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Research Article

Decompositions of Single-Valued Neutrosophic Continuity via Primalization

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Abstract: This paper introduces new single-valued neutrosophic closure operators based on single-valued neutrosophic primals, laying the foundation for single-valued neutrosophic primal topologies. This framework establishes a novel class of topological spaces, termed single-valued neutrosophic primal topological spaces. Within this context, classical topological concepts are redefined to align with the properties of single-valued neutrosophic primal set-topologies. The study explores the basic structure of these topologies, including the construction of bases, and examines relationships between various single-valued neutrosophic primal and single-valued neutrosophic topologies. A significant contribution of this work is the introduction of single-valued neutrosophic primal continuous functions, supported by counterexamples to illustrate their implications and nuances.

Keywords: Single-valued neutrosophic primal, single-valued neutrosophic (closure, interior, per-primal interior, α-primal interior, β-primal closure, β-primal interior) operators, single-valued neutrosophic primal α-continuity, single-valued neutrosophic primal continuity

MSC: 54A40, 54B99

1. Introduction

Neutrosophic set theory has a powerful influence since this modern section of philosophy is presented as a study of neutrality's origin, nature, and scope. The concept of neutrosophy was initiated by Smarandache [1] in 1999 as a new mathematical approach that corresponds to the degree of indeterminacy (uncertainty, etc.). The notion of continuity is important in fuzzy topology and fuzzy topology in the Šostak sense and all branches of mathematics and quantum physics (see [2–4]). We must state that this subject has been researched by physicists and others. El-Naschie has shown that the notion of fuzzy topology in the Šostak sense has very important applications in quantum particle physics especially in both string theory and ε^{∞} theory [5, 6].

Nowadays, the theory of neutrosophy has become used in many branches of mathematics. More precisely, this theory has achieved exceptional progress in the field of topological spaces. Salama et al. [7, 8] published their works of neutrosophic topological spaces, following the method of Chang [9] in the situation of fuzzy topological spaces $(\Omega, \mathfrak{T})$. Later, the category of neutrosophic (NSet) and neutrosophic crisp (NCSet) sets NSet (H) and NCSet have been studied by

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Hur et al. [10, 11]. Smarandache [12] provided an application for the idea of neutrosophic topology on the non-standard interval. Individuals can simply discover that the fuzzy topology familiarized by Chang is a crisp group of fuzzy subsets.

Šostak [13] determined that Chang's style is crisp and so he redefined the idea of fuzzy topology, frequently mentioned as smooth fuzzy topology, as a mapping from the group of all fuzzy subsets of Ω to $[0,\ 1]$. Fang Jin-ming et al. [14] and Zahran et al. [15] investigated the notion of foundation as a function from an appropriate collection of fuzzy subsets of Ω to $[0,\ 1]$ and decomposition of fuzzy continuity, fuzzy ideal continuity and fuzzy ideal α-continuity. Saber et al. [16] found a parallel theory in the context of fuzzy ideal topological space in Šostak sense.

The concept of a single-valued neutrosophic set was introduced by Wang [17]. Gayyar [18] presented the idea of fuzzy neutrosophic topological spaces in a Šostak sense. Kim [19]. Presented the notion of the foundation for an ordinary single-valued neutrosophic topology. The idea of a stratified single-valued neutrosophic soft filter (*stratified svns-filter*) and stratified single-valued neutrosophic soft quasi uniformity (*stratified svnsq-uniformity*) was initially proposed by Alsharari et al. [20]. The new concept of single-valued neutrosophic soft topology (*svnft*) is defined to discuss the topological structure of *svns-set* by Saber et al. [21]. Several authors [22–26] posted their efforts for the idea of single-valued neutrosophic topological spaces $(\Omega,\ \mathfrak{T}^{\varrho\varsigma\varphi})$. The concept of a single-valued neutrosophic open local function $(\Pi^{\star}_{\varkappa}(\mathfrak{T}^{\varrho\varsigma\varphi},\ \mathfrak{P}^{\varrho\varsigma\varphi}))$ for a single-valued neutrosophic primal topological space $(\Omega,\ \mathfrak{T}^{\varrho\varsigma\varphi}, \mathfrak{P}^{\varrho\varsigma\varphi})$ have been studied by Alsharari et al. [27]. The concept of n-cylindrical fuzzy neutrosophic topological spaces was first introduced by Kumari et al. [28]. Subsequently, Kungumaraj et al. [29] extended this work by presenting neutrosophic topological vector spaces. Jana et al. [30] further contributed to the field by introducing Pythagorean fuzzy topological spaces.

The theory of neutrosophic sets is widely regarded as a generalization of fuzzy sets, intuitionistic fuzzy sets, and rough sets. It serves as a vital mathematical framework for addressing uncertainty. This paper introduces the concept of single-valued neutrosophic Primals in the sense of Sostak, which extends the ideas presented in [31, 32].

Motivated by these foundational concepts, this work focuses on single-valued neutrosophic Primals as defined in Sostak's framework. Neutrosophy, and particularly neutrosophic sets, provides a flexible and general framework that extends intuitionistic fuzzy sets, fuzzy sets, and classical sets from both a mathematical and philosophical perspective.

In this article, we define several new notions in the context of single-valued neutrosophic primal topological spaces, including r-single-valued neutrosophic primal open, r-single-valued neutrosophic semi-primal open, r-single-valued neutrosophic pre-primal open, r-single-valued neutrosophic α-primal open, r-single-valued neutrosophic β-primal open, r-single-valued neutrosophic strongly β-primal open and r-single-valued neutrosophic δ-primal open. Additionally, we present new decompositions of single-valued neutrosophic continuity within these spaces and explore their relationships with other types of mappings.

A neutrosophic set is a generalized formal framework that extends the concepts of classical sets, fuzzy sets, interval-valued fuzzy sets, intuitionistic fuzzy sets, and interval intuitionistic fuzzy sets from a philosophical perspective. These sets have various applications across multiple domains. For instance, in Geographical Information Systems (GIS), they are used to model spatial regions with indeterminate boundaries under uncertainty (see [33]). Additionally, neutrosophic sets have potential applications in superstring theory and ε^{∞} space-time modeling (see [34]). They are also relevant in control engineering, particularly in average consensus for multi-agent systems with uncertain topologies, time-varying delays, and random noisy environments (see [35]).

2. Preliminaries

This section gives readers an in-depth understanding of the basic concepts and methods used in single-valued neutrosophic (for short, *svn-set*) and single-valued neutrosophic primal (for short, *svn-primal*) theories, setting the groundwork for the later development of single-valued neutrosophic operators $P\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}$, $\alpha\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}$, $\beta\mathfrak{P}\mathfrak{C}\mathfrak{L}_{\mathfrak{T}^{\varrho\varsigma\varphi}}$, $\beta\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}$: $\pounds^{\Omega}\times\pounds_0\to\pounds^{\Omega}$. Conventionally, $\pounds^{\Omega}$ denotes the family of all *svn-sets*, defined on Ω and $(\Omega,\ \mathfrak{T}^{\varrho\varsigma\varphi}, \mathfrak{P}^{\varrho\varsigma\varphi})$ denotes the single-valued neutrosophic primal topological space (for short, *svnpts*). Here, $\pounds=[0,\ 1]$, $\pounds_0=(0,\ 1]$ and for any $\sigma\in\pounds$ and $u\in\Omega$, $\overline{\sigma}(u)=\sigma$.

We begin by introducing the following abbreviations used throughout this manuscript:

n-set	neutrosophic set
svn-set	single-valued neutrosophic set
svn-primal	single-valued neutrosophic primal
svnt	single-valued neutrosophic topology
svnpts	single-valued neutrosophic primal topological spaces
$\Pi^{\star}_{\varkappa}(\mathfrak{T}^{\varrho\varsigma\varphi},\ \mathfrak{P}^{\varrho\varsigma\varphi})$	single-valued neutrosophic primal open local function
$(\Omega,\ \mathfrak{T}^{\varrho},\ \mathfrak{T}^{\varsigma},\ \mathfrak{T}^{\varphi})$	single-valued neutrosophic topological paces.

Definition 1 [12] Let Ω be a non-empty set. An *n*-set on Ω is defined as

$$\Pi = \{\langle u,\ \varrho_{\Pi}(u),\ \varsigma_{\Pi}(u),\ \varphi_{\Pi}(u)\rangle \mid u \in \Omega,\ \varrho_{\Pi}(u),\ \varsigma_{\Pi}(u),\ \varphi_{\Pi}(u) \in \rfloor^{-}0,\ 1^{+}\lfloor\},$$

where representing $(\varphi_{\Pi}(u))$ the degree of nonmembership, $(\varsigma_{\Pi}(u))$ the degree of indeterminacy and $(\varrho_{\Pi}(u))$ degree of membership; for each $u \in \Omega$ to the set Π.

Definition 2 [17] Let Ω be a space of points (objects) with a generic element in Ω denoted by u. Then Π is called an *svn-set* in Ω if Π has the form $\Pi = \{\langle u,\ \varrho_{\Pi}(u),\ \varsigma_{\Pi}(u),\ \varphi_{\Pi}(u)\rangle \mid u \in \Omega\}$ where $\varrho_{\Pi} : \Omega \to \pounds$, $\varsigma_{\Pi} : \Omega \to \pounds$ and $\varphi_{\Pi} : \Omega \to \pounds$. In this case, φ_{Π}, ς_{Π} and ϱ_{Π} are called the falsity membership function, indeterminacy membership function and truth membership function, respectively.

An *svn-set* Π on Ω is called a null *svn-set* (for short, $\overline{0}$), if $\varrho_{\Pi}(u) = 0$, $\varsigma_{\Pi}(u) = 1$ and $\varphi_{\Pi}(u) = 1$, $\forall\ u \in \Omega$.

An *svn-set* Π on Ω is called a absolute *svn-set* (for short, $\overline{1}$), if $\varrho_{\Pi}(u) = 1$, $\varsigma_{\Pi}(u) = 0$ and $\varphi_{\Pi}(u) = 0$, $\forall\ u \in \Omega$.

Remark 1 To clarify the relationship between intuitionistic fuzzy sets (*if-sets*), neutrosophic sets (*n-sets*), and single-valued neutrosophic sets (*svn-sets*), it is important to confirm that both neutrosophic sets and single-valued neutrosophic sets generalize the concept of intuitionistic fuzzy sets. This relationship can be summarized as follows:

In intuitionistic fuzzy sets (IFS), the representation of paraconsistent (contradictory but coexistent), dialetheist (true contradictions), or incomplete information is not feasible. This limitation highlights a fundamental distinction between *if-sets*, which are based on IFS principles, and *n-sets*, which extend to neutrosophic logic. To illustrate this distinction, the neutrosophic cube.

In technical applications, only the classical interval $[0,\ 1]$ is used as the range for neutrosophic parameters (truth, indeterminacy, and falsity). This limitation defines the technical neutrosophic cube $ABCDEFGH$. By contrast, the extended neutrosophic cube $A'B'C'D'E'F'G'H'$ is used in fields such as philosophy, where differentiating between absolute and relative notions is necessary in Figure 1.

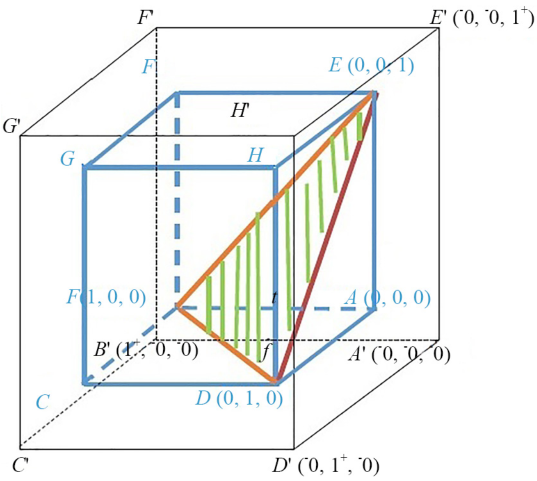

Figure 1. The neutrosophic cube

Definition 3 [17] Let $\Pi = \{\langle u,\ \varrho_{\Pi}(u),\ \varsigma_{\Pi}(u),\ \varphi_{\Pi}(u)\rangle \mid u \in \Omega\}$ be an *svn-set* on Ω. The complement of the set Π (Π^c) is defined as follows:

$$\varrho_{\Pi^c}(u) = \varphi_{\Pi}(u),\ \varsigma_{\Pi^c}(u) = [\varsigma_{\Pi}]^c(u),\ \varphi_{\Pi^c}(u) = \varrho_{\Pi}(u).$$

Definition 4 [22] Let $\Pi,\ \Upsilon \in \pounds^{\Omega}$, then,

(1) Π is said to be contained in Υ, denoted by $\Pi \subseteq \Upsilon$, if, for each $u \in \Omega$,

$$\varrho_{\Pi}(u) \le \varrho_{\Upsilon}(u),\ \varsigma_{\Pi}(u) \ge \varsigma_{\Upsilon}(u),\ \varphi_{\Pi}(u) \ge \varphi_{\Upsilon}(u).$$

(2) Π is said to be equal to Υ, denoted by $\Pi = \Upsilon$, if $\Pi \subseteq \Upsilon$ and $\Upsilon \subseteq \Pi$.

Definition 5 [24] Let $\Pi,\ \Upsilon \in \pounds^{\Omega}$. Then,

(a) $\Pi \wedge \Upsilon$ is an *svn-set*, if $\forall\ u \in \Omega$,

$$\Pi \wedge \Upsilon = \langle(\varrho_{\Pi} \wedge \varrho_{\Upsilon})(u),\ (\varsigma_{\Pi} \vee \varsigma_{\Upsilon})(u),\ (\varphi_{\Pi} \vee \varphi_{\Upsilon})(u)\rangle$$

where,

$$(\varrho_{\Pi} \wedge \varrho_{\Upsilon})(u) = \varrho_{\Pi}(u) \wedge \varrho_{\Upsilon}(u), \quad (\varsigma_{\Pi} \vee \varsigma_{\Upsilon})(u) = \varsigma_{\Pi}(u) \vee \varsigma_{\Upsilon}(u),$$

$$(\varphi_{\Pi} \vee \varphi_{\Upsilon})(u) = \varphi_{\Pi}(u) \vee \varphi_{\Upsilon}(u),$$

for every $u \in \Omega$.

(b) $\Pi \vee \Upsilon$ is an *svn-set*, if $\forall\ u \in \Omega$,

$$\Pi \vee \Upsilon = \langle(\varrho_{\Pi} \vee \varrho_{\Upsilon})(u),\ (\varsigma_{\Pi} \wedge \varsigma_{\Upsilon})(u),\ (\varphi_{\Pi} \wedge \varphi_{\Upsilon})(u)\rangle.$$

Definition 6 [23] An svnts is an ordered $(\Omega,\ \mathfrak{T}^{\varrho},\ \mathfrak{T}^{\varsigma},\ \mathfrak{T}^{\varphi})$ where $\mathfrak{T}^{\varrho} : \pounds^{\Omega} \to \pounds$, $\mathfrak{T}^{\varsigma} : \pounds^{\Omega} \to \pounds$ and $\mathfrak{T}^{\varphi} : \pounds^{\Omega} \to \pounds$ are a mappings fulfill specific conditions:

(SVNT1) $\mathfrak{T}^{\varrho}(\overline{0}) = \mathfrak{T}^{\varrho}(\overline{1}) = 1$ and $\mathfrak{T}^{\varsigma}(\overline{0}) = \mathfrak{T}^{\varsigma}(\overline{1}) = \mathfrak{T}^{\varphi}(\overline{0}) = \mathfrak{T}^{\varphi}(\overline{1}) = 0$,

(SVNT2) $\mathfrak{T}^{\varrho}(\Pi \wedge \Upsilon) \ge \mathfrak{T}^{\varrho}(\Pi) \wedge \mathfrak{T}^{\varrho}(\Upsilon)$, $\mathfrak{T}^{\varsigma}(\Pi \wedge \Upsilon) \le \mathfrak{T}^{\varsigma}(\Pi) \vee \mathfrak{T}^{\varsigma}(\Upsilon)$,
$\mathfrak{T}^{\varphi}(\Pi \wedge \Upsilon) \le \mathfrak{T}^{\varphi}(\Pi) \vee \mathfrak{T}^{\varphi}(\Upsilon)$, for every $\Pi,\ \Upsilon \in \pounds^{\Omega}$,

(SVNT3) $\mathfrak{T}^{\varrho}(\bigvee_{j\in J}(\Pi_j) \ge \bigwedge_{j\in J}\mathfrak{T}^{\varrho}(\Pi_j)$,
$\mathfrak{T}^{\varsigma}(\bigvee_{j\in J}(\Pi_j) \le \bigvee_{j\in J}\mathfrak{T}^{\varsigma}(\Pi_j)$,
$\mathfrak{T}^{\varphi}(\bigvee_{j\in J}(\Pi_j) \le \bigvee_{j\in J}\mathfrak{T}^{\varphi}(\Pi_j)$, for every $\Pi_j \in \pounds^{\Omega}$.

Usually in this paper, we will use the abbreviation $\mathfrak{T}^{\varrho\varsigma\varphi}$ for $(\mathfrak{T}^{\varrho},\ \mathfrak{T}^{\varsigma},\ \mathfrak{T}^{\varphi})$ without any ambiguity.

