

Research Article

Optimizing Computer Engineering Problems Using CODAS Method with Bipolar Complex Fuzzy Soft Frank Aggregation Operators

Abdul Jaleel¹, Tahir Mahmood¹, Dragan Pamucar^{2*}, Muhammad Iftikhar¹

¹Department of Mathematics and Statistics, International Islamic University Islamabad, Pakistan

²Széchenyi István University, Győr, Hungary

E-mail: pamucar.dragan@sze.hu

Received: 11 March 2025; **Revised:** 27 May 2025; **Accepted:** 4 July 2025

Abstract: In this article, we start by explaining why Bipolar Complex Fuzzy Soft Set (BCFSS) is better than existing approaches by highlighting its abilities to enhance Computer Engineering (CpE) by removing ambiguity more effectively than Fuzzy Set (FS), Fuzzy Soft Set (FSS), Complex Fuzzy Soft Set (CFSS), Bipolar Fuzzy Set (BFS), Bipolar Fuzzy Soft Set (BFSS), and Bipolar Complex Fuzzy Set (BCFS). This will improve effectiveness as well as clarity. Make sure that those new to fuzzy sets properly understand the key issue in CpE. Reveal first the BCFSS's new strategy and then detail how it differs from existing approaches, pointing out the overlooked features of previous approaches that justified your research. Enhance the preparation concept link by explaining how BCFSS enhances CpE Decision-Making (DM) in situations when conventional models are inadequate when paired with Frank Aggregation Operator (AO) (Frank Arithmetic Aggregation Operator (FAAO) and Frank Geometric Aggregation Operator (FGAO)). Moreover, keeping in mind the effectiveness of the "Combinative Distance-based Assessment (CODAS) method" we developed this method in the framework of BCFSSs and used this method and develop AOs to solve some decision-making problems related to CpE. Our sentence structure should also be refined to provide smooth transitions between ideas and to enhance readability and navigation. Lastly, provide comparison studies that highlight the improvement's superiority over current methods to show that it is feasible.

Keywords: bipolar complex fuzzy soft sets, CODAS method, computer engineering, frank aggregation operators

MSC: 68T27, 68T37, 03B52, 03E52

1. Introduction

Computer Engineering (CpE) is an interdisciplinary field that merges Computer Science (CS) and Electrical Engineering (EE) to develop and innovate computer hardware and software systems. Because of their special combination of expertise in CS and EE, computer engineers can combine hardware and software systems with ease. From creating the software that runs on microprocessors to designing them, they are involved in many facets of computing. The subject is always changing, and new developments like Artificial Intelligence (AI) and quantum computing are becoming more and more well-known. Computer engineers continue to be at the forefront of innovation as technology develops, pushing the limits of what is conceivable with computers and other electronic devices. A solid background in mathematics, physics,

and programming is necessary for anyone hoping to work in CpE. A bachelor's degree in computer engineering or a similar discipline is typically held by computer engineers. Further, we discuss the requirement of BCFSS in CpE. Because of their capacity to manage ambiguity and uncertainty in data, BCFSS are crucial in CpE. Computer engineers are essential in creating algorithms that use BCFSS to process and interpret data, which helps with DM in cases where the data is unclear or lacking. Additionally, they incorporate BCFSS into software, guaranteeing precise and efficient calculations with user-friendly interfaces and reliable back-end systems. Computer engineers create systems to effectively handle the complicated computations required to handle massive datasets with BCFSS, guaranteeing prompt and accurate results. Furthermore, BCFSS can be incorporated into AI and machine learning models to improve their capacity to manage imprecise and uncertain data; this calls for specific expertise in fuzzy logic and CpE. Computer engineers also focus on performance and optimization to make sure BCFSS systems function well, especially when dealing with big and complicated information. This partnership between CpE and BCFSS is an important area of research and application since it improves problem-solving skills in a variety of domains. It encompasses a broad range of topics, including:

- **Hardware Engineering:** Designing and testing individual microcontrollers, microprocessors, personal computers, and supercomputers.
- **Software Engineering:** Creating and improving software and firmware for various applications, from embedded systems to large-scale operating systems.
- **Developing Very-Large-Scale Integration (VLSI) Design:** VLSI chips, which are the building blocks of modern electronic devices.
- **Embedded Systems:** Integrating computing systems into other devices, often with real-time computing constraints.
- **Robotics:** Applying computer-engineering principles to design and control robots.
- **Computer Networks:** Designing and managing networks that connect computers and other devices.
- **Signal Processing:** Analyzing and manipulating signals, like audio, video, and sensor data, using digital systems.

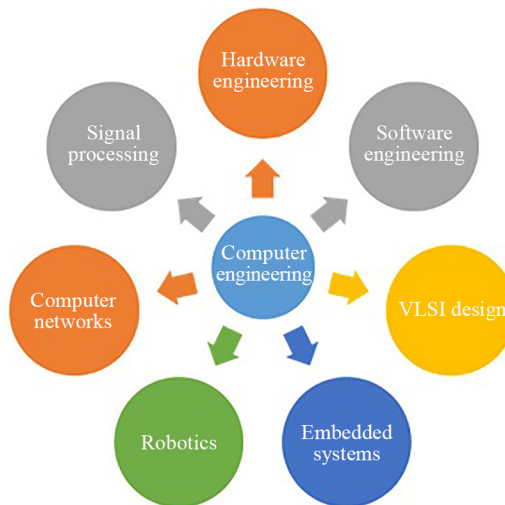


Figure 1. Computer engineering logo

The contribution of CODAS with BCFSS. All these years, research on CODAS in distributed environments has been carried out in two phases, first dealing with its theoretical aspects and mathematical formulations, and subsequently with practical implementations. The early research studies focused on establishing CODAS as a strong tool for DM, and its dominance over traditional methodologies in managing uncertainty and fuzzy information. Follow-up studies have investigated its efficacy in highly complicated Multi-Criteria Decision-Making (MCDM) problems, especially in areas of CpE. Later work has utilized fuzzy CODAS in logistics optimization and industrial condition monitoring, verifying its applicability to real-world problems. While these efforts have pushed the field further, research is still needed to

