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Research Article

# Irreducible Ordinary Characters in Blocks of Finite Groups

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**Abstract:** Many years ago, the classification of all finite simple groups was accomplished after a very long time working on the problem. After such completion, attention was turned to other aspects of studying finite groups, one such aspect being the study of blocks of finite groups. Richard Brauer conjectured that if $D \in \delta(B)$ is a defect group of a block $B \in Bl(G)$ of a group $G$ and $d(B)$ is the defect of $B \in Bl(G)$, then $k(B) \leq |D| = p^{d(B)}$. This has generally come to be known as Brauer's $k(B)$-conjecture and is obviously true for blocks of defect 0. The object in this paper is to study the irreducible ordinary characters in a block of a finite group and prove that if $G$ is any finite group, then for any block $B \in Bl(G)$ of $G$ with defect group $D \in \delta(B)$ and defect $d(B)$, it is indeed true that $k(B) \leq |D| = p^{d(B)}$. This result therefore will enhance the body of knowledge in the study of blocks of finite groups and ultimately contribute to the overall ongoing study of finite groups, whose ultimate goal is to classify finite groups.

***Keywords***: blocks of characters, defect groups of blocks, defect of a block, irreducible ordinary characters, irreducible Brauer characters

**MSC:** 20C15, 20C20, 20D20

## 1. Introduction

Many years ago, the classification of all finite simple groups was accomplished after a very long time working on the problem. After such completion, attention was turned to other aspects of studying finite groups, one such aspect being the study of blocks of finite groups. Such a study is intended to contribute to the ongoing study of finite groups whose ultimate goal is to classify finite groups.

Let $G$ be a finite group, $Irr(G)$ the set of all the irreducible ordinary characters of $G$, $IBr(G)$ the set of all the irreducible Brauer characters of $G$ and $S = Irr(G) \cup IBr(G)$ such that $S$ gets partitioned into disjoint subsets called blocks of $G$, where the set of all blocks of $G$ is denoted by $Bl(G)$.

If $\chi \in Irr(G)$ and $\phi \in IBr(G)$, then $\chi \in Irr(B)$ and $\phi \in IBr(B)$ for some block $B \in Bl(G)$ of $G$. For $\chi \in \mathrm{Irr}(B)$ for some block $B \in Bl(G)$ of $G$, we define $d(\chi)$, $d(B)$, $h(\chi)$ which are the defect of $\chi \in Irr(B)$, the defect of $B \in Bl(G)$ and the height of $\chi \in Irr(B)$ respectively as follows (cf. [1 (5.5, 5.6, 5.7), 2 (Proposition 13.5.15)]):


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$$p^{d(\chi)} = (|G|/\chi(1))_p$$

$$d(B) = \max\{d(\chi) \mid \chi \in Irr(B)\} \tag{1}$$

$$h(\chi) = d(B) - d(\chi)$$

Richard Brauer posed as [3, Problem 20] the following question: *Is it true that a p-block B of defect d consists of at most $p^d$ ordinary characters?* This question has generally come to be known as Brauer's $k(B)$-conjecture. In [4] Brauer mentioned a conjecture that the decomposition numbers $d_{ij}$ corresponding to the block $B \in Bl(G)$ of defect $d(B)$ lie below a bound uniquely determined by $p$ and $d(B)$. He (Brauer) furthermore drew an equivalent statement concerning the Cartan invariants $c_{ij}$ of $B \in Bl(G)$.

Brauer then posed the following question in [4]: Is it true that $c_{ij} \leq p^d$ for the Cartan invariants of a block $B$ of defect $d$? This very same question posed in [4] also appears as [3, Problem 22]. According to Brauer and Feit in [5], if $B \in Bl(G)$ is a block of $G$ with $D \in \delta(B)$ cyclic of order $p^{d(B)}$ and $n = l(B)$, then $n \mid p-1$. If $d(B) = 1$ and $n = p-1$, then for $B \in Bl(G)$, $k(B) = p = n+1$, thus satisfying Brauer's $k(B)$-conjecture.

