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Research Article

Study of Burger Equation Using q-HAM with Yang-Abdel-Cattani Derivative

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Abstract: In order to solve the generalized Burgers equation, this research work introduces one of the most recent operators in fractional calculus. An easier-to-understand version of the problem can be obtained by using the Yang-Abdel-Cattani fractional operator. The generalized Burgers equation's result is obtained analytically effectively using the q-homotopy analysis method (q-HAM). A visual study is also acquired to demonstrate the operation of the technique. By employing this approach to solve the generalized Burgers problem, this study advances the field of nonlinear differential equations.

Keywords: Burger equation, Yang-Abdel-Cattani derivative, fractional operator, fixed point condition, homotopy, q-homotopy, q-HAM, HAM

MSC: 34A34, 26A33

1. Introduction

After classical calculus, fractional calculus is a logical progression. In a variety of scientific areas, it has recently gained in prominence and respect. The increasing number of applications for fractional calculus reveals how it offers more precise mathematical representations of everyday things. A simulation model is used to draw any natural or physical phenomenon whose shape is important for comprehending the problem. Due to widespread research initiatives, the literature on fractional calculus is rapidly expanding. Numerous literary disciplines, including as biology, engineering, fluid mechanics, heat conduction, viscoelasticity, astronomy, and electricity have been influenced by fractional calculus [1–10]. The calculus of fractional order therefore has an effect on each and every branch of science and technology.

In order to understand many scientific ideas [11–28], fractional differential equations-which may be linear or nonlinear-are employed. Differential equations of fractional order frequently have resolved the issues. Numerous original numerical and analytical approaches are defined to tackle such challenges [29, 30]. The conventional homotopy analysis method (HAM), a potent semi-analytical method for solving nonlinear differential equations, is extended by the q-homotopy analysis method (q-HAM). The approach is especially helpful for handling complicated initial value problems (IVPs) and boundary value problems (BVPs), when more conventional approaches like perturbation theory may not work or be ineffective. The homotopy parameter is generalized by the "q" in q-HAM, which gives the series

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solution's convergence behavior greater latitude. The Burger equation was initially presented by Bateman citebat in 1915, which was then reviewed by Burgers citebur. The Burger equation is given in its usual form:

$$\frac{\partial v}{\partial t}+af(v)\frac{\partial v}{\partial p}=c\frac{\partial^2 v}{\partial p^2}. \tag{1}$$

Here, a may be some constant. In many areas of physical processes, including boundary layer behaviour, shock weave creation, turbulence, the weather issue, mass transportation, traffic flow, and acoustic transmission, Burgers' equation-a nonlinear partial differential equation of second order-is employed. A interesting area of mathematics is fractional calculus, which deals with integrals and derivatives of any order. Contrary to traditional definitions for derivatives and integrals, fractional integrals and derivatives of order $\xi > 0$ can be interpreted in a variety of ways. Samko, Riemann, Caputo, Kilbas, and other individuals had a significant impact on the theory of singular kernels in fractional calculus. Integrals and derivatives with kernels without singularity were studied by researchers Miller-Ross, Atangana-Baleanu, Wiman, Yang, and others. We have two different types of differential equations to deal with in modelling. The differential equation might be either (i) a linear fractional differential equation or (ii) a nonlinear fractional differential equation. It is clear that linear fractional differential equation solutions are much simpler than nonlinear fractional differential equation solutions. There aren't many methods in the literature to solve models that have nonlinear fractional differential equations. It is evident that numerical methods that provide approximations are more effective for resolving these issues. Methods of perturbation, such as the homotopy perturbation technique, are available. The homotopy analysis method (HAM) is one of the finest ways to solve differential equations, both linear and nonlinear. The q-HAM is an effective method for resolving nonlinear issues. The n and h auxiliary parameters make up this method. The q-HAM is converted into HAM when $n = 1$. We can apply the Laplace transformation of fractional derivatives and integration in this study.

