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Research Article

Neutrosophic Lindley Distribution: Simulation, Application, and Comparative Study

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Abstract: Classical statistical methods are commonly applied in distribution theory across various disciplines; however, they often fall short in addressing uncertainty, imprecision, and indeterminacy. In situations when classical distributions fail, such as when there is uncertainty, ambiguity, or missing information, the neutrosophic lindley distribution (NLiD) is important because it models indeterminate data. The classical Lindley distribution is extended by incorporating neutrosophic notions, providing flexibility for methods of ambiguous inference. Applications involving reliability analysis, risk management, and other domains with internal data uncertainties are especially well-suited for NLiD. This paper introduces the neutrosophic Lindley distribution (NLiD) to incorporate imprecision into the statistical framework. We derive key properties of the NLiD, including survival, hazard, and reverse hazard functions, as well as the odds ratio, Mills ratio, mean, and variance. Additionally, we explore entropy measures such as Neutrosophic Renyi, Neutrosophic Tsallis, and Neutrosophic Arimoto entropies, complemented by a simulation study and graphical analysis. Maximum likelihood estimation is employed to estimate the distribution parameters, with simulation validating the accuracy of these estimates. Our findings reveal that the proposed distribution can exhibit symmetric, left-skewed, and right-skewed characteristics. An empirical evaluation using a on dioxin consumption in food demonstrates that the proposed model is effective and practical for real-world application.

***Keywords*:** classical statistics, imprecise data, distribution, simulation, application

MSC: 65L05, 34K06, 34K28

1. Introduction

[1] introduced one parameter Lindley distribution. This distribution is a mixture of exponential (ϑ) and gamma (2, ϑ) distributions. [2] derived many properties of this distribution and explored that this distribution is better than the exponential distribution in many cases. The CDF and the PDF of the neutrosophic Lindley distribution are defined as:

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$$F(X) = 1 - \left[1 + \frac{\vartheta x}{1+\vartheta}\right] e^{-\vartheta x} \tag{1}$$

$$f(x) = \frac{\vartheta^2}{(1+\vartheta)}(1+x)e^{-\vartheta x} \tag{2}$$

Neutrosophic statistics is a branch of statistics that focuses on handling incomplete and uncertain data by applying neutrosophic reasoning. It was introduced in 1995 by Smarandache. Neutronosophy, which allows for the representation of ambiguity, contradiction, and uncertainty, was developed by [3] through the extension of fuzzy logic. According to [4]. It is a novel field of philosophy that is promoted as a fuzzy logic generalization. It is commonly assumed when using classical statistics that the data are unambiguous, in which case each observation is assigned a particular value. However, information from the real world sometimes includes missing or confusing details. Classical distributions can only represent determinate data with exact values, they are not appropriate for situations where information is ambiguous or lacking, such as reliability analysis in systems with unpredictable failure times. Neutrosophic statistics offers a paradigm for managing unclear, inadequate, and inconsistent data to get beyond these limitations. Interval statistics is a neutrosophic generalization that also examines fuzzy interval sets. When data is known or predictable, neutrophilic statistics transform into classical statistics. This allows for the handling of all possible scenarios when working with statistical data, particularly when dealing with ambiguous and imprecise statistical data. According to [5], many data sets are more ambiguous, nondeterministic, or confusing than determinate data; in these circumstances, it is preferable to use neutrosophic statistical approaches. Recent years have seen an interest in the study of neutrosophic probability distributions among some scholars like Neutrosophic Weibull distribution [6], Neutrosophic Log-Logistic Distribution [7], Neutrosophic exponential distribution [8], Neutrosophic beta distribution [9], Neutrosophic gamma distribution [10], and Neutrosophic generalized Pareto distribution [11]. [12] presented the negative binomial distribution under neutrosophic statistics and the algorithm. More information can be seen in [13–16]. Existing neutrosophic distributions use degrees of indeterminacy to expand classical models, but they are not flexible enough to explain complicated survival data that contains both determinate and indeterminate components. By combining the adaptability of the Lindley distribution with neutrosophic concepts, the Neutrosophic Lindley Distribution (NLiD) fills these gaps and improves its suitability for uncertainty and reliability.

The motivation for this new approach to the Lindley distribution arises from the limitations of the neutrosophic Lindley distribution introduced by [17]. Alanaz's methodology assumes that each observation is uncertain and lies within a specific range, where the uncertainty or indeterminacy is defined by two values corresponding to a single observation of the random variable. For example, if the random variable represents the lifetime of a product, an observation might ambiguously indicate that the lifetime falls between 20-25 hours. This range-based uncertainty applies consistently to all observations. [17] addressed this scenario by modeling the neutrosophic random variable $X_N \in (X_L, X_U)$, where it follows the Lindley distribution. In contrast, our proposed methodology targets a different form of uncertainty or indeterminacy, which is often encountered in real-world applications. Specifically, this approach addresses cases where ambiguity exists only in the upper limit of the recorded observation. For instance, when recording a product's lifetime, we may know the product lasts at least 20 hours but remain uncertain about the exact upper limit. Here, the lower limit is known, while the uncertainty pertains to the upper limit. To accommodate this type of neutrosophic data, we propose a novel neutrosophic Lindley distribution. In this model, the neutrosophic random variable is expressed as $X_N = X_L + I_N X_L = (1 + I_N)X_L$, where I_N represents the indeterminate factor that determines the upper limit of the observation.

