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Research Article

Novel Group Acceptance Sampling Plan for Modified Power Exponential Distribution with Application to Quality and Reliability Analysis

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Abstract: This study introduces a group acceptance sampling plan (GASP) utilizing a truncated life test in which the lifespan of an object follows a modified power exponential distribution. The median is used as a quality measure for various constraint design criteria, including consumer and producer risks, as well as the minimum group size required for a specified acceptance number and test termination time. The optimized values are presented in tables and graphs. In addition, we illustrate our findings using real-world datasets. Furthermore, we compared different distributions under the GASP framework and compared GASP with the ordinary sampling plan (OSP) approach. Simulations were performed using the estimated parametric values. Future research is recommended to further enhance the efficiency and quality control procedures.

***Keywords*:** producer risk, optimized values, quality index, sampling plan, GASP

MSC: 65L05, 34K06, 34K28

1. Introduction

In recent years, there has been increasing emphasis on the enhancement, measurement, and monitoring of the quality of products, services, and procedures. The recognition of a robust correlation among productivity, reputation, quality, and confidence in a brand's image is the driving force behind this trend. Consequently, organizations in a variety of sectors have been investing in quality management systems, process improvement initiatives, and customer feedback mechanisms to guarantee that their products and services satisfy the expectations and requirements of their clients. Currently, companies are considering the implementation of statistical quality control (SQC) procedures, which are critical for improving their market competitiveness. Quality control (QC) has undergone a transformation from its initial definition, which predominantly entailed the adaptation of production to a standardized model to satisfy customer demands. It is now implemented in a variety of industrial and service sectors in addition to manufacturing procedures. To identify and eradicate sources of variability and guarantee consistent adherence to quality standards, these methods are implemented to supervise and regulate the quality of products, services, and procedures as well as to statistically analyze

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the data collected during production or service delivery. SQC also employs statistical tools and methodologies to analyze data and make informed decisions based on objective evidence, rather than relying on subjective judgments [1–4].

Several researchers have developed GASP strategies to maximize product efficacy while reducing time and expenses, particularly when an item's lifetime follows specific distributions (see [4–9]). Aslam et al. [10] have performed several studies on GASP. These studies explored the fundamental principles of GASP and the process of selecting items with minimal risk to consumers and producers. Aslam and Jun [11] proposed GASP for lifetime data following a Weibull distribution. Aslam et al. [12] presented a GASP for resubmitted lots using the Burr XII distribution. Aslam et al. [5] developed a GASP for the generalized Pareto distribution. Ameeq et al. [13] employed the alpha power transformation inverted perk distribution, and Ahsan-ul-Haq et al. [2] developed a GASP for a one shape parameter power-inverted Nadarajah-Haghighi distribution using the median as a quality index. Al-Omari [14] presented GASP for the Garima distribution, Ameeq et al. [15] presented the truncated exponential logarithmic distribution to test the reliability of quality control using the median as quality index.

There is a growing interest in developing distributions with potential parameters using baseline distributions and compounding techniques. However, recent studies that introduced new parameter techniques have had a significant impact on the shape and convergent validity of models, as noted in the comprehensive review [13, 16–22]. As in our research experience, no previous studies have been conducted on the Modified Power Exponential Distribution (MPoE), as evidenced by the lack of literature on GASP. We considered the MPoE distribution because of its flexibility in modeling real-world data, particularly for quality control and reliability analysis. This distribution effectively captures the asymmetric and skewed behaviors often observed in electronic component quality characteristics, such as resistance or failure rates. Moreover, the median-based approach of the MPoE distribution provides a more robust quality index than traditional mean-based methods, which can be highly sensitive to outliers. Its ability to model extreme events makes it particularly suitable for acceptance sampling plans (ASP), ensuring that quality standards are met while optimizing inspection efficiency [4, 15].

The remainder of this paper is organized as follows. Section 2 defines the structure of the MPoE distribution, including cdf, pdf, and qf. Section 3 presents the conceptual framework of the MPoE distribution. In Section 4, the properties are derived and Section 5 describes and illustrates of an example where, in Section 6, the application is performed, and the simulation study is performed in Section 7. In Section 8, we present our findings and discuss future work.

2. Modified power exponential distribution

Recently [23] proposed the modified power family of distributions in their most recent article, and we will use a particular sub-model of this family, the modified power exponential distribution (MPoE). This distribution is characterized by its cumulative distribution function (CDF), probability density function (PDF), and quantile function (QF), which depend on two parameters: the shape parameter γ and the scale parameter α. Specifically, the CDF, PDF, and QF of the MPoE distribution with shape parameter γ and scale parameter α are given by:

The cumulative distribution function (CDF) is given by:

$$F(t) = \alpha^{-e^{-\gamma t}} \left(1 - e^{-\gamma t}\right), \tag{1}$$

where $\alpha > e^{-1}$ and γ, $t > 0$.

The probability density function (PDF) is given by:

$$f(t) = \gamma e^{-\gamma t} \alpha^{-e^{-\gamma t}} + \gamma e^{-\gamma t} \alpha^{-e^{-\gamma t}} \log(\alpha) \left(1 - e^{-\gamma t}\right). \tag{2}$$

As shown in Figures 1 and 2 with different parameter values that are: (a) $\alpha = 0.42$, $\gamma = 0.65$, (b) $\alpha = 0.44$, $\gamma = 0.7$, (c) $\alpha = 4.04$, $\gamma = 1.88$, (d) $\alpha = 0.72$, $\gamma = 1.52$, (e) $\alpha = 6.64$, $\gamma = 0.65$, (f) $\alpha = 1.73$, $\gamma = 1.5$.