Theorem 1 [22] Let $(\Omega,\ \mathfrak{T}^{\varrho\varsigma\varphi})$ be an *svnts*. Then, for every $\Pi \in \pounds^{\Omega}$ and $\varkappa \in \pounds_0$ we define the operator $\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}} : \pounds^{\Omega} \times \pounds_0 \to \pounds^{\Omega}$ as follows:

$$\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa) = \bigwedge\{\Upsilon \in \pounds^{\Omega} \mid \Pi \leq \Upsilon,\ \ \mathfrak{T}^{\varrho}(\Upsilon^{c})) \geq r,\ \ \mathfrak{T}^{\varsigma}(\Upsilon^{c})) \leq 1-\varkappa,\ \ \mathfrak{T}^{\varphi}(\Upsilon^{c})) \leq 1-\varkappa\}.$$

For any $\Pi,\ \Upsilon \in \pounds^{\Omega}$ and $\varkappa,\ s \in \pounds_0$, the operator $\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}$ satisfies the following conditions:
($\mathfrak{CL}_1$) $\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\bar{0},\ \varkappa) = \bar{0}$.
($\mathfrak{CL}_2$) $\Pi \leq \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)$.
($\mathfrak{CL}_3$) $\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ r) \vee \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon,\ \varkappa) = \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi \vee \Upsilon,\ \varkappa)$.
($\mathfrak{CL}_4$) $\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa) \leq \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon,\ s)$ if $\varkappa \leq s$.
($\mathfrak{CL}_5$) $\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa) = \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)$.

Definition 7 [23] Let $(\Omega,\ \mathfrak{T}^{\varrho\varsigma\varphi})$ be an *svnts*. Then, for every $\Pi \in \pounds^{\Omega}$ and $\varkappa \in \pounds_0$ we define the operator $int_{\mathfrak{T}^{\varrho\varsigma\varphi}} : \pounds^{\Omega} \times \pounds_0 \to \pounds^{\Omega}$ as follows:

$$int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa) = \bigvee\{\Upsilon \in \pounds^{\Omega} \mid \Pi \geq \Upsilon,\ \ \mathfrak{T}^{\varrho}(\Upsilon)) \geq r,\ \ \mathfrak{T}^{\varsigma}(\Upsilon)) \leq 1-\varkappa,\ \ \mathfrak{T}^{\varphi}(\Upsilon)) \leq 1-\varkappa\}.$$

Definition 8 [27] Let $\Omega \neq \phi$. A mapping $\mathfrak{P}^{\varrho},\ \mathfrak{P}^{\varsigma},\ \mathfrak{P}^{\varphi} : \pounds^{\Omega} \to \pounds$ is called a svn-primal on Ω, if it satisfies the following conditions:
($\mathfrak{P}_1$) $\mathfrak{P}^{\varrho}(\bar{1}) = 0,\ \mathfrak{P}^{\varsigma}(\bar{1}) = 1,\ \mathfrak{P}^{\varphi}(\bar{1}) = 1$ and $\mathfrak{P}^{\varrho}(\bar{0}) = 1,\ \mathfrak{P}^{\varsigma}(\bar{0}) = 0,\ \mathfrak{P}^{\varphi}(\bar{0}) = 0$.
($\mathfrak{P}_2$) $\mathfrak{P}^{\varrho}(\Pi \wedge \Upsilon) \leq \mathfrak{P}^{\varrho}(\Pi) \vee \mathfrak{P}^{\varrho}(\Upsilon),\ \mathfrak{P}^{\varsigma}(\Pi \wedge \Upsilon) \geq \mathfrak{P}^{\varsigma}(\Pi) \wedge \mathfrak{P}^{\varsigma}(\Upsilon),\ \mathfrak{P}^{\varphi}(\Pi \wedge \Upsilon) \geq \mathfrak{P}^{\varphi}(\Pi) \wedge \mathfrak{P}^{\varphi}(\Upsilon),\ \forall\, \Pi,\ \Upsilon \in \pounds^{\Omega}$.
($\mathfrak{P}_3$) If $\Pi \leq \Upsilon$, then $\mathfrak{P}^{\varrho}(\Upsilon) \leq \mathfrak{P}^{\varrho}(\Pi),\ \mathfrak{P}^{\varsigma}(\Upsilon) \geq \mathfrak{P}^{\varsigma}(\Pi),\ \ \mathfrak{P}^{\varphi}(\Upsilon) \geq \mathfrak{P}^{\varphi}(\pi),\ \forall\, \Pi,\ \Upsilon \in \pounds^{\Omega}$.
Sometimes, we will write $\mathfrak{P}^{\varrho\varsigma\varphi}$ for $(\mathfrak{P}^{\varrho},\ \mathfrak{P}^{\varsigma},\ \mathfrak{P}^{\varphi})$.

If $\mathfrak{P}^{\varrho\varsigma\varphi}$ and $\mathfrak{P}^{\star\varrho\varsigma\varphi}$ are svn-primals on Ω, then "$\mathfrak{P}^{\varrho\varsigma\varphi}$ is finer than $\mathfrak{P}^{\star\varrho\varsigma\varphi}$ or ($\mathfrak{P}^{\star\varrho\varsigma\varphi}$ is coarser than $\mathfrak{P}^{\varrho\varsigma\varphi}$)" denoted by $\mathfrak{P}^{\varrho\varsigma\varphi} \leq \mathfrak{P}^{\star\varrho\varsigma\varphi}$ if and only if

$$\mathfrak{P}^{\varrho}(\Pi) \leq \mathfrak{P}^{\star\varrho}(\Upsilon),\ \ \mathfrak{P}^{\varsigma}(\Pi) \geq \mathfrak{P}^{\star\varsigma}(\Upsilon),\ \ \mathfrak{P}^{\varphi}(\Pi) \geq \mathfrak{P}^{\star\varphi}(\Upsilon),\ \forall,\ \Pi,\ \Upsilon \in \pounds^{\Omega}.$$

The triable $(\Omega,\ \mathfrak{T}^{\varrho\varsigma\varphi},\ \mathfrak{P}^{\varrho\varsigma\varphi})$ is called a *svnpts*.

Definition 9 [27] Let $(\Omega,\ \mathfrak{T}^{\varrho\varsigma\varphi},\ \mathfrak{P}^{\varrho\varsigma\varphi})$ be a *svnpts*, for each $\varkappa \in \pounds_0$ and $\Pi \in \pounds^{\Omega}$. Then, the single-valued neutrosophic primal open local function $\Pi^{\star}_{\varkappa}(\mathfrak{T}^{\varrho\varsigma\varphi},\ \mathfrak{P}^{\varrho\varsigma\varphi})$ of Π is the union of all *svn-points* $u_{r,\ t,\ s}$ such that if $\Upsilon \in \mathscr{Q}_{\mathscr{T}^{\varrho\varsigma\varphi}}(u_{r,\ t,\ s},\ \varkappa)$ and $\mathfrak{P}^{\varrho}(\Theta) \geq \varkappa,\ \mathfrak{P}^{\varsigma}(\Theta) \leq 1-\varkappa,\ \mathfrak{P}^{\varphi}(\Theta) \leq 1-\varkappa$, then there is at least one $u \in \Omega$, for which $\varrho_{\Pi}(u) + \varrho_{\Upsilon}(u) - 1 > \varrho_{\Theta}(u)$, $\varsigma_{\Pi}(u) + \varsigma_{\Upsilon}(u) - 1 \leq \varsigma_{\Theta}(u)$, $\varphi_{\Pi}(u) + \varphi_{\Upsilon}(u) - 1 \leq \varphi_{\Theta}(u)$.

In this article, we will write $\Pi^{\star}_{\varkappa}$ for $\Pi^{\star}_{\varkappa}(\mathfrak{T}^{\varrho\varsigma\varphi},\ \mathfrak{P}^{\varrho\varsigma\varphi})$ without any ambiguity.

Lemma 1 [27] Let $(\Omega,\ \mathfrak{T}^{\varrho\varsigma\varphi},\ \mathfrak{P}^{\varrho\varsigma\varphi})$ be a *svnpts* and $\Pi \in \pounds^{\Omega}$; we can define

$$\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ r) = \Pi \cup \Pi^{\star}_{\varkappa},\quad \mathfrak{I}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa) = \Pi \wedge ((\Pi^{c})^{\star}_{\varkappa})^{c}.$$

Clearly, $\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}$ is an *svn-closure operator* and $(\mathfrak{T}^{\varrho}(\mathfrak{P}^{\varrho}),\ \mathfrak{T}^{\varsigma}(\mathfrak{P}^{\varsigma}),\ \mathfrak{T}^{\varphi}(\mathfrak{P}^{\varphi}))$ is the *svnt* generated by $\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}$, i.e.,

$$\mathfrak{T}^{\star}(\mathfrak{I})(\Pi) = \bigvee\{\varkappa \mid\ \ \mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi^{c},\ \varkappa) = \Pi^{c}\}.$$

Now, if

$\mathfrak{P}^{\varrho\varsigma\varphi} = \mathfrak{P}_0^{\varrho\varsigma\varphi}$, $\forall\ \Pi \in \pounds^{\Omega}$, then $\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa) = \Pi \cup \Pi^{\star}_{\varkappa} = \Pi \vee \mathfrak{C}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa) = \mathfrak{C}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)$. So, $\mathfrak{T}^{\varrho\star}(\mathfrak{P}_0^{\varrho}) = \mathfrak{T}^{\varrho}$, $\mathfrak{T}^{\varsigma\star}(\mathfrak{P}_0^{\varsigma}) = \mathfrak{T}^{\varsigma}$ and $\mathfrak{T}^{\varphi\star}(\mathscr{P}_0^{\varphi}) = \mathfrak{T}^{\varphi}$.

Theorem 2 [27] Let $(\Omega,\ \mathfrak{T}^{\varrho\varsigma\varphi})$ be an *svnts* and $\mathfrak{P}_1^{\tau\pi\sigma}$, $\mathfrak{P}_2^{\varrho\varsigma\varphi}$ be two *svn-primals* on Ω. Then, for each $\Pi,\ \Upsilon \in \pounds^{\Omega}$ and $\varkappa \in \pounds_0$, we have

(1) If $\Pi \leq \Upsilon$, then $\Pi^{\star}_{\varkappa} \leq \Upsilon^{\star}_{\varkappa}$.

(2) If $\mathfrak{P}_1^{\varrho} \leq \mathfrak{P}_2^{\varrho}$, $\mathfrak{P}_1^{\varsigma} \geq \mathfrak{P}_2^{\varsigma}$ and $\mathfrak{P}_1^{\varphi} \geq \mathfrak{P}_2^{\varphi}$, then $\Pi^{\star}_{\varkappa}(\mathfrak{P}_1^{\varrho\varsigma\varphi},\ \mathfrak{T}^{\varrho\varsigma\varphi}) \geq \Pi^{\star}_{\varkappa}(\mathfrak{P}_2^{\varrho\varsigma\varphi},\ \mathfrak{T}^{\varrho\varsigma\varphi})$.

(3) $\Pi^{\star}_{\varkappa} = \mathfrak{C}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi^{\star}_{\varkappa},\ \varkappa) \leq \mathfrak{C}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)$.

(4) If $(\Pi^{\star}_{\varkappa})^{\star}_{\varkappa} \leq \Pi^{\star}_{\varkappa}$.

(5) If $\mathfrak{P}^{\varrho}(\Theta) \geq \varkappa$, $\mathfrak{P}^{\varsigma}(\Upsilon) \leq 1-\varkappa$ and $\mathfrak{P}^{\varphi}(\Upsilon) \leq 1-\varkappa$, then $(\Pi \vee \Upsilon)^{\star}_{\varkappa} = \Pi^{\star}_{\varkappa} \vee \Upsilon^{\star}_{\varkappa} = \Pi^{\star}_{\varkappa}$.

(6) $(\Pi^{\star}_{\varkappa} \vee \Upsilon^{\star}_{\varkappa}) = (\Pi \vee \Upsilon)^{\star}_{\varkappa}$.

(7) If $\mathfrak{P}^{\varrho}(\Theta) \geq \varkappa$, $\mathfrak{P}^{\varsigma}(\Upsilon) \leq 1-\varkappa$ and $\mathfrak{P}^{\varphi}(\Upsilon) \leq 1-\varkappa$, then, $(\Upsilon \wedge \Pi^{\star}_{\varkappa}) \leq (\Upsilon \wedge \Pi)^{\star}_{\varkappa}$.

(8) $(\Pi^{\star}_{\varkappa} \wedge \Theta^{\star}_{\varkappa}) \geq (\Pi \wedge \Upsilon)^{\star}_{\varkappa}$.

3. Several types of single-valued neutrosophic primal sets

Within the correctness mathematical harmony, the single-valued neutrosophic primal (*svn-primal*) is the result of the combination of neutrosophic logic with primal set theory. These primals provide an integrative structure to deal with uncertainty by serving as crucial bridges between the fields of soft computing and neutrosophic research. Precisely defined, these *svn-primals* reveal their core, from compatibility conditions controlling intersection operations to basic characteristics capturing limit conditions. A central theorem emphasizes group interaction by giving an orderly approach to combining separate primals into an integrated unit. Through our study of this mathematical setting, the *svn-primals* not only reveal their complexities but also open up fresh prospects for the construction of single-valued neutrosophic (closure, interior, per-primal interior, α-primal interior, β-primal closure, β-primal interior) operators (for short, $\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}$, $\mathfrak{I}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}$, $P\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}$, $\alpha\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}$, $\beta\mathfrak{P}\mathfrak{C}\mathfrak{L}_{\mathfrak{T}^{\varrho\varsigma\varphi}}$, $\beta\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}} : \pounds^{\Omega} \times \pounds_0 \to \pounds^{\Omega}$) respectively, showing their important impact on precisely and effectively understanding uncertainty. We begin this section by defining the notions of $\varkappa$-single-valued neutrosophic [primal, semi-primal, pre-primal, α-primal, β-primal, strongly-β-primal, regular primal, δ-primal, $\beta^{\star}$-primal, $P^{\star}$-primal, strongly pre-primal] open sets (for short, *$\varkappa$-svnpo*, *$\varkappa$-svnspo*, *$\varkappa$-svnppo*, *$\varkappa$-svnαpo*, *$\varkappa$-svnβpo*, *$\varkappa$-svnsβpo*, *$\varkappa$-svnrpo*, *$\varkappa$-svnδpo*, *$\varkappa$-svn$\beta^{\star}$po*, *$\varkappa$-svn$P^{\star}$po*, *$\varkappa$-svnsppo*) respectively.

Definition 10 Let $(\Omega,\ \mathfrak{T}^{\varrho\varsigma\varphi},\ \mathfrak{P}^{\varrho\varsigma\varphi})$ be a *svnpts* and $\varkappa \in \pounds_0$. Then $\Pi \in \pounds^{\Omega}$ is called:

(1) *$\varkappa$-svnpo* if $\Pi \leq int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi^{\star}_{\varkappa},\ \varkappa)$.

(2) *$\varkappa$-svnspo* if $\Pi \leq \mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa)$.

(3) *$\varkappa$-svnppo* if $\Pi \leq int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa)$.

(4) *$\varkappa$-svnαpo* if $\Pi \leq int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa),\ \varkappa)$.

(5) *$\varkappa$-svnβpo* if $\Pi \leq \mathfrak{C}\mathfrak{L}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa),\ \varkappa)$.

(6) *$\varkappa$-svnsβpo* if $\Pi \leq \mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa),\ \varkappa)$.

(7) *$\varkappa$-svnrpo* if $\Pi \leq int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa)$.

(8) *$\varkappa$-svnδpo* if $int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ r),\ r) \leq \mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa)$.

The complement of *$\varkappa$-svnpo* (resp, *$\varkappa$-svnspo*, *$\varkappa$-svnppo-set*, *$\varkappa$-svnαpo*, *$\varkappa$-svnβpo*, *$\varkappa$-svnsβpo*, *$\varkappa$-svnδpo*) are called *$\varkappa$-svnpc* (resp, *$\varkappa$-svnspc*, *$\varkappa$-svnppc*, *$\varkappa$-svnαpc*, *$\varkappa$-svnβpc*, *$\varkappa$-svnsβpc*, *$\varkappa$-svnδpc*).