A single-parameter Lindley distribution is famous for life data analysis due to its flexibility and simplicity. The distribution that most accurately represents lifetime data based on an interval set of numbers for ambiguous parameters is the Lindley distribution. Due to its practical approach, we constructed the Neutrosophic Lindley model which can be applied to uncertain, ambiguous, and interval data. The presentation of the article is as follows: section 2 presents the construction of Neutrosophic Lindley distribution (NLiD) along with graphical representation, section 3 depicts some

properties for the NLiD, section 4 shows the Neutrosophic entropies, section 5 presents the simulation study for the entropies and maximum likelihood estimation of the NLiD, section 6 discussed the applications of the NLiD, section 6 shows the conclusion of the study.

2. Development of neutrosophic lindley distribution

The following is the conventional form for neutrosophic numbers, which is based on classical statistics.

$$X_N = E + I$$

Two components make up data: E represents the precise or determined data, while I represent the uncertain, inexact, or indeterminate portion of the data. To identify the neutrosophic random variable, use a subscript N, such as X_N. The neutrosophic variable is defined as $X_N = X_L + I_N X_U$, $I_N[X_L,\ X_U]$ where X_L is the determined and $I_N X_U$ is the indetermined part. $I_N \in (X_L,\ X_U)$ is represented as an undetermined interval. Assume that a random variable $X_N = X_L + I_N X_U$ follows a Neutrosophic Lindley distribution with one parameter $\vartheta_N \in [\vartheta_L,\ \vartheta_U]$. The CDF and PDF for the Neutrosophic Lindley distribution are:

$$F(x) = 1 - \left[1 + \frac{\vartheta(1+I_N)x}{1+\vartheta}\right] e^{-\vartheta(1+I_N)x} \tag{3}$$

$$f(x) = \frac{\vartheta^2}{(1+\vartheta)}(1+I_N)\left[1+(1+I_N)x\right] e^{-\vartheta(1+I_N)x} \tag{4}$$

Theorem 1 Consider that the neutrosophic random variable $X_N \in (1+I_N)X_L$, where X_N follows the PDF given in (4), is a valid PDF.

Proof. Consider the PDF given in (4), then integrating it over the domain of X_N as follows:

$$\frac{\vartheta^2}{(1+\vartheta)}(1+I_N)\int_0^\infty [1+(1+I_N)x]\, e^{-\vartheta(1+I_N)x}\, dx = 1$$

Let $\vartheta(1+I_N)x = u$, the limits remain the same. Substituting, we get:

$$\frac{\vartheta}{(1+\vartheta)}\int_0^\infty \left[e^{-u} + \frac{1}{\vartheta}ue^{-u}\right] du = 1$$

Hence the integral is equal to one which shows that the function given in (4) is a valid PDF.

Theorem 2 Let the random variable $X_N \in (1+I_N)X_L$ follows the PDF given in (4) whose CDF in (3) is a valid distribution function.

Proof. A CDF is a valid function if it fulfills the properties $F(-\infty) = 0$ and $F(+\infty) = 1$. Let the CDF in (3) and

$$\lim_{x\to 0} F(x) = 1 - \left[1 + \frac{\vartheta(1+I_N)0}{1+\vartheta}\right] e^{-\vartheta(1+I_N)0} = 0,$$

$$\lim_{x\to\infty} F(x) = 1 - \frac{1}{\infty} = 1.$$

Hence, proved that (3) is a valid CDF.

The Probability Density Function (PDF) and Cumulative Distribution Function (CDF) are shown graphically. Furthermore, a visualization of the simulation findings for neutrophilic entropies has been produced. These plots demonstrate the impact of neutrosophic factors in the study and offer a thorough understanding of the behavior and features of the distributions in question. Figure 1 demonstrates that for different Neutrosophic values of the parameter appropriate for the skewed data sets, the NLiD is highly tailed.

