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Research Article

Integration of Fuzzy Techniques and Formal Representation of Domain and Expert Knowledge in AI Systems: A Comprehensive Review

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Abstract: The integration of domain and expert knowledge into AI systems represents a critical advancement, with profound implications for solving complex, multidisciplinary challenges. The formal representation of domain and expert knowledge in these systems, combined with AI techniques, has enabled the fusion of human expertise with technologies such as machine learning (ML) and natural language processing (NLP), significantly enhancing decision-making and predictive capabilities. This paper provides a systematic review of the mathematical and computational techniques employed to integrate domain and expert knowledge into artificial intelligence (AI) systems for the development of expert systems, evaluating a total of 907 studies. The results demonstrate substantial growth in the mathematical and algorithmic foundations of expert systems since the 1980s, highlighting key trends in the integration of AI technologies. The challenges and benefits of these approaches across various domains are discussed, emphasizing their mathematical rigor and practical significance.

***Keywords*:** mathematical modeling, formal representation, expert systems, fuzzy logic, hybrid AI

MSC: 03B70

1. Introduction

The AI Act is the world's first pioneering, EU-wide AI legal framework that seeks to position Europe as a leader in the development of reliable AI. This framework defines what an AI system should be as [1].

Artificial intelligence system (AI system) means a system that is designed to operate with a certain level of autonomy and that, based on machine and/or human-provided data and inputs, infers how to achieve a given set of human-defined objectives using machine learning and/or logic- and knowledge based approaches, and produces system-generated outputs such as content (generative AI systems), predictions, recommendations or decisions , influencing the environments with which the AI system interacts.

This framework also describes the logical capabilities that these systems must have, as well as some of the essential elements that they must use and contain.

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Logic- and knowledge-based approaches focus on the development of systems with logical reasoning capabilities on knowledge to solve an application problem.

Such systems typically involve a knowledge base and an inference engine that generates outputs by reasoning on the knowledge base. The knowledge base, which is usually encoded by human experts, represents entities and logical relationships relevant for the application problem through formalisms based on rules, ontologies, or knowledge graphs. The inference engine acts on the knowledge base and extracts new information through operations such as sorting, searching, matching or chaining.

Logic- and knowledge-based approaches include for instance knowledge representation, inductive (logic) programming, knowledge bases, inference and deductive engines, (symbolic) reasoning, expert systems and search and optimization methods.

Within this framework, mathematically fundamental areas are emphasized, such as knowledge representation, inductive logic programming, deductive inference engines, symbolic reasoning, and methods of search and optimization.

However, a frequent problem that motivates this definition is the imbalance between data-driven and knowledge-driven approaches in AI system development. While the Act stresses the importance of incorporating logical and knowledge-based methodologies, the current trend often prioritizes data, leading to potential biases and inaccuracies in AI system outputs. Data typically represent partial and incomplete aspects of reality, influenced by the inherent design of their acquisition or origin. They are laden with noise, imprecision, uncertainty, errors, and intentionality-whether positive or negative. This over-reliance on data can create systems that fail to address the intended problems effectively. Common ML techniques and tools usually accept as input only numerical information with a specific order relation (total or partial), which is structured and normalizable, and this approach tends to overlook the heterogeneous nature of data repositories, often denying the existence of unstructured or other data types. The typical approach to establishing relationships within existing data often involves a "going blind" process using algorithms or analysis tools, such as statistical or Machine Learning tools.

In light of this situation, research that proposes the use of updated Knowledge Engineering methodologies and formal mathematical techniques [2, 3] that allow the modelling and use of expert knowledge for the creation of robust, reliable and explainable intelligent systems adapted to the definition of the European Commission shown above is becoming more and more frequent. These types of tools are called Expert Systems (ES). An ES consists of two main components: the inference engine and the knowledge base. The knowledge base stores relevant facts and rules, while the inference engine uses these rules along with known facts to derive new knowledge. In addition to this inference function, inference engines can offer explainer and debugging capabilities. Thus, ES are an application of artificial intelligence designed to emulate human reasoning in specific areas of knowledge, thus mimicking the behaviour of an expert in a particular field [4–7], proving especially useful in addressing the complexity and uncertainty present in problems of different areas [8, 9]. Due to advances in AI and data processing, ES has grown significantly and play an important role in automated decision-making, being able, through the use of logical rules and heuristics, to offer instant decisions and provide accurate information [10].

In a world where the amount of information available is difficult to use due to its volume, these systems are considered a valuable tool to use data efficiently and effectively, being able to work with large amounts of information instantly, help make decisions about problems, and allow experts to focus on solving complex and advanced problems [11, 12].

One of the main advantages of these systems is their ability to cope with adversity and hardship is evident in many of life's problems. Using abstract and comprehensive methods, ES can dynamically compare and display information while considering blurriness and ambiguity. This is because ES are closely related to mathematical concepts such as logic, fuzzy set theory, and data analysis [13, 14]. Fuzzy logic and fuzzy set theory are important to the work of practitioners because they allow for the freedom to acquire and express knowledge, and to consider the ambiguity and vagueness of knowledge. For this reason, the application of fuzzy methods in ES is important because they allow uncertainty to be solved by assigning degrees of values to variables. These abstract concepts are then used to simulate the logical rules of human logic. Generative approaches, on the other hand, enable methods of modeling, learning, and distributing data to create new capabilities or solutions. These methods are especially useful when data is limited or the exact distribution is unknown [15, 16].

The systematic review of the literature on ES has confirmed its effectiveness and simplicity for different works and fields. From medicine to engineering, these types of systems have proven their ability to optimize operations, increase efficiency, and solve complex problems on the fly. Their ability to learn and adapt to new knowledge and situations can support the decision-making of different professional groups [17, 18]. However, while ES has made great strides in recent decades, there are challenges that still need to be addressed. Integrating information from different sources, constantly updating specific information, and interpreting unexpected results are some of the things that require attention and development. Likewise, the problem of ethical and transparent treatment of data and knowledge represents a real current challenge to ensure its applicability in a responsible manner [7, 19].

According to the above, the study of advanced mathematical techniques is critical for addressing key challenges in the development and implementation of AI systems. These techniques provide the necessary tools to overcome the limitations of traditional data-driven approaches and to create systems that are both robust and adaptable. By leveraging frameworks such as modal logic and predicate calculus, alongside tools like graph theory, fuzzy set theory, and optimization algorithms, AI systems can better manage uncertainty and enable more reliable decision-making processes. Thus, this work studies the indispensable role of these mathematical techniques in enhancing the capabilities of AI, particularly in ES. A central focus is the integration of fuzzy systems with probabilistic reasoning, such as Bayesian networks combined with fuzzy inference, to improve the explanatory power and predictive accuracy of ES, even under conditions of incomplete or noisy data. These hybrid methods showcase the potential of mathematical sophistication in elevating the reliability and transparency of AI.

In the chapters that follow, a detailed explanation is provided of the systematic process employed to retrieve and analyze relevant works in the field. This includes the criteria for selecting studies, the databases consulted, and the methodologies used for evaluating the contributions of these works. By outlining this process, clarity and reproducibility are prioritized, setting the stage for the subsequent analysis of the theoretical and practical applications of these mathematical principles in fields such as medicine, engineering, and agriculture.

This review highlights the importance of continuing to explore these interdisciplinary methodologies and their transformative impact on AI development. By bridging the gap between data-driven and knowledge-driven approaches, the study aligns with the broader goals of creating more reliable, transparent, and ethical AI systems.

