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Research Article

Automatic Recognition of Human Psychological State Based on EEG Data

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Abstract: The paper deals with the problem of binary classification of Electroencephalography (EEG) data using ordinary personal computers at making computation in real time. Eleven different criteria of similarity of two Multivariate Time Series (MTS) were used for this purpose. Basis on computation results of 32 dimensional EEG signals was established the advantages of the considered methods over each other. Methods “ascending eigenvalue-weighted difference between eigenvector matrices”, “getting into the confidence regions of the linear trends of MTS” and of the method which is obtained by the union of previous two methods gave better results by classification accuracy than others.

***Keywords*:** EEG, MTS, binary classification, accuracy of classification, covariance matrix, eigenvector, eigenvalue

MSC: 62P10, 62P15, 62H30, 68T10

1. Introduction

Mathematical statistics methods are increasingly used to solve practical problems. Among them are problems that need to be solved in real time, with minimal or no human intervention. The last belongs to the problems of machine learning and artificial intelligence, in the set of theoretical foundations of which the methods of mathematical statistics are included too. In recent decades, the share of problems in medicine and psychology has been increasing among the mentioned problems. Their successful solution largely depends on advancements in these scientific fields and the development of relevant technical tools. This class of problems includes the problem discussed in the proposed work such as automatic emotion recognition based on Electroencephalographic (EEG) data set. The solution of the stated problem is principally available because of the relationship between emotions and EEG variations [1]. In this work it is noted that “Many studies suggest that emotional states are associated with electrical activity that is produced in the central nervous system. Brain activity can be detected through its electrical signals by sensing its variations, locations, and functional interactions using EEG devices. EEG signals have excellent temporal resolution and are a direct measurement of neuronal activity. These signals cannot be manipulated or simulated to fake an emotional state, so they provide reliable information.

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The challenge is to decode this information and map it to specific emotions". Moreover, that "Human emotions can be recognized from facial expressions, speech, behavior (gesture/posture) or physiological signals. However, the first three methods can be ineffective since humans may involuntarily or deliberately conceal their real emotions (so-called social masking). The use of physiological signals can lead to more objective and reliable emotion recognition. Compared with peripheral neurophysiological signals, Electroencephalogram (EEG) signals respond to fluctuations of affective states more sensitively and in real time and thus can provide useful features of emotional states" [2]. Time signals of EEG are Multivariate Time Series (MTS) obtained from different points of the human brain in a certain period of time, by means of which the psychological state of a person is classified.

This problem is also relevant in solving many practical problems. For instance, MTS datasets are common in various multimedia, medical and financial applications [3]. "In military and aerospace applications, the high-risk functional state of soldiers and pilots/astronauts can be detected in real time. Emotion recognition can also be applied to public transportation, for example to enhance driving safety by monitoring the emotional state of the driver in real time to prevent dangerous driving under extreme emotional conditions" is mentioned in [2]. Many different methods have been proposed and investigated to solve the problem under consideration (see, for example, [2–12]). For example, a similarity measure for MTS datasets, *Eros* (*E*xtended F*ro*benius norm), which is based on Principal Component Analysis (PCA) is considered in [3]. Along with this, the following traditional similarity measures for MTS datasets, such as Euclidean Distance (ED), Dynamic Time Warping (DTW), Weighted Sum Singular Value Decomposition (WSSVD) and PCA similarity factor Software Process and Capability Maturity Assessment (SPCA) are considered there. Also the following methods were applied for solving the considered problem: Support Vector Machine (SVM) [7], Multilayer Perceptron (MLP) [8], Hidden Markov Model (HMM) [6, 9], Convolutional Neural Network (CNN) [10, 11], *k*-Nearest Neighbors (*k*NN) [2, 12] and many others. However, due to the complexity of the EEG signals, there is no particularly effective analysis method. The problem of determination what distinguishes time series in that set from other time series obtained from the same source is considered in [13]. An incremental algorithm for identifying distinctive subsequences in multivariate, real-valued time series is described and evaluated with data from two very different sources: the response of a set of bandpass filters to human speech and the sensors of a mobile robot.

Especially many works in the discussed direction have been published in recent years (see, for example, [14–20]) and their number is increasing. The reason for this lies in the relevance of the problem and the difficulty of solving it, which is caused by the large dimension and low quality of the data. Low quality is due to false signals arising from objective and subjective reasons, which are added to EEG time signals during their recording and distort them. "The EEG signals are usually mixed with a number of other signals (such as Electrooculography (EOG), Electromyography (EMG), and Electrocardiography (ECG)), noises, interferences, or artifacts" as it is noted in [2]. Based on this, the automatic classification of emotional states with high reliability using EEG is quite a challenging problem. The current results do not meet the existing requirements in terms of both high classification accuracy and promptness.

The paper [1] reviews such machine learning methods for classifying human emotional states using EEG data as: Linear Discriminant Analysis (LDA), Bayesian Linear Discriminant Analysis (BLDA), Support Vector Machine, Multilayer Perceptron, Convolutional Neural Network, Hidden Markov Model, *k*-Nearest Neighbors and many others. The disadvantages of these methods are highlighted here, including the low quality of the obtained results, which is caused by the linearity of the algorithms, the multidimensionality of the observed processes, and their non-stationarity. This non-stationarity complicates the accurate estimation of the relevant covariance matrices and increases the demand for computational power. Also, the following methods for processing EEG signals are considered in [2]: *k*-Nearest Neighbor (*k*NN), Naïve Bayesian (NB), Support Vector Machine (SVM) and Random Forest (RF). Power spectrum estimation methods which are divided into classical and modern methods are considered there as well, where it is noted that the Autoregressive (AR), Moving Average (MA) and Autoregressive Moving Average (ARMA) models are commonly used in modern spectral estimation. Because spectral estimation requires the assumption that the EEG signal is stationary but the measured EEG signal is typically non-stationary in nature, in modern methods time-frequency analysis methods are applied. A similarity measure *Eros* for *k*NN searches in MTS databases is proposed in [21].

In recent years, some research has shown that the human brain is a nonlinear dynamic system, and EEG signals can be considered as the output of such a system [2]. Therefore, nonlinear analysis has been widely used in the analysis of

EEG signals [22–24]. In particular, deep learning models to classify two emotional categories (positive and negative) of EEG data from a 62-channel electrode cap were considered in [25]. To remove eye-movement artifacts, authors also recorded the Electrooculogram (EOG). A pressure sensor was employed to record the response from the subjects in the experiments. They used a 10 s hint before each clip and 20 s rest after each clip. The authors investigated a Deep Belief Network (DBN), DBN integrated with a Hidden Markov Model (DBN-HMM), *k*NN, SVM and Graph regularized Extreme Learning Machine (GELM) methods and achieved the accuracies of classification 87.62%, 86.91%, 85.67%, 84.08%, and 69.66%, respectively.