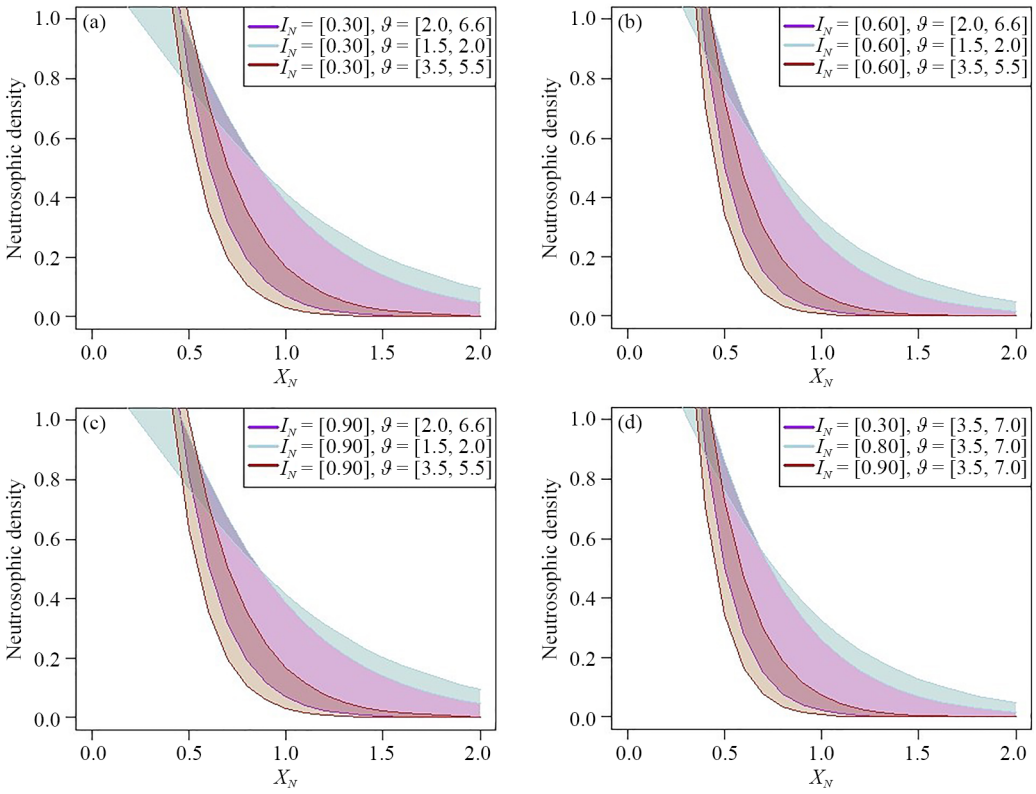

Figure 1. Density plots for the NLiD with different parameter's values

Figure 2 demonstrates the applicability of Neutrosophic entropies when real data contains uncertainties or unpredictability. For the Neutrosophic data sets, the values of the NRE, NTE, and MAE will lie within the lower and higher ranges.

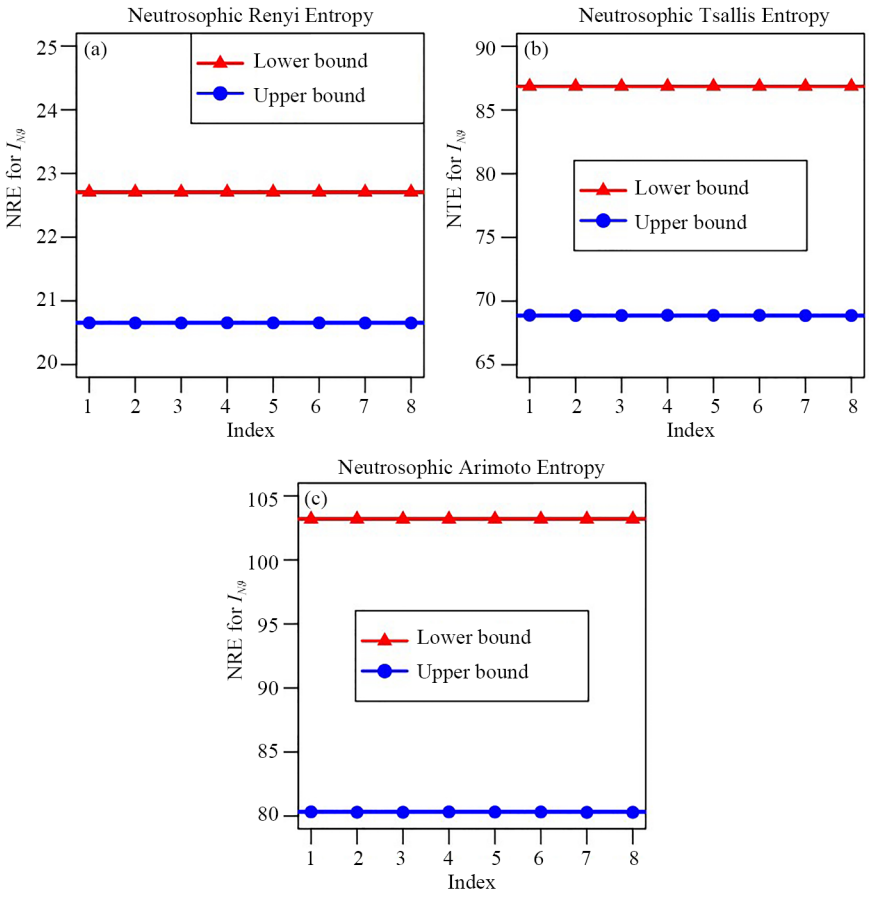

Figure 2. The neutrosophic entropies plot for NLiD with $I_{(N_\vartheta)}$, $\alpha = -0.92$, $[1.62132,\ 0.84459]$

Figure 3 illustrates how well the Neutrosophic Lindley Distribution models dioxin intake data. The theoretical distribution fits most observations well, as seen by its restricted alignment with the actual data, especially in the center range.

