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Research Article

Markov Modelling of Small Cell Base Station Sleep Strategies with Disaster and Repairable Server

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Abstract: A network of Base Station (BS) is required in order to function mobile phones and other mobile devices. A large number of BSs will be deployed in advanced networks namely Long-Term Evolution (LTE-A) to achieve high data rates which result in heavy energy usage in cellular networks. The energy consumption can be reduced by BS sleep strategy. A Markov model (MM) is proposed for the strategy with disaster and repairable server, as a system may encounter a disaster at any stage. The proposed model efficiently calculates the energy saving factor with disaster and the results shows that the amount of power saved in BS increases with the number of light sleep cycles. The expressions for steady-state probabilities (SSP) and transient probabilities are derived . Moreover for the proposed model, mean and variance are computed. Numerical illustration on energy saving factor with disaster is presented to show the effectiveness.

***Keywords*:** long-term evolution advanced, disaster, markov model, standby, continuous time markov chain

MSC: 60K30, 90B22, 60K25

1. Introduction

Queueing systems play a major role in system modelling and analysis of performance measures in various telecommunication networks. As a result of increasing smart phones, there is a huge demand for high speed data access and the operators are struggling to fulfill them. This can be solved by increasing the number of small cell BS which results in enormous energy consumption. A simple and cost effective way of reducing energy consumption in BS is implementation of sleep strategies as it does not require any kind of replacement in existing hardware components in the network.

About sixty to eighty percent of the energy provided to the whole cellular network is consumed by the BS. Energy saving mechanisms like discontinuous reception (DRX) and discontinuous transmission (DTX) are followed by LTE-A networks which momentarily switch off the devices while still connected to network providing minimum throughput. By enabling sleep modes in BSs one can exploit the BSs power saving potential and during these sleep modes switching off some of the inactive hardware components of BSs can result in huge difference in energy saving under low load circumstances.

The aim of the paper is to propose a MM for BS sleep strategy and to compute the energy saving measure with disaster and repairable server. Many studies have used various models but MM is one such simple model through which

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one can evaluate performance measures of the BS. The proposed model can be used to evaluate the power saving measures of various parts of the telecommunication networks.

The organization of this paper is in the following way. The Preliminaries are given in section 2. Section 3 comprises of the description of the MM. The transient and steady-state analysis is given in section 4 and 5 respectively. The performance measures of the proposed MM is given in section 6. Section 7 presents the evaluation of energy saving factor with disaster and numerical illustration is given in section 8. Section 9 deals with the results and discussion of the proposed model. Finally the conclusion and the future plan is presented in section 10.

2. Preliminaries

2.1 *Stochastic modelling*

Stochastic modelling is more appreciable as they are close to real world situations where things happen randomly. Under different circumstances it forecast the probability of outcomes using random variables. One always choose stochastic models over deterministic models for real life applications. It is useful in framing decisions for the future, based on the present events. Thus it helps a decision maker to make quality decisions.

2.2 *Markov model (MM)*

MM is a kind of stochastic modelling where the future behaviour relies only on the present state. In other words, the past has no influence over the future events that are about to occur. This model is used to model the probabilities and transition rates among different states. It is being used in modelling computer hardware and software systems, bioinformatics, modelling languages and speech recognition.

2.3 *Continuous-time markov chains (CTMC)*

A continuous stochastic process is called a CTMC if it satisfies the Markov property i.e., the present state behaviour cannot be influenced by its past. CTMC has a wide range of applications which includes developing models for promoting health in cancer patients, to study the relationship between components in Uninterruptible Power Supply (UPS) and to improve customer interactions in call center.

2.4 *Literature review*

A queueing decision model has been proposed to maximize the utilization of energy efficiency based on the BS sleeping technique [1]. In [2] investigated the transient behaviour of non-homogeneous systems, which opened the door for thinking about systems with varying service rates. In [3] a single server queue with degrading service rate which undergoes vacation when there is no client to be served was discussed. The motivation of our study is the sleep strategies of small cell BS [4] and the energy saving factor which has been computed using stochastic modeling in LTE-A networks on the basis of various periods of DRX mechanism [5]. In [6] proposed a practically applicable Switching-On/Off based Energy Saving (SWES) algorithm to turn power off/on BS to save energy without affecting the network. The base stations performance has been studied in [7, 8] which is concerned with minimising power consumption in networks. A type of vacation schedule which has numerous adaptive vacation with the condition that K is designated positive numeral rather than a random variable. This kind of vacation is covered in [9]. The time dependent solution of an $M/M/1$ queue with catastrophes is obtained in [10]. A study on $M/M/2$ queue with catastrophes using probability generating function (PGF) is carried out in [11]. In [12], a study on power conservation using different methods which includes base-station on-off switching is investigated. An Markov Decision Processes (MDP) based algorithm to improve energy efficiency has been proposed in [13] by switching between different modes of base-station which reduces the latency in 5G networks.

In general, when a disaster occurs in the system, it undergoes a breakdown and no service will be provided for the waiting customers. To provide service, the system has to undergo a process of repair in order to accept new customers. A queueing system with disaster and a server which is repairable has been studied by a number of authors. A single server queue with disaster and a server which is repairable is studied under a multi-phase environment in [14]. In [15], a study involving variety of customers under working vacation strategy in an $M/G/1$ retrial queue is found. A time dependent solution is studied for a multi-server Markovian queuing model with catastrophes using PGF with the properties of Bessel function in [16]. A Reinforcement Learning algorithm is being used to take decision regarding turning on/off base-station [17]. In a LTE network, a base-station that can be turned off depending on the load and an auction scheme to enable energy gains have been proposed [18]. In [19] investigated non homogeneous systems with disaster and repairable server taking into account two and three processors respectively. The fundamentals of the research are from [20, 21]. The telecommunication switching basics are from [22]. The basics of stochastic and random process are from [23, 24].

3. Description of the model

For the proposed MM, an $M/M/1$ queueing system with disaster and repairable server is considered. Let the arrival rate be λ, at which the users enter the system following a poisson process and μ be the exponentially distributed service rate. Let α be the exponentially distributed parameter which denotes the time spent in idle state r before entering into first light sleep (standby) state l. Let β and γ be the exponentially distributed parameters which denotes the time spent in state l and state d respectively. The disaster may happen at any state of the queueing system in this proposed model at a rate of ξ and it follows a Poisson process. All the users are removed when there occurs a disaster and the server ends up in failure state. The server is repaired instantly and the time at which the server is repaired is exponentially distributed with mean η^{-1}. We propose a markov model for the different stages of sleep in a small cell BS. The time spent in each states are independent of each other and follows an exponential distribution and thus it results in a CTMC. Let $S(t)$ be the different states of the server at any time $t \geq 0$ and is given as follows:

$$S(t) = \begin{cases} r, & \text{the system is idle state} \\ w, & \text{the system is in regular working state} \\ l, & \text{the system is in working vacation} \\ d, & \text{the system is in vacation} \\ f, & \text{the system is in breakdown and repair} \end{cases}$$

