Contemporary Mathematics
https://ojs.wiserpub.com/index.php/CM/

Research Article

Prey and Predators in Mathematical Model Under Caputo Fractional Derivative

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Received: 12 October 2024; **Revised:** 18 November 2024; **Accepted:** 16 January 2025

Abstract: The main concentration of ours as writers of this paper is finding the relation between two predators and one prey in a fractional mathematical model governed by Caputo fractional derivative (CFD). To achieve the above-mentioned goal the fractional reduced transform method (FRTM) has been utilized. When we use the CFD there will be no need to use Adomian's polynomials to calculate the nonlinear terms. To prove the reliability of CFD method, we have compared the outcomes of the fractional derivative orders with the ordinary derivative order index $0 < \omega < 1$. For further confirmation, we have presented our findings by showing graphs simulating the series solution. Moreover, we have also compared the error estimation with Rung Kutta and different orders. The Matrix Laboratory (MATLAB) software package was used as a tool for the above-mentioned process.

Keywords: fractional reduced differential transform method (FRDTM), CFD, prey predator model, MATLAB program

MSC: 34A08, 65Lxx

1. Introduction

Applied mathematics uses the mathematical model to explain natural phenomena and predicts system behavior over a given period. The study of mathematical modeling in ecology and economics is compelling due to the numerous elements influencing the existence of populations, the equilibrium of organisms, and their interactions within ecosystems [1]. The predator-prey population model is one mathematical model that elucidates the phenomenon. The link between predator species and their prey is robust; predators cannot survive without prey. Additionally, it functions as a regulatory controller of predator-prey populations. Interactions among species within an ecosystem can induce changes in their population dynamics. This contact can yield a positive, negative, or neutral effect on the species involved. Excessive predation on prey populations, combined with limited prey abundance or minimal population expansion from the onset of the prey population, is one cause of population extinction [2].

Numerous published studies have developed a model of Lotka-Volterra by incorporating certain assumptions. Srinivasu et al. have analyzed the Lotka-Volterra model along with the harvesting control system [3]. Kar [4] has analyzed the Lotka-Volterra model by incorporating the impact of time delay on selective harvesting. Didiharyono [5] has conducted a stability analysis of a one-prey, two-predator model with Holling-type III functional response and

DOI: https://doi.org/10.37256/cm.6220255892

harvesting. Didiharyono et al. examined the stability analysis of a model featuring two predator populations and one prey population in fisheries, with constant harvesting efforts [6]. Kunal et al. have studied the best way to regulate effort in a stage-structured prey-predator fisheries model that includes harvesting [7]. Li and Kaitai have investigated the necessary conditions for the development of stable, positive, steady-state solutions in a one-predator, and two-prey system [8]. Abd-Elhameed et al. looked into Tau and collocation spectral methods, did a convergence analysis, made a new operational matrix for the derivative of modified Chebyshev polynomials, and gave examples that show how their algorithm can be used in different Emden-Fowler equation situations [9]. Abd-Elhameed et al. studied how to solve fractional Riccati differential equations using spectral methods that use shifted Chebyshev polynomials of the second order. Abd-Elhameed et al. have investigated the solution for fractional Riccati differential equations utilizing spectral methods based on the shifted Chebyshev polynomials of the second kind, formulating an approximation for the fractional derivatives of these polynomials [10]. The fractional reduced differential transform method (FRDTM) was tested by Abdallah et al. [11] to see how well it works with a nonlinear mutualism model that includes fractional diffusion. Hadžiabdić et al. have studied the Lotka-Volterra model with two predators and their prey [12]. Noori et al. tested the convergence of the reduced differential transform method for different classes of differential equations [13]. In brief, using the Caputo derivative in fractional-order predator-prey models is a good way to include memory effects, nonlocal interactions, and complexity in ecological systems. Even though there are some problems, ongoing research could lead to better understanding of how predators and prey interact and better ways to protect them. This would require more model building, parameter estimates, and real-world applications. Where we hypothesize that the use of FRDTM is better than a domain's decomposition method.

The authors utilized Caputo's fractional derivatives (CFD) to depict the solution as a rapidly converging infinite series, thereby tackling nonlinearities. This method reduced the need for costly computations such as Adomian's polynomials. The study seeks to examine the stability of predator-prey models in populations and their harvesting techniques across all three categories of predators and prey. The study aims to analyze the stability of predator and prey models in populations, as well as their harvesting methods across all three groups of predators and prey. It will do this by comparing different fractional orders and the Runge-Kutta method using the MATLAB software package. The usage of Fractional Reduced Differential Transform Method (FRDTM) is not mainly practised as a domain decomposition method. Thus, this study is expected to highlight on that.