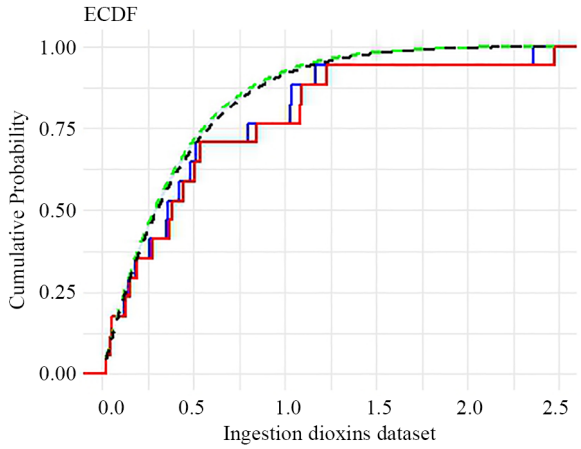

Figure 3. Empirical cumulative distribution function with real life dataset

3. Properties for the neutrosophic lindley distribution

In the context of reliability engineering, the NLiD can simulate the lifespan of industrial machinery, such as turbines or engines that operate under unpredictable environmental circumstances. For example, variations in temperature or material use may inject uncertainty into failure probability, which the NLiD efficiently captures, providing a more reliable survival prediction than classical distributions. Similarly, risk analysis in healthcare is a good fit for the NLiD's hazard function. Think about the situation when uncertainty is introduced by genetic variability or incomplete clinical data while forecasting the survival duration of patients receiving experimental therapies. Because of its capacity to account for these uncertainties, the NLiD is a useful tool for risk assessment and decision-making in the healthcare industry. This section presents several statistical features for the NLiD, including the elasticity, moments, mills ratio, odd function, hazard rate function, and reliability function. The survival function of the neutrosophic Lindley distribution is derived as:

$$S(x) = 1 - F(x)$$

$$S(x) = \left[1 + \frac{\vartheta(1+I_N)x}{1+\vartheta}\right] e^{-\vartheta(1+I_N)x} \tag{5}$$

The hazard function of the proposed model is:

$$h(x) = \frac{f(x)}{S(x)}$$

$$h(x) = \frac{\vartheta^2[1+(1+I_N)x]}{[1+\vartheta+\vartheta(1+I_N)x]} \tag{6}$$

The reverse hazard function is obtained as:

$$\varphi_F(x) = \frac{f(x)}{F(x)}$$

$$\varphi_F(x) = \frac{\vartheta^2[1+(1+I_N)x]e^{-\vartheta(1+I_N)x}}{(1+\vartheta)[(1+\vartheta)+\vartheta(1+I_N)x]e^{-\vartheta(1+I_N)x}} \tag{7}$$

The odd ratio of the NLiD is:

$$O(x) = \frac{F(x)}{S(x)}$$

$$O(x) = \frac{(1+\vartheta)\left(1-e^{-\vartheta(1+I_N)x}\right)+\vartheta\left[(1+I_N)x\right]e^{-\vartheta(1+I_N)x}}{[(1+\vartheta)+\vartheta(1+I_N)x]e^{-\vartheta(1+I_N)x}} \tag{8}$$

The Mills ratio of the proposed distribution is derived as:

$$M(x) = \frac{R(x)}{f(x)}$$

$$M(x) = \frac{1+\vartheta+\vartheta(1+I_N)x}{\vartheta^2\left[1+(1+I_N)x\right]} \tag{9}$$

As the neutrosophic random variable is defined as $X_N = (1+I_N)X_L$, and if we apply expectation on using the properties of expectation, we get:

$$E(X_N) = E[(1+I_N)X_L] = (1+I_N)E(X_L)$$

Similarly,

$$E(X_N)^r = E[(1+I_N)X_L]^r = (1+I_N)E(X_L)^r$$

Where the $E(X)$ is the mean of the classical Lindley distribution and $E(X_L)^r$ is the r-th moments of the Lindley distribution. Additionally, applying the variance and properties of variance on the neutrosophic random variable $X_N = (1+I_N)X_L$ as:

$$\mathrm{Var}(X_N) = \mathrm{Var}[(1+I_N)X_L] = (1+I_N)^2\mathrm{Var}(X_L)$$

Using the above properties of expectation and variance, the mean and variance of the Neutrosophic Lindley distribution are derived as:

$$\mu = (1+I_N)\frac{\vartheta+2}{\vartheta(\vartheta+1)} \tag{10}$$

$$\sigma^2 = (1+I_N)^2\frac{6\vartheta-\vartheta^2+4}{\vartheta^2(\vartheta+1)} \tag{11}$$

4. Neutrosophic entropies for lindley distribution

This section derives three types of Neutrosophic entropy measurements for the Lindley distribution: Rényi, Arimoto, and Tsallis entropies. A simulation analysis is carried out to determine the effectiveness and efficiency of the suggested entropies.

4.1 *Neutrosophic rényi entropy (NRE)*

Due to indeterminacy, this measure works well for measuring uncertainty in situations where data is represented in interval forms. Neutosophic Rényi entropy, for example, might offer a more accurate measure of uncertainty in

reliability engineering by taking into consideration the indeterminate character of the data while evaluating the lifespan of components with unpredictable failure times [18].