analyze how to leverage them as part of other fuzzy-based methods to make decisions more accurately. Their work expands this research by applying CODAS to BCFSSs and Fuzzy Aggregation Operators (FAOs), proposing a new method that enhances computational efficiency and decision reliability to CpE issues. This enhancement makes their work a notable contribution, overcoming earlier shortcomings and expanding the scope of fuzzy CODAS applicability in technical specialty areas. The CpE logo is shown in Figure 1.

Significantly, new ideas like BCFSSs are discussed. BCFSSs were presented by Mahmood et al. [1], who also investigated how they might be used in DM. Additionally, they defined the BCFSSs' extended union, intersection, restricted union, and complement. Trigonometric Similarity Measures (TSMs) are further developed by Mahmood et al. [2] inside the BCFSS framework, incorporating functions such as cosine, tangent, and cotangent for use in medical diagnostics and pattern recognition. The Aggregation Operators (AOs) in the context of BCFSSs are emphasized with the determination of the medical robotic system by Mahmood et al. [3]. The use of BCFSS Dombi AOs in agricultural robotics systems is then illustrated by Jaleel [4]. BCFSS Power Dombi AOs are examined and used by Jaleel et al. [5] for Artificial Intelligence (AI) robot selection. Furthermore, Jaleel et al. [6] defined the extended union, intersection, restricted union, intersection, and complement of Interval-Valued BCFSSs (IVBCFSSs) and discussed their uses in DM.

1.1 Motivation and contribution

A fundamental idea in this field is BCFSSs, which are an extension of both bipolar fuzzy soft sets and complex fuzzy soft sets. Firstly, the researchers can get great results by using the Bipolar Complex Fuzzy Soft Set Aggregation Operator (BCFSFAO), which offers a strong framework for analyzing and summarizing ambiguous data. After this, to successfully handle ambiguity and imprecision, the Soft Frank AO combines geometric and averaging aggregation techniques. These sophisticated FS theories have the potential to greatly improve DM in the fields of computer and Software Engineering (SE). To get the best results, for example, these AOs can be subjected to the Combinative Distance-based Assessment (CODAS) technique. The details of AOs are especially well-resolved by CODAS, guaranteeing accurate and trustworthy outcomes. FS theories and SE techniques can be used to create novel solutions in fields including data analysis, AI, and system optimization.

1.2 Paper structure

In this manuscript, we proposed BCFSFAO, which has two types Bipolar Complex Fuzzy Soft Set Fuzzy Weighted Arithmetic Aggregation Operator (BCFSFWAAO) and Bipolar Complex Fuzzy Soft Set Fuzzy Weighted Geometric Aggregation Operator (BCFSFWGAO) in Section 2, we have discussed the literature review, which consists of SE application, in subsection 2.1 the introduction of the CODAS method, and in subsection 2.2 the preliminaries of BCFSS. Section 3, discussed the problem statement. Section 4, demonstrated the proposed work like BCFSFAO with the algorithm of CODAS method. Section 5, demonstrated linguistic terms with numerical examples and ranking alternatives graph. Section 6, introduced a comparative study with an associated graph. In Section 7, sensitivity analysis. In Section 8, the conclusion of this manuscript with a graph. In Section 9, the associated references.

2. Literature review

Software is an actionable program or group of actionable programs that provide the necessary functionality. Engineering is the process of making something that meets a demand or affordably solves a problem. There are seven types of Software engineering.

- **Front-End Engineer:** The User Interface (UI) and User Experience (UX) of apps are the domains of front-end engineers. They use technologies like HyperText Markup Language (HTML), Cascading Style Sheets (CSS), and JavaScript to create components that are interactive and beautifully attractive.

- **Back-End Engineer:** The focus of back-end engineers is server-side programming. They create the databases, APIs, and logic that drive apps. Python, Ruby, and Java are common languages.

- **Full Stack Engineer:** Full stack engineers possess skills in both front-end and back-end development. They can work on the server-side code and User Interface (UI) of an application.

- **Software Engineer in Test (Quality Assurance (QA) Engineer):** Through the creation and application of test cases, QA engineers guarantee the ability of the software. They make sure the product satisfies quality requirements by locating and reporting faults.

- **Software Development Engineer in Test (SDET):** SDETs integrate testing knowledge with development abilities. To verify the functionality of the software, they create computerized tests and tools.

- **Development and Operations (DevOps) Engineer:** DevOps engineers fill the gap between development and operations. They highlight deployment pipelines, computerization, and continuous combination.

- **Security Engineer:** Software system security is the area of capability for security engineers. They recognize weak points, put security measures in place, and keep away from attacks.

The software engineering is shown in Figure 2.

