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Research Article

Improved Passive Synchronization for Complex Dynamical Networks with Randomly Occurring Distributed Coupling Time-Varying Delays via State Feedback Control

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Abstract: The improved synchronization of complex dynamical networks under passivity performance is the main topic of this paper. To distinguish between simple and complex dynamics in real-world scenarios, the effect of randomly occurring time-varying delays is specifically considered. The study is based on the development of a feedback controller and a Lyapunov-Krasovskii functional (LKF) with novel integral terms. Finally, a useful example is provided to illustrate the effectiveness of the proposed approach.

Keywords: complex networks, synchronization, randomly occurring coupling delays, passivity, linear matrix inequality

MSC: 65L05, 34K06, 34K28

1. Introduction

Complex systems are prevalent in nature and everyday life, with most of them frequently modeled as complex networks [1–4]. Such networks can effectively represent real-world systems, including communication networks, which comprise interconnected and interdependent components designed for transmitting and receiving signals.

Among the various topics in complex network research, synchronization has received significant attention [5–7]. The synchronization of dynamical components is widely regarded as one of the most fundamental properties of complex networks, leading to extensive investigations in the literature [8–10]. Moreover, numerous synchronization-related challenges have substantial practical implications [11].

Passivity plays a crucial role in understanding and analyzing dynamic behaviors. It has been extensively studied by researchers and is considered a powerful tool for assessing system stability. Passivity-based approaches have found applications in diverse fields, including signal processing, complexity analysis, stability assessment, and fuzzy control (see, for example, references [12–15]). Ren et al. [16] introduced various passivity concepts, significantly contributing to the advancement of passive control in dynamical systems [17, 18].

Synchronization of stochastic complex networks with time delays are recently investigated in [19–22].

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However, to the simplest of the authors' knowledge, the passivity synchronization of complex networks subject to randomly occurring coupling time-varying delays still remains challenging. Building on these insights, this study explores state feedback control for achieving passivity in complex networks with delays.

The main contribution of this paper is listed below:

1. State feedback control with randomly occurring distributed coupling time-varying delays are considered for complex dynamical networks.

2. The application of stochastic analysis often leads to the derivation of delay-dependent conditions necessary for ensuring synchronization while maintaining sufficient passivity performance, such as disturbance attenuation.

3. Synchronization controllers are developed using linear matrix inequalities (LMIs) to achieve the tolerable condition for the complex dynamical networks.

4. At terminally, numerical example is displayed to emphasize the efficacy of the derived theoretical results.

Notations: Throughout this paper, ✠ denotes the elements below the main diagonal of a symmetric of a symmetric block matrix, $\mathfrak{I}$ denotes the identity matrix with appropriate dimensions. $\mathfrak{R}^n$ represents the n dimensional Euclidean space and $\mathfrak{R}^{n\times m}$ is the set of all $n\times m$ real matrices. $\mathfrak{P}>0$ means $\mathfrak{P}$ is real symmetric and positive definite, $Pr\{\beta\}$ means the occurrence probability of the event β, and $diag\{a, b, ..., z\}$ indicates the block-diagonal matrix with $a, b, ..., z$ in the diagonal entries. $\mathfrak{E}\{x\}$ means the expectation of the stochastic variable x. The notation $\mathfrak{A}\otimes\mathfrak{B}$ stands for the Kronecker product of matrices $\mathfrak{A}$ and $\mathfrak{B}$.

2. Problem description

Consider the class networks with $\mathbf{O}$ nodes that contain delays and randomly appearing nodes:

$$\begin{cases} \dot{\mathfrak{p}}_\eta(\iota) = \mathfrak{A}\mathfrak{p}_\eta(\iota) + \mathfrak{B}\mathfrak{f}(\iota, \mathfrak{p}_\eta(\iota)) + \mathfrak{C}\int_{\iota-\mathfrak{S}(\iota)}^{\iota}\mathfrak{f}(\mathfrak{p}(\mathfrak{s}))d\mathfrak{s} + (1-\Phi(\iota))\sum_{\psi=1}^{\mathbf{O}}\mathbf{b}_{\eta\psi}\Gamma\mathfrak{p}_\psi(\iota) \\ \qquad + \Phi(\iota)\sum_{\psi=1}^{\mathbf{O}}\mathbf{b}_{\eta\psi}\Gamma\mathfrak{p}_\psi(\iota-\wp(\iota)) + \mathbf{u}_\eta(\iota) + \mathbf{w}_\eta(\iota), \\ \mathfrak{p}_\eta(\iota) = {}_\eta\iota, \quad \eta = 1, 2, ..., \mathbf{O}, \ \iota\in[-max(\widehat{\wp}, \widehat{\mathfrak{S}}), 0], \end{cases} \tag{1}$$

where $\mathfrak{A}$ be a constant matrix and $\mathfrak{B}$, $\mathfrak{C}$ are the weight matrices, $\mathfrak{p}_\eta(\iota) = (\mathfrak{p}_{\eta 1}(\iota), \mathfrak{p}_{\eta 2}(\iota), ..., \mathfrak{p}_{\eta n}(\iota))^T \in \mathfrak{R}^n$ be the state vector associated ηth node, $\mathbf{u}_\eta(\iota)\in\mathfrak{R}^n$ the control input, $\mathbf{w}_\eta(\iota)$ is the disturbance $\mathfrak{S}(\iota)$ and $\wp(\iota)$ represent delays that satisfies

$$0 \le \wp(\iota) \le \widehat{\wp}, \quad 0 \le \mathfrak{S}(\iota) \le \widehat{\mathfrak{S}}, \tag{2}$$