The Renyi entropy of the neutrosophic Lindley distribution is derived as:

$$R_v = \frac{1}{1-v}\frac{\vartheta^{2v}}{\vartheta(\vartheta+1)}\log\int_0^\infty [1+(1+I_N)x]^v e^{-\vartheta(1+I_N)x}\,dx, \quad v>0, v\neq 1$$

4.2 *Neutrosophic tsallis entropy (NTE)*

According to [19], Neutrosophic In clustering algorithms that deal with imprecise or ambiguous data, Tsallis entropy is very helpful. By efficiently controlling the inherent uncertainty in pixel categorization, this entropy measure can enhance clustering performance in image processing, for instance, when segmenting pictures with ambiguous borders. The neutrosophic tallis entropy will be:

$$T_v = \frac{1}{v-1}\left[1-\frac{\vartheta^{2v}}{(\vartheta+1)}\int_0^\infty [1+(1+I_N)x]^v e^{-\vartheta(1+I_N)x}\,dx\right], \quad v>0, v\neq 1$$

4.3 *Neutrosophic arimoto entropy (NAE)*

Neutrosophic Arimoto entropy's classical form is well-known for improving information processing in resource-constrained systems, but its specific applications are less well-documented. Bringing it to the neutrosophic realm might improve decision-making in situations when data is not fully or partially trustworthy, such as some data compression settings or communication networks. The neutrosophic arimoto entropy is obtained as:

$$A_v = \frac{v}{1-v}\left[\left(\frac{\vartheta^{2v}}{\vartheta(\vartheta+1)}\int_0^\infty [1+(1+I_N)x]^v e^{-\vartheta(1+I_N)x}\,dx\right)^{\frac{1}{v}}-1\right], \quad v>0, v\neq 1$$

There are no closed forms of neutrosophic entropy measures. So, the numerical values of these Neutrosophic entropy measures are calculated in Table 1.

Table 1. Neutrosophic Renyi, Tsallis, and Arimoto entropies for $v=0.9$ with $I_\vartheta = -0.92,\ [1.62132,\ 0.84459]$

I_ϑ	ϑ	NRE	NTE	NAE
$-0.92,\ [1.62132,\ 0.84459]$	[1.62142, 0.84525]	[22.7066, 20.6551]	[86.8578, 68.8932]	[103.1883, 80.3209]
	[1.62142, 0.84462]	[22.7066, 20.6522]	[86.8578, 68.8704]	[103.1883, 80.2922]
	[1.62132, 0.84459]	[22.7064, 20.6521]	[86.8563, 68.8693]	[103.1864, 80.2908]
	[1.62135, 0.84520]	[22.7065, 20.6549]	[86.8568, 68.8914]	[103.1870, 80.3186]
	[1.62153, 0.84502]	[22.7068, 20.6540]	[86.8594, 68.8849]	[103.1904, 80.3104]
	[1.62143, 0.84507]	[22.7066, 20.6543]	[86.8579, 68.8867]	[103.1885, 80.3127]
	[1.62133, 0.84446]	[22.7065, 20.6515]	[86.8565, 68.8646]	[103.1866, 80.2849]
	[1.62155, 0.84457]	[22.7068, 20.6520]	[86.8597, 68.8686]	[103.1907, 80.2899]

5. Simulation

This section represents the simulation study, firstly for the proposed entropies and secondly for ML estimated of NLiD. A simulation study is used to determine the parameters' performance as well as the effectiveness and performance of the suggested Neutrosophic entropies for the Lindley distribution.

5.1 *Simulation for neutrosophic entropies*

The simulation research is essential for verifying how neutrosophic Rényi, Tsallis, and Arimoto entropies behave under different parameter settings. In contrast to conventional measurements, it shows their adaptability, efficiency, and flexibility in capturing uncertainty. This demonstrates their applicability in the actual world for tasks like risk management and reliability analysis. For the simulation of entropy measures, firstly the random numbers of 10,000 iterations with a sample size of $n = 1,000$ are generated and the neutrosophic parameter value. This process is repeated eight times to find the eight different values of the parameter. These values are used to find the numerical values of the entropies such as NRE, NTE and NAE. The efficiency of the entropies is also discussed in the graphical representation.

According to the results in Tables 1 and 2 the neutrosophic entropies are more flexible and adaptable than their classical equivalents when it comes to capturing uncertainty. The differences in neutrosophic entropy values between various ranges of $I_{N\vartheta}$ and ϑ demonstrate their capacity to simulate complicated systems with uncertain and conflicting inputs. This contrasts with the static character of traditional entropies, which are unable to capture these subtleties. These distinctions highlight the usefulness of neutrosophic entropy measures in solving real-world problems and are especially useful in applications like reliability analysis, clustering ambiguous data, and resource optimization in uncertain contexts.

Table 2. Classical Renyi, Tsallis, and Arimoto entropies for $v = 0.7$

ϑ	NRE	NTE	NAE
1.72967	1.45896	1.830405	2.027078
1.72987	1.45896	1.830405	2.027078
1.73002	1.45896	1.830405	2.027078
1.73005	1.45896	1.830405	2.027078
1.72990	1.45896	1.830405	2.027078
1.72997	1.45896	1.830405	2.027078
1.72993	1.45896	1.830405	2.027078
1.73002	1.45896	1.830405	2.027078

5.2 *Simulation for the MLE*

This simulation study uses the Neutrosophic mean square error (MSE_N), Neutrosophic bias, and Neutrosophic average bias (AB_N) to assess the performance of the Neutrosophic maximum likelihood estimators for the NLiD.