The term "q-HAM" refers to an extension of the homotopy analysis method (HAM) for solving nonlinear differential equations. By applying the q-HAM, the interval of convergence of HAM is lengthened, assuming it exists. In comparison to the convergence of the homotopy analysis technique (HAM), the q-homotopy analysis method is more accurate. A more versatile version of HAM called the q-homotopy analysis technique (q-HAM) was suggested by M. A. El-Tawil and S. N. Huseen. The homotopy parameter q and its values range from 0 to $(1/n)$, where $(n > 1)$ is the value for the auxiliary parameter h. In the area of mathematical modelling, it has been shown that the Yang-Abdel-Cattani fractional operator (YAC operator) produced superior outcomes. Numerous advancements in a variety of fields, including chemical science, physical science, medical research, and many more, have resulted from the discovery of fractional calculus.

Now suppose the Burger equation be:

$$\frac{\partial v}{\partial t}+af(v)\frac{\partial v}{\partial p}=c\frac{\partial^2 v}{\partial p^2} \tag{2}$$

with

$$v(p, 0) = g(p),\ \ p \in \Omega \tag{3}$$

A more manageable form of the problem can be obtained by reducing general fractional derivative into different operators, further. The generalised Burgers equation is shown in Equation (2). We solve Equation (2) using Yang-Abdel-Cattani derivative and the familiar homotopy analysis technique to get a roughly solution with condition (3).

The seven sections that make up this article's structure are as follows: The pre-requisites are defined in Section 2. We go into the solution's existence and uniqueness in Section 3 of this article. We go through the HAM's stages and how to apply them to the generalised Burgers problem in segment 4. The convergence analysis is covered in portion 5. In part 6, the HAM was illustrated using a simple example. In addition, we examine how this article ends in Section 7.

2. Pre-requisites

This section gives some background information on the Yang-Abdel-Cattani derivative, a recently developed fractional operator and about Laplace transformation. Here is the description of both in more detail [31–33]:

2.1 *Yang-Abdel-Cattani differential operator*

Consider $\phi \in Y(0, \infty)$, then Yang-Abdel-Cattani differential operator of exponent α of ϕ with parameters (α, δ, n), $\alpha \geq 0$, $\delta > 0$ where $n \in I^{+}$, is expressed as:

$$ {}_{0}^{YAC}D_{t}^{\alpha}\phi(t) = \int_{0}^{t} R_{\alpha}\left[-\delta(t-\zeta)^{\alpha}\right]\phi^{(n)}(\zeta)d\zeta; \quad t > 0 \tag{4} $$

here R is a fractional exponent (in the Rabotnov sense). Since the Yang-Abdel-Cattani derivative comprises a non-singular kernel and produces results more quickly than other derivatives, we employed it in this situation.

2.2 *Yang-Abdel-Cattani integral operator*

Yang-Abdel-Cattani integral of $g(t)$ of exponent α is:

$$ {}_{a}^{YAC}I_{0}^{\alpha}g(t) = \int_{0}^{t} \phi_{\alpha}\left[-\delta(t-\tau)^{\alpha}\right]g(\tau)d\tau. \tag{5} $$

2.3 *Laplace transform*

Suppose, Laplace change of function $F(t)$ be denoted by $L\{F(t)\}$ and is expressed as:

$$ L\{F(t)\} = \int_{0}^{\infty} e^{-st}F(t)dt, \quad s > 0 \tag{6} $$

where e^{-st} is kernal of transform and 's' is transform variable and is a complex number. This transformation has the main advantage over alternatives because it breaks down complicated systems into algebraic ones, which are by far the simplest to solve.