where $\widehat{\wp}$, μ and $\widehat{\mathfrak{S}}$ are known constants. $\Gamma\in\mathfrak{R}^{n\times n}$ is coupling matrix and $\mathbf{B} = (\mathbf{b}_{\eta\psi})_{\mathbf{O}\times\mathbf{O}} \in \mathfrak{R}^{\mathbf{O}\times\mathbf{O}}$ represents an outer-coupling configuration matrix. If easy to implement η and ψ ($\eta\neq\psi$), then $\mathbf{b}_{\eta\psi}\neq 0$; otherwise, $\mathbf{b}_{\eta\psi}=0$, $\mathbf{b}_{\eta\eta}$ are given by $\mathbf{b}_{\eta\eta} = -\sum_{\psi=1,\,\psi\neq\eta}^{\mathbf{O}}\mathbf{b}_{\eta\psi}$, $\eta = 1, 2, ..., \mathbf{O}$.

$\Phi(\iota)$ is the Bernoulli random variable convenient randomly occurring coupling delay and satisfying:

$$\Phi(\mathrm{t}) = \begin{cases} 1, & \textit{coupling delay happens}, \\ 0, & \textit{coupling delay does not happens}, \end{cases} \tag{3}$$

with

$$Prob\{\Phi(\iota)=1\}=\widehat{\Phi},$$

$$Prob\{\Phi(\iota)=0\}=1-\widehat{\Phi},$$

where $\widehat{\Phi}\in[0,\,1]$ is a known constant.

According to the expectation of Bernoulli function, we have:

$$\mathfrak{E}\{\Phi(\iota)-\widehat{\Phi}\}=0,$$

$$\mathfrak{E}\{(\Phi(\iota)-\widehat{\Phi})^2\}=\widehat{\Phi}(1-\widehat{\Phi}),$$

Assumption 1: For given matrices $\mathfrak{R}_1$ and $\mathfrak{S}_1$ and f

$$[\mathfrak{f}(\mathfrak{x})-\mathfrak{f}(\mathfrak{y})-\mathfrak{R}_1(\mathfrak{x}-\mathfrak{y})]^T[\mathfrak{f}(\mathfrak{x})-\mathfrak{f}(\mathfrak{y})-\mathfrak{S}_1(\mathfrak{x}-\mathfrak{y})]\leq 0,\quad \forall\mathfrak{x},\,\mathfrak{y}\in\mathfrak{R}^n. \tag{4}$$

The system unforced isolate node is given by

$$\dot{\mathfrak{s}}(\iota)=\mathfrak{A}\mathfrak{s}(\iota)+\mathfrak{B}\mathfrak{f}(\iota,\,\mathfrak{s}(\iota)), \tag{5}$$

we define the synchronization error $\rho_\eta(\iota)=\mathfrak{p}(\iota)-\mathfrak{s}(\iota)$. Then, (1) is

$$\dot{\rho}_\eta(\iota)=\mathfrak{A}\rho_\eta(\iota)+\mathfrak{B}\widetilde{\mathfrak{f}}(\iota,\,\rho_\eta(\iota))+\mathfrak{C}\int_{\iota-\mathfrak{I}(\iota)}^{\iota}\widetilde{\mathfrak{f}}(\rho(\mathfrak{s}))d\mathfrak{s}+(1-\Phi(\iota))\sum_{\psi=1}^{\mathbf{O}}\mathbf{b}_{\eta\psi}\Gamma\rho_\psi(\iota)$$

$$+\Phi(\iota)\sum_{\psi=1}^{\mathbf{O}}\mathbf{b}_{\eta\psi}\Gamma\rho_\psi(\iota-\wp(\iota))+\mathbf{u}_\eta(\iota)+\mathbf{w}_\eta(\iota),\quad \eta=1,\,2,\,....,\,\mathbf{O}, \tag{6}$$

where $\widetilde{\mathfrak{f}}(\iota,\,\rho_\eta(\iota))=\mathfrak{f}(\iota,\,\mathfrak{p})-\mathfrak{f}(\iota,\,\mathfrak{s}(\iota))$.

The controllers are made in the following manner:

$$\mathbf{u}_\eta(\iota)=\mathfrak{K}_\eta(\iota)\rho_\eta(\iota),\quad \eta=1,\,2,\,....,\,\mathbf{O}. \tag{7}$$

Then by using (6) and (7),

$$\dot{\rho}_\eta(\iota) = (\mathfrak{A} + \mathfrak{K}_\eta)\rho_\eta(\iota) + \mathfrak{B}\widetilde{\mathfrak{f}}(\iota, \rho_\eta(\iota)) + \mathfrak{C}\int_{\iota-\mathfrak{I}(\iota)}^{\iota} \widetilde{\mathfrak{f}}(\rho(\mathfrak{s}))d\mathfrak{s} + (1-\Phi(\iota))\sum_{\psi=1}^{\mathbf{O}} \mathbf{b}_{\eta\psi}\Gamma\rho_\psi(\iota)$$

$$+\Phi(\iota)\sum_{\psi=1}^{\mathbf{O}} \mathbf{b}_{\eta\psi}\Gamma\rho_\psi(\iota - \wp(\iota)) + \mathbf{w}_\eta(\iota), \quad \eta = 1, 2, ..., \mathbf{O}, \tag{8}$$

Then system (8) is

$$\dot{\rho}(\iota) = (\mathfrak{I}\otimes\mathfrak{A} + \mathfrak{K})\rho(\iota) + \mathfrak{B}\widetilde{\mathfrak{F}}(\rho(\iota)) + \mathfrak{C}\int_{\iota-\mathfrak{I}(\iota)}^{\iota} \widetilde{\mathfrak{F}}(\rho(\mathfrak{s}))d\mathfrak{s} + (1-\Phi(\iota))(\mathbf{B}\otimes\Gamma)\rho(\iota)$$

$$+\Phi(\iota)(\mathbf{B}\otimes\Gamma)\rho(\iota - \wp(\iota)) + \mathbf{w}(\iota), \tag{9}$$