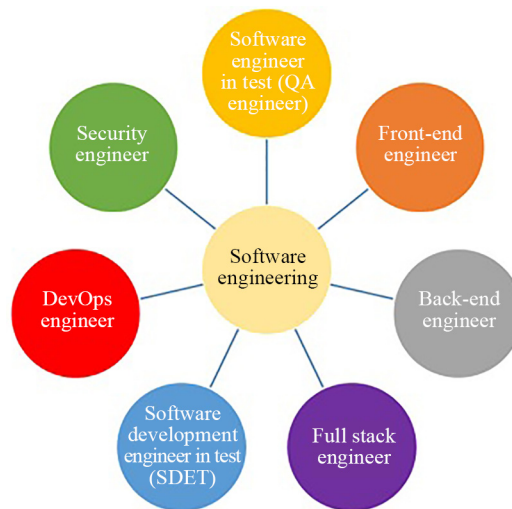


Figure 2. Software engineering logo

2.1 CODAS

Combinative Distance-based Assessment (CODAS), which stands for Combinative Distance-based Assessment, is an MCDM method. It is particularly useful for solving problems where a decision needs to be made based on various contradictory criteria. Here is a brief introduction to the CODAS method:

- **Purpose:** CODAS is designed to evaluate a set of alternatives based on multiple criteria and rank them in order of preference.

- **Process:** It involves calculating the Euclidean distance of each alternative from the Negative-Ideal Solution (NIS). The NIS represents the worst performance on each criterion.

- **Weighting:** Each criterion has assigned a weight based on its importance, which is used in the distance calculation.

- **Ranking:** Alternatives are ranked according to their distances from the NIS, with the smallest distance indicating the most preferred alternative.

The CODAS method is appreciated for its simplicity and effectiveness in various fields, such as engineering, business, and management, where complex DM is a regular task. It helps decision-makers to accurately compare different options and make informed choices. Please note that the search results also included information about the musical term “coda”, which is unrelated to the CODAS method in DM. If you’re looking for information on the musical coda, I can provide that as well. Many researchers have used the CODAS in fuzzy-related environments. Ghorabae et al. [7] initiated a fuzzy extension of the CODAS approach. After this, Simic et al. [8] demonstrated the Picture Fuzzy (PF) extension of the

CODAS method. Moreover, Bolturk [9] presented Pythagorean Fuzzy (PyF) CODAS and its use to provide assortment in a manufacturing firm. Furthermore, Karagoz et al. [10] initiated a novel Intuitionistic Fuzzy (IF) MCDM-based CODAS approach. Peng and Garg [11] demonstrated algorithms for Interval-Valued FSSs (IVFSSs) in emergency DM based on Weighted Distance-Based Approximation (WDBA) and CODAS with new information measures. Various approaches are used to address the issues of complexity and imprecision. These tactics include more cutting-edge methods like artificial intelligence and machine learning as well as more conventional mathematical approaches. Every approach has advantages and disadvantages, and the particulars of the issue at hand frequently influence the strategy that is chosen. For example, heuristic approaches may be more appropriate for cases when an exact solution is not possible, but statistical methods can efficiently manage big datasets. For this, Zadeh [12] initiated FS that contains Membership Grade (MG) belonging to the closed interval $[0, 1]$. Fuzzy Set (FS) deals with problems that have a single characteristic that is both imprecise and complex. Everyday routines produce more problems. However, most of them are not focusing only on FS. There are a greater number of problems with two dimensions. For tackling these dilemmas Ramot et al. [13] initiated Complex FS (CFS). Moreover, Rahman [14] initiated probability Complex Fuzzy Aggregation Operator (CFAO) with application in DM. Hussain et al. [15] demonstrated picture FS with Schweizer-Sklar AO. All those problems that involve two dimensions are handled from above. Let's discuss those difficulties in greater detail. They have two parts. These conundrums were considered and Intuitionistic FS (IFS) was started. The IFS, which is a further expansion of FS, was started by Atanassov [16]. The IFS contains two value functions while the FS only contains one. One is MG and 2nd is non-MG (nMG). It addresses the two aspects dilemmas mentioned above. The second dimension problem in complexity and imprecision is caused by a lack of IF information. To address this, Garg and Rani [17] developed unique AOs and a rating system for Complex IFSs (CIFSs) and how to apply them to the DM process. CIFS contains phase and amplitude terms of MG and nMG. The problems that the CIFSs are solving have two dimensions and two functions. Following this, Ali [18] explained DM methods using Complex Intuitionistic Fuzzy Power Interactive Aggregation Operators (CIFPIAOs) and their uses. The above cutting-edge ideas have addressed a wide range of challenges. Here, the uncertainties and complexity have given rise to another problem. The intricacy and imprecision of the problems are not more affected by the FS's absence of positive and negative features. The lack of a negative side is posing further challenges for the FS. Fuzzy relations and Bipolar FS (BFS) were proven by Zhang [19]. In this article, experts indulge in negative aspects and positive aspects. BFS is containing Positive MG (PMG) and Negative MG (NMG). Jana et al. [20] initiated Bipolar Fuzzy Dombi Aggregation Operators (BFDAOs) and their application in the Multi-Attribute Decision-Making (MADM) procedure. This has addressed the negative aspects of the complex and ambiguous problems. Currently, the BFS that is addressing the two-dimensional problems requires additional information. Mahmood and Rehman [21] started Bipolar CFS (BCFS) as a result. BCFS contains the phase and amplitude terms of PMG and NMG. Moreover, Naeem et al. [22] demonstrated BCF Power Frank AO (BCFPFAO). As FS theory evolved, several related concepts emerged, such as IFS, CFS, CIFS, BFS, and BCFS. These advanced frameworks have been instrumental in addressing the persistent issues of imprecision and complexity in various fields. Each of these extensions builds upon the foundational principles of FSSs, offering more nuanced ways to handle uncertainty and vagueness. Despite these advancements, the inherent intricacy and imprecision of real-world problems often reveal further challenges. This is where the concept of Soft Sets (SS) becomes particularly valuable. SSs, introduced by Molodtsov [23], offer a highly flexible and generalized framework for dealing with uncertainty. Unlike fuzzy sets, which rely on membership functions, SSs use parameterized families of sets, making them adaptable to a wide range of applications without the need for precise mathematical formulations. SSs are especially useful in scenarios where the parameters themselves are uncertain or where the relationships between parameters are complex. This makes them a powerful tool for DM, data analysis, and other areas where traditional methods may fall short. The Maji et al. [24] indicated a few SS actions. A few novel operations over SS were demonstrated by Ali et al. [25]. Most complex and imprecise dilemmas were resolved by SS. Use FS in conjunction with SS for further extension. It is more potent when compared to FS and SS independently. This was supported by Cagman et al. [26] demonstration of FSS Theory (FSST) and its applications. To categorize trends in renewable energy, Ahmmad [27] introduced the application of novel entropy measurements in the framework of q -rung Orthopair FSSs. FSS applies accuracy and complexity to one-dimensional issues. Still, additional two-dimensional puzzles are generated. Thirunavukarasu et al. [28] introduced the idea of Complex FSS (CFSS) with applications to address these conundrums. Currently, MG the single functional

