) \right\} \\ &= m, \end{aligned}$$

and

$$F_{\bar{y}_{(j)}}(s_0) = inf_{r\in(\lambda\circ\mu)}\ F_{\bar{y}_{(j)}}(r)$$

$$\geq \vee\left\{inf_{r\in(\lambda\circ\mu)}\ F_{\bar{y}_{(j)}}(r),\ F_{\bar{y}_{(j)}}(\mu)\right\}$$

$$= n.$$

This indicates that r_0 belongs to $\mathfrak{X}_\rho$, s_0 belongs to $\mathfrak{Y}_\sigma$ and $\mathfrak{Z}_\omega$. Consequently, $(\lambda\circ\mu)\cap\mathfrak{X}_v \neq \emptyset$, $(\lambda\circ\mu)\cap\mathfrak{Y}_m \neq \emptyset$ and $(\lambda\circ\mu)\cap\mathfrak{Z}_n \neq \emptyset$.Then

$$T_{\bar{y}_{(j)}}(\lambda) \leq v = \wedge\left\{sup_{r\in(\lambda\circ\mu)}\ T_{\bar{y}_{(j)}}(r),\ T_{\bar{y}_{(j)}}(\mu)\right\},$$

$$I_{\bar{y}_{(j)}}(\lambda) \geq m = \vee\left\{inf_{r\in(\lambda\circ\mu)}\ I_{\bar{y}_{(j)}}(r),\ I_{\bar{y}_{(j)}}(\mu)\right\},$$

and

$$F_{\bar{y}_{(j)}}(\lambda) \geq n = \vee\left\{inf_{r\in(\lambda\circ\mu)}\ F_{\bar{y}_{(j)}}(r),\ F_{\bar{y}_{(j)}}(\mu)\right\}.$$

As a result, $(\bar{y},\ R)$ is an FNSSH BCK-ideal of P. □

Theorem 7 If $(\bar{y},\ R)$ is an FNSS over P and the nonempty sets $\mathfrak{X}_\rho$, $\mathfrak{Y}_\sigma$ and $\mathfrak{Z}_\omega$ in (3.8) are strong hyper BCK-ideals of P for every $(\rho,\ \sigma,\ \omega) \in [0,\ 1]\times[0,\ 1]\times[0,\ 1]$, then $(\bar{y},\ R)$ is an FNSSH BCK-ideal of P.

Proof. Assume $\mathfrak{X}_\rho$, $\mathfrak{Y}_\sigma$ and $\mathfrak{Z}_\omega$ in (3.8) are non-empty strong hyper BCK-ideals of P for every $(\rho,\ \sigma,\ \omega) \in [0,\ 1]\times[0,\ 1]\times[0,\ 1]$. Therefore, $\mathfrak{X}_\rho$, $\mathfrak{Y}_\sigma$ and $\mathfrak{Z}_\omega$ are hyper BCK ideals of P. Thus, according to Theorem 1, $(\bar{y},\ R)$ is an FNSH BCK-ideal of P. For any element λ in P, the set $\lambda\circ\lambda \subseteq \lambda\circ P \ll \{\lambda\}$ for all $\lambda\in P$. Equation (3.3) states that

$$T_{\bar{y}_{(j)}}(r) \leq T_{\bar{y}_{(j)}}(\lambda),$$

$$I_{\bar{y}_{(j)}}(r) \geq I_{\bar{y}_{(j)}}(\lambda),$$

and

$$F_{\bar{y}_{(j)}}(r) \geq F_{\bar{y}_{(j)}}(\lambda),\ \forall\ r\in(\lambda\circ\lambda).$$

Thus,

$$T_{\bar{y}_{(j)}}(\lambda) \leq inf_{r\in(\lambda\circ\lambda)} T_{\bar{y}_{(j)}}(r),$$

$$I_{\bar{y}_{(j)}}(\lambda) \geq sup_{r\in(\lambda\circ\lambda)} I_{\bar{y}_{(j)}}(r),$$

$$F_{\bar{y}_{(j)}}(\lambda) \geq sup_{r\in(\lambda\circ\lambda)} F_{\bar{y}_{(j)}}(r).$$