Despite the rather high accuracy of the obtained results, the mentioned methods of machine learning require a lot of time and computing resources, which is unacceptable in many practical cases. Therefore, the development of algorithms for the solution of the mentioned problem, which allows it to be solved in real time using modern personal computers, remains relevant today and is a very necessary and demanding problem. The results obtained by us in this direction are presented in the present article. In particular, emotional data description and their pre-processing results are given in Item 2. In Item 3 the used criteria and algorithms of their realization are presented. Computation results of processing of EEG signals using examined criteria are described in Item 4. A short discussion of the obtained results is offered in Item 5 and a conclusion is given in Item 6.

2. Data description and their pre-processing

We consider only binary classification of emotion based on 32-dimensional EEG signals. Decisions about the emotional state of a person can be one from two “high” and “low”. Initial information for making decisions is EEG signals written down in Excel files with 32 columns and with several tens of thousands of rows. Information is recorded by means of sensors attached to special helmets while watching a certain video for several minutes. The sensors register 256 values of the signals in a second. The evaluation of the resulting emotional state is carried out by the person under the experiment himself. The Electroencephalographic (EEG) data used in this paper are downloaded from the website http://www.eecs.qmul.ac.uk/mmv/datasets/deap/download.html. Thus we have the initial information as 32 dimensional MTS with different lengths of observations for different tested persons, given in Excel files. The results of the observation of three individuals (for whom the most complete data were provided) were selected for the research below, with conditional numbers 1, 27 and 30. Unfortunately, the results of the observation for the rest of the individuals turned out to be very imperfect. For each person mentioned, the results of 18 experiments are given, which are presented in two sets of “high” and “low” and which correspond to the emotional perception of the signals provided by these persons. Our goal is to develop-select such a method based on this information, which will recognize the emotional state of a person automatically in real time with the highest possible accuracy. The file numbers corresponding to the first person are one or two-digit numbers, of 27^{th} person are four-digit numbers with the first two digits being 33 or 34, and the 30^{th} person are four-digit numbers with the first two digits being 37 or 38.

In order to select the appropriate methods for the preliminary investigation of MTS and to achieve the set goal, a preliminary investigation of the observation results was done by descriptive statistics methods, which was carried out using the statistical package SPSS and MATLAB. In particular, time series trends and random components were separated, covariance matrices were calculated, hypotheses about the linearity of the separated trends were tested, random components were tested for normality, and processes stationarity was tested. The research results showed that MTSs are non-stationary time series with linear trends, the coefficients of which differ significantly even for the same parameters of different experiments (for example, see Figure 1), parameters of MTS are quite strongly correlated with each other (for example, see Table 1), random components are usually not normally distributed (see Figure 2).

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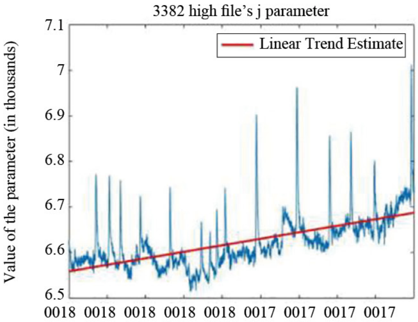
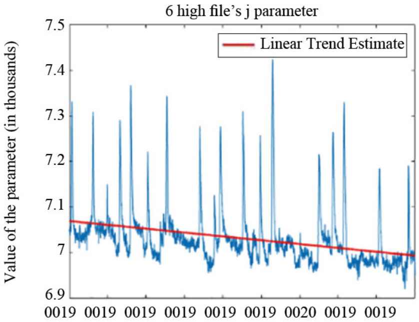
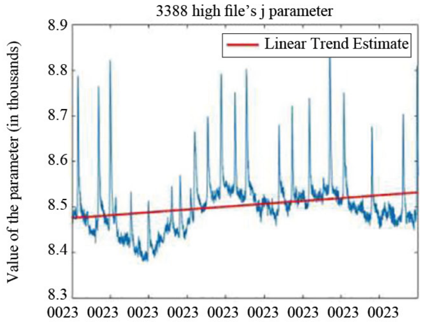
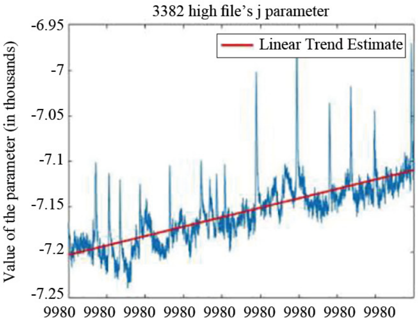
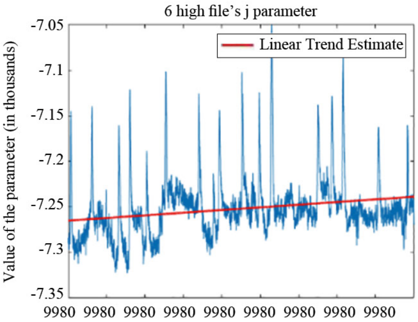
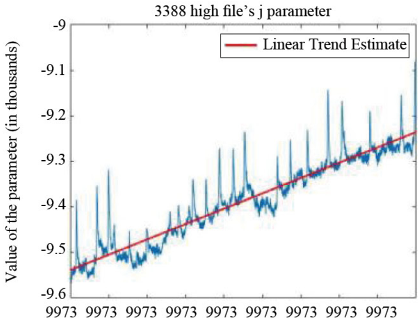
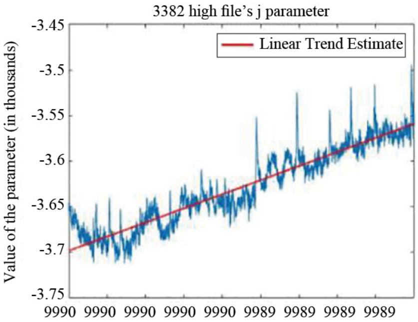
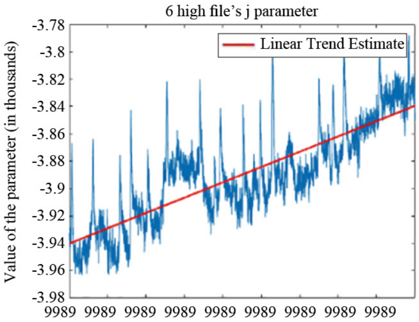
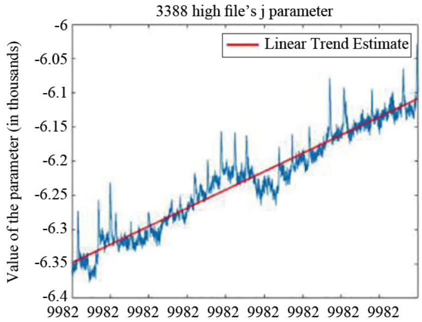
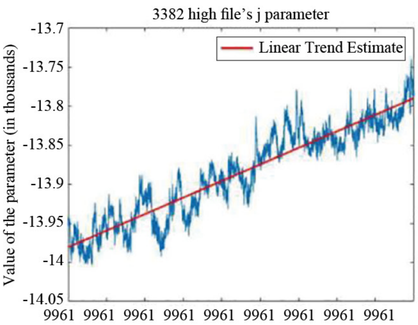
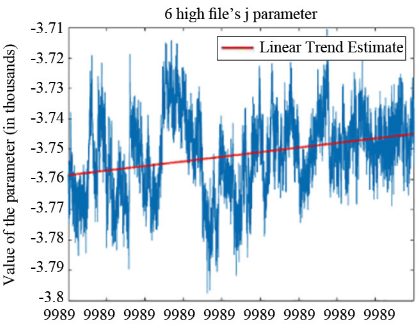
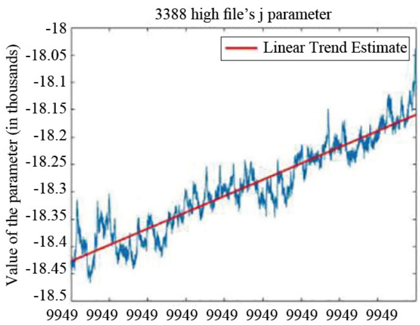
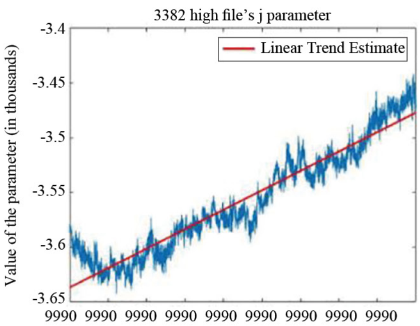
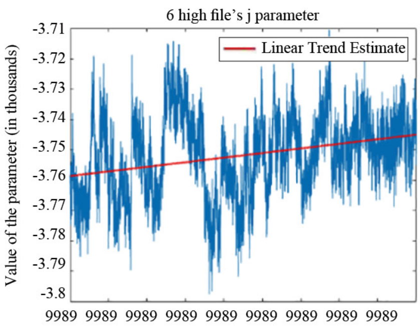
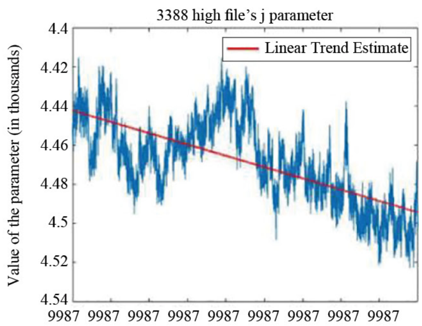
1			
2			
3			
4			
5			