After the above definition, we get the next drawing:

$$\varkappa - svn\delta po$$

$$\Uparrow$$

$$\varkappa\text{-}svno \Rightarrow \varkappa\text{-}svn\alpha po \Rightarrow \varkappa\text{-}svnspo \Rightarrow \varkappa\text{-}svnso$$

$$\varkappa\text{-}svnpo \Rightarrow \quad \Downarrow \qquad \Downarrow \qquad \Downarrow$$

$$\varkappa\text{-}svnro \Rightarrow \varkappa\text{-}svnrpo \Rightarrow \varkappa\text{-}svnppo \Rightarrow \varkappa\text{-}svn\beta po \Rightarrow \varkappa\text{-}svn\beta o$$

$$\Downarrow$$

$$\varkappa\text{-}svnspo \Rightarrow \varkappa\text{-}svns\beta po \Rightarrow \varkappa\text{-}svn\beta po.$$

Example 1 Presume that $\Omega = \{u_1, u_2, u_3\}$; define the *svn-sets* $\Phi_1, \Phi_2, \Phi_3, \Upsilon \in £^{\Omega}$ as follows

$$\Phi_1 = \langle (0.5, 0.5, 0.5), (0.5, 0.5, 0.5), (0.5, 0.5, 0.5) \rangle,$$

$$\Phi_2 = \langle (0.4, 0.4, 0.4), (0.5, 0.5, 0.5), (0.5, 0.5, 0.5) \rangle,$$

$$\Phi_3 = \langle (0.4, 0.4, 0.4), (0.4, 0.4, 0.4), (0.4, 0.4, 0.4) \rangle,$$

$$\Upsilon = \langle (0.1, 0.1, 0.1), (0, 0, 0), (0, 0, 0) \rangle.$$

Define the mapping, $\mathfrak{T}^{\varrho} : £^{\Omega} \to £$, $\mathfrak{T}^{\varsigma} : £^{\Omega} \to £$, $\mathfrak{T}^{\varphi} : £^{\Omega} \to £$ and $\mathfrak{P}^{\varrho} : £^{\Omega} \to £$, $\mathfrak{P}^{\varsigma} : £^{\Omega} \to £$, $\mathfrak{P}^{\varphi} : £^{\Omega} \to £$ as follows:

$$\mathfrak{T}^{\varrho}(\Pi) = \begin{cases} 1, & \text{if } \Pi = \overline{0}, \\ 1, & \text{if } \Pi = \overline{1}, \\ \frac{1}{2}, & \text{if } \Pi = \{\Phi_1, \Phi_2\}, \\ 0, & \text{otherwise}, \end{cases} \qquad \mathfrak{P}^{\varrho}(\Pi) = \begin{cases} 1, & \text{if } \Pi = \overline{0}, \\ \frac{1}{4}, & \text{if } \Pi = \Upsilon, \\ \frac{1}{2}, & \text{if } \overline{0} < \Pi < \Upsilon, \\ 0, & \text{otherwise}, \end{cases}$$

$$\mathfrak{T}^{\varsigma}(\Pi)=\begin{cases}0, & \text{if } \Pi=\overline{0},\\ 0, & \text{if } \Pi=\overline{1},\\ \frac{1}{2}, & \text{if } \Pi=\{\Phi_2,\ \Phi_3\},\\ 1, & \text{otherwise},\end{cases}\quad \mathfrak{P}^{\varsigma}(\Pi)=\begin{cases}0, & \text{if } \Pi=\overline{0},\\ \frac{3}{4}, & \text{if } \Pi=\Upsilon,\\ \frac{1}{2}, & \text{if } \overline{0}<\Xi<\Upsilon,\\ 1, & \text{otherwise},\end{cases}$$

$$\mathfrak{T}^{\varphi}(\Pi)=\begin{cases}0, & \text{if } \Pi=\overline{0},\\ 0, & \text{if } \Pi=\overline{1},\\ \frac{1}{4}, & \text{if } \Pi=\{\Phi_2,\ \Phi_3\},\\ 1, & \text{otherwise},\end{cases}\quad \mathfrak{P}^{\varphi}(\Pi)=\begin{cases}0, & \text{if } \Pi=\overline{0},\\ \frac{2}{3}, & \text{if } \Pi=\Upsilon,\\ \frac{1}{2}, & \text{if } \overline{0}<\Pi<\Upsilon,\\ 1, & \text{otherwise}.\end{cases}$$

Suppose that $\Theta=\langle(0.3,\ 0.3,\ 0.3),\ (0.3,\ 0.3,\ 0.3),\ (0.3,\ 0.3,\ 0.3)\rangle$, then, Θ is $\frac{1}{2}$-*svnppo* but it is not $\frac{1}{2}$-*svnspo*.

Example 2 Consider $\Omega=\{u_1,\ u_2,\ u_3\}$; define the *svns* $\Phi_1,\ \Phi_2,\ \Phi_3,\ \Upsilon\in £^{\Omega}$ as follows

$$\Phi_1=\langle(0.3,\ 0.3,\ 0.3),\ (0.3,\ 0.3,\ 0.3),\ (0.3,\ 0.3,\ 0.3)\rangle,$$

$$\Phi_2=\langle(0.5,\ 0.5,\ 0.5),\ (0.5,\ 0.5,\ 0.5),\ (0.5,\ 0.5,\ 0.5)\rangle,$$

$$\Phi_3=\langle(0.6,\ 0.6,\ 0.6),\ (0.6,\ 0.6,\ 0.6),\ (0.6,\ 0.6,\ 0.6)\rangle,$$

$$\Upsilon=\langle(0.3,\ 0.3,\ 0.3),\ (0,\ 0,\ 0),\ (0,\ 0,\ 0)\rangle.$$

Define the mapping, $\mathfrak{T}^{\varrho}:£^{\Omega}\to £$, $\mathfrak{T}^{\varsigma}:£^{\Omega}\to £$, $\mathfrak{T}^{\varphi}:£^{\Omega}\to £$ and $\mathfrak{P}^{\varrho}:£^{\Omega}\to £$, $\mathfrak{P}^{\varsigma}:£^{\Omega}\to £$, $\mathfrak{P}^{\varphi}:£^{\Omega}\to £$ as follows:

$$\mathfrak{T}^{\varrho}(\Pi)=\begin{cases}1, & \text{if } \Pi=\overline{0},\\ 1, & \text{if } \Pi=\overline{1},\\ \frac{1}{2}, & \text{if } \Pi=\Phi_3,\\ 0, & \text{otherwise},\end{cases}\quad \mathfrak{P}^{\varrho}(\Pi)=\begin{cases}1, & \text{if } \Pi=\overline{0},\\ \frac{1}{4}, & \text{if } \Pi=\Upsilon,\\ \frac{1}{2}, & \text{if } \overline{0}<\Pi<\Upsilon,\\ 0, & \text{otherwise},\end{cases}$$

$$\mathfrak{T}^{\varsigma}(\Pi)=\begin{cases}0, & \text{if } \Pi=\overline{0},\\ 0, & \text{if } \Pi=\overline{1},\\ \frac{1}{2}, & \text{if } \Pi=\Phi_2,\\ 1, & \text{otherwise,}\end{cases}\quad \mathfrak{P}^{\varsigma}(\Pi)=\begin{cases}0, & \text{if } \Pi=\overline{0},\\ \frac{3}{4}, & \text{if } \Pi=\Upsilon,\\ \frac{1}{2}, & \text{if } \overline{0}<\Xi<\Upsilon,\\ 1, & \text{otherwise,}\end{cases}$$

$$\mathfrak{T}^{\varphi}(\Pi)=\begin{cases}0, & \text{if } \Pi=\overline{0},\\ 0, & \text{if } \Pi=\overline{1},\\ \frac{1}{4}, & \text{if } \Pi=\Phi_1,\\ 1, & \text{otherwise,}\end{cases}\quad \mathfrak{P}^{\varphi}(\Pi)=\begin{cases}0, & \text{if } \Pi=\overline{0},\\ \frac{2}{3}, & \text{if } \Pi=\Upsilon,\\ \frac{1}{2}, & \text{if } \overline{0}<\Pi<\Upsilon,\\ 1, & \text{otherwise.}\end{cases}$$

Suppose that $\Theta=\langle(0.4,\ 0.4,\ 0.4),\ (0.4,\ 0.4,\ 0.4),\ (0.4,\ 0.4,\ 0.4)\rangle$, then, Θ is $\frac{1}{2}$-*svnspo* but it is neither $\frac{1}{2}$-*svnppo* nor $\frac{1}{2}$-*svnpo*.

Lemma 2 Let $(\Omega,\ \mathfrak{T}^{\varrho\varsigma\varphi},\ \mathfrak{P}^{\varrho\varsigma\varphi})$ be a *svnpts*, $\mathfrak{T}^{\varrho}(\Upsilon)\geq\varkappa$, $\mathfrak{T}^{\varsigma}(\Upsilon)\leq 1-\varkappa$, $\mathfrak{T}^{\varphi}(\Upsilon)\leq 1-\varkappa$ and $r\in \pounds_0$. Then $\Upsilon\wedge \mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)\leq \mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi\wedge\Upsilon,\ \varkappa)$.

Proof. Let $\mathfrak{T}^{\varrho}(\Upsilon)\geq\varkappa$, $\mathfrak{T}^{\varsigma}(\Upsilon)\leq 1-\varkappa$ and $\mathfrak{T}^{\varphi}(\Upsilon)\leq 1-\varkappa$. Then by Theorem 2 (7),

$$\Upsilon\wedge \mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ r)=\Upsilon\wedge(\Pi\vee\Pi^{\star}_{r})=(\Upsilon\wedge\Pi)\vee(\Upsilon\wedge\Pi^{\star}_{\varkappa})$$

$$\leq(\Upsilon\wedge\Pi)\vee(\Upsilon\wedge\Pi)^{\star}_{\varkappa}=\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon\wedge\Pi,\ \varkappa).$$

□

Lemma 3 Let $(\Omega,\ \mathfrak{T}^{\varrho\varsigma\varphi},\ \mathfrak{P}^{\varrho\varsigma\varphi})$ be a *svnpts*, for every $\Pi\in\pounds^{\Omega}$ and $r\in\pounds_0$. Then Π is *svnβpo* iff $\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ r)=\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa),\ \varkappa)$.

Proof. Let Π be an $\varkappa$-*svnβpo*. Then $\Pi\leq\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(Cl^{\star}(\Pi,\ \varkappa),\ \varkappa),\ \varkappa)$ and thus,

$$\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)\leq\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{CL}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa),\ \varkappa)$$

$$\leq\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa),\ \varkappa)\leq\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa).$$

Hence, $\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ r)=\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa),\ \varkappa)$. □

Theorem 3 Let $(\Omega,\ \mathfrak{T}^{\varrho\varsigma\varphi},\ \mathfrak{P}^{\varrho\varsigma\varphi})$ be a *svnpts*, for every $\varkappa\in\pounds_0$ and $\Pi,\ \Upsilon\in\pounds^{\Omega}$. Then the subsequent statements are holds:

(1) If Υ is *r-svnβpo* and $\mathfrak{T}^{\varrho}(\Pi)\geq\varkappa$, $\mathfrak{T}^{\varsigma}(\Pi)\leq 1-\varkappa$, $\mathfrak{T}^{U\varphi}(\Pi)\leq 1-\varkappa$, then $\Pi\wedge\Upsilon$ is $\varkappa$-*svnβpo*.

(2) If Π is *ϰ-svnspo* and Υ is *ϰ-svnαpo*, then $\Pi \wedge \Upsilon$ is *ϰ-svnspo*.
(3) If Π is *ϰ-svnppo* and Υ is *ϰ-svnαpo*, then $\Pi \wedge \Upsilon$ is *ϰ-svnppo*.
(4) If Π and Υ are *ϰ-svnαpo*, then $\Pi \wedge \Upsilon$ is *ϰ-svnαpo*.
(5) If Υ is *ϰ-svnspo* and $\mathfrak{T}^{\varrho}(\Pi) \geq \varkappa$, $\mathfrak{T}^{\varsigma}(\Pi) \leq 1-\varkappa$, $\mathfrak{T}^{U\varphi}(\Pi) \leq 1-\varkappa$, then $\Pi \wedge \Upsilon$ is *ϰ-svnspo*.
(6) If Υ is *ϰ-svnppo* and $\mathfrak{T}^{\varrho}(\Pi) \geq \varkappa$, $\mathfrak{T}^{\varsigma}(\Pi) \leq 1-\varkappa$, $\mathfrak{T}^{U\varphi}(\Pi) \leq 1-\varkappa$, then $\Pi \wedge \Upsilon$ is *ϰ-svnppo*.
Proof. (1) Let Υ be a *ϰ-svnβpo* and $\mathfrak{T}^{\varrho}(\Pi) \geq r\varkappa$, $\mathfrak{T}^{\varsigma}(\Pi) \leq 1-\varkappa$, $\mathfrak{T}^{U\varphi}(\Pi) \leq 1-\varkappa$, then by Theorem 2 (7),

$$\Upsilon \wedge \Pi \leq \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon,\ \varkappa),\ \varkappa),\ \varkappa) \wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)$$

$$= \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}([\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon,\ \varkappa) \wedge \Pi],\ \varkappa),\ \varkappa)$$

$$= \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}([(\Upsilon \vee \Upsilon^{\star}_{\varkappa}) \wedge \Pi],\ \varkappa),\ \varkappa)$$

$$= \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}([(\Upsilon \wedge \Pi) \vee (\Upsilon^{\star}_{\varkappa} \wedge \Pi)],\ \varkappa),\ \varkappa)$$

$$\leq \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}([(\Upsilon \wedge \Pi) \vee (\Upsilon \wedge \Pi)^{\star}_{\varkappa}],\ \varkappa),\ \varkappa)$$

$$= \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon \wedge \Pi,\ \varkappa),\ \varkappa),\ \varkappa).$$

This show that $\Pi \wedge \Upsilon$ is *ϰ-svnβpo*.
(2) Let Π is *ϰ-svnspo* and Υ is *ϰ-svnαpo*, then by using Lemma 1, we have

$$\Pi \wedge \Upsilon \leq \mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa) \wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon,\ \varkappa),\ \varkappa),\ \varkappa)$$

$$\leq \mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa) \wedge \mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon,\ \varkappa),\ \varkappa)$$

$$\leq \mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}([int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa) \wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon,\ \varkappa)],\ \varkappa),\ \varkappa)$$

$$\leq \mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi \wedge \Upsilon,\ \varkappa),\ \varkappa).$$

This show that $\Pi \wedge \Upsilon$ is *ϰ-svnspo*.
(3) and (4) are similar to (2).
(5) and (6) are similar to (1). □

Example 3 Consider $\Omega = \{u_1,\ u_2,\ u_3\}$; define the *svn-sets* $\Phi_1,\ \Phi_2,\ \Pi_1,\ \Pi_2,\ \Upsilon \in \pounds^{\Omega}$ as follows

$$\Phi_1 = \langle (0.3,\ 0.3,\ 0.3),\ (0.2,\ 0.2,\ 0.2),\ (0.2,\ 0.2,\ 0.2) \rangle,$$

$$\Phi_2 = \langle (0.2,\ 0.2,\ 0.2),\ (0.2,\ 0.2,\ 0.2),\ (0.2,\ 0.2,\ 0.2) \rangle,$$

$$\Pi_1 = \langle (0.3,\ 0.3,\ 0.3),\ (0.6,\ 0.6,\ 0.6),\ (0.6,\ 0.6,\ 0.6) \rangle,$$

$$\Pi_2 = \langle (0.4,\ 0.4,\ 0.4),\ (0,\ 0,\ 0),\ (0,\ 0,\ 0) \rangle,$$

$$\Upsilon = \langle (0.2,\ 0.2,\ 0.2),\ (0,\ 0,\ 0),\ (0,\ 0,\ 0) \rangle.$$

We define, $\mathfrak{T}^{\varrho} : \pounds^{\Omega} \to \pounds$, $\mathfrak{T}^{\varsigma} : \pounds^{\Omega} \to \pounds$, $\mathfrak{T}^{\varphi} : \pounds^{\Omega} \to \pounds$ and $\mathfrak{P}^{\varsigma} : \pounds^{\Omega} \to \pounds$, $\mathfrak{P}^{\varphi} : \pounds^{\Omega} \to \pounds$ as follows:

$$\mathfrak{T}^{\varrho}(\Pi) = \begin{cases} 1, & \text{if } \Pi = \overline{0}, \\ 1, & \text{if } \Pi = \overline{1}, \\ \frac{1}{2}, & \text{if } \Pi = \{\Phi_1, \Phi_2\}, \\ 0, & \text{otherwise}, \end{cases} \qquad \mathfrak{P}^{\varrho}(\Pi) = \begin{cases} 1, & \text{if } \Pi = \overline{0}, \\ \frac{1}{4}, & \text{if } \Pi = \Upsilon, \\ \frac{1}{2}, & \text{if } \overline{0} < \Pi < \Upsilon, \\ 0, & \text{otherwise}, \end{cases}$$

$$\mathfrak{T}^{\varsigma}(\Pi) = \begin{cases} 0, & \text{if } \Pi = \overline{0}, \\ 0, & \text{if } \Pi = \overline{1}, \\ \frac{1}{3}, & \text{if } \Pi = \{\Phi_1, \Phi_2\}, \\ 1, & \text{otherwise}, \end{cases} \qquad \mathfrak{P}^{\varsigma}(\Pi) = \begin{cases} 0, & \text{if } \Pi = \overline{0}, \\ \frac{3}{4}, & \text{if } \Pi = \Upsilon, \\ \frac{1}{2}, & \text{if } \overline{0} < \Xi < \Upsilon, \\ 1, & \text{otherwise}, \end{cases}$$

$$\mathfrak{T}^{\varphi}(\Pi) = \begin{cases} 0, & \text{if } \Pi = \overline{0}, \\ 0, & \text{if } \Pi = \overline{1}, \\ \frac{1}{4}, & \text{if } \Pi = \{\Phi_1, \Phi_2\}, \\ 1, & \text{otherwise}, \end{cases} \qquad \mathfrak{P}^{\varphi}(\Pi) = \begin{cases} 0, & \text{if } \Pi = \overline{0}, \\ \frac{2}{3}, & \text{if } \Pi = \Upsilon, \\ \frac{1}{2}, & \text{if } \overline{0} < \Pi < \Upsilon, \\ 1, & \text{otherwise}. \end{cases}$$

(1) $\Pi_1 \wedge \Pi_2$ is $\frac{1}{2}$-*svnpso* but Π_1 it is not $\frac{1}{2}$-*svnpαo* and Π_2 is not $\frac{1}{2}$-*svnpso*.