Let the number of users be $N(t)$ at any time '$t \geq 0$'. Then $X(t) = N(t),\ S(t),\ t \geq 0$ represents CTMC with the following state space given by $S = n,\ j;\ n \geq 0;\ j = r,\ w,\ l,\ d,\ f$ where $l = 1,\ 2,\ \ldots\ L$. The state $(0,\ r)$ denotes the power off state of the system i.e., idle. The state $(n,\ w)$denotes that the system is in working state with n users where $n = 1,\ 2\ \ldots$. The state $(0,\ l)$ denotes that the system is in lth standby (light sleep) cycle. The state $(n,\ d)$ denotes the deep sleep state of the system with n number of users. Assuming initially the system is waiting for users i.e., it is in idle state. When an user arrives in idle state, the queueing system shifts to working state $(1,\ w)$ offering service. on arrival of another user, it moves to $(n+1,\ w)$ state while the previous user is still in service and the system is already occupied by n users. The system moves from state to $(n-1,\ w)$ after completely offering service everytime. The system shifts to $(0,\ 1)$ state when no user arrives in idle state. Then it continues shifting to state $(0,\ l)$ until no user arrives in state $(0,\ l-1)$ where $l = 1,\ 2,\ \ldots\ L$. The system shifts to working state $(1,\ w)$ if any user arrives in state $(0,\ l)$. If no user arrives in $(0,\ l)$ then it shifts to $(0,\ d)$. On arrival of any user in deep sleep $(n,\ d)$ state it moves to state $(n+1,\ d)$ when the system has already n users. If n users arrive during deep sleep state, after the deep sleep state is completed the system moves to state $(n,\ w)$.

3.1 *State transition diagram*

The state transition diagram for the proposed MM is given in the following Figure 1.

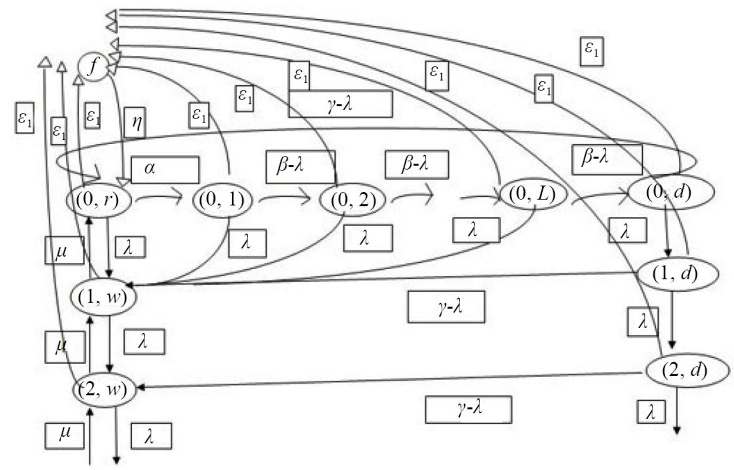

Figure 1. State transition diagram of the system

Let the probability with 'n' users at time 't' in the system be $Pn,\ j\,(t) = P\{N(t) = n,\ S(t) = j\}, n \geq 0;\ j = r,\ w,\ l,\ d,\ f;$ where 'j' denotes the state of the server. We assume that the system has no users initially and it is represented by $P_{0,\ r}(0) = 1$. Now, $P_{n,\ j}(t)$ satisfies the forward Kolmogorov equations given below.

$$P'_f(t) = -\eta P_f(t) + \xi\left(1 - P_f(t)\right) \tag{1}$$

$$P'_{0,\ r}(t) = -(\lambda + \alpha + \xi)P_{0,\ r}(t) + \mu P_{1,\ w}(t) + \eta P_f(t) \tag{2}$$

$$\begin{aligned} P'_{1,\ w}(t) &= -(\lambda + \mu + \xi)P_{1,\ w}(t) + \lambda P_{0,\ r}(t) + \mu P_{2,\ w}(t) \\ &\quad + \lambda\left(P_{0,\ 1}(t) + P_{0,\ 2}(t) + \ldots + P_{0,\ L}(t)\right) + (\gamma - \lambda)P_{1,\ d}(t) \end{aligned} \tag{3}$$

$$\begin{aligned} P'_{n,\ w}(t) &= -(\lambda + \mu + \xi)P_{n,\ w}(t) + \lambda P_{n-1,\ w}(t) + \mu P_{n+1,\ w}(t) \\ &\quad + (\gamma - \lambda)P_{n,\ d}(t),\ n \geq 2 \end{aligned} \tag{4}$$

$$P'_{0,\ 1}(t) = -(\beta + \xi)P_{0,\ 1}(t) + \alpha P_{0,\ r}(t) \tag{5}$$

$$P'_{0,\ l}(t) = -(\beta + \xi)P_{0,\ 1}(t) + (\beta - \lambda)P_{0,\ l-1}(t),\ l = 2,\ 3,\ \ldots\ L \tag{6}$$

$$P'_{0,\ d}(t) = -(\gamma + \xi)P_{0,\ d}(t) + (\beta - \lambda)P_{0,\ L}(t) \tag{7}$$

$$P'_{n,\ d}(t) = -(\gamma + \xi)P_{n,\ d}(t) + \lambda P_{n-1,\ d}(t) \tag{8}$$

4. Transient analysis

Now, we discuss about the time dependent analysis of the proposed MM.

Theorem 1 The transient probability of system being in l is denoted by $P_{0,\ l}(t)$ where $l = 1,\ 2,\ \ldots\ L$ and is given by

Proof. Let $f_{n,\ j}(s)$ denote the laplace-transform of $P_{n_{nj}}(t)$ and taking laplace-transform of (5)

$$(5) \Rightarrow P'_{0,\ 1}(t) = -(\beta+\xi)P_{0,\ 1}(t) + \alpha P_{0,\ r}(t)$$

Applying Laplace-transform

$$sf_{0,\ 1}(s) = -(\beta+\xi)f_{0,\ 1}(s) + \alpha f_{0,\ r}(s)$$

$$f_{0,\ 1}(s) = \frac{\alpha}{(s+\beta+\xi)} f_{0,\ r}(s)$$

on inversion

$$P_{0,\ 1}(t) = \alpha e^{-(\beta+\xi)t} * P_{0,\ r}(t) \tag{9}$$

$$(6) \Rightarrow P'_{0,\ l}(t) = -(\beta+\xi)P_{0,\ l}(t) + (\beta-\lambda)P_{0,\ l-1}(t) \text{ where } l = 2,\ 3,\ \ldots\ L$$

Applying Laplace transform

$$sf_{0,\ l}(s) = -(\beta+\xi)f_{0,\ l}(s) + (\beta-\lambda)f_{0,\ l-1}(s)$$

$$f_{0,\ l}(s) = \frac{(\beta-\lambda)}{(s+\beta+\xi)} f_{0,\ l-1}(s)$$

recursively

$$f_{0,\ l}(s) = \frac{(\beta-\lambda)^{l-1}}{(s+\beta+\xi)^{l-1}} f_{0,\ l-1}(s),\ l = 2,\ 3,\ \ldots\ L$$

$$\text{from}(9)\ f_{0,\ l}(s) = \frac{\alpha(\beta-\lambda)^{l-1}}{(s+\beta+\xi)^{l}} f_{0,\ r}(s),\ l = 1,\ 2,\ \ldots\ L$$

on inversion

$$P_{0,\ l}(t) = \alpha(\beta-\lambda)^{l-1} e^{-(\beta+\xi)t} \frac{t^{l-1}}{l-1!} * P_{0,\ r}(t),\ l = 1,\ 2,\ \ldots\ L \tag{10}$$