2.4 *Laplace change of Yang-Abdel-Cattani differential operator*

Suppose $\phi \in Y^{1,n}(0, \infty) \cap C^{n-1}([0, \infty))$, $n \in N$ then Laplace change of Yang-Abdel-Cattani operator is given as:

$$L\{^{YAC}D_t^{\mu,\,\lambda,\,n}\phi\}(s)=\frac{1}{s^{\mu+1}\left(1+\lambda s^{-(\mu+1)}\right)}\times\left[s^nL\{\phi\}(s)-\sum_{r=1}^{n}s^{n-r}\phi^{(r-1)}(0)\right],\ s>0 \tag{7}$$

3. Logical solution of Burger equation using *q*-HAM

Now, we are going to get solution of Burger equation [34–46] by analytical approach. Here, we are going to discuss the results of a particular case of Burger equation by taking $f(v)=v$. In this case, the burger equation becomes

$$\frac{\partial v}{\partial t}+av\frac{\partial v}{\partial p}=c\frac{\partial^2 v}{\partial p^2} \tag{8}$$

with initial conditions

$$v(p,\,0)=p^2,\ \ p\in\Omega \tag{9}$$

Now, replacing $\frac{\partial v}{\partial t}$ by Yang-Abdel-Cattani derivative, we obtain

$$^{YAC}D_t^{\mu}v(t)+av\frac{\partial v}{\partial p}=c\frac{\partial^2 v}{\partial p^2} \tag{10}$$

Further, taking Laplace both sides in above equation, we have

$$L\left\{^{YAC}D_t^{\mu}v(t)+av\frac{\partial v}{\partial p}\right\}=L\left\{c\frac{\partial^2 v}{\partial p^2}\right\} \tag{11}$$

or,

$$L\{^{YAC}D_t^{\mu}v(t)\}=L\left\{c\frac{\partial^2 v}{\partial p^2}-av\frac{\partial v}{\partial p}\right\} \tag{12}$$

or,

$$\frac{1}{(1+s^{\mu+1})}\cdot[sL\{v(t)\}-v(0)]=L\left\{c\frac{\partial^2 v}{\partial \wp^2}-av\frac{\partial v}{\partial \wp}\right\} \tag{13}$$

or,

$$sL\{v(t)\} - v(0) = \left(1 + s^{\mu+1}\right).L\left\{c\frac{\partial^2 v}{\partial \wp^2} - av\frac{\partial v}{\partial \wp}\right\} \tag{14}$$

or,

$$sL\{v(t)\} = v(0) + \left(1 + s^{\mu+1}\right).L\left\{c\frac{\partial^2 v}{\partial \wp^2} - av\frac{\partial v}{\partial \wp}\right\} \tag{15}$$

or,

$$L\{v(t)\} = \frac{v(0)}{s} + \left(\frac{1}{s} + s^{\mu}\right).L\left\{c\frac{\partial^2 v}{\partial \wp^2} - av\frac{\partial v}{\partial \wp}\right\} \tag{16}$$

Now, we define non-linear operator $\aleph$ like:

$$\begin{aligned} \aleph\left[\psi\left(\wp, t, q\right)\right] = {} & \frac{v(0)}{s} + \left(\frac{1}{s} + s^{\mu}\right).L\left\{c\psi_{pp}\left(\wp, t, q\right) - a\psi\left(\wp, t, q\right)\psi_{p}\left(\wp, t, q\right)\right\} \\ & - L\left\{\psi\left(\wp, t, q\right)\right\} \end{aligned} \tag{17}$$

Now, to solve problem, we are going to use q-homotopy method to get homotopy in following form:

$$(1 - nq)L\left[\psi\left(\wp, t, q\right) - v_0(\wp, t)\right] = hnq\aleph\left[\psi\left(\wp, t, q\right)\right] \tag{18}$$

Putting, $q = 0$ and $q = 1/n$ in above condition (18)

$$\psi\left(\wp, t, 0\right) = v_0(\wp, t) \tag{19}$$

and

$$\psi\left(\wp, t, 1/n\right) = v(\wp, t) \tag{20}$$

Again using Taylor series expansion, we have

$$v\left(\wp, t, q\right) = v_0(\wp, t) + \sum_{m=1}^{\infty} v_m\left(\wp, t\right)q^m \tag{21}$$

here

$$v_m(\wp, t) = \left(\frac{1}{m!} \frac{\partial^m}{\partial q^m} \psi(\wp, t, q) \right)_{q=0} \tag{22}$$