(2) $\Pi_1 \wedge \Pi_2$ is $\frac{1}{2}$-*svnpso* but $\mathfrak{T}^{\varrho}(\Pi_1) \geq \frac{1}{2}$, $\mathfrak{T}^{\varsigma}(\Pi) \leq \frac{1}{2}$, $\mathfrak{T}^{U\varphi}(\Pi) \leq \frac{1}{2}$, and Π_2 is not $\frac{1}{2}$-*svnpso*.

Lemma 4 Let $(\Omega,\ \mathfrak{T}^{\varrho\varsigma\varphi},\ \mathfrak{P}^{\varrho\varsigma\varphi})$ be a *svnpts*, for every $\varkappa \in \pounds_0$ and $\Pi,\ \Upsilon \in \pounds^{\Omega}$. Then the subsequent statements hold:

(1) Any union of $\varkappa$-βpo sets is $\varkappa$-βpo.

(2) Any intersection of $\varkappa$-βpc sets is $\varkappa$-βpc.

Proof. (1) Assume that $\{\Pi_j\}_{j\in J}$ is a collection of $\varkappa$-*svn*βpo sets. Then for each $j \in J$ we have. $\Pi_j \leq \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi_j,\ \varkappa),\ \varkappa),\ \varkappa)$. Thus,

$$\bigvee_{j\in J}\Pi_j \le \bigvee_{j\in J}\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi_i,\ \varkappa),\ \varkappa),\ \varkappa)$$

$$\le \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\bigvee_{j\in J}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi_i,\ \varkappa),\ \varkappa)),\ \varkappa)$$

$$\le \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\bigvee_{i\in\Gamma}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi_j,\ \varkappa)),\ \varkappa),\ \varkappa)$$

$$= \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\bigvee_{j\in J}(\Pi\vee(\Pi_j)^{\star}_{\varkappa}),\ \varkappa),\ \varkappa)$$

$$= \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}([\bigvee_{j\in J}\Pi\vee\bigvee_{j\in J}(\Pi_j)^{\star}_{\varkappa}],\ \varkappa),\ \varkappa)$$

$$\le \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}([\bigvee_{j\in J}\Pi\vee(\bigvee_{j\in J}\Pi_j)^{\star}_{\varkappa}],\ \varkappa),\ \varkappa)$$

$$= \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\bigvee_{j\in J}\Pi,\ \varkappa),\ \varkappa),\ \varkappa).$$

Therefore, $\bigvee_{j\in J}\pi_j$ is $\varkappa$-*svnβpo* sets.

(2) Likewise, using related reasoning, we can determine that $\bigwedge_{j\in J}\pi_j$ is $\varkappa$-*svnβpc* sets. □

Theorem 4 Let $(\Omega,\ \mathfrak{T}^{\varrho\varsigma\varphi},\ \mathfrak{P}^{\varrho\varsigma\varphi})$ be a *svnpts*, for each $\varkappa\in\pounds_0$ and $\Pi,\ \Upsilon\in\pounds^{\Omega}$, we define the operators $P\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}$, $\alpha\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}$, $\beta\mathfrak{PCL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}$, $\beta\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}:\pounds^{\Omega}\times\pounds_0\to\pounds^{\Omega}$ as follows:

$$P\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)=\bigvee\{\Upsilon\in\pounds^{\Omega}:\ \Upsilon\le\Pi,\ \Upsilon\ is\ \varkappa-svnppo\}.$$

$$\alpha\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)=\bigvee\{\Upsilon\in\pounds^{\Omega}:\ \Upsilon\le\Pi,\ \Upsilon\ is\ \varkappa-svn\alpha po\}.$$

$$\beta\mathfrak{PCL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)=\bigwedge\{\Upsilon\in\pounds^{\Omega}\mid\Pi\le\Upsilon,\ \Upsilon\ is\ \varkappa-svn\beta pc\}.$$

$$\beta\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)=\bigvee\{\Upsilon\in\pounds^{\Omega}\mid\Upsilon\le\Pi,\ \Upsilon\ is\ \varkappa-svn\beta po\}.$$

Then,

(1) Π is $\varkappa$-*svnppo* iff $P\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)=\Pi$.

(2) Π is $\varkappa$-*svnαpo* iff $\alpha\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)=\Pi$.

(3) $P\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)=\Pi\wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa)$.

(4) $\alpha\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)=\Pi\wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa),\ \varkappa)$.

(5) $\beta\mathfrak{PCL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\underline{0},\ \varkappa)=\underline{0}$.

(6) $\Pi\le\beta\mathfrak{PCL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)$.

(7) $\beta\mathfrak{PCL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi^c,\ \varkappa) = (\beta\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa))^c$.
(8) $int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa) \leq \beta\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa) \leq \Pi \leq \beta\mathfrak{PCL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa) \leq \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)$.
(9) $\beta\mathfrak{PCL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa) \vee \beta\mathfrak{PCL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon,\ \varkappa) \leq \beta\mathfrak{PCL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)(\Pi \vee \Upsilon,\ r\varkappa)$.
(10) $\beta\mathfrak{PCL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\beta\mathfrak{PCL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa) = \beta\mathfrak{PCL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)$.
(11) Π is **$\varkappa$-svnβpc**, iff $\Pi = \beta\mathfrak{PCL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)$.
(12) $P\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa) = \Pi \wedge \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa),\ \varkappa)$.
Proof. By the relevant definitions $P\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}$ and $\alpha\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}$, we can easily prove (1) and (2).
(3) Since

$$\Pi \wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa) \leq int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa) = iint_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa),\ \varkappa)$$

$$= int_{\mathfrak{T}^{\varrho\varsigma\varphi}}([\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ r) \wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}\, int_{\tau}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa)],\ \varkappa)$$

$$\leq int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}([\Pi \wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa)],\ \varkappa),\ \varkappa).$$

$\Pi \wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa)$ is *$\varkappa$-svnppo* contained in Π. So, $\Pi \wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa) \leq P\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)$. Since $P\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)$ is *$\varkappa$-svnppo*, we have

$$P\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa) \leq \Pi \wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(P\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa),\ \varkappa).$$

Thus, $P\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa) = \Pi \wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa)$.
(4) Similarly to (3).
(5) $\Rightarrow$ (9) and (11) are easily proved from the definitions of $\beta\mathfrak{PCL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}$ and $\beta\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}$.
(10) form (8) we only show $\beta\mathfrak{PCL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\beta\mathfrak{PCL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa) \leq \beta\mathfrak{PCL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)$.
Assume that $\beta\mathfrak{PCL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa) \not\geq \beta\mathfrak{PCL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\beta\mathfrak{PCL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa)$. There exist $u \in \Omega$ and $z \in \pounds_0$ such that

$$\beta\mathfrak{PCL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)(u) < z < \beta\mathfrak{PCL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\beta\mathfrak{PCL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa)(u). \tag{1}$$

Since $\beta\mathfrak{PCL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)(u) < z$, by definition $\beta\mathfrak{PCL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}$ there exists *$\varkappa$-svnβpc* set $\Upsilon \in \pounds^{\Omega}$ with $\Pi \leq \Upsilon$ such that

$$\beta\mathfrak{PCL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)(u) \leq \Upsilon(u) < z.$$

Since $\Pi \leq \Upsilon$, we have $\beta\mathfrak{PCL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa) \leq \Upsilon$. Again, by the definition of $\beta\mathfrak{PCL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}$, we have $\beta\mathfrak{PCL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\beta\mathfrak{PCL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa) \leq \Upsilon$. Thus, $\beta\mathfrak{PCL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\beta\mathfrak{PCL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa)(u) \leq \Upsilon(u) < z$. It is a contradiction for equation (1).
(12) Since

$$\pi \wedge \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa),\ \varkappa)$$

$$\leq \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa),\ \varkappa)$$

$$= \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}([int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa) \wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa)],\ \varkappa)$$

$$= \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}([\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa) \wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa)],\ \varkappa),\ \varkappa)$$

$$\leq \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}([\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa) \wedge \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa),\ \varkappa)],\ \varkappa),\ \varkappa)$$

$$= \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}([\Pi \wedge \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(Cl^{\star}(\Pi,\ \varkappa),\ \varkappa),\ \varkappa)],\ \varkappa),\ \varkappa),\ \varkappa).$$

Hence, $\Pi \wedge \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa),\ \varkappa)$ is $\varkappa$-*svn*β*po* set contained in π and so $\Pi \wedge \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa),\ \varkappa) \leq P\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)$.

Since $P\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)$ is *r*-$\varkappa$-*svn*β*po* set,

$$P\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa) \leq \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(P\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa),\ \varkappa),\ \varkappa)$$

$$\leq \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa),\ \varkappa).$$

So, $\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa),\ \varkappa) \geq P\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)$. Thus,

$$\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa),\ \varkappa) = P\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa).$$

□

Definition 11 Let $(\Omega,\ \mathfrak{T}^{\varrho\varsigma\varphi},\ \mathfrak{P}^{\varrho\varsigma\varphi})$ be a *svnpts*, for each $\Pi,\ \Upsilon \in \pounds^{\Omega}$, $\varkappa \in \pounds_0$ and $u_{r,\ t,\ s} \in svn-point(\Omega)$. Then,

(1) Π is called $\varkappa$-single valued neutrosophic $Q_{\mathfrak{T}^{\varrho\varsigma\varphi}}$-neighborhood of $u_{r,\ t,\ s}$ ($\mathscr{Q}_{\mathscr{T}\tau\pi\sigma}(u_{r,\ t,\ s},\ \varkappa)$ if $u_{r,\ t,\ s}\ q\ \Pi$ with $\mathfrak{T}^{\varrho}(\Pi) \geq \varkappa$, $\mathfrak{T}^{\varsigma}(\Pi) \leq 1-\varkappa$ and $\mathfrak{T}^{\varphi}(\Pi) \leq 1-\varkappa$.

(2) $u_{r,\ t,\ s}$ is called $\varkappa$-single valued neutrosophic $\delta\mathfrak{P}$-cluster point ($\varkappa$-*svn*$\delta\mathfrak{P}$-*cluster point*) of Π if for any $\Upsilon \in \mathscr{Q}_{\mathscr{T}\tau\pi\sigma}(u_{r,\ t,\ s},\ \varkappa)$, we have $\Pi\ q\ int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon,\ \varkappa),\ \varkappa)$.

(3) $\delta\mathfrak{P}$-closure ($\mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}}$) operator is a mapping $\mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}} : \pounds^{\Omega} \times \pounds_0 \to \pounds^{\Omega}$ defined as:

$$\mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa) = \bigvee\{u_{r,\ t,\ s} \in svn-point(\Omega) \mid\ u_{r,\ t,\ s}\ is\ \varkappa - svn\delta\mathfrak{P} - cluster\ \ point\ of\ \Pi\}.$$

(4) Π is called $\varkappa$-single valued neutrosophic $\mathfrak{R}_{\mathfrak{T}^{\varrho\varsigma\varphi}}$-neighborhood of $u_{r,\ t,\ s}$ ($\mathscr{R}_{\mathscr{T}\tau\pi\sigma}(u_{r,\ t,\ s},\ \varkappa)$ if $u_{r,\ t,\ s}\ q\ \Pi$ and Π is $\varkappa$-*svnro*.

(5) $u_{r,\ t,\ s}$ is called $\varkappa$-single valued neutrosophic δ-cluster point ($\varkappa$-*svn*δ-*cluster point*) of Π if for any $\Upsilon \in \mathscr{R}_{\mathscr{T}\tau\pi\sigma}(u_{r,\ t,\ s},\ \varkappa)$, we have $\Pi\ q\ \Upsilon$.

(6) δ-closure ($\mathscr{D}^{\delta}_{\mathfrak{T}^{\varrho\varsigma\varphi}}$) operator is a mapping $\mathscr{D}^{\delta}_{\mathfrak{T}^{\varrho\varsigma\varphi}} : \pounds^{\Omega} \times \pounds_0 \to \pounds^{\Omega}$ defined as:

$$\mathscr{D}^{\delta}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa) = \bigvee\{u_{r,\ t,\ s} \in svn-point(\Omega) \mid\ u_{r,\ t,\ s}\ is\ \varkappa-svn\delta-cluster\ \ point\ of\ \Pi\}.$$

Theorem 5 Let $(\Omega,\ \mathfrak{T}^{\varrho\varsigma\varphi},\ \mathfrak{P}^{\varrho\varsigma\varphi})$ be a *svnpts*, for each $\varkappa \in \pounds_0$ and $\Pi,\ \Upsilon \in \pounds^{\Omega}$. Then the subsequent statements hold:
(1) If $\Pi \le \Upsilon$, then $\mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa) \le \mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon,\ \varkappa)$.
(2) $\Pi \le \mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)$.
(3) $\mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa) = \bigwedge\{\Upsilon \in \pounds^{\Omega} \mid \Pi \le \Upsilon,\ \Upsilon$ is $\varkappa$-*svnrpc*$\}$.
(4) $int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa)$ is $\varkappa$-*svnrpo*.
(5) $\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ r) \le \mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)$.
Proof. (1) and (2): Definition 11, provide a clear and simple explanation.

(3) Put $\mathscr{H} = \bigwedge\{\Upsilon \in \pounds^{\Omega} \mid \Pi \le \Upsilon,\ \Upsilon$ is $\varkappa$-*svnrpc*$\}$. Assume that $\mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa) \not\ge \mathscr{H}$, then there exist $u \in \Omega$ and $r,\ t,\ s \in \pounds_0$ such that

$$\mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho}}(\Pi,\ \varkappa)(u) < r \le \varrho_{\mathscr{H}}(u), \quad \mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varsigma}}(\Pi,\ \varkappa)(u) \ge t > \mathscr{H}(u), \quad \mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varphi}}(\Pi,\ \varkappa)(u) \ge s > \mathscr{H}(u).$$

Then $u_{r,\ t,\ s}$ is not $\varkappa$-*svn*$\delta\mathfrak{P}$-*cluster point* of Π. So, there exists $\Upsilon \in \mathscr{Q}_{\mathscr{T}\tau\pi\sigma}(u_{r,\ t,\ s},\ \varkappa)$ such that $\Pi \le [int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon,\ \varkappa),\ \varkappa)]^c$. Hence,

$$\Pi \le [int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon,\ \varkappa),\ \varkappa)]^c = \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{I}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon^c,\ \varkappa),\ \varkappa).$$

Since $\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{I}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon^c,\ \varkappa),\ \varkappa)$ is $\varkappa$-*svnrpc*, we have

$$\mathscr{H}(u) \le (\mathfrak{CL}_{\mathfrak{T}^{\varrho}}(\mathfrak{I}^{\star}_{\mathfrak{T}^{\varrho}}(\Upsilon^c,\ \varkappa),\ \varkappa))(u) < r, \quad \mathscr{H}(u) \ge \mathfrak{CL}_{\mathfrak{T}^{\varsigma}}(\mathfrak{I}^{\star}_{\mathfrak{T}^{\varsigma}}(\Upsilon^c,\ \varkappa),\ \varkappa)(u) \ge t,$$

$$\mathscr{H}(u) \ge \mathfrak{CL}_{\mathfrak{T}^{\varsigma}}(\mathfrak{I}^{\star}_{\mathfrak{T}^{\varphi}}(\Upsilon^c,\ \varkappa),\ \varkappa)(u) \ge s.$$

It is a contradiction. Hence, $\mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa) \ge \mathscr{H}$.
Presume that $\mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa) \not\le \mathscr{H}$, then there exists $\varkappa$-*svn*$\delta\mathfrak{P}$-*cluster point* $v_{r_1,\ t_1,\ s_1} \in svn-point(\Omega)$ of Π, such that

$$\mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho}}(\Pi,\ \varkappa)(v) > r_1 \ge \varrho_{\mathscr{H}}(v), \quad \mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varsigma}}(\Pi,\ \varkappa)(v) \le t_1 < \mathscr{H}(v), \quad \mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varphi}}(\Pi,\ \varkappa)(v) \le s_1 < \mathscr{H}(v).$$

By the definition of $\mathscr{H}$, there exists $\varkappa$-*svnpc* $\Upsilon \in \pounds^{\Omega}$ with $\Pi \le \Upsilon$ such that

$$\mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho}}(\Pi,\ \varkappa)(v) > r_1 > \Upsilon(v) \ge \mathscr{H}(v),\ \mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varsigma}}(\Pi,\ \varkappa)(v) \le t_1 < \Upsilon(v) \le \mathscr{H}(v),$$

$$\mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varphi}}(\Pi,\ \varkappa)(v) \le s_1 < \Upsilon(v) \le \mathscr{H}(u).$$

Then $\Upsilon^c \in \mathscr{Q}_{\mathscr{T}\tau\pi\sigma}(v_{r_1, t_1, s_1}, \varkappa)$. So, $\Pi \leq \Upsilon \leq [int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon^c, \varkappa), \varkappa)]^c$. Therefore,

$$\Pi \, \overline{q} \, int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon^c, \varkappa), \varkappa).$$

Thus, v_{r_1, t_1, s_1} is not *$\varkappa$-svn$\delta\mathfrak{P}$-cluster point* of Π that is,

$$\mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho}}(\Pi, \varkappa)(v) < r_1, \quad \mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varsigma}}(\Pi, \varkappa)(v) \geq t_1, \quad \mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varphi}}(\Pi, \varkappa)(v) \geq s_1.$$

It is a contradiction. Hence, $\mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi, \varkappa) \leq \mathscr{H}$.