The qth quantile function y_q of the MPoE distribution using Equation (1) is given by:

$$y_q = -\frac{1}{\gamma}\log\left[1 - W_0\left(\frac{\alpha\log(\alpha)q}{\log(\alpha)}\right)\right] \tag{3}$$

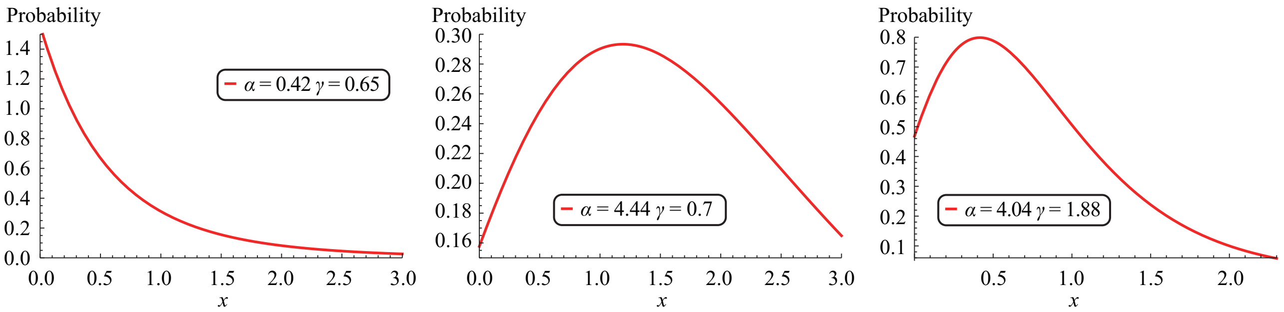


Figure 1. MPoE distribution pdf plots for some parametric values

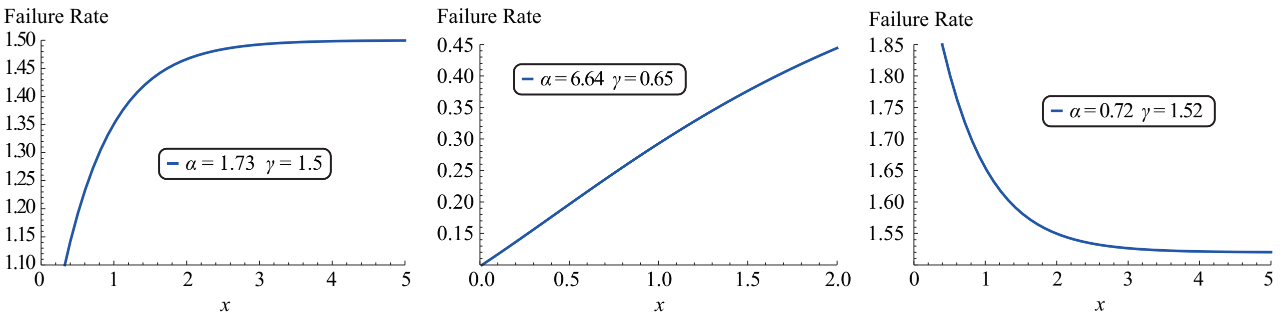


Figure 2. MPoE distribution hrf plots for some parametric values

3. Operating procedure for GASP

It is now possible to collect GASP parameters by utilizing the MPoE distribution framework. The process of utilizing a group acceptance plan involves the following steps, as briefly outlined below, while selecting the design parameters, as outlined in [10]:

i. The allocation of sample size n for a lot with a selected number of groups g and an allocation of predetermined r items to each group can be calculated using the formula $n = g \times r$.

ii. Selecting a group's acceptance number (c) and establishing the experiment's duration (t_o).

iii. To count the number of failures in every group and conduct an experiment simultaneously for every group (g).

iv. Accept the lot if, by the end of the trial, there are no more than d failures across all groups ($c = 0$).

v. If there are (c) or more failures in any group, suspend the experiment and discard the entire lot.

Thus, the proposed GASP for the MPoE distribution is defined for a given r and described by two design parameters $(g,\ c)$. In contrast to the median life, the CDF of the MPoE distribution is given in Equation (1) and is dependent on α median life, which is expressed in Equation (3). In the following equations, the likelihood of accepting a lot is expressed:

$$P_a(p) = \left[\sum_{j=0}^{c} \binom{r}{j} p^j (1-p)^{r-j} \right]^g, \tag{4}$$

p insinuates the number of probabilities an item in a group that would not succeed before t_0, and this is obtained by adding Equation (3) to Equation (1). Using Equation (3), we put

$$\Psi = -\frac{1}{\Gamma} \log \left[1 - W_0 \left(\frac{\alpha \log(\alpha) q}{\log(\alpha)} \right) \right] \tag{5}$$

and

$$\phi = \log \left[1 - W_0 \left(\frac{\alpha \log(\alpha) q}{\log(\alpha)} \right) \right] \tag{6}$$

Let $\Gamma = -\frac{\phi}{\Psi}$ and $\tilde{t}_0 = a_1 \Psi_0$. However, the standard of quality can be measured by comparing the actual lifespan of the product to its expected lifespan $\frac{\Psi}{\Psi_0}$ by putting $\Gamma = -\frac{\phi}{\Psi}$ and $t_0 = a_1 \Psi_0$ in Equation (1), and failure probability can be expressed as:

$$P = \alpha^{-e^{-\Gamma t}} \left(1 - e^{-\Gamma t} \right) \tag{7}$$

$$P = \alpha^{-e^{-\left(-\frac{\phi}{\Psi}\right)(a_1 \Psi_0)}} \left(1 - e^{-\left(-\frac{\phi}{\Psi}\right)(a_1 \Psi_0)} \right) \tag{8}$$

Above equation also written as,

$$P = \alpha^{-e^{a_1 \phi \left(\frac{\Psi}{\Psi_0}\right)^{-1}}} \left(1 - \alpha^{a_1 \phi \left(\frac{\Psi}{\Psi_0}\right)^{-1}} \right) \tag{9}$$

where a_1 is given and $r_2 = \frac{\Psi}{\Psi_0}$. Now, our current failure probabilities are p_1 and p_2, which are employed to convey risk to the producer and consumer, where the probability of rejecting a good lot is referred to as producer risk, whereas the likelihood of accepting a good campaign is referred to as consumer risk. In order to concurrently validate the next two equations for a given value α, a_1, we evaluate the value of g and c respectively.