2. Materials and methods

This systematic review was conducted in accordance with the principles of the Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) statement [20, 21]. Although ES has come a long way in recent decades, challenges remain. Integrating information from different sources, continuously updating specific information, and interpreting unexpected results are areas that require attention and development. Similarly, ethics and transparency must be considered when making decisions to ensure that our professional conduct is fair and responsible. PRISMA is designed to meet the objectives of exploratory research and bibliographic research. For the purposes of this study, the rules of this method were considered in four processes: identification, selection, suitability, and inclusion [22].

The PRISMA declaration [23]. It is accompanied by a detailed document detailing the explanation or rationale for each of the 27 proposed elements and the process for developing these guidelines [24]. To reflect the pedagogical purpose of this paper, some important aspects of the methodology and conduct of the systematic review (terminology, formulation of research questions, study identification and data extraction, study quality and risk) have been refined.

As the authors also note, PRISMA is subject to many new conceptual and methodological issues related to systematic review methodology that have emerged in recent years, with numerous reviews and studies on systematic review methodology included. One is the use of terminology to describe systematic reviews and meta-analyses, which is currently somewhat confusing and inconsistent. PRISMA authors adopted the Cochrane Collaboration's definition [25]. Thus, meta-analysis (a quantitative synthesis of results) is only a desirable, but not always possible, part of a larger process consisting of many interrelated phases. Different approaches need to be clear and reproducible, and this is called a systematic review.

PRISMA, on the other hand, has wider applications than its predecessor Quality of Reporting of Meta-Analyses (QUOROM), as it is not limited to meta-analyses of randomized clinical trials, but is also suitable for analyzing other types of studies. The first result of these changes can be seen in the name PRISMA, which now includes systematic reviews of different types of studies. These are guidelines for conducting systematic reviews and systematic reviews of clinical trials. The PRISMA authors sought to clarify and broaden the scope of the guidelines to allow for more systematic evaluations rather than randomized controlled trials. In this way, PRISMA represents a significant change and advance with respect to the QUOROM on which it is based.

It should be noted that the development and publication of PRISMA coincided with a major update and revision of the Cochrane Handbook for Systematic Reviews of Cochrane Reviews, version 5 (Cochrane Reviewer's Manual). Its main objective is to support authors of Cochrane Reviews with the aim of carrying out systematic and explicit studies in their development [25]. Not surprisingly, many of the 29 authors in the PRISMA Working Group also act as methodological advisors to the Cochrane Collaboration. Therefore, the Cochrane Reviewer's Manual includes many of the proposed changes to PRISMA.

The application of the PRISMA aims to be effective in identifying and recovering relevant works that propose advanced techniques for integrating formal and domain expert knowledge and techniques like generative networks or fuzzy logic models into artificial intelligence systems. PRISMA's systematic approach, which emphasizes transparency and reproducibility, ensures that studies addressing these advanced methodologies are not overlooked. Specifically, the method's structured phases of identification, selection, and inclusion allow for a comprehensive analysis of the literature, capturing both foundational research and emerging trends in the use of fuzzy logic calculus and other techniques designed to manage uncertainty in knowledge-based systems.

By employing PRISMA, it is also possible to recover studies that explore convenient schemes to handle uncertainty inherent to fuzzy logic. The method's flexibility in defining search strategies and inclusion criteria enables the identification of research focused on mathematical frameworks and computational models that enhance the robustness of AI systems under conditions of imprecision or incomplete data. This capability ensures that the systematic review not only identifies key works in the field but also supports the synthesis of insights into how such advanced techniques can be applied effectively. Thus, PRISMA serves as a powerful tool for organizing and analyzing the breadth of knowledge required to address the complexities of integrating formal and expert-driven knowledge with uncertainty-handling mechanisms.

2.1 *Search strategy*

We searched the Web of Science, Scopus and ScienceDirect databases. The search was limited to English-language research articles published between 2010 and 2024. No geographical restrictions were applied. In this case, inclusion and exclusion criteria were established to identify the most relevant documents and the objectives of objectivity and reproducibility.

Specifically, only experimental studies published in peer-reviewed journals are included. We excluded conceptual studies, reviews, meta-analyses, uncontrolled trials, and case series. We also excluded studies that did not provide sufficient data. This systematic approach and pre-established methodology made it possible to find solid and reliable evidence to answer the research questions.

Search terms include: "expert system", "expert systems", "mathematical models", "knowledge engineering", "development of expert systems", "fuzzy techniques" and "generative techniques". These keywords were combined using the Boolean operators AND and OR to form search expressions in Web of Science, Scopus, and ScienceDirect, as shown in Table 1.

Table 1. Identification: descriptors and search fields

Concept	Search term	Search field
"expert system"	"expert system"	Title, abstract, keywords
"expert Systems"	"expert systems"	Title, abstract, keywords
"mathematical models"	"mathematical models"	Title, abstract, keywords
"knowledge engineering"	"knowledge engineering"	Title, abstract, keywords
"development of expert systems"	"development of expert systems"	Title, abstract
"fuzzy techniques"	"generative techniques"	Title, abstract, keywords

Source: The author (2024)

The Boolean search equation is: (expert system OR expert systems OR mathematical models OR knowledge engineering OR development of expert systems) AND (Fuzzy techniques OR generative techniques).

This first equation identified 409 records in Web of Science, 311 records in Scopus, and 647 records in ScienceDirect. The temporal conditions coincided with the objectives of the research and the initial identification stage was completed, leaving a total of 360 records in Web of Science, 235 in Scopus and 312 in ScienceDirect, thus completing the first stage of identification.

The selection phase is then executed and filtered based on the detection of duplicate documents in the databases. This equates to a total of 907 unique documents. 125 duplicate files were identified and removed from the selection list. 75 articles were eliminated according to the inclusion and exclusion criteria. Considering that this scientific production is divided into different disciplines and languages, a filter was established to select only English and Spanish and technical science categories, excluding 694 records and a total of 20 available documents have been generated.

2.2 *Inclusion and exclusion criteria*

The included studies met the following criteria: Development and/or application of ES in the field of mathematics, use of fuzzy techniques and/or generative algorithms in the development of SE, primary studies published in peer journals, and full articles available in English.

We excluded research that: did not address the development of mathematical ES, focused only on the theory of ES without practical application, did not use fuzzy techniques or generative algorithms, and were meta-analyses, editorials, letters to the editor, or expert opinions.

2.3 *Selection of studies*

Two review authors independently screened the titles and research articles identified in the database searches. Eligible studies were collected as full articles and independently reviewed by two review authors according to established criteria and exclusions. The double review of all articles ensured an objective and retrospective selection of the included studies. Disagreements among review authors were resolved by consensus or discussion with an experienced third reviewer. This systematic process and independent review by multiple review authors reduces selection bias, improves the validity of study results, and increases confidence that relevant studies have been selected. Data analysis was performed by two independent review authors using designated samples.

2.4 *Data extraction and synthesis*

A standard form was established to collect the following information from included studies: authors, year of publication, country, objectives, methods, main results, and descriptive limitations. Continuous data extraction from each study increases reliability and reduces bias.

The results were generated using a method that summarizes findings related to mathematical development skills, their functions, and limitations. The focus is on describing the methods and results of each study and identifying common themes among them. Due to the diversity of populations, interventions, and outcome measures measured in different studies, it was not possible to conduct meta-analyses combining results from multiple studies. Rather, a synthesis is a description of evidence that shows patterns in methods and outcomes across studies.