The population size (B, M, N) of prey, the first predator, and the second predator are considered in the nonlinear differential equation system [6] as in Equation (1).

$$\begin{cases} \dfrac{dB}{dt} = aB\left(1-\dfrac{B}{k}\right) - \beta BM - \gamma BN - E_1B, \\[2ex] \dfrac{dM}{dt} = -bM + \beta BM - dM - E_2M, \\[2ex] \dfrac{dN}{dt} = -cN + \gamma BN + dM - E_3N, \\[2ex] B(0) = B_0, \quad M(0) = M_0, \quad N(0) = N_0, \end{cases} \tag{1}$$

where a is the intrinsic growth rate in the prey; b denotes the natural death of M; c denotes the natural death of N; d denotes the conversion rate, M is a variable with positive parameter; β is the interaction of M with B; γ is the interaction of N with B; and E_i is harvesting efforts for $i = 1, 2, 3$. Now, we use the preceding model (1) to extend under CFD as follows:

Consider $\alpha_1 = a - E_1$, $\alpha_2 = b + d + E_2$, $\alpha_3 = c + E_3$, $\delta = \alpha/k$

$$\begin{cases} D_t^{\omega} B(t) = \alpha_1 B(t) - \delta B^2 - \beta B(t) M(t) - \gamma B(t) N(t), \\ D_t^{\omega} M(t) = -\alpha_2 M(t) + \beta M(t) B(t), \\ D_t^{\omega} N(t) = -\alpha_3 N(t) + \gamma N(t) B(t) + d M(t), \\ B(0) = B_0, \quad M(0) = M_0, \quad N(0) = N_0. \end{cases} \quad 0 < \omega < 1 \tag{2}$$

Furthermore, we intend to utilize the FRTM to derive the related numerical findings of the examined model. Conversely, we also obtained the relevant numerical results for the examined model using the CFD method. We additionally have simulated the series solution to graphically demonstrate our findings. We have structured the paper as follows plus the initial section. In Section 2, we have provided some pertinent definitions of fractional calculus. In Section 3, we have investigated equilibrium point to check the stability. In Section 4, we have talked about how to resolve the fractional prey and predator model using the FRTM. We also have shown and talked about the graphical representation of the numerical solution in Section 5. In Section 6, we compare the results with the Runge-Kutta method and assess the convergence and error related to the numerical conduct that is provided. We have concluded our study in Section 7.

2. Preliminaries

In this section, we recap some useful definitions of fractional calculus and Sumudu transform which are required to introduce the proposed method.

Definition 1 The Caputo fractional differential operator is defined as:

$$c_{D_t^r} f(t) = \frac{1}{\Gamma(1-r)} \int_a^t \frac{f^{'}(\tau)}{(t-\tau)^r} d\tau, \tag{3}$$

where Γ is the gamma function, $0 < r \leq 1$, $a \in [-\infty, t)$, $f \in H^1(a, b)$ and $b > a$.

Definition 2 The Mittag-Leffler Function

Suppose $\alpha > 0$, $\beta > 0$, then the Mittag-Leffler function is defined by

$$E_{\alpha, \beta}(t) = \sum_{k=0}^{\infty} \frac{t^k}{\Gamma(\alpha k + \beta)}. \tag{4}$$

3. Lotka-Volterra model

We have examined the Lotka-Volterra's model, which features one prey and two predators [12]. The following differential equations make up the framework:

$$\begin{cases} B_t(t) = \alpha_1 B(t) - B(t)M(t) - B(t)N(t), \\ \\ M_t(t) = -\alpha_2 M(t) + M(t)B(t), \\ \\ N_t(t) = -\alpha_3 N(t) + N(t)B(t), \\ \\ B(0) = B_0, \quad M(0) = M_0, \quad N(0) = N_0, \end{cases} \tag{5}$$

where $\boldsymbol{B}(\boldsymbol{t}) \geq \mathbf{0}$ represents prey, $\boldsymbol{M}(\boldsymbol{t}) \geq \mathbf{0}$, $\boldsymbol{N}(\boldsymbol{t}) \geq \mathbf{0}$ represents predators and $\boldsymbol{\alpha_1}$, $\boldsymbol{\alpha_2}$, $\boldsymbol{\alpha_3}$ are positive parameters. The equilibrium points are defined as:

$$f(t) = \begin{bmatrix} \alpha_1 B(t) - B(t)M(t) - B(t)N(t) \\ -\alpha_2 M(t) + M(t)B(t) \\ -\alpha_3 N(t) + N(t)B(t) \end{bmatrix}. \tag{6}$$

when $\boldsymbol{f}(\boldsymbol{t}) = \mathbf{0}$, we obtain the equilibrium points $\boldsymbol{t_1} = (\mathbf{0}, \mathbf{0}, \mathbf{0})^{\boldsymbol{T}}$, $\boldsymbol{t_2} = (\boldsymbol{\alpha_2}, \boldsymbol{\alpha_1}, \mathbf{0})^{\boldsymbol{T}}$ & $\boldsymbol{t_3} = (\boldsymbol{\alpha_3}, \mathbf{0}, \boldsymbol{\alpha_1})^{\boldsymbol{T}}$, now we compute the Jacobian matrix of partial derivatives:

$$J(t) = \begin{bmatrix} \alpha_1 - M - N & -B & -B \\ M & -\alpha_2 + B & 0 \\ N & 0 & -\alpha_3 + B \end{bmatrix}. \tag{7}$$

$$J(t_1) = \begin{bmatrix} \alpha_1 & 0 & 0 \\ 0 & -\alpha_2 & 0 \\ 0 & 0 & -\alpha_3 \end{bmatrix}. \tag{8}$$