$$
\text{minimize: } c \text{ and } g
$$

$$
\text{subject to: } P_a\left(p_1 \mid \frac{\Psi}{\Psi_0}=r_1\right) \leq \delta,
$$

$$
P_a\left(p_2 \mid \frac{\Psi}{\Psi_0}=r_2\right) \geq 1-\gamma,
$$

$$
\text{where } g,\ r \in \mathbb{Z}^+,\ 0 \leq c < r.
$$

where, $\mathbb{Z}^+ = \{1,\ 2,\ 3,\ \ldots\}$ symbolizes the group of whole numbers. In this instance, the acceptability zone is determined by:

$$
P_a\left(p_1 \mid \frac{\Psi}{\Psi_0}=r_1\right) = \left[\sum_{j=0}^{c}\binom{r}{j} p_1^j (1-p)^{r-j}\right]^g \leq \delta, \tag{10}
$$

$$
P_a\left(p_2 \mid \frac{\Psi}{\Psi_0}=r_2\right) = \left[\sum_{j=0}^{c}\binom{r}{j} p_2^j (1-p)^{r-j}\right]^g \geq 1-\gamma, \tag{11}
$$

The two-point approach in designing GASP considers the consumer's and producer's perspectives, ensuring an acceptable balance between producer risk (probability of rejecting a good lot) and consumer risk (probability of accepting a bad lot). This approach evaluates probabilities at two critical points to optimize the design parameters, as outlined in Equations (10) and (11).

Where r_1 and r_2 are the median ratios of consumer risk and producer risk. The probabilities that should be utilized in Equations (10) and (11) are as follows:

$$
P_1 = \alpha^{-e^{a_1\phi}}\left(1-e^{a_1\phi}\right) \tag{12}
$$

$$
P_2 = \alpha^{-e^{a_1\phi\left(\frac{\Psi}{\Psi_0}\right)^{-1}}}\left(1-e^{a_1\phi\left(\frac{\Psi}{\Psi_0}\right)^{-1}}\right) \tag{13}
$$

4. Properties

4.1 *Moment generating function (MGF)*

The moment generating function (MGF) of a random variable X is defined as:

$$
M_X(t) = \mathbb{E}[e^{tX}] = \int_0^\infty e^{tX} f(x)\mathrm{d}x.
$$

Substituting the PDF of the MPoE distribution:

$$M_X(t) = \int_0^{\infty} e^{tX} \left(\gamma e^{-\gamma x} \alpha^{-e^{-\gamma x}} + \gamma e^{-\gamma x} \alpha^{-e^{-\gamma x}} \log(\alpha)(1 - e^{-\gamma x}) \right) \mathrm{d}x.$$

Using Gamma function properties, after transformation and substitution

$$M_X(t) = \gamma \Gamma\left(\frac{t}{\gamma} + 1\right) \alpha^{-\Gamma\left(\frac{t}{\gamma}+1\right)} + \gamma \log(\alpha) \left[\Gamma\left(\frac{t}{\gamma} + 1\right) - \Gamma\left(\frac{t}{\gamma} + 2\right)\right].$$

Thus, the final expression for the MGF in terms of the Gamma function is:

$$M_X(t) = \gamma \Gamma\left(\frac{t}{\gamma} + 1\right) \left[\alpha^{-\Gamma\left(\frac{t}{\gamma}+1\right)} + \log(\alpha) \left(1 - \frac{t}{\gamma + 1}\right)\right].$$

4.2 *Shannon entropy*

The Shannon entropy is given by:

$$H(X) = -\int_0^{\infty} f(x) \log f(x) \mathrm{d}x.$$

Substituting the PDF, we get:

$$H(X) = -\int_0^{\infty} \left(\gamma e^{-\gamma x} \alpha^{-e^{-\gamma x}} + \gamma e^{-\gamma x} \alpha^{-e^{-\gamma x}} \log(\alpha)(1 - e^{-\gamma x}) \right) \log f(x) \mathrm{d}x.$$

4.3 *Order Statistics*

Let $X_1, X_2, \ldots, X_n$ be a random sample from the MPoE distribution with cumulative distribution function (CDF) and probability density function (PDF) given by:

$$F(t) = \alpha^{-e^{-\gamma t}} \left(1 - e^{-\gamma t}\right),$$

$$f(t) = \gamma e^{-\gamma t} \alpha^{-e^{-\gamma t}} + \gamma e^{-\gamma t} \alpha^{-e^{-\gamma t}} \log(\alpha) \left(1 - e^{-\gamma t}\right).$$

The kth order statistic $X_{k:n}$ in a random sample of size n has the probability density function given by:

$$f_{X_{k:n}}(t) = \frac{n!}{(k-1)!(n-k)!} [F(t)]^{k-1} [1 - F(t)]^{n-k} f(t).$$

Substituting $F(t)$ and $f(t)$.
Using the given CDF and PDF, we substitute:

$$F(t)^{k-1} = \left[\alpha^{-e^{-\gamma t}} \left(1 - e^{-\gamma t}\right)\right]^{k-1},$$

$$(1 - F(t))^{n-k} = \left[1 - \alpha^{-e^{-\gamma t}} \left(1 - e^{-\gamma t}\right)\right]^{n-k},$$

$$f(t) = \gamma e^{-\gamma t} \alpha^{-e^{-\gamma t}} + \gamma e^{-\gamma t} \alpha^{-e^{-\gamma t}} \log(\alpha) \left(1 - e^{-\gamma t}\right).$$

Thus, the PDF of the kth order statistic is:

$$f_{X_{k:n}}(t) = \frac{n!}{(k-1)!(n-k)!} \left[\alpha^{-e^{-\gamma t}} (1 - e^{-\gamma t})\right]^{k-1}$$

$$\times \left[1 - \alpha^{-e^{-\gamma t}} (1 - e^{-\gamma t})\right]^{n-k}$$

$$\times \left[\gamma e^{-\gamma t} \alpha^{-e^{-\gamma t}} + \gamma e^{-\gamma t} \alpha^{-e^{-\gamma t}} \log(\alpha)(1 - e^{-\gamma t})\right].$$

This formula provides the probability density function of the kth order statistic for the MpoE distribution.