value that SS and FSS contain is present. Maji [29] provided an Intuitionistic FSS (IFSS) demonstration for the two functional values. MG and nMG are two features of IFSS. Additionally, Kumar and Bajaj [30] denoted concepts that are two-dimensional, such as Complex IFSSs (CIFSSs). The two-dimensional and two-aspect categories are both included in the CIFSS. Due to its lack of a negative element, the FSS is faced with numerous complex and imprecise challenges. Bipolar FSS (BFSS) with its uses was identified for this by Riaz and Tehrim [31]. Following this, a strong Aggregation Operator (AO) for the MCDM approach with the BFS environment was indicated by the expansion of BFSS by Jana et al. [32]. Additionally, the negative aspect of the BFSS. Bipolar Fuzzy Dombi (BFD)-prioritized AOs were identified by Jana et al. [33] in MADM. Subsequently, Jana and Pal [34] provided an example of how bipolar IFSSs might be applied to DM problems. The need for BCFSS to deal with complicated and vague data, such as parameters, fuzzy data, and positive and negative attributes, is another motivation that pushes us. The concept is prompted by the bipolarity of parameters, which is preceded by the complex fuzziness of the data. Three notions of parameterization, complex fuzziness, and bipolarity are those that make modeling a problem simpler when these three factors are available when combined. Merging BCFS and SS, the BCFSS combines popular concepts such as BCFS, BFSS, BFS, FS, SS, and many more. While we will demonstrate in this research through the use of two actual implementations, the idea generated here is a significant approach to managing uncomfortable and tricky real-world scenarios and is an important contribution to the DM process.

2.2 Preliminaries

There is demonstrated the fundamental definitions of BCFSSs with their properties:

Definition 1 [1] Let \mathcal{U} be the universal set, f be the set of attributes, and $g \subseteq f$, then the pair (B, g) is called BCFSS over \mathcal{U} , where $B : g \rightarrow \text{BCFS}(\mathcal{U})$, and $\text{BCFS}(\mathcal{U})$ is the accumulating of all BCFSs of U . It is defined as

$$\begin{aligned}(B, g) &= B(g_v) = \{(e_u, (Q_B^+, R_B^-)) \mid \forall e_u \in \mathcal{U}, \& g_v \in f\} \\ &= \{(e_u, (\mathcal{E}_B^+ + \iota P_B^+, \mathcal{E}_B^- + \iota P_B^-)) \mid \forall e_u \in \mathcal{U}, \& g_v \in f\}.\end{aligned}\tag{1}$$

For easiness in this article, we apply

$$(B, g) = B_{g_{uv}} = (Q_{uv}^+, R_{uv}^-) = (\mathcal{E}_{uv}^+ + \iota P_{uv}^+, \mathcal{E}_{uv}^- + \iota P_{uv}^-); (u = 1, 2, \dots, p, v = 1, 2, \dots, q)$$

as a Bipolar Complex Fuzzy Soft Number (BCFSN).

Definition 2 [1] The complement of a BCFSN $(B, g) = B_{g_{uv}} = (Q_{uv}^+, R_{uv}^-) = (\mathcal{E}_{uv}^+ + \iota P_{uv}^+, \mathcal{E}_{uv}^- + \iota P_{uv}^-)$ is designated and defined as

$$(B, g)^c = (B_{g_{uv}})^c = \{(1 - \mathcal{E}_{uv}^+ + i(1 - P_{uv}^+), -1 - \mathcal{E}_{uv}^- + i(-1 - P_{uv}^-))\}.$$

Definition 3 [1] The restricted intersection of two BCFSNs (B, g) and (G, v) over \mathcal{U} is a BCFSN (I_R, D) , where $D = \alpha \cap \beta \neq \emptyset$ and $I_R : D \rightarrow \text{BCFS}(U)$ is deliberated as $I_R(\wp) = \alpha(\wp) \cap \beta(\wp)$ and designated by

$$(I_R, D) = (B, g) \cap_R (C, \gamma).$$

Definition 4 [1] The restricted union of two BCFSNs (B, g) and (C, γ) over \mathcal{U} is a BCFSN (ψ_R, D) , where $D = \alpha \cap \beta \neq \emptyset$ and $\psi_R : D \rightarrow \text{BCFS}(\mathcal{U})$ is deliberated as $\psi_R(\wp) = \alpha(\wp) \cup \beta(\wp)$ and designated by

$$(I_R, D) = (B, g) \cup_R (G, \gamma).$$

Where \cup_R represents restricted union.