Definition 1 [23] If there is a scalar $\gamma > 0$ then (9) is stochastically passive, that $\iota_\pi \geq 0$

$$2\int_0^{\iota_\pi} \mathfrak{E}\Big\{\ell^T(\mathfrak{s})\mathbf{w}(\theta)\Big\}d\theta \geq -\gamma\int_0^{\iota_\pi} \mathfrak{E}\Big\{\mathbf{w}^T(\theta)\mathbf{w}(\theta)\Big\}d\theta \tag{10}$$

Lemma 1 [24] For any matrix $\begin{bmatrix} \mathfrak{M} & \mathfrak{T} \\ \maltese & \mathfrak{M} \end{bmatrix} \geq 0$, scalars $\widehat{\wp} > 0$, $\wp(\iota) > 0$, $0 \leq \wp(\iota) \leq \widehat{\wp}$, vector function $\dot{\rho}(\iota + \cdot) : [-\widehat{\wp}, 0] \to \mathfrak{R}^n$ then the concerned integrations are

$$-\widehat{\wp}\int_{\iota-\widehat{\wp}}^{\iota} \dot{\rho}^T(\alpha)\mathfrak{M}\dot{\rho}(\alpha)d\alpha \leq \varpi^T(\iota)\Omega\varpi(\iota), \tag{11}$$

where

$$\varpi(\iota) = \Big[\rho^T(\iota) \;\; \rho^T(\iota - \wp(\iota)) \;\; \rho^T(\iota - \widehat{\wp})\Big]^T,$$

$$\Omega = \begin{bmatrix} -\mathfrak{M} & \mathfrak{M} - \mathfrak{T} & \mathfrak{T} \\ \maltese & -2\mathfrak{M} + \mathfrak{T} + \mathfrak{T}^T & -\mathfrak{T} + \mathfrak{M} \\ \maltese & \maltese & -\mathfrak{M} \end{bmatrix}.$$

Lemma 2 [25] The matrices are $\mathfrak{Z}$, $\mathfrak{W}$ and $\mathfrak{Y}$ such that $\mathfrak{Y} > 0$, $\begin{bmatrix} \mathfrak{W} & \mathfrak{Z} \\ \mathfrak{Z}^T & -\mathfrak{Y} \end{bmatrix} < 0$, holds if and only if, $\mathfrak{W} + \mathfrak{Z}^T\mathfrak{Y}^{-1}\mathfrak{Z} < 0$.

Lemma 3 [26] Any constant matrix $\mathfrak{W} \in \mathfrak{R}^{n\times n}$, $\mathfrak{W}^T = \mathfrak{W} > 0$, scalars δ and ε with $\delta > \varepsilon$ and vector $\rho : [\varepsilon, \delta] \to \mathfrak{R}^n$, then,

$$-(\delta-\varepsilon)\int_{\varepsilon}^{\delta}\rho^{T}(s)\mathfrak{W}\rho(s)d\mathfrak{s}\leq-\Big(\int_{\varepsilon}^{\delta}\rho(\mathfrak{s})d\mathfrak{s}\Big)^{T}\mathfrak{W}\Big(\int_{\varepsilon}^{\delta}\rho(\mathfrak{s})d\mathfrak{s}\Big). \tag{12}$$

3. Main results

Theorem 1 Assumption 1 is true, scalars Φ, $\widehat{\wp}$, h and ε are positive, matrices $\mathfrak{R}_1$, $\mathfrak{S}_1$ the system (9) is stochastically passive, $\exists$ matrices $\mathfrak{P}>0$, $\mathfrak{Q}>0$, $\mathfrak{H}>0$, $\mathfrak{U}>0$, matrix $\mathfrak{M}$ and a constant $\gamma>0$ hence, the subsequent LMIs are true:

$$\chi:=\begin{bmatrix}\chi_1 & \chi_2\\ \maltese & \chi_3\end{bmatrix},$$

$$\chi_1:=\begin{bmatrix}\chi_{11} & \Phi\mathfrak{P}(\mathbf{B}\otimes\Gamma)+\mathfrak{H}-\mathfrak{M} & \mathfrak{M} & \mathfrak{B}\mathfrak{P}-\varepsilon\overline{\mathfrak{S}}_{\Lambda_1}\\ \maltese & -2\mathfrak{H}+\mathfrak{M} & -\mathfrak{M}\mathfrak{H} & 0\\ \maltese & \maltese & -\mathfrak{Q}-\mathfrak{H} & 0\\ \maltese & \maltese & \maltese & -\varepsilon\mathfrak{I}+\mathfrak{I}^2\mathfrak{U}\end{bmatrix},$$

$$\chi_{11}:=2\mathfrak{P}((\mathfrak{I}\otimes\mathfrak{A})+\mathfrak{K}+(1-\Phi)(\mathbf{B}\otimes\Gamma))+\mathfrak{Q}-\mathfrak{H}-\varepsilon\overline{\mathfrak{R}}_{\Lambda_1},$$

$$\chi_2:=\begin{bmatrix}\mathfrak{P}-\mathfrak{I} & \mathfrak{C}\mathfrak{P} & \chi_{21} & \chi_{22}\\ 0 & 0 & \widehat{\wp}\Phi(\mathbf{B}\otimes\Gamma)^T\mathfrak{P} & \chi_{23}\\ 0 & 0 & 0 & 0\\ 0 & 0 & \widehat{\wp}\mathfrak{B} & 0\\ -\gamma\mathfrak{I} & 0 & \widehat{\wp}\mathfrak{I} & 0\end{bmatrix},$$

