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Research Article

Analysis of a Queue Subject to Discouraged Arrivals, Customer Impatience, and Self-Switching Server Dynamics

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Abstract: This study examines a single-server queue approach that discourages arrivals, accounts for impatient customers, and includes a self-switching server is considered. A queue with a single self-switching server, subjected to discouraged arrivals and customer impatience, is analyzed. Customers arrive at a rate of $\lambda/(n+1)$, where λ denotes a positive constant and n corresponds to the total number of customers in the system. The server provides service at a rate of μ_1 until the size of the system is smaller than K. Once the system size reaches K, the server switches to a faster service rate of μ_2. After joining the queue, customers become impatient and abandon the system at a rate of ξ until the server switches to the fast service rate. There is no impatience once the server starts serving at the faster rate. The steady-state probability distribution of this system is determined, and its performance metrics, such as expected wait time and the number of customers likely to join the queue or system, are provided. Numerical illustrations are also included.

***Keywords*:** discouraged arrival, customer impatience, server switch random, single server queue

MSC: 60K25, 60K30, 90B22

1. Introduction

Queueing models are used in telecommunication, traffic control, healthcare sectors, businesses, banks, post offices, and amusement parks. They are very much useful for reducing congestion and dedicated to provide good service. Queues with discouraged arrivals and impatient customers have more applications in practical situations. Discouraged arrivals refer to potential customers or arrivals who are capable of making a decision to enter the queue or not, because of the lengthy queue or busy server. Discouraged arrivals can occur for several reasons, such as long wait times, high queue lengths, or the perceived inconvenience of waiting in the queue. The presence of discouraged arrivals can have significant implications for queueing models, as they can affect the accuracy of the model and the efficiency of the system. On the other hand, impatient customers are those who are not willing to wait a long time to receive service or attention and may leave the queue or the service before their turn arrives. Impatience arises for various reasons, such as boredom, frustration, urgency, or a lack of information about the expected waiting time. Impatient customers are a common feature of service

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systems. Impatient customers are typically modelled as customers who quits the system when they are not provided with service in an epoch of time. The modelling of impatient customers is important because it can impact the overall performance of the queueing system.

A self-switching server refers to a scenario where the server can switch to more than one mode. In queueing system, a self-switching server can increase system efficiency by saving wait time and minimizing the queues. Building on the discussion of discouraged arrivals and impatience, we now present a specific model incorporating these behaviors. In this model, arrivals are inversely proportional to the queue length, and customers leave if they become impatient. The server adjusts its rate based on thresholds in the queue.

Considering the above concepts, a Markovian queuing model for single-server having discouraged arrivals, customer impatience and self-server switch is generated. The solution for the steady-state condition is found.

The following describes the way the article is structured: Section 2 provides the review as the literature. Section 3 generates the queueing model. Section 4: The model's governing equations are obtained. In Section 5, special examples for the queuing models are defined, and the governing equations are solved constantly to obtain the stationary probability. Section 6 derives measures of effectiveness. The seventh part is a list of the numerical illustrations. The final portion contains the conclusion.