Theorem 2 The transient probability of system being in d is denoted by $P_{n,\, d}(t)$ where $l = 1,\ 2,\ \dots\ L$; $n \geq 0$ and is given by

Proof.

$$(7) \Rightarrow P'_{0,\, d}(t) = -(\gamma+\xi)P_{0,\, d}(t) + (\beta-\lambda)P_{0,\, L}(t)$$

Applying Laplace Transform

$$sf_{0,\, d}(s) = -(\gamma+\xi)f_{0,\, d}(s) + (\beta-\lambda)f_{0,\, L}(s)$$

$$f_{0,\, d}(s) = \frac{(\beta-\lambda)}{(s+\gamma+\xi)}f_{0,\, L}(s)$$

$$f_{0,\, d}(s) = \frac{(\beta-\lambda)}{(s+\gamma+\xi)}\frac{\alpha(\beta-\lambda)^{L-1}}{(s+\beta+\xi)^{L}}f_{0,\, r}(s) = \frac{\alpha(\beta-\lambda)^{L}}{(s+\gamma+\xi)(s+\beta+\xi)^{L}}f_{0,\, r}(s)$$

on inversion

$$P_{0,\, d}(t) = \alpha(\beta-\lambda)^{L}e^{-(\gamma+\xi)t} * e^{-(\beta+\xi)t}\frac{t^{L-1}}{L-1!} * P_{0,\, r}(t) \tag{11}$$

$$(8) \Rightarrow P'_{n,\, d}(t) = -(\gamma+\xi)P_{n,\, d}(t) + \lambda P_{n-1,\, d}(t)$$

Applying Laplace Transform

$$sf_{n,\, d}(s) = -(\gamma+\xi)f_{n,\, d}(s) + \lambda f_{n-1,\, d}(s)$$

$$f_{n,\, d}(s) = \frac{\lambda}{(s+\gamma+\xi)}f_{n-1,\, d}(s),\ n = 1,\ 2\ \dots$$

$$f_{n,\, d}(s) = \left(\frac{\lambda}{(s+\gamma+\xi)}\right)^{i}f_{0,\, d}(s) = \frac{\alpha\lambda^{i}(\beta-\lambda)^{L}}{(s+\gamma+\xi)^{i+1}(s+\beta+\xi)^{L}}f_{0,\, r}(s),\ i = 0,\ 1\ \dots$$

on inversion

$$P_{n,\, d}(t) = \alpha\lambda^{i}(\beta-\lambda)^{L}e^{-(\gamma+\xi)t}\frac{t^{i}}{i!} * e^{-(\beta+\xi)t}\frac{t^{L-1}}{L-1!} * P_{0,\, r}(t),\ n \geq 0 \tag{12}$$

Theorem 3 The transient probability $P_{n,\, w}(t)$ denotes the system being in state w, where $n \geq 1$ is given by

Proof. Consider a generating function with time dependent probabilities representing the working states, we obtain the following by the solving the above partial differential equation

$$G(z,\ t) = -\left(\alpha + \frac{\mu}{z} - \mu\right) \int_0^t P_{0,\ r}(u) e^{\left(\lambda z + \frac{\mu}{z} - (\lambda + \mu + \xi)\right)(t-u)} du$$

$$+ (\gamma - \lambda) \int_0^t \sum_{n=0}^{\infty} P_{n,\ d}(u) z^n e^{\left(\lambda z + \frac{\mu}{z} - (\lambda + \mu + \xi)\right)(t-u)} du$$

$$+ \lambda z \int_0^t \sum_{l=1}^{N} P_{0,\ l}(u) e^{\left(\lambda z + \frac{\mu}{z} - (\lambda + \mu + \xi)(t-u)\right.} du + \eta \int_0^t P_f(u) + e^{-\left((\lambda + \mu + \xi) - \left(\lambda z + \frac{\mu}{z}\right)\right)t}$$

if $x = 2\sqrt{\lambda\mu}$ and $y = \sqrt{\dfrac{\lambda}{\mu}}$ then

$$e^{\left(\lambda z + \frac{\mu}{z}\right)(t-u)} = \sum_{n=-\infty}^{\infty} (yz)^n I_n(x(t-u))$$

Where $I_n(.)$ denotes the modified bessel-function of first kind.

$$G(z,\ t) = -\left(\alpha + \frac{\mu}{z} - \mu\right) \int_0^t P_{0,\ r}(u) e^{-(\lambda + \mu + \xi)(t-u)} \left[\sum_{n=-\infty}^{\infty} (yz)^n I_n(x(t-u))\right] du$$

$$+ (\gamma - \lambda) \int_0^t \sum_{n=0}^{\infty} P_{n,\ d}(u) z^n e^{-(\lambda + \mu + \xi)(t-u)} \left[\sum_{n=-\infty}^{\infty} (yz)^n I_n(x(t-u))\right] du$$

$$+ \lambda z \int_0^t \sum_{l=1}^{N} P_{0,\ l}(u) e^{-(\lambda + \mu + \xi)(t-u)} \left[\sum_{n=-\infty}^{\infty} (yz)^n I_n(x(t-u))\right] du$$

$$+ \eta \int_0^t P_f(u) + e^{-(\lambda + \mu + \xi)t} \sum_{n=-\infty}^{\infty} (yz)^n I_n(x(t)))$$

For $n = 1,\ 2,\ \ldots$, we find the following by comparing the coefficients of z^n on both sides of the equation

$$P_{n,\,w}(t) = -\int_0^t P_{0,\,r}(u)e^{-(\lambda+\mu+\xi)(t-u)}y^n\left[(\alpha-\mu)I_n(.)+\mu y I_{n+1}(.)\right]du$$

$$+(\gamma-\lambda)\int_0^t \sum_{i=0}^{n} P_{i,\,d}(u)e^{-(\lambda+\mu+\xi))(t-u)}y^{n-i}I_{n-i}(.)du$$

$$+(\gamma-\lambda)\int_0^t \sum_{i=1}^{\infty} P_{n+i,\,d}(u)e^{-(\lambda+\mu+\xi))(t-u)}y^{-i}I_{-i}(.)du$$

$$+\lambda\int_0^t \sum_{l=1}^{N} P_{0,\,l}(u)e^{-(\lambda+\mu+\xi))(t-u)}y^{n-1}I_{n-1}(.)du$$

$$+\eta\int_0^t P_f(u)e^{-(\lambda+\mu+\xi))(t-u)}y^n I_n(.)du+e^{-(\lambda+\mu+\xi)t}y^n I_n(x(t)) \tag{13}$$