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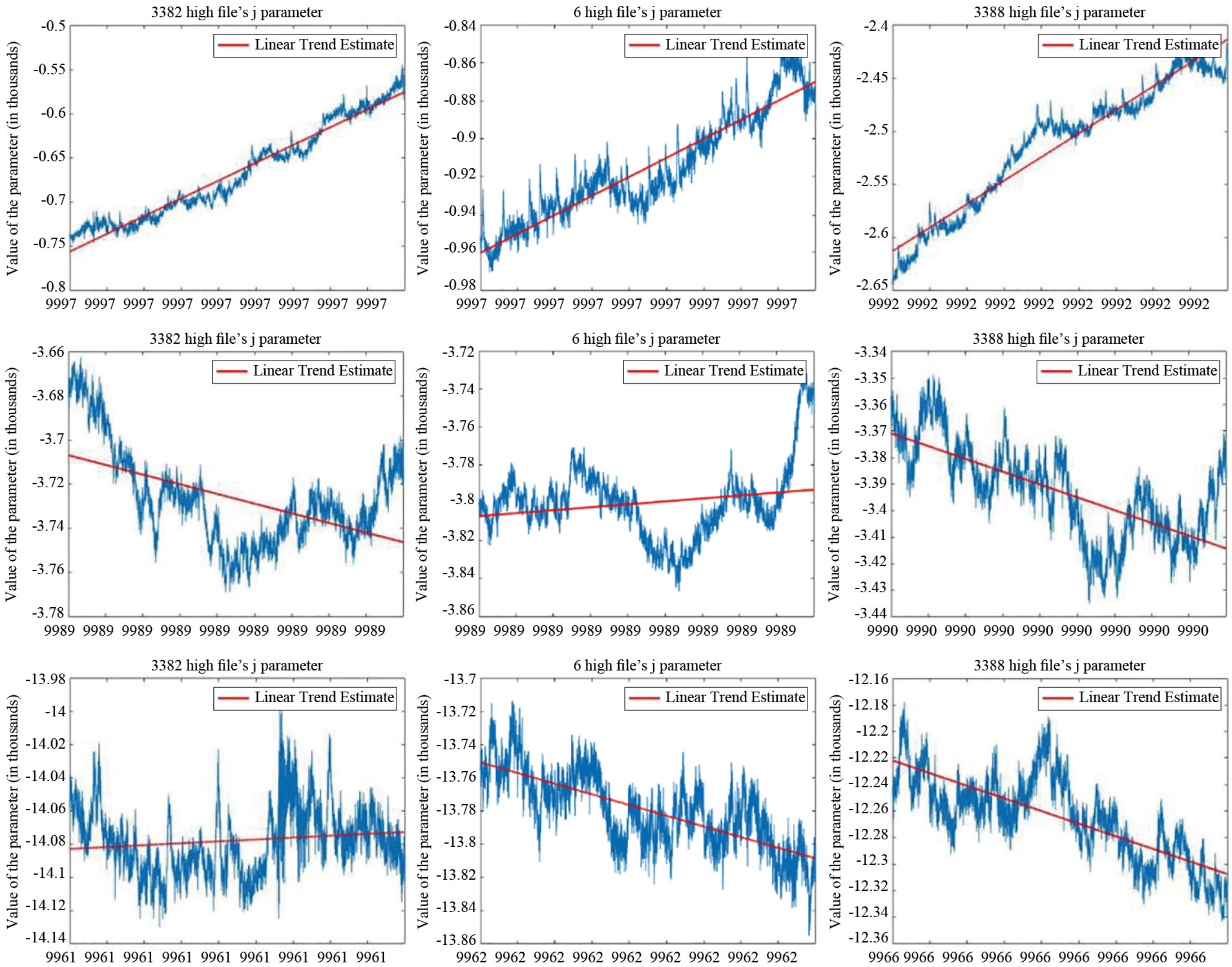
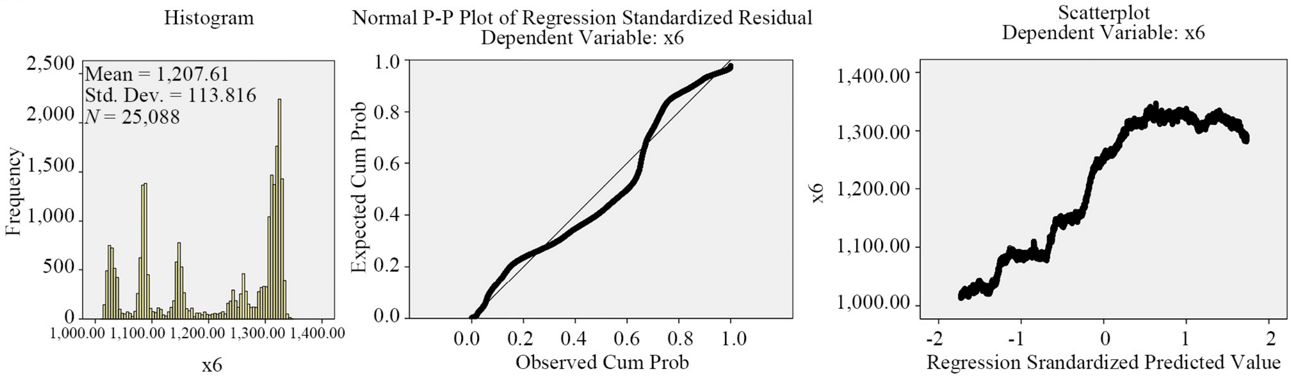