5. Discussion with illustrative example

In electronic component quality control, consider an acceptance sampling plan (ASP) characterized by parameters g, c, and $P_a(p)$, representing the acceptance number, sample size, and probability of acceptance, respectively. Suppose $g = 4$, $c = 10$, and $P_a(p) = 95\%$. This implies that in a sample of ten components, the batch is accepted if four or fewer defective items are found; otherwise, it is rejected. Engineers monitor critical quality parameters, such as resistance, and use the MPoE distribution to determine the median as a quality index. The ASP necessitates routine batch sampling, ensuring a systematic assessment of production quality. This method provides several advantages: reduced inspection costs by sampling a smaller portion of each batch, enhanced overall product quality by maintaining at least 95% compliance with standards, efficient process monitoring using a robust median-based approach, and informed decision-making based on predefined acceptance criteria. The values of g, c, and $P_a(p)$ can be adjusted according to industry and product specifications, offering flexibility for diverse manufacturing applications.

The GASP are designed for various parametric values of consumer risk (0.25, 0.10, 0.05, and 0.01) and r_2 (2, 4, 6, and 8), r (5, 10), and a_1 (5, 10). The findings show that decreasing consumer risk tends to increase the number of groups and that as r_2 rises, the number of groups eventually declines. However, with a constant g and c, the likelihood of accepting a lot eventually increases. Tables 1 and 2 show the GASP with minimum g and c; therefore, when we take consumer risk 0.25 and $g = 25$, $r_2 = 4$, and $c = 2$, there should be required units of $(25 \times 4) = 100$ to conduct a life test. In contrast, when r_2 increases to 8, a total of $(5 \times 1) = 5$ units will be required to pass a life test on it, and the minimum required group will also be 5 for a consumer risk of 0.25. Under the MPoE distribution to consider GASP, and the median is used as a quality index, the OC values increase as g decreases and the true median lifetime increases, as shown graphically in Figure 3. Further shows that g and c tend to decrease as the true median life increases, while the OC values tend to increase gradually. Thus, much is accepted at these points. Consequently, accepting the lot for $r = 10$ and in Figure 3 would be better because it would require fewer groups than for $r = 5$.

In this study, we illustrate Tables 1 and 2 with relevant examples, providing comprehensive insights as detailed in [15]. Consider a scenario where the lifespan of a bulb is tested using the MPoE distribution with $\alpha = 1.05$ and an average lifespan of 2,000 cycles. Consumers face risks of 25% and 5%, corresponding to average lifetimes of 3,000 and 5,000 cycles, respectively.

In Table 3-10, To assess whether the average lifespan exceeds the recommended value, a researcher designs an experiment with 1,000 cycles and 10 units per group. Given the parameters $m_0 = 3,000$ cycles, $\alpha = 1.05$, $\delta = 0.25$, $r = 10$, and $a_1 = 1$, the mean lifespan must surpass the prescribed threshold if no more than one unit in each of the 21 groups fails before completing 1,000 cycles ($c = 2$, 42 units tested). For a scenario assuming an average lifespan of 5,000 cycles, the researcher organizes 21 groups with two units each. If one unit fails within 1,000 cycles ($a_1 = 0.5$), the test concludes with 95% confidence that the lifespan exceeds 5,000 cycles, leading to lot acceptance. Figure 3 presents the operating characteristic (OC) curves under various test conditions. The OC values increase with higher median lifetimes, indicating improved reliability as the number of groups and acceptance criteria are adjusted. These graphs effectively illustrate the impact of different testing parameters on the reliability assessment. (a) $\delta = 0.01$, $a_1 = 1$, $r = 10$, $\alpha = 1.05$, (b) $\delta = 0.10$, $a_1 = 0.5$, $r = 5$, $\alpha = 1.75$, (c) $\delta = 0.01$, $a_1 = 1$, $r = 10$, $\alpha = 1.05$. Figure 4 and Figure 5 showed MPoE distribution for data set I and set II. Figure 6 showed Plots of descriptive analysis for (a) Data set I and (b) Data set II.

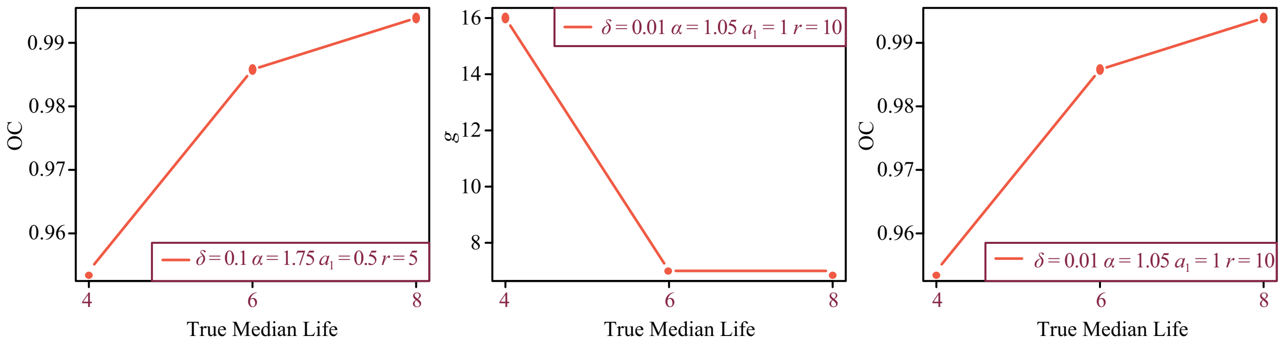


Figure 3. Illustration of OC curve values taken from Tables 1 and 2

Table 1. GASP displaying minimal g and c for $\alpha = 1.05$

		$r = 5$						$r = 10$					
		$a_1 = 0.5$			$a_1 = 1$			$a_1 = 0.5$			$a_1 = 1$		
δ	r_2	g	c	$P_a(p)$	g	c	$P_a(p)$	g	c	$P_a(p)$	g	c	$P_a(p)$
0.25	2	-	-	-	-	-	-	247	5	0.9589	-	-	-
	4	25	2	0.9670	25	3	0.9874	12	3	0.9852	3	3	0.9586
	6	25	2	0.9895	6	2	0.9820	4	2	0.9825	3	3	0.9895
	8	5	1	0.9667	2	1	0.9509	4	2	0.9921	2	2	0.9738
0.10	2	-	-	-	-	-	-	-	-	-	-	-	-
	4	376	3	0.9863	40	3	0.9799	20	3	0.9754	8	4	0.9854
	6	42	2	0.9825	9	2	0.9731	7	2	0.9695	4	3	0.9861
	8	42	2	0.9924	9	2	0.9880	7	2	0.9862	2	2	0.9738
0.05	2	-	-	-	-	-	-	-	-	-	-	-	-
	4	489	3	0.9822	52	3	0.9739	26	3	0.9682	11	4	0.9800
	6	54	2	0.9775	11	2	0.9672	9	2	0.9610	5	3	0.9826
	8	54	2	0.9902	11	2	0.9853	9	2	0.9823	3	2	0.9609
0.01	2	-	-	-	-	-	-	-	-	-	-	-	-
	4	751	3	0.9728	80	3	0.9601	39	3	0.9526	16	4	0.9710
	6	83	2	0.9657	80	3	0.9912	39	3	0.9892	7	3	0.9757
	8	83	2	0.9849	17	2	0.9774	13	2	0.9745	7	3	0.9913