The eigenvalues are $\lambda_1 = \alpha_1$, $\lambda_2 = -\alpha_2$ & $\lambda_3 = -\alpha_3$, then t_1 is a saddle point, so the point is unstable.

$$J(t_2) = \begin{bmatrix} 0 & -\alpha_2 & -\alpha_2 \\ \alpha_1 & 0 & 0 \\ 0 & 0 & -\alpha_3 + \alpha_2 \end{bmatrix}. \tag{9}$$

The eigenvalues are $\lambda_1 = \alpha_2 - \alpha_3$, $\lambda_2 = i\sqrt{\alpha_1 \alpha_2}$ & $\lambda_3 = -i\sqrt{\alpha_1 \alpha_2}$, then t_2 is a non-hyperbolic point, so the point is marginal stable.

$$J(t_3) = \begin{bmatrix} 0 & -\alpha_3 & -\alpha_3 \\ 0 & -\alpha_2 + \alpha_3 & 0 \\ \alpha_1 & 0 & 0 \end{bmatrix}. \tag{10}$$

The eigenvalues are $\lambda_1 = \alpha_3 - \alpha_2$, $\lambda_2 = i\sqrt{\alpha_1 \alpha_3}$ & $\lambda_3 = -i\sqrt{\alpha_1 \alpha_3}$, then t_2 is a non-hyperbolic point, so the point is marginal stable.

4. Fractional reduced differential transform method

Fractional reduced differential transform method (FRDTM) is an iteration method, suppose $u(t, x_1, x_2, \ldots, x_n)$ be analytical and continuously differentiable with respect to $n+1$ variables $t, x_1, x_2, .., x_n$ in the domain of interest; then FRDTM in n dimensions for the following differential equation:

$$D_t^r u + Lu + N(u) = 0, \tag{11}$$

where D_t^r is differential operator with respect to time, L differential operator with respect to variables $x_1, x_2, \ldots, x_n$ and $N(u)$ is nonlinear term.

$$u_{k+1} = \frac{\Gamma(kr+1)}{\Gamma(r(k+1)+1)}\left[-L(u_k) - \sum_{j=0}^{k} N(u_j)N(u_{k-j})\right]. \tag{12}$$

The approximate solution is given by:

$$u(t, x_1, x_2, \ldots, x_n) = \sum_{k=0}^{\infty} u_k t^{\omega k} = u_0 + u_1 t^{\omega} + u_2 t^{2\omega} + \ldots. \tag{13}$$

The primary perspective of FRTDM about the solutions of nonlinear models is determining the power series expansion at its initial time t_0,

$$u(x, t) = \sum_{k=0}^{\infty} c_k(x)(t-t_0)^k. \tag{14}$$

Definition 3 If $\phi_k(x, t) = c_k(x)(t-t_0)^k$, then the series solution $\sum_{k=0}^{\infty} \phi_k(x, t)$, stated in Equation (14), $\forall k \in N \cup \{0\}$.

(i) It is convergence if $\exists 0 < \lambda < 1$, such that $\|\phi_{k+1}\| \leq \lambda \|\phi_k\|$.

(ii) It is divergent if $\exists \lambda > 1$, such that $\|\phi_{k+1}\| > \lambda \|\phi_k\|$.

Proof. See [13]. □

5. Numerical results

5.1 *Solution steps*

First, applying the FRTM of (2) gives:

$$B_{k+1} = \frac{\Gamma(k\omega+1)}{\Gamma(\omega(k+1)+1)}\left\{\alpha_1 B_k - \delta\sum_{j=0}^{k} B_j B_{k-j} - \beta\sum_{j=0}^{k} B_j M_{k-j} - \gamma\sum_{j=0}^{k} B_j N_{k-j}\right\}, \tag{15}$$

$$M_{k+1}=\frac{\Gamma(k\omega+1)}{\Gamma(\omega(k+1)+1)}\left\{-\alpha_2 M_k+\beta\sum_{j=0}^{k} M_j B_{k-j}\right\} \tag{16}$$

$$N_{k+1}=\frac{\Gamma(k\omega+1)}{\Gamma(\omega(k+1)+1)}\left\{-\alpha_3 N_k+\gamma\sum_{j=0}^{k} N_j B_{k-j}+dM_k\right\} \tag{17}$$

$$B(t)=\sum_{k=0}^{\infty} B_k t^{\omega k}, \quad M(t)=\sum_{k=0}^{\infty} M_k t^{\omega k}\ , \quad N(t)=\sum_{k=0}^{\infty} N_k t^{\omega k}. \tag{18}$$

5.2 *Simulation results*

In this subsection, we consider numerical findings with the following parameter values: $\alpha_1=0.0001$, $\alpha_2=0.0001$, $\alpha_3=0.0001$, $\beta=0.003$, $\gamma=0.005$, $d=0.005$ and $\delta=0.005$ starting with the initial approximations $B(0)=4$, $M(0)=2$, $N(0)=2$ we obtain.

Elapsed time is 1.782126 s.

Table 1 indicates the estimated solution of prey $B(t)$. Figure 1 shows the different between ordinary order with multiple fractional orders. We get the order of fractional $w=0.2$ as the best one because the prey rate is low compared to the rest, and Figure 2 shows the surface presentation of prey.