Definition 5 [1] The extended union of two BCFSNs (B, g) and (C, γ) over \mathcal{U} is a BCFSN (ψ_E, D) , where $D = \alpha \cup \beta$

$$\psi_E = \begin{cases} \alpha(\wp) & \text{if } \wp \in \alpha - \beta \\ \beta(\wp) & \text{if } \wp \in \beta - \alpha \\ \alpha(\wp) \cup \beta(\wp) & \text{if } \wp \in \alpha \cap \beta. \end{cases}$$

And designated by $(\psi_E, D) = (B, g) \cup_E (C, \gamma)$.

Definition 6 [1] The extended intersection of two BCFSNs (B, g) and (C, γ) over \mathcal{U} is a BCFSN (I_E, D) , where $D = \alpha \cap \beta$,

$$I_E = \begin{cases} \alpha(\wp) & \text{if } \wp \in \alpha - \beta \\ \beta(\wp) & \text{if } \wp \in \beta - \alpha \\ \alpha(\wp) \cap \beta(\wp) & \text{if } \wp \in \alpha \cap \beta. \end{cases}$$

And designated by $(I_E, D) = (B, g) \cap_E (C, \gamma)$.

3. Problem statement

First, with a specific focus on software engineering, hardware engineering, Very-Large-Scale Integration (VLSI) chips, embedded systems, robotics, computer networks, and signal processing, the goal is to overcome the difficulties in evaluating renewable CpE resources. Because of their differing qualities, it is difficult to choose which of the previously stated possibilities the greatest and worst CpE source is. Regarding the method for examining the BCFSS technique based on Frank and algebraic norms. To indicate which AOs, such as Bipolar Complex Fuzzy Soft Set Fuzzy Aggregation (BCFSFA), Bipolar Complex Fuzzy Soft Set Fuzzy Weighted Arithmetic Aggregation (BCFSFWA), Bipolar Complex Fuzzy Soft Set Fuzzy Geometric Aggregation (BCFSFG), and Bipolar Complex Fuzzy Soft Set Fuzzy Weighted Geometric Aggregation (BCFSFWG), are for Bipolar Complex Fuzzy Soft Set Numbers (BCFSNs). Moreover, to use BCFSNs using the CODAS technique. Using MADM approaches to solve issues with renewable CpE (SE) resources. To relate the Software Engineering (SE) applications here. Bipolar Complex Fuzzy Soft Set Aggregation Operators (BCFSAOs) should be incorporated into decision support systems to manage complicated, uncertain data in SE scenarios. It is also possible to use Bipolar Complex Fuzzy Soft (BCFS) aggregation techniques in project evaluation to rate software development projects according to their scalability, security, and performance.

4. Proposed work

The indulging of BCFSS and associated consequences and proposed work BCFSFAO with CODAS method.

4.1 Bipolar complex fuzzy soft sets

There are many notions but the BCFSS is one of them that is a very fruitful and advanced notion. The BCFSS is demonstrated by Mahmood et al. [1]. Here is demonstrating restricted union, intersection, extended union, intersection, and complement as well. Their applications are initiated in the DM dilemmas. The succeeding designated the primary operations for BCFSNs.

Definition 7 Suppose $B_g = (\mathfrak{L}^+ + \mathfrak{I}P^+, \mathfrak{L}^- + \mathfrak{I}P^-)$, $B_{g_{uv}} = (\mathfrak{L}_{11}^+ + \mathfrak{I}P_{11}^+, \mathfrak{L}_{11}^- + \mathfrak{I}P_{11}^-)$, $F_{g_{12}} = (\mathfrak{L}_{12}^+ + \mathfrak{I}P_{12}^+, \mathfrak{L}_{12}^- + \mathfrak{I}P_{12}^-)$ be the BCFSNs over the universal set U , then the below operations are nominated as trails:

$$B_{g_{11}} \oplus B_{g_{12}} = (\mathfrak{L}_{11}^+ + \mathfrak{L}_{12}^+ - \mathfrak{L}_{11}^+ \mathfrak{L}_{12}^+ + \mathfrak{I}(P_{11}^+ + P_{12}^+ - P_{11}^+ P_{12}^+), -(\mathfrak{L}_{11}^- \mathfrak{L}_{12}^-) + \mathfrak{I}(P_{11}^- P_{12}^-))$$

$$B_{g_{11}} \otimes B_{g_{12}} = (\mathfrak{L}_{11}^+ \mathfrak{L}_{12}^+ + \mathfrak{I}(P_{11}^+ P_{12}^+), (\mathfrak{L}_{11}^- + \mathfrak{L}_{12}^- + \mathfrak{L}_{11}^- \mathfrak{L}_{12}^-) + \mathfrak{I}(P_{11}^- + P_{12}^- + P_{11}^- P_{12}^-))$$

$$\sigma B_g = (1 - (1 - \mathfrak{L}^+)^{\sigma}) + \mathfrak{I}(1 - (1 - P^+)^{\sigma}), -|\mathfrak{L}^-|^{\sigma} + \mathfrak{I}(-|P^-|^{\sigma})$$

$$B_g^{\sigma} = ((\mathfrak{L}^+)^{\sigma} + \mathfrak{I}(P^+)^{\sigma}, -1 + (1 + \mathfrak{L}^-)^{\sigma} + \mathfrak{I}(-1 + (1 + P^-)^{\sigma})).$$