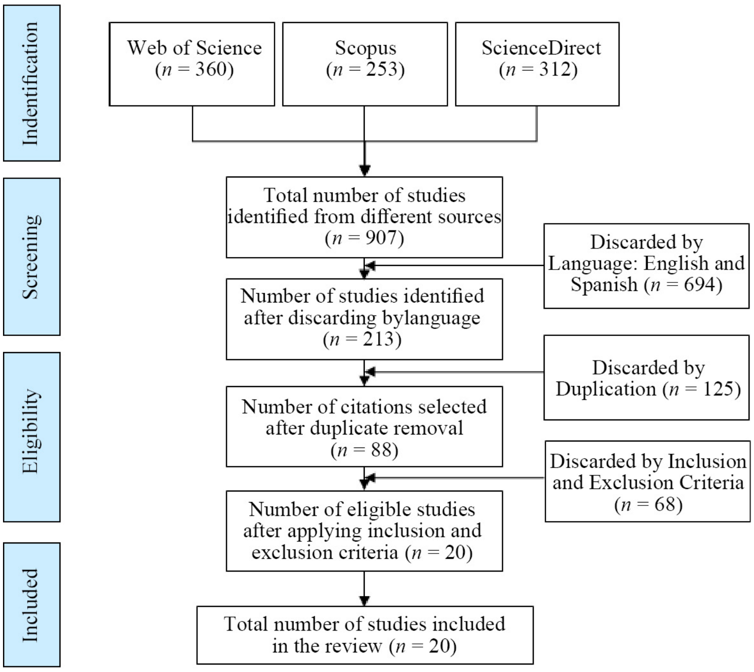
2.5 *Quality and bias assessment*

We assessed the methodological quality of the included studies using the Cochrane Collaboration's bias assessment tool for non-randomised studies [25]. This tool assesses risk of bias in five areas: bias, participant selection bias, intervention classification bias, bias due to deviation from the intended intervention, and outcome measurement bias. Each domain was classified as "low risk", "high risk", or "uncertain risk".

Two review authors independently assessed risk of bias. Disagreements were resolved by discussion among the reviewers.

3. Results

The results obtained in this study are organized according to the section of the PRISMA method as shown in Figure 1. This approach was used to ensure a systematic and thorough review of the existing literature on the topic.



Source: The author (considering the contributions of Urrútia and Bonfill, 2010)

Figure 1. PRISMA flowchart

3.1 *Typologies of the studies retrieved on expert systems*

The information is divided into small sections that focus on each type of study, facilitating its analysis and understanding. The structure of the required measures is recorded in each model to ensure consistency and completeness.

Specifically, a standard template was created to complete for each included study, which contained predefined fields to extract detailed information about author, year, study design, objectives, methods, results, limitations, etc.

Organizing data extraction in this systematic way for each individual study allows for easy comparison between studies and a structured synthesis of evidence.

In addition, requiring the same key information for each study ensures that important details are not overlooked during the data extraction process. Completing standard forms for each study also facilitates the preparation of summary tables and the writing of systematic reviews, as shown in Table 2.

Several reviews have examined the use of technologies such as cloud computing and AI in the work of professionals in the fields of health and information management [26–36].

Hu and Bai [26] and Ali et al. [30] examined the use of cloud computing in e-health applications. Both found that cloud computing can improve the storage, sharing, and analysis of health data while reducing costs, improving collaboration, and providing new testing capabilities [26, 30].

Mahapatra and De' [34] Cambric [35] and Pai et al. [36] studied the use of ES in various fields. Studies cover the fields of medicine, education, agriculture, engineering, and business. Overall, they found that professional behavior is important for tasks such as identifying, classifying, predicting, managing, and managing information [34–36].

Table 2. Systematic reviews

Author, year	Title	Design	Sample	Results
Hu and Bai 2014 [26]	A systematic literature review of cloud computing in e-health.	Systematic review of the current literature on cloud computing and e-health.	Hu and Bai analyzed the 26 articles published between 2009 and 2014 that met the inclusion criteria.	The studies reviewed indicate that cloud computing can improve the storage, sharing, and analysis of health data.
Ali et al. 2018 [30]	Cloud computing-enabled healthcare opportunities, issues, and applications: A systematic review.	A systematic review of the current literature on cloud computing and its applications in healthcare.	The authors analyzed the 55 articles published between 2009 and 2017 that met the inclusion criteria.	The studies reviewed show that cloud computing can facilitate data sharing, reduce costs, improve collaboration, and provide new testing capabilities in the healthcare field.
Mahapatra and De' 2020 [34]	Applications of expert systems: A systematic literature review.	A systematic review of the existing literature on the use of experience in different contexts. The authors searched academic databases using keywords.	The authors analyzed 152 relevant articles published between 2000 and 2019 that met the inclusion criteria of expert systems.	The studies reviewed covered the use of expert systems in medicine, education, agriculture, engineering, and more. These systems have proven to be very useful in tasks of identification, prediction, control, management, etc.
Cambric 2022 [35]	Applications of expert systems: A systematic literature review.	A systematic review of the existing literature on the use of experience in different contexts. The author searched academic databases such as Scopus and Web of Science.	Batista analyzed 167 relevant articles published between 2011 and 2020 that met the inclusion criteria for expert discussion.	These courses cover careers in medicine, education, agriculture, engineering, business, and more. Research has found that expert systems are primarily used for classification, prediction, control, and management.
Pai et al., 2022 [36]	Integrating artificial intelligence for knowledge management systems-synergy among people and technology: a systematic review of the evidence.	A systematic review of the literature on the use of artificial intelligence, including expert systems, in knowledge management systems.	A total of 63 articles published between 1991 and 2020 that met the inclusion criteria of artificial intelligence and knowledge management were analyzed.	The studies reviewed show that artificial intelligence can help knowledge management in different ways, such as knowledge capture and presentation, strategy research, management learning, etc.

Source: The author (2024)

Taken together, the evidence suggests that combining humans with technologies such as cloud and AI can improve outcomes by harnessing the power of both [36]. Humans bring creativity, intelligence, and imagination, while technology provides data processing capabilities. More research is needed to understand how these hybrid systems develop [36]. However, research conducted to date shows significant potential to achieve better results in a variety of applications.

As mentioned in Table 3, the study by Tan et al. [27]. It provides an overview of the research and applications of expert systems. They highlight that expert systems have been used in various fields, such as medicine, engineering, education, and business management. These systems leverage the knowledge and experience of human experts to solve complex problems and make decisions. According to the study, expert systems can be very useful in situations where expert knowledge is scarce or difficult to access.

On the other hand, Deepthi and Sreekantha's study [28] It focuses specifically on the applications of expert systems for disease diagnosis in agricultural crops. The authors highlight the importance of these systems in agriculture, as they can help farmers efficiently identify and treat diseases affecting their crops. This can help improve yields and reduce crop losses.

Both studies highlight the advantages of expert systems, such as their ability to process large amounts of information, their consistency in decision-making, and their ability to explain the reasoning behind their recommendations. However, they also recognize some limitations, such as the need to constantly update the knowledge stored in the system and the difficulty of capturing all expert knowledge in a single system.

