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Research Article

Observer-Based Proportional Derivative Control for Takagi-Sugeno Fuzzy Singular Systems with Time Delay

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Abstract: This paper studies an observer-based control problem for Takagi-Sugeno fuzzy singular system with time delay via proportional derivative control scheme. To achieve this, we design a new proportional derivative controller to transform the original problem into the stabilization problem of the new augmented system. In addition, asymmetric Lyapunov-Krasovskii functional and membership-function-dependent method are used to reduce the conservatism. The admissible sufficient conditions of the closed loop system and the calculation methods of observer gain and controller gain are given by using linear matrix inequality. Finally, two examples are given to verify the advantages of the method.

Keywords: Takagi-Sugeno fuzzy systems, observer-based control, asymmetric Lyapunov-Krasovskii functional method, membership-function-dependent approach, proportional derivative control

MSC: 65L05, 34K06, 34K28

1. Introduction

Singular systems originated in the 1960s, and applications of such systems including circuits, economic systems, demography, and robotics, have been extensively and intensively studied since the 1970s [1]. Singular systems often involve algebraic equations and differential equations. When the system has infinite poles, strong impulse behavior occurs, which leads to system anomalies. In [2], the matrix rank condition is defined for singular systems. If the system has a unique solution and no impulse behavior, the singular system is stable. Based on these definitions, control theories such as passive control, time-delay dependent control and robust control have been developed. In time-delay dependent control, Lyapunov-Krasovskii functional (LKF) method is a key tool for analyzing these systems, and treating delay terms with inequalities has also been extensively studied, such as the auxiliary integral inequality [3], Free-weighted matrix inequality [4] and Wirtinger inequality [5]. In the study [6], a new LKF is introduced and an enhanced stability criterion is given using the extended mutually convex matrix inequalities and the auxiliary function integral inequalities. However, in the past studies of time-delay system, matrices in LKF are often required to be positive definite and symmetric. In [7], a new stability condition is given, which does not require that a matrix in LKF must be positive definite, but also needs to be symmetric. Recently, in [8], a new asymmetric LKF method has been developed and the level of conservatism of the results has been reduced. In [9], the asymmetric LKF method is extended to Takagi-Sugeno (T-S) fuzzy systems with time

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delay. Similarly, [10] uses this method to study the admissibility and dissipative properties of singular system with time delay. In addition, the admissibility and stabilization properties of T-S fuzzy singular system with time delay are analyzed using asymmetric LKF methods in [11]. The asymmetric LKF is also used in [12], and the sliding mode control design does not require the control coefficient matrix of all linear subsystems to be the same, which reduces the conservatism of the method.

T-S fuzzy model is mainly used for fuzzy modeling the complex nonlinear systems. By dividing the global nonlinear system into several simple linear relationships and making fuzzy inference and decision on the output of these models, the model can accurately represent the complex nonlinear relationships [13]. Since this model has been rigorously demonstrated to have the ability to approximate nonlinear systems with high precision on a given set [14], it has been extensively studied. The study of T-S fuzzy time-delay systems based on Lyapunov stability theory has made great progress [15, 16]. To solve the problem of membership function handling in T-S fuzzy systems, a new membership function association LKF is proposed in [17–19], which effectively reduces the conservatism. In [20], the Takagi-Sugeno fuzzy modeling method is adopted, two fuzzy rules are used to represent the target system, and it is generalized to a finite number of fuzzy rules.

In the stability analysis of T-S fuzzy singular systems, [21, 22] propose a Proportional Derivative (PD) control method. The application of differential feedback in PD controller can effectively eliminate pulse behavior and reduce complexity. The observer-based PD fuzzy control problem for T-S fuzzy singular systems is studied in [23]. However, it is a pity that the time-delay problem is not discussed in the above research. If you use traditional methods as in [21, 22], the delay term cannot be dealt with. However, the problem of time delay is very common in real life, so how to ensure the stability of the system under the state of time delay is inevitable. In [24], a new idea is proposed to solve the stability problem of uncertain singular systems. By transforming the problem into an augmented uncertain singular system, the sufficient and necessary conditions of linear matrix inequalities are given. Inspired by the same idea, we construct a new PD controller using augmented time-delay systems, and effectively solve the PD control problem of time-delay systems.

Therefore, on the above research basis, we propose an observer-based PD controller design method. Firstly, an augmented singular system with time delay is designed. Then, the sufficient conditions are transformed into Linear Matrix Inequality (LMI) form using the Schur complement and asymmetric LKF functional method. In fuzzy system processing, a new LMI stability condition with time delay is proposed by using switching idea and membership function association method. The main highlights of this paper can be summarized as follows:

(1) Based on the asymmetric LKF method, a new observer-based control design is proposed, relaxing the requirements of positive definiteness and symmetry of the matrices involved in LKF.

(2) A PD controller is designed by using a new augmented singular system with time delay to solve the PD control problem in T-S fuzzy singular system with time delay.

1.1 *Notations*

In the context of this paper, we utilize the notion $\mathbb{R}^n$ to signify the n-dimensional Euclidean space, $\mathbb{R}^{n\times m}$ is the set of $n\times m$ real matrices. The 'T' appended to the matrix A represents its transpose. Within block symmetric matrices, the symbol '$*$' signifies a block matrix that is symmetric to its counterpart. I and 0 are identity and zero matrices of compatible dimensions, respectively. $P<0$ $(P>0)$ means that P is a negative definite (positive definite) matrix.

2. Problem formulation and preliminaries

The section discusses the control problem of nonlinear singular systems with time delay using the T-S fuzzy model. The nonlinear singular system with time delay is decomposed into multiple linear singular subsystems with time delay through fuzzy rules. Consider a nonlinear singular system with time delay, which can be described by the following T-S fuzzy time delay model with r plant rules.