Figure 1. The first eight parameters of MTS of three different experiments

(a)



(b)

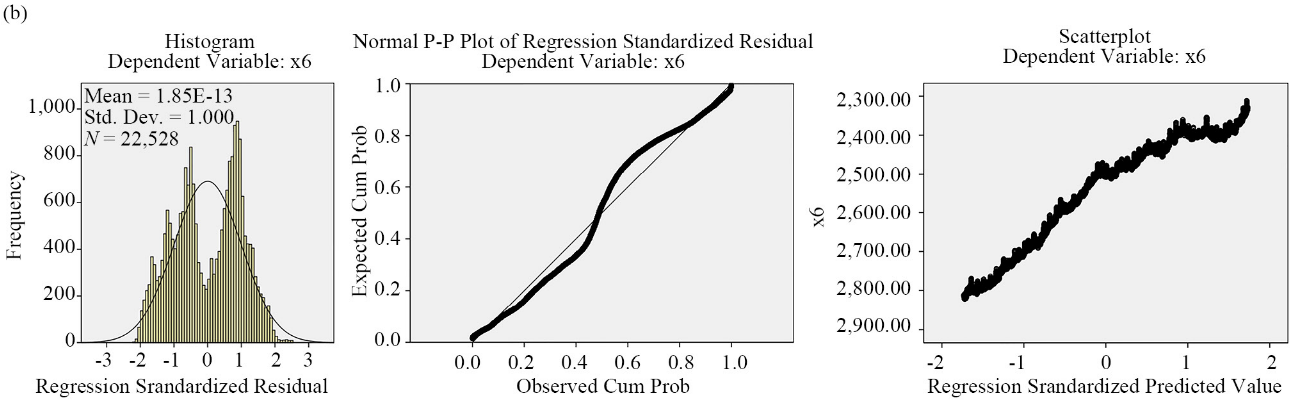


Figure 2. Illustration of non-normality of the random component of the parameters: (a) parameter 6 of the file "12_high"; (b) parameter 6 of the file "3388_high"

N-the numbers of the parameters of MTS (8 from 32); 3382_high, 3388_high and 3396_low-the numbers of the experiments with estimation of the emotions as "high" and "low" by the person being investigated.

Table 1. Correlations at the same moments of time between the first eight parameters of MTS (on example of the file "12_high")

Correlations		Parameters							
		x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
x_1	P.C.	1	0.626**	0.447**	−0.331**	0.831**	0.923**	0.166**	0.879**
	Sig. (2-tailed)		0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Covariance	8,086.73	4,393.67	2,902.34	−1,026.79	2,590.69	4,541.33	489.45	5,482.74
	N	15,360	15,360	15,360	15,360	15,360	15,360	15,360	15,360
x_2	P.C.	0.626**	1	0.901**	0.025**	0.792**	0.647**	0.518**	0.746**
	Sig. (2-tailed)	0.000		0.000	0.002	0.000	0.000	0.000	0.000
	Covariance	4,393.669	6,089.954	5,082.318	68.126	2,144.894	2,759.940	1,327.212	4,034.707
	N	15,360	15,360	15,360	15,360	15,360	15,360	15,360	15,360
x_3	P.C.	0.447**	0.901**	1	0.283**	0.614**	0.375**	0.554**	0.532**
	Sig. (2-tailed)	0.000	0.000		0.000	0.000	0.000	0.000	0.000
	Covariance	2,902.363	5,082.318	5,224.768	704.254	1,539.772	1,481.832	1,314.985	2,665.354
	N	15,360	15,360	15,360	15,360	15,360	15,360	15,360	15,360
x_4	P.C.	−0.331**	0.025**	0.283**	1	0.009	−0.430**	0.313**	−0.156**
	Sig. (2-tailed)	0.000	0.002	0.000		0.250	0.000	0.000	0.000
	Covariance	−1,026.719	68.126	704.254	1,186.972	11.090	−810.062	354.507	−371.671
	N	15,360	15,360	15,360	15,360	15,360	15,360	15,360	15,360
x_5	P.C.	0.831**	0.792**	0.614**	0.009	1	0.868**	0.396**	0.949**
	Sig. (2-tailed)	0.000	0.000	0.000	0.250		0.000	0.000	0.000
	Covariance	2,590.685	2,144.894	1,539.772	11.090	1,203.199	1,646.710	451.143	2,282.860
	N	15,360	15,360	15,360	15,360	15,360	15,360	15,360	15,360
x_6	P.C.	0.923**	0.647**	0.375**	−0.430**	0.868**	1	0.225**	0.928**
	Sig. (2-tailed)	0.000	0.000	0.000	0.000	0.000		0.000	0.000
	Covariance	4,541.333	2,759.940	1,481.832	−810.062	1,646.710	2,990.400	403.458	3,518.219
	N	15,360	15,360	15,360	15,360	15,360	15,360	15,360	15,360

Table 1. (cont.)

Correlations		Parameters							
		x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
x_7	P.C.	0.166**	0.518**	0.554**	0.313**	0.396**	0.225**	1	0.345**
	Sig. (2-tailed)	0.000	0.000	0.000	0.000	0.000	0.000		0.000
	Covariance	489.449	1,327.212	1,314.985	354.507	451.143	403.458	1,079.796	786.321
	N	15,360	15,360	15,360	15,360	15,360	15,360	15,360	15,360
x_8	P.C.	0.879**	0.746**	0.532**	−0.156**	0.949**	0.928**	0.345**	1
	Sig. (2-tailed)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	
	Covariance	5,482.737	4,034.707	2,665.354	−371.671	2,282.860	3,518.219	786.321	4,809.602
	N	15,360	15,360	15,360	15,360	15,360	15,360	15,360	15,360

P.C.-Pearson Correlation; x_1, ..., x_8-the first eight parameters of the file "12_high"
**.-Correlation is significant at the 0.01 level (2-tiled)
*.-Correlation is significant at the 0.05 level (2-tiled)

MTSs clustering by the eigenvectors of the corresponding covariance matrices using the MATLAB program *clustergram (A)*, where A is a 32×32 dimensional covariance matrix, shows (see Figure 3) that the 54 MTSs corresponding to the three individuals under consideration are grouped into clusters with the number significantly greater than two. Which indicates that the classification of the MTSs into two "high" and "low" groups, based on the available information, is quite a difficult problem.