In R software, a Monte Carlo simulation with different sample sizes and fixed values of the Neutrosophic parameter ϑ value with I_N is conducted. The NLiD is used to make an imprecise dataset with Neutrosophic parametric value, and the simulation is replicated $N = 10,000$ times with sample sizes of $n = 50,\ 100,\ 300,\ 500$, respectively. The performance of the Neutrosophic Maximum Likelihood estimators is then computed and shown in Tables 1-4.

Table 3. Parameter's estimated values, average bias, mean standard error, and mean relative estimates for $\vartheta_N = [0.5,\ 1.2]$ and $I_N = 0.58$ calculated from ϑ_N

$I_N, \vartheta_N = 0.58\ [0.5,\ 1.2]$				
Sizes	Estimated value	AB_N	MSE_N	MRE_N
50	[4.8003, 1.8115]	[4.3003, 0.6115]	[18.4971, 0.3789]	[8.6006, 0.5096]
100	[4.8007, 1.8088]	[4.3007, 0.6088]	[18.4982, 0.3732]	[8.6014, 0.5074]
300	[4.7999, 1.8088]	[4.2999, 0.6088]	[18.4900, 0.3714]	[8.5998, 0.5073]
500	[4.8000, 1.8083]	[4.3000, 0.6083]	[18.4905, 0.3706]	[8.6000, 0.5069]

Table 4. Parameter's estimated values, average bias, mean standard error, and mean relative estimates for $\vartheta_N = [0.5,\ 1.2]$ and $I_N = 0$

$I_N, \vartheta_N = 0\ [0.5,\ 1.2]$				
Sizes	Estimated value	AB_N	MSE_N	MRE_N
50	[10.6542, 3.3297]	[10.1542, 2.1297]	[103.1099, 4.5383]	[20.3085, 1.7747]
100	[10.6544, 3.3298]	[10.1544, 2.1298]	[103.1120, 4.5371]	[20.3087, 1.7748]
300	[10.6543, 3.3297]	[10.1543, 2.1297]	[103.1104, 4.5361]	[20.3086, 1.7748]
500	[10.6544, 3.3292]	[10.1544, 2.1292]	[103.1115, 4.5336]	[20.3087, 1.7743]

The algorithm of the Neutrosophic simulation has the following steps:

1. Specify the starting parameter combinations, sample sizes, iterations, and probability density function (PDF).
2. Put into practice an optimization technique that maximizes the probability function to estimate parameters.
3. Calculate the indeterminacy value I_N and generate random samples for every sample size. Determine performance measures like bias and error and carry out parameter estimation repeatedly.
4. Create tables that compile and summarize the data for every combination of sample size and parameters.

The following parameter values are used:

• In Table 1: $\vartheta = [0.5,\ 1.2]$ with $I_N = 0.58$
• In Table 2: $\vartheta = [0.5,\ 1.2]$ with $I_N = 0$ (no indeterminacy)
• In Table 3: $\vartheta = [1.8,\ 3.5]$ with $I_N = 0.49$
• In Table 4: $\vartheta = [1.8,\ 3.5]$ with $I_N = 0$ (no indeterminacy)

The formulas for the Neutrosophic average bias and Neutrosophic mean square error are:

$$AB_N = \frac{1}{N}\sum_{i=1}^{N}\left(\hat{\theta}_{Ni} - \theta_N\right).$$

and

$$MSE_N = \frac{1}{N}\sum_{i=1}^{N}\left(\hat{\theta}_{Ni} - \theta_N\right)^2.$$

Monte Carlo simulations with different sample sizes are run, and Neutrosophic parameters are set using R studio. With sample sizes of $n = 50,\ 100,\ 300$, and 500, in that sequence, the simulation applies a Neutrosophic probability density function for $N = 10{,}000$ runs. Tables 1, 2, 3, and 4 display the NML estimator's performance.

When compared to the classical tables in 4 and 5, the neutrosophic tables in 3 and 6 show better performance. Even in cases of indeterminacy $I_N = 0$, the neutrosophic technique consistently yields reduced mean square error (MSE) and average bias (AB), indicating greater accuracy and precision in parameter estimation. Furthermore, stability and dependability are maintained by the neutrosophic MLE method's robustness across a variety of parameter values and sample sizes. This demonstrates how well it captures uncertainty, which makes it a superior option for practical applications requiring ambiguous or unpredictable data.