Remark: A large sample length in needed cells contains hyphens (-)

Table 2. GASP displaying minimal g and c for $\alpha = 1.75$

		$r = 5$						$r = 10$					
		$a_1 = 0.5$			$a_1 = 1$			$a_1 = 0.5$			$a_1 = 1$		
δ	r_2	g	c	$P_a(p)$	g	c	$P_a(p)$	g	c	$P_a(p)$	g	c	$P_a(p)$
0.25	2	-	-	-	-	-	-	175	5	0.9662	-	-	-
	4	21	2	0.9722	4	3	0.9609	10	3	0.9876	2	3	0.9696
	6	21	2	0.9914	4	2	0.9876	4	2	0.9829	1	2	0.9714
	8	5	1	0.9676	2	1	0.9509	4	2	0.9924	1	2	0.9868
0.10	2	-	-	-	-	-	-	-	-	-	-	-	-
	4	35	2	0.9541	26	3	0.9855	16	3	0.9803	3	3	0.9548
	6	35	2	0.9858	7	2	0.9785	6	2	0.9744	3	3	0.9892
	8	35	2	0.9939	7	2	0.9906	6	2	0.9887	2	2	0.9738
0.05	2	-	-	-	-	-	-	-	-	-	-	-	-
	4	380	3	0.9862	34	3	0.9810	21	3	0.9742	7	4	0.9856
	6	45	2	0.9817	8	2	0.9754	8	2	0.9660	4	3	0.9856
	8	45	2	0.9922	8	2	0.9893	8	2	0.9849	2	2	0.9738
0.01	2	-	-	-	-	-	-	-	-	-	-	-	-
	4	583	3	0.9789	52	3	0.9711	32	3	0.9610	11	4	0.9775
	6	69	2	0.9721	13	2	0.9604	11	2	0.9536	5	3	0.9821
	8	69	2	0.9880	13	2	0.9827	11	2	0.9793	3	2	0.9609

Remark: A large sample length in needed cells contains hyphens (-)

Table 3. Optimized values for g, c, and $Pa(p)$

r_2	2	4	6	8
g	-	16	7	7
c	-	4	3	3
$Pa(p)$	-	0.9710	0.9757	0.9913
g	-	35	35	35
c	-	2	2	2
$Pa(p)$	-	0.9541	0.9858	0.9939

Comments: The cells with (-) indicate that a larger sample size is required

6. Applications

6.1 *Data set I*

Table 4. Survival periods (months) of 121 breast cancer patients taken from [24]

0.3	0.3	4.0	5.0	5.6	6.2	6.3	6.6	6.8	7.4	7.5	8.4
8.4	10.3	11.0	11.8	12.2	12.3	13.5	14.4	14.4	14.8	15.5	15.7
16.2	16.3	16.5	16.8	17.2	17.3	17.5	17.9	19.8	20.4	20.9	21.0
21.0	21.1	23.0	23.4	23.6	24.0	24.0	27.9	28.2	29.1	30.0	31.0
31.0	32.0	35.0	35.0	37.0	37.0	37.0	38.0	38.0	38.0	39.0	39.0
40.0	40.0	40.0	41.0	41.0	41.0	42.0	43.0	43.0	43.0	44.0	45.0
45.0	46.0	46.0	47.0	48.0	49.0	51.0	51.0	51.0	52.0	54.0	55.0
56.0	57.0	58.0	59.0	60.0	60.0	60.0	61.0	62.0	65.0	65.0	67.0
67.0	68.0	69.0	78.0	80.0	83.0	88.0	89.0	90.0	93.0	96.0	103.0
105.0	109.0	109.0	111.0	115.0	117.0	125.0	126.0	127.0	129.0	129.0	139.0
154.0											

Table 5. The maximum likelihood estimates (MLE) with their standard error for two parameters of data sets II

$\hat{\alpha}$	$\hat{\beta}$	Kolmogorov-smirnov test (K-s)	p-Value
7.7496 (4.189)	0.1802 (0.0191)	0.05247	0.9537

Table 6. GASP for $\hat{\alpha} = 3.4323$, $\hat{\beta} = 0.0326$, showing minimum g and c

		$r = 5$						$r = 10$					
		$a_1 = 0.5$			$a_1 = 1$			$a_1 = 0.5$			$a_1 = 1$		
δ	r_2	g	c	$P_a(p)$	g	c	$P_a(p)$	g	c	$P_a(p)$	g	c	$P_a(p)$
0.25	2	-	-	-	780	4	0.9736	280	4	0.9621	39	5	0.9776
	4	81	2	0.9833	10	2	0.9816	11	2	0.9755	2	2	0.9646
	6	11	1	0.9654	3	1	0.9606	3	1	0.9600	2	2	0.9893
	8	11	1	0.9807	3	1	0.9781	3	1	0.9773	1	1	0.9698
0.10	2	-	-	-	-	-	-	-	-	-	65	5	0.9626
	4	134	2	0.9725	16	2	0.9707	18	2	0.9603	7	3	0.9869
	6	134	2	0.9922	16	2	0.9918	18	2	0.9882	3	2	0.9841
	8	18	1	0.9685	5	1	0.9637	5	1	0.9624	3	2	0.9933
0.05	2	-	-	-	-	-	-	-	-	-	84	5	0.9523
	4	174	2	0.9644	120	2	0.9635	100	3	0.9889	9	3	0.9832
	6	174	2	0.9898	20	2	0.9898	23	2	0.9850	4	2	0.9788
	8	23	1	0.9600	6	1	0.9567	23	2	0.9937	4	2	0.9910
0.01	2	-	-	-	-	-	-	-	-	-	-	-	-
	4	-	-	-	187	3	0.9894	153	3	0.9831	13	3	0.9758
	6	267	2	0.9845	31	2	0.9842	35	2	0.9773	6	2	0.9684
	8	267	2	0.9936	31	2	0.9934	35	2	0.9904	6	2	0.9866

Remark: The cells with hyphens (-) indicate that a large sample size is required.