Plant rule i $(i=1,\ 2,\ \ldots,\ r)$: IF $\zeta_1(t)$ is μ_{i1} and $\cdots$ and $\zeta_p(t)$ is μ_{ip}, Then

$$E\dot{x} = A_i x(t) + A_{\tau i} x(t-\tau) + B_i u(t),$$

$$y_p(t) = C_{pi} x(t),$$

$$y_d(t) = C_{di} \dot{x}(t),$$

$$x(t) = \phi(t),\ \forall t \in [-\tau,\ 0],\ i = 1,\ 2,\ \ldots,\ r. \tag{1}$$

In (1), $x(t) \in \mathbb{R}^n$ represents the state vector, while $u(t) \in \mathbb{R}^m$ signifies the control input. $y_p(t) \in \mathbb{R}^p$ is output vector and $y_d(t) \in \mathbb{R}^d$ is the derivative output vector. The system matrices, including $A_i \in \mathbb{R}^{n\times n}$, $A_{\tau i} \in \mathbb{R}^{n\times n}$, $C_{di} \in \mathbb{R}^{d\times n}$, $C_{pi} \in \mathbb{R}^{p\times n}$ and $B_i \in \mathbb{R}^{n\times m}$ are known constants. E is a singular matrix with rank $l \le n$. The fuzzy sets are denoted by μ_{ij}. The premise variables are defined as $\zeta(t) = [\zeta_1(t),\ \zeta_2(t),\ \ldots,\ \zeta_p(t)]$, and the constant time delay is denoted by $\tau > 0$. The initial condition of the system is indicated by $\phi(t)$.

Utilizing fuzzy blending techniques, the comprehensive model from (1) is derived as follows;

$$E\dot{x}(t) = \sum_{i=1}^{r} h_i(\zeta(t)) A_i x(t) + \sum_{i=1}^{r} h_i(\zeta(t)) A_{\tau i} x(t-\tau) + \sum_{i=1}^{r} h_i(\zeta(t)) B_i u(t),$$

$$y_p(t) = \sum_{i=1}^{r} h_i(\zeta(t)) C_{pi} x(t),\ y_d(t) = \sum_{i=1}^{r} h_i(\zeta(t)) C_{di} \dot{x}(t),$$

$$x(t) = \phi(t),\ \ \forall t \in [-\tau,\ 0], \tag{2}$$

where $h_i(\zeta(t))$ is membership function of fuzzy rule i, given by,

$$h_i(\zeta(t)) = \frac{\omega_i(\zeta(t))}{\sum_{i=1}^{r} \omega_i(\zeta(t))},\ \omega_i(\zeta(t)) = \prod_{j=1}^{r} \mu_{ij}(\zeta(t)),$$

and where function $\mu_{ij}(\zeta(t)) \ge 0$ signifies the level of membership that $\zeta_j(t)$ possesses within μ_{ij}. It is evident that $\omega_i(\zeta(t))$ is the product of $\mu_{ij}(\zeta(t))$. It is easy to see that,

$$0 \le h_i(\zeta(t)) \le 1,\ \sum_{i=1}^{r} h_i(\zeta(t)) = 1.$$

For simplicity, we introduce the notation h_i to replace $h_i(\zeta(t))$ and define $X(h) = \sum_{i=1}^{r} h_i(\zeta(t)) X_i$, where X_i are matrices with appropriate dimension.

Next, based on the system described in equation (2), we introduce an augmented system,

$$\bar{E}\dot{z}(t) = \sum_{i=1}^{r} h_i \bar{A}_i z(t) + \sum_{i=1}^{r} h_i \bar{A}_{\tau i} z(t-\tau) + \sum_{i=1}^{r} h_i \bar{B}_i u(t),$$

$$y = \sum_{i=1}^{r} h_i \bar{C}_i z(t), \tag{3}$$

where

$$\bar{E} = \begin{bmatrix} I_n & 0 \\ 0 & 0 \end{bmatrix},\ z(t) = \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix},\ \bar{A}_i = \begin{bmatrix} 0 & I_n \\ A_i & -E \end{bmatrix},\ \bar{A}_{\tau i} = \begin{bmatrix} 0 & 0 \\ A_{\tau i} & 0 \end{bmatrix},\ \bar{B}_i = \begin{bmatrix} 0 \\ B_i \end{bmatrix},\ \bar{C}_i = \begin{bmatrix} C_{pi} & 0 \\ 0 & C_{di} \end{bmatrix}.$$

In [24] it is proved that the Proportional and Derivative State Feedback (PDSF) normalization and stabilization problem of (2) is equivalent to the proportional state feedback stabilization problem of (3).

Based on the Parallel Distributed Compensation (PDC) and PD control scheme, the following PD fuzzy controller is proposed as,

$$u(t) = \sum_{i=1}^{r} h_i \{ \begin{bmatrix} K_{pi} & -K_{di} \end{bmatrix} \hat{z}(t) + \begin{bmatrix} K_{\tau pi} & -K_{\tau di} \end{bmatrix} \hat{z}(t-\tau) \}, \tag{4}$$

where $\hat{z}(t)$ is the state estimation of $z(t)$, K_{pi}, K_{di} are memoryless controller gains and $K_{\tau pi}$, $K_{\tau di}$ are memory controller gains.

For the system (3), an observer-based controller is designed as follows,

$$\bar{E}\dot{\hat{z}}(t) = \sum_{i=1}^{r} h_i(\zeta(t))[\bar{A}_i \hat{z}(t) + \bar{A}_{\tau i}\hat{z}(t-\tau) + \bar{B}_i u(t) + L_i(y(t) - \hat{y}(t))],$$

$$\hat{y} = \sum_{i=1}^{r} h_i(\zeta(t)) \bar{C}_i \hat{z}(t),$$

$$u(t) = \sum_{i=1}^{r} h_i \{ \bar{K}_i \hat{z}(t) + \bar{K}_{\tau i} \hat{z}(t-\tau) \},$$

$$\hat{z}(t) = \xi(t),\ \forall t \in [-\tau,\ 0], \tag{5}$$

where $\hat{y}(t)$ is the output of the observer, L_i is the observer gain matrix, and $\bar{K}_i = \begin{bmatrix} K_{pi} & -K_{di} \end{bmatrix}$, $\bar{K}_{\tau i} = \begin{bmatrix} K_{\tau pi} & -K_{\tau di} \end{bmatrix}$, $\xi(t)$ is the initial condition for the system.