(4) Let $\Upsilon \in \pounds^{\Omega}$ and $\Upsilon = int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi, \varkappa), \varkappa)$. Then, we have

$$\begin{aligned} int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon, \varkappa), \varkappa) &= int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi, \varkappa), \varkappa), \varkappa), \varkappa) \\ &\leq int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi, \varkappa), \varkappa), \varkappa) \\ &= int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi, \varkappa), \varkappa) = \Upsilon. \end{aligned}$$

Since $\Upsilon = int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon, \varkappa) \leq int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon, \varkappa), \varkappa)$, we have $int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon, \varkappa), \varkappa) = \Upsilon$.

(5) Consider there exists $\Pi \in \pounds^{\Omega}$ and $\varkappa \in \pounds_0$ such that $\mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varphi}}(\Pi, \varkappa) \nleq \mathscr{D}^{\delta}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi, \varkappa)$, then there exist $u \in \Omega$ and $r, t, s \in \pounds_0$ such that

$$\mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho}}(\Pi, \varkappa)(u) \geq r > \mathscr{D}^{\delta}_{\mathfrak{T}^{\varrho}}(\Pi, \varkappa)(u), \; \mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varsigma}}(\Pi, \varkappa)(u) < t \leq \mathscr{D}^{\delta}_{\mathfrak{T}^{\varsigma}}(\Pi, \varkappa)(u),$$

$$\mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varphi}}(\Pi, \varkappa)(u) < s \leq \mathscr{D}^{\delta}_{\mathfrak{T}^{\varphi}}(\Pi, \varkappa)(u).$$

Since $\mathscr{D}^{\delta}_{\mathfrak{T}^{\varrho}}(\Pi, \varkappa)(u) < r$, $\mathscr{D}^{\delta}_{\mathfrak{T}^{\varsigma}}(\Pi, \varkappa)(u) \leq t$, $\mathscr{D}^{\delta}_{\mathfrak{T}^{\varphi}}(\Pi, \varkappa)(u) \leq s$ we have $u_{r, t, s}$ is not *$\varkappa$-svnδ-cluster point* of Π. So, there exists $\Upsilon \in \mathscr{R}_{\mathscr{T}\tau\pi\sigma}(u_{r, t, s}, \varkappa)$, $\Pi \leq \Upsilon^c$ and by using definition 10, we get, Υ^c is *$\varkappa$-svnrc*. Hence,

$$\mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho}}(\Pi, \varkappa)(v) \leq \Upsilon^c(u) < r_1, \; \mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varsigma}}(\Pi, \varkappa)(v) \geq \Upsilon^c(u) \geq t_1, \; \mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varphi}}(\Pi, \varkappa)(v) \geq \Upsilon^c(v) \geq s_1.$$

It is a contradiction. Thus $\mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varphi}}(\Pi, \varkappa) \leq \mathscr{D}^{\delta}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi, \varkappa)$.

On the other hand, assume that $\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi, \varkappa) \nleq \mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi, \varkappa)$, then there exist $u \in \Omega$ and $r, t, s \in \pounds_0$ such that

$$\mathfrak{CL}_{\mathfrak{T}^{\varrho}}(\Pi,\ \varkappa)(u) \geq r > \mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho}}(\Pi,\ \varkappa)(u),$$

$$\mathfrak{CL}_{\mathfrak{T}^{\varsigma}}(\Pi,\ \varkappa)(u) < t \leq \mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varsigma}}(\Pi,\ \varkappa)(u),$$

$$\mathfrak{CL}_{\mathfrak{T}^{\varphi}}(\Pi,\ \varkappa)(u) < s \leq \mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varphi}}(\Pi,\ \varkappa)(u).$$

Since $\mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho}}(\Pi,\ \varkappa)(u) < r,\ \mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varsigma}}(\Pi,\ \varkappa)(u) \leq t,\ \mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varphi}}(\Pi,\ \varkappa)(u) \leq s$, we have $u_{r,\ t,\ s}$ is not $\varkappa$-*svn*$\delta\mathfrak{P}$-*cluster point* of Π. So, there exists $\Upsilon \in \mathscr{Q}_{\mathscr{T}\tau\pi\sigma}(u_{r,\ t,\ s},\ \varkappa)$, $\Pi \leq [int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon,\ \varkappa),\ \varkappa)]^{c}$. Thus,

$$\mathfrak{CL}_{\mathfrak{T}^{\varrho}}(\Pi,\ \varkappa)(u) \leq ([int_{\mathfrak{T}^{\varrho}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho}}(\Upsilon,\ \varkappa),\ \varkappa)]^{c}) < r,$$

$$\mathfrak{CL}_{\mathfrak{T}^{\varsigma}}(\Pi,\ \varkappa)(u) \geq ([int_{\mathfrak{T}^{\varsigma}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varsigma}}(\Upsilon,\ \varkappa),\ \varkappa)]^{c}) \geq t,$$

$$\mathfrak{CL}_{\mathfrak{T}^{\varphi}}(\Pi,\ \varkappa)(u) \geq ([int_{\mathfrak{T}^{\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varphi}}(\Upsilon,\ \varkappa),\ \varkappa)]^{c}) \geq s.$$

It is a contradiction. Thus $\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa) \leq \mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)$. □

Lemma 5 Let $(\Omega,\ \mathfrak{T}^{\varrho\varsigma\varphi},\ \mathfrak{P}^{\varrho\varsigma\varphi})$ be a *svnpts* and Π, Υ is $\varkappa$-*svns*β*po*. Then, $\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa) = \mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)$.

Proof. We show that $\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa) \geq \mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)$. Presume that, $\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa) \not\geq \mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)$, then there exists $u \in \Omega$ and $r,\ t,\ s \in \pounds_0$ such that

$$\mathfrak{CL}_{\mathfrak{T}^{\varrho}}(\Pi,\ \varkappa)(u) \leq r < \mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho}}(\Pi,\ \varkappa)(u),\quad \mathfrak{CL}_{\mathfrak{T}^{\varsigma}}(\Pi,\ \varkappa)(u) > t \geq \mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varsigma}}(\Pi,\ \varkappa)(u),$$

$$\mathfrak{CL}_{\mathfrak{T}^{\varphi}}(\Pi,\ \varkappa)(u) > s \geq \mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varphi}}(\Pi,\ \varkappa)(u).$$

By using definition of $\mathfrak{CL}_{\mathfrak{T}^{\varphi}}$ there exists $\Upsilon \in \pounds^{\Omega}$ with $\mathfrak{T}^{\varrho}(\Upsilon^{c}) \geq \varkappa$, $\mathfrak{T}^{\varsigma}(\Upsilon^{c}) \leq 1-\varkappa$, $\mathfrak{T}^{\varphi}(\Upsilon^{c}) \leq 1-\varkappa$ and $\Pi \leq \Upsilon$ such that

$$\mathfrak{CL}_{\mathfrak{T}^{\varrho}}(\Pi,\ \varkappa)(u) \leq \Upsilon(u) \leq r < \mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho}}(\Pi,\ \varkappa)(u),\quad \mathfrak{CL}_{\mathfrak{T}^{\varsigma}}(\Pi,\ \varkappa)(u) \geq \Upsilon(u) > t \geq \mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varsigma}}(\Pi,\ \varkappa)(u),$$

$$\mathfrak{CL}_{\mathfrak{T}^{\varphi}}(\Pi,\ \varkappa)(u) \geq \Upsilon(u) > s \geq \mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varphi}}(\Pi,\ \varkappa)(u),$$

Then $\Upsilon^{c} \in \mathscr{Q}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(u_{r,\ t,\ s},\ \varkappa)$ and $\Pi^{c} \geq \Upsilon^{c}$ implies

$$\mathfrak{I}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi^{c},\ \varkappa) \geq \mathfrak{I}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon^{c},\ \varkappa) = (\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon,\ \varkappa))^{c} \geq (\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon,\ \varkappa))^{c} = \Upsilon^{c}.$$

Then, $\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)\overline{q}\Upsilon^{c}$ implies that

$$\Upsilon^{c} \leq [\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)]^{c} \Rightarrow \mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon^{c},\ r) \leq [\mathfrak{I}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa)]^{c}$$

$$\Rightarrow int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon^{c},\ \varkappa),\ \varkappa) \leq [\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{I}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa),\ \varkappa)$$

$$\Rightarrow int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon^{c},\ \varkappa),\ \varkappa) \leq [\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa),\ \varkappa)]^{c}.$$

Hence, $int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon^{c},\ \varkappa),\ \varkappa)\overline{q}\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa),\ \varkappa)$. Since Π is $\varkappa$-*svnsβpo*, $int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon^{c},\ \varkappa),\ \varkappa)\overline{q}\Pi$. So, $u_{r,\ t,\ s}$ is not $\varkappa$-*svn*$\delta\mathfrak{P}$-*cluster point* of Π. Thus, $\mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho}}(\Pi,\ \varkappa)(u) < r$, $\mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varsigma}}(\Pi,\ \varkappa)(u) \geq t$ and $\mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varphi}}(\Pi,\ \varkappa)(u) \geq s$. It is a contradiction. Therefore, $\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa) \geq \mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)$. By Theorem 5 (5), $\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa) = \mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)$. □

Definition 12 Let $(\Omega,\ \mathfrak{T}^{\varrho\varsigma\varphi},\ \mathfrak{P}^{\varrho\varsigma\varphi})$ be a *svnpts*, for each $\varkappa \in \pounds_0$ and $\Pi,\ \Upsilon \in \pounds^{\Omega}$. Then

(1) Π is said to be $\varkappa$-*svnB*^{P}-*set* [resp, $\varkappa$-*svnC*^{P}-*set*, , $\varkappa$-*svnsB*^{P}-*set*, $\varkappa$-*svnS*$^{P}_{\alpha}$-*set*, $\varkappa$-*svnB*$^{\star P}$-*set*] if $\Pi = \Upsilon \wedge \Theta$ where $\mathfrak{T}^{\varrho}(\Upsilon) \geq \varkappa$, $\mathfrak{T}^{\varsigma}(\Upsilon) \leq 1-\varkappa$, $\mathfrak{T}^{\varphi}(\Upsilon) \leq 1-\varkappa$ and $int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta,\ \varkappa) = int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta,\ \varkappa),\ \varkappa)$ [resp, $int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta,\ \varkappa) = int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta,\ \varkappa),\ \varkappa),\ \varkappa)$, $int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta,\ \varkappa) = int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta,\ \varkappa),\ \varkappa)$, $int_{\tau}(\Theta,\ \varkappa) = int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta,\ \varkappa),\ \varkappa),\ \varkappa)$, $int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta,\ \varkappa) = \mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathscr{D}^{\delta}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta,\ \varkappa),\ \varkappa),\ \varkappa)$].

(2) Π is said to be $\varkappa$-*svnβ***po* if $\Pi \leq \mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathscr{D}^{\delta}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta,\ \varkappa),\ \varkappa),\ \varkappa)$.

(3) Π is said to be $\varkappa$-*svnP***po* if $\Pi \leq int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta,\ \varkappa),\ \varkappa)$.

(4) Π is said to be $\varkappa$-*svnsppo* if Π both $\varkappa$-*svnppo* and $\varkappa$-*svnC*^{P}-*set*.

Remark 2 After the above definition, we get the next drawing:

$$\varkappa - svnB^{\star P}\text{-}set \Rightarrow \varkappa - svnsB^{P}\text{-}set \Rightarrow \varkappa - svnB^{P}\text{-}set$$

$$\Uparrow$$

$$\varkappa - svno \Rightarrow \varkappa - svnsppo \Rightarrow \varkappa - svnppo \Rightarrow \varkappa - svns\beta po$$

$$\Downarrow \qquad\qquad\qquad\qquad \Downarrow \qquad\qquad \Downarrow$$

$$\varkappa - svnS^{P}_{\alpha}\text{-}set \qquad\qquad \varkappa - svnP^{\star}p \Rightarrow \varkappa - svn\beta^{\star}P.$$

Example 4 Consider $\Omega = \{u_1,\ u_2,\ u_3\}$; define the *svn-sets* $\Phi_1,\ \Phi_2,\ \Theta,\ \Upsilon \in \pounds^{\Omega}$ as follows

$$\Phi_1 = \langle (0.7,\ 0.7,\ 0.7),\ (0.7,\ 0.7,\ 0.7),\ (0.7,\ 0.7,\ 0.7) \rangle,$$

$$\Phi_2 = \langle (03,\ 0.3,\ 0.3),\ (0.7,\ 0.7,\ 0.7),\ (0.7,\ 0.7,\ 0.7) \rangle,$$

$$\Theta = \langle (0.4,\ 0.4,\ 0.4),\ (0.4,\ 0.4,\ 0.4),\ (0.4,\ 0.4,\ 0.4) \rangle,$$

$$\Upsilon = \langle (0.3,\ 0.3,\ 0.3),\ (0,\ 0,\ 0),\ (0,\ 0,\ 0) \rangle.$$

We define, $\mathfrak{T}^{\varrho} : \pounds^{\Omega} \to \pounds$, $\mathfrak{T}^{\varsigma} : \pounds^{\Omega} \to \pounds$, $\mathfrak{T}^{\varphi} : \pounds^{\Omega} \to \pounds$ and $\mathfrak{P}^{\varsigma} : \pounds^{\Omega} \to \pounds$, $\mathfrak{P}^{\varphi} : \pounds^{\Omega} \to \pounds$ as follows:

$$\mathfrak{T}^{\varrho}(\Pi) = \begin{cases} 1, & \text{if } \Pi = \overline{0}, \\ 1, & \text{if } \Pi = \overline{1}, \\ \frac{1}{2}, & \text{if } \Pi = \{\Phi_1,\ \Phi_2\}, \\ 0, & \text{otherwise}, \end{cases} \qquad \mathfrak{P}^{\varrho}(\Pi) = \begin{cases} 1, & \text{if } \Pi = \overline{0}, \\ \frac{1}{4}, & \text{if } \Pi = \Upsilon, \\ \frac{1}{2}, & \text{if } \overline{0} < \Pi < \Upsilon, \\ 0, & \text{otherwise}, \end{cases}$$

$$\mathfrak{T}^{\varsigma}(\Pi) = \begin{cases} 0, & \text{if } \Pi = \overline{0}, \\ 0, & \text{if } \Pi = \overline{1}, \\ \frac{1}{3}, & \text{if } \Pi = \{\Phi_1,\ \Phi_2\}, \\ 1, & \text{otherwise}, \end{cases} \qquad \mathfrak{P}^{\varsigma}(\Pi) = \begin{cases} 0, & \text{if } \Pi = \overline{0}, \\ \frac{3}{4}, & \text{if } \Pi = \Upsilon, \\ \frac{1}{2}, & \text{if } \overline{0} < \Xi < \Upsilon, \\ 1, & \text{otherwise}, \end{cases}$$

$$\mathfrak{T}^{\varphi}(\Pi) = \begin{cases} 0, & \text{if } \Pi = \overline{0}, \\ 0, & \text{if } \Pi = \overline{1}, \\ \frac{1}{4}, & \text{if } \Pi = \{\Phi_1,\ \Phi_2\}, \\ 1, & \text{otherwise}, \end{cases} \qquad \mathfrak{P}^{\varphi}(\Pi) = \begin{cases} 0, & \text{if } \Pi = \overline{0}, \\ \frac{2}{3}, & \text{if } \Pi = \Upsilon, \\ \frac{1}{2}, & \text{if } \overline{0} < \Pi < \Upsilon, \\ 1, & \text{otherwise}. \end{cases}$$

Θ is $\frac{1}{2}$-*svnB*^{p}-*set*, but it is not $\frac{1}{2}$-*svns*β*po*.