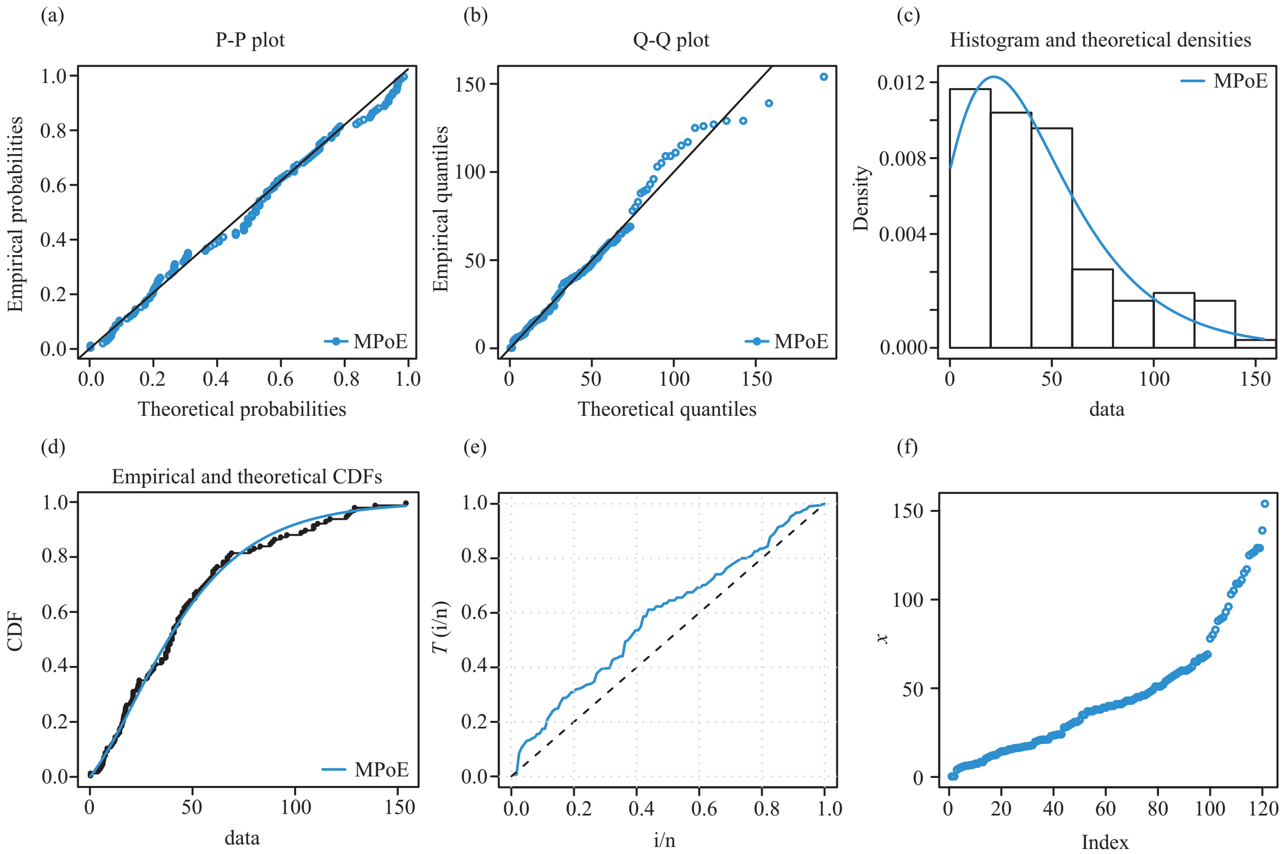


Figure 4. MPoE distribution for data set I presented graphically with (a) Estimate the density (b) Estimate the cdf (c) P-P plot (d) Q-Q plot (e) TTT plot (f) Index plot

6.2 *Data set 2*

Table 7. Waiting times (min) of 100 bank customers data taken from [25]

0.8	0.8	1.3	1.5	1.8	1.9	1.9	2.1	2.6	2.7	2.9	3.1
3.2	3.3	3.5	3.6	4.0	4.1	4.2	4.2	4.3	4.3	4.4	4.4
4.6	4.7	4.7	4.8	4.9	4.9	5.0	5.3	5.5	5.7	5.7	6.1
6.2	6.2	6.2	6.3	6.7	6.9	7.1	7.1	7.1	7.1	7.4	7.6
7.7	8.0	8.2	8.6	8.6	8.6	8.8	8.8	8.9	8.9	9.5	9.6
9.7	9.8	10.7	10.9	11.0	11.0	11.1	11.2	11.2	11.5	11.9	12.4
12.5	12.9	13.0	13.1	13.3	13.6	13.7	13.9	14.1	15.4	15.4	17.3
17.3	18.1	18.2	18.4	18.9	19.0	19.9	20.6	21.3	21.4	21.9	23.0
27.0	31.6	33.1	38.5								