The coefficients of z^{-n} are compared for $n = 1,\ 2,\ 3\ \ldots.$

$$0 = -\int_0^t P_{0,\,r}(u)e^{-(\lambda+\mu+\xi)(t-u)}y^{-n}\left[(\alpha-\mu)I_{-n}(.)+\mu y I_{-n+1}(.)\right]du$$

$$+(\gamma-\lambda)\int_0^t \sum_{i=0}^{n} P_{i,\,d}(u)e^{-(\lambda+\mu+\xi))(t-u)}\times y^{-n-i}I_{-n-i}(.)du$$

$$+\lambda\int_0^t \sum_{l=1}^{N} P_{0,\,l}(u)e^{-(\lambda+\mu+\xi)(t-u)}\times y^{-n-1}I_{-n-1}(.)du+e^{-(\lambda+\mu)t}y^{-n}I_{-n}(x(t)) \tag{14}$$

By utilising the modified bessel-function attribute for $n \geq 1,\ I_n(.) = I_{-n}(.)$

$$P_{n,\,w}(t) = -\int_0^t P_{0,\,r}(u)e^{-(\lambda+\mu+\xi)(t-u)}\mu y^{n+1}\times\left[I_{n+1}(.)-I_{n-1}(.)\right]du$$

$$+(\gamma-\lambda)\int_0^t \sum_{i=0}^{n} P_{i,\,d}(u)e^{-(\lambda+\mu+\xi)(t-u)}\times y^{n-i}\left[I_{n-i}(.)-I_{n+i}(.)\right]du$$

$$+(\gamma-\lambda)\int_0^t \sum_{i=0}^{n} P_{n+i,\,d}(u)e^{-(\lambda+\mu+\xi)(t-u)}\times y^{-i}\left[I_i(.)-I_{2n+i}(.)\right]du$$

$$\lambda\int_0^t \sum_{l=1}^{N} P_{0,\,l}(u)\times e^{-(\lambda+\mu+\xi)(t-u)}y^{n-1}\left[I_{n-1}(.)-I_{n+1}(.)\right]du \tag{15}$$

Utilising $\frac{2n}{xt}I_n(xt) = I_{n-1}(xt) - I_{n+1}(xt)$ in (15) gives the time-dependent probability of working state $P_{n,\, w}(t),\ n \geq 1$ as follows

$$P_{n,\, w}(t) = \mu y^{n+1} P_{0,\, r}(t) \times [I_{n+1}(xt) - I_{n-1}(xt)]\, e^{-(\lambda+\mu+\xi))(t)}$$

$$+ (\gamma - \lambda) \sum_{n=0}^{\infty} P_{i,\, d}(t) y^{n-i} \times [I_{n-i}(xt) - I_{n+i}(xt)]\, e^{-(\lambda+\mu+\xi)(t)}$$

$$+ (\gamma - \lambda) \int_0^t \sum_{i=1}^{\infty} P_{n+i,\, d}(t) y^{-i} \times [I_i(xt) - I_{2n+i}(xt)]\, e^{-(\lambda+\mu+\xi)(t)}$$

$$+ \lambda \sum_{l=1}^{N} P_{0,\, l}(t) y^{n-1} \times [I_{n-1}(.) - I_{n+1}(.)]\, e^{-(\lambda+\mu+\xi)(t)}$$

5. Steady-state analysis of the proposed model

In this section, we present the SSP of the proposed model. After a certain period of time the probability $P_{n,\, j}(t)$ becomes independent of time and thus we have $\underset{t\to\infty}{\text{Lt}} P_{n,\, j}(t) = \pi_{n,\, j}$ then $\underset{t\to\infty}{\text{Lt}} P'_{n,\, j}(t) = 0$.

Thus equations (1)-(8) is given as follows

$$0 = -\eta \pi_f + \xi\left(1 - \pi_f\right) \tag{16}$$

$$0 = -(\lambda + \alpha + \xi)\pi_{0,\, r} + \mu \pi_{1,\, w} + \eta \pi_f \tag{17}$$

$$0 = -(\lambda + \mu + \xi)\pi_{1,\, w} + \lambda \pi_{0,\, r} + \mu \pi_{2,\, w}$$

$$+ \lambda\left(\pi_{0,\, 1} + \pi_{0,\, 2} + \ldots + \pi_{0,\, L}\right) + (\gamma - \lambda)\pi_{1,\, d} \tag{18}$$

$$0 = -(\lambda + \mu + \xi)\pi_{n,\, w} + \lambda \pi_{n-1,\, w} + \mu \pi_{n+1,\, w} + (\gamma - \lambda)\pi_{n,\, d} \tag{19}$$

$$0 = -(\beta + \xi)\pi_{0,\, 1} + \alpha \pi_{0,\, r} \tag{20}$$

$$0 = -(\beta + \xi)\pi_{0,\, l} + (\beta - \lambda)\pi_{0,\, l-1},\ l = 2,\ 3,\ \ldots\ L \tag{21}$$

$$0 = -(\gamma + \xi)\pi_{0,\, d} + (\beta - \lambda)\pi_{0,\, L} \tag{22}$$

$$0 = -(\gamma + \xi)\pi_{n,\, d} + \lambda \pi_{n-1,\, d} \tag{23}$$

The SSP exist for the following conditions $\lambda \leq \beta,\ \lambda < \mu$ and $\lambda < \gamma$.

Theorem 4 The SSP of system being in different states for $n \geq 0,\ l \leq l \leq L$ are given below:

Proof. (20)

$$(\beta+\xi)\pi_{0,\ 1}=\alpha\pi_{0,\ r}$$

$$\pi_{0,\ 1}=\left(\frac{\alpha}{\beta+\xi}\right)\pi_{0,\ r}$$

1. The SSP of system being in state (0, l) is given by

$$(21)\Rightarrow 0=-(\beta+\xi)\pi_{0,\ l}+(\beta-\lambda)\pi_{0,\ l-1}$$

$$\pi_{0,\ l}=\frac{(\beta-\lambda)^{l-1}}{(\beta+\xi)^{l-1}}\pi_{0,\ 1};\ l=2,\ 3,\ \ldots.\ L$$

$$\pi_{0,\ l}=\frac{\alpha(\beta-\lambda)^{l-1}}{(\beta+\xi)^{l-1}(\beta+\xi)}\pi_{0,\ r}$$

$$\pi_{0,\ l}=\frac{\alpha(\beta-\lambda)^{l-1}}{(\beta+\xi)^{l}}\pi_{0,\ r};\ 1\leq l\leq L \tag{24}$$