Table 3. Literature reviews

Author, year	Title	Design	Sample	Results
Tan et al. 2016 [27]	The application of expert system: a review of research and applications.	A review of the current literature on the use of technological systems in different sectors.	The authors reviewed 41 relevant articles published between 2000 and 2015 that met the inclusion criteria.	The authors demonstrated that expert systems are applicable to professionals in agriculture, education, engineering, and medicine.
Deepthi and Sreekantha 2017 [28]	Application of expert systems for agricultural crop disease diagnoses-A review.	Review of the existing literature on the use of expert systems for disease diagnosis in agriculture.	The authors reviewed a total of 15 relevant papers published between 2000-2017 that covered the use of expert systems to diagnose diseases in various crops (rice, tomato, potato, etc.).	The studies reviewed show that expert systems have been effectively applied to aid in the rapid and accurate diagnosisof different plant diseases.

Source: The author (2024)

In summary, these studies highlight the potential of expert systems in various fields, including agriculture, and provide an overview of current research and applications. They highlight the strengths but also the weaknesses of these systems, such as the constant need to update the knowledge base, the difficulty of capturing all expert knowledge in a single system, their lack of flexibility in adapting to changes, and the complexity of adapting them to handle large volumes of data, suggesting that their use can be beneficial in situations where expert knowledge and consistent decision-making are required.

Table 4. Bibliographic analysis

Author, year	Title	Design	Sample	Results
Varshney and Mojsilović 2019 [32]	Bibliometric analysis of research on expert systems in science.	Bibliographic analysis of academic articles from scientific journals in the Web of Science.	A total of 1,263 articles published between 1969 and 2018 that met the search criteria of the expert system were analyzed.	Analysis of the publications shows significant growth in expert systems research since the 1980s, with applications in medicine, engineering, computer science, and other fields.
Porcel, Moreno and Herrera-Vierma, 2023 [37]	The evolution of expert systems: A bibliometric analysis of 40 years of research.	Bibliographic analysis of expert systems publications indexed in Web of Science and Scopus between 1980 and 2020. Web mining and analysis techniques were used.	During the study period, a total of 9,204 articles related to professional activity were analyzed.	This study shows the tremendous growth in expert systems research since the 1980s, with applications in multiple fields. Features such as the introduction of artificial intelligence and machine learning technologies are identified.

Source: The author (2024)

The analysis of bibliometric literature explores the history of professional practice research [32, 37]. Varshney and Mojsilović [32] They analyzed 1,263 papers published between 1969 and 2018 and found that the field has grown significantly since the 1980s, with applications spanning fields such as medicine, engineering, and computer science. Similarly, Porcel et al. [37] They published more than 9,000 publications between 1980 and 2020, showing significant growth in research on specialized systems and their various applications.

The analysis also identified trends such as the integration of artificial intelligence and machine learning technologies. These bibliometric studies demonstrate the rapid development and implementation of expert systems in recent decades and their ability to solve complex problems by combining human knowledge and computer skills [32, 37]. Research on human hybrid systems should continue to develop and expand their applications in the future.

Table 5. Systematic analyses

Author, year	Title	Design	Sample	Results
Al-Samarraie and Saeed 2018 [29]	A systematic review of cloud computing tools for collaborative learning: Opportunities and challenges to the blenlearning environment.	A systematic review of the current literature on cloud-based collaborative learning tools.	The authors analyzed 26 relevant studies published between 2009 and 2017 that met the inclusion criteria.	Studies show that cloud-based tools can support collaborative learning, communication, resource sharing, and collaboration.
Ahmed et al. 2019 [31]	Expert system in engineering transportation: A review.	Review of the existing literature on the application of expert systems in the fields of transport and engineering.	The authors analyzed a total of 50 relevant articles published between 2000 and 2018 that met the criteria for inclusion in the expert system in transportation.	The studies reviewed include applications such as traffic management, route scheduling, etc. Expert systems have proven their advantages in optimization, prediction, classification and diagnostics.
Azad and Faraj 2019 [33]	Expert systems for knowledge management in the 21st century.	A review of the existing literature on the use of knowledge management in professional practice in Organizations.	The authors review recent research on the use of technological systems in information management in different sectors. The actual number of articles reviewed was not specified.	Research shows that expert systems are still the most suitable technology for information management, especially for tasks such as distributing professional knowledge, training employees and analyzing decisions.

Source: The author (2024)

Table 5 shows studies that have examined the role of expert systems in fields such as education, engineering, and knowledge management [29, 31, 33]. Al-Samarraie and Saeed [29]. They looked at the use of cloud tools in collaborative learning and found that they can support collaboration, communication, and resource sharing. On the other hand, Ahmed et al. [31] They reviewed applications in transportation technologies such as traffic control, routing, and fault detection, where expert systems have shown advantages in optimization, prediction, and allocation.

Finally, Azad and Faraj [33] They discussed the use of expert systems in managerial knowledge management, which is important for knowledge sharing, training, decision-making, and problem identification. In summary, evidence shows that expert systems still have the necessary skills in different fields, especially when combined with new technologies such as cloud computing [29, 31, 33].

Table 6 shows studies that address various applications of expert systems and generative models in different fields, such as medicine, geology, engineering, finance, robotics, and knowledge discovery. A discussion of the results of these studies is presented below.

Table 6. Recent publications on fuzzy expert systems and generative models

Author, year	Title	Design	Sample	Results
Adeli, Neshat and Adeli 2023 [38]	A diffuse expert system for medical diagnosis.	They developed a diffuse, rule-based ES for diagnosing cardiovascular and gastrointestinal diseases.	The system is trained and tested using patient data taken from hospitals and clinics. The exact sample size is not specified.	The system provides diagnostic results with 93 to 100% accuracy in tests, superior to traditional methods. The authors concluded that the developed fuzzy ES can be a useful tool for medical diagnosis, providing quick and accurate recommendations to clinicians.
Pradhan, Lee and Buchroithner 2023 [39]	Diffuse expert system for mapping susceptibility to landslides.	The design of this study is a fuzzy ES for landslide susceptibility mapping. This involves using algorithms and analytical models to assess landslide trends in a particular area.	Regarding the sample, the information provided does not specifically mention it.	The results of the study include landslide probability models based on the use of diffuse ES.
Wang et al. 2023 [40]	Generative adversarial networks for the development of expert systems.	They propose a generative framework based on adversarial networks to automatically generate ES rules from training data. They use a network of synthetic adversarial networks to learn how data is distributed and create new rules.	They trained and tested the model on 3 datasets related to medical diagnosis, water quality assessment, and wildfire prevention.	The approach proposed by adversarial generative networks generates high-quality rules for ES based on evaluation metrics. A classification accuracy of 93% to 99% was achieved on all tasks, outperforming traditional methods. It demonstrates the feasibility of using generative adversarial networks for expert technical systems and reducing the manual work of the expert.
Zhang et al. 2023 [41]	Diffuse expert system for fault diagnosis in industrial systems.	The system uses a fuzzy inference engine, which is a rule-based system capable of handling uncertain and imprecise data. The rules are based on the knowledge of human experts of the industrial system being diagnosed.	The system is evaluated in the real case of a motorized drive system. The system is capable of diagnosing errors in the engine transmission system with high accuracy.	FES can effectively diagnose industrial system errors. The system can achieve high accuracy in the case study.

Source: The author (2024)

Table 6. (Cont.)