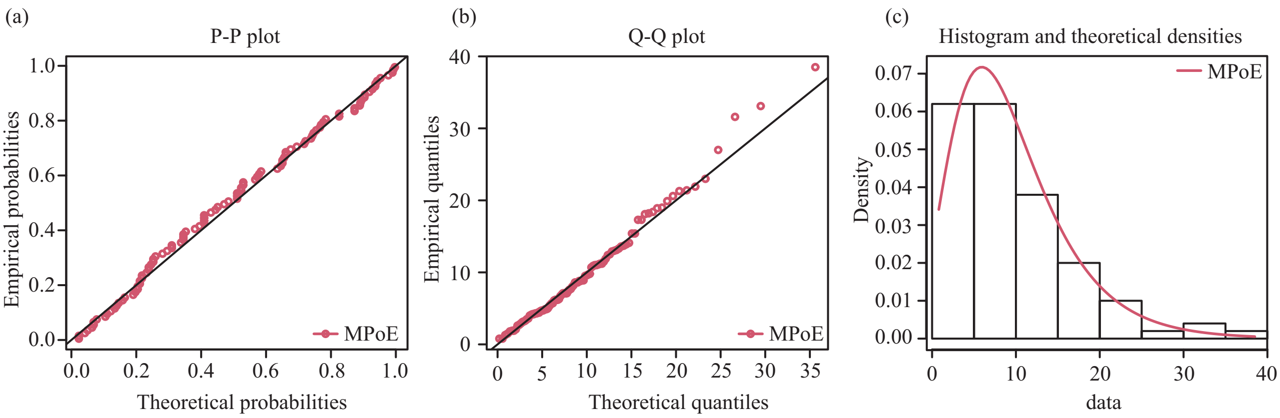
Table 8. The maximum likelihood estimates (MLE) with their standard error for two parameters of MPoE distribution

$\hat{\alpha}$	$\hat{\beta}$	Kolmogorov-smirnov test (K-S)	p-Value
3.4323 (1.3315)	0.0326 (0.0035)	0.055248	0.8539

Table 9. GASP for $\hat{\alpha} = 7.7496$, $\hat{\beta} = 0.1802$, showing minimum g and c

		$r = 5$						$r = 10$					
		$a_1 = 0.5$			$a_1 = 1$			$a_1 = 0.5$			$a_1 = 1$		
δ	r_2	g	c	$P_a(p)$	g	c	$P_a(p)$	g	c	$P_a(p)$	g	c	$P_a(p)$
0.25	2	74	3	0.9669	17	4	0.9792	15	4	0.9761	2	5	0.9509
	4	3	1	0.9568	2	2	0.9831	2	2	0.9878	1	3	0.9873
	6	3	1	0.9834	1	1	0.9704	1	1	0.9768	1	2	0.9830
	8	3	1	0.9914	1	1	0.9854	1	1	0.9877	1	2	0.9939
0.10	2	-	-	-	29	4	0.9647	24	4	0.9620	-	-	-
	4	19	2	0.9888	2	2	0.9831	4	2	0.9757	1	3	0.9873
	6	5	1	0.9724	1	1	0.9704	2	1	0.9542	1	2	0.9830
	8	5	1	0.9856	1	1	0.9854	2	1	0.9756	1	2	0.9939
0.05	2	-	-	-	37	4	0.9552	31	4	0.9512	-	-	-
	4	25	2	0.9853	3	2	0.9748	5	2	0.9697	2	3	0.9748
	6	7	1	0.9616	3	2	0.9947	2	1	0.9542	1	2	0.9830
	8	7	1	0.9799	2	1	0.9710	2	1	0.9756	1	2	0.9939
0.01	2	-	-	-	-	-	-	183	5	0.9739	-	-	-
	4	-	-	-	4	2	0.9665	7	2	0.9579	2	3	0.9748
	6	38	2	0.9948	4	2	0.9930	7	2	0.9896	2	2	0.9664
	8	10	1	0.9715	2	1	0.9710	4	1	0.9518	2	2	0.9878

Remark: The cells with hyphens (-) indicate that a large sample size is required



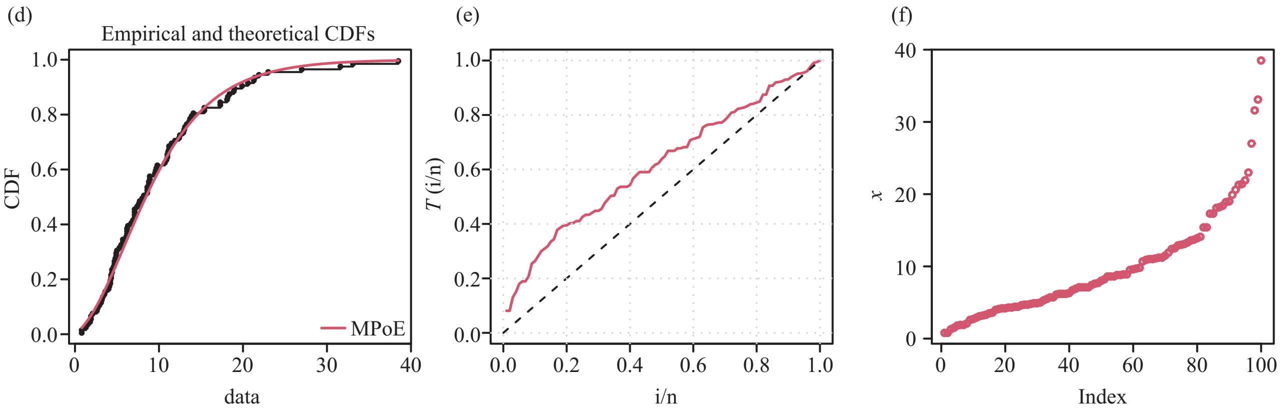


Figure 5. MPoE distribution for data set II presented graphically with (a) Estimate the density (b) Estimate the cdf (c) P-P plot (d) Q-Q plot(e) TTT plot (f) Index plot

Table 10. Descriptive statistics

Data sets	Mean	Median	Mode	Max	Min	Q1	Q3	Var.	Std.
Data set I	46.32	40.0	37, 38, 40, 41, 43, 51	60	0.3	17.4	60.5	1,244.44	35.27
Data set II	9.877	8.1	7.1	38.5	0.8	4.65	13.05	52.37	7.236

Table 11. Comparison among GASP follows distribution

Distribution	Parameters			$a_1 = 0.5, r = 5$			$a_1 = 0.5, r = 10$		
		δ	r_2	g	c	$P_a(p)$	g	c	$P_a(p)$
Alpha power transformation inverted perks distribution (APTIP)	Two	0.25	4	69	2	0.9759	9	2	0.9668
		0.1	4	114	2	0.9604	64	3	0.9859
		0.05	4	2004	3	0.9879	83	3	0.9817
		0.01	4	3080	3	0.9815	127	3	0.9722
New compounded three parametric weibull distribution	Three	0.25	4	5	1	0.9548	4	2	0.9876
		0.1	4	44	2	0.9872	7	2	0.9784
		0.05	4	58	2	0.9832	9	2	0.9723
		0.01	4	88	2	0.9746	14	2	0.9572
Marshall-olkin kumaraswamy exponential (MOKw-E)	Three	0.25	4	41	3	0.9852	3	3	0.9693
		0.1	4	67	3	0.9760	13	4	0.9840
		0.05	4	88	3	0.9686	17	4	0.9792
		0.01	4	134	3	0.9525	26	4	0.9683
Modified power exponential distribution	Two	0.25	4	3	1	0.9568	2	2	0.9878
		0.1	4	19	2	0.9888	4	2	0.9757
		0.05	4	25	2	0.9853	5	2	0.9697
		0.01	4	38	2	0.9777	7	2	0.9579