Table 1. Numerical results of prey

Time	$\omega=0.9$	$\omega=0.8$	$\omega=0.5$	$\omega=0.2$
0.0000	4.0000	4.0000	4.0000	4.0000
1.0000	3.8453	3.8453	3.8453	3.8453
2.0000	3.7100	3.7140	3.7178	3.7216
3.0000	3.5825	3.5913	3.6000	3.6084
4.0000	3.4601	3.4744	3.4883	3.5019
5.0000	3.3418	3.3619	3.3814	3.4003
6.0000	3.2266	3.2527	3.2780	3.3025
7.0000	3.1141	3.1465	3.1777	3.2078
8.0000	3.0037	3.0425	3.0798	3.1157
9.0000	2.8953	2.9406	2.9841	3.0258
10.0000	2.7883	2.8404	2.8902	2.9378

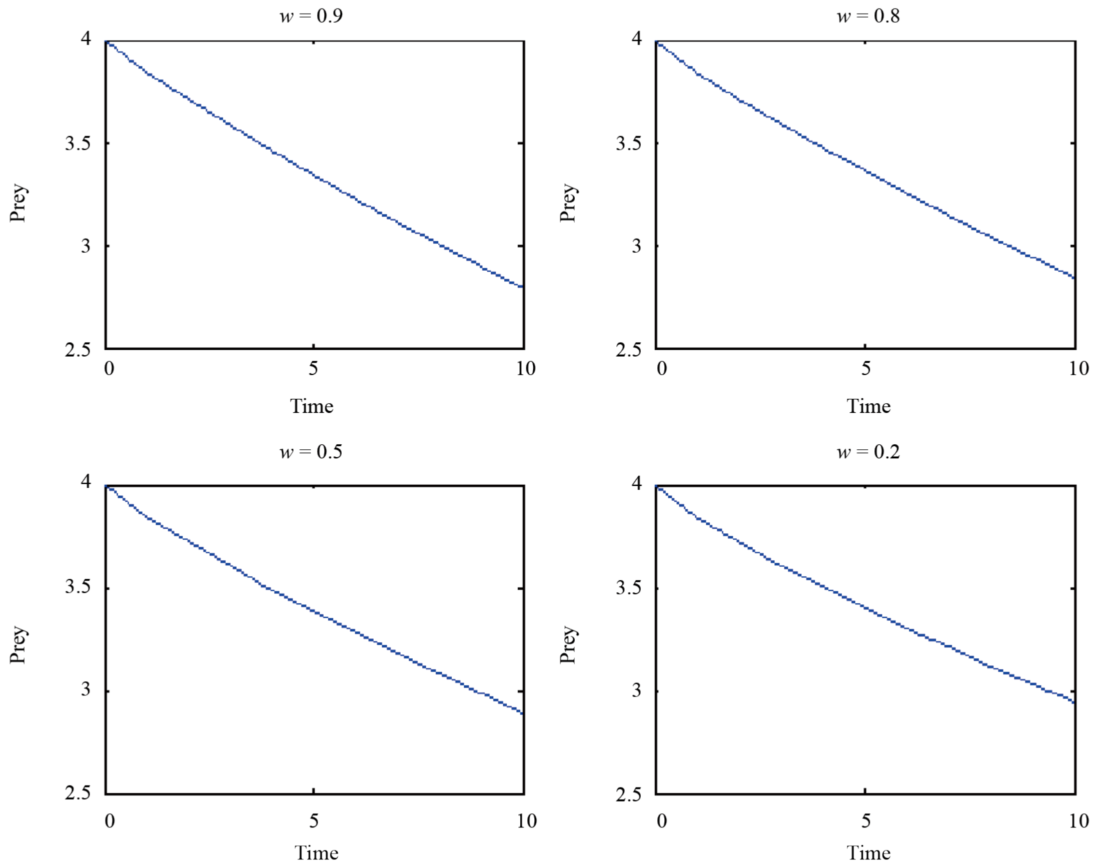

Figure 1. Graphical presentation of prey with different orders

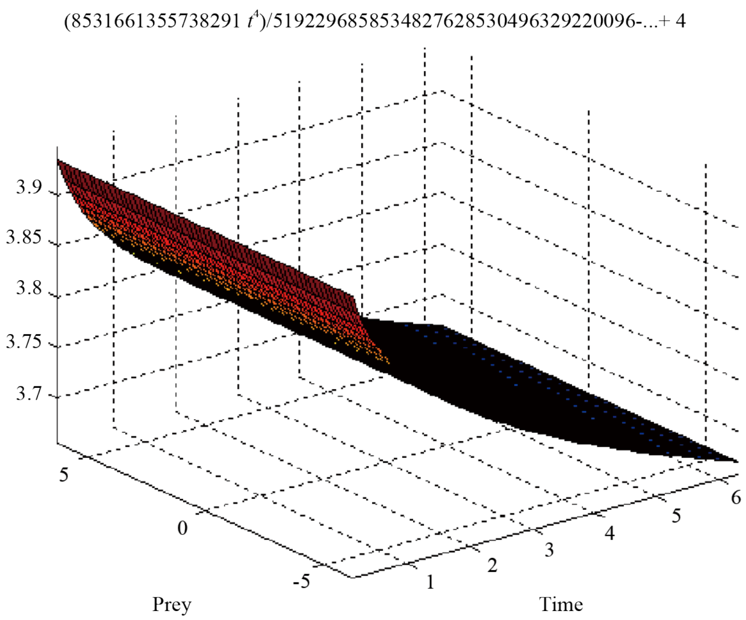

Figure 2. Surface presentation of prey

Elapsed time is 0.420441 s.