Next, we define the observer error as

$$e(t) = z(t) - \hat{z}(t). \tag{6}$$

Combing (3) and (5), we obtain the closed-loop system as

$$\tilde{E}\dot{\chi}(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j(\zeta(t))[\tilde{A}_{ij}\chi(t)+\tilde{A}_{\tau ij}\chi(t-\tau)], \tag{7}$$

where

$$\tilde{E}=\begin{bmatrix}\bar{E} & 0\\ 0 & \bar{E}\end{bmatrix},\ \tilde{A}_{ij}=\begin{bmatrix}\bar{A}_i+\bar{B}_i\bar{K}_j & L_i\bar{C}_i\\ 0 & \bar{A}_i-L_i\bar{C}_i\end{bmatrix},\ \tilde{A}_{\tau ij}=\begin{bmatrix}\bar{A}_{\tau i}+\bar{B}_i\bar{K}_{\tau j} & 0\\ 0 & \bar{A}_{\tau i}\end{bmatrix},\ \chi(t)=\begin{bmatrix}\hat{z}(t)\\ e(t)\end{bmatrix}.$$

In the context of this study, the system (7) is considered acceptable if it is impulse free, regular and asymptotically stable [25]. Furthermore, as considered in [17–19], we assume the membership function $h_i(\zeta(t))(i=1,\ 2,\ ...r)$ is differentiable. Here are some lemmas that will be used in this paper.

Lemma 1 [26] Given $R>0$, for any continuously differential function $\delta(\xi)$, the following inequality is feasible in $[\zeta_1,\ \zeta_2]\rightarrow\mathbb{R}^n$,

$$(\zeta_2-\zeta_1)\int_{\zeta_1}^{\zeta_2}\delta^T(\xi)R\delta(\xi)d\xi\geq\left(\int_{\zeta_1}^{\zeta_2}\delta^T(\xi)d\xi\right)R\left(\int_{\zeta_1}^{\zeta_2}\delta(\xi)d\xi\right).$$

Lemma 2 [5] Given $R>0$, for any continuously differential function $\delta(\xi)$, the following inequality is feasible in $[\zeta_1,\ \zeta_2]\rightarrow\mathbb{R}^n$,

$$\int_{\zeta_1}^{\zeta_2}\delta^T(\xi)R\delta(\xi)d\xi\geq\frac{1}{(\zeta_2-\zeta_1)}\left(\int_{\zeta_1}^{\zeta_2}\delta^T(\xi)d\xi\right)R\left(\int_{\zeta_1}^{\zeta_2}\delta(\xi)d\xi\right)+\frac{3}{(\zeta_2-\zeta_1)}\Phi^TR\Phi,$$

where $\Phi=\int_{\zeta_1}^{\zeta_2}\delta(\xi)d\xi-\frac{2}{(\zeta_2-\zeta_1)}\int_{\zeta_1}^{\zeta_2}\int_{\zeta_1}^{\theta}\delta(\xi)d\xi d\theta.$

Lemma 3 [27] For the singular system $E\dot{x}(t)=Ax(t)$, the pair$(E,\ A)$ is impulse free and regular and asymptotically stable if there exists a matrix P such that

$$EP=P^TE\geq 0,$$

$$AP^T+PA^T<0.$$

Lemma 4 [28] For full rank matrix $C\in\mathbb{R}^{m\times n}$ with $rank(C)=m$, suppose that the Singular Value Decomposition (SVD) for C is described as $C=O\begin{bmatrix}S & 0\end{bmatrix}V^T$, with O and V being orthogonal matrices, and satisfying $O\cdot O^T=I$ and $V\cdot V^T=I$, and S being a diagonal matrix. Assume $X>0$.

Then, there exists nonsingular matrix $\hat{X}$ such that $CX=\hat{X}C$ if and only if the following condition holds:

$$X=V\begin{bmatrix}X_{11} & 0\\ 0 & X_{22}\end{bmatrix}V^T,$$

where $0 < X_{11} \in \mathbb{R}^{m\times m}$, $0 < X_{22} \in \mathbb{R}^{(n-m)\times(n-m)}$. In this case, $\hat{X}$ can be chosen as

$$\hat{X} = OSX_{11}S^{-1}O^{-1}.$$

3. Main results

In this section, we study the observer-based PD control for T-S fuzzy singular systems with time delay, and propose a new theorem.

Theorem 1 Given $\tau > 0$, system (7) is admissible if there exist matrices $Q_i = \begin{bmatrix} Q_{1i} & Q_{2i} \end{bmatrix}$ with $0 < Q_{1i} \in R^{4n\times 4n}$, $Q_{2i} \in R^{4n\times 4n}$, $0 < M_{1i} \in R^{4n\times 4n}$, $0 < M_{2i} \in R^{4n\times 4n}$, $W_{j\rho} \in R^{m\times n}(\rho = 1,\ 2,\ 3,\ 4)$, $G_{i\rho} \in R^{n\times p}(\rho = 1,\ 3)$, $G_{i\rho} \in R^{n\times d}(\rho = 2,\ 4)$ and $P = \begin{bmatrix} X & 0 \\ 0 & X \end{bmatrix}$, $X = \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix} > 0$, such that for $i,\ j = 1,\ 2,\ \ldots,\ r$, $i \le j$,

$$\tilde{E}P = P^T\tilde{E} \ge 0, \tag{8}$$

$$\Phi_i = \begin{bmatrix} Q_{1i} & \frac{1}{2}Q_{2i} \\ * & M_{2i} \end{bmatrix} > 0, \tag{9}$$

$$\dot{M}_1(h) \le 0,\ \dot{M}_2(h) \le 0,\ \dot{\Phi}(h) \le 0, \tag{10}$$

$$\Omega_{ij} + \Omega_{ji} < 0, \tag{11}$$

where

$$\Omega_{ij} = \begin{bmatrix} \Omega_{11ij} & \Omega_{12ij} & \Omega_{13i} & \Omega_{14ij} \\ * & \Omega_{22i} & \Omega_{23i} & \Omega_{24ij} \\ * & * & \Omega_{33i} & 0 \\ * & * & * & \Omega_{44i} \end{bmatrix}, \tag{12}$$