It is however trivially true that $1 \leq k(B) \leq |Irr(G)|$. According to Brauer and Feit in [5, Theorem 1], for $B \in Bl(G)$, it follows that $k(B) \leq \frac{1}{4}p^{2d(B)} + 1$. By [6, Lemma IV.4.21], if $B \in Bl(G)$ contains a unique irreducible Brauer character, then $k(B) \leq p^{d(B)}$ with equality holding when every decomposition number for $B \in Bl(G)$ is 1, every irreducible character has height 0 and the unique Cartan invariant is $p^{d(B)}$.

Robinson and Thompson in [7] provided an affirmative answer to the question for blocks of $p$-solvable groups for finitely many primes. According to [5, Theorem 3], if a block $B \in Bl(G)$ contains irreducible ordinary characters of $G$ of positive height, then $k(B) < \frac{1}{2}p^{2d(B)-2}$. It is furthermore asserted in [5, Theorem 1*] that if the defect group of a block $B \in Bl(G)$ is cyclic, then

$$k(B) \leq \begin{cases} p^{d(B)} & \text{for } d(B) \in \{0,\, 1,\, 2\} \\ \\ p^{2d(B)-2} & \text{for } d(B) > 2 \end{cases} \tag{2}$$

If $p = 2$, $B \in Bl(G)$ with $d(B) \in \{0, 1, 2\}$, then all $D \in \delta(B)$ are cyclic with $l(B) \geq 1$ and since $l(B) \mid p-1$, $l(B) = 1$. For $d(B) = 1$, $D \in \delta(B)$ is cyclic with $l(B) = p-1$ so that $k(B) = 2 \leq p^{d(B)}$. However by [8, Theorem 1.6], we obtain that $k(B) \leq |D| 2^{[4d(B)]/3} 3^{[10d(B)]/9}$. According to Navarro in [9, Theorem 11.1], if $|G|_p = p$, where $p$ is odd, $P \in Syl_p(G)$, $N = N_G(P)$, $C = C_G(P)$, $e = [N\colon C]$, $t = \frac{p-1}{e}$, then $k(B_0) = e + t$.

The object in this paper is to study the irreducible ordinary characters in a block of a finite group and prove that if $G$ is any finite group, then for any block $B \in Bl(G)$ of $G$ with defect group $D \in \delta(B)$ and defect $d(B)$, it is true that $k(B) \leq |D| = p^{d(B)}$. Therefore, this result will enhance the body of knowledge in the study of blocks of finite groups and ultimately contribute to the overall study of finite groups.

In §2 we give preliminaries, in §3 we discuss the irreducible ordinary characters, in §4 we give concluding remarks and in §5 we give a declaration on the conflict of interest.

Throughout, all our groups $G$ are finite, $p$ is a prime that divides the order $|G|$ of $G$, $B_0 \in Bl(G)$ is the principal block of $G$ and $B \in Bl(G)$ is any block of $G$ unless otherwise specified to the contrary, $\delta(G)$ is the set of defect groups of $B \in Bl(G)$, $Syl_p(G)$ is the set of Sylow $p$-subgroups of $G$, $|G|_p$ is the $p$-part of the order $|G|$ of $G$ and $1_G \in G$ is the identity element of the finite group $G$.

## 2. Preliminaries

It is proven in [7, Theorem 1] that for a $p$-solvable group $G$ for $p > 5^{30}$, $B \in Bl(G)$, $D \in \delta(B)$, $k(B) \leq |D|$. For $G$ of odd order, [8, Theorem 2.7] gives that $k(B) \leq p^{d(B)}3^{[d(B)-1]/2}$. By [10, Theorem 1], if $B \in Bl(G)$ is a block of a $p$-solvable group $G$ with $D \in \delta(B)$ nonabelian, then $k(B) < |D|$ while by [11], if $l(B) \leq 2$ or $k(B) - l(B) \leq 2$, then $k(B) \leq p^{d(B)}$. Furthermore [11, Corollary 7, Corollary 7*] give that for $p = 2$ and $l(B) = 3$ or $k(B) - l(B) = 3$, $k(B) \leq 2^{d(B)}$.

**Proposition 1** Let $B \in Bl(G)$ with the highest defect, $D \in \delta(B)$ and $\chi \in Irr(B)$. Then $[\chi(1_G)]_p \leq |D|$.