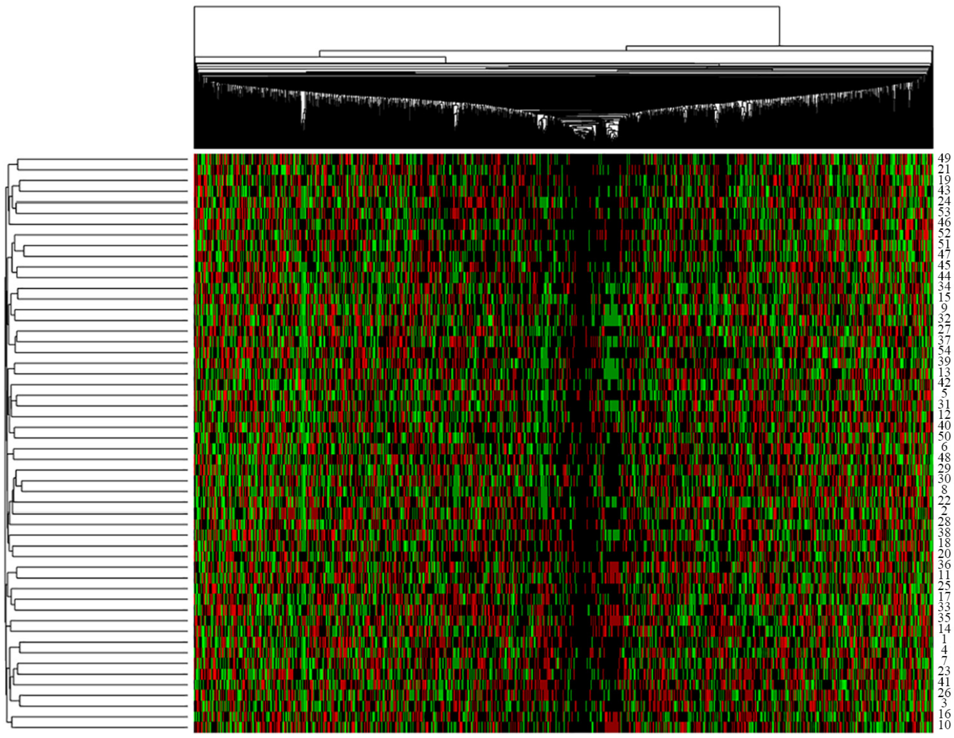


Figure 3. The results of clustering using eigenvectors of MTS covariance matrices

The conditional numbers of MTSs corresponding to the considered three persons are given from the right side.

MTSs with the numbers 1-21 correspond to the "High" condition and with the numbers 22-54 correspond to the "low" condition. MTSs of the "High" condition are distributed as follows: 1-8 belong to the 27 person, 9-15-to the first person and 16-21-to the 30 person. MTSs of the "low" condition are distributed as follows: 22-31 belong to the 27 person, 32-42-to the first person and 43-54-to the 30 person.

In order to maximally exclude from the observational results uncontrollable distortion factors related to the psychological state of the patient at the beginning and at the end of the experiment, and at the same time to examine all the used methods and data under equal conditions, all MTSs were reduced to the same length by excluding an equal number of initial and final observations. As a result, the dimensions of the corresponding MATLAB areas of the Excel files left for further processing are $32 \times 9{,}216$, where 32 is the number of parameters, and 9,216 is the number of observations of these parameters. The algorithm of such transformation is given in Appendix as Algorithm 1.

3. Used criteria and algorithms of their realization

Different similarity measures of Linear Univariate Time Series (LUTS) and Linear Multivariate Time Series (LMTS) are given in [26]. Considering many different distance metrics, the following inference is made: "the similarity measure functions of Linear Multivariate Time Series (LMTS) based on the norm distance of covariance matrix and nonlinear multivariable time series based on kernel function are reasonable and practical". Because LMTS are considered below, criteria based on the norm distance of the covariance matrix are basically applied for classifying EEG data.

For solving the stated problem, i.e., for classification into two emotional categories ("high" and "low") of EEG data obtained as MTS, we used eleven different criteria of similarity of MTS such as: Weighted Sum Singular Value Decomposition (WSSVD) [3, 27], Ascending Eigenvalue-Weighted Difference between Eigenvector Matrices (AEWDEM), Not Weighed Divergence Between Eigenvector Matrices (NWDEM), Not Weighed Divergence Between Covariance Matrices (NWDCM), Generalized Variance (GV) [28], Descending Eigenvalue-Weighted Difference between Eigenvector Matrices (DEWDEM), Principal Components Analysis Similarity Factor (PCASF) [3], *Eros* (*E*xtended F*ro*benius norm) [3], Dynamic Time Warping (DTW) [3, 29, 30], Getting into the Confidence Regions of the Linear Trends of MTS (GCALT) and union of two AEWDEM and GCRLT methods. The last two criteria are developed by us to solve the problem under consideration. Formulas for realizing these criteria and a brief explanation of their essence are given in Table 2.

Table 2. Criteria used for classification of MTS files

N	Criteria	Formulae	Explanation
1.	Weighted Sum Singular Value Decomposition (WSSVD)	$Q_1 = B_1^T \times B_1$, $Q_2 = B_2^T \times B_2$ Singular Value Decompositions (SVD): $Q_1 = V_1 \times A_1 \times V_1^T$, $Q_2 = V_2 \times A_2 \times V_2^T$ Columns representation of V_1 and V_2 are $V_1 = [e_1,\ e_2,\ \ldots,\ e_m]$ and $V_2 = [f_1,\ f_2,\ \ldots,\ f_m]$, where $A_1 = diag\,[c_1,\ c_2,\ \ldots,\ c_m]$, $A_2 = diag\,[d_1,\ d_2,\ \ldots,\ d_m]$. The similarity of B_1 and B_2 is defined as: $\theta(B_1,\ B_2) = \min(\theta_1(A_1,\ A_2),\ \theta_2(A_1,\ A_2))$, where $\theta_1(B_1,\ B_2) = (\sum_{i=1}^m c_i\,(e_i \cdot f_i))\,/\,(\sum_{i=1}^m \lvert c_i \rvert)$, $\theta_2(B_1,\ B_2) = (\sum_{i=1}^m d_i\,(e_i \cdot f_i))\,/\,(\sum_{i=1}^m \lvert d_i \rvert)$ and $x \cdot y$ is the inner product of the two vectors x and y.	B_1 and B_2 are covariance matrices of MTS to be distinguished;

Table 2. (cont.)