Table 2 reflects the approximate solution of first predator $M(t)$. Figure 3 signifies the different between ordinary order with multiple fractional orders. We get the order of fractional $w = 0.9$ is the best one because the first predator rate is grown compared to the rest, and Figure 4 shows the surface presentation of first predator.

Table 2. Numerical results of first predator

Time	$\omega = 0.9$	$\omega = 0.8$	$\omega = 0.5$	$\omega = 0.2$
0.0000	2.0000	2.0000	2.0000	2.0000
1.0000	2.0260	2.0260	2.0260	2.0260
2.0000	2.0498	2.0491	2.0484	2.0477
3.0000	2.0731	2.0714	2.0698	2.0683
4.0000	2.0962	2.0935	2.0908	2.0882
5.0000	2.1194	2.1154	2.1115	2.1078
6.0000	2.1427	2.1373	2.1322	2.1272
7.0000	2.1661	2.1593	2.1528	2.1465
8.0000	2.1898	2.1814	2.1734	2.1658
9.0000	2.2137	2.2036	2.1940	2.1850
10.0000	2.2378	2.2260	2.2148	2.2042

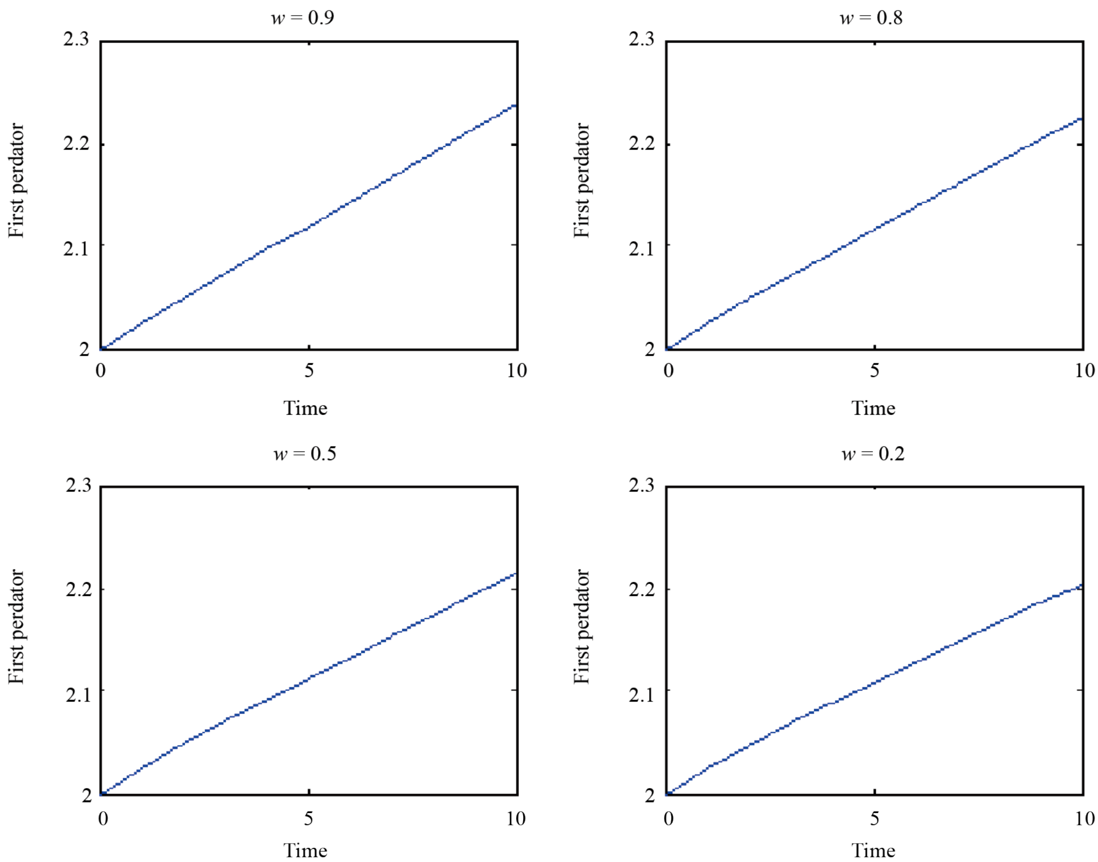

Figure 3. Graphical presentation of first predator with different orders

$29\ t^{43/2})/1517710072051350836655829614705874145814380343009484000977978445108518972 8$