2. Literature review

There are many articles that imply the ideas of impatience and discouragement. Haight [1], provides valuable insights into how customer impatience affects system performance. Its mathematical rigor and practical relevance make it an important work in the field of queueing theory, especially for industries where customer abandonment plays a significant role in service quality and efficiency. Rasheed and Manoharan [2], presents a unique and promising model for analyzing queueing systems with discouraged arrivals and self-regulating servers. Its theoretical contributions are significant, offering insights into system performance under varying conditions. However, the model's assumptions and lack of scientific evaluation limit its immediate practical use. Kumar and Sharma [3], offers a valuable theoretical contribution to queueing theory by incorporating multiple realistic phenomena into a unified model. It represents a well-structured mathematical framework for analysing the system's behavior under varying conditions. While the model's assumptions are helpful for simplicity, they may not fully capture the complexity of real-world systems. Kumar and Sharma [4], the mathematical analysis is comprehensive, and the results provide valuable insights for managing service systems where customer arrivals and behavior are impacted by queue length and waiting time. Medhi [5], enhances the understanding of customer behavior in queueing systems and offers practical tools for managing service processes where customer impatience and system capacity are critical factors. Som and Seth [6], presents a comprehensive stochastic model addressing complex customer behaviors in queueing systems and develop a multi-server, finite capacity Markovian queueing model that incorporates encouraged arrivals, reverse reneging, feedback customers, and retention strategies for reneged customers. Premalatha and Thiagarajan [7] provides important contributions to queueing theory, particularly in the areas of service management and operations research. While the model is solid from the mathematical perspective, its practical usefulness could be improved by conducting scientific conclusion and expanding it to include service distributions other than the exponential one. Ammar et al. [8] examines the transient behavior of an M/M/1/N queueing system with discouraged arrivals and reneging. The authors introduce a matrix-based approach to model the system and compute key performance metrics over time, offering effective observations into the system's dynamics before reaching steady state. While the M/M/1/N queue is a well-known model, the inclusion of discouraged arrivals, where customers depart if the system becomes too filled, and reneging, where customers leave after waiting too long, adds practical relevance to the study, especially in real-world applications like call centers and service stations. Rao et al. [9], by including customer behavior, this model goes above the traditional M/M/1/N queue, where customers may be encouraged or discouraged from joining the queue based on characteristics like system overload in the system. Additionally, the model features breaking, allowing customers to leave if they are not served within a specified time or if the service quality does not meet their expectations. Ayyappan and Thilagavathy [10], provides an in-depth analysis of a non-preemptive priority queueing model that includes several complexities, such as classical retrial policies, breakdowns and repairs, customer discouragement, single vacations, standby servers, negative arrivals, and impatient customers. The model centers around a MAP (1), MAP (2)/PH/1 queue, combining Markov Arrival Processes (MAP) and Phase-type (PH) service distributions. The analysis addresses various real-world service conditions, emphasizing the management of customer priorities, system reliability, and the handling of customer impatience.

Medhi and Choudhury [11], analyzes a single-server finite buffer queue that includes discouraged arrivals and the retention of rending customers. This model explores a queueing system in which customers may choose not to join if the queue is too full and where those who initially leave due to long waits may return. This approach is especially relevant for service systems where customer arrival behaviors are affected by congestion and where managing customers who leave and later return is essential. Apice et al. [12], examines a queueing system with multiple servers handling incoming customers. If the system is busy, customers may depart and return later for service. The arrival rate of customers is influenced by the number of available sources, which can fluctuate over time. The study seeks to calculate key performance metrics, including system utilization, waiting times, and queue lengths, while accounting for the dynamic nature of source availability. Pravina et al. [13], this paper introduces a server that operates in distinct phases, and customers may leave the queue if their impatience threshold is exceeded, representing a more realistic customer behavior in many service systems. It investigates how customer impatience and the server's differentiated phases affect the key performance metrics such as waiting times, queue lengths, and system utilization. Ramesh and Ganesh [14], it offers a more adaptable and realistic approach for analyzing queueing systems with uncertain arrival rates, service times, and system parameters. Although the model is mathematically sound and provides useful performance insights, it would benefit from real-world examination, exploration of more complex systems, and the inclusion of transient analysis to improve its relevance in dynamic settings. Jain and Singh [15], provides that balking occurs when customers decide not to join the queue if it appears too lengthy, reneging takes place when customers leave the queue because of extended waiting periods, and the addition of servers helps reduce system delay. Premalatha and Thiagarajan [16], this paper is designed to reflect realistic service systems where customers may choose not to join the queue when it is too full, leave the queue if they wait too long, and return if allowed by the system. Additionally, the paper introduces controllable arrival rates, allowing for the adjustment of the arrival based on system overloading, providing more flexibility and control over the system's operation. Kumar et al. [17], explores the performance analysis of queueing systems affected by scheduled maintenance and temporary server shutdowns. Using Markovian processes, matrix-analytic techniques, and Laplace Steltjes transforms, the authors develop a probabilistic model to evaluate key performance metrics, including queue length, waiting time, and server utilization. The study highlights the trade-offs between system availability and maintenance scheduling, emphasizing that while maintenance minimizes unexpected failures, excessive downtime can lead to inefficiencies. Udayabaskaran and Pravina [18], analyzes the system is based on the traditional M/M/1 queue, but it introduces a server that operates in three distinct modes, each affecting its service rate. The focus of the study is to examine the transient behavior of the system, especially how it performs during non-steady-state conditions. This is crucial for understanding how the system behaves during periods of fluctuating traffic or system initialization. The paper derives essential performance metrics during the transient phase, offering deeper insights into the system's behavior before reaching steady state. Wang et al. [19], examine various queueing theory models and methods that address the behavior of customers who may abandon the system if their wait becomes too long. Impatience plays a crucial role in many service systems, such as call centers, retail, healthcare, and telecommunications, where customers' tolerance for waiting is limited. The paper compiles existing research on queueing systems involving impatient customers, providing insights into different strategies for managing impatience and enhancing system performance. Yang and Wu [20], provides the server experiences working breakdowns, which cause interruptions in the service, and the customers exhibit impatience, with the possibility of them leaving the system if their wait is too long, though they may return later. This combination of breakdowns and impatient customers is common in manufacturing systems, call centers, and other service-oriented industries.