N	Criteria	Formulae	Explanation
2.	Ascending Eigenvalue-Weighted Difference between Eigenvector Matrices (AEWDEM)	$WB = \sum_{i=1}^{m} \frac{b(i,\ i)}{b_1} \sum_{j=1}^{m} (W_1(i,\ j) - W_2(i,\ j))^2$, $b_1 = \sum_{i=1}^{m} b(i,\ i)$.	Used MATLAB codes $[a,\ b,\ c] = eig(\text{cov\_}matrix)$ and $[V,\ D,\ W] = svd(\text{cov\_}matrix)$, where cov_$matrix$ is covariance matrix of MTS under investigation; c is the right eigenvector matrix; a is the left eigenvector matrix (the number of columns is the rank of the covariance matrix); b is a diagonal $m \times m$ matrix of the eigenvalues λ_i, where $\lambda_1 \geq \ldots \geq \lambda_r \geq 0$. The eigenvalues and the corresponding eigenvectors are sorted in non-increasing order.
3.	Not Weighed Divergence Between Eigenvector Matrixes (NWDEM)	$WE = \sum_{i=1}^{m} \sum_{j=1}^{m} (W_1(i,\ j) - W_2(i,\ j))^2$.	Used MATLAB code $[V,\ D,\ W] = svd(\text{cov\_}matrix)$.
4.	Not Weighed Divergence Between Covariance Matrixes (NWDCM)	$WC = \sum_{i=1}^{m} \sum_{j=1}^{m} (A_1(i,\ j) - A_2(i,\ j))^2$.	A_1 and A_2 are covariance matrices of MTS to be distinguished.
5.	Generalized Variance (GV)	$WM = \dfrac{\left\lvert {A_1}^{1/2} \right\rvert \cdot \left\lvert {A_2}^{1/2} \right\rvert}{\left\lvert ({A_1}^{1/2} + {A_1}^{1/2})/2 \right\rvert}$.	A_1 and A_2 are covariance matrices of MTS to be distinguished. This criterion is developed for comparison on the similarity of two normal population covariance matri¬ces.
6.	Descending Eigenvalue-Weighted Difference between Eigenvector Matrices (DEWDEM)	$WD = \sum_{i=1}^{m} \frac{D(i,\ i)}{D_1} \sum_{j=1}^{m} (W_1(i,\ j) - W_2(i,\ j))^2$, $D_1 = \sum_{i=1}^{m} D(i,\ i)$.	Used MATLAB code $[V,\ D,\ W] = svd(\text{cov\_}matrix)$, were cov_$matrix$ is covariance matrix of MTS under investigation; matrix V's columns are the corresponding right eigenvectors, so that $A * V = V * D$; D is diagonal matrix of eigenvalues; full matrix W's columns are the corresponding left eigenvectors, so that $W' * A = D * W'$.
7.	Principal Components Analysis Similarity Factor (PCASF)	$S_{PCA}(A_1,\ A_2) = trace(PC^T CP^T) = \sum_{i=1}^{k} \sum_{j=1}^{k} \cos^2 \theta_{ij}$, where P and C are the matrices that contain the first k principal components of A_1 and A_2, respectively, θ_{ij} is the angle between the ith principal component of A_1 and the jth principal component of A_2.	A_1 and A_2 are covariance matrices of MTS to be distinguished.
8.	*Eros* (*E*xtended F*ro*beniu*s* norm)	$Eros(A,\ B,\ \omega) = \sum_{i=1}^{n} \omega_i \lvert < a_i,\ b_i > \rvert = \sum_{i=1}^{n} \omega_i \lvert \cos \theta_i \rvert$, where $< a_i,\ b_i >$ is the inner product of a_i and b_i, ω is a weight vector which is based on the eigenvalues of the MTS dataset, $\sum_{i=1}^{m} \omega_i = \sum_{i=1}^{m} D(i,\ i)/D_1 = 1$ $(D_1 = \sum_{i=1}^{m} D(i,\ i))$ and θ_i is the angle between a_i and b_i. The range of $Eros$ is between 0 and 1, with 1 being the most similar.	A and B are covariance matrices of MTS to be distinguished. V_A and V_B are two right eigenvector matrices by applying SVD program to the covariance matrices, A and B, respectively. $V_A = [a_1,\ \ldots,\ a_m]$ and $V_B = [b_1,\ \ldots,\ b_m]$, where a_i and b_i are column orthonormal vectors of size m.

Table 2. (cont.)

N	Criteria	Formulae	Explanation
9.	Dynamic Time Warping (DTW)	$dist = dtw(X,\ Y)$ stretches two matrices $X_{m\times n_1}$ and $Y_{m\times n_2}$ by repeating their columns onto a common set of instants such that distance, the sum of the Euclidean distances between corresponding columns, is smallest. In that case, X and Y must have the same number of rows.	Used MATLAB codes $dist = dtw(X,\ Y),\ X_{m\times n_1} = A^T_{n_1\times m},$ $Y_{m\times n_2} = B^T_{n_2\times m},$ where A and B are two matrix of MTS.

10.	Getting into the Confidence Regions of the Linear Trends of MTS (GCRLT)	**Input**: C, $S1$, $S2$- MTSs corresponding to a controlled, standard 1 and standard 2 states, respectively.
		for $i = 1$ to 32 do
		Isolate: trC_i (trends of controlled MTS)
		$trS1_i$ (trends of MTS, corresponding to the "high" condition standard)
		$trS2_i$ (trends of MTS, corresponding to the "low" condition standard)
		Compute: $\Delta trS1_i$ (deviations from the trends of the "high" condition standard with the given confidence probability)
		$\Delta trS2_i$ (deviations from the trends of the "low" condition standard with the given confidence probability)
		end for i
		$I1 = 0;$
		$I2 = 0;$
		for $i = 1$ to 32 do
		if $trC_i \in (trS1_i - \Delta trS1_i;\ trS1_i + \Delta trS1_i)$
		$I1 = I1 + 1;$
		end if
		if $trC_i \in (trS2_i - \Delta trS2_i; trS2_i + \Delta trS2_i)$
		$I2 = I2 + 1;$
		end if
		end for i
		if $I1 > I2$
		else
		end if
		In more details see Algorithm 3 of Appendix.
			Attribution of the trends of the control MTS is controlled by the facts of falling within the confidence regions of the trends of the corresponding time series of the two MTSs fixed as standards, and a deci¬sion is made in favor of the standard, i.e., of the corresponding state, for which this condition is more satisfied.

11.	Union of two methods AEWDEM and GCRLT	-	For the control MTS, the pro¬ximity to the two standard states ("high" and "low") of the corresponding MTSs is tested simultaneously by the second and tenth criteria, and a decision is made in favor of the state for which the total criterion is more satisfied.

To make it easier to refer to the discussed criteria and to reduce the volume of the article, we basically will refer to these methods according to their numbers given in Table 2.

It is known that Time Series (TS) consists of two main components: deterministic and stochastic. Therefore, the similarity of TS can be defined on their basis. It is known that three types of transformation should be considered for similarity measures. They are shift, scale and time warping [3]. It is clear that using the correlation matrix instead of the covariance matrix would lead to the scale transformation, and if the scale for two MTSs is different, then they should be considered different. Therefore, the first eight criteria are based on the use of covariance matrices. They (except for the fourth and fifth criteria) use covariance matrices not directly but through the computation of their eigenvectors and eigenvalues using weighted-sum Singular Value Decomposition (SVD program realized in MATLAB). SVD works directly on an aggregation of several parameters of MTS and it performs dimension reduction due to its capability to linearly transform a given dataset into rotations with an optimal set of magnitudes. Finally, it functions as a similarity measure by comparing corresponding eigenvectors weighted by their respective eigenvalues. The order of computing these criteria is as follows: computing the covariance matrices of the two MTSs, then computing the corresponding eigenvectors and eigenvalues, and then computing the criteria of interest (see Algorithm 2 of the Appendix as an example). The fourth criterion uses not weighted Euclidean distance between covariance matrices. The fifth method uses a special criterion of similarity of two normal population covariance matrices, introduced in [28].