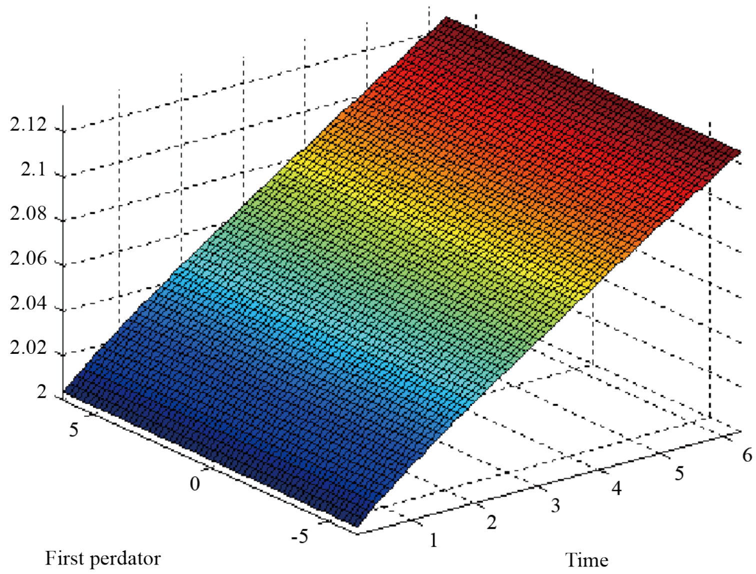

Figure 4. Surface presentation of first predator

Elapsed time is 0.422298 s.

Table 3 shows the approximate solution of second predator $N(t)$. Figure 5 showing the variance between ordinary order and multiple fractional orders. We get the order of fractional $w = 0.9$ as the best one because the second predator rate is grown compared to the rest, and Figure 6 shows the surface presentation of second predator.

Table 3. Numerical results of second predator

Time	$\omega = 0.9$	$\omega = 0.8$	$\omega = 0.5$	$\omega = 0.2$
0.0000	2.0000	2.0000	2.0000	2.0000
1.0000	2.0553	2.0553	2.0553	2.0553
2.0000	2.1068	2.1053	2.1038	2.1023
3.0000	2.1584	2.1547	2.1511	2.1476
4.0000	2.2106	2.2043	2.1983	2.1924
5.0000	2.2638	2.2546	2.2457	2.2372
6.0000	2.3184	2.3058	2.2937	2.2821
7.0000	2.3744	2.3580	2.3424	2.3276
8.0000	2.4321	2.4115	2.3921	2.3736
9.0000	2.4916	2.4664	2.4427	2.4204
10.0000	2.5529	2.5227	2.4944	2.4679

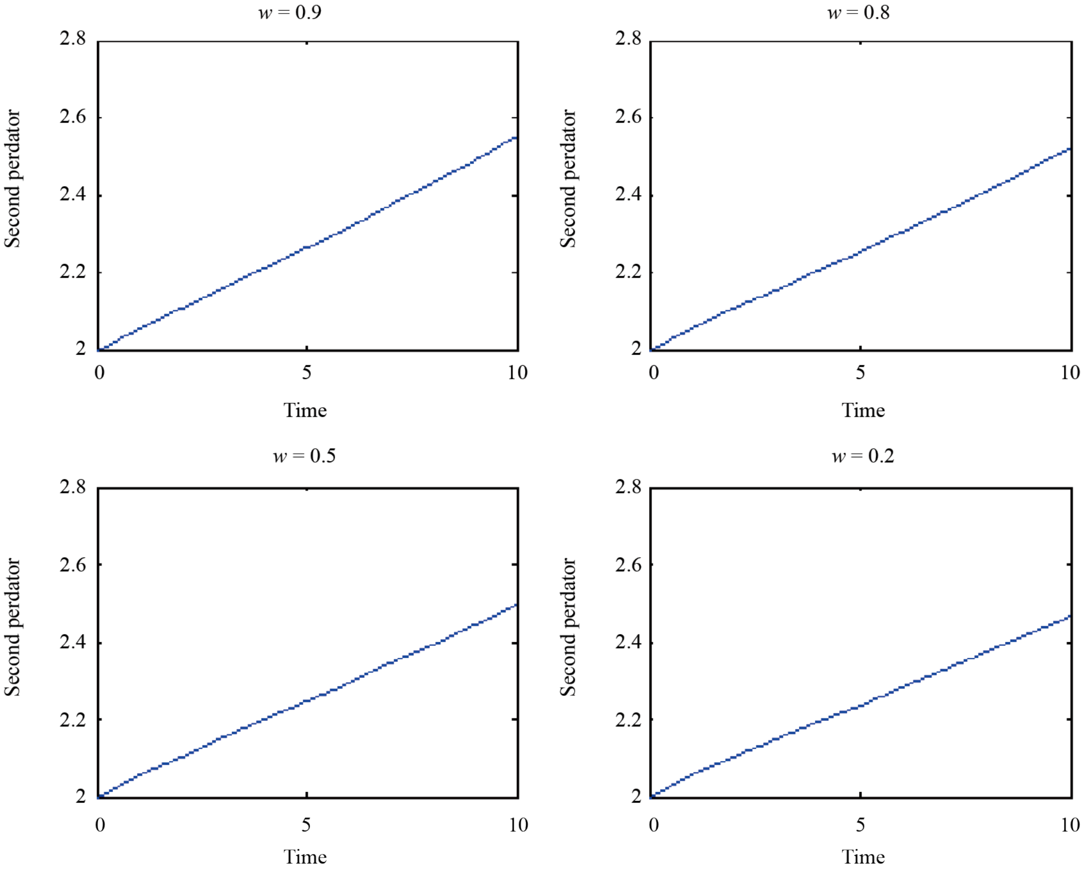

Figure 5. Graphical presentation of second predator with different orders

$2207\, t^{43/2})/23158417847463239084714197001737581570653996933128112807891516801582625$