Let

$$\wedge \left\{ sup_{r\in(\lambda\circ\mu)} T_{\bar{y}_{(j)}}(r),\ T_{\bar{y}_{(j)}}(\mu) \right\} = \rho,$$

$$\vee \left\{ inf_{r\in(\lambda\circ\mu)} I_{\bar{y}_{(j)}}(r),\ I_{\bar{y}_{(j)}}(\mu) \right\} = \sigma,$$

and

$$\vee \left\{ inf_{r\in(\lambda\circ\mu)} F_{\bar{y}_{(j)}}(r),\ F_{\bar{y}_{(j)}}(\mu) \right\} = \omega.$$

Then,

$$sup_{r\in(\lambda\circ\mu)} T_{\bar{y}_{(j)}}(r) \leq \rho,\ \ T_{\bar{y}_{(j)}}(\mu) \leq \rho,$$

$$inf_{r\in(\lambda\circ\mu)} I_{\bar{y}_{(j)}}(r) \geq \sigma,\ \ I_{\bar{y}_{(j)}}(\mu) \geq \sigma,$$

$$inf_{r\in(\lambda\circ\mu)} F_{\bar{y}_{(j)}}(r) \geq \omega,\ \ F_{\bar{y}_{(j)}}(\mu) \geq \omega.$$

Since $|\lambda\circ\mu| < \infty$ for every $\lambda,\ \mu \in P$, there exists $s \in \lambda\circ\mu$ such that

$$T_{\bar{y}_{(j)}}(s) \leq \rho,\ \ T_{\bar{y}_{(j)}}(\mu) \leq \rho,$$

$$I_{\bar{y}_{(j)}}(s) \geq \sigma,\ \ I_{\bar{y}_{(j)}}(\mu) \geq \sigma,$$

$$F_{\bar{y}_{(j)}}(s) \geq \omega,\ \ F_{\bar{y}_{(j)}}(\mu) \geq \omega.$$

Hence,

$$(\lambda \circ \mu) \cap \mathfrak{X}_\rho \neq \emptyset,\ \mu \in \mathfrak{X}_\rho,$$

$$(\lambda \circ \mu) \cap \mathfrak{Y}_\sigma \neq \emptyset,\ \mu \in \mathfrak{Y}_\sigma,$$

$$(\lambda \circ \mu) \cap \mathfrak{Z}_\omega \neq \emptyset,\ \mu \in \mathfrak{Z}_\omega.$$

We have $\lambda \in \mathfrak{X}_\rho \cap \mathfrak{Y}_\sigma \cap \mathfrak{Z}_\omega$. Consequently,

$$T_{\bar{y}_{(j)}}(\lambda) \leq \rho = \wedge \left\{ sup_{r \in (\lambda \circ \mu)} T_{\bar{y}_{(j)}}(r),\ T_{\bar{y}_{(j)}}(\mu) \right\},$$

$$I_{\bar{y}_{(j)}}(\lambda) \geq \sigma = \vee \left\{ inf_{r \in (\lambda \circ \mu)} I_{\bar{y}_{(j)}}(r),\ I_{\bar{y}_{(j)}}(\mu) \right\},$$

and

$$F_{\bar{y}_{(j)}}(\lambda) \geq \omega = \vee \left\{ inf_{r \in (\lambda \circ \mu)} F_{\bar{y}_{(j)}}(r),\ F_{\bar{y}_{(j)}}(\mu) \right\}.$$

As a result $(\bar{y},\ R)$ is an FNSSH BCK-ideal of P. □

4. Application

In medical decision-making, the imprecise and fuzzy data present in patient information (symptoms, lab results, and risk factors) can best be managed using FNSS. The hyper BCK-algebra structure easily captures the antecedent between symptoms and diagnoses, while the Fermatean neutrosophic sets take care of the truth, indeterminacy-membership, and degree of non-membership corresponding diagnoses. The soft structure enables a free combination of medical cases to be grouped while hyper BCK-ideal can isolate specific patterns of symptoms that lead to certain diagnoses thus enhancing overall diagnosis and treatment.