Definition 8 The score function of a BCFSN

$$B_{g_{uv}} = (\mathfrak{L}_{uv}^+ + \mathfrak{I}P_{uv}^+, \mathfrak{L}_{uv}^- + \mathfrak{I}P_{uv}^-)$$

is defined as

$$S_B(B_{g_{uv}}) = \frac{1}{4} (2 + \mathfrak{L}_{uv}^+ + P_{uv}^+ + \mathfrak{L}_{uv}^- + P_{uv}^-). \quad (2)$$

It is held as $S_B(B_{g_{uv}}) \in [0, 1]$.

Definition 9 The accuracy function of a BCFSN

$$B_{g_{uv}} = (\mathfrak{L}_{uv}^+ + \mathfrak{I}P_{uv}^+, \mathfrak{L}_{uv}^- + \mathfrak{I}P_{uv}^-)$$

is defined as

$$H_B(B_{g_{uv}}) = \frac{\mathfrak{L}_{uv}^+ + P_{uv}^+ - \mathfrak{L}_{uv}^- - \mathfrak{I}P_{uv}^-}{4}. \quad (3)$$

It is held as $H_B(B_{g_{uv}}) \in [0, 1]$.

Here, the suggested AOs based on Fermatean Trapezoidal Number (FTN) and Fermatean Trapezoidal Complex Number (FTCN). BCFSFWA and BCFSFWG operators for aggregation of an assortment of BCFSNs are existing in this section.

Definition 10 For the assortment of Bipolar Complex Fuzzy Soft Set Numbers (BCFSNs)

$$\mathbb{B}_{uv} = (Q_{uv}, R_{uv}) = (\mathcal{E}_{uv}^+ + \imath P_{uv}^+, \mathcal{E}_{uv}^- + \imath P_{uv}^-); u = 1, 2, \dots, p, v = 1, 2, \dots, q.$$

Then Bipolar Complex Fuzzy Soft Set Fuzzy Weighted Arithmetic Aggregation (BCFSFWA) operator is demarcated as

$$\begin{aligned} & \text{BCFSFWA}(\mathbb{B}_{g_{11}}, \mathbb{B}_{g_{12}}, \dots, \mathbb{B}_{g_{pq}}) \\ &= \bigoplus_{v=1}^q w_v \left(\bigoplus_{u=1}^p g_u \mathbb{B}_{g_{uv}} \right) \\ &= \left(\begin{array}{l} 1 - \log_{\odot} \left(1 + \prod_{v=1}^q \left(\prod_{u=1}^p \left(\odot^{1-\mathcal{E}_{uv}^+} - 1 \right)^{g_u} \right)^{w_v} \right) \\ + \imath \left(1 - \log_{\odot} \left(1 + \prod_{v=1}^q \left(\prod_{u=1}^p \left(\odot^{1-P_{uv}^+} - 1 \right)^{g_u} \right)^{w_v} \right) \right), \\ - \log_{\odot} \left(1 + \prod_{v=1}^q \left(\prod_{u=1}^p \left(\odot^{1-\mathcal{E}_{uv}^-} - 1 \right)^{g_u} \right)^{w_v} \right) \\ + \imath \left(- \log_{\odot} \left(1 + \prod_{v=1}^q \left(\prod_{u=1}^p \left(\odot^{1-P_{uv}^-} - 1 \right)^{g_u} \right)^{w_v} \right) \right) \end{array} \right). \end{aligned} \quad (4)$$

Where $\odot > 1$ and Weight Vectors (WVs) of experts and attributes are designated by $w_v, g_u > 0$,

$$\sum_{u=1}^p g_u = 1 \text{ and } \sum_{v=1}^q w_v = 1.$$

Where $\odot > 1$, and the weight vectors of experts and attributes are designated by $w_v, g_u > 0$, satisfying $\sum_{u=1}^p g_u = 1$ and $\sum_{v=1}^q w_v = 1$.

Definition 11 For the assortment of BCFSNs

$$\mathbb{B}_{uv} = (Q_{uv}, R_{uv}) = (\mathcal{E}_{uv}^+ + \imath P_{uv}^+, \mathcal{E}_{uv}^- + \imath P_{uv}^-); u = 1, 2, \dots, p, v = 1, 2, \dots, q.$$