2. The SSP of system being in state $(n,\ d)$ is given by

$$(21)\Rightarrow 0=-(\gamma+\xi)\pi_{0,\ d}+(\beta-\lambda)\pi_{0,\ L}$$

$$\pi_{0,\ d}=\frac{(\beta-\lambda)}{(\gamma+\xi)}\pi_{0,\ L}$$

$$(22)\ l=L,\ \pi_{0,\ L}=\frac{\alpha(\beta-\lambda)^{L-1}}{(\beta+\xi)^{L}}\pi_{0,\ r}$$

$$\pi_{0,\ d}=\frac{(\beta-\lambda)}{(\gamma+\xi)}\frac{\alpha(\beta-\lambda)^{L-1}}{(\beta+\xi)^{L}}\pi_{0,\ r}$$

$$\pi_{0,\ d}=\frac{\alpha(\beta-\lambda)^{L}}{(\gamma+\xi)(\beta+\xi)^{L}}\pi_{0,\ r}$$

$$(23)\Rightarrow 0=-(\gamma+\xi)\pi_{n,\ d}+\lambda\pi_{n-1,\ d}$$

$$\pi_{n,\ d}=\frac{\lambda}{(\gamma+\xi)}\pi_{n-1,\ d},\ n=1,\ 2\ \ldots$$

$$\pi_{n,\,d} = \frac{\lambda^n}{(\gamma+\xi)^n}\pi_{0,\,d},\ n = 1,\ 2\ \ldots$$

from $\pi_{0,\,d}$

$$\pi_{n,\,d} = \frac{\lambda^n}{(\gamma+\xi)^n}\frac{\alpha(\beta-\lambda)^L}{(\gamma+\xi)(\beta+\xi)^L}\pi_{0,\,r}$$

$$\pi_{n,\,d} = \frac{\alpha\lambda^n(\beta-\lambda)^L}{(\gamma+\xi)^{n+1}(\beta+\xi)^L}\pi_{0,\,r},\ n = 0,\ 1,\ 2\ \ldots \tag{25}$$

3. The SSP of the system in state f is given by

$$(16) \Rightarrow 0 = -\eta\pi_f + \xi\left(1-\pi_f\right)$$

$$\pi_f = \frac{\xi}{\xi+\eta}$$

4. The SSP of the system in state $(n,\ w)$, where $n \geq 1$ is given by
For $n \geq 1$, we have the following by applying laplace-transform in (15)

$$f_{n,\,w}(s) = \frac{\mu y^{n+1}}{\sqrt{k^2-x^2}}\left[\left(\frac{k-\sqrt{k^2-x^2}}{x}\right)^{n+1} - \left(\frac{k-\sqrt{k^2-x^2}}{x}\right)^{n-1}\right] f_{0,\,r}(s)$$

$$+ \frac{(\gamma-\lambda)}{\sqrt{k^2-x^2}}\sum_{i=0}^{n} f_{i,\,d}y^{n-i}\left[\left(\frac{k-\sqrt{k^2-x^2}}{x}\right)^{n-i} - \left(\frac{k-\sqrt{k^2-x^2}}{x}\right)^{n+i}\right]$$

$$+ \frac{(\gamma-\lambda)}{\sqrt{k^2-x^2}}\sum_{i=1}^{\infty} f_{n+i,\,d}(s)y^{-i}\left[\left(\frac{k-\sqrt{k^2-x^2}}{x}\right)^{i} - \left(\frac{k-\sqrt{k^2-x^2}}{x}\right)^{2n+i}\right]$$

$$+ \lambda\sum_{l=1}^{N} f_{0,\,l}(s)y^{n-1}\left[\left(\frac{k-\sqrt{k^2-x^2}}{x}\right)^{n-1} - \left(\frac{k-\sqrt{k^2-x^2}}{x}\right)^{n+1}\right]$$

where

$$k=s+\lambda+\mu+\xi,\ x=2\sqrt{\lambda\mu},\ y=\sqrt{\frac{\lambda}{\mu}},\ c=\sqrt{k^2-x^2},\ b=\frac{k-\sqrt{k^2-x^2}}{x}$$

$$f_{n,\ w}(s)=\frac{\mu y^{n+1}}{\sqrt{k^2-x^2}}\left[\left(\frac{k-\sqrt{k^2-x^2}}{x}\right)^{n+1}-\left(\frac{k-\sqrt{k^2-x^2}}{x}\right)^{n-1}\right]f_{0,\ r}(s)$$

$$+\frac{(\gamma-\lambda)}{\sqrt{k^2-x^2}}\sum_{i=0}^{n}y^{n-i}\frac{\alpha\lambda^i(\beta-\lambda)}{(s+\gamma)^i(s+\gamma+\xi)(s+\beta)^N}\left[\left(\frac{k-\sqrt{k^2-x^2}}{x}\right)^{n-i}-\left(\frac{k-\sqrt{k^2-x^2}}{x}\right)^{n+i}\right]f_{0,\ r}(s)$$

$$+\frac{(\gamma-\lambda)}{\sqrt{k^2-x^2}}\sum_{i=1}^{\infty}y^{-i}\frac{\alpha\lambda^{n+i}(\beta-\lambda)^N}{(s+\gamma)^{n+i}(s+\gamma+\xi)(s+\beta)^N}\left[\left(\frac{k-\sqrt{k^2-x^2}}{x}\right)^{i}-\left(\frac{k-\sqrt{k^2-x^2}}{x}\right)^{2n+i}\right]f_{0,\ r}(s)$$

$$+\lambda\sum_{l=1}^{N}y^{n-1}\frac{\alpha(\beta-\lambda)^{l-1}}{(s+\beta)^l}\left[\left(\frac{k-\sqrt{k^2-x^2}}{x}\right)^{n-1}-\left(\frac{k-\sqrt{k^2-x^2}}{x}\right)^{n+1}\right]f_{0,\ r}(s)$$

By using the properties of laplace transform, the steady state probability $\pi_{n,\ w}$ can be obtained for $n=1,\ 2,\ \dots$

For steady state we know that $\frac{\lambda}{\mu}<1$ and $\lim_{t\to\infty}P_{n,\ w}(t)=\pi_{n,\ w}=\lim_{s\to 0}sf_{n,\ w}(s)$

$$\pi_{n,\ w}=\left\{\begin{array}{l}\frac{\mu y^{n+1}}{c}\left[b^{n+1}-b^{n-1}\right]+\sum_{i=0}^{n}\frac{(\gamma-\lambda)y^{n-i}}{c}\times\frac{\alpha\lambda^i(\beta-\lambda)^L}{(\gamma+\xi)^{i+1}(\beta+\xi)^L}\left[b^{n-i}-b^{n+i}\right]+\\ \sum_{i=1}^{\infty}\frac{(\gamma-\lambda)y^{-i}}{c}\times\frac{\alpha\lambda^{n+i}(\beta-\lambda)^L}{(\gamma+\xi)^{n+i+1}(\beta+\xi)^L}\left[b^{i}-b^{2n+i}\right]+\lambda\sum_{l=1}^{L}\frac{y^{n-1}}{c}\times\frac{\alpha(\beta-\lambda)^{l-1}}{(\beta+\xi)^l}\left[b^{n-1}-b^{n+1}\right]\end{array}\right\}\pi_{0,\ r}$$

$$\pi_{n,\ w}=A_n\pi_{0,\ r}, \tag{26}$$

where

$$A_n=\left\{\frac{\mu y^{n+1}}{c}\left[b^{n+1}-b^{n-1}\right]+\sum_{i=0}^{n}\frac{(\gamma-\lambda)y^{n-i}}{c}\times\frac{\alpha\lambda^i(\beta-\lambda)^L}{(\gamma+\xi)^{i+1}(\beta+\xi)^L}\left[b^{n-i}-b^{n+i}\right]\right.$$

$$\left.+\sum_{i=1}^{\infty}\frac{(\gamma-\lambda)y^{-i}}{c}\times\frac{\alpha\lambda^{n+i}(\beta-\lambda)^L}{(\gamma+\xi)^{n+i+1}(\beta+\xi)^L}\left[b^{i}-b^{2n+i}\right]+\lambda\sum_{l=1}^{L}\frac{y^{n-1}}{c}\times\frac{\alpha(\beta-\lambda)^{l-1}}{(\beta+\xi)^l}\left[b^{n-1}-b^{n+1}\right]\right\}$$