Assume $v_0(\wp, t)$, h, n are chosen such that the series (21) meets at $q = 1/n$, so

$$v(\wp, t, q) = v_0(\wp, t) + \sum_{m=1}^{\infty} v_m(\wp, t)(1/n)^m \tag{23}$$

Now, consider the vectors $\overline{v_r} = \{v_0, v_1, v_2, \ldots v_r\}$ and differentiating the Equation (18) m times w.r.t. q then putting $q = 0$ and fractionating the equation by $m!$, we obtain

$$L[u_m(\wp, t) - x_m u_{m-1}(\wp, t)] = h\Re_m u_{m-1}(\wp, t). \tag{24}$$

here

$$R_m u_{m-1} = L\{v_{m-1}\} - \frac{v(\wp, 0)}{s}\left(1 - \frac{x_m}{n}\right) + \left(\frac{1}{s} + s^{\mu}\right).L\left\{c\frac{\partial^2 v_{m-1}}{\partial \wp^2} - av_{m-1}\frac{\partial v_{m-1}}{\partial \wp}\right\} \tag{25}$$

with

$$x_m = \begin{cases} 0, & m \leq 1 \\ n, & \text{otherwise.} \end{cases} \tag{26}$$

Using reverse Laplace of Equation (24), we get

$$u_m(\wp, t) = x_m u_{m-1}(\wp, t) + hL^{-1}[\Re_m u_{m-1}(\wp, t)]. \tag{27}$$

Now, from the Equation (26), we obtain

$$u(\wp, t) = \lim_{n \to \infty} \sum_{m=0}^{\infty} u_m(\wp, t)(1/n)^m \tag{28}$$

Because of the factor $\left(\frac{1}{n}\right)^m$, above series rapidly converges to exact solution. Applying q-HAM, we get the values of various coefficients and then putting those values in Equation (28), we get our solution of the given Burger equation.

4. Convergence of method

In this segment, we will show the convergence of the q-HAM method.

Theorem 1 Consider $G_m(\wp,\ t,\ h,\ n)$ and $g(\wp,\ t)$ be expressed in Banach space $\mathscr{B}$ then solution in series form generated by q-HAM converges for a prescribed value of h, if

$$\| g_{m+1} \| \leq \frac{\alpha}{n} \| g_m \|, \quad \forall [g_0 \in \mathscr{B}], \tag{29}$$

where $0 < \alpha < n$.

Proof. Let $G_0 = g_0 \in \mathscr{B}$. Define sequence $\{G_m\}_{m=0}^{\infty}$ as below

$$\begin{aligned}
&G_0 = g_0,\\
&G_1 = g_0 + g_1\left(\frac{1}{n}\right),\\
&G_2 = g_0 + g_1\left(\frac{1}{n}\right) + g_2\left(\frac{1}{n}\right)^2,\\
&\vdots\\
&G_m = g_0 + g_1\left(\frac{1}{n}\right) + g_2\left(\frac{1}{n}\right)^2 + \ldots + g_m\left(\frac{1}{n}\right)^m.
\end{aligned} \tag{30}$$

Now, we establish that $\{G_m\}_{m=0}^{\infty}$ is Cauchy sequence in Banach space $\mathscr{B}$. Hence, we have

$$\begin{aligned}
\| G_{m+1} - G_m \| = \| g_{m+1} \| &\leq \frac{\alpha}{n} \| g_m \|\\
&\leq \left(\frac{\alpha}{n}\right)^2 \| g_{m-1} \|\\
&\vdots\\
&\leq \left(\frac{\alpha}{n}\right)^{m+1} \| g_0 \|.
\end{aligned} \tag{31}$$