**Proof.** Suppose that $[\chi(1_G)]_p = p^n$ and $|D| = p^m$, where $D \in \delta(B)$ and of course $m = d(B)$. If $|G|_p = p^a$, we have that $m, n \leq a$. Since $m$ is the highest defect of $B$, we obtain that $m = a$ yielding $n \leq m$ as required, which thus completes the proof. □

We can see from Proposition 1 above that if $B \in Bl(G)$ with the highest defect and $\chi \in Irr(B)$, then the height $h(\chi)$ of $\chi \in Irr(B)$ satisfies $h(\chi) = 0$ if and only if $[\chi(1_G)]_p = 1$. If $B \in Bl(G)$ has positive defect $d(B)$, $\chi \in Irr(B)$ with $\nu([\chi(1_G)]_p) = m$ and $\nu(|G|_p) = a$, then $h(\chi) = 0$.

## 3. Irreducible ordinary characters

Alghamdi proved in [12, Theorem 1.4] that the Ordinary Weight Conjecture implies Brauer's $k(B)$-Conjecture in the case $B \in Bl(G)$, $D \in \delta(B)$ an extra-special $p$-group of order $p^3$ and exponent $p$ under the assumption that $N_G(D, b_D)/DC_G(D)$ is isomorphic to $C_2 \times C_{p-1}$.

The authors proved in [13, Theorem 2, 14] that any defect group $D \in \delta(B)$ where $B \in Bl(G)$ is a block of $G$, is the intersection of two Sylow $p$-subgroups of $G$ (which may be equal). According to [15, Theorem 8] every block which is not of the highest kind contains more families of ordinary characters than it contains modular characters and blocks of the highest kind are discussed by Brauer and Nesbitt in [16].

**Proposition 2** [5] For any finite group $G$ and $B \in Bl(G)$ a block of $G$, $k(B) \leq p^{[d(B)(d(B)+1)]/2}$.

**Lemma 1** If $B \in Bl(G)$ is a block of $G$, $D \in \delta(B)$ a defect group of $B \in Bl(G)$ such that $d(B) \in \{0, 1, 2\}$, then $k(B) \leq |D|$.

**Proof.** For $d(B) = 0$, the result follows by [9 (Theorem 3.18), 17 (Theorem 3.6.29)]. Otherwise the overall result follows by [5, Theorem 1*] and the proof is complete. □

**Proposition 3** Let $G$ be an abelian group and $B \in Bl(G)$ a block of $G$ with defect group $D \in \delta(B)$. Then $k(B) \leq |D|$.

**Proof.** The result follows immediately by [18, Theorem 3] and moreover all the irreducible characters in $B \in Bl(G)$ will be linear and $l(B) = 1$. Also by the normality of $D \in \delta(B)$ in $G$ and the triviality of $D'$, the result follows immediately by [19, Theorem A, Theorem 2.3] thus completing the proof. □

**Proposition 4** Let $B \in Bl(G)$ be a block of $G$ with defect group $D \in \delta(B)$ abelian. Then $k(B) \leq |D|$.

**Proof.** The case $G$ solvable is addressed in [20]. For $D \in \delta(B)$ cyclic, the result follows by [5 (Theorem 4), 21]. If $\mu(B) = 1$, then the result follows by [2, 22 (Theorem 2.2)]. The case $G$ abelian is addressed in Proposition 3 above. If the inertial index of $B \in Bl(G)$ is 1, then $B \in Bl(G)$ is nilpotent and the result follows by [2, Theorem 13.5.17]. Since $D \in \delta(B)$ is abelian, we obtain from [23] that $k(B) = k_0(B)$ and so by [11, Theorem 15] we get that $k(B) \leq p^{d(B)}\sqrt{l(B)}$ and $k(B) \leq p^{d(B)}\sqrt{k(B) - l(B)}$. By [17, Corollary 3.6.38] if $d(B) > 0$, then $k(B) > l(B)$ and so for $l(B) = 1$, the result follows by [2 (Theorem 13.5.11), 11 (Theorem 15)]. Since $k(B) = k_0(B)$, for $l(B) = 1$, the result follows by [2, Proposition 13.5.5]. When $l(B) \in \{2, 3\}$, the result follows by [2, Theorem 13.5.7]. If all $\chi \in Irr(B)$ have the same degree, then $B \in Bl(G)$ becomes separable over its center and the result follows by [2, Theorem 13.5.11, 24, Proposition 1]. If $D \in \delta(B)$ satisfies the conditions of [2, Theorem 13.5.11], then $k(B)$ is the number of conjugacy classes of $D$ and so the result follows. By Lemma 1 above, the result follows for $d(B) \in \{0, 1, 2\}$ and thus for $d(B) > 1$, applying induction yields the desired result and the proof is complete. □