Example 5 Consider $\Omega = \{u_1,\ u_2,\ u_3\}$; define the *svn-sets* $\Phi_1,\ \Phi_2,\ \Theta,\ \Upsilon \in \pounds^{\Omega}$ as follows:

$$\Phi_1 = \langle(0.7,\ 0.7,\ 0.7),\ (0.7,\ 0.7,\ 0.7),\ (0.7,\ 0.7,\ 0.7)\rangle,$$

$$\Phi_2 = \langle(0.6,\ 0.6,\ 0.6),\ (0.7,\ 0.7,\ 0.7),\ (0.7,\ 0.7,\ 0.7)\rangle,$$

$$\Theta = \langle(0.4,\ 0.4,\ 0.4),\ (0.4,\ 0.4,\ 0.4),\ (0.4,\ 0.4,\ 0.4)\rangle,$$

$$\Upsilon = \langle(0.5,\ 0.5,\ 0.5),\ (0,\ 0,\ 0),\ (0,\ 0,\ 0)\rangle.$$

We define, $\mathfrak{T}^{\varrho} : £^{\Omega} \to £$, $\mathfrak{T}^{\varsigma} : £^{\Omega} \to £$, $\mathfrak{T}^{\varphi} : £^{\Omega} \to £$ and $\mathfrak{P}^{\varsigma} : £^{\Omega} \to £$, $\mathfrak{P}^{\varphi} : £^{\Omega} \to £$ as follows:

$$\mathfrak{T}^{\varrho}(\Pi) = \begin{cases} 1, & \text{if } \Pi = \overline{0}, \\ 1, & \text{if } \Pi = \overline{1}, \\ \frac{1}{2}, & \text{if } \Pi = \{\Phi_1,\ \Phi_2\}, \\ 0, & \text{otherwise}, \end{cases} \qquad \mathfrak{P}^{\varrho}(\Pi) = \begin{cases} 1, & \text{if } \Pi = \overline{0}, \\ \frac{1}{4}, & \text{if } \Pi = \Upsilon, \\ \frac{1}{2}, & \text{if } \overline{0} < \Pi < \Upsilon, \\ 0, & \text{otherwise}, \end{cases}$$

$$\mathfrak{T}^{\varsigma}(\Pi) = \begin{cases} 0, & \text{if } \Pi = \overline{0}, \\ 0, & \text{if } \Pi = \overline{1}, \\ \frac{1}{3}, & \text{if } \Pi = \{\Phi_1,\ \Phi_2\}, \\ 1, & \text{otherwise}, \end{cases} \qquad \mathfrak{P}^{\varsigma}(\Pi) = \begin{cases} 0, & \text{if } \Pi = \overline{0}, \\ \frac{3}{4}, & \text{if } \Pi = \Upsilon, \\ \frac{1}{2}, & \text{if } \overline{0} < \Xi < \Upsilon, \\ 1, & \text{otherwise}, \end{cases}$$

$$\mathfrak{T}^{\varphi}(\Pi) = \begin{cases} 0, & \text{if } \Pi = \overline{0}, \\ 0, & \text{if } \Pi = \overline{1}, \\ \frac{1}{4}, & \text{if } \Pi = \{\Phi_1,\ \Phi_2\}, \\ 1, & \text{otherwise}, \end{cases} \qquad \mathfrak{P}^{\varphi}(\Pi) = \begin{cases} 0, & \text{if } \Pi = \overline{0}, \\ \frac{2}{3}, & \text{if } \Pi = \Upsilon, \\ \frac{1}{2}, & \text{if } \overline{0} < \Pi < \Upsilon, \\ 1, & \text{otherwise}. \end{cases}$$

Θ is $\frac{1}{2}$-*svnP***p-set*, but it is not *svnppo*.

Theorem 6 Let $(\Omega,\ \mathfrak{T}^{\varrho\varsigma\varphi},\ \mathfrak{P}^{\varrho\varsigma\varphi})$ be a *svnpts*. Then the following properties are holds:

(1) If Π is $\varkappa$-*svnC*^{P}*-set*, then $\alpha\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa) = int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)$.

(2) If Π is $\varkappa$-*svn*δ*po*, then $\alpha\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa) = P\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)$.

(3) Π is $\varkappa$-*svnαpo* iff Π is both $\varkappa$-*svnspo* and $\varkappa$-*svnppo*.

Proof. (1) Presume that Π is $\varkappa$-*svnC^P-set*, then $\Pi = \Upsilon \wedge \Theta$ where $\mathfrak{T}^{\varrho}(\Upsilon) \geq \varkappa$, $\mathfrak{T}^{\varsigma}(\Upsilon) \leq 1-\varkappa$, $\mathfrak{T}^{\varphi}(\Upsilon) \leq 1-\varkappa$ and $int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta,\ \varkappa) = int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta,\ \varkappa),\ \varkappa),\ \varkappa)$. Now,

$$\Pi \leq \Theta \Rightarrow int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta,\ \varkappa) = int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa),\ \varkappa) \leq int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta,\ \varkappa)$$

$$= int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta,\ \varkappa),\ \varkappa),\ \varkappa)$$

$$= int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta,\ \varkappa).$$

Hence, by Theorem 4 (4), we have

$$\alpha\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa) = \Pi \wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta,\ \varkappa),\ \varkappa),\ \varkappa) \leq \Pi \wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta,\ \varkappa)$$

$$= \Upsilon \wedge (\Theta \wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta,\ \varkappa)) = int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon,\ \varkappa) \wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta,\ \varkappa)$$

$$= int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon \wedge \Theta,\ r) = int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa).$$

Since, $\alpha\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa) \geq int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)$, we have $\alpha\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa) \geq int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)$.

(2) Clear and straightforward from (1).

(3$\Rightarrow$) Definition 10, provide a clear and simple explanation.

($\Leftarrow$) Suppose that Π is $\varkappa$-*svnspo* and $\varkappa$-*svnppo*. Then, we have

$$\Pi \leq int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta,\ \varkappa),\ \varkappa) \leq int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa),\ \varkappa),\ \varkappa)$$

$$= int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa),\ \varkappa).$$

This show that Π is $\varkappa$-*svnαpo*. □

Theorem 7 Let $(\Omega,\ \mathfrak{T}^{\varrho\varsigma\varphi},\ \mathfrak{P}^{\varrho\varsigma\varphi})$ be a *svnpts*, for each $\varkappa \in \pounds_0$ and $\Pi \in \pounds^{\Omega}$. Then, the following statements are equivalent.

(1) For any $\Pi \in \pounds^{\Omega}$ and $\varkappa \in \pounds_0$, $\mathfrak{T}^{\varrho}(\Pi) \geq \varkappa$, $\mathfrak{T}^{\varsigma}(\Pi) \leq 1-\varkappa$ and $\mathfrak{T}^{\varphi}(\Pi) \leq 1-\varkappa$.

(2) π is $\varkappa$-*svnαpo* and $\varkappa$-*svnsB^p-set*.

(3) Π is $\varkappa$-*svn$P^{\star}$po* and $\varkappa$-*svnsB^p-set*.

(4) Π is $\varkappa$-**svnαpo** and $\varkappa$-*svn$B^{\star p}$-set*.

(5) Π is $\varkappa$-**svnsβpo** and $\varkappa$-*svn$B^{\star p}$-set*.

(6) Π is $\varkappa$-**svn$\beta^{\star}$po** and $\varkappa$-*svn$B^{\star p}$-set*.

(7) Π is $\varkappa$-**svnppo** and $\varkappa$-*svnsB^p-set*.

(8) Π is $\varkappa$-**svnppo** $\varkappa$-*svn$B^{\star p}$-set*.

(9) Π is $\varkappa$-**svn$P^{\star}$po** and $\varkappa$-*svn$B^{\star p}$-set*.

(10) Π is $\varkappa$-**svnppo** and $\varkappa$-*svnB^p-set*.

(11) Π is $\varkappa$-**svnsppo** and $\varkappa$-**svnδpo**.

(12) Π is $\varkappa$-**svnαpo** and $\varkappa$-*svn*S_{α}^{p}-*set*.

Proof. $(1)\Rightarrow(2)\Rightarrow(3)$, $(1)\Rightarrow(4)\Rightarrow(5)\Rightarrow(6)$, $(1)\Rightarrow(7)$, $(1)\Rightarrow(8)\Rightarrow(9)$, $(1)\Rightarrow(10)$, $(1)\Rightarrow(11)$ and $(1)\Rightarrow(12)$, are easily proved by Lemma 2 and Definition 10.

$(3)\Rightarrow(1)$: suppose that Π is $\varkappa$-*svnsB*^{p}-*set*. Then $\Pi=\Upsilon\wedge\Theta$ where $\mathfrak{T}^{\varrho}(\Upsilon)\geq\varkappa$, $\mathfrak{T}^{\varsigma}(\Upsilon)\leq 1-\varkappa$, $\mathfrak{T}^{\varphi}(\Upsilon)\leq 1-\varkappa$ and $int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta,\ \varkappa)=int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta,\ \varkappa),\ \varkappa)$, Since Π is $\varkappa$-*svnP***po*, we have

$$\Pi\leq int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa)=int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon\wedge\Theta,\ \varkappa),\ \varkappa)$$

$$\leq int_{\mathfrak{T}^{\varrho\varsigma\varphi}}[\mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon,\ \varkappa)\wedge\mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta,\ \varkappa),\ \varkappa]$$

$$=int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon,\ \varkappa),\ \varkappa)\wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta,\ \varkappa),\ \varkappa)$$

$$=int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon,\ \varkappa),\ \varkappa)\wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta,\ \varkappa).$$

Hence,

$$\Pi=\Upsilon\wedge\Theta=[\Upsilon\wedge\Theta]\wedge\Upsilon\leq[int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon,\ \varkappa),\ \varkappa)\wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta,\ \varkappa)]\wedge\Upsilon$$

$$=[int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon,\ \varkappa),\ \varkappa)\wedge\Upsilon]\wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta,\ \varkappa)$$

$$=int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon,\ \varkappa)\wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta,\ \varkappa)=int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon\wedge\Theta,\ \varkappa)$$

$$=int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa).$$

So, $\Pi=int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)$. Thus, $\mathfrak{T}^{\varrho}(\Pi)\geq\varkappa$, $\mathfrak{T}^{\varsigma}(\Pi)\leq 1-\varkappa$, $\mathfrak{T}^{\varphi}(\Pi)\leq 1-\varkappa$.

$(6)\Rightarrow(1)$: Assume that Π is $\varkappa$-*svnB*$^{\star p}$-*set*. Then $\Pi=\Upsilon\wedge\Theta$ where $\mathfrak{T}^{\varrho}(\Upsilon)\geq\varkappa$, $\mathfrak{T}^{\varsigma}(\Upsilon)\leq 1-\varkappa$, $\mathfrak{T}^{\varphi}(\Upsilon)\leq 1-\varkappa$ and $int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta,\ \varkappa)=\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathscr{D}^{\delta}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta,\ \varkappa),\ \varkappa),\ \varkappa)$. Since Π is $\varkappa$-*svn*$\beta^{\star}$*po*, we have

$$\Pi\leq\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathscr{D}^{\delta}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa),\ \varkappa)=\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathscr{D}^{\delta}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon\wedge\Theta,\ \varkappa),\ \varkappa),\ \varkappa)$$

$$\leq\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}([int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathscr{D}^{\delta}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon,\ \varkappa)\wedge\mathscr{D}^{\delta}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta,\ \varkappa),\ \varkappa)],\ \varkappa)$$

$$=\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}([int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathscr{D}^{\delta}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon,\ \varkappa),\ \varkappa)\wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathscr{D}^{\delta}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta,\ \varkappa),\ \varkappa)],\ \varkappa)$$

$$\leq\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathscr{D}^{\delta}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon,\ \varkappa),\ \varkappa),\ \varkappa)\wedge\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathscr{D}^{\delta}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta,\ \varkappa),\ \varkappa),\ \varkappa)$$

$$=\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathscr{D}^{\delta}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon,\ \varkappa),\ \varkappa),\ \varkappa)\wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta,\ \varkappa).$$

This implies that

$$\Pi = \Upsilon \wedge \Theta = [\Upsilon \wedge \Theta] \wedge \Upsilon \leq [\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathscr{D}^{\delta}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon, \varkappa), \varkappa), \varkappa) \wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta, \varkappa)] \wedge \Upsilon$$

$$= [\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathscr{D}^{\delta}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon, \varkappa), \varkappa), \varkappa) \wedge \Upsilon] \wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta, \varkappa) = int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon \wedge \Theta, \varkappa)$$

$$= int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi, \varkappa).$$

So, $\mathfrak{T}^{\varrho}(\Pi) \geq \varkappa$, $\mathfrak{T}^{\varsigma}(\Pi) \leq 1 - \varkappa$, $\mathfrak{T}^{\varphi}(\Pi) \leq 1 - \varkappa$.

(7) $\Rightarrow$ (1): Since Π is $\varkappa$-*svnppo* and $\varkappa$-*svnP***po*, by (3), we have $\mathfrak{T}^{\varrho}(\Pi) \geq \varkappa$, $\mathfrak{T}^{\varsigma}(\Pi) \leq 1 - \varkappa$, $\mathfrak{T}^{\varphi}(\Pi) \leq 1 - \varkappa$.

(9) $\Rightarrow$ (1): Since, Π is $\varkappa$-*svnsB*$^{\star p}$-*set* and $\varkappa$-*svnsB*^{p}-*set* and by (3), $\mathfrak{T}^{\varrho}(\Pi) \geq \varkappa$, $\mathfrak{T}^{\varsigma}(\Pi) \leq 1 - \varkappa$, $\mathfrak{T}^{\varphi}(\Pi) \leq 1 - \varkappa$.

(10) $\Rightarrow$ (1): Presume that Π is $\varkappa$-*svnppo* and $\varkappa$-*svnB*^{p}-*set*. Then $\Pi \leq int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi, \varkappa), \varkappa)$ and $\Pi = \Upsilon \wedge \Theta$ where $\Theta = int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta, \varkappa), \varkappa)$ and $\mathfrak{T}^{\varrho}(\Pi) \geq \varkappa$, $\mathfrak{T}^{\varsigma}(\Pi) \leq 1 - \varkappa$, $\mathfrak{T}^{\varphi}(\Pi) \leq 1 - \varkappa$. Thus,

$$\Upsilon \wedge \Theta \leq int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon \wedge \Theta, \varkappa), \varkappa)$$

$$\leq int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon, \varkappa), \varkappa) \wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta, \varkappa), \varkappa)$$

$$= int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon, \varkappa), \varkappa) \wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta, \varkappa).$$

Therefore,

$$\Upsilon \wedge \Theta = [\Upsilon \wedge \Theta] \wedge \Upsilon \leq int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon, \varkappa), \varkappa) \wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta, \varkappa) \wedge \Upsilon$$

$$= int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon, \varkappa), \varkappa) \wedge \Upsilon \wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta, \varkappa)$$

$$= \Upsilon \wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta, \varkappa) = int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon \wedge \Theta, \varkappa).$$

Thus, $\mathfrak{T}^{\varrho}(\Pi) \geq \varkappa$, $\mathfrak{T}^{\varsigma}(\Pi) \leq 1 - \varkappa$, $\mathfrak{T}^{\varphi}(\Pi) \leq 1 - \varkappa$.

(11) $\Rightarrow$ (1): consider that Π is $\varkappa$-*svnsppo* and $\varkappa$-*svn*δ*po*. Then by Theorem 6 (1),

$$\alpha\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi, \varkappa) = P\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi, \varkappa) = \Pi.$$

By Theorem 6 (2), $\alpha\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi, \varkappa) = int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi, \varkappa)$. Hence, $\Pi = int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi, \varkappa)$.

(12) $\Rightarrow$ (1): Assume that Π is $\varkappa$-*svnS*$^{p}_{\alpha}$-*set*, then $\Pi = \Upsilon \wedge \Theta$ where $\mathfrak{T}^{\varrho}(\Pi) \geq \varkappa$, $\mathfrak{T}^{\varsigma}(\Pi) \leq 1 - \varkappa$, $\mathfrak{T}^{\varphi}(\Pi) \leq 1 - \varkappa$ and $int_{\tau}(\Theta, \varkappa) = int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta, \varkappa), \varkappa), \varkappa)$. Since Π is $\varkappa$-*svn*α*po*, $\Pi \leq int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi, \varkappa), \varkappa), \varkappa) = int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon \wedge \Theta, \varkappa), \varkappa), \varkappa)$. It implies that

$$\Pi = \Upsilon \wedge [\Upsilon \wedge \Theta] \leq \Upsilon \wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon \wedge \Theta, \varkappa), \varkappa), \varkappa)$$

$$= \Upsilon \wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}([int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon, \varkappa) \wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta, \varkappa)], \varkappa), \varkappa), \varkappa)$$

$$= \Upsilon \wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}([int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon, \varkappa)$$

$$\wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta, \varkappa), \varkappa), \varkappa), \varkappa)], \varkappa), \varkappa), \varkappa)$$

$$\leq \Upsilon \wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}([\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon, \varkappa), \varkappa), \varkappa)$$

$$\wedge \mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta, \varkappa), \varkappa), \varkappa), \varkappa), \varkappa), \varkappa)], \varkappa)$$

$$= \mathscr{B} \wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon, \varkappa), \varkappa), \varkappa), \varkappa)$$

$$\wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathscr{C}, \varkappa), \varkappa), \varkappa), \varkappa), \varkappa), \varkappa)\varkappa), \varkappa), \varkappa), \varkappa)$$

$$\leq \Upsilon \wedge \mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon, \varkappa), \varkappa), \varkappa)$$

$$\wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta, \varkappa), \varkappa), \varkappa), \varkappa), \varkappa), \varkappa), \varkappa)$$

$$= \Upsilon \wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\tau}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta, \varkappa), \varkappa), \varkappa), \varkappa), \varkappa), \varkappa), \varkappa)$$

$$= \Upsilon \wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int^{\delta\mathfrak{P}}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta, \varkappa), \varkappa), \varkappa), \varkappa)$$

$$= \Upsilon \wedge int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Theta, \varkappa) = int_{\tau}(\Upsilon \wedge \Theta, \varkappa) = int_{\tau}(\Pi, \varkappa).$$

Hence, $\mathfrak{T}^{\varrho}(\Pi) \geq \varkappa$, $\mathfrak{T}^{\varsigma}(\Pi) \leq 1-\varkappa$, $\mathfrak{T}^{\varphi}(\Pi) \leq 1-\varkappa$.