Then BCFSFWG operator is demarcated as

$$\begin{aligned}
& BCFSFWG(\mathbb{B}_{g_{11}}, \mathbb{B}_{g_{12}}, \dots, \mathbb{B}_{g_{pq}}) \\
&= \bigotimes_{v=1}^q w_v \left(\bigotimes_{u=1}^p g_u \mathbb{B}_{g_{uv}} \right) \\
&= \left(\begin{array}{l} \log_{\mathbb{O}} \left(1 + \prod_{v=1}^q \left(\prod_{u=1}^p \left(\mathbb{O}^{1-\mathcal{E}_{uv}^+} - 1 \right)^{g_u} \right)^{w_v} \right) \\ + \iota \left(\log_{\mathbb{O}} \left(1 + \prod_{v=1}^q \left(\prod_{u=1}^p \left(\mathbb{O}^{1-P_{uv}^+} - 1 \right)^{g_u} \right)^{w_v} \right) \right), \\ -1 + \log_{\mathbb{O}} \left(1 + \prod_{v=1}^q \left(\prod_{u=1}^p \left(\mathbb{O}^{1-\mathcal{E}_{uv}^-} - 1 \right)^{g_u} \right)^{w_v} \right) \\ + \iota \left(-1 + \log_{\mathbb{O}} \left(1 + \prod_{v=1}^q \left(\prod_{u=1}^p \left(\mathbb{O}^{1-P_{uv}^-} - 1 \right)^{g_u} \right)^{w_v} \right) \right) \end{array} \right). \tag{5}
\end{aligned}$$

Where $\mathbb{O} > 1$ and WVs of experts and attributes are designated by w_v , $g_u > 0$, $\sum_{u=1}^p g_u = 1$, and $\sum_{v=1}^q w_v = 1$.

4.2 Algorithm of CODAS

Let $u = \{u_1, u_2, \dots, u_p\}$ be the set of experts, and $v = \{v_1, v_2, \dots, v_q\}$ be the set of attributes. Suppose

$$g = (g_1, g_2, \dots, g_p) \text{ and } w = (w_1, w_2, \dots, w_q)$$

indicate respectively the Weight Vectors (WVs) of the p engineers u_p and the q attributes v_q , such that

$$g_p > 0, g \in [0, 1], \text{ and } \sum_{u=1}^p g_u = 1,$$

$$w_q > 0, w \in [0, 1], \text{ and } \sum_{v=1}^q w_v = 1.$$

This data would be processed through the following steps:

Step 1 Make the BCFS decision matrix $M_s(B_{g_{uv}})$. For each decision-maker, calculate the average BCFS decision matrix $M_s(B_{g_{uv}})$ as follows:

$$M_s = [B_{g_{uv}}]_{p \times q} = \left[\begin{array}{ccc} \left(\begin{array}{l} \mathcal{E}_{11}^{+s} + \iota P_{11}^{+s} \\ \mathcal{E}_{11}^{-s} + \iota P_{11}^{-s} \end{array} \right) & \dots & \left(\begin{array}{l} \mathcal{E}_{1q}^{+s} + \iota P_{1q}^{+s} \\ \mathcal{E}_{1q}^{-s} + \iota P_{1q}^{-s} \end{array} \right) \\ \vdots & \ddots & \vdots \\ \left(\begin{array}{l} \mathcal{E}_{p1}^{+s} + \iota P_{p1}^{+s} \\ \mathcal{E}_{p1}^{-s} + \iota P_{p1}^{-s} \end{array} \right) & \dots & \left(\begin{array}{l} \mathcal{E}_{pq}^{+s} + \iota P_{pq}^{+s} \\ \mathcal{E}_{pq}^{-s} + \iota P_{pq}^{-s} \end{array} \right) \end{array} \right]. \tag{6}$$

Step 2 Combine BCFS matrices from Step 1 to obtain a single BCFS decision matrix $M = [B_{g_{uv}}]_{p \times q}$ by utilizing the BCFSFWA or BCFSFWG operators as:

$$M = [B_{g_{uv}}]_{p \times q} = \begin{bmatrix} \begin{pmatrix} \mathfrak{f}_{11}^+ + \mathfrak{I}P_{11}^+ \\ \mathfrak{f}_{11}^- + \mathfrak{I}P_{11}^- \end{pmatrix} & \dots & \begin{pmatrix} \mathfrak{f}_{1q}^+ + \mathfrak{I}P_{1q}^+ \\ \mathfrak{f}_{1q}^- + \mathfrak{I}P_{1q}^- \end{pmatrix} \\ \vdots & \ddots & \vdots \\ \begin{pmatrix} \mathfrak{f}_{p1}^+ + \mathfrak{I}P_{p1}^+ \\ \mathfrak{f}_{p1}^- + \mathfrak{I}P_{p1}^- \end{pmatrix} & \dots & \begin{pmatrix} \mathfrak{f}_{pq}^+ + \mathfrak{I}P_{pq}^+ \\ \mathfrak{f}_{pq}^- + \mathfrak{I}P_{pq}^- \end{pmatrix} \end{bmatrix}. \quad (7)$$

The final aggregated decision matrix is given by:

$$M = \bigoplus_{s=1}^{\tau} M_s. \quad (8)$$

Where each entry is a Bipolar Complex Fuzzy Soft Number (BCFSN).

Step 3 The normalization of decision matrix $M = [B_{g_{uv}}]_{p \times q}$ is required, as there are cost and benefits types of attributes. Thus, in this step, the BCF matrix $M = [B_{g_{uv}}]_{p \times q}$ would be normalized by the assistance of underneath Eq.

$$B_{g_{uv}}^s = \begin{cases} \left(\mathfrak{f}_{uv}^{(+s)} + \mathfrak{I}P_{uv}^{(+s)}, \mathfrak{f}_{uv}^{(-s)} + \mathfrak{I}P_{uv}^{(-s)} \right) & \text{if } v \in \mathbb{D}_B \\ \left(1 - \mathfrak{f}_{uv}^{(+s)} + \mathfrak{I}(1 - P_{uv}^{(+s)}), -1 + \mathfrak{f}_{uv}^{(-s)} + \mathfrak{I}(-1 + P_{uv}^{(-s)}) \right) & \text{if } v \in \mathbb{D}_C. \end{cases} \quad (9)$$

Observe that \mathbb{D}_B is the gathering of benefits and \mathbb{D}_C is a gathering of cost types of attributes.