For each $m,\ r \in N$ with $m > r$, and using triangle inequality, we get

$$\begin{aligned}
\|G_m - G_r\| &= \|(G_m - G_{m-1}) + (G_{m-1} - G_{m-2}) + \ldots + (G_{r+1} - G_r)\| \\
&\leq \|G_m - G_{m-1}\| + \|G_{m-1} - G_{m-2}\| + \ldots + \|G_{r+1} - G_r\| \\
&\leq \left(\frac{\alpha}{n}\right)^m \|g_0\| + \left(\frac{\alpha}{n}\right)^{m-1} \|g_0\| + \ldots + \left(\frac{\alpha}{n}\right)^{r+1} \|g_0\| \\
&\leq \left(\frac{\alpha}{n}\right)^{r+1} \left(\left(\frac{\alpha}{n}\right)^{m-r-1} + \left(\frac{\alpha}{n}\right)^{m-r-2} + \ldots + \left(\frac{\alpha}{n}\right) + 1 \right) \|g_0\| \\
&\leq \left(\frac{\alpha}{n}\right)^{r+1} \left(\frac{1 - \left(\frac{\alpha}{n}\right)^{m-r}}{1 - \left(\frac{\alpha}{n}\right)} \right) \|g_0\|.
\end{aligned} \tag{32}$$

Since $0 < \alpha < n$, we have $1 - \left(\frac{\alpha}{n}\right)^{m-r} < 1$. Then

$$\|G_m - G_r\| \leq \left(\frac{\left(\frac{\alpha}{n}\right)^{r+1}}{1 - \frac{\alpha}{n}} \right) \|g_0\|. \tag{33}$$

Considering that $\| g_0 \| < \infty$, we have

$$\lim_{\wp \to m} \| G_m - G_r \| = 0. \tag{34}$$

So, $\{G_m\}_{m=0}^{\infty}$ is the Cauchy sequence in Banach space $\mathscr{B}$ and we know that each Cauchy sequence converges. Therefore, series solution of the q-HAM is convergent.

Theorem 2 According to q-HAM, the series solution's greatest absolute truncate error is given as

$$\|y(\wp, t) - Y_k\| \leq \left(\frac{\left(\frac{\alpha}{n}\right)^{k+1}}{1 - \left(\frac{\alpha}{n}\right)} \right) \|g_0\|. \tag{35}$$

Proof. It reflects from inequality (32) in Theorem 1. For $m \geq k$, we have

$$\|G_m - G_k\| \leq \left(\frac{\alpha}{n}\right)^{k+1} \left(\frac{1 - \left(\frac{\alpha}{n}\right)^{m-k}}{\left(1 - \frac{\alpha}{n}\right)} \right) \|g_0\|. \tag{36}$$

As $m \to \infty$, $G_m \to g(x, t)$, and $1 - \left(\frac{\alpha}{n}\right)^{m-k} < 1$ (since $0 < \alpha < n$). Thus,

$$\|u(\wp) - Y_k\| \leq \left(\frac{\left(\frac{\alpha}{n}\right)^{k+1}}{\left(1 - \frac{\alpha}{n}\right)} \right) \|g_0\| . \tag{37}$$

Remark 1 Here, in this procedure, we have option to freely choose h and n values that will aid in the convergence of the outcome.

5. Example

Equation (8), which represents this particular case of the generalised Burgers equation, will be employed in this section to solve the generalised Burgers equation using the previously mentioned mathematical strategy of the q-HAM.