4. Decompositions of single-valued neutrosophic continuity

The focus now switches to single-valued neutrosophic [pre- primal, B^p, α-primal, strongly pre-primal, δ-primal, $P^{\star}$-primal, $\beta^{\star}$-primal, strongly β-primal, strongly B^{P}, $B^{\star P}$, S^{P}_{α}, B^{P}]-continuous (for short, *svnpp-continuous*, *svn*B^p-*continuous*, *svn*α*p-continuous*, *svnspp-continuous*, *svn*δ*p-continuous*, *svn*$P^{\star}$*p-continuous*, *svn*$\beta^{\star}$*p-continuous*, *svns*β*p-continuous*, *svns*B^{P}*-continuous*, *svn*$B^{\star P}$*-continuous*, *svns* S^{P}_{α} -continuous, *svn*B^{P}*-continuous*), as we continue our exploration of advanced neutrosophic theories.

Definition 13 Let $f : (\Omega, \mathfrak{T}^{\varrho\varsigma\varphi}) \to (\Xi, \mathfrak{F}^{\varrho\varsigma\varphi})$ be a mapping and $\varkappa \in \pounds_0$. Then f is said to be *svn-continuous* if $\mathfrak{F}^{\varrho}(\Pi) \leq \mathfrak{T}^{\varrho}(f^{-1}(\Pi))$, $\mathfrak{F}^{\varsigma}(\Pi) \geq \mathfrak{T}^{\varsigma}(f^{-1}(\Pi))$ and $\mathfrak{F}^{\varphi}(\Pi) \geq \mathfrak{T}^{\varphi}(f^{-1}(\Pi))$ for each $\mathfrak{F}^{\varrho}(\Pi) \geq \varkappa$, $\mathfrak{F}^{\varsigma}(\Pi) \leq 1-\varkappa$, $\mathfrak{F}^{\varphi}(\Pi) \leq 1-\varkappa$.

Definition 14 A mapping $f:(\Omega,\ \mathfrak{T}^{\varrho\varsigma\varphi},\ \mathfrak{P}^{\varrho\varsigma\varphi})\rightarrow(\Xi,\ \mathfrak{F}^{\varrho\varsigma\varphi})$ is said to be *svnpp-continuous*, (resp, *svnB^{p}-continuous*, *svn αp-continuous*, *svnspp-continuous*, *svnδp-continuous*, *svnP*p-continuous*, *svn$\beta^{\star}$p-continuous*, *svnsβp-continuous*, *svns B^{P}-continuous*, *svn$B^{\star P}$-continuous*, *svns S_{α}^{P}* -continuous, *svnB^{P}-continuous*) if $f^{-1}(\Pi)$ is $\varkappa$-*svnsppo* (resp, $\varkappa$-*svnB^{P}-set*, $\varkappa$-*svnαpo*, $\varkappa$-*svnsppo*, $\varkappa$-*svnδpo*, $\varkappa$-*svnsP*po*, $\varkappa$-*svn$\beta^{\star}$po*, $\varkappa$-*svnsβpo*, $\varkappa$-*svnsB^{P}-set*, $\varkappa$-*svnB$^{\star P}$-set*, $\varkappa$-*svnsS$_{\alpha}^{p}$-set*, $\varkappa$-*svnβpo*), for every $\mathfrak{F}^{\varrho}(\Pi)\geq\varkappa$, $\mathfrak{F}^{\varsigma}(\Pi)\leq 1-\varkappa$, $\mathfrak{F}^{\varphi}(\Pi)\leq 1-\varkappa$.

Remark 3 After the above definition, we get the next drawing:

$$\begin{array}{ccccc} & & \textit{svnP}^{\star}\textit{p-continuous} & \Rightarrow & \textit{svn}\beta^{\star}\textit{p-continuous}, \\ & & \Uparrow & & \Uparrow \\ \textit{svn}\delta\textit{p-continuous} & \Rightarrow & \textit{svnpp-continuous} & \Rightarrow & \textit{svns}\beta\textit{p-continuous} \\ \Uparrow & & \Uparrow & & \\ \textit{svn-continuous} & \Rightarrow & \textit{svnspp-continuous} & & \\ \Downarrow & & & & \\ \textit{svnB}^{\star P}\textit{-continuous} & \Rightarrow & \textit{svnsB}^{P}\textit{-continuous} & \Rightarrow & \textit{svnB}^{P}\textit{-continuous}. \end{array}$$

None of these implications is reversible as shown in the following examples:

Example 6 Consider $\Omega=\{u_1,\ u_2,\ u_3\}$; define the *svn-sets* $\Phi_1,\ \Phi_2,\ \Phi_3,\ \Upsilon\in\pounds^{\Omega}$ as follows

$$\Phi_1=\langle(0.7,\ 0.7,\ 0.7),\ (0.7,\ 0.7,\ 0.7),\ (0.7,\ 0.7,\ 0.7)\rangle,$$

$$\Phi_2=\langle(03,\ 0.3,\ 0.3),\ (0.7,\ 0.7,\ 0.7),\ (0.7,\ 0.7,\ 0.7)\rangle,$$

$$\Phi_3=\langle(0.5,\ 0.5,\ 0.5),\ (0.5,\ 0.5,\ 0.5),\ (0.5,\ 0.5,\ 0.5)\rangle,$$

$$\Theta=\langle(0.1,\ 0.1,\ 0.1),\ (0,\ 0,\ 0),\ (0,\ 0,\ 0)\rangle.$$

We define, $\mathfrak{T}^{\varrho}:\pounds^{\Omega}\rightarrow\pounds$, $\mathfrak{T}^{\varsigma}:\pounds^{\Omega}\rightarrow\pounds$, $\mathfrak{T}^{\varphi}:\pounds^{\Omega}\rightarrow\pounds$, $\mathfrak{F}^{\varrho}:\pounds^{\Omega}\rightarrow\pounds$, $\mathfrak{F}^{\varsigma}:\pounds^{\Omega}\rightarrow\pounds$, $\mathfrak{F}^{\varphi}:\pounds^{\Omega}\rightarrow\pounds$ and $\mathfrak{P}^{\varrho}:\pounds^{\Omega}\rightarrow\pounds$, $\mathfrak{P}^{\varsigma}:\pounds^{\Omega}\rightarrow\pounds$, $\mathfrak{P}^{\varphi}:\pounds^{\Omega}\rightarrow\pounds$ as follows:

$$\mathfrak{T}^{\varrho}(\Pi) = \begin{cases} 1, & \text{if } \Pi = \overline{0}, \\ 1, & \text{if } \Pi = \overline{1}, \\ \dfrac{3}{4}, & \text{if } \Pi = \{\Phi_1, \Phi_2\}, \\ 0, & \text{otherwise}, \end{cases} \qquad \mathfrak{F}^{\varrho}(\Pi) = \begin{cases} 1, & \text{if } \Pi = \overline{0}, \\ 1, & \text{if } \Pi = \overline{1}, \\ \dfrac{2}{3}, & \text{if } \Pi = \Phi_3, \\ 0, & \text{otherwise}, \end{cases}$$

$$\mathfrak{T}^{\varsigma}(\Pi) = \begin{cases} 0, & \text{if } \Pi = \overline{0}, \\ 0, & \text{if } \Pi = \overline{1}, \\ \dfrac{1}{4}, & \text{if } \Pi = \{\Phi_1, \Phi_2\}, \\ 1, & \text{otherwise}, \end{cases} \qquad \mathfrak{F}^{\varsigma}(\Pi) = \begin{cases} 1, & \text{if } \Pi = \overline{0}, \\ 1, & \text{if } \Pi = \overline{1}, \\ \dfrac{1}{3}, & \text{if } \Pi = \Phi_3, \\ 0, & \text{otherwise}, \end{cases}$$

$$\mathfrak{T}^{\varphi}(\Pi) = \begin{cases} 0, & \text{if } \Pi = \overline{0}, \\ 0, & \text{if } \Pi = \overline{1}, \\ \dfrac{1}{3}, & \text{if } \Pi = \{\Phi_1, \Phi_2\}, \\ 1, & \text{otherwise}, \end{cases} \qquad \mathfrak{P}^{\varphi}(\Pi) = \begin{cases} 0, & \text{if } \Pi = \overline{0}, \\ 0, & \text{if } \Pi = \overline{1}, \\ \dfrac{1}{4}, & \text{if } \Pi = \Phi_3, \\ 1, & \text{otherwise}, \end{cases}$$

$$\mathfrak{P}^{\varrho}(\Pi) = \begin{cases} 1, & \text{if } \Pi = \overline{0}, \\ \dfrac{1}{4}, & \text{if } \Pi = \Theta, \\ \dfrac{1}{2}, & \text{if } \overline{0} < \Pi < \Theta, \\ 0, & \text{otherwise}, \end{cases} \qquad \mathfrak{P}^{\varsigma}(\Pi) = \begin{cases} 0, & \text{if } \Pi = \overline{0}, \\ \dfrac{3}{4}, & \text{if } \Pi = \Theta, \\ \dfrac{1}{2}, & \text{if } \overline{0} < \Xi < \Theta, \\ 1, & \text{otherwise}, \end{cases}$$

$$\mathfrak{P}^{\varphi}(\Pi) = \begin{cases} 0, & \text{if } \Pi = \overline{0}, \\ \dfrac{2}{3}, & \text{if } \Pi = \Theta, \\ \dfrac{1}{2}, & \text{if } \overline{0} < \Xi < \Theta, \\ 1, & \text{otherwise}. \end{cases}$$

(1) The identity mapping $f : (\Omega,\ \mathfrak{T}^{\varrho\varsigma\varphi}) \to (\Xi,\ \mathfrak{F}^{\varrho\varsigma\varphi})$ is *svnB*^{P}*-continuous* but it is not *svn-continuous*.
(2) The identity mapping $f : (\Omega,\ \mathfrak{T}^{\varrho\varsigma\varphi}) \to (\Xi,\ \mathfrak{F}^{\varrho\varsigma\varphi})$ is *svn*β*p-continuous* but it is not *svn*α*p-continuous*.
Example 7 Consider $\Omega = \{u_1,\ u_2,\ u_3\}$; define the *svn-sets* $\Phi_1,\ \Phi_2,\ \Phi_3,\ \Upsilon \in £^{\Omega}$ as follows

$$\Phi_1 = \langle (0.6,\ 0.6,\ 0.6),\ (0.6,\ 0.6,\ 0.6),\ (0.6,\ 0.6,\ 0.6) \rangle,$$

$$\Phi_2 = \langle (0.2,\ 0.2,\ 0.2),\ (0.6,\ 0.6,\ 0.6),\ (0.6,\ 0.6,\ 0.6) \rangle,$$

$$\Phi_3 = \langle (0.5,\ 0.5,\ 0.5),\ (0.5,\ 0.5,\ 0.5),\ (0.5,\ 0.5,\ 0.5) \rangle,$$

$$\Theta = \langle (0.1,\ 0.1,\ 0.1),\ (0,\ 0,\ 0),\ (0,\ 0,\ 0) \rangle.$$

We define, $\mathfrak{T}^{\varrho} : £^{\Omega} \to £$, $\mathfrak{T}^{\varsigma} : £^{\Omega} \to £$, $\mathfrak{T}^{\varphi} : £^{\Omega} \to £$, $\mathfrak{F}^{\varrho} : £^{\Omega} \to £$, $\mathfrak{F}^{\varsigma} : £^{\Omega} \to £$, $\mathfrak{F}^{\varphi} : £^{\Omega} \to £$ and $\mathfrak{P}^{\varrho} : £^{\Omega} \to £$, $\mathfrak{P}^{\varsigma} : £^{\Omega} \to £$, $\mathfrak{P}^{\varphi} : £^{\Omega} \to £$ as follows:

$$\mathfrak{T}^{\varrho}(\Pi) = \begin{cases} 1, & \text{if } \Pi = \overline{0}, \\ 1, & \text{if } \Pi = \overline{1}, \\ \frac{3}{4}, & \text{if } \Pi = \{\Phi_1,\ \Phi_2\}, \\ 0, & \text{otherwise}, \end{cases} \qquad \mathfrak{F}^{\varrho}(\Pi) = \begin{cases} 1, & \text{if } \Pi = \overline{0}, \\ 1, & \text{if } \Pi = \overline{1}, \\ \frac{2}{3}, & \text{if } \Pi = \Phi_3, \\ 0, & \text{otherwise}, \end{cases}$$

$$\mathfrak{T}^{\varsigma}(\Pi) = \begin{cases} 0, & \text{if } \Pi = \overline{0}, \\ 0, & \text{if } \Pi = \overline{1}, \\ \frac{1}{4}, & \text{if } \Pi = \{\Phi_1,\ \Phi_2\}, \\ 1, & \text{otherwise}, \end{cases} \qquad \mathfrak{F}^{\varsigma}(\Pi) = \begin{cases} 1, & \text{if } \Pi = \overline{0}, \\ 1, & \text{if } \Pi = \overline{1}, \\ \frac{1}{3}, & \text{if } \Pi = \Phi_3, \\ 0, & \text{otherwise}, \end{cases}$$

$$\mathfrak{T}^{\varphi}(\Pi) = \begin{cases} 0, & \text{if } \Pi = \overline{0}, \\ 0, & \text{if } \Pi = \overline{1}, \\ \frac{1}{3}, & \text{if } \Pi = \{\Phi_1,\ \Phi_2\}, \\ 1, & \text{otherwise}, \end{cases} \qquad \mathfrak{P}^{\varphi}(\Pi) = \begin{cases} 0, & \text{if } \Pi = \overline{0}, \\ 0, & \text{if } \Pi = \overline{1}, \\ \frac{1}{4}, & \text{if } \Pi = \Phi_3, \\ 1, & \text{otherwise}, \end{cases}$$

$$\mathfrak{P}^{\varrho}(\Pi)=\begin{cases}1, & \text{if } \Pi=\overline{0},\\ \frac{1}{4}, & \text{if } \Pi=\Theta,\\ \frac{1}{2}, & \text{if } \overline{0}<\Pi<\Theta,\\ 0, & \text{otherwise,}\end{cases}\qquad \mathfrak{P}^{\varsigma}(\Pi)=\begin{cases}0, & \text{if } \Pi=\overline{0},\\ \frac{3}{4}, & \text{if } \Pi=\Theta,\\ \frac{1}{2}, & \text{if } \overline{0}<\Xi<\Theta,\\ 1, & \text{otherwise,}\end{cases}$$

$$\mathfrak{P}^{\varphi}(\Pi)=\begin{cases}0, & \text{if } \Pi=\overline{0},\\ \frac{2}{3}, & \text{if } \Pi=\Theta,\\ \frac{1}{2}, & \text{if } \overline{0}<\Xi<\Theta,\\ 1, & \text{otherwise.}\end{cases}$$

(1) The identity mapping $f:(\Omega,\ \mathfrak{T}^{\varrho\varsigma\varphi})\to(\Xi,\ \mathfrak{F}^{\varrho\varsigma\varphi})$ is *svn*$\beta^{\star}$*p-continuous*, but it is not *svnP***p-continuous*.
(2) The identity mapping $f:(\Omega,\ \mathfrak{T}^{\varrho\varsigma\varphi})\to(\Xi,\ \mathfrak{F}^{\varrho\varsigma\varphi})$ is *svns*β*p-continuous* but it is not *svnpp-continuous*.

Theorem 8 Let $f:(\Omega,\ \mathfrak{T}^{\varrho\varsigma\varphi},\ \mathfrak{P}^{\varrho\varsigma\varphi})\to(\Xi,\ \mathfrak{F}^{\varrho\varsigma\varphi})$ be a mapping. Then the following statements are equivalent:
(1) f is *svn-continuous*.
(2) f is *svn*α*p-continuous* and *svn*$B^{\star P}$*-continuous*.
(3) f is *svnpp-continuous* and *svn*$B^{\star P}$*-continuous*.
(4) f is *svnP***p-continuous* and *svn*$B^{\star P}$*-continuous*.
(5) f is *svns*β*p-continuous* and *svn*$B^{\star P}$*-continuous*.
(6) f is *svns*$\beta^{\star}$*p-continuous* and *svn*$B^{\star P}$*-continuous*.
(7) f is *svns*α*p-continuous* and *svns*B^{P}*-continuous*.
(8) f is *svnpp-continuous* and *svns*B^{P}*-continuous*.
(9) f is *svnpp-continuous* and *svns*B^{P}*-continuous*.
(10) f is *svnpp-continuous* and *svn*B^{P}*-continuous*.
(11) f is *svnspp-continuous* and *svn*δ*p-continuous*.
(12) f is *svn*δ*p-continuous* and *svn*S^{P}_{α}*p-continuous*.