Table 5. Parameter's estimated values, average bias, mean standard error, and mean relative estimates for $\vartheta_N = [1.8,\ 3.5]$ and $I_N = 0$ that calculated from ϑ_N

$I_N, \vartheta_N = 0\ [1.8,\ 3.5]$				
Sizes	Estimated value	AB_N	MSE_N	MRE_N
50	[2.2299, 1.4332]	[0.4299, 2.0668]	[0.1887, 4.2807]	[0.2388, 0.5905]
100	[2.2290, 1.4311]	[0.4290, 2.0689]	[0.1860, 4.2850]	[0.2383, 0.5911]
300	[2.2281, 1.4274]	[0.4281, 2.0726]	[0.1839, 4.2972]	[0.2378, 0.5922]
500	[2.2278, 1.4273]	[0.4278, 2.0727]	[0.1834, 4.2970]	[0.2377, 0.5922]

Table 6. Parameter's estimated values, average bias, mean standard error, and mean relative estimates for $\vartheta_N = [1.8,\ 3.5]$ and $I_N = 0.49$ that calculated from ϑ_N

$I_N, \vartheta_N = 0.49\ [1.8,\ 3.5]$				
Sizes	Estimated value	AB_N	MSE_N	MRE_N
50	[1.4295, 1.0416]	[0.3705, 2.4584]	[0.1435, 6.0579]	[0.2058, 0.7024]
100	[1.4271, 1.0328]	[0.3729, 2.4672]	[0.1422, 6.0935]	[0.2072, 0.7049]
300	[1.4257, 1.0278]	[0.3743, 2.4722]	[0.1412, 6.1140]	[0.2080, 0.7064]
500	[1.4251, 1.0274]	[0.3748, 2.4726]	[0.1411, 6.1151]	[0.2082, 0.7065]

6. Applications

To identify the efficiency and adaptability of the NLiD when the data contains complex or unclear values, this section offers a real-world application based on one dataset. A few model selection criteria are used to check the performance of the proposed distribution. The following dataset is:

Average daily ingestion of dioxins: This data is the yearly absorption of dioxins from the typical diet for the years 1998 to 2015 is given in the yearly Report on Environmental Statistics by [20]. It includes an imprecise estimate of the total amount of dioxin consumed from dietary samples. We considered the daily ingestion of dioxins data from [21].

[0.80, 2.06] [0.51, 1.26] [0.12, 1.47] [0.36, 1.47] [1.04, 1.46] [0.35, 1.19] [0.42, 0.65] [0.02, 0.91]
[0.04, 1.05] [2.36, 2.44] [1.029, 1.62] [1.17, 2.26] [0.05, 0.96] [0.48, 1.35] [0.18, 1.18] [0.26, 1.24]
[0.14, 0.80]

Table 7 shows the descriptive statistics for the dioxin intake dataset using the neutrosophic Lindley distribution. The variance shows significant variability in the dataset, whereas the mean values, which fall between 0.6887 and 0.7115, show the central tendency. A flatter distribution than a normal curve is indicated by the negative kurtosis, while skewness values near 0 imply that the data is roughly symmetric. These figures show that the model successfully captures the features of the dataset.

Table 7. Descriptive statistics for the proposed model with dataset

Descriptives	Ingestion of dioxins
Mean	[0.6887, 0.7115]
Variance	[0.3243, 0.3448]
Median	[0.2823, 0.2973]
First Quartile	[0.1192, 0.1257]
Third Quartile	[0.5517, 0.5802]
Skewness	[−0.0548, −0.0500]
Kurtosis	[−0.8476, −0.7604]

Tables 8 and 9 show the estimates and modeling of the proposed density on the ingestion of dioxins dataset, the goodness of fit criteria and p-value shows that the NLiD is showing a good fit on the ingestion of dioxins dataset. The dataset is in interval form therefore the classical Lindley distribution is not suitable for the model.

Table 8. ML estimates and standard errors of the proposed distribution for ingestion of dioxins dataset

Distribution	ϑ	Standard Error
N Lindley	[3.0168, 2.8884]	[0.5554, 0.5297]
Lindley	3.0168	0.5554

Table 9. Model selection criteria for ingestion of dioxins dataset

Models	LL	AIC	BIC	CAIC	HQIC	KS-value	P-value
N Lindley $I_N = [0.05]$	[−1.0638, −2.1032]	[4.1277, 6.2063]	[4.9609, 7.0395]	[4.3943, 6.473]	[4.2105, 6.2891]	[0.1672, 0.1661]	[0.669, 0.6761]
Lindley	−1.0638	4.1277	4.9609	4.3943	4.2105	0.1672	0.669

7. Conclusions

In this research, a new model named Neutrosophic Lindley distribution is proposed to apply to ambiguous and uncertain datasets. This model is helpful for the survival and reliability of datasets for indeterminacies. Different properties in terms of Neutrosophic are discussed here such as Neutrosophic survival function, Neutrosophic hazard function, Neutrosophic reverse hazard function, Neutrosophic odd ratio, Neutrosophic mills ratio, Neutrosophic mean, and Neutrosophic variance. The Neutrosophic entropies for the proposed model are derived and discussed with the help of simulations study for both classical and Neutrosophic Lindley distribution. The shape of this distribution is left skewed, right skewed, and symmetric. The maximum likelihood method is used to find the parametric values. A simulation study is also conducted here to check the applicability of estimators for the small, medium, and large sample sizes and find that as the sample size increases, the mean square error and mean relative error are going to decrease. The proposed model Neutrosophic Lindley distribution is applied to a real-life data set where the dataset has uncertain values. Particularly NLiD is modeled on the daily ingestion of dioxins in dietary. Dioxin is toxic and unhealthy for humans. It causes cancer and further developmental issues such as weaker immune systems and disorders of hormones. The proposed NLiD shows flexibility over the dioxin intake dataset. The proposed density would be chosen to conduct a detailed analysis of food inspections and human nutrition. Overall, the Lindley is considered for this research due to its significant applications in many fields of life particularly in real life situations where the data is uncertain or ambiguous then the classical Lindley is not a suitable model to apply rather its Neutrosophic form named as Neutrosophic Lindley would be applied.