$$\Omega_{11ij} = \Lambda_{ij} + \Lambda_{ij}^T + Q_{1i} - 4\bar{E}^T M_{1i}\bar{E} + \tau^2 M_{2i},$$

$$\Omega_{12ij} = \Lambda_{ij} - 2\bar{E}^T M_{1i}\bar{E},$$

$$\Omega_{13i} = \frac{1}{2}Q_{2i}^T + \frac{6}{\tau}\bar{E}M_{1i}\bar{E}^T,$$

$$\Omega_{14ij} = \Lambda_{ij}^T,$$

$$\Omega_{22i} = -Q_{1i} - 4\bar{E}M_{1i}\bar{E}^T,$$

$$\Omega_{23i} = -\frac{1}{2}Q_{2i} + \frac{6}{\tau}\bar{E}M_{1i}\bar{E}^T,$$

$$\Omega_{24ij} = \Lambda_{\tau ij}^T,$$

$$\Omega_{33i} = -\frac{12}{\tau^2}\bar{E}M_{1i}\bar{E}^T - M_{2i},$$

$$\Omega_{44i} = -2\tau^{-2}P^T + \tau^{-2}M_{1i},$$

$$\Lambda_{ij} = \begin{bmatrix} 0 & X_2 & G_{i1}C_{pi} & G_{i2}C_{di} \\ A_iX_1 + B_iW_{j1} & -EX_2 - B_iW_{j2} & G_{i3}C_{pi} & G_{i4}C_{di} \\ 0 & 0 & -G_{i1}C_{pi} & X_2 - G_{i2}C_{di} \\ 0 & 0 & AX_1 - G_{i3}C_{pi} & -EX_2 - G_{i4}C_{di} \end{bmatrix},$$

$$\Lambda_{\tau ij} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ A_{\tau ij}X_1 + B_iW_{j3} & -B_iW_{j4} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & A_{\tau ij}X_1 & 0 \end{bmatrix}.$$

In this case, the controller gains and the observer gains are derived as

$$K_{pi} = W_{i1}X_1^{-1},\ K_{di} = W_{i2}X_2^{-1},\ K_{\tau pi} = W_{i3}X_1^{-1},\ K_{\tau di} = W_{i4}x_2^{-1},$$

$$L_i = \begin{bmatrix} G_{i1}OSX_1^{-1}S^{-1}O^{-1} & G_{i2}OSX_2^{-1}S^{-1}O^{-1} \\ G_{i3}OSX_1^{-1}S^{-1}O^{-1} & G_{i4}OSX_2^{-1}S^{-1}O^{-1} \end{bmatrix},$$

where O and S are given in Lemma 4 and $i = 1, 2, \ldots, r$.

Proof. Given the positive definite properties of P and M_{1i}, we have

$$(P - M_{1i})M_{1i}^{-1}(P^T - M_{1i}) \geq 0.$$

So we get

$$-\tau^{-2}PM_{1i}^{-1}P^T \leq -2\tau^{-2}P^T + \tau^{-2}M_{1i},$$

and we have

$$\widehat{\Omega}_{ij} = \begin{bmatrix} \Omega_{11ij} & \Omega_{12ij} & \Omega_{13i} & \Omega_{14ij} \\ * & \Omega_{22i} & \Omega_{23i} & \Omega_{24ij} \\ * & * & \Omega_{33i} & 0 \\ * & * & * & -\tau^{-2}PM_{1i}^{-1}P^T \end{bmatrix} < \Omega_{ij}.$$

For $X_1 = V \begin{bmatrix} X_{11} & 0 \\ 0 & X_{22} \end{bmatrix} V^T$, according to Lemma 4, there exists $\hat{X}_1 = OSX_{11}S^{-1}O^{-1}$ satisfying $C_{pi}X_1 = \hat{X}_1 C_{pi}$, and we can use the same method to get $C_{di}X_2 = \hat{X}_2 C_{di}$. Letting $\check{P} = P^{-1}$ and $\check{M}_{1i} = P^{-T}M_{1i}P^{-1}$, and multiplying $\widehat{\Omega}_{ij}$ from the left and the right by $diag\begin{bmatrix} \check{P}^T & \check{P}^T & \check{P}^T & \check{M}_{1i}^T \end{bmatrix}$ and its transpose, respectively, it yields

$$\check{\Omega}_{ij} = \begin{bmatrix} \check{\Omega}_{11ij} & \check{\Omega}_{12ij} & \check{\Omega}_{13i} & A_{ij}^T \check{M}_{1i} \\ * & \check{\Omega}_{22i} & \check{\Omega}_{23i} & A_{\tau ij}^T \check{M}_{1i} \\ * & * & \check{\Omega}_{33i} & 0 \\ * & * & * & -\tau^{-2}\check{M}_{1i} \end{bmatrix}, \tag{13}$$

where

$$\check{\Omega}_{11ij} = \check{P}^T A_{ij} + A_{ij}^T \check{P} + \check{Q}_{1i} - 4E^T \check{M}_{1i} E + \tau^2 \check{M}_{2i},$$

$$\check{\Omega}_{12ij} = \check{P}^T A_{\tau ij} - 2E^T \check{M}_{1i} E,$$

$$\check{\Omega}_{13i} = \frac{1}{2}\check{Q}_{2i}^T + 6\tau^{-1} E \check{M}_{1i} E^T,$$

$$\check{\Omega}_{22i} = -\check{Q}_{1i} - 4E^T \check{M}_{1i} E,$$

$$\check{\Omega}_{23i} = -\frac{1}{2}\check{Q}_{2i} + 6\tau^{-1} E^T \check{M}_{1i} E,$$

$$\check{\Omega}_{33i} = -12\tau^{-2} E^T \check{M}_{1i} E - \check{M}_{2i}.$$

We get $\check{\Omega}_{ji}$ in the same way. According to the condition (11), we get $\check{\Omega}_{ij} + \check{\Omega}_{ji} < 0$.