$$\chi_{21}:=\widehat{\wp}((\mathfrak{I}\otimes\mathfrak{A})+\mathfrak{K}+(1-\Phi)(\mathbf{B}\otimes\Gamma))^T\mathfrak{P},$$

$$\chi_{22}:=\widehat{\wp}\sqrt{\Phi(1-\Phi)}(\mathbf{B}\otimes\Gamma)^T\mathfrak{P},$$

$$\chi_{23}:=-\widehat{\wp}\sqrt{\Phi(1-\Phi)}(\mathbf{B}\otimes\Gamma)^T\mathfrak{P},$$

$$\chi_3:=\begin{bmatrix}-\mathfrak{U} & 0 & 0\\ \maltese & \mathfrak{H}-2\mathfrak{P} & 0\\ \maltese & \maltese & \mathfrak{H}-2\mathfrak{P}\end{bmatrix},$$

with

$$\overline{\mathfrak{R}}_{\Lambda_1} = \frac{\mathfrak{R}^T_{\Lambda_1} + \mathfrak{S}^T_{\Lambda_1} + \mathfrak{S}^T_{\Lambda_1} + \mathfrak{R}_{\Lambda_1}}{2}, \ \overline{\mathfrak{S}}_{\Lambda_1} = -\frac{\mathfrak{R}^T_{\Lambda_1} + \mathfrak{S}^T_{\Lambda_1}}{2}, \ \mathfrak{R}_{\Lambda_1} = diag_{\mathbf{O}}(\mathfrak{R}_1), \ \mathfrak{S}_{\Lambda_1} = diag_{\mathbf{O}}(\mathfrak{S}_1)$$

Proof. Define the L-K functional

$$\mathfrak{V}(\rho(\imath), \imath) = \mathfrak{V}_1(\rho(\imath), \imath) + \mathfrak{V}_2(\rho(\imath), \imath) + \mathfrak{V}_3(\rho(\imath), \imath) + \mathfrak{V}_4(\rho(\imath), \imath), \tag{13}$$

where

$$\mathfrak{V}_1(\rho(\imath), \imath) = \rho^T(\imath)\mathfrak{P}\rho(\imath), \tag{14}$$

$$\mathfrak{V}_2(\rho(\imath), \imath) = \int_{\imath-\widehat{\wp}}^{\imath} \rho^T(\mathfrak{s})\mathfrak{Q}\rho(\mathfrak{s})d\mathfrak{s}, \tag{15}$$

$$\mathfrak{V}_3(\rho(\imath), \imath) = \widehat{\wp}\int_{-\widehat{\wp}}^{0}\int_{\imath+\mathfrak{s}}^{\imath} \dot{\rho}^T(\theta)\mathfrak{H}\dot{\rho}(\theta)d\theta d\mathfrak{s}, \tag{16}$$

$$\mathfrak{V}_4(\rho(\imath), \imath) = \mathfrak{I}\int_{-\mathfrak{I}}^{0}\int_{\imath+\mathfrak{s}}^{\imath} \widetilde{\mathfrak{F}}^T(\rho(\theta))\mathfrak{U}\widetilde{\mathfrak{F}}(\rho(\theta))d\theta d\mathfrak{s}. \tag{17}$$

Applying $\mathfrak{L}$, then

$$\mathfrak{L}\mathfrak{V}(\rho(\imath), \imath) = \mathfrak{L}\mathfrak{V}_1(\rho(\imath), \imath) + \mathfrak{L}\mathfrak{V}_2(\rho(\imath), \imath) + \mathfrak{L}\mathfrak{V}_3(\rho(\imath), \imath) + \mathfrak{L}\mathfrak{V}_4(\rho(\imath), \imath), \tag{18}$$

where

$$\begin{aligned}\mathfrak{L}\mathfrak{V}_1(\rho(\imath), \imath) = 2\rho^T(\imath)\mathfrak{P}\Big[&(\mathfrak{I}\otimes\mathfrak{A} + \mathfrak{K} + (1-\Phi)(\mathbf{B}\otimes\Gamma))\rho(\imath) + \mathfrak{B}\widetilde{\mathfrak{F}}(\rho(\imath)) + \mathfrak{C}\int_{\imath-\mathfrak{I}(\imath)}^{\imath}\widetilde{\mathfrak{F}}(\rho(\mathfrak{s}))d\mathfrak{s} \\ &+ \Phi(\mathbf{B}\otimes\Gamma)\rho(\imath-\wp(\imath)) + \mathbf{w}(\imath)\Big],\end{aligned} \tag{19}$$

$$\mathfrak{L}\mathfrak{V}_2(\rho(\imath), \imath) = \rho^T(\imath)\mathfrak{Q}\rho(\imath) - \rho^T(\imath-\widehat{\wp})\mathfrak{Q}\rho(\imath-\widehat{\wp}), \tag{20}$$

$$\mathfrak{L}\mathfrak{V}_3(\rho(\imath), \imath) = \widehat{\wp}^2\dot{\rho}^T(\imath)\mathfrak{H}\dot{\rho}(\imath) - \widehat{\wp}\int_{\imath-\widehat{\wp}}^{\imath}\dot{\rho}^T(\mathfrak{s})\mathfrak{H}\dot{\rho}(\mathfrak{s})d\mathfrak{s}, \tag{21}$$

$$\mathfrak{L}\mathfrak{V}_4(\rho(\imath), \imath) = \mathfrak{I}^2\widetilde{\mathfrak{F}}^T(\rho(\imath))\mathfrak{U}\widetilde{\mathfrak{F}}(\rho(\imath)) - \mathfrak{I}\int_{-\mathfrak{I}}^{\imath}\widetilde{\mathfrak{F}}^T(\rho(s))\mathfrak{U}\widetilde{\mathfrak{F}}(\rho(s))d\mathfrak{s}. \tag{22}$$