The results obtained by the seventh criterion (PCASF) significantly depend on the number of linear combinations of the initial parameters that are left for classification, which in turn is determined by the share of variance caused by these parameters (in relation to the total variance determined by the initial parameters). Thus the accuracy of classification of PCA depends on the number of eigenvectors left for classification. The maximum accuracy of classification obtained by us with this method does not exceed 66.67%. However, when the total share of the variances corresponding to the left vectors in the total number of variances is equal to 10%, 98.5% and 99%, the PCA method gives a correct decision with 66.67% in 4 cases, one case and 2 cases, respectively.

The ninth criterion (DTW) uses directly two matrices of MTSs for the establishment of their similarity. It stretches these MTS matrices by repeating their columns onto a common set of instants such that distance, the sum of the Euclidean distances between corresponding columns, is smallest.

The tenth criterion uses the trends of two MTSs for their comparison. In particular, two TS of two different MDTs are compared as follows. If a trend of TS of one MTS belongs to the confidence region of a trend of the same TS of another MTS, then it is believed that these two TSs are similar. In such a manner a controlled MTS is compared with two MTSs which are chosen as the standards of “high” and “low” conditions and a decision is made in favor of the state to which the maximum number of coinciding corresponds. The algorithm of this criterion is given in Appendix (see Algorithm 3). For the aim of reducing the influence of the random component on the quality of the isolated trends and building their confidence regions, we act as follows. Since the registration of time series by sensors is carried out at such a high frequency (256 observations in 1 s) that the obtained results can be considered as continuous time series, we discretize these series in time. In particular, we discretize the time axis with an interval equal to 1 s, and we consider the observation results obtained in each second as the set of observations obtained at a given moment in time. Thus, to isolate the trend, we have 36 (9,216 : 256 = 36) points on the time axis separated by 1 s from each other, and in each of them 256 observation results are given. This allows us to construct the confidence areas of trends, singled out by the method of least squares, with high accuracy [31]. We call this method as Getting Into the Confidence Regions of the Linear Trends (GCRLT).

The eleventh criterion is the union of two methods: AEWDEM and GCRLT. Like the previous ten criteria, a decision is made in favor of the state to which the minimum sum of these criteria corresponds.

4. Computation results

The methodology of investigating the emotional state of a person based on EEG signals consists of two stages: verification and testing. At the verification stage, the so-called “standard” MTSs are selected, that is, the corresponding MTSs for each “high” and “low” state are selected, which reflect the corresponding state as perfectly as possible. At the testing stage, the rest of the MTSs are compared to the selected standards with the similarity criteria given in Table 2 in order to classify them. We make the decision that the MTS corresponds to the “high” or “low” state depending on which standard the tested MTS is closer to with the given criterion.

According to the methodology mentioned above, the corresponding MTSs classification of EEG signals was carried out in different scenarios: (1) MTSs selected to represent “high” and “low” states, i.e., the so-called standards and classifiable MTS belong to the same person; (2) MTSs selected to represent “high” and “low” states, that is, the so-called standard MTSs belong to one group of persons, and classifiable MTSs belong to another group of persons.

For all considered cases, the 2nd, 10th and 11th methods gave us the best results in terms of the classification accuracy obtained for specific standards and the number of standards for which the classification was carried out with high accuracy. In particular, the accuracy of the classification by the mentioned methods for the first person is: 93.75%-the second method; 75%-10th and 11th methods. For the 27th person: 75%-the second method, as well as 75%-the 10th and 11th methods;

For the 30th person: 81.25%-2nd method, also 81.25%-10th and 11th methods. Methods 10 and 11 gave mainly the same results for the same standards. Out of the calculated cases only once (for one standard) the 11th method gave us 81.25%, and the 10th method-75%.

Method 2 for individuals 1 and 27 gave us higher accuracies more times than methods 10 and 11, and for individual 30, on the contrary, methods 10 and 11 outperformed method 2.

For the second scenario, when at the verification stage, based on the MTSs corresponding to the 1st and 27th persons, the standards corresponding to the "high" and "low" states were selected by the methods given in Table 2, and the MTSs corresponding to the 30th person was tested, the best result was given by the 11th method, which for 9 selected standards classified with high accuracy 7 times. Namely, 1 time-by 83.33%, 1 time-by 77.78% and 5 times-by 66.67%. The 10th method classified with high accuracy also 7 times. Namely: 1 time-by 83.33% and 6 times-by 66.67%. The third result was given by method 2, which classified 4 times with high accuracy. Namely: 1 time-by 83.33% and 3 times-by 66.67%. For the sake of objectivity, it should be noted that the 2nd method gave us the best results on "its" standards, that is, on the standards that were selected for it at the verification stage. The 11th method performed the correct classification on its standards with an accuracy equal to 77.78%, the 10th method performed the correct classification on its standards with an accuracy equal to 66.67%. However, as we mentioned above, for certain standards, both of these methods made the correct classification with an accuracy equal to 83.33%. In terms of reliability, method 11 is by far the best, as it correctly classified 7 out of 9 cases with the accuracy of more than 66.67%.

5. Discussion

As mentioned above, the emotional state classification problem has many practical applications, which need to be solved under conditions of limited computing and time resources. Therefore, to solve this problem, we deliberately avoided modern Neural Network and non-linear methods of machine learning. As we saw Ascending Eigenvalue-Weighted Difference between Eigenvector Matrices (AEWDEM), Getting into the confidence regions of the linear trends (GCRLT) and a combination of these methods gave us the best results for the processed data. We believe that for other similar data, these methods will give analogous results especially as the last method uses complete information about MTS such as trends and covariance matrices. For showing this fact, it is necessary to test the discussed and other similar methods on each specific type of data, and based on the obtained results, to make a well-founded conclusion. Therefore, we plan to review the mentioned and other similar methods for other types of EEG data and to offer recommendations for the practical use of these methods based on the obtained results.

6. Conclusions

Binary classification of EEG data using ordinary personal computers at making computation in real time is considered. For this purpose here is considered eleven different criteria of similarity of MTS such as: Weighted Sum Singular Value Decomposition (WSSVD), Ascending Eigenvalue-Weighted Difference between Eigenvector Matrices (AEWDEM), Not Weighed Divergence Between Eigenvector Matrices (NWDEM), Not Weighed Divergence Between Covariance Matrices (NWDCM), Generalized Variance (GV), Descending Eigenvalue-Weighted Difference between Eigenvector Matrices (DEWDEM), Principal Components Analysis Similarity Factor (PCASF), *Eros* (*E*xtended F*ro*beniu*s* norm), Dynamic Time Warping (DTW), Getting into the Confidence Regions of the Linear Trends of MTS (GCALT) and union of two AEWDEM and GCRLT methods. They were applied to the concrete data, in particular, to 32 dimensional EEG signals. Computing results show the superiority of AEWDEM, GCALT methods and the method, obtained by the union of these two methods in comparison with considered other methods.