Now, we choose LKF candidate as follows

$$V_t = V_{t1} + V_{t2} + V_{t3}, \tag{14}$$

where

$$V_{t1} = \chi(t) \tilde{E} \check{P} \chi(t),$$

$$V_{t2}=\int_{t-\tau}^{t}\chi^T(\theta)\check{Q}(h)\left[\chi^T(\theta)\quad \textstyle\int_\theta^t\chi^T(\xi)d\xi\right]^T d\theta,$$

$$V_{t3}=\tau\int_{t-\tau}^{t}\int_{\theta}^{t}\left[\dot{\chi}^T(\xi)\tilde{E}^T\check{M}_1(h)\tilde{E}\dot{\chi}(\xi)+\chi^T(\xi)\check{M}_2(h)\chi(\xi)\right]d\xi d\theta.$$

Through $M_{2i}>0$, we have $\check{M}_2(h)>0$. According to Lemma 1, we have

$$\tau\int_{\theta}^{t}\chi^T(\xi)\check{M}_2(h)\chi(\xi)d\xi\geq\int_{\theta}^{t}\chi^T(\xi)d\xi\check{M}_2(h)\int_{\theta}^{t}\chi(\xi)d\eta. \tag{15}$$

Based on (15), we deduce

$$\int_{t-\tau}^{t}\chi^T(\theta)\check{Q}(h)\left[\chi^T(\theta)\quad \textstyle\int_\theta^t\chi^T(\xi)d\xi\right]^T d\theta+\tau\int_{t-\tau}^{t}\int_{\theta}^{t}\left[\chi^T(\xi)\check{M}_2(h)\chi(\xi)\right]d\xi d\theta$$

$$\geq\int_{t-\tau}^{t}\begin{bmatrix}\chi(\theta)\\ \int_\theta^t\chi(\xi)d\xi\end{bmatrix}^T\check{\Phi}(h)\begin{bmatrix}\chi(\theta)\\ \int_\theta^t\chi(\xi)d\xi\end{bmatrix}d\theta,$$

where

$$\check{\Phi}(h)=\begin{bmatrix}\check{Q}_1(h) & \frac{1}{2}\check{Q}_2(h)\\ * & \check{M}_2(h)\end{bmatrix}>0.$$

Since $\Phi_i>0$ in (9), it follows that $\check{\Phi}(h)>0$. Furthermore, considering the fact that $M_{1i}>0$, we can conclude that $\check{M}_1(h)>0$, which implies $V_t>0$.

Taking the time-derivative of V_t, we have

$$\begin{aligned}\dot{V}_t=&\,2\dot{\chi}^T(t)\tilde{E}^T\check{P}\chi(t)+\chi^T(t)\check{Q}_1(h)\chi(t)+\int_{t-\tau}^{t}\chi^T(\xi)d\xi\check{Q}_2(h)\chi(t)\\ &-\chi^T(t-\tau)\check{Q}(h)\left[\chi^T(t-\tau)\quad \textstyle\int_{t-\tau}^t\chi^T(\xi)d\xi\right]^T+\int_{t-\tau}^{t}\chi^T(\theta)\dot{\check{Q}}(h)\left[\chi^T(\theta)\quad \textstyle\int_\theta^t\chi^T(\xi)d\xi\right]^T d\theta\\ &+\tau^2\chi^T(t)\check{M}_2(h)\chi(t)+\tau^2\dot{\chi}^T(t)\bar{E}^T\check{M}_1(h)\tilde{E}\dot{\chi}(t)\\ &-\tau\int_{t-\tau}^{t}\left[\dot{\chi}^T(\xi)\tilde{E}^T\check{M}_1(h)\tilde{E}\dot{\chi}(\xi)+\chi^T(\xi)\check{M}_2(h)\chi(\xi)\right]d\xi\end{aligned}$$

$$+\tau\int_{t-\tau}^{t}\int_{\theta}^{t}\left[\dot{\chi}^T(\xi)\tilde{E}\dot{\check{M}}_1(h)\bar{E}\dot{\chi}(\xi)+\chi(\xi)^T\dot{\check{M}}_2(h)\chi(\xi)\right]d\xi d\theta.$$

Considering the constraints in (10), we have

$$\dot{V}_t\leq 2\dot{\chi}^T(t)\tilde{E}^T\check{P}(h)\chi(t)-\chi^T(t-\tau)\check{Q}(h)\left[\chi^T(t-\tau)\quad \int_{t-\tau}^{t}\chi^T(\xi)d\xi\right]^T+\chi^T(t)\check{Q}_1(h)\chi(t)$$

$$+\int_{t-\tau}^{t}\chi^T(\xi)d\xi\check{Q}_2(h)\chi(t)+\tau^2\dot{\chi}^T(t)\tilde{E}^T\check{M}_1(h)\tilde{E}\dot{\chi}(t)+\tau^2\chi^T(t)\check{M}_2(h)\chi(t)$$

$$-\tau\int_{t-\tau}^{t}\left[\dot{\chi}^T(\xi)\tilde{E}^T\check{M}_1(h)\tilde{E}\dot{\chi}(\xi)+\chi^T(\xi)\check{M}_2(h)\chi(\xi)\right]d\xi.$$

By leveraging Lemma 1 and Lemma 2, we establish upper bounds for $-\tau\int_{t-\tau}^{t}\chi^T(\xi)\check{M}_2(h)\chi(\xi)d\xi$ and $-\tau\int_{t-\tau}^{t}\dot{\chi}^T(\xi)\tilde{E}^T\check{M}_1(h)\tilde{E}\dot{\chi}(\xi)d\xi$ in $\dot{V}_t$. This leads to the following inequality

$$\dot{V}_t\leq\vartheta^T\left(\Xi(h)+\tau^2\Psi_i^T\tilde{M}_{1i}^{-1}\Psi_i\right)\vartheta, \tag{16}$$

where,

$$\vartheta^T=\left[\chi^T(t)\quad \chi^T(t-\tau)\quad \int_{t-\tau}^{t}\chi^T(\xi)d\xi\right],$$

$$\Xi_{ij}=\begin{bmatrix}\check{\Omega}_{11ij} & \check{\Omega}_{12ij} & \check{\Omega}_{13i}\\ * & \check{\Omega}_{22i} & \check{\Omega}_{23i}\\ * & * & \check{\Omega}_{33i}\end{bmatrix},$$

$$\Psi=\left[\tilde{M}_{1i}\bar{A}_{ij}\quad \tilde{M}_{1i}\bar{A}_{\tau ij}\quad 0\right].$$

According to the inequality (11) and the Schur complement lemma, for $i=1,\ 2,\ \dots,\ r$, it follows that $\Xi_{ij}+\Xi_{ji}+2\tau^2\Psi_i^T\tilde{Z}_{1i}^{-1}\Psi_i<0$, so we have $\dot{V}_t\leq 0$. Thus, the closed-loop system (7) is admissible.