We obtain from [23, Proposition 3.1] that if $D \in \delta(B)$ is abelian, then every irreducible $FG$-module in $B \in Bl(G)$ has vertex $D \in \delta(B)$. By [2], $k(B) = 1$ if and only if $|D| = 1$ and $k(B) = 2$ if and only if $|D| = 2$. Furthermore by [22, Theorem 2.2], for all finite groups $G$, $B \in Bl(G)$ a block of $G$ and $D \in \delta(B)$ a defect group of $B \in Bl(G)$ with defect $d(B)$ such that $\mu(B) = 1$, $k(B) = |D| = p^{d(B)}$.

**Corollary 1** Let $B \in Bl(G)$ be a large vertex block and $D \in \delta(B)$ a defect group of $B \in Bl(G)$. Then $k(B) \leq |D|$.

**Proof.** Since $B \in Bl(G)$ is a large vertex block, by [23] we get that $D \in \delta(B)$ is abelian and so the result follows by Proposition 4 above. □

A large vertex block $B \in Bl(G)$ with defect group $D \in \delta(B)$ and inertial index 1 is separable over its center and satisfies $k(B) = |D|$, $l(B) = 1$. If $G$ is abelian, then all its blocks are separable over their centers and satisfy $k(B) = |D|$, $l(B) = 1$ for every block $B \in Bl(G)$ of $G$.

**Proposition 5** Let $B \in Bl(G)$ be a block of $G$ with defect group $D \in \delta(B)$ which is normal in $G$. Then $k(B) \leq |D|$.

**Proof.** Since $D \in \delta(B)$ is normal in $G$, we have by [6 (Corollary III.6.9), 17 (Theorem 5.2.8(i)), 25 (Lemma 1), 26 (Lemma 87.26)] that $D$ is contained in a defect group of every block of $G$. By [19, Theorem A, Theorem 2.3], we obtain that $k(B)/k_0(B) \leq k(D')$ and $k(B)/l(B) \leq k(D)$, where $k(D')$ and $k(D)$ are the numbers of irreducible ordinary characters of $D'$ and $D$ respectively and the desired result is obtained in the case $l(B) = 1$. If $D \in \delta(B)$ is abelian, then by [23], [18] $B \in Bl(G)$ becomes a large vertex block and thus the result follows immediately by Corollary 1 above. Again for $D \in \delta(B)$ abelian, the result follows by [19, Theorem A, Theorem 2.3] since $D'$ is trivial and $k(B) = k_0(B)$. Otherwise for $D \in \delta(B)$ nonabelian, by [18, Theorem 9] $B \in Bl(G)$ will possess representations of strictly positive height and so by [5, Theorem 3], $k(B) < \frac{1}{2}p^{2d(B)-2}$. Moreover $B \in Bl(G)$ will be of strictly positive defect and so by [11, Proposition 10], $k_0(B) \geq 2$. Since $D \in \delta(B)$ is nonabelian, $d(B) \geq 3$ and thus applying induction on this $d(B)$, yields that $p^{d(B)} \leq \frac{1}{2}p^{2d(B)-2}$. From this we then obtain that $k(B) \leq p^{d(B)}$ so that the result follows immediately completing the proof. □

According to [1], $D \in \delta(B)$ a defect group of a block $B \in Bl(G)$ of $G$ which is normal in $G$ in Proposition 5 above is such that $D = O_p(G)$.

**Proposition 6** Let $B_0 \in Bl(G)$ be the principal block of $G$ and $D \in \delta(B_0)$ its defect group. Then $k(B_0) \leq |D|$.