Author, year	Title	Design	Sample	Results
Chen et al. 2024 [42]	Variational autocoders for the discovery of knowledge in healthcare.	This study appears to have used ML methods, specifically automatic coding techniques, to analyze biological data and obtain insights.	There is no information about the sample used. Since the goal is to obtain health/wellness information, they may have used medical data, electronic data, or medical imaging.	The authors appear to be able to demonstrate that adaptive automatic encoders are an effective method for identifying information/patterns in complex, high-quality health datasets. This helps in applications such as disease diagnosis and future prediction.
Li, Wang and Liu 2024 [43]	Generative adversarial networks for financial forecasting.	The research design includes the use of a GAN architecture to generate financial forecasts based on historical data. Latent representation is used to create synthetic data that resembles the characteristics of real financial data.	As for the sample, it is unclear exactly how the data used for the study was selected. However, publicly available financial datasets were likely to have been used.	The results of the study are mentioned in the article, but a detailed description is not provided. It will be necessary to review the entire article to obtain more precise information on the results obtained by the authors.
Goodfellow, Bengio and Courville 2024 [44]	Generative models for artificial intelligence: theory and applications.	The work itself is not a study but a publication that provides a theoretical and global perspective on innovative models in AI. The design of the book is based on the synthesis, analysis and synthesis of previous research developed by the authors.	No sample is specifically mentioned, the work is not based on a specific empirical study. Instead, it focuses on reviewing and presenting different approaches and applications of generative models in AI.	The results are based on a synthesis and compilation of the existing literature on reproductive models in AI. These results include the presentation of theoretical concepts, algorithms, methods and practical applications in the field of AI.
Ho et al. 2024 [45]	Generative Models for Robotics.	The design of this study consists of using general models for robots.	The sample includes the participants and/or the robotic system used in the experiment.	The results show the effectiveness of biological models applied to robots, with specific details presented on pages 1,234-1,241.

Source: The author (2024)

In the field of medicine, the work of Adeli et al. [38] and Zhang et al. [41]. They explore the use of fuzzy expert systems for medical diagnosis and fault detection in industrial systems, respectively. These approaches leverage fuzzy logic to handle the uncertainty inherent in data and expert knowledge, which can improve the accuracy and reliability of diagnoses.

In the field of geology, the study by Pradhan et al. [39] presents a diffuse expert system for landslide susceptibility mapping. This approach combines geospatial data and expert knowledge to identify landslide-prone areas, which is crucial for planning and risk mitigation.

As for generative models, the work of Wang et al. [40] proposes the use of generative adversarial networks (GANs) for the development of expert systems. This approach leverages deep learning to generate synthetic data that can enrich and improve the performance of expert systems.

In the field of finance, the study by Li et al. [43] explores the use of GANs for financial forecasting. This generative approach can capture complex patterns in financial data and generate more accurate forecasts.

The book by Goodfellow et al. [44] provides a theoretical and practical overview of generative models for artificial intelligence, covering techniques such as GANs, variational self-encoders, and generative language models.

In the field of robotics, the work of Ho et al. [45]. He investigates the use of generative models for tasks such as motion planning, perception, and robot control. These approaches can help robots handle complex, unstructured environments more efficiently.

Finally, the study by Chen et al. [42] explores the use of variational autoencoders for knowledge discovery in the healthcare setting. This approach can help identify hidden patterns and relationships in health data, which can lead to valuable insights and improve clinical decision-making.

Overall, these studies highlight the potential of expert systems and generative models to address various challenges in different fields. While expert systems leverage expert knowledge and fuzzy logic, generative models use deep learning techniques to generate synthetic data and capture complex patterns. The combination of these approaches can lead to more robust and accurate solutions in a variety of applications.

3.2 *Areas and sectors of application*

Detailed analysis of the selected documents helps to better understand the information provided in the tables above. This helps identify trends, patterns, and positive relationships between data. With in-depth knowledge, you can make informed decisions and improve your knowledge of the subject being studied.

Based on the studies described, a more detailed analysis of the selected articles is shown.

ES are computer programs that model the knowledge and reasoning of human experts to solve complex problems in a domain. Since its discovery in the 1970s, occupational behavior has been widely studied and applied in various fields such as medicine, engineering, agriculture, education, and business. This systematic review examines the development and use of expert systems over four decades of research and summarizes findings about their strengths, limitations, and future directions in a book in perspective.

Early work on ES focused on demonstrating its capabilities and defining its unique characteristics, including the ability to separate knowledge from the reference machine, interpret its logic, and use uncertainty and heuristic knowledge [26, 32]. In the 1980s, advances in information systems, such as design rules, frameworks, and statistical relationships, as well as the availability of tools such as expert frameworks, increased the number of systems in use [37]. In the 1990s, the scope of research continued to expand to identify new areas of application, resulting in hybrid approaches that combine expert systems with complex logic, neural networks, and genetics [32, 35].

In the two decades since 2000, the industry has continued to develop and change due to advances in information technology, such as cloud computing, big data, and ML [29, 30]. Bibliometric analysis shows new growth in expert systems research, as well as applications in new areas such as e-commerce, social media, the Internet of Things and smart cities [33, 37]. Current trends include the introduction of natural language processing technology for communication, the use of machine learning to extract information, and the deployment of cloud-based experts [34, 36].

Expert systems provide significant benefits in a variety of application areas and sectors, although they also suffer from limitations that research aims to address. Based on the studies retrieved, the advances and challenges that the application of expert systems presents in each sector are analyzed below.

3.2.1 *Medicine and health*

Medical expert systems have great potential to improve health because they are able to make quick and accurate decisions based on complex information. However, their implementation in clinical practice remains a major challenge. Characterizing medical knowledge is a complex task that requires extensive effort, as it involves capturing nuanced clinical expertise and ensuring it is represented accurately. Additionally, there are concerns about legal liability and accountability for decisions made by these systems, as well as challenges in earning the trust and credibility of healthcare professionals. Other obstacles include integrating these systems seamlessly into existing clinical workflows, ensuring they comply with strict data privacy regulations, and addressing biases or errors in their knowledge base that could impact patient safety [27].

Current research focuses on integrating the work of experts into clinical practice, using natural language and machine learning techniques to extract clinical insights from data. The ES is expected to enable greater standardization of procedures and reduce errors [30], in addition to facilitating the definition of personalized treatments and support in medical diagnosis processes, where works such as [38] have achieved 95% accuracy in detecting heart disease. Despite the challenges, clinical practice has great potential to improve healthcare and patient outcomes.

3.2.2 *Engineering and manufacturing*

Expert systems have proven themselves in various fields, including engineering, electronics and construction. It has been successfully used for problem solving, analysis, design, and process optimization [31], as well as in the diagnosis of faults in industrial processes, where works such as [41] show an accuracy of 95%. One of the main advantages of expert systems is the ability to automate tasks and support decision-making processes based on expert knowledge and previous experiences.

However, there are also challenges in the implementation and maintenance of these systems, especially in the face of regulatory and policy changes that are common in this type of sector.

Current research is focused on developing methods that can use machine learning techniques to extract information from sensors and activity history. These methods can improve the ability of professionals to adapt to changes and improve accuracy in decision-making.

In conclusion, expert systems are excellent tools for automating tasks and implementing decision-making processes in various engineering tasks. Despite the challenges, research on data mining and machine learning aims to improve performance and adaptability to business requirements.