Sharma and Maheswar [21], investigates customers may decide not to proceed with the queue if it appears extremely extensive or crowded; this is referred to as "discourage arrivals". This model extends traditional queueing systems, where customer arrival rates are usually assumed to be independent of the system's current state. Sudhesh and Azhagappan [22], the M/M/∞ queue represents a system with an infinite number of servers, where the service times and inter-arrival times are exponentially distributed. The system also includes additional tasks that the servers must handle, as well as customers who may leave the system due to impatience if their wait times exceed a threshold. Kumar et al. [23], offers a solid foundation for understanding patient flow management in healthcare systems and presents actionable solutions to reduce patient dissatisfaction and optimize service delivery. Tóth and Sztrik [24], this paper is highly relevant to telecommunication networks, call centers, and other service systems where retries, server failures, and customer impatience are common. The paper offers valuable insights into the system performance under these challenging conditions, providing useful metrics for optimizing system design and service delivery. Kim and Yoo [25], provides that highlights the importance of considering customer impatience in service system design and offers useful analytical techniques for evaluating system performance under these conditions. The paper's clear focus on key performance metrics such as waiting time and queue length provides practical value for optimizing service delivery and reducing customer abandonment. Pravina et al. [26], this paper addresses a complex and important issue in queueing theory modeling a single-server system with multi-phase operations that includes disaster recovery and repair processes. While the theoretical contributions are commendable, offering a deeper exploration of real-world situations where downtime, recovery, and repair processes affect system performance, there are several areas that deserve further critical assessment.

In many practical situations in queueing system discouragement and impatient customers alone, when there is long waiting period for service, it is certain that the server will work quicker even though the queue size is long. Various extensions of customer impatience in single server queue are taken care of. Further, the customer's impatience will have high negative impacts on losing the potential customers in any business firms. Having the impacts of customer's impatience in mind, the theory of self-switching server is studied in this paper.

2.1 *Definitions*

In queueing theory, Poisson processes, exponential distributions, and Laplace transforms play significant roles in modeling and analyzing systems involving customer arrivals, service times, and system behavior. Here's a brief explanation of their significance.

2.1.1 *Poisson process*

The Poisson process is a mathematical model used to describe random events occurring over time, typically used to model arrivals in a queue. In queueing theory, the Poisson process models the random arrival of customers or jobs to the system, where the number of arrivals in a given time period follows a Poisson distribution.

2.1.2 *Exponential distribution*

The exponential distribution describes the time between events in a Poisson process. It is widely used to model service times or waiting times in queueing systems. In queueing systems, both the arrival rate and the service rate are key to determining system performance metrics such as wait times and queue lengths.

2.1.3 *Laplace transforms*

The Laplace transform is a mathematical tool used to convert functions from the time domain into the complex frequency domain. It is used to simplify the analysis of differential equations, which are often encountered in queueing models. Laplace transforms are used to find the probability generating functions of queueing models, which can then be analyzed to determine performance metrics like average queue length, waiting time, and system utilization.