5. Comparison analysis

This paper compares the neutrosophic hyper BCK-ideals in hyper BCK algebras and the Fermatean neutrosophic soft structure hyper BCK algebras aiming at providing a method of dealing with uncertainty or indeterminacy in algebraic structures. Neutrosophic hyper BCK-ideals use the concept of neutrosophic set I earlier employed to define ideals in BCK-algebras and generalize it to hyper BCK-algebras, so that truth, indeterminacy, and falsity can all be expressed through different membership functions. On the other hand, Fermatean neutrosophic soft structures extend Fermatean neutrosophic sets by including soft set theory as well as real numbers lying in an interval [0, 1] and symbol 'N' to make up for the degrees of different parameters for membership, non-membership, and indeterminacy. Although neutrosophic hyper BCK-ideals form basic building blocks of hyper BCK-algebras and their applications are mainly in theoretical studies and decision-making, optimization, Fermatean neutrosophic soft structures are lenient only for modeling many interconnected mechanisms. Each framework contributes to the study of hyper BCK-algebras while incorporating different paradigms, approaches to handling uncertainty, and particular applications outside the domain of hyper BCK-algebras but within the more general context of algebraic structures.

6. Conclusion

The concepts of FNSH BCK-ideals, FNSWH BCK-ideal, FNSs-WH BCK-ideals, and NSSH BCK-ideals, as well as their properties and relationships, have been introduced and investigated. Characterizations of FNSH BCK-ideals and FNSWH BCK-ideals have been studied, and requirements for an FNSWH BCK-ideal to be an FNSs-WH BCK-ideal have been developed. In addition, requirements have been established for an FNSS to match the criteria for being an FNSSH BCK-ideal. Future research on FNSS and its ideals in hyper BCK-algebras may investigate applications in image processing, pattern recognition, and decision-making, among other domains. Furthermore, more studies might concentrate on creating effective methods and algorithms for analyzing FNSS inside the framework of hyper BCK algebras to improve their efficacy and suitability for use in practical situations. This makes huge decisions, where there is either little information to guide the choice, or where different factors must be considered according to their importance, making it a sound mathematical basis for medical expert systems.

Conflict of interest

The authors declare that they have no conflicts of interest.

References

[1] Molodtsov D. Soft set theory-first results. *Computers and Mathematics with Applications*. 1999; 37(4): 19-31.

[2] Jun YB. Soft BCK/BCI-algebras. *Computers and Mathematics with Applications*. 2008; 56(5): 1408-1413.

[3] Jun YB, Park CH. Applications of soft sets in ideal theory of BCK/BCI-algebras. *Information Sciences*. 2008; 178(11): 2466-2475.

[4] Maji K P, Biswas R, Roy AR. Applfuzzy soft sets. *Journal of Fuzzy Mathematics*. 2001; 9: 589-602.

[5] Maji PK, Biswas R, Roy AR. Intuitionistic fuzzy soft sets. *Journal of Fuzzy Mathematics*. 2001; 9: 677-692.

[6] Jun YB, Lee KJ, Park CH. Fuzzy soft set theory applied to BCK/BCI-algebras. *Computers and Mathematics with Applications*. 2010; 59(9): 3180-3192.

[7] Sahar AM, Alkouri AU, Adeeb TG, Bataihah A. Bipolar complex fuzzy soft sets and their application. *International Journal of Fuzzy System Applications*. 2021; 11(1): 1-23.

[8] Qamar MA, Ahmad AG, Hassan N. On *Q*-neutrosophic soft fields. *Neutrosophic Sets and Systems*. 2020; 32: 80-93. Available from: https://doi.org/10.5281/zenodo.3723603.

[9] Marty F. Sur une generalization de la notion de groupe. In: *Proceedings of the 8th Congrès des Mathématiciens Scandinaves*. Stockholm, Sweden: Almqvist and Wiksell; 1934. p.45-49.

[10] Ameri R. On categories of hypergroups and hypermodules. *Journal of Discrete Mathematical Sciences and Cryptography*. 2003; 6(2-3): 121-132.

[11] Ameri R, Rosenberg IG. Congruences of multialgebras. *Journal of Multiple-Valued Logic and Soft Computing*. 2009; 15(5): 525-536.

[12] Ameri R, Zahedi MM. Hyper algebraic systems. *Italian Journal of Pure and Applied Mathematics*. 1999; 6: 21-32.

[13] Corsini P. In: Aviani. (ed.) *Prolegomena of Hypergroup Theory*. Aviani Editore, Tricesimo; 1994.

[14] Corsini P, Leoreanu V. *Applications of Hyperstructure Theory*. Kluwer: Dordrecht and The Netherlands; 2003.

[15] Leoreanu-Fotea V, Davvaz B. Join *n*-spaces and lattices. *Journal of Multiple Valued Logic and Soft Computing*. 2009; 15(5): 421-432.