Remark 1 In Theorem 1, $\dot{M}_1(h)\leq 0$, $\dot{M}_2(h)\leq 0$, $\dot{\Phi}(h)\leq 0$ in (10) can be achieved when

$$\begin{cases}\dot{h}_u\leq 0:\ \Phi_u-\Phi_r\geq 0,\ M_{1u}-Z_{1r}\geq 0,\ M_{2u}-Z_{2r}\geq 0;\\ \dot{h}_u>0:\ \Phi_u-\Phi_r<0,\ M_{1u}-M_{1r}<0,\ M_{2u}-M_{2r}<0,\end{cases} \tag{17}$$

where $u=1,\ 2,\ \dots,\ r-1$. There are in total 2^{r-1} situations ($S_l,\ l=1,\ 2,\ \dots,\ 2^{r-1}$) ensuring $\dot{\Phi}(h)\leq 0$, $\dot{M}_1(h)\leq 0$ and $\dot{M}_2(h)\leq 0$ simultaneously.

Remark 2 In the study of observer-based control problems, previous studies often require LKF to be symmetric, as documented in [29–32]. In Theorem 1, the asymmetric LKF method is first applied to observer-based control problem for T-S fuzzy singular system with time delay. By making $Q_{2i} = 0$ and $Q_2(h) = 0$ in Theorem 1, LKF can be reduced to be symmetry and the method proposed in this section will be degraded to the method in [31]. Obviously, asymmetric LKF reduces conservatism by removing the constraint symmetric matrix. In this paper, we use relatively simple LKF, but the results are still better than results in [29] and especially Lemma 5 of [31].

Remark 3 Although the observer-based fuzzy control problem discussed by PD control scheme is widely studied, it is difficult for traditional methods to solve the PD control problem of time-delay systems. Current research lacks PD control-related time delay problems, as in [23]. Therefore we introduce a new augmented system with time delay, transform the problem into the corresponding stabilization problem of an augmented system with time delay, and give the sufficient and necessary conditions of the problem in the form of linear matrix inequalities.

Remark 4 We analyze the computational complexity by Theorem 1, where there are 14 unknown matrices. The number of unknowns is calculated as $40n^2 + 6n + mn + np + nd$. Obviously, after the use of asymmetric LKF, the complexity of the calculation is increased, but the conservatism is reduced.

4. Numerical examples

In the section, two examples are given to verify the reduction of conservativeness of our methods numerically.

Example 1 Consider a nonlinear single-species bio-economic system without disturbance, which is borrowed from [33]. Nonlinear single-species bio-economic system refers to the resource utilization and economic activities of a single species (such as a certain fish, forest or economic animal) in an ecological economic system, and these activities show nonlinear characteristics.

$$\dot{x}_1(t) = \left(-\frac{\alpha\beta}{r_2} - \frac{\eta c}{p}\right)x_1(t) + \alpha x_2(t) - \frac{c}{p}x_3(t) - \eta x_1^2(t)$$

$$-x_1(t)x_3(t) + d_{11}x_1(t-\tau)$$

$$\dot{x}_2(t) = \beta x_1(t) - r_2 x_2(t)$$

$$0 = p\left(\frac{\alpha\beta}{r_2} - r_1 - \beta - \frac{\eta c}{p}\right)x_1(t) + px_1(t)x_3(t) + u(t)$$

where $x_1(t) \in [-l,\ l]$, $(l > 0)$, $\beta = 0.5$, $\alpha = 0.15$, $\eta = 0.01$, $r_1 = 0.2$, $p = 1$, $c = 40$, $r_2 = 0.1$, $l = 10$, $d_{11} = -0.1$. The following is a fuzzy model of a nonlinear single-species bioeconomic system, which is established based on the fuzzy blending method.

$$E\dot{x}(t) = A_i x(t) + A_{\tau i} x(t-\tau) + B_i u(t),$$

where

$$E=\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix},\ A_1=\begin{bmatrix}-\frac{\alpha\beta}{r_2}-\frac{\eta c}{p}+\eta l & \alpha & -\frac{c}{p}+l\\ \beta & -r_2 & 0\\ p(\frac{\alpha\beta}{r_2}-r_1-\beta-\frac{\eta c}{p}) & 0 & -pl\end{bmatrix},$$

$$A_2=\begin{bmatrix}-\frac{\alpha\beta}{r_2}-\frac{\eta c}{p}+\eta l & \alpha & -\frac{c}{p}+l\\ \beta & -r_2 & 0\\ p(\frac{\alpha\beta}{r_2}-r_1-\beta-\frac{\eta c}{p}) & 0 & pl\end{bmatrix},\ A_{d1}=\begin{bmatrix}d_{11}&0&0\\0&0&0\\0&0&0\end{bmatrix},$$

$$A_{d2}=\begin{bmatrix}d_{11}&0&0\\0&0&0\\0&0&0\end{bmatrix},\ B_1=B_2=\begin{bmatrix}0.6\\1\\0\end{bmatrix},$$

$$C_{d1}=C_{d2}=\begin{bmatrix}-0.5&1&-1\\0&1&0\end{bmatrix},\ C_{p1}=C_{p2}=\begin{bmatrix}-0.5&1&-1\\0&1&0\end{bmatrix}.$$

The membership function has the following form

$$h_1(x(t))=\frac{1}{2}(1-x_1(t)/l),\ h_2(x(t))=1-h_1(x(t)).$$

With theorem 1, by using the LMI tool in Matlab, we can calculate that under $S_1:\{\dot{h}_1\leq 0:\ P_1\geq P_2,\ M_{11}\geq M_{12},\ M_{21}\geq M_{22},\ \Theta_1\geq\Theta_2\}$, the MAD (maximum allowable time delay) τ_1 is 0.4877, and under $S_2:\{\dot{h}_1>0:\ P_1<P_2,\ M_{11}<M_{12},\ M_{21}<M_{22},\ \Theta_1<\Theta_2\}$, the MAD τ_2 is 0.3829. Assuming that both S_1 and S_2 of the system are possible, the MAD τ_M obtained from Theorem 1 is $\tau_M=min\{\tau_1,\ \tau_2\}=0.3829$. We choose the initial condition $x(t)=[0.3,\ 0.2,\ 0.1]$ with $t\in[-\tau,\ 0]$ and $e(t)=[0.5,\ 0.2,\ 0.1]$. When $\tau=0.2$, the trajectory of state variables of the closed-loop system is shown in Figure 1 and the error is shown in Figure 2.