Since

$$\sum_{l=1}^{L} \pi_{0,\ l} + \sum_{n=0}^{\infty} \pi_{n,\ d} + \sum_{n=0}^{\infty} \pi_{n,\ w} = 1$$

substituting (24), (25) and (26) in above equation, we have

$$\pi_{0,\ r} + \sum_{n=1}^{\infty} A_n \pi_{0,\ r} + \sum_{l=1}^{L} \frac{\alpha(\beta-\lambda)^{l-1}}{(\beta+\xi)^l} \pi_{0,\ r} + \sum_{n=0}^{\infty} \frac{\alpha\lambda^i(\beta-\lambda)^L}{(\gamma+\xi)^{n+1}(\beta+\xi)^L} \pi_{0,\ r} = 1$$

$$\pi_{0,\ r} = \frac{1}{1+\sum_{n=1}^{\infty} A_n + \sum_{l=1}^{L} \frac{\alpha(\beta-\lambda)^{l-1}}{(\beta+\xi)^l} + \sum_{n=0}^{\infty} \frac{\alpha\lambda^i(\beta-\lambda)^L}{(\gamma+\xi)^{n+1}(\beta+\xi)^L}}$$

where $\pi_{0,\ r}$ exists for $\lambda \leq \beta$, $\lambda < \mu$ and $\lambda < \gamma$.

6. Analysis of the performance measures

The performance measures of the proposed MM is presented in this section.

6.1 *Time-dependent performance metrics*

Let $P\{N(t),\ S(t) = LS(t)\}$ denote the probability of the system in state l and state d at time '$t > 0$'. Then

$$P\{LS(t)\} = \sum_{l=1}^{L} P_{0,\ l}(t)$$

$$= \alpha e^{-(\beta+\xi)t} \left\{ \sum_{l=1}^{L} \frac{(t(\beta-\lambda))^{l-1}}{(l-1)!} \right\} * P_{0,\ r}(t)$$

The probability of the system in state d at time '$t > 0$' is given by $P\{N(t),\ S(t) = D(t)\}$ as follows

$$P\{D(t)\} = \sum_{n=0}^{\infty} P_{n,\ d}(t)$$

$$= \alpha(\beta-\lambda)^L e^{(\lambda-(\gamma+\xi))t} * e^{-(\beta+\xi)t} \frac{t^{L-1}}{(L-1)!} * P_{0,\ r}(t)$$

6.2 *Performance measures of steady state*

Let LS_M denote the SSP for l state and D_M denote the SSP for d state. Then the performance metrics of the steady state are given as follows:

$$LS_M = \sum_{l=1}^{L} \pi_{0,\ l} = \frac{\alpha\left((\beta+\xi)^L - (\beta-\lambda)^L\right)}{(\lambda+\xi)(\beta+\xi)^L}\pi_{0,\ r}$$

$$D_M = \sum_{n=0}^{\infty} \pi_{n,\ d} = \frac{\alpha(\beta-\lambda)^L}{(\gamma+\xi-\lambda)(\beta+\xi)^L}\pi_{0,\ r}$$

Table 1. Steady-state probability of state I for different values of β and γ

L	$\beta = 1.5,\ \gamma = 2$	$\beta = 2,\ \gamma = 2.2$
7	0.5419	0.5002
8	0.5529	0.5189
9	0.5599	0.5328
10	0.5648	0.5433
11	0.5680	0.5511
12	0.5699	0.5570
13	0.5714	0.5614
14	0.5724	0.5646
15	0.5720	0.5670
16	0.5734	0.5687
17	0.5736	0.5702
18	0.5738	0.5712
19	0.5738	0.5719
20	0.5738	0.5724

In Table 1, the Steady-state probability of system being in state l with respect to maximum number of working vacations L for different values of β and γ are calculated. We find that as the number of L increases, so does the SSP. It is observed that the probability increases the value of β decreases.

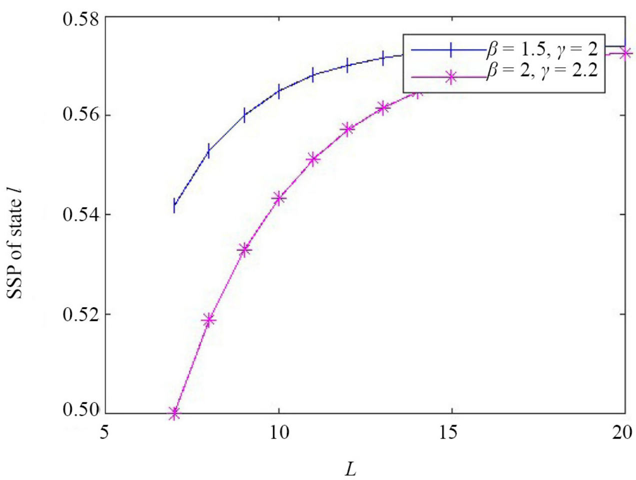

Figure 2. SSP of state l with respect to L for different values of β and γ

Figure 2 Shows the the behaviour of SSP of state l with respect to L for different values of β and γ. We observer that SSP of state l increases with increases in number of working vacations L.

Table 2. Steady-state probability of state d for different values of β and γ

L	$\beta = 1.5,\ \gamma = 2$	$\beta = 2,\ \gamma = 2.2$
7	0.0107	0.021
8	0.007	0.016
9	0.004	0.0122
10	0.0031	0.0026
11	0.0022	0.0068
12	0.0014	0.0051
14	0.00048	0.0039
15	0.00048	0.0022
16	0.00024	0.0014
17	0.00024	0.0009

In Table 2, the Steady-state probability of system being in state d with respect to maximum number of working vacations L for different values of β and γ are calculated. We find that the SSP of state d decreases with an increase in L.

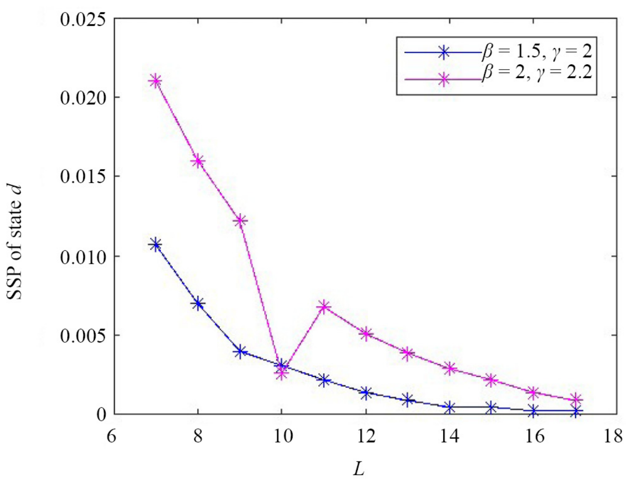

Figure 3. SSP of state d with respect to L for different values of β and γ

Figure 3 Shows the the behaviour of SSP of state d with respect to L for different values of β and γ. We observe that as the number of working vacations L increases, the SSP of state d decreases.