Proof. Follows from Theorem 7 and Definition 13. □

Theorem 9 Let $f:(\Omega,\ \mathfrak{T}^{\varrho\varsigma\varphi},\ \mathfrak{P}^{\varrho\varsigma\varphi})\to(\Xi,\ \mathfrak{F}^{\varrho\varsigma\varphi})$ be a mapping. Then the following statements are equivalent:
(1) f is *svn*α*p-continuous*.
(2) For every $\mathfrak{F}^{\varrho}(\Pi^{c})\geq\varkappa$, $\mathfrak{F}^{\varsigma}(\Pi^{c})\leq 1-\varkappa$, $\mathfrak{F}^{\varphi}(\Pi^{c})\leq 1-\varkappa$, $f^{-1}(\Pi)$ is *svn*α*pc*.
(3) $\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{I}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(f^{-1}(\Pi),\ \varkappa),\ \varkappa),\ \varkappa)\leq f^{-1}(\mathfrak{CL}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa))$, for every $\Pi\in\pounds^{\Xi}$.
(4) $\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{I}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon,\ \varkappa),\ \varkappa),\ \varkappa)\leq f^{-1}(\mathfrak{CL}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(f(\Upsilon),\ \varkappa))$, for every $\Upsilon\in\pounds^{\Omega}$.

Proof. (1) $\Rightarrow$ (2): Definition 14., provide a clear and simple explanation.

(2) $\Rightarrow$ (3): For any $\Pi\in\pounds^{\Xi}$, $\varkappa\in\pounds_0$. Since, $\mathfrak{F}^{\varrho}([\mathfrak{CL}_{\mathfrak{F}^{\varrho}}(\Pi,\ \varkappa)]^{c})\geq\varkappa$, $\mathfrak{F}^{\varsigma}([\mathfrak{CL}_{\mathfrak{F}^{\varsigma}}(\Pi,\ \varkappa)]^{c})\leq 1-\varkappa$, $\mathfrak{F}^{\varphi}([\mathfrak{CL}_{\mathfrak{F}^{\varphi}}(\Pi,\ \varkappa)]^{c})\leq 1-\varkappa$, by (2) $f^{-1}(\mathfrak{CL}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\Pi,\ r))$ is $\varkappa$-*svn*α*pc*. Therefore, $[f^{-1}(\mathfrak{CL}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa))]^{c}$ is is $\varkappa$-*svn*α*po*. Thus,

$$[\mathfrak{CL}_{\mathfrak{F}^{\varrho}}(\Pi,\ \varkappa)]^c \leq int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\underline{1}-f^{-1}(\mathfrak{CL}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)),\ \varkappa),\ \varkappa),\ \varkappa)$$

$$= [\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{I}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(f^{-1}(\mathfrak{CL}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)),\ \varkappa),\ \varkappa),\ \varkappa)]^c.$$

Thus, $f^{-1}(\mathfrak{CL}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa) \geq \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{I}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(f^{-1}(\Pi),\ \varkappa),\ \varkappa),\ \varkappa)$.

(3) $\Rightarrow$ (4): For any $\Upsilon \in \pounds^{\Omega}$ and $\varkappa \in \pounds_0$. By (3), we have,

$$\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{I}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon,\ \varkappa),\ \varkappa),\ \varkappa) \leq \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{I}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(f^{-1}f(\Upsilon),\ \varkappa),\ \varkappa),\ \varkappa)$$

$$\leq f^{-1}(\mathfrak{CL}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(f(\Upsilon),\ \varkappa)),$$

and hence $f(\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{I}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Upsilon,\ \varkappa),\ \varkappa),\ \varkappa)) \leq \mathfrak{CL}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(f(\Upsilon),\ \varkappa)$.

(4) $\Rightarrow$ (1): Presume that $\mathfrak{F}^{\varrho}(\Theta) \geq \varkappa$, $\mathfrak{F}^{\varsigma}(\Theta) \leq 1-\varkappa$, and $\mathfrak{F}^{\varphi}(\Theta) \leq 1-\varkappa$, then, by (4),

$$f(\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{I}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(f^{-1}(\Theta^c),\ \varkappa),\ \varkappa),\ \varkappa)) \leq \mathfrak{CL}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(ff^{-1}(\Theta^c),\ r)$$

$$\leq \mathfrak{CL}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\Theta^c,\ \varkappa) = \Theta^c.$$

Thus, $\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{I}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(f^{-1}(\Theta^c),\ \varkappa),\ \varkappa),\ \varkappa) \leq f^{-1}(\Theta^c) = [f^{-1}(\Theta)]^c$. Consequently, we have $f^{-1}(\Theta) \leq int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(f^{-1}(\Theta),\ \varkappa),\ \varkappa),\ \varkappa)$. This show that $f^{-1}(\Theta)$ is *svnαpo* and f is *svnαp-continuous*. □

Theorem 10 Let $f : (\Omega,\ \mathfrak{T}^{\varrho\varsigma\varphi},\ \mathfrak{P}^{\varrho\varsigma\varphi}) \to (\Xi,\ \mathfrak{F}^{\varrho\varsigma\varphi})$ be a *svnαp-continuous*. Then,

(1) $f(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)) \leq \mathfrak{CL}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(f(\Pi),\ \varkappa)$, for any $\Pi \in \pounds^{\Omega}$ is *$\varkappa$-svnppo* and $\varkappa \in \pounds_0$.

(2) $\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(f^{-1}(\Upsilon),\ \varkappa) \leq f^{-1}(\mathfrak{CL}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\Upsilon,\ \varkappa))$, for every $\Upsilon \in \pounds^{\Xi}$ is *$\varkappa$-svnppo* and $\varkappa \in \pounds_0$.

Proof. (1) Assume that $\Pi \in \pounds^{\Omega}$ is *$\varkappa$-svnppo*. Then $\Pi \leq int_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa)$. Thus, by Theorem 9, we have

$$f(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)) \leq f(\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ r)) \leq f(\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa),\ \varkappa))$$

$$\leq f(\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{I}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa),\ )) \leq \mathfrak{CL}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(f(\Pi),\ \varkappa).$$

(2) Assume that $\Upsilon \in \pounds^{\Xi}$ is *$\varkappa$-svnppo*, then $\Upsilon \leq int_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\Upsilon,\ \varkappa),\ \varkappa)$. Thus, by Theorem 9, we have

$$\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(f^{-1}(\Upsilon),\ \varkappa) \leq \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(f^{-1}(\Upsilon),\ \varkappa) \leq \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(f^{-1}(int_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\Upsilon,\ \varkappa),\ \varkappa),\ \varkappa))$$

$$\leq \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(int_{\mathfrak{T}^{\varrho\varsigma\varphi}}(f^{-1}(int_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\Upsilon,\ \varkappa),\ \varkappa)),\ \varkappa),\ \varkappa),\ \varkappa),\ \varkappa)$$

$$\leq \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{I}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(f^{-1}(int_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\Upsilon,\ \varkappa),\ \varkappa)),\ \varkappa),\ \varkappa),\ \varkappa)$$

$$\leq f^{-1}(\mathfrak{CL}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(int_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\Upsilon,\ \varkappa),\ \varkappa),\ \varkappa)) \leq f^{-1}(\mathfrak{CL}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)).$$

□

Definition 15 Let $f : (\Omega,\ \mathfrak{T}^{\varrho\varsigma\varphi}) \to (\Xi,\ \mathfrak{F}^{\varrho\varsigma\varphi})$ be a mapping and $\varkappa \in £_0$. Then f is said to be *svn-open* if $\mathfrak{F}^{\varrho}(f(\Pi)) \leq \mathfrak{T}^{\varrho}(\Pi)$, $\mathfrak{F}^{\varsigma}(f(\Pi)) \geq \mathfrak{T}^{\varsigma}(\Pi)$ and $\mathfrak{F}^{\varphi}(f(\Pi)) \geq \mathfrak{T}^{\varphi}(\Pi)$ for each $\mathfrak{T}^{\varrho}(\Pi) \geq \varkappa$, $\mathfrak{T}^{\varsigma}(\Pi) \leq 1-\varkappa$, $\mathfrak{T}^{\varphi}(\Pi) \leq 1-\varkappa$.

Definition 16 A mapping $f : (\Omega,\ \mathfrak{T}^{\varrho\varsigma\varphi}) \to (\Xi,\ \mathfrak{F}^{\varrho\varsigma\varphi},\ \mathfrak{P}^{\varrho\varsigma\varphi})$ is called *svnαp-open* (resp. *svnsp-open*, *svnpp-open*, *svnβp-open*) if $f(\Pi)$ is *ϰ-svnαpo* (resp. *ϰ-svnspo*, *ϰ-svnppo*, *ϰ-svnβpo*) for every $\mathfrak{T}^{\varrho}(\Pi) \geq \varkappa$, $\mathfrak{T}^{\varsigma}(\Pi) \leq 1-\varkappa$, $\mathfrak{T}^{\varphi}(\Pi) \leq 1-\varkappa$.

Remark 4 From the above definitions we obtain the following diagram:

svn-open ⇒ *svnαp-open* ⇒ *svnpp-open*

⇓ ⇓

svnsp-open ⇒ *svnβp-open*

Theorem 11 A mapping $f : (\Omega,\ \mathfrak{T}^{\varrho\varsigma\varphi}) \to (\Xi,\ \mathfrak{F}^{\varrho\varsigma\varphi},\ \mathfrak{P}^{\varrho\varsigma\varphi})$ is called *svnαp-open* iff it is both *svnsp-open* and *svnpp-open*.

Proof. Theorem 6 (3), providing a clear and simple explanation. □

Theorem 12 A mapping $f : (\Omega,\ \mathfrak{T}^{\varrho\varsigma\varphi}) \to (\Xi,\ \mathfrak{F}^{\varrho\varsigma\varphi},\ \mathfrak{P}^{\varrho\varsigma\varphi})$ is called *svnαp-open* iff for every $\Pi \in £^{\Xi}$ and any $\mathfrak{T}^{\varrho}(\Upsilon^c) \geq \varkappa$, $\mathfrak{T}^{\varsigma}(\Upsilon^c) \leq 1-\varkappa$, $\mathfrak{T}^{\varphi}(\Upsilon^c) \leq 1-\varkappa$ such that $f^{-1}(\Pi) \leq \Upsilon$, there exists $\Theta \in £^{\Xi}$ is *ϰ-svnαpc* containing Π such that $f^{-1}(\Theta) \leq \Upsilon$.

Proof. Straightforward. □

Theorem 13 Let $f : (\Omega,\ \mathfrak{T}^{\varrho\varsigma\varphi}) \to (\Xi,\ \mathfrak{F}^{\varrho\varsigma\varphi},\ \mathfrak{P}^{\varrho\varsigma\varphi})$ be a called *svnαp-open*. Then,

(1) $f^{-1}(\mathfrak{CL}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(int^{\star}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\mathfrak{CL}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa),\ \varkappa),\ \varkappa)) \leq \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(f^{-1}(\Pi),\ \Pi)$, for every $\Pi \in £^{\Xi}$.

(2) $f^{-1}(\mathfrak{C}^{\star}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\Pi,\ \varkappa)) \leq \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(f^{-1}(\Pi),\ \varkappa)$ for every *ϰ-svnppo* and $\Pi \in £^{\Xi}$.

Proof. (1) Let $\Pi \in £^{\Xi}$ then, $\mathfrak{T}^{\varrho}([\mathfrak{CL}_{\mathfrak{T}^{\varrho}}(f^{-1}(\Pi),\ \varkappa)]^c) \geq \varkappa$, $\mathfrak{T}^{\varsigma}([\mathfrak{CL}_{\mathfrak{T}^{\varsigma}}(f^{-1}(\Pi),\ \varkappa)]^c) \leq 1-\varkappa$ and $\mathfrak{T}^{\varphi}([\mathfrak{CL}_{\mathfrak{T}^{\varphi}}(f^{-1}(\Pi),\ \varkappa)]^c) \leq 1-\varkappa$. By Theorem 12, there exists *ϰ-svnαpc* $\Upsilon \in £^{\Xi}$ with $f^{-1}(\Upsilon) \leq \mathfrak{CL}_{\mathfrak{T}^{\varrho}}(f^{-1}(\Pi),\ \varkappa)$. Since Υ^c is *ϰ-svnαpo*, we have $f^{-1}(\Upsilon^c) \leq f^{-1}(int_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\mathfrak{C}^{\star}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(int_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\Upsilon^c,\ \varkappa),\ \varkappa),\ \varkappa))$ and

$$[f^{-1}(\Upsilon)]^c \leq f^{-1}([\mathfrak{CL}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\mathfrak{I}^{\star}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\mathfrak{CL}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\Upsilon,\ \varkappa),\ \varkappa),\ \varkappa))$$

$$\leq [f^{-1}(\mathfrak{CL}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\mathfrak{I}^{\star}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\mathfrak{CL}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\Upsilon,\ \varkappa),\ \varkappa),\ \varkappa)]^c).$$

Therefore, $f^{-1}(\mathfrak{CL}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\mathfrak{I}^{\star}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\mathfrak{CL}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\Upsilon, \varkappa), \varkappa), \varkappa)) \leq f^{-1}(\Upsilon) \leq \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(f^{-1}(\Pi), \varkappa)$. Hence, $f^{-1}(\mathfrak{CL}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\mathfrak{I}^{\star}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\mathfrak{CL}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\Pi, \varkappa), \varkappa), \varkappa)) \leq \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(f^{-1}(\Pi), \varkappa)$.

(2) We can determine that by means of a related reasoning. □

Theorem 14 Let $f : (\Omega, \mathfrak{T}^{\varrho\varsigma\varphi}) \to (\Xi, \mathfrak{F}^{\varrho\varsigma\varphi}, \mathfrak{P}^{\varrho\varsigma\varphi})$ be a mapping. Then,

(1) If f is $\varkappa$-*svnppo*, then $f^{-1}(\mathfrak{CL}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\mathfrak{I}^{\star}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\Pi, \varkappa), \varkappa)) \leq \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(f^{-1}(\Pi), \varkappa)$, for every $\Pi \in \pounds^{\Xi}$ and $\varkappa \in \pounds_0$.

(2) If f is $\varkappa$-*svnspo*, then $f^{-1}(\mathfrak{I}^{\star}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\mathfrak{CL}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\Pi, \varkappa), \varkappa)) \leq \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(f^{-1}(\Pi), \varkappa)$, for every $\Pi \in \pounds^{\Xi}$ and $\varkappa \in \pounds_0$.

(3) If f is $\varkappa$-*svn*β*po*, then $f^{-1}(int_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\mathfrak{CL}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\mathfrak{I}^{\star}_{\mathfrak{F}^{\varrho\varsigma\varphi}}(\Pi, \varkappa), \varkappa), \varkappa)) \leq \mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}(f^{-1}(\Pi), \varkappa)$, for every $\Pi \in \pounds^{\Xi}$ and $\varkappa \in \pounds_0$.

Proof. The proof is quite similar to that of Theorem 13. □

5. Conclusion

In conclusion, this study explored the complex domain of *svn* closure operators ($\mathfrak{C}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}$, $\mathfrak{I}^{\star}_{\mathfrak{T}^{\varrho\varsigma\varphi}}$, $P\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}$, $\alpha\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}}$, $\beta\mathfrak{P}\mathfrak{CL}_{\mathfrak{T}^{\varrho\varsigma\varphi}}$, $\beta\mathfrak{P}int_{\mathfrak{T}^{\varrho\varsigma\varphi}} : \pounds^{\Omega} \times \pounds_0 \to \pounds^{\Omega}$). We devel-oped basic theorems that clarify the connections between finer and coarser primals by investigating svn- primals, cluster points, and limit points. These theorems offer important new insights into the patterns of continuous of single valued neutrosophic primal systems. The newly presented ideas of svns-limit and svns-cluster points proved to be crucial in describing the behavior of these systems. The results of this research open up new possibilities for practical applications in several fields and strengthen the theoretical foundations of neutrosophic soft systems. Novel approaches to computational intelligence and decision support systems are made possible by the established theorems and insights, which offer scholars and practitioners a sophisticated knowledge of continuous in single valued neutrosophic primal topological spaces. In summary, the work described here represents a major advancement in the field of neutrosophic primal structures, providing a useful foundation for future study.

6. Discussion of future work

Boundedness in topological spaces is a well-established concept fundamental to topological analysis (see [36]). The collection of bounded sets forms an ideal, a notion extended by bornology, which generalizes this concept. In fuzzy set theory, this generalization is further enriched through fuzzy bornology (see [37–39]).

Extending these ideas, future research could explore boundedness in single-valued neutrosophic topological spaces by investigating:

The collection of bounded single-valued neutrosophic soft sets; The concept of boundedness within neutrosophic soft topological spaces.

Conflict of interest

The author declares that there is no conflict of interest regarding the publication of this article.

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