The single parameter neutrosophic Lindley distribution is a generalization of the classical Lindley distribution that incorporates uncertainty or indeterminacy in observed data. The Lindley distribution is widely used in reliability, survival analysis, and other applied fields due to its simplicity and flexibility. However, it has several limitations that restrict its effectiveness in real-life applications: It is a single-parameter model, limiting its ability to accommodate various data shapes. The Lindley distribution lacks the capacity to model heavy-tailed data accurately, as it assumes an exponential decay in the tail. The distribution assumes unimodality and cannot represent datasets with multiple peaks. In the presence of neutrosophic data, this work can be extended to generalizations of the Lindley distribution.

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Conflict of interest

The authors declare no competing financial interest.

References

[1] Lindley DV. Fiducial distributions and Bayes' theorem. *Journal of the Royal Statistical Society Series B (Methodological)*. 1958; 20(1): 102-107.

[2] Ghitany ME, Atieh B, Nadarajah S. Lindley distribution and its application. *Mathematics and Computers in Simulation*. 2008; 78(4): 493-506.

[3] Smarandache F. *A Unifying Field in Logics: Neutrosophic Logic Philosophy*. USA: American Research Press; 1999.

[4] Zadeh LA. Fuzzy sets. *Information and Control*. 1965; 8(3): 338-353.

[5] Alhabib R, Ranna MM, Farah H, Salama AA. Some neutrosophic probability distributions. *Neutrosophic Sets and Systems*. 2018; 22: 30-38. Available from: https://doi.org/10.5281/zenodo.2160478.

[6] Hamza Alhasan KF, Smarandache F. Neutrosophic Weibull distribution and neutrosophic family Weibull distribution. *Neutrosophic Sets and Systems*. 2019; 28(1): 15.

[7] Rao GS. Neutrosophic log-logistic distribution model in complex alloy metal melting point applications. *International Journal of Computational Intelligence Systems*. 2023; 16(1): 48.

[8] Duan WQ, Khan Z, Gulistan M, Khurshid A. Neutrosophic exponential distribution: Modeling and applications for complex data analysis. *Complexity*. 2021; 2021(1): 1-8.

[9] Khan Sherwani RA, Naeem M, Aslam M, Raza MA, Abbas S. Neutrosophic beta distribution with properties and applications. *Neutrosophic Sets and Systems*. 2021; 41(1): 12.

[10] Khan Z, Al-Bossly A, Almazah MM, Alduais FS. On statistical development of neutrosophic gamma distribution with applications to complex data analysis. *Complexity*. 2021; 2021(1): 1-8.

[11] Eassa NI, Zaher HM, El-Magd NAA. Neutrosophic generalized pareto distribution: Infinite study. *Mathematics and Statistics*. 2023; 11(5): 827-833.

[12] Aslam M. The neutrosophic negative binomial distribution: Algorithms and practical application. *REVSTAT Statistical Journal*. 2024. Available from: https://orcid.org/0000-0003-0644-1950.

[13] Aslam M. Neutrosophic *F*-test for two counts of data from the poisson distribution with application in climatology. *Stats*. 2022; 5(3): 773-783.

[14] Aslam M, Rao GS, Khan N, Ahmad L. Two-stage sampling plan using process loss index under neutrosophic statistics. *Communications in Statistics-Simulation and Computation*. 2022; 51(6): 2831-2841.

[15] Al Aita A, Aslam M. Analysis of covariance under neutrosophic statistics. *Journal of Statistical Computation and Simulation*. 2023; 93(3): 397-415.

[16] Aslam M. The *t*-test of a regression coefficient for imprecise data. *Hacettepe Journal of Mathematics and Statistics*. 2024; 53(4): 1130-1140.

[17] Alanaz MM, Mustafa MY, Algamal ZY. Neutrosophic lindley distribution with application for alloying metal melting point. *International Journal of Neutrosophic Science*. 2023; 21(4): 65-71.

[18] Sherwani RAK, Arshad T, Albassam M, Aslam M, Abbas S. Neutrosophic entropy measures for the Weibull distribution: Theory and applications. *Complex Intelligent Systems*. 2021; 7(6): 3067-3076.

[19] Li Q, Ma Y, Smarandache F, Zhu S. Single-valued neutrosophic clustering algorithm based on tsallis entropy maximization. *Axioms*. 2018; 7(3): 57.

[20] Muzembo BA, Iwai-Shimada M, Isobe T, Arisawa K, Shima M, Fukushima T, et al. Dioxins levels in human blood after implementation of measures against dioxin exposure in Japan. *Environmental Health and Preventive Medicine*. 2019; 24(1): 6.

[21] Khan Z, Almazah MMA, Odhah OH, Alshandari HM. Generalized pareto model: Properties and applications in neutrosophic data modelling. *Mathematical Problems in Engineering*. 2022; 2022: 1-11. Available from: https://doi.org/10.1155/2022/3686968.