6.3 *Mean number of users*

The mean number of users in the system is denoted by $E[U_M]$, the mean number of users in working state and standby state is denoted by $E[U_{W_M}]$ and $E[U_{LS_M}]$ respectively. Then,

$$E[U_M] = E[U_{W_M}] + E[U_{LS_M}]$$

where

$$E\left[U_{W_M}\right]=\sum_{n=0}^{\infty} n\pi_{n,\ w}$$

$$E\left[U_{LS_M}\right]=\sum_{n=0}^{\infty} n\pi_{n,\ d}$$

Let the expected system size at any $t>0$ is given by $M(t)$,

$$M(t)=\sum_{n=0}^{\infty} nP_{n,\ w}(t)+\sum_{n=0}^{\infty} nP_{n,\ d}(t)$$

7. Energy saving factor with disaster

The energy consumption is reduced when the base station is in standby mode and deep sleep mode. This sleep strategy paves way for saving power. The energy saving factor with disaster E_f is calculated using the steady state probabilities for $n=0,\ 1,\ 2,\ \ldots;\ j=r,\ w,\ l,\ d,\ f$, and is given by,

$$E_f=\sum_{l=1}^{L} \pi_{0,\ l}+\sum_{n=0}^{\infty} \pi_{n,\ d}$$

$$=\left\{\frac{\alpha\left((\beta+\xi)^L-(\beta-\lambda)^L\right)}{(\lambda+\xi)(\beta+\xi)^L}+\frac{\alpha(\beta-\lambda)^L}{(\gamma+\xi-\lambda)(\beta+\xi)^L}\right\}\pi_{0,\ r}$$

8. Numerical illustration

We compute the energy saving factor with disaster where the values of the parameters are $\lambda=0.5,\ \mu=1.5,\ \alpha=1.2,\ \xi=0.01$. We discuss the above for different values of β, γ as they represent the time spent in states l, d respectively. The values are plotted in the graphs below.

Table 3. E_f for $\beta=1,\ \gamma=1.8$

L	E_f
7	0.5726
8	0.5734
9	0.5741
10	0.5748
11	0.5748
12	0.5748
13	0.5751
14	0.5751
15	0.5751
16	0.5751

In Table 3, the energy saving factor with respect to L for $\beta = 1$ and $\gamma = 1.8$ is calculated. As L increases, we find that the factor also increases. With an increase in the L, the factor also stabilises. At this point, for $L \geq 13$, the conservation factor gets close its steady-state.

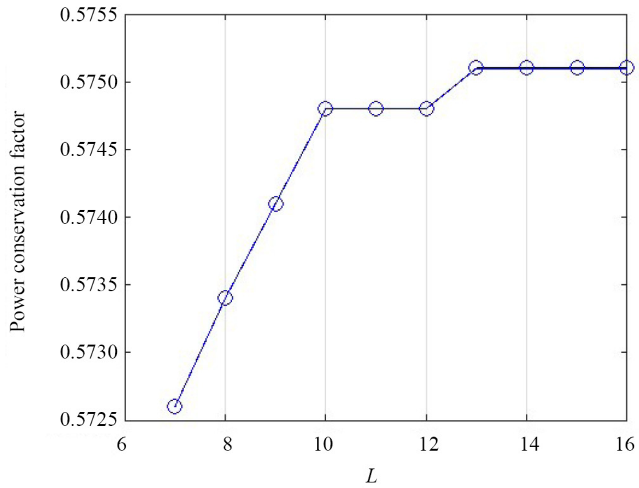

Figure 4. The values of E_f with respect to L for $\beta = 1$ and $\gamma = 1.8$

We observe from Figure 4. that the factor increases with increase in maximum number of L. Also, we find the power conservation factor stabilizes for $L \geq 13$.

Table 4. E_f for $\beta = 1.5$, $\gamma = 2$

L	E_f
7	0.5536
8	0.5609
9	0.5655
10	0.5690
11	0.5712
12	0.5724
13	0.5734
14	0.5738
16	0.5743
17	0.5743
18	0.5748
19	0.5748

In Table 4, the conservation factor with respect to L for $\beta = 1.5$ and $\gamma = 2$ is calculated.

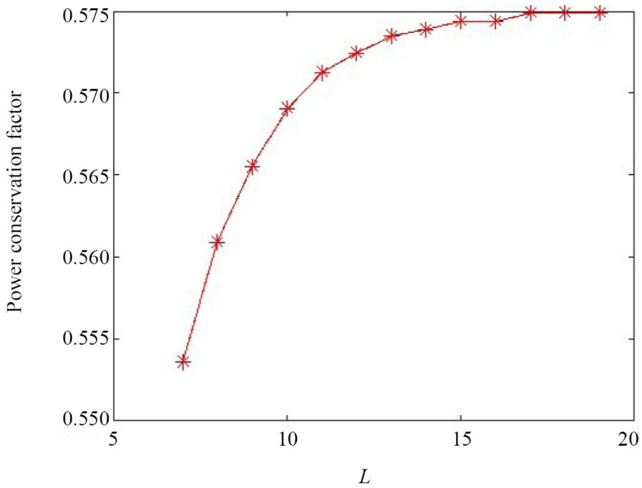

Figure 5. The values of E_f with respect to L for $\beta = 1.5$ and $\gamma = 2$

We observe from Figure 5. that the factor increases with increase in maximum number of L. Also, we find the conservation factor stabilizes for $L \geq 17$.

Table 5. E_f for $\beta = 2,\ \gamma = 2.2$

L	E_f
7	0.5230
8	0.5365
9	0.5460
10	0.5740
11	0.5589
12	0.5631
13	0.5663
14	0.5702
15	0.5712
16	0.5731
17	0.5736
18	0.5738
19	0.5743
20	0.5743
21	0.5746
22	0.5746
24	0.5748
25	0.5748
26	0.5748

In Table 5, the power conservation factor with respect to L for $\beta = 2$ and $\gamma = 2.2$ is calculated.

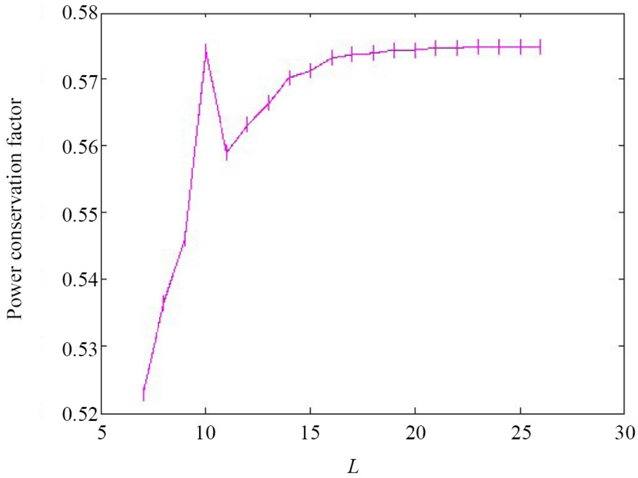

Figure 6. The values of E_f with respect to L for $\beta = 2$ and $\gamma = 2.2$

We observe from Figure 6 that the factor increases with increase in maximum number of sleep cycles L. Also, we find the power conservation factor stabilizes for $L \geq 23$.