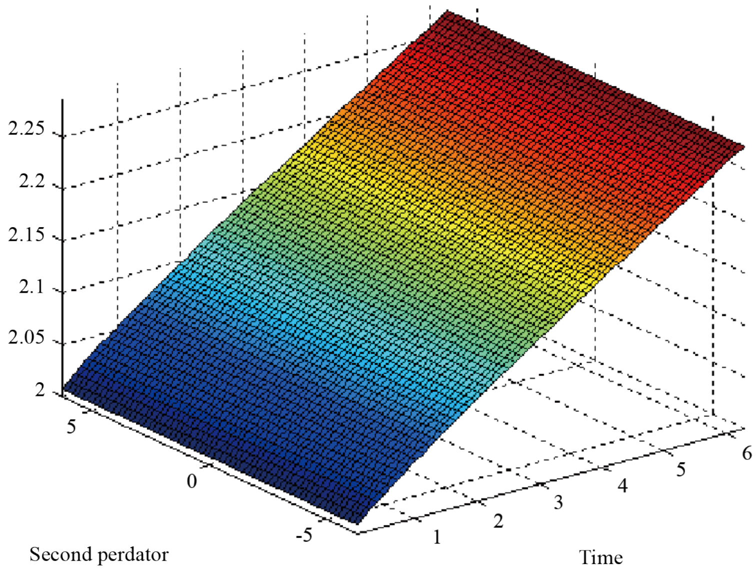

Figure 6. Surface presentation of second predator

6. Convergence and error analysis

This section evaluates the convergence and error associated with the presented numerical conduct and compares the findings with the Runge-Kutta method.

Table 4 illustrates the error comparison between Rung Kutta method and prey $B(t)$. Figure 7 explains the error comparison.

Table 4. Error estimation

N	$\omega = 0.9$	$\omega = 0.8$	$\omega = 0.5$	$\omega = 0.2$
2	0.1353	0.1313	0.1275	0.1237
4	0.1224	0.1169	0.1117	0.1065
6	0.1152	0.1092	0.1034	0.0978
10	0.1070	0.1002	0.0024	0.9120

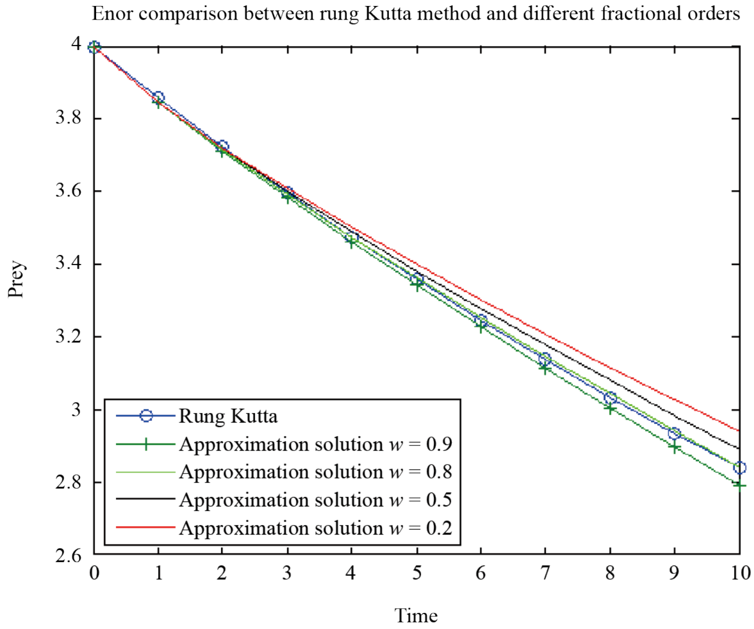

Figure 7. Estimation error between Rung Kutta and different orders of prey

Table 5 shows the error comparison between Rung Kutta method and first predator $M(t)$. Figure 8 shows the error comparison.

Table 5. Error comparison between Rung Kutta method (RKM) and FRDTM when $N = 10$

RKM	FRDTM			
	$\omega = 0.9$	$\omega = 0.8$	$\omega = 0.5$	$\omega = 0.2$
9.58×10^{-2}	10.7×10^{-2}	10.02×10^{-2}	2.4×10^{-3}	9.12×10^{-2}

Table 6 shows the error estimation, Table 7 shows the error comparison between Rung Kutta method (RKM) and FRDTM.