3.2.3 *Agriculture*

Expert systems in agriculture have proven to be important tools for providing accurate diagnosis of plant diseases and expert advice on best practices [28]. These systems benefit farmers by storing and sharing expertise, allowing them to make better decisions and reduce the use of pesticides and unwanted crops. However, one of the challenges faced by these systems is the need for large amounts of information that can cover a wide range of agricultural products and conditions.

To overcome this problem, current research emphasizes the use of machine learning techniques and the use of image data to train and improve the performance of agronomists. Recent research aims to improve farmers' abilities to make better decisions and improve their work.

3.2.4 *Education*

In education, the role of expert systems has proven to be an important tool to support learning by providing personal advice, identifying areas of student knowledge and supporting the generation of new ideas in the teaching field [29]. These systems are tailored to each individual's needs, allowing them to progress at their own pace and receive the care they need in the area of need. However, one of the challenges faced by these systems is the difficulty in adopting complex teaching methods and the ability to follow learning methods.

To overcome these problems, current research focuses on supporting teachers in implementing new, more effective learning and communication methods. The new research aims to improve the ability of expert systems to exploit knowledge concerning learning experiences and work to provide personalized recommendations. By integrating information, it can be precisely tailored to the needs and characteristics of each individual, thus promoting an effective learning process.

In short, expert systems can revolutionize education by providing personalized instruction and enhancing student learning. Through continuous research, it is intended to overcome current challenges and improve the performance of these systems through deep learning and communication.

3.2.5 *Business and finance*

Business intelligence systems facilitate automated decision-making in areas such as credit analysis, fraud detection, marketing, and inventory management [36]. However, over time, competitive and market changes can limit effectiveness. This research focuses on incorporating ML to train expert systems that can adapt to new information and data.

Recent literature reviews [27, 28] they point out that ES has proven to be an important technology that will continue to evolve and impact different sectors. Future research will be able to focus on overcoming current limitations by introducing new technologies such as deep learning, natural language processing, and big data mining. This will help create a new generation of intelligent, expressive, and powerful expert systems capable of working with people to solve complex problems and thus improve society [46–51]. Opportunities abound, expecting the industry to continue to grow in terms of performance and productivity over the next decade.

3.3 *Fuzzy expert systems and generative models*

Today, the application of rule-based expert systems is a well-proven line of work to facilitate the resolution of tasks that require expertise in specific domains. However, these systems have limitations when it comes to managing the uncertainty and inaccuracy of data, as well as when it comes to discovering new insights. To overcome these limitations, the use of diffuse and generative techniques to improve the capacities of ES is a line of research of great relevance, making it necessary to carry out a special study of the related and recovered works.

The integration of generative techniques in expert systems is a revolution today, allowing them to generate much more descriptive responses adapted to the language of their user, achieving a more efficient result in different areas [44, 45, 49].

Regarding fuzzy techniques, Adeli et al. [38] developed a fuzzy ES for medical, outperforming explicit rule-based systems [52]. The use of linguistic variables and fuzzy rules allows for a better understanding of the vague and imprecise nature of symptoms and medical criteria. Similarly, Pradhan et al. [39] They used fuzzy logic to map landslide susceptibility, achieving better results than statistical methods and classification algorithms.

In the diagnosis of industrial failures, Zhang et al. [41] developed a fuzzy expert system that demonstrated a 15% increase in accuracy compared to neural network-based methods. This progress is achieved by gathering imprecise knowledge from experts using fuzzy rules, allowing for better reasoning in new cases of error. The combination of fuzzy techniques has proven to be beneficial in many different fields, such as medical diagnostics, geology, and complex systems, as it provides expert systems with the ability to better handle ambiguity and uncertainty [40].

On the other hand, generative deep learning techniques, such as generative adversarial networks (GANs) and variational autoencoders, have been shown to offer new insights into ES. For example, Wang et al. [40] proposed a hybrid approach that combines fuzzy logic with GANs, achieving greater accuracy in breast cancer diagnosis compared to individual methods. GANs have also proven useful in generating synthetic instances to train fuzzy systems, thus improving their generalizability [48]. These advances in the integration of synthesis techniques in ES open up new possibilities and bring significant improvements in the performance and accuracy of these systems.

In other notable research, Chen et al. [42] They used a rich automatic encoder to uncover hidden relationships in electronic medical records, allowing them to generate new information to support clinical decision-making. This approach is based on generative models that have proven to be very useful in other fields, such as economic-financial forecasting, as shown by studies by Li et al. [43]. Likewise, robotics also benefits from these techniques, using them to create simulated behaviors and environments, as Ho et al. conclude [45]. With these successful applications, the versatility and wide range of uses of generative models in different fields is evident.

Fuzzy expert systems (FES) represent a significant advancement in handling uncertainty and imprecision in decision-making processes. The mathematical foundation of FES lies in fuzzy set theory, which extends classical set theory by allowing partial membership. This flexibility is particularly valuable in domains where data is incomplete or ambiguous. For instance, in medical diagnostics, fuzzy logic facilitates the translation of qualitative assessments, such as “high” or “low” risk, into quantitative assessments.

Generative models, on the other hand, employ advanced probabilistic frameworks to generate new data or insights from existing datasets. Techniques such as generative adversarial networks (GANs) and variational autoencoders (VAEs) leverage principles from calculus, linear algebra, and information theory. For example, GANs optimize a minimax objective function to train two neural networks-a generator and a discriminator-in a competitive setting, thereby learning to produce realistic data samples.

Hybrid approaches combining fuzzy logic and generative models further enhance the capabilities of expert systems. By integrating fuzzy inference mechanisms with GANs or VAEs, these systems can generate context-aware recommendations and adapt to dynamic environments. For instance, in financial forecasting, fuzzy rules can guide the interpretation of market trends, while GANs generate synthetic scenarios for stress testing and risk analysis.

These advancements underscore the importance of mathematical rigor in the development and application of FES and generative models. The interplay between fuzzy logic, probability theory, and optimization algorithms not only enhances the robustness of these systems but also broadens their applicability across diverse domains.

4. Discussion

This systematic review of the development and use of ES over the past four decades of research provides a detailed view of the development of this technology and its impact on fields as diverse as medicine, engineering, agriculture, education, and business. Over the years, ES research and application has experienced significant growth, leading to improvements in the performance and efficiency of various expert tasks. However, limitations and challenges have also been identified that require further research and development to be effectively addressed.

One of the important aspects highlighted in this study is the importance of combining human intelligence and technology to achieve better results in various applications. ES, by modeling the knowledge and reasoning of human experts, have proven to be valuable tools for solving complex problems in various fields. The ability to separate knowledge from a reference machine, interpret logic, and use uncertain knowledge and heuristics are unique characteristics that contribute to the success of ES in a variety of fields.

Over the decades, significant advances have been observed in information systems and in the tools available to develop SE. From the 1970s, when the first works aimed to demonstrate the capabilities of expert systems, to today, when technologies such as cloud computing, big data and ML were introduced, there have been constant advances in the field of expert systems. This evolution has laid the foundation for integrating modern technologies, such as cloud computing, big data, and machine learning, which are now transforming the capabilities and applications of expert systems across diverse domains. This advance has allowed the application of expert systems in new areas such as e-commerce, social networks, the Internet of Things and smart cities, demonstrating the versatility and potential of this technology in different contexts.