By using Lemma 2.2

$$-\widehat{\wp}\int_{\imath-\widehat{\wp}}^{\imath}\dot{\rho}^{T}(\mathfrak{s})ds\mathfrak{H}\dot{\rho}(\mathfrak{s})d\mathfrak{s}\leq\begin{bmatrix}\rho(\imath)\\ \rho(\imath-\wp(\imath))\\ \rho(\imath-\widehat{\wp})\end{bmatrix}^{T}\begin{bmatrix}-\mathfrak{H} & \mathfrak{H}-\mathfrak{M} & \mathfrak{M}\\ \maltese & -2\mathfrak{H}+\mathfrak{M}+\mathfrak{M}^{T} & -\mathfrak{M}+\mathfrak{H}\\ \maltese & \maltese & -\mathfrak{H}\end{bmatrix}\begin{bmatrix}\rho(\imath)\\ \rho(\imath-\wp(\imath))\\ \rho(\imath-\widehat{\wp})\end{bmatrix}. \quad (23)$$

By using Lemma 2.4 the above integral terms will become

$$-\mathfrak{I}\int_{-\mathfrak{I}}^{\imath}\widetilde{\mathfrak{F}}^{T}(\rho(s))\mathfrak{U}\widetilde{\mathfrak{F}}(\rho(s))d\mathfrak{s}\leq-\Big(\int_{\imath-\mathfrak{I}(\imath)}^{\imath}\widetilde{\mathfrak{F}}^{T}(\rho(s))d\mathfrak{s}\Big)^{T}\mathfrak{U}\Big(\int_{\imath-\mathfrak{I}(\imath)}^{\imath}\widetilde{\mathfrak{F}}(\rho(s))d\mathfrak{s}\Big). \quad (24)$$

Moreover, one has

$$\mathfrak{L}\{\widehat{\wp}^{2}\dot{\rho}^{T}(\imath)\mathfrak{H}\dot{\rho}(\imath)\}=\widehat{\wp}^{2}\varsigma^{T}(\imath)\Big\{\zeta_{1}^{T}\mathfrak{H}\zeta_{1}+\Phi(1-\Phi)\zeta_{2}^{T}\mathfrak{H}\zeta_{2}\Big\}, \quad (25)$$

where

$$\zeta_{1}=\Big[\widehat{\wp}(\mathfrak{I}\otimes\mathfrak{A})+\mathfrak{K}+(1-\Phi)(\mathbf{B}\otimes\Gamma)\ \ \widehat{\wp}\Phi(\mathbf{B}\otimes\Gamma)\ \ 0\ \ \widehat{\wp}\mathfrak{B}\ \ \widehat{\wp}\mathfrak{I}\ \ 0\Big],$$

$$\zeta_{2}=\Big[\widehat{\wp}(\mathbf{B}\otimes\Gamma)\ \ -\widehat{\wp}(\mathbf{B}\otimes\Gamma)\ \ 0\ \ 0\ \ 0\ \ 0\Big],$$

$$\varsigma(\imath)=\Big[\rho^{T}(\imath)\ \ \rho^{T}(\imath-\wp(\imath))\ \ \rho^{T}(\imath-\widehat{\wp})\ \ \widetilde{\mathfrak{F}}(\rho(\imath))\ \ \mathbf{w}(\imath)\ \ \Big(\int_{\imath-\mathfrak{I}(\imath)}^{\imath}\widetilde{\mathfrak{F}}(\rho(s))d\mathfrak{s}\Big)^{T}\Big]^{T}.$$

From (4), for any ε the nonlinear functions $\widehat{\mathfrak{F}}(\ell(\imath))$ satisfy

$$-\varepsilon\begin{bmatrix}\rho(\imath)\\ \widehat{\mathfrak{F}}(\rho(\imath))\end{bmatrix}^{T}\begin{bmatrix}\overline{\mathfrak{R}}_{\Lambda_1} & \overline{\mathfrak{S}}_{\Lambda_1}\\ \maltese & \mathfrak{I}\end{bmatrix}\begin{bmatrix}\rho(\imath)\\ \widehat{\mathfrak{F}}(\rho(\imath))\end{bmatrix}\geq 0, \quad (26)$$

where $\overline{\mathfrak{R}}_{\Lambda_1}$, $\overline{\mathfrak{S}}_{\Lambda_1}$ are also defined in.

The following expression is supplied after recalling the last target to demonstrate nonattendance of the passivity property:

$$\mathfrak{J}(\iota_\pi) := 2\mathfrak{E}\Big\{\int_0^{\iota_\pi} \rho^T(\iota)\mathbf{w}(\iota)d\iota\Big\} \geq -\mathrm{\gamma}\mathfrak{E}\Big\{\int_0^{\iota_\pi} \mathbf{w}^T(\iota)\mathbf{w}(\iota)d\iota\Big\},$$

$$\Rightarrow -2\mathfrak{E}\Big\{\int_0^{\iota_\pi} \rho^T(\iota)\mathbf{w}(t)dt\Big\} - \mathrm{\gamma}\mathfrak{E}\Big\{\int_0^{\iota_\pi} \mathbf{w}^T(\iota)\mathbf{w}(t)d\iota\Big\} \leq 0,$$

$$\mathfrak{J}(\iota_\pi) := \mathfrak{E}\Big\{\int_0^{\iota_\pi} -2\rho^T(\iota)\mathbf{w}(\iota) - \mathrm{\gamma}\mathbf{w}^T(t)\mathbf{w}(\iota)\Big\}. \tag{27}$$