We will solve the specific instance of the generalised Burger equation provided by equation (8) and with starting constraint given by (9) in this section.

$$v_0 = u_0 = \wp^2,$$

Now using above relation, we obtain

$$\frac{\partial}{\partial \wp} v_0(\wp, t) = 2\wp$$

also

$$\frac{\partial^2}{\partial \wp^2} v_0(\wp, t) = 2,$$

Using aforementioned values and equations (25), (26) and (27), we get the numerical result of the equation. After using the method mentioned in segment (3), we get following values of the coefficients

$$v_0 = \wp^2 \tag{38}$$

$$v_1 = 2h\left(c - a\wp^3\right)\left(t + \frac{t^{-\mu}}{\Gamma(1-\mu)}\right) \tag{39}$$

similarly, we found the value of v_2, we have

$$
\begin{aligned}
v_2 = {} & 2nh\left(c - a\wp^3\right)\left(t + \frac{t^{-\mu}}{\Gamma(1-\mu)}\right) + 2h^2\left(c - a\wp^3\right)\frac{t^{2-\mu}}{\Gamma(3-\mu)} \\
& - 12ach^2\wp\left(\frac{t^{3-\mu}}{\Gamma(4-\mu)} + \frac{t^{2-2\mu}}{\Gamma(3-2\mu)}\right) + 12a^2h^3\wp^2\left[\frac{t^3}{6} + \frac{\Gamma(1-2\mu)}{(\Gamma(1-\mu))^2}\cdot\frac{t^{1-2\mu}}{\Gamma(2-2\mu)}\right. \\
& \left. + \frac{2\Gamma(2-\mu)}{\Gamma(1-\mu)}\cdot\frac{t^{2-\mu}}{\Gamma(3-\mu)} + \frac{2t^{2-\mu}}{\Gamma(3-\mu)} + \frac{\Gamma(1-2\mu)}{(\Gamma(1-\mu))^2}\cdot\frac{t^{-3\mu}}{\Gamma(1-3\mu)} + \frac{2\Gamma(2-\mu)}{\Gamma(1-\mu)}\cdot\frac{t^{1-2\mu}}{\Gamma(2-2\mu)}\right] \\
& \vdots
\end{aligned}
\tag{40}
$$

Similarly, find the remaining terms, and utilising relation (28), we can quickly determine the nearby answer.

$$
\begin{aligned}
v = {} & \wp^2 + 2h\left(c - a\wp^3\right)\left(t + \frac{t^{-\mu}}{\Gamma(1-\mu)}\right) + 2nh\left(c - a\wp^3\right)\left(t + \frac{t^{-\mu}}{\Gamma(1-\mu)}\right) \\
& + 2h^2\left(c - a\wp^3\right)\frac{t^{2-\mu}}{\Gamma(3-\mu)} - 12ach^2\wp\left(\frac{t^{3-\mu}}{\Gamma(4-\mu)} + \frac{t^{2-2\mu}}{\Gamma(3-2\mu)}\right) \\
& + 12a^2h^3\wp^2\left[\frac{t^3}{6} + \frac{\Gamma(1-2\mu)}{(\Gamma(1-\mu))^2}\cdot\frac{t^{1-2\mu}}{\Gamma(2-2\mu)} + \frac{2\Gamma(2-\mu)}{\Gamma(1-\mu)}\cdot\frac{t^{2-\mu}}{\Gamma(3-\mu)}\right. \\
& \left. + \frac{2t^{2-\mu}}{\Gamma(3-\mu)} + \frac{\Gamma(1-2\mu)}{(\Gamma(1-\mu))^2}\cdot\frac{t^{-3\mu}}{\Gamma(1-3\mu)} + \frac{2\Gamma(2-\mu)}{\Gamma(1-\mu)}\cdot\frac{t^{1-2\mu}}{\Gamma(2-2\mu)}\right] + \ldots
\end{aligned}
\tag{41}
$$

We now pick $a = 0.5$, $n = 1$, $c = 1$ and $h = -0.1$ to plot numerical results for outcomes. The graphs for $\mu = 0.6$, 0.8, and 0.95 have been displayed.

6. Conclusion

In this work, we used Yang-Abdel-Cattani fractional operator to analyze the generalized Burgers equation. We solve several well-known instances using the HAM. We determine the defied problem's outcomes and talk about its outcomes as well. Finally, we exhibit the graph of those cases (Figure 1-4) to demonstrate the effectiveness of the operator.