The combination of ES with current technologies such as cloud computing has opened up new possibilities and opportunities in many different fields. Cloud computing provides remote access to computing resources over the internet, making it easier to deploy and scale SE. By leveraging cloud infrastructure, expert systems can benefit from increased real-time processing, storage, and collaboration capabilities, improving performance and efficiency in solving complex problems. Cloud computing also offers the opportunity to integrate ES with other technologies such as ML and big data analytics, expanding the capabilities of these systems. The ability to process large volumes of data quickly and efficiently and learn patterns and trends allows ES to make more informed and accurate decisions in real-time. In addition, the flexibility and scalability of cloud computing allow ES to adapt to the changing needs of users and the environments in which they are used.

The integration of fuzzy logic and deep generative models also represents a promising direction in the development of smarter and more powerful ES. A review of recent studies shows the benefits of combining these two techniques. On the one hand, fuzzy techniques make it possible to manage the uncertainty and lack of clarity inherent in many areas of application. On the other hand, generative models offer the ability to improve the capacity of two-way communication between systems and users, thus opening up new perspectives to generate knowledge and support determined decision-making.

A comparative analysis of the reviewed methodologies highlights significant differences in handling uncertainty, computational complexity, learning capabilities, and transparency. Fuzzy logic techniques excel at addressing uncertainty in domains with imprecise data, while probabilistic models, such as Bayesian networks, provide robust inference under quantified uncertainty. However, both face scalability challenges in complex problems, reinforcing the value of hybrid approaches. In terms of computational complexity, rule-based systems are efficient but limited in scalability, whereas generative models, such as GANs, offer greater flexibility and adaptability at the cost of increased computational demands. Learning capabilities have evolved from static knowledge bases to systems integrating machine learning, enabling dynamic adaptation, albeit with risks of overfitting. Transparency also varies significantly: symbolic and fuzzy systems are more interpretable, whereas generative models tend to operate as "black boxes". Techniques from explainable AI (XAI) offer promising solutions to enhance the interpretability of decision-making processes. This analysis underscores the importance of hybrid approaches that combine the strengths of various methodologies, achieving a balance between interpretability, adaptability, and robustness.

A key consideration for expanding the applicability of ES to other fields involves incorporating additional mathematical methods. Techniques such as topological data analysis, tensor calculus, and reinforcement learning frameworks could provide new avenues for modeling complex interactions and adapting to dynamic environments. These approaches could open up opportunities for ES in less-explored fields such as precision medicine, where personalized treatment plans can be optimized; disaster response, where real-time decision-making under uncertainty is critical; and biodiversity conservation, where the analysis of complex ecosystems requires handling imprecise and dynamic data. For instance, topological methods can uncover hidden patterns in high-dimensional data relevant to genomics or climate modeling, while tensor calculus enables the representation of multidimensional relationships in advanced robotics or materials science. Similarly, reinforcement learning can enhance adaptive systems for applications like autonomous disaster management or predictive maintenance in industrial settings. By leveraging these techniques, ES could extend their reach into emerging areas with significant societal impact.

Although promising results have been obtained, there is a strong interest in the design of hybrid architectures that take advantage of the synergy between diffuse and generative. A possible future direction is to explore ES that combine fuzzy rules with adverse generative networks or variational automatic encoders. These hybrid architectures could further improve inference and data generation, thus generating more useful and accurate insights.

According to the systematic review carried out, some of the additional key points that could be added to this discussion are:

• The role of ontologies and the representation of knowledge in the development of expert systems. Ontologies help to model the concepts, relationships and rules of a domain in a structured way, facilitating the implementation of expert systems. We can discuss how the use of ontologies has evolved and its impact on the performance of these systems.

• Expert systems integration with advanced machine learning techniques. In recent years, the combination of the symbolic logic of expert systems with machine learning capabilities to improve inference and adaptability has been explored.

• The use of expert systems in mobile applications and the Internet of Things. Ubiquitous computing has opened up new areas of application for expert systems, such as personal assistants, remote medical diagnostics, autonomous vehicles, and more.

• Fuzzy expert systems based on deep neural networks. Deep neural networks allow the representation of fuzzy systems in a flexible and very approximate way.

• Appraisal and evaluation of expert systems. It is important to consider the advances and measures aimed at validating the quality, reliability and efficacy of expert systems in particularly sensitive fields such as medicine.

These points are currently the main lines of research and work, with the fundamental objective of improving the capacity and accuracy of systems that are experts in specific problems. Advances in the incorporation of this type of emerging technologies will contribute to the future development of more sophisticated and powerful expert systems.

This discussion delves deeper into the mathematical frameworks underpinning the findings of this review, highlighting their relevance and impact across diverse applications of Expert Systems (ES). A key takeaway is the role of logic-based reasoning, probabilistic modeling, and optimization algorithms in advancing the capabilities of ES. These mathematical tools not only provide theoretical rigor but also practical efficiency, enabling the systems to handle real-world complexities effectively.

The advancements in expert systems are deeply rooted in their mathematical frameworks, which provide the theoretical foundation for their reasoning, adaptability, and efficiency. These frameworks enable ES to tackle real-world challenges with precision and scalability. Logical frameworks, such as propositional and predicate calculus, form the basis of knowledge representation in ES. These frameworks allow for precise formalization of domain knowledge, ensuring that the systems can perform consistent and interpretable reasoning. The extension of these logical models through fuzzy set theory addresses the inherent uncertainty and vagueness in many application domains, such as medical diagnosis and environmental monitoring.

Probabilistic methods, including Bayesian networks and Markov models, are instrumental in enhancing the predictive and inferential capabilities of ES. These methods facilitate decision-making under uncertainty by quantifying the likelihood of various outcomes. This approach has been particularly impactful in fields like finance, where stochastic models are employed to forecast market trends, and in engineering, where reliability analysis relies on probabilistic assessments.

Optimization algorithms, derived from calculus and linear programming, play a critical role in resource allocation and decision-making within ES. These algorithms ensure that the systems can operate efficiently, even under constraints such as limited computational resources or incomplete data. For example, linear optimization models have been used to optimize scheduling in industrial operations, while non-linear programming techniques have enabled the design of adaptive learning systems in education.

The integration of generative models with ES represents a frontier in this field, combining the creative potential of deep learning with the structured reasoning of expert systems. Techniques like Generative Adversarial Networks (GANs) and Variational Autoencoders (VAEs) leverage principles of optimization and information theory to generate realistic data and explore new scenarios.

The challenges and limitations identified in this review also highlight opportunities for future research. The mathematical complexity of hybrid systems, which integrate fuzzy logic, probabilistic reasoning, and generative techniques, requires the development of more robust and scalable algorithms.

Among these challenges, symbolic reasoning stands out for its unique data processing methods, which require distinct mathematical tools compared to data-driven approaches. Symbolic reasoning offers a distinct approach to data processing in Expert Systems (ES), relying on structured representations such as rules, ontologies, and logic-based frameworks. Unlike data-driven methods, symbolic reasoning emphasizes the explicit encoding of knowledge, enabling systems to draw inferences using logical operations like modus ponens and backward chaining. Mathematically, this involves techniques such as propositional and predicate logic, set theory, and graph-based algorithms for managing relationships within knowledge bases. These methods are particularly advantageous in domains requiring high interpretability, as they allow users to trace the reasoning path explicitly. However, symbolic approaches also face challenges, such as scalability and the integration of heterogeneous data sources, which require innovative mathematical solutions like constraint satisfaction problems and description logics. Addressing these limitations is key to enhancing the adaptability and reliability of symbolic reasoning in modern ES.