[16] Leoreanu-Fotea V, Davvaz B. *N*-hypergroups and binary relations. *European Journal of Combinatorics*. 2008; 29(5): 1207-1218.

[17] Pelea C. On the direct product of multialgebras. *Studia Universitatis Babes-Bolyai Mathematica*. 2003; 48(2): 93-98.
[18] Pickett HE. Homomorphisms and subalgebras of multialgebras. *Pacific Journal of Mathematics*. 2001; 21(2): 327-342.
[19] Schweigert D. Congruence relations of multialgebras. *Discrete Mathematics*. 1985; 53: 249-253. Available from: https://doi.org/10.1016/0012-365X(85)90145-1.
[20] Serafimidis K, Kehagias A, Konstantinidou M. The *L*-fuzzy hypermodules. *Italian Journal of Pure and Applied Mathematics*. 2002; 12: 83-90.
[21] Vougiouklis T. *Hyperstructures and Their Representations*. USA: Hadronic Press; 1994.
[22] Kousar S, Shaheen S, Kausar N, Pamucar D, Simic V, Salman MA. On the construction of congruences over generalized fuzzy *G*-acts. *International Journal of Computational Intelligence Systems*. 2024; 17(1): 242.
[23] Palanikumar M, Kausar N, Ozbilge E, Ozbilge E. Extending the concepts of complex interval valued neutrosophic subbisemiring of bisemiring. *International Journal of Neutrosophic Science*. 2024; 23(4): 117-135.
[24] Palanikumar M, Kausar N, Garg H, Kadry S, Kim J. Robotic sensor based on score and accuracy values in *q*-rung complex diophatine neutrosophic normal set with an aggregation operation. *Alexandria Engineering Journal*. 2024; 77: 149-164. Available from: https://doi.org/10.1016/j.aej.2023.06.064.
[25] Mahapatra BS, Bera MB, Mondal MK, Smarandache F, Mahapatra DGS. Health insurance provider selection through novel correlation measure of neutrosophic sets using TOPSIS. *Contemporary Mathematics*. 2024; 5(4): 4497-4522.
[26] Rath AK, Ranadive AS, Pamucar D, Marinkovic D. Support based essential and core based superfluous fuzzy modules. *Contemporary Mathematics*. 2024; 5(4): 4367-4383.
[27] Jun YB, Zahedi MM, Xin XL, Borzooei RA. On hyper BCK-algebras. *Italian Journal of Pure and Applied Mathematics*. 2000; 8: 127-136.
[28] Jun YB, Xin XL. Scalar elements and hyper atoms of hyper BCK-algebras. *Scientiae Mathematicae*. 1999; 2(3): 303-309.
[29] Jun YB, Xin XL, Zahedi MM, Roh EH. Strong hyper BCK-ideals of hyper BCK-algebras. *Scientiae Mathematicae Japonicae*. 2000; 51(3): 493-498.
[30] Jun YB, Shim WH. Fuzzy implicative hyper BCK-ideals of hyper BCK-algebras. *International Journal of Mathematics and Mathematical Sciences*. 2002; 29(2): 63-70.
[31] Jun YB, Xin XL. Fuzzy hyper BCK-ideals of hyper BCK-algebras. *Scientiae Mathematicae Japonicae*. 2001; 53: 353-360.
[32] Davvaz B, Cristea I. *Fuzzy Algebraic Hyperstructures*. Cham, Switzerland: Springer; 2015.
[33] Zail SH, Abed MM, Faisal AS. Neutrosophic BCK-algebra and Ω-BCK-algebra. *International Journal of Neutrosophic Science*. 2022; 19(3): 8-15.
[34] Santhakumar S, Sumathi IR, Mahalakshmi J. A novel approach to the algebraic structure of neutrosophic super hyper algebra. *Neutrosophic Sets and Systems*. 2024; 60(1): 39.
[35] Hamidi M. Extended BCK-ideal based on single-valued neutrosophic hyper BCK-ideals. *Bulletin of the Section of Logic*. 2023; 52(4): 411-440.
[36] Borzooei RA, Kim HS, Jun YB, Ahn SS. MBJ-neutrosophic subalgebras and filters in BE-algebras. *AIMS Mathematics*. 2022; 7(4): 6016-6033.
[37] Xin X, Borzooei RA, Bakhshi M, Jun YB. Intuitionistic fuzzy soft hyper BCK-algebras. *Symmetry*. 2019; 11(3): 399.