3. Model description

The customers will enter the queueing system with discouragement, which follows the Poisson process with rate $\frac{\lambda}{n+1}$. There is a single server, and when K customers are present, the mode of service changes from standard mode to fast mode. In this instance, service is provided in normal mode; meanwhile, if K customers are present in the system, the service provider immediately switches to fast mode. Service rates in normal and fast phases provide an exponential distribution having a mean of $\frac{\mathbf{1}}{\boldsymbol{\mu_1}}$ and $\frac{\mathbf{1}}{\boldsymbol{\mu_2}}$, the service rate in fast mode is greater than that in standard mode ($\boldsymbol{\mu_2} > \boldsymbol{\mu_1}$). When the server is working normally, customers might become impatient and will abandon the system corresponding to an exponential distribution having a mean of $\frac{\mathbf{1}}{\xi}$. Customers exhibit impatience and may abandon the queue at a rate of ξ while the server is in the slow service mode. Once the server switches to the fast service rate $\boldsymbol{\mu_2}$, customers no longer abandon the system due to impatience.

3.1 *Notations*

1. $\mathscr{P}r(A)$: event A's Probability.
2. $\pi_n = \lim\limits_{t\to\infty} p_n(t)$: Steady state probability in 'n' customers in the system.
3. $\int_0^t f(u)g(t-u)\mathrm{d}u : f(t)g(t)$ Convolution of $f(t)$ and $g(t)$ in Laplace Transform.
4. $\int_0^\infty e^{-st} f(t)\mathrm{d}t = f^*\ (s)$: Laplace transform of $f(t)$.

4. Governing equations

Let $\{X(t);\ t > 0\}$ be the number of customers present in the system at time t in Figure 1, we denote the probability that there are n customers in the system at time t. $P_n(t) = \ Pr\{X(t) = n\},\ n = 0,\ 1,\ 2,\ \ldots$

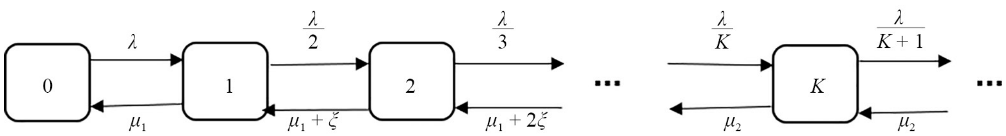

Figure 1. Transition diagram

Below differential-difference equations are used to govern the system:
Apply probabilistic laws, we have

$$p(0,\ t) = e^{-\lambda t} + \ \mu_1\ p(1,\ t)\ \ e^{-\lambda t} \tag{1}$$

$$p(n,\ t) = \left[\frac{\lambda}{n} p(n-1,\ t) + (\mu_1 + n\xi)\ p(n+1,\ t)\right]\ e^{-\left(\frac{\lambda}{n+1} + \ \mu_1\ \ + (n-1)\xi\right)t},\ n = 1,\ 2,\ \ldots,\ K-2 \tag{2}$$

$$p(K-1,\ t) = \left[\frac{\lambda}{K-1} p(K-2,\ t) + \mu_2\ p(K,\ t)\right]\ e^{-\left(\frac{\lambda}{K} + \ \mu_1\ \ + (K-2)\xi\right)t} \tag{3}$$

$$p(n,\, t) = \left[\frac{\lambda}{n} p(n-1,\, t) + \mu_2\ p(n+1,\, t)\right]\ e^{-\left(\frac{\lambda}{K+1} + \mu_2\ \right)t},\ n\ \geq\ K. \tag{4}$$

Taking Laplace transform Equations (1)-(4), we get

$$(s+\lambda)p^*\ (0,\ s) = 1 + \mu_1 p^*\ (1,\ s) \tag{5}$$

$$\left(s + \frac{\lambda}{n+1} + \mu_1\ \ + (n-1)\xi\right) p^*\ (n,\ s) = \ \frac{\lambda}{n} p^*\ (n-1,\ s) +\ (\mu_1 + n\xi)\ p^*\ (n+1,\ s),\ n = 1,\ 2,\ \ldots,\ K-2 \tag{6}$$

$$\left(s + \frac{\lambda}{K} + \mu_1\ \ + (K-2)\xi\right) p^*\ (K-1,\ s) = \ \frac{\lambda}{K-1} p^*\ (K-2,\ s) +\ \mu_2 p^*\ (K,\ s) \tag{7}$$

$$\left(s + \frac{\lambda}{n+1} + \mu_2\ \ \right) p^*\ (n,\ s) = \ \frac{\lambda}{n} p^*\ (n-1,\ s) +\ \mu_2 p^*\ (n+1,\ s),\ n \geq K. \tag{8}$$