Another critical aspect of advancing Expert Systems (ES) is the incorporation of mathematical tools to address ethical and operational challenges. Formal verification methods provide a rigorous framework for ensuring the correctness and reliability of ES, particularly in high-stakes domains such as medicine and finance. These techniques can validate that the system's behavior aligns with predefined specifications, reducing the risk of errors and unintended consequences. Additionally, fairness-aware algorithms are essential for mitigating biases that may arise from training data or rule definitions. By quantifying and addressing potential disparities, these algorithms ensure equitable outcomes and enhance the societal trust in ES. Together, these approaches not only reinforce the technical robustness of ES but also align their development with broader ethical and regulatory standards.

Additionally, ethical considerations, such as fairness and transparency, demand the incorporation of formal verification methods to ensure the reliability and accountability of these systems. ES deployed in sensitive areas, such as medicine and finance, must also address critical issues such as data privacy, informed consent, and potential biases. These challenges require robust mechanisms for ensuring that decisions are not only technically sound but also socially responsible. For example, fairness-aware algorithms can help mitigate biases embedded in training datasets, while rigorous compliance with data privacy regulations ensures the ethical handling of sensitive information. Furthermore, explainable AI techniques can enhance transparency, enabling stakeholders to scrutinize and trust the decision-making processes. By addressing these dimensions, ES can align with both societal expectations and regulatory frameworks, fostering broader acceptance and adoption.

In summary, ES has evolved significantly thanks to the integration of modern technologies and robust mathematical foundations. Despite challenges in terms of scalability, interpretability, and ethics, recent advances in fuzzy logic, generative models, and optimization techniques offer a promising path forward. By addressing these limitations and exploring new areas, ES can continue to play a crucial role in solving complex problems with high social impact.

5. Conclusions

After the study of the recovered works, it is possible to point out the application of ES in various fields such as agriculture, education, business and finance with the aim of improving decision-making and solving complex problems is a line of research and work of great current interest. Some of the work has shown that the combination of fuzzy logic and generative models produces promising results in creating smarter and more powerful ES. In addition, a significant growth has been detected in ES research and application in sectors such as those mentioned, where its use has been shown to be key to improving the efficiency and effectiveness of professionals during the execution of certain tasks.

The integration of technologies such as cloud computing and AI into the implementation of ES has helped identify the advantages and challenges of its application in different fields. ES has proven to be valuable tools for improving decision-making and efficiency in various tasks, showing significant potential for use in a multitude of applications.

The main conclusions drawn from the selected research highlighting the key points and implications of their findings are as follows.

1. Importance of fuzzy ES and generative models: Fuzzy ES and generative models have proven to be powerful tools for solving complex problems and making decisions in multiple fields. The combination of fuzzy logic, which is able to handle the uncertainty and imprecision of data, with generative models, which are able to improve interaction and its responses, has yielded promising results in the creation of smarter and more powerful ES. This integration allows solving problems that are difficult to solve with traditional methods, showing great potential to improve performance and efficiency.

2. Applications in various fields: ES has found applications in a variety of fields, such as the following, demonstrating their versatility and usefulness in solving complex problems and facilitating decision-making.

In agriculture, fuzzy expert systems and biological models are used to improve the efficiency of agronomic decision-making, such as crop management, pest control, and production optimization. These systems allow agronomists and farmers to make more informed decisions, based on accurate data and detailed analysis, thus helping to improve the productivity and sustainability of the agricultural industry.

In the field of education, ES has proven to be valuable tools for providing one-on-one counselling to students, identifying areas of knowledge where they need support, and furthering their learning more effectively. These systems are tailored to each student's individual needs, allowing them to progress at their own pace and receive the attention they need in specific areas. The integration of communication and deep learning techniques into expert systems aims to improve the understanding of the student's learning experience and provide personalized recommendations to optimize the process of their learning.

In business and finance, ES plays an important role in automated decision-making in areas such as credit analysis, fraud detection, marketing, and inventory management. These systems enable organizations to optimize their operations, identify market opportunities, and effectively reduce financial risk. Integrating technologies such as ML and natural language processing into a company's ES aims to improve their ability to adapt to new data and information, allowing them to remain effective in their competitive and changing environment. These technologies allow expert systems to effectively analyze large volumes of data, identify patterns and trends, and make decisions based on relevant and up-to-date information. In particular, ML plays a key role in improving the ability of expert systems to learn from data and adapt their models based on available information. Using ML algorithms, expert systems can identify complex correlations and relationships in data, allowing them to improve predictions and optimize their recommendations. Natural language processing (NLP), on the other hand, allows ES to understand and process human language effectively, allowing them to interact with users in a more natural and effective way. By integrating NLP capabilities into enterprise expert systems, it will be easier to communicate with users, extract information from documents, and generate automated reports, improving the efficiency and productivity of business operations.

In short, the application of Expert Systems in different areas is currently successfully demonstrating a great capacity to facilitate the execution of certain tasks in a more automated way and with a high reliability rate. The combination and integration of emerging technologies in its development is a key element to improve its performance and ease of use. That is why, although the existence of the first ES dates back several decades, current technology, and the possibilities that its application offers to improve their capabilities, propose a wide range of lines of research that may result in a new generation of these systems with previously unknown capabilities in the near future.

The findings of this systematic review underline the pivotal role of mathematical methodologies in the evolution and application of Expert Systems (ES). These systems, grounded in logic, probability, and optimization, have demonstrated transformative potential across various domains. This section synthesizes the scientific contributions and outlines future research directions.

One of the most critical insights is the synergy between fuzzy logic and generative models. The mathematical frameworks underpinning these techniques, such as fuzzy set theory and adversarial optimization, have enabled ES to navigate uncertainty and generate novel solutions. For example, fuzzy logic's capacity to model im-precise knowledge has proven indispensable in fields like medicine and agriculture, where data is often in-complete or ambiguous.

Another significant conclusion is the importance of probabilistic reasoning in enhancing the decision-making capabilities of ES. Bayesian networks and Markov models, derived from foundational probability theory, pro-vide robust frameworks for making informed predictions under uncertainty. These methods have seen wide-spread success in domains such as finance, where they are used to model risk and forecast market trends, and in engineering, where they enhance system reliability analysis.

The integration of optimization algorithms into ES has also emerged as a key factor in improving their efficiency and scalability. Techniques from linear programming, integer optimization, and evolutionary algorithms have facilitated resource allocation and adaptive learning in sectors ranging from industrial automation to education. The mathematical rigor of these methods ensures that ES can operate effectively even in resource-constrained environments.

Looking forward, the future of ES research lies in the development of hybrid models that integrate these mathematical techniques. The combination of fuzzy logic with probabilistic reasoning and generative models promises to create systems that are not only accurate but also adaptable and explainable. For instance, hybrid approaches could leverage fuzzy rules for interpretability, probabilistic models for uncertainty quantification, and generative models for scenario simulation.

Furthermore, this review highlights the need for greater emphasis on the ethical and transparent deployment of ES. Mathematical tools such as formal verification methods and fairness-aware algorithms can play a crucial role in ensuring that these systems operate responsibly and equitably.

In conclusion, the mathematical advancements discussed in this review not only enhance the capabilities of Expert Systems but also expand their applicability across complex and dynamic domains. By continuing to integrate and refine these methodologies, researchers can unlock the full potential of ES, driving innovation and addressing global challenges with scientific precision and rigor.

Conflict of interest

The authors declare no competing financial interest.

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