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Conflict of interest

Authors Kartlos J. Kachiashvili, Joseph K. Kachiashvili and Vakhtang V. Kvaratskhelia declare that they have no financial interests.

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Appendix

Algorithm 1 Transforming Excel files of MTS to MATLAB arrays with standard length

Require:

A set of Excel files (“high”/“low” conditions)

N: Number of files

M: Standard length

m: Number of parameters (columns)

Ensure:

$A1_i$, $i = 1, \ldots, N$: MATLAB arrays with standard length

for $i = 1$ **to** N **do**

$t(i) \leftarrow \text{readtable}(\text{‘Excelfile(i)’})$

$A(i) \leftarrow \text{table2array}(t(i))$

$AA(i) \leftarrow \text{length}(A(i)(:, 1))$

$SLL(i) \leftarrow (AA(i) - M)/2$

$SLH(i) \leftarrow AA(i) - SLL(i)$

$SL(i) \leftarrow SLH(i) - SLL(i)$

$SSL1(i) \leftarrow SSL(i) + 1$

for $j = 1$ **to** m **do**

$j1 \leftarrow 0$

for $k = SSL1(i)$ **to** $SLH(i)$ **do**

$j1 \leftarrow j1 + 1$

$A1(i)(j1, j) \leftarrow A(i)(k, j)$

end for

end for

end for

Algorithm 2 Classification using AEWDEM method

Require:

Standard1 (high), Standard2 (low)

MATLAB arrays $A1(i)$, $i = 1, \ldots, N$

Ensure:

Classification results

for $i = 1$ **to** N **do**

$A1_{\text{cov}}(i) \leftarrow \text{cov}(A1(i))$

end for

$ASH \leftarrow \text{Standard1}$

$ASL \leftarrow \text{Standard2}$

$[V, D, W] \leftarrow \text{svd}(ASH)$

$[V1, D1, W1] \leftarrow \text{svd}(ASL)$

for $k = 1$ **to** $N - 2$ **do**

$[V2, D2, W2] \leftarrow \text{svd}(A1_{\text{cov}}(k))$

$b_1 \leftarrow 0$

for $i = 1$ **to** m **do**

$b_1 \leftarrow b_1 + b(i, i)$

end for

for $i = 1$ **to** m **do**

for $j = 1$ **to** m **do**

$zH \leftarrow (W(i,\ j) - W2(i,\ j))^2$
$zL \leftarrow (W1(i,\ j) - W2(i,\ j))^2$
end for
$wH \leftarrow (b(i,\ i)/b_1) \cdot zH(i)$
$wL \leftarrow (b(i,\ i)/b_1) \cdot zL(i)$
end for
if $wH > wL$ **then**
print "File k: Low"
else
print "File k: High"
end if
end for

Algorithm 3 Isolation of the linear trends of the parameters of MTS by computing the averages of observations obtained in several neighboring seconds and testing of getting into the confidence region of a standard files.

Input: $AS_{n_1 \times m}$-standard file 1, $BS_{n_2 \times m}$-standard file 2, $W_{n_3 \times m}$ - control file.

Output: *A decision about belonging of the control file to "high" or "low" condition.*

% *The reduction of the files sizes up to standard length by computing the average of observations obtained during*
% *several seconds*
$t1 \leftarrow lenght\ (AS(:,\ 1));$
$t2 \leftarrow lenght\ (AS(1,\ :));$
$pt \leftarrow 1;$
$h \leftarrow pt*256;$
$for\ i = 1\ to\ t1\ do$
$for\ j = 1\ to\ t2\ do$
$kix((j-1)/h);$
$k1 \leftarrow k+1;$
$A1(k1,\ i) \leftarrow AS(j,\ i);$
$B1(k1,\ i) \leftarrow BS(j,\ i);$
end for j
end for i
$A1 \leftarrow A1/h;$
$B1 \leftarrow B1/h;$
% $Computation\ of\ the\ variances\ of\ the\ matrices\ AS\ and\ BS$
$for\ i = 1\ to\ t1\ do$
$for\ j = 1\ to\ t2\ do$
$k \leftarrow fix((j-1)/h);$
$k1 \leftarrow k+1;$
$\mathrm{var}A(k1,\ i) \leftarrow (AS(j,\ i) - A1(k1,\ i))\hat{}2;$
$\mathrm{var}B(k1,\ i) \leftarrow (BS(j,\ i) - B1(k1,\ i))\hat{}2;$
$end\ for\ j$
$end\ for\ i$
$\mathrm{var}A \leftarrow \mathrm{var}A/h;$
$\mathrm{var}B \leftarrow \mathrm{var}B/h;$
% $The\ reduction\ of\ the\ size\ up\ to\ standard\ length$
$tW1 \leftarrow length(W(:,\ 1));$
$tW2 \leftarrow length(W(1,\ :));$

```
kkW1/h;
for i = 1 to tW1 do
  for j = 1 to tW2 do
    k ← fix((j − 1)/h);
    k1 ← k + 1;
    W1(k1, i) ← W(j, i) ;
  end for j
end for i
W1 ← W1/h;
% Computation of the variances of the matrix W
for i = 1 to tW1 do
  for j = 1 to tW2 do
    k ← fix((j − 1)/h);
    k1 ← k + 1;
    var W(k1, i) ← (W(j, i) − W1(k1, i))^2;
  end for j
end for i
var W ← var W/h;
% Testing the condition that a trend of the control file belongs to the
% confidence region of standard file "AS" withprobability equal to 0.99
for i = 1 to tW1 do
  for j = 1 to k1 do
    deltASH(j, i) ← (2.32634787404084 ∗ sqrt(var A(j, i)))/sqrt(h);
    deltASL(j, i) ← (2.32634787404084 ∗ sqrt (var B(j, i)))/sqrt (h);
  end for j
end for i
for i = 1 to tW1 do
  for j = 1 to k1 do
    trHH ← A1(j, i) + deltSH(j, i);
    trHL ← A1(j, i) − deltSH(j, i);
    trLH ← B1(j, i) + deltSL(j, i);
    trLL ← B1(j, i) − deltSL(j, i);
  end for j
end for i
for i = 1 to tW1 do
  for j = 1 to k1 do
  if W1(j, i) <= trHH(j, i)
      IH ← 0;
    else if trHL <= W1(j, i)
      IH ← 1;
    else
      IH ← 0;
    end if
  end for j
end for i
for i = 1 to tW1 do
  for j = 1 to k1 do
    if W1(j, i) <= trLH(j, i)
```

```
        IL ← 0;
    else if trLL <= W1(j, i)
        IL ← 1;
    else
        IL ← 0;
    end if
    end for j
end for i
if IH > IL
    disp ('The "low" condition has place')
else
    disp ('The "high" condition has place')
end if
```