Table 6. Error estimation

N	$\omega = 0.9$	$\omega = 0.8$	$\omega = 0.5$	$\omega = 0.2$
2	0.0238	0.0231	0.0224	0.0217
4	0.0231	0.0221	0.0210	0.0199
6	0.0233	0.0.219	0.0207	0.0194
10	0.0241	0.0224	0.0208	0.0192

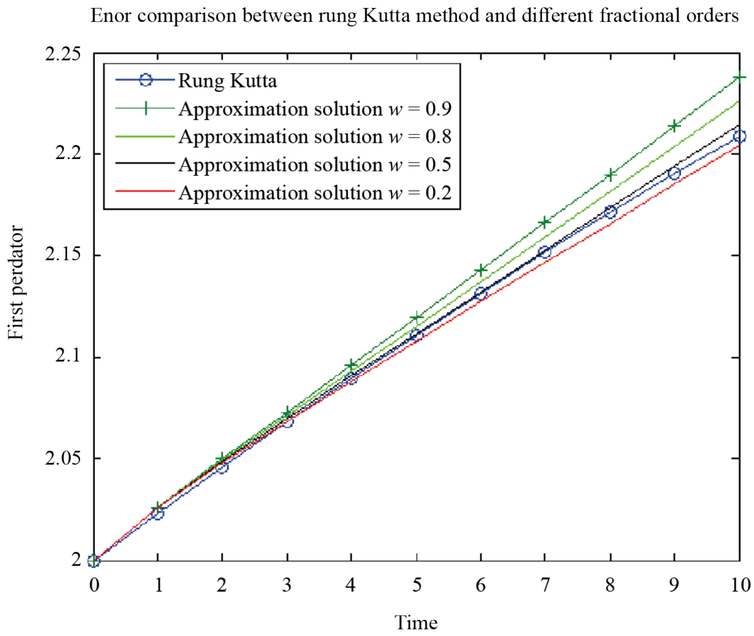

Figure 8. Estimation errors between Rung Kutta and different orders of first predator

Table 7. Error comparison between Rung Kutta method (RKM) and FRDTM when $N = 10$

RKM	FRDTM			
	$\omega = 0.9$	$\omega = 0.8$	$\omega = 0.5$	$\omega = 0.2$
1.86×10^{-2}	2.41×10^{-2}	2.24×10^{-2}	2.08×10^{-3}	1.92×10^{-2}

Table 8 shows the error comparison between Rung Kutta method and first predator $M(t)$. Figure 9 shows the error comparison. Table 9 shows the error comparison between Rung Kutta method (RKM) and FRDTM when $N = 10$.

Table 8. Error estimation

N	$\omega = 0.9$	$\omega = 0.8$	$\omega = 0.5$	$\omega = 0.2$
2	0.0515	0.0500	0.0505	0.0470
4	0.0522	0.0477	0.0472	0.0448
6	0.0546	0.0512	0.0480	0.0449
10	0.0613	0.0563	0.0517	0.0475

Table 9. Error comparison between Rung Kutta method (RKM) and FRDTM when $N = 10$

RKM	FRDTM			
	$\omega = 0.9$	$\omega = 0.8$	$\omega = 0.5$	$\omega = 0.2$
4.59×10^{-2}	2.41×10^{-2}	2.24×10^{-2}	2.08×10^{-3}	1.92×10^{-2}

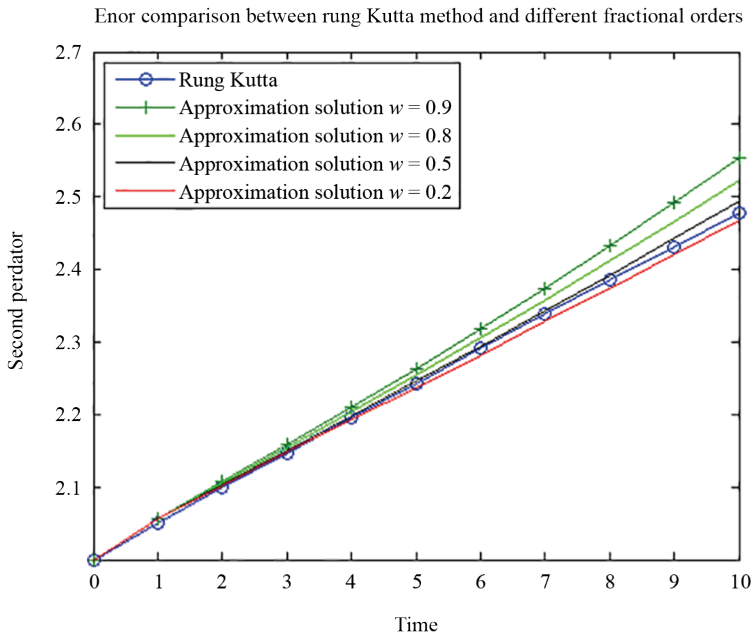

Figure 9. Estimation errors between Rung Kutta and different orders of second predator

7. Conclusions

This study introduces a nonlinear mathematical model examined through the FRDTM to derive an analytical approximate solution to a prey-predator model incorporating the Caputo fractional derivative. The FRDTM serves as a robust mathematical instrument for addressing various forms of linear and nonlinear fractional partial differential equations (PDEs), eliminating Adomian's polynomials necessity in the computation of nonlinear terms. Our findings show the fractional derivative of the prey-predator model which yields greater accuracy compared to the ordinary derivative order. Based on the numerical results, we ascertain the method's efficacy in resolving diverse types of nonlinear fractional differential equations. We encourage researchers to use FRDTM because its derivation mathematical models are better than a domain's decomposition method and that will lead to the protection of the environment. The results of this study will be fruitful if they are used in the biological environment field such as animal reserves.

Conflict of interest

The authors conclude that the conflict of interest is zero pertaining to the publishing of this paper.

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