5. Steady state probabilities

Taking $\lim_{t\to\infty} p_n(t) = \ \pi_n$ and therefore $\frac{\mathrm{d}p_n}{\mathrm{d}t} = 0$ as $t \to \infty$, hence we get balance equations,

$$\lambda \pi_0 = \mu_1 \pi_1 \tag{9}$$

$$\left(\frac{\lambda}{n+1} + \mu_1 + (n-1)\xi\right) \pi_n = \frac{\lambda}{n} \pi_{n-1} +\ (\mu_1 + n\xi)\ \pi_{n+1},\ 1 \leq n \leq\ K-2 \tag{10}$$

$$\left(\frac{\lambda}{K} + \mu_1 + (K-2)\xi\right) \pi_{K-1} = \frac{\lambda}{K-1} \pi_{K-2} +\ \mu_2 \pi_K \tag{11}$$

$$\left(\frac{\lambda}{n+1} + \mu_2\right) \pi_n = \frac{\lambda}{n} \pi_{n-1} +\ \mu_2 \pi_{n+1},\ n \geq K. \tag{12}$$

By an iterative procedure, equations from (9)-(12) yield

$$\pi_n = \begin{cases} \Pi_{r=1}^{n} \left[\dfrac{\lambda}{r(\mu_1 + (r-1)\xi\ )}\right] \pi_0, & 1 \leq n \leq K-1 \\ \left[\left(\dfrac{\lambda}{\mu_2}\right)^{n-K+1} \dfrac{1}{(K)_{n-K+1}}\right] \Pi_{r=1}^{K-1} \left[\dfrac{\lambda}{r(\mu_1 + (r-1)\xi\ )}\right] \pi_0, & n \geq K, \end{cases} \tag{13}$$

where $(K)_i = K(K+1)(K+2)\cdots(K+i-1)$.

By using total probability law, we get

$$\pi_0 = \left[1 + \sum_{n=1}^{K-1} \Pi_{r=1}^{n} \left[\frac{\lambda}{r(\mu_1 + (r-1)\xi\ )}\right] + \sum_{n=K}^{\infty} \left[\left(\frac{\lambda}{\mu_2}\right)^{n-K+1} \frac{1}{(K)_{n-K+1}}\right] \Pi_{r=1}^{K-1} \left[\frac{\lambda}{r(\mu_1 + (r-1)\xi\ )}\right]\right]^{-1}. \tag{14}$$

Hence, the probabilities for the system's steady states for each state are established explicitly.

5.1 *Special case*

1. When $\xi = 0$, this model reduces Discouraged Arrivals and Self-Regulatory Serversin the Markovian Queueing System as discussed by Rasheed and Manoharan [24] with

$$\pi_n = \begin{cases} \dfrac{1}{n!} {r_1}^n \pi_0, & 1 \le n \le K-1 \\ \\ \dfrac{1}{n!} {r_1}^{K-1} {r_2}^{n-K+1} \pi_0, & n \ge K, \end{cases} \tag{15}$$

$$\pi_0 = \left[1 + \sum_{n=1}^{K-1} \frac{1}{n!} {r_1}^n + \left(\frac{r_1}{r_2}\right)^{K-1} \left\{e^{r_2} - \sum_{n=0}^{K-1} \frac{{r_2}^n}{n!}\right\}\right]^{-1} \tag{16}$$

where $r_1 = \dfrac{\lambda}{\mu_1}$ and $r_2 = \dfrac{\lambda}{\mu_2}$.

2. When there is no discouragement, no impatience and no server switch, then Equations (15) and (16) becomes

$$\pi_n = \left(\frac{\lambda}{\mu}\right)^n \pi_0.$$

And $\pi_0 = 1 - \dfrac{\lambda}{\mu}$.