**Proof.** Trivially we have that $1 \leq k(B) \leq |Irr(G)|$ for any block $B \in Bl(G)$ of $G$. The principality of $B_0 \in Bl(G)$ gives that $d(B_0)$ is the highest defect so that $d(B_0) = \nu(|G|_p)$ and thus $D \in \delta(B_0)$ is a Sylow $p$-subgroup of $G$. For $G$ $p$-nilpotent with $D \in \delta(B_0)$ abelian, the result follows by [2, Theorem 13.5.11, Theorem 13.5.16]. However by Proposition 1 above, for all $\chi \in Irr(B)$, $[\chi(1_G)]_p \leq |D|$. Moreover by [17, Corollary 3.6.38] if $d(B) > 0$, then $k(B) > l(B)$. For $G$ and $D \in \delta(B_0)$ nonabelian, $B_0 \in Bl(G)$ will possess representations of strictly positive height and so by [5, Theorem 3], $k(B) < \frac{1}{2}p^{2d(B)-2}$. Hence from the proof of Proposition 5 above, the result follows immediately. □

**Corollary 2** Let $B \in Bl(G)$ be a block of $G$ containing a linear character and $D \in \delta(B)$ its defect group. Then $k(B) \leq |D|$.

**Proof.** The case $B_0 \in Bl(G)$ being the principal block of $G$ is addressed in Proposition 6 above. Otherwise $B \in Bl(G)$ will have the highest defect so that its defect group $D \in \delta(B) \in Syl_p(G)$ is a Sylow $p$-subgroup of $G$. Hence the result follows immediately from the proof of Proposition 6 above. □

**Remark 1** We have from Proposition 6 and Corollary 2 above that $D \in \delta(B)$ a defect group of $B \in Bl(G)$ a block of $G$ is such that $D \in Syl_p(G)$ is a Sylow $p$-subgroup of $G$. According to [27], a finite group $G$ such that every block of $G$ has the highest defect is said to be **full defective** e.g. abelian groups are full defective. Thus for $G$ a full defective group, we have by Proposition 6, Corollary 2 above for all $B \in Bl(G)$ a block of $G$ with defect group $D \in \delta(B)$, that $k(B) \leq |D|$.

We now prove in the proposition following here below the main result in this paper that the number of irreducible ordinary characters in any block of a finite group $G$ is at most the order of a defect group of that block.

**Proposition 7** Let $G$ be a finite group. Then for all blocks $B \in Bl(G)$ of $G$ with defect group $D \in \delta(B)$, $k(B) \leq |D|$.

**Proof.** According to [28, Theorem 64.3], we obtain that $k(B) \leq p^{2d(B)}$. If all $\chi \in Irr(B)$ have the same degree, then the result follows by [2, Theorem 13.5.15, Theorem 13.5.16]. By [2, Proposition 13.5.5], we have that $k(B) \leq |D|l(B)$ and so for $l(B) = 2$, the result follows by [2, Theorem 13.5.7]. When $d(B) = 0$, the result follows by [9, Theorem 3.18]. For $d(B) = 1$, we obtain by Proposition 2 above (cf. [5]) that $k(B) \leq p = |D|$ and so the result follows. Moreover for $d(B) \in \{0, 1, 2\}$, the result follows by [5, Theorem 1*], Lemma 1 above. If $B \in Bl(G)$ contains a unique irreducible Brauer character, then the result follows by [6, Corollary IV.4.21]. Since the result holds for $d(B) \in \{0, 1, 2\}$, supposing that $k(B) \leq p^k$ for some $k \in \mathbb{N}$, from this induction hypothesis we obtain that $k(B) \leq p^{k+1}$ so that induction gives the desired result and the proof is complete. □

It can be proven by induction that actually $p^{d(B)} \leq p^{[d(B)(d(B)+1)]/2}$.

## 4. Concluding remarks

For any finite group $G$, any block $B \in Bl(G)$ of $G$ with defect group $D \in \delta(B)$ and defect $d(B)$, it is indeed true that $k(B) \leq |D| = p^{d(B)}$ and this is immediately true in the case of blocks of defect 0. Thus Brauer's $k(B)$-conjecture is in fact a further improvement on Proposition 2 above, [2 (Proposition 13.5.5), 28 (Theorem 62.2)] etc.

After the completion of the classification of all finite simple groups, among aspects of studying finite groups, is the study of blocks of finite groups. In that pursuit therefore, this result will indeed enhance the body of knowledge in the study of blocks of finite groups and ultimately contribute to the overall study of finite groups whose ultimate goal is to classify finite groups.

## Conflict of interest

The author hereby declares that there is no conflict of interest.

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