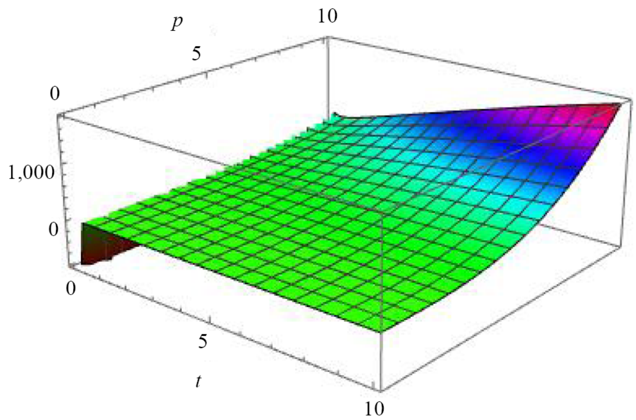

Figure 1. Demonstration of v for $\mu = 0.6$

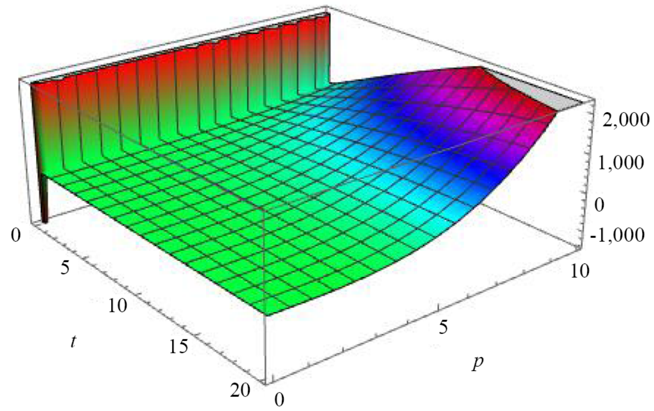

Figure 2. Representation of v for $\mu = 0.8$

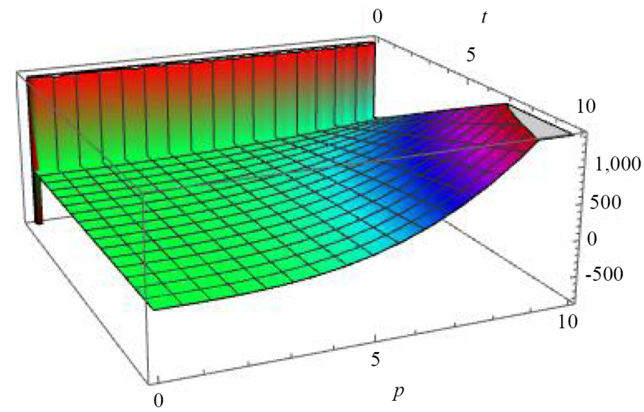

Figure 3. Figure of v for $\mu = 0.95$

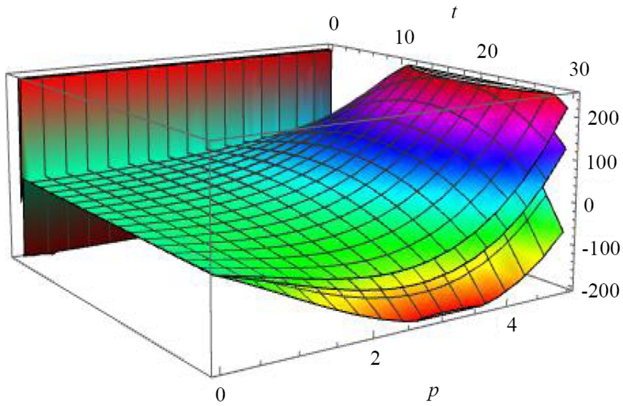

Figure 4. Figure of v for μ= 0.6, 0.8 and 0.95

Disclosure

Availability of statistics is already cited in article.

Conflict of interest

According to researchers, there are no conflicts of interest to disclose about the article that is being presented.

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