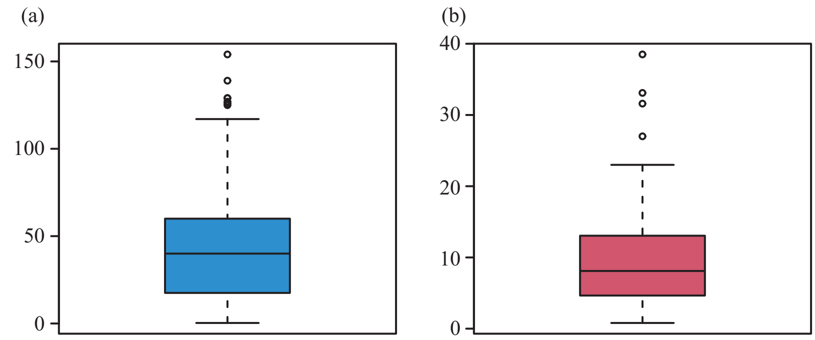


Figure 6. Plots of descriptive analysis for (a) Data set I and (b) Data set II

In Table 11, a comparison is made between distributions with known shapes and scale parameters with fixed r_2, where $a_1 = 0.5$, and $r = (5,\ 10)$. However, δ (producer risk) was also used in the three distributions mentioned above, and we observed that when we have $r_2 = 4$, $g = 69$, and $c = 2$, 138 units are required to put a life test on it. As we move forward and δ decreases, the values of g and c also increase for the APTIP distribution. Compared to APTIP, the new compounded three-parametric Weibull distribution is also suitable, but when we compare it with the MOKw-E distribution, we need more units compared to the NCTPW distribution. Finally, in the MPoE distribution, the required minimum units are required to conduct a life test on it to save time and cost.

7. Simulation

In Figure 7 using the estimated parameter combinations, we generated 1,000 samples from MPoE with sizes of (50, 75, 100, 150, 250, and 500). The simulation results are listed in Table 12. While the mean square errors (MSE) of the estimators are arranged in decreasing order, Table 12 demonstrates that the estimates for these sample sizes are highly consistent and, more specifically, are close to the genuine parameter values. Bias and MSE both exhibit a declining tendency as the sample size increases.

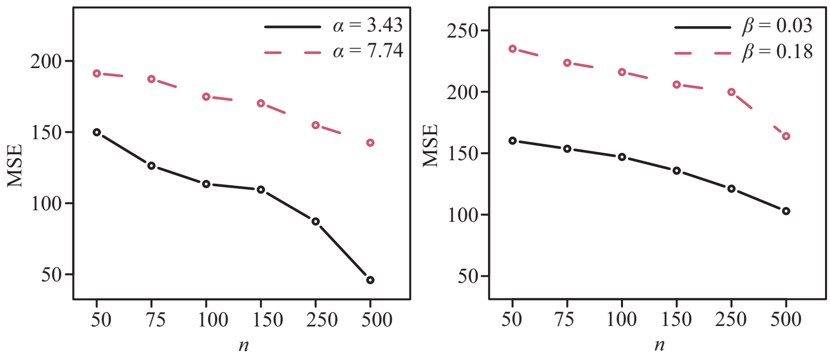


Figure 7. MSE plot for the parameter value using different sample sizes

Table 12. Bias and MSE values for different $\hat{\alpha}$ and $\hat{\beta}$ estimates

	$\hat{\alpha} = 3.43$		$\hat{\beta} = 0.03$	
n	*Bias*	*MSE*	*Bias*	*MSE*
50	−0.147	149.85	−0.127	160.21
75	−0.600	126.35	−0.576	153.66
100	−0.723	113.45	−0.687	146.98
150	−0.845	109.56	−0.867	135.87
250	−0.897	87.12	−0.967	121.12
500	−0.945	45.98	−0.998	102.87
	$\hat{\alpha} = 7.74$		$\hat{\beta} = 0.18$	
n	*Bias*	*MSE*	*Bias*	*MSE*
50	−0.884	191.28	−0.621	235.06
75	−0.720	187.34	−0.723	223.66
100	−0.563	174.92	−0.587	216.08
150	−0.608	170.23	−0.617	205.87
250	−0.817	154.87	−0.742	199.82
500	−0.915	142.56	−0.998	163.87

8. Conclusions

In this study, we proposed a GASP based on a modified power exponential distribution, with the median serving as a quality index. We considered consumer risk and tested the termination criteria when determining important design parameters, such as sample size (n) and acceptance number (c). We also observed as the median life expectancy increased and the number of groups (g) decreased, the operational characteristic (OC) values increased, as shown in the tables and graphs. Our research also indicates that it is advantageous to accept a batch of products with low values of g and c, as this minimizes inspection costs and time. The results of previous studies [26] are consistent with this conclusion. Furthermore, by comparing several GASP follow-up distributions, we found that our distribution had the lowest values for g and c, whereas r_2 remained constant. A comparison between GASP and OSP was also performed, which demonstrated the superiority of GASP. Furthermore, the simulation study shows that with an increase in sample size, Bias and MSE decreases, which can be helpful in the assessment of risk modeling variability, optimization of sampling parameters, cost-benefit analysis, sensitivity analysis, and dynamic modeling. These results are particularly useful for novice researchers who wish to expand on this subject by incorporating fuzzy-logic and double-acceptance sampling plans into their examination of quality and dependability. Essentially, our proposed method offers real advantages in improving quality control procedures and decision making in production applications.

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Conflict of interest

We, all the authors, have no conflict of interest.

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