Additionally, observe that it is true for the matrices $\mathfrak{P}$ and $\mathfrak{H}$.

$$-\mathfrak{P}\mathfrak{H}^{-1}\mathfrak{P} < \mathfrak{H} - 2\mathfrak{P} \tag{28}$$

Finally, using Lemma 2.3 and equivalent transformations, combining (27) and (36), it is possible to deduce that

$$\mathfrak{E}\Big\{\mathfrak{L}\mathfrak{V}(\rho(\iota),\iota) - 2\rho^T(\iota)\mathbf{w}(\iota) - \mathrm{\gamma}\mathbf{w}^T(\iota)\mathbf{w}(\iota)\Big\} \leq \mathfrak{E}\Big\{\varsigma^T(\iota)\chi\varsigma(\iota)\Big\} < 0. \tag{29}$$

where $\varsigma(\iota)$ is defined in Equ. (33).

If we integrate the two sides of (37) with regard to t across the range of 0 to t_π, we have

$$2\mathfrak{E}\Big\{\int_0^{\iota_\pi} \rho^T(\iota)\rho(\iota)d\iota\Big\} \geq \mathfrak{E}\Big\{\int_0^{\iota_\pi} \mathfrak{L}\mathfrak{V}(\rho(\iota),\iota) - \mathrm{\gamma}\rho^T(\iota)\mathbf{w}(\iota)d\iota\Big\} \geq -\mathrm{\gamma}\mathfrak{E}\Big\{\int_0^{\iota_\pi} \mathbf{w}^T(\iota)\mathbf{w}(\iota)d\iota\Big\}. \tag{30}$$

Thus, according to Definition 2.1, the ensuing synchronization of CDNs (9) is stochastically passive, concluding the argument. □

Theorem 2 Assumption 1 true, for given scalars Φ, $\widehat{\wp}$, $\mathfrak{I}$ and ε are positive, matrices $\mathfrak{R}_1$, $\mathfrak{S}_1$ the system (9) is stochastically passive, $\exists$ matrices $\mathfrak{P} = diag\{\mathfrak{P}_1, \mathfrak{P}_2, ..., \mathfrak{P}_{\mathbf{O}}\} > 0$, $\mathfrak{Q} = diag\{\mathfrak{Q}_1, \mathfrak{Q}_2, ..., \mathfrak{Q}_{\mathbf{O}}\} > 0$, $\mathfrak{H} = diag\{\mathfrak{H}_1, \mathfrak{H}_2, ..., \mathfrak{H}_{\mathbf{O}}\} > 0$, $\mathfrak{U} = diag\{\mathfrak{U}_1, \mathfrak{U}_2, ..., \mathfrak{U}_{\mathbf{O}}\} > 0$, matrix $\mathfrak{D} = diag\{\mathfrak{D}_1, \mathfrak{D}_2, ..., \mathfrak{D}_{\mathbf{O}}\} > 0$, $\mathfrak{M}$ and a constant $\mathrm{\gamma} > 0$ then following LMIs holds:

$$\chi := \begin{bmatrix} \chi_1 & \chi_2 \\ \maltese & \chi_3 \end{bmatrix},$$

$$\chi_1 := \begin{bmatrix} \chi_{11} & \beta\mathfrak{P}(\mathbf{B}\otimes\Gamma)+\mathfrak{H}-\mathfrak{M} & \mathfrak{M} & \mathfrak{B}\mathfrak{P}-\varepsilon\overline{\mathfrak{S}}_{\Lambda_1} \\ \maltese & -2\mathfrak{H}+\mathfrak{M} & -\mathfrak{M}+\mathfrak{H} & 0 \\ \maltese & \maltese & -\mathfrak{Q}-\mathfrak{H} & 0 \\ \maltese & \maltese & \maltese & -\varepsilon\mathfrak{I}+h^2\mathfrak{U} \end{bmatrix},$$

$$\chi_{11} := 2\mathfrak{P}(\mathfrak{I}\otimes\mathfrak{A})+2\mathfrak{D}+(1-\Phi)(\mathbf{B}\otimes\Gamma)+\mathfrak{Q}-\mathfrak{H}-\varepsilon\overline{\mathfrak{R}}_{\Lambda_1},$$

$$\chi_2 := \begin{bmatrix} \mathfrak{P}-\mathfrak{I} & \mathfrak{C}\mathfrak{P} & \chi_{21} & \chi_{22} \\ 0 & 0 & \widehat{\wp}\Phi(\mathbf{B}\otimes\Gamma)^T\mathfrak{P} & \chi_{23} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \widehat{\wp}\mathfrak{B} & 0 \\ -\gamma\mathfrak{I} & 0 & \widehat{\wp}\mathfrak{I} & 0 \end{bmatrix},$$

$$\chi_{21} := \widehat{\wp}(\mathfrak{I}\otimes\mathfrak{A})^T\mathfrak{P}+\widehat{\wp}\mathfrak{K}^T\mathfrak{P}+\mathfrak{P}(1-\Phi)(\mathbf{B}\otimes\Gamma)^T\mathfrak{P},$$

$$\chi_{22} := \widehat{\wp}\sqrt{\Phi(1-\Phi)}(\mathbf{B}\otimes\Gamma)^T\mathfrak{P},$$