Hence, proved that the model will be reduced to simple M/M/1/∞ queueing model.

6. Stationary measures of effectiveness

6.1 *Expected number of customers in the system*

The average number of customers in the system is denoted by Ls. It helps in understanding how well the system performs in an M/M/1 queue. Knowing the expected number of customers helps determine if the server is operating at optimal efficiency.

$$Ls = \sum_{n=0}^{\infty} n\pi_n = \sum_{n=1}^{K-1} n\pi_n + \sum_{n=K}^{\infty} n\pi_n$$

$$Ls = \left[\sum_{n=1}^{K-1} n\Pi_{r=1}^{n}\left[\frac{\lambda}{r(\mu_1+(r-1)\xi\ )}\right]\pi_0, + \sum_{n=K}^{\infty} n\left[\left(\frac{\lambda}{\mu_2}\right)^{n-K+1}\frac{1}{(K)_{n-K+1}}\right]\Pi_{r=1}^{K-1}\left[\frac{\lambda}{r(\mu_1+(r-1)\xi\ )}\right]\pi_0\right].$$

6.2 *Expected number of customers in the queue*

Let average number of customers in the system be denoted by Lq. It provides insights into how efficiently a system is running. If this number is too high, it suggests that the system is overloades, leading to longer wait times and potentially frustuated customers.

$$Lq = Ls\ (1-\pi_0)$$

$$Lq = \left[\sum_{n=1}^{K-1} n\Pi_{r=1}^{n}\left[\frac{\lambda}{r(\mu_1+(r-1)\xi\ )}\right]\pi_0, + \sum_{n=K}^{\infty} n\left[\left(\frac{\lambda}{\mu_2}\right)^{n-K+1}\frac{1}{(K)_{n-K+1}}\right]\Pi_{r=1}^{K-1}\left[\frac{\lambda}{r(\mu_1+(r-1)\xi\ )}\right]\pi_0\right] - 1 + \pi_0.$$

6.3 *Average impatience rate*

The number of customers, those departed the system according to their impatience, is denoted by $E[I]$. It helps that measures the tendency of customers to leave a queue before receiving service. This metric is particularly useful in situations where customers may abandon the queue if their wait becomes too long. Then it is given by

$$E[I] = \xi\sum_{n=1}^{K-1} n\pi_n = \xi\sum_{n=1}^{K-1} n\Pi_{r=1}^{n}\left[\frac{\lambda}{r(\mu_1+(r-1)\xi\ )}\right]\pi_0.$$

7. Numerical illustration

7.1 *Steady state probabilities*

Assuming the following values for the parameters of the system: $\lambda = 15$; $\mu_1 = 20$; $\mu_2 = 30$; $K = 20$ arrived the steady-state probabilities by using (9). In Table 1, we have tabulated the probability distribution:

Table 1. Steady state probability distribution

N	π_n	N	π_n
1	0.3785	6	0.4953
2	0.4799	7	0.4953
3	0.4940	8	0.4953
4	0.4952	9	0.4953
5	0.4953	10	0.4953

7.2 *Mean count of the customers arriving in the system and queue (Ls & Lq)*

Take the values of parameters are $\lambda = 15$; $\mu_1 = 20$; $\mu_2 = 30$; $K = 20$ and varying π_n from 0.1 to 1.0, determining the expected count of customers arriving in the system and queue for state probabilities. Table 2 has the values, and Figure 2 depicts the variation:

Table 2. Mean count of the customers Ls and Lq

π_n	Ls	π_n	Lq
0.1	0.3785	0.6	0.0860
0.2	0.5813	0.7	0.1283
0.3	0.6236	0.8	0.1331
0.4	0.6284	0.9	0.1334
0.5	0.6287	1.0	0.1335
0.6	0.6287	0.6	0.1335
0.7	0.6287	0.7	0.1335
0.8	0.6287	0.8	0.1335
0.9	0.6287	0.9	0.1335
1.0	0.6287	1.0	0.1335