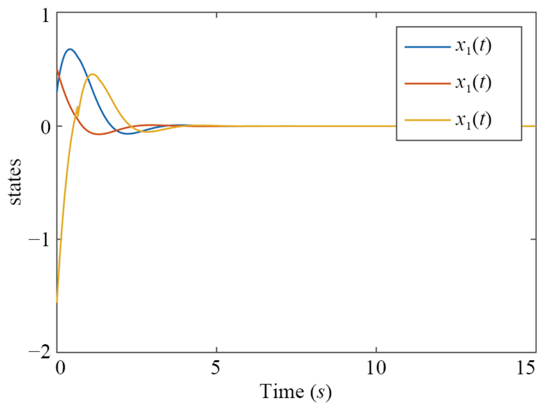

Figure 1. Evolution of the $x(t)^T=[x_1^T,\ x_2^T,\ x_3^T]$

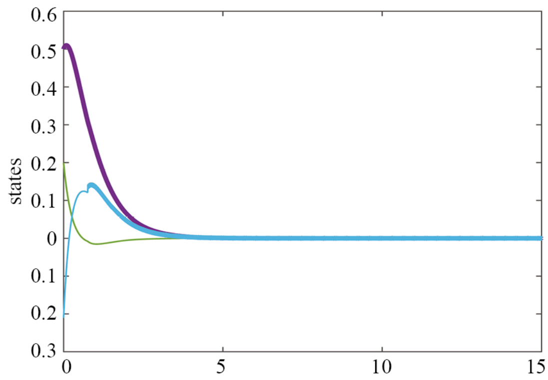

Figure 2. Evolution of the $e(t)^T$

Table 1. Maximum τ_M obtained through various methods (Example 2)

Approaches	τ_M
[23]	null
Theorem 1	0.5846
Theorem 1 ($Q_2 = 0$)	0.5479

Example 2 Consider singular T-S fuzzy system with time delay (2) with

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},\ A_1 = \begin{bmatrix} 1.5 & 0.2 \\ 0 & -1.1 \end{bmatrix},\ A_2 = \begin{bmatrix} 1.6 & 0.3 \\ 0 & -1.2 \end{bmatrix},\ A_{\tau 1} = \begin{bmatrix} -2 & 0.2 \\ 0.6 & 0.5 \end{bmatrix},\ A_{\tau 2} = \begin{bmatrix} -2.2 & 0.1 \\ 0.5 & 0.6 \end{bmatrix},$$

$$B_1 = B_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix},\ C_{d1} = C_{d2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ C_{p1} = C_{p2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

We let

$$h_1 = \frac{1}{1+e^{-2x_1}},\ h_2 = \frac{e^{-2x_2}}{1+e^{-2x_2}}.$$

With Theorem 1, by using the LMI tool in Matlab, we can calculate that under $S_1 : \{\dot{h}_1 \leq 0 :\ P_1 \geq P_2,\ M_{11} \geq M_{12},\ Z_{21} \geq M_{22},\ \Theta_1 \geq \Theta_2\}$, the MAD τ_1 is 0.5479, and under $S_2 : \{\dot{h}_1 > 0 :\ P_1 < P_2,\ M_{11} < M_{12},\ M_{21} < M_{22},\ \Theta_1 < \Theta_2\}$, the MAD τ_2 is 0.5846. Assuming that both S_1 and S_2 of the system are possible, the MAD τ_M obtained from Theorem 1 is $\tau_M = min\{\tau_1,\ \tau_2\} = 0.5479$. In order to further demonstrate the role of asymmetric terms in reducing the conservatism of the results, Theorem 1 for $Q_2 = 0$ is also verified, and the MAD is 0.4452. If you use the treatment in [23], it is obvious that it cannot deal with time-delay systems.

5. Conclusions

The observer-based PD control problem of T-S fuzzy singular systems with time delay is studied in this paper. Firstly, a new asymmetric LKF is designed to obtain the observer-based control design method. Moreover, we design a PD controller by using a new augmented singular system with time delay, which solve the PD control problem in singular systems with time delay. Meanwhile, the information of the membership function is used to design LKF, which reduces the conservatism of the result. Finally, two examples are used to illustrate the effectiveness of the new method. The research results of this paper have great significance for PD control of time-delay systems, but there are some limitations in the present results, because the P matrix in the LKF functional has certain limitations.

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Conflict of interest

The authors declare no competing financial interest.

References

[1] Rosenbrock HH. Structural properties of linear dynamical systems. *International Journal of Control*. 1974; 20(2): 191-202.

[2] Dai L. *Singular Control Systems*. Heidelberg: Springer; 1989.

[3] Park PG, Lee WI, Lee SY. Auxiliary function-based integral inequalities for quadratic functions and their applications to time-delay systems. *Journal of the Franklin Institute*. 2015; 352(4): 1378-1396.

[4] Zeng HB, He Y, Wu M, She J. Free-matrix-based integral inequality for stability analysis of systems with time-varying delay. *IEEE Transactions on Automatic Control*. 2015; 60(10): 2768-2772.

[5] Seuret A, Gouaisbaut F. Wirtinger-based integral inequality: Application to time-delay systems. *Automatica*. 2013; 49(9): 2860-2866.

[6] Lian Z, He Y, Wu M. Stability and stabilization for delayed fuzzy systems via reciprocally convex matrix inequality. *Fuzzy Sets and Systems*. 2020; 402(10): 124-141.