$$\chi_{23} := -\widehat{\wp}\sqrt{\Phi(1-\Phi)}(\mathbf{B}\otimes\Gamma)^T\mathfrak{P},$$

$$\chi_3 := \begin{bmatrix} -\mathfrak{U} & 0 & 0 \\ \maltese & \mathfrak{H}-2\mathfrak{P} & 0 \\ \maltese & \maltese & \mathfrak{H}-2\mathfrak{P} \end{bmatrix},$$

with

$$\overline{\mathfrak{R}}_{\Lambda_1} = \frac{\mathfrak{R}^T_{\Lambda_1}+\mathfrak{S}^T_{\Lambda_1}+\mathfrak{S}^T_{\Lambda_1}+\mathfrak{R}_{\Lambda_1}}{2},\ \overline{\mathfrak{S}}_{\Lambda_1} = -\frac{\mathfrak{R}^T_{\Lambda_1}+\mathfrak{S}^T_{\Lambda_1}}{2},\ \mathfrak{R}_{\Lambda_1} = diag_{\mathbf{O}}(\mathfrak{R}_1),\ \mathfrak{S}_{\Lambda_1} = diag_{\mathbf{O}}(\mathfrak{S}_1),$$

it through the desired controllers can be represented as $\mathfrak{K}_\eta = \mathfrak{P}_\eta^{-1}\mathfrak{D}_\Phi$.

Proof. $\mathfrak{D} = \mathfrak{P}\mathfrak{K}$. The method of derivation approaches that used in Theorem 3.1. So it easy omitted here. □

Remark 1 Additionally, if we take into consideration the delayed CDNs in (1) without distributed time-varying delays and $\mathfrak{B} = \mathfrak{I}$, then

$$\begin{cases} \dot{\mathfrak{p}}_{\eta}(\iota) = \mathfrak{A}\mathfrak{p}_{\eta}(\iota) + \mathfrak{f}(\iota, \mathfrak{p}_{\eta}(\iota)) + (1-\Phi(\iota))\sum_{\psi=1}^{\mathbf{O}} \mathbf{b}_{\eta\psi}\Gamma\mathfrak{p}_{\psi}(\iota) + \Phi(\iota)\sum_{\psi=1}^{\mathbf{O}} \mathbf{b}_{\eta\psi}\Gamma\mathfrak{p}_{\psi}(\iota - \wp(\iota)) \\ \qquad + \mathbf{u}_{\eta}(\iota) + \mathbf{w}_{\eta}(\iota), \\ \mathfrak{p}_{\eta}(\iota) = {}_{\eta}(\iota), \quad \iota \in [-\widehat{\wp}, 0]. \end{cases} \tag{31}$$

Then, using a similar concept to the proof of Theorem 3.2 that follows, we reach at 3.3.

Corollary 1 Under Assumption 1 holds true, the system (9) is stochastically passive considering that the given scalars Φ, $\widehat{\wp}$, $\mathfrak{I}$, and ε are positive, matrices $\mathfrak{R}_1$, $\mathfrak{S}_1$, if it contains matrices $\mathfrak{P} = diag\{\mathfrak{P}_1, \mathfrak{P}_2, ..., \mathfrak{P}_{\mathbf{O}}\} > 0$, $\mathfrak{Q} = diag\{\mathfrak{Q}_1, \mathfrak{Q}_2, ..., \mathfrak{Q}_{\mathbf{O}}\} > 0$, $\mathfrak{H} = diag\{\mathfrak{H}_1, \mathfrak{H}_2, ..., \mathfrak{H}_{\mathbf{O}}\} > 0$, $\mathfrak{U} = diag\{\mathfrak{U}_1, \mathfrak{U}_2, ..., \mathfrak{U}_{\mathbf{O}}\} > 0$, matrix $\mathfrak{D} = diag\{\mathfrak{D}_1, \mathfrak{D}_2, ..., \mathfrak{D}_{\mathbf{O}}\} > 0$, $\mathfrak{M}$ and a constant $\gamma > 0$ such that the resulting LMIs are true:

$$\chi := \begin{bmatrix} \chi_1 & \chi_2 \\ \maltese & \chi_3 \end{bmatrix},$$

$$\chi_1 := \begin{bmatrix} \chi_{11} & \Phi\mathfrak{P}(\mathbf{B}\otimes\Gamma) + \mathfrak{H} - \mathfrak{M} & M & \mathfrak{P} - \varepsilon\overline{\mathfrak{S}}_{\Lambda_1} \\ \maltese & -2\mathfrak{H} + \mathfrak{M} & -\mathfrak{M} + \mathfrak{H} & 0 \\ \maltese & \maltese & -\mathfrak{Q} - \mathfrak{H} & 0 \\ \maltese & \maltese & \maltese & -\varepsilon\mathfrak{I} \end{bmatrix},$$

$$\chi_{11} := 2\mathfrak{P}(\mathfrak{I}\otimes\mathfrak{A}) + 2\mathfrak{D} + (1-\Phi)(\mathbf{B}\otimes\Gamma) + \mathfrak{Q} - \mathfrak{H} - \varepsilon\overline{\mathfrak{R}}_{\Lambda_1},$$

$$\chi_2 := \begin{bmatrix} \mathfrak{P} - \mathfrak{I} & \chi_{21} & \chi_{22} \\ 0 & \widehat{\wp}\Phi(\mathbf{B}\otimes\Gamma)^T\mathfrak{P} & \chi_{23} \\ 0 & 0 & 0 \\ 0 & \widehat{\wp}\mathfrak{I} & 0 \end{bmatrix},$$