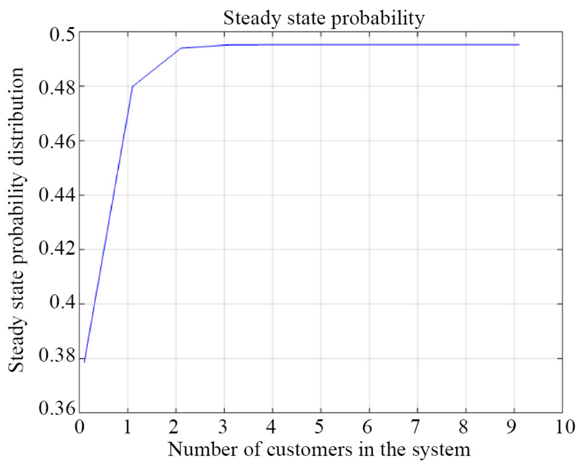

Figure 2. Steady state probability

Table 2 and Figure 3 demonstrate that as the state probability π_n increases from 0.1 to 1.0, the expected number of customers in the system Ls stabilizes at 0.6287, while the number in the queue Lq levels off at 0.1335 from $\pi_n = 0.6$ onward. This indicates that beyond a certain point, the system becomes more efficient in handling customer arrivals and queue management. The transition to faster service rates reduces customer impatience and stabilizes queue size, demonstrating the system's optimal performance with diminishing increases in queue length as arrival probabilities rise.

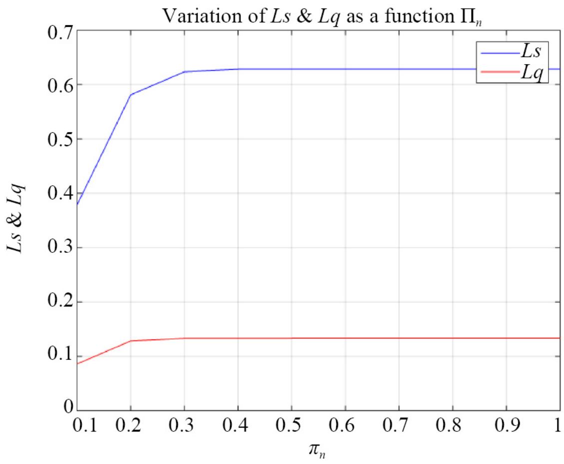

Figure 3. Variation of Ls & Lq as a function π_n

7.3 *Mean count of customers in the system against* ξ

Assume $\lambda = 15$; $\mu_1 = 20$; $\mu_2 = 30$; $K = 20$ and varying ξ from 1 to 10 calculating the system's mean customer count's steady-state probabilities. Table 3 tabulates it, and Figure 3 illustrates the variation:

Table 3. Mean count of customer arrivals in the system Ls

ξ	Ls	ξ	Ls
1	0.72490	6	0.64812
2	0.70444	7	0.63792
3	0.68724	8	0.62872
4	0.67245	9	0.6204
5	0.65954	10	0.6128

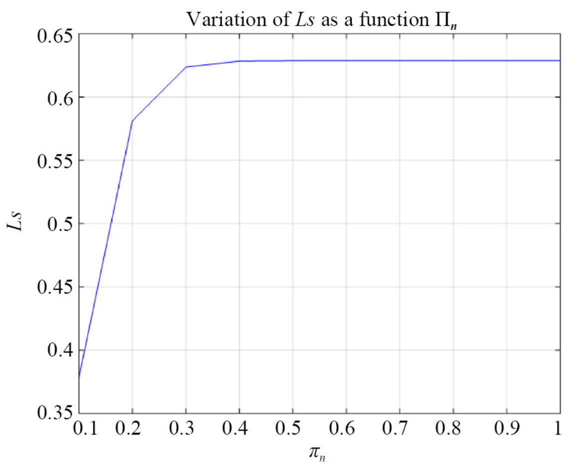

Figure 4. Variation of Ls as a function π_n

Table 3 and Figure 4 illustrate the impact of varying ξ from 1 to 10 on the mean number of customers in the system Ls under the given parameters. The results indicate a steady decline in Ls, decreasing from 0.72490 at $\xi = 1$ to 0.6128 at $\xi = 10$. This trend suggests that increasing ξ enhances system performance by reducing congestion. However, the rate of reduction diminishes at higher values, indicating that beyond a certain threshold, further increases in ξ offer marginal benefits. Optimally, values between 6 and 8 provide a balance between efficiency and stability.