9. Results and discussion

From the Tables 3, 4 and 5, we find that as the values of β and γ increases, there is an increase in the number of L to reach the steady-state value. Comparatively the power conservation factor for $\beta = 1$ and $\gamma = 1.8$ obtained from the proposed model with breakdown and repairable server using the Markov model. We observed that the factor is atleast 1.79% more in [5]. This percentage variation is because of the breakdown involved in the proposed model. Hence by varying the values of β and γ, we analysed the behaviour of the conservation factor and found the maximum number of L to be administered to reach the steady-state value.

10. Conclusion and future work

The total energy consumption of all BSs would result in heavy energy usage in cellular networks. We have proposed a MM for the BS sleep cell strategy. Also, We have shown that the power saving factor with disaster be easily computed using the MM and we find that the power saving factor with disaster attains stability with the increasing number of L. Hence, through the proposed MM the amount of power saved in BS is increased in the light sleep mode. The proposed model is known for its simplicity and is useful in analysing transition between states. Thus the proposed model is used in decision making problems that involves risk and continuous over time. Several business applications use Markov models, such as marketing's ability to predict customer brand switching, human resources ability to predict employee retention, manufacturing's ability to predict a machine's time to failure, and finance's ability to predict stock prices. We have a plan to extend this work by using the semi-markov modelling.

Conflict of interest

The authors declare no competing financial interest.

References

[1] Gbenga-Ilori A, Ladipo O, Alamu O. Queueing decision model for throughput maximization in green communications networks. *International Journal of Engineering Business Management*. 2019; 11(1): 1847979019867246. Available from: https://doi.org/10.1177/1847979019867246.

[2] Ammar SI. Transient behavior of a two-processor heterogeneous system with catastrophes, server failures and repairs. *Applied Mathematical Modelling*. 2014; 38(7-8): 2224-2234. Available from: https://doi.org/10.1016/j.apm.2013.10.033.

[3] Ayyappan G, Meena S. Phase type queueing model of server vacation, repair and degrading service with breakdown, starting failure and close-down. *Reliability: Theory & Applications*. 2023; 18(1): 464-483.

[4] Liu C, Natarajan B, Xia H. Small cell base station sleep strategies for energy efficiency. *IEEE Transactions on Vehicular Technology*. 2016; 65(3): 1652-1661.

[5] Dharmaraja S, Aggarwal A, Sudhesh R. Analysis of energy saving in user equipment in LTE-A using stochastic modelling. *Telecommunication Systems*. 2022; 80(1): 123-140.

[6] Oh E, Son K, Krishnamachari B. Dynamic base station switching-on/off strategies for green cellular networks. *IEEE Transactions on Wireless Communications*. 2013; 12(5): 2126-2136.

[7] Nafea HB, Sallam MM, Zaki FW. Study of DRX sleep mode performance on virtual base station energy saving in 5G networks. *Wireless Personal Communications*. 2021; 118(4): 3251-3270.

[8] Holtkamp H, Auer G, Bazzi S, Haas H. Minimizing base station power consumption. *IEEE Journal on Selected Areas in Communications*. 2014; 32(2): 297-306.

[9] Khan IE, Paramasivam R. Performance study of an M/M/1 retrial queueing system with balking, dissatisfied customers, and server vacations. *Contemporary Mathematics*. 2023; 4(3): 467-483.

[10] Kumar BK, Arivudainambi D. Transient solution of an M/M/1 queue with catastrophes. *Computers & Mathematics with Applications*. 2000; 40(10-11): 1233-1240. Available from: https://doi.org/10.1016/S0898-1221(00)00234-0.

[11] Krishna Kumar B, Pavai Madheswari S. Transient behaviour of the M/M/2 queue with catastrophes. *Statistica*. 2002; 62(1): 129-136. Available from: https://doi.org/10.6092/issn.1973-2201/1287.

[12] Feng M, Mao S, Jiang T. Base station ON-OFF switching in 5G wireless networks: Approaches and challenges. *IEEE Wireless Communications*. 2017; 24(4): 46-54. Available from: https://doi.org/10.1109/MWC.2017.1600353.

[13] Islam N, Alazab A, Agbinya J. Energy efficient and delay aware 5G multi-tier network. *Remote Sensing*. 2019; 11(9): 1019. Available from: https://doi.org/10.3390/rs11091019.

[14] Paz N, Yechiali U. An M/M/1 queue in random environment with disasters. *Asia-Pacific Journal of Operational Research*. 2014; 31(03): 1450016. Available from: https://doi.org/10.1142/S021759591450016X.

[15] Rajadurai P. A study on M/G/1 retrial queueing system with three different types of customers under working vacation policy. *International Journal of Mathematical Modelling and Numerical Optimisation*. 2018; 8(4): 393-417. Available from: https://doi.org/10.1504/IJMMNO.2018.094550.

[16] Kumar R. A transient solution to the M/M/c queuing model equation with balking and catastrophes. *Croatian Operational Research Review*. 2017; 8(2): 577-591. Available from: https://doi.org/10.17535/crorr.2017.0037.

[17] Chakrabarty R, Singh S, Rajinikanth E. Reduction in energy loss using automatic base station switching operation in cellular network. *World Engineering & Applied Sciences Journal*. 2015; 6(2): 124-131.

[18] Sadiwala RC. Energy efficiency for long term evolution cellular networks. *International Journal of Engineering Research & Technology*. 2013; 2(10): 1968-1982.

[19] Sudhesh R, Savitha P. Transient behaviour of three-heterogeneous servers queue with system disaster and server repair. *Rairo Operations Research*. 2017; 51(4): 965-983. Available from: https://doi.org/10.1051/ro/2017018.

[20] Shortle JF, Thompson JM, Gross D, Harris CM. *Fundamentals of Queueing Theory*. 5th ed. USA: John Wiley & Sons; 2018. Available from: https://doi.org/10.1002/9781119453765.

[21] Bolch G, Greiner S, De Meer H, Trivedi KS. *Queueing Networks and Markov Chains: Modeling and Performance Evaluation With Computer Science Applications*. USA: John Wiley & Sons; 2006.

[22] Viswanathan T, Bhatnagar M. *Telecommunication Switching Systems and Networks*. 5th ed. PHI Learning Pvt. Ltd; 2015.

[23] Medhi J. *Stochastic Models in Queueing Theory*. Amsterdam, Netherlands: Elsevier; 2003.

[24] Veerarajan T. *Probability, Statistics and Random Processes*. New York: Tata McGraw-Hill; 2008.