$$\chi_{21} := \widehat{\wp}(\mathfrak{I}\otimes\mathfrak{A})^T\mathfrak{P} + \widehat{\wp}\mathfrak{K}^T\mathfrak{P} + \mathfrak{P}(1-\Phi)(\mathbf{B}\otimes\Gamma)^T\mathfrak{P},$$

$$\chi_{22} := \widehat{\wp}\sqrt{\Phi(1-\Phi)}(\mathbf{B}\otimes\Gamma)^T\mathfrak{P},$$

$$\chi_{23} := -\widehat{\wp}\sqrt{\Phi(1-\Phi)}(\mathbf{B}\otimes\Gamma)^T\mathfrak{P},$$

$$\chi_3 := \begin{bmatrix} -\gamma\mathfrak{I} & \widehat{\wp}\mathfrak{I} & 0 \\ \maltese & \mathfrak{H} - 2\mathfrak{P} & 0 \\ \maltese & \maltese & \mathfrak{H} - 2\mathfrak{P} \end{bmatrix},$$

with

$$\overline{\mathfrak{R}}_{\Lambda_1} = \frac{\mathfrak{R}_{\Lambda_1}^T + \mathfrak{S}_{\Lambda_1}^T + \mathfrak{S}_{\Lambda_1}^T + \mathfrak{R}_{\Lambda_1}}{2}, \ \overline{\mathfrak{S}}_{\Lambda_1} = -\frac{\mathfrak{R}_{\Lambda_1}^T + \mathfrak{S}_{\Lambda_1}^T}{2}, \ \mathfrak{R}_{\Lambda_1} = diag_{\mathbf{O}}(\mathfrak{R}_1), \ \mathfrak{S}_{\Lambda_1} = diag_{\mathbf{O}}(\mathfrak{S}_1),$$

moreover, the desired gains from the controllers might be presented as $\mathfrak{K}_\eta = \mathfrak{P}_\eta^{-1}\mathfrak{D}_\eta$, $\eta = 1, 2, ..., \mathbf{O}$.

4. Numerical example

Example 1 Consider the (1) with 4-nodes, 2-dimensional nodes. So far, one has $\mathbf{O} = 4$, $n = 2$. Following are the parameters:

$$\mathfrak{A} = \begin{bmatrix} 2.5 & -0.75 \\ 1.5 & -0.5 \end{bmatrix}, \ \mathfrak{B} = \begin{bmatrix} -3.5 & -4 \\ -2.5 & -3.9 \end{bmatrix}, \ \mathfrak{C} = \begin{bmatrix} -3.5 & -1.48 \\ 2.5 & -3.9 \end{bmatrix}.$$

The inner-coupling Γ and the network topology $\mathbf{B}$ matrices are given as

$$\Gamma = \begin{bmatrix} 0.2 & -1.5 \\ 1.4 & -1.3 \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} -2 & 2 & 0 & 0 \\ 0 & -2 & 2 & 0 \\ 0 & 0 & -2 & 2 \\ 2 & 0 & 0 & -2 \end{bmatrix}.$$

The functions $\mathfrak{f}(\cdot)$ is

$$\mathfrak{f}(\mathfrak{p}_\eta(\iota)) = \begin{bmatrix} -0.5\mathfrak{p}_{\eta 1}(\iota) + tanh(0.2\mathfrak{p}_{\eta 1}(\iota)) + 0.2\mathfrak{p}_{\eta 2}(\iota)) \\ 0.95\mathfrak{p}_{\eta 2}(\iota)) - tanh(0.75\mathfrak{p}_{\eta 2}(\iota)) \end{bmatrix}, \quad \eta = 1, 2, 3, 4.$$

It is evident that $\mathfrak{f}(\cdot)$ is satisfied,

$$\mathfrak{R}_1 = \begin{bmatrix} -0.3 & 0 \\ 0 & 0.2 \end{bmatrix}, \ \mathfrak{S}_1 = \begin{bmatrix} -0.5 & 0 \\ 0 & 0.2 \end{bmatrix}.$$

The specified delays are used as $\wp(\iota) = 1.95 + 0.05 sin(10\iota)$, $\widehat{\wp} = 0.2$, $\Phi = 0.5$, $\mathfrak{I} = 0.1$, $\varepsilon = 0.9$. So that Table 1 lists the largest permissible upper limitations of $\widehat{\wp}$ for various Φ values.

Table 1. Maximum allowable bounds of $\widehat{\wp}$ with different values of Φ for Example 4.1

$\widehat{\wp}$	0.1	0.2	0.3	0.4	0.5	0.7	0.9
Theorem 3.2	2.2220	2.7154	2.8124	2.8345	3.9212	3.9218	3.9240

The resulting controller gain matrices are

$$\mathfrak{K}_1 = \begin{bmatrix} -625.8544 & 399.8733 \\ 334.1447 & -223.2991 \end{bmatrix}, \ \mathfrak{K}_2 = \begin{bmatrix} -399.3295 & 263.8947 \\ 232.6038 & -163.0190 \end{bmatrix}, \ \mathfrak{K}_3 = \begin{bmatrix} -399.3295 & 263.8947 \\ 232.6038 & -163.0190 \end{bmatrix}.$$

5. Conclusion

In this study, we propose a novel approach for improving passive synchronization in complex dynamical networks (CDNs) with time-varying distributed coupling delays that occur randomly. In particular, the randomly occurring coupling delays are modeled using a Bernoulli random variable. By developing a feedback controller and utilizing a Lyapunov-Krasovskii functional (LKF) with integral terms, we derive synchronization criteria in the form of linear matrix inequalities (LMIs). Finally, a numerical example is presented to demonstrate the effectiveness of the proposed solutions.

Conflict of interest

The authors declare no competing financial interest.

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