7.4 *Mean count of customers in the queue against ξ*

Assume $\lambda = 15$; $\mu_1 = 20$; $\mu_2 = 30$; $K = 20$ and varying ξ from 1 to 10 calculating the system's mean customer count's steady-state probabilities. Table 3 tabulates it, and Figure 3 illustrates the variation

Table 4. Average number of customers waiting in queue (Lq)

ξ	Lq	ξ	Lq
1	0.20336	6	0.14719
2	0.18811	7	0.13994
3	0.17545	8	0.13345
4	0.16468	9	0.12760
5	0.15537	10	0.12228

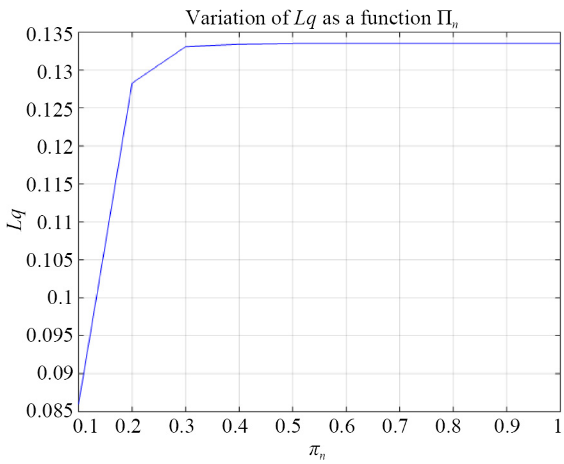

Figure 5. Variation of Lq as a function π_n

Table 4 and Figure 5 show that as ξ increases from 1 to 10, the average number of customers in the queue Lq steadily declines from 0.20336 to 0.12228, improving queue management and reducing wait times. However, the rate of reduction diminishes at a higher ξ values, indicating limited benefits beyond a certain point. An optimal range of 6 to 8 ensures efficiency while maintaining system stability. While higher ξ enhances performance, a balanced approach is essential for effective queue management.

7.5 *Sensitive analysis*

We have investigated the behaviour of the customers who are expected to be in the queue and in the system based on the different switch point value K by the below sensitivity analysis. With Table 5, we understand that increase in switch point value K will result in discouraged arrival with impatient customers in single server and self-switching server tends to result in single server discouraged arrival with impatient customers without switching the server. If $K \geq 5$, $\pi_0 = 0.5047$, where π_0 is the probability of discouraged arrival model of the single server with impatient customers without switching. Similarly, if $K \geq 7$, $Ls = 0.6287$, where Ls is the length of system discouraged arrival model of the single server with impatient customers without switching the server and if $K \geq 6$, $Lq = 0.1335$, where Lq of the discouraged arrival model of the single server with impatient customers without switching the server. Hereby, though the value of switch point increases it has zero effects.

Table 5. Sensitive analysis

K	π_0	(Ls)	(Lq)
2	0.5068	0.6267	0.1335
3	0.5028	0.6392	0.1419
4	0.5044	0.6312	0.1355
5	0.5047	0.6290	0.1337
6	0.5047	0.6288	0.1335
7	0.5047	0.6287	0.1335
8	0.5047	0.6287	0.1335

8. Conclusions

This paper presents a comprehensive analysis of a Markovian queueing system with a single self-switching server, subject to discouraged arrivals, customer impatience, and server switching dynamics. The study examines the complex interactions between various system behaviors, including the rate of customer arrivals, which is inversely proportional to the queue length, customer impatience leading to abandonment, and the switching of server modes from a normal service rate to a faster service rate once the queue reaches a specific threshold. The findings show that the introduction of discouraged arrivals, customer impatience, and self-switching server behavior significantly impacts system performance metrics, such as waiting times, queue lengths, and the likelihood of customer abandonment. Specifically, the transition to a faster service mode when the queue reaches a certain size enhances service delivery and mitigates the adverse effects of customer impatience.

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Conflict of interest

The authors declare that they have no conflicts of interest, whether in whole or in part, regarding the content of this article.

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