[7] Xu S, Lam J, Zhang B, Zou Y. New insight into delay-dependent stability of time-delay systems. *International Journal of Robust and Nonlinear Control*. 2015; 25(7): 961-970.

[8] Sheng Z, Lin C, Chen B, Wang Q. Asymmetric Lyapunov-Krasovskii functional method on stability of time-delay systems. *International Journal of Robust and Nonlinear Control*. 2021; 31(7): 2847-2854.

[9] Sheng Z, Lin C, Chen B, Wang QG. An asymmetric Lyapunov-Krasovskii functional method on stability and stabilization for T-S fuzzy systems with time delay. *IEEE Transactions on Fuzzy Systems*. 2022; 30(6): 2135-2140.

[10] Fu X, Sheng Z, Lin C, Chen B. New results on admissibility and dissipativity analysis of descriptor time-delay systems. *Applied Mathematics and Computation*. 2022; 419: 126860. Available from: https://doi.org/10.1016/j.amc.2021.126860.

[11] Wang H, Sheng Z, Lin C, Chen B. Asymmetric Lyapunov-Krasovskii functional method for admissibility analysis and stabilization of T-S fuzzy singular systems with time delay. *International Journal of Systems Science*. 2022; 53(14): 2998-3009.

[12] Kavikumar R, Kaviarasan B, Lee YG, Kwon OM, Sakthivel R, Choi SG. Robust dynamic sliding mode control design for interval type-2 fuzzy systems. *Discrete and Continuous Dynamical Systems-S*. 2022; 15(7): 1839-1858.

[13] Takagi T, Sugeno M. Fuzzy identification of systems and its application to modelling and control. *IEEE Transactions on Systems, Man, and Cybernetics*. 1985; SMC-15(1): 116-132.

[14] Zhang QL, Zhu BY. *Analysis and Control for T-S Fuzzy Descriptor Systems*. Bei Jing: National Defence Industry Press; 2011.

[15] Feng Z, Zheng WX. Improved stability condition for Takagi-Sugeno fuzzy systems with time-varying delay. *IEEE Trans Cybern*. 2017; 47(3): 661-670.

[16] Li ZC, Yan HC, Zhang H, Peng Y, Park JH, He Y. Stability analysis of linear systems with time-varying delay via intermediate polynomialbased functions. *Automatica*. 2020; 113(1): 108756.

[17] Wang L, Lam HK. A new approach to stability and stabilization analysis for continuous-time Takagi-Sugeno fuzzy systems with time delay. *IEEE Transactions on Fuzzy Systems*. 2017; 26(4): 2460-2465.

[18] Wang L, Liu J, Lam HK. Further study on stabilization for continuous-time Takagi-Sugeno fuzzy systems with time delay. *IEEE Transactions on Cybernetics*. 2021; 51(11): 5637-5643.

[19] Wang L, Lam HK. New stability criterion for continuous-time Takagi-Sugeno fuzzy systems with time-varying delay. *IEEE Transactions on Cybernetics*. 2019; 49(4): 1551-1556.

[20] Kavikumar R, Kaviarasan B, Kwon OM, Sakthivel R. Event-triggered finite-time admissibilization of bio-economic singular semi-Markovian jump fuzzy systems. *Journal of the Franklin Institute*. 2023; 360(16): 12055-12075.

[21] Mu Y, Zhang H, Sun S, Ren J. Robust non-fragile proportional plus derivative state feedback control for a class of uncertain Takagi-Sugeno fuzzy singular systems. *Journal of the Franklin Institute*. 2019; 356(12): 6208-6225.

[22] Chang WJ, Lian KY, Su CL, Tsai MH. Multi-constrained fuzzy control for perturbed T-S fuzzy singular systems by proportional-plus-derivative state feedback method. *Journal of the Franklin Institute*. 2021; 23(2): 1972-1985.

[23] Ku CC, Chang WJ, Huang YM. Robust observer-based fuzzy control via proportional derivative feedback method for singular Takagi-Sugeno fuzzy systems. *International Journal of Fuzzy Systems*. 2022; 24(11): 3349-3365.

[24] Lin C, Wang QG, Lee TH. Robust normalization and stabilization of uncertain descriptor systems with norm-bounded perturbations. *IEEE Transactions on Automatic Control*. 2005; 50(4): 515-520.

[25] Xu S, Lam J. *Robust Control and Filtering of Singular Systems*. Heidelberg: Springer; 2006.

[26] Gu K, Kharitonov VL, Chen J. *Stability of Time-Delay Systems*. Heidelberg: Springer; 2003.

[27] Masubuchi I, Kamitane Y, Ohara A, Suda N. H∞ control for descriptor systems: A matrix inequalities approach. *Automatica*. 1997; 33(4): 669-673.

[28] Peng C, Ma S, Xie X. Observer-based non-pdc control for networked T-S fuzzy systems with an event-triggered communication. *IEEE Transactions on Cybernetics*. 2017: 47(8): 2279-2287.

[29] Ullah R, Li Y, Aslam MS, Sheng A. Event-triggered dissipative observer-based control for delay dependent T-S fuzzy singular systems. *IEEE Access*. 2020; 8: 134276-134289. Available from: https://doi.org/10.1109/ACCESS.2020.3011281.

[30] Shi R, Shi G, Cui Y. Observer-based control for uncertain T-S fuzzy systems with process disturbances and time-delays. *International Journal of Systems Science*. 2020; 51(16): 3213-3224.

[31] Zhang Q, Li R, Ren J. Robust adaptive sliding mode observer design for T-S Fuzzy descriptor systems with time-varying delay. *IEEE Access*. 2018; 6: 46002-46018. Available from: https://doi.org/10.1109/ACCESS.2018.2865618.

[32] Chang WJ, Tsai MH, Pen CL. Observer based fuzzy controller design for nonlinear discrete time singular systems through proportional derivative feedback scheme. *Applied Sciences*. 2021; 11(6): 2833.

[33] Feng Z, Zhang H, Lam HK. New results on dissipative control for a class of singular Takagi-Sugeno fuzzy systems with time delay. *IEEE Transactions on Fuzzy Systems*. 2022; 30(7): 2466-2475.