Step 4 Calculate BCFS weighted normalized decision matrix. The BCFS-weighted normalized performance values ($B_{g_{uv}}$) calculated as follows:

$$B_{g_{uv}}^W = w_v \circledast B_{g_{uv}}. \quad (10)$$

Where, w_v indicates the BCFS weight of v th attributes, and $0 < w_v < 1$.

Step 5 Determine BCFS negative-ideal solution as follows:

$$(\widetilde{NS}) = [(\widetilde{ns})_v]_{1 \times p} \quad (11)$$

$$(\widetilde{ns})_v = \min_{\tilde{g}\tilde{v}} B_{\tilde{g}\tilde{v}}^{(n, W)} \quad (12)$$

$$(\widetilde{ns})_v = \min B_{g_{uv}}^W = \min(Q_{uv}^{(s, W)}, R_{uv}^{(s, W)}) = \left(\min \mathfrak{f}_{uv}^{(+s, W)} + \mathfrak{I} \min \mathfrak{P}_{uv}^{(+s, W)}, \min \mathfrak{f}_{uv}^{(-s, W)} + \mathfrak{I} \min \mathfrak{P}_{uv}^{(-s, W)} \right).$$

Step 6 Calculate the BCFS Euclidean (BCFSED_{*ū*}) and BCFS Hamming (BCFSHD_{*ū*}) the distance of the alternatives from the BCFS negative ideal solution is shown as follows:

$$\text{BCFSED}_{\bar{u}} \left(B_{g, \bar{u}v}^{(s, W)} - \widetilde{ns}_v \right) = \sqrt{\frac{1}{4v} \sum_{v=1}^q \left(\left(\log_O \left(1 + \prod_{\bar{u}=1}^p \left(O^{1-f_{\bar{u}v}^{(+s, W)}} - 1 \right)^{g_{\bar{u}}} \right) - \widetilde{ns}_v \right)^2 + \right.} \quad (13)$$

$$\left. \left(\log_O \left(1 + \prod_{\bar{u}=1}^p \left(O^{1-f_{\bar{u}v}^{(+s, W)}} - 1 \right)^{g_{\bar{u}}} \right) - \widetilde{ns}_v \right)^2 + \right. \\ \left. \left(-\log_O \left(1 + \prod_{\bar{u}=1}^p \left(O^{1-f_{\bar{u}v}^{(-s, W)}} - 1 \right)^{g_{\bar{u}}} \right) - \widetilde{ns}_v \right)^2 + \right. \\ \left. \left(-\log_O \left(1 + \prod_{\bar{u}=1}^p \left(O^{1-f_{\bar{u}v}^{(-s, W)}} - 1 \right)^{g_{\bar{u}}} \right) - \widetilde{ns}_v \right)^2 \right)$$

$$\text{BCFSHD}_{\bar{u}} \left(B_{g_{\bar{u}v}}^{(s, W)} - \widetilde{ns}_v \right) = \frac{1}{4v} \sum_{v=1}^q \left(\left(\log_O \left(1 + \prod_{\bar{u}=1}^p \left(O^{1-f_{\bar{u}v}^{(+s, W)}} - 1 \right)^{g_{\bar{u}}} \right) - \widetilde{ns}_v + \right. \right. \\ \left. \log_O \left(1 + \prod_{\bar{u}=1}^p \left(O^{1-f_{\bar{u}v}^{(+s, W)}} - 1 \right)^{g_{\bar{u}}} \right) - \widetilde{ns}_v + \right. \\ \left. \left(-\log_O \left(1 + \prod_{\bar{u}=1}^p \left(O^{1-f_{\bar{u}v}^{(-s, W)}} - 1 \right)^{g_{\bar{u}}} \right) - \widetilde{ns}_v \right) \right. \\ \left. \left(-\log_O \left(1 + \prod_{\bar{u}=1}^p \left(O^{1-f_{\bar{u}v}^{(-s, W)}} - 1 \right)^{g_{\bar{u}}} \right) - \widetilde{ns}_v \right) \right) \right). \quad (14)$$

Step 7 Determine Relative Assessment (RA) matrix, by using equations (15) and (16) shown as follows:

$$RA = [P_{\bar{u}k}]_{p \times l} \quad (15)$$

$$P_{\bar{u}k} = (\text{BCFSFED}_{\bar{u}} - \text{BCFSFED}_k) + \mathcal{T}(\text{BCFSFED}_{\bar{u}} - \text{BCFSFED}_k) \times (\text{BCFSFHD}_{\bar{u}} - \text{BCFSFHD}_k). \quad (16)$$

Where, $k \in \{1, 2, \dots, l\}$ and \mathcal{T} is the threshold function that is given equation (17) and Θ is a threshold parameter of this function which can be set by DM. Defined as follows:

$$\mathcal{T}(y) = \begin{cases} 1 & \text{if } |y| \geq \Theta \\ 0 & \text{if } |y| < \Theta. \end{cases} \quad (17)$$

The beginning (Θ) of this function can be set by the decision-maker. In this study, we use $\Theta = 0.02$ for the calculations.

Step 8 Compute the Assessment Score (AS_{*ū*}) each alternative is shown as follows:

$$AS_{\bar{u}} = \sum_{k=1}^l P_{\bar{u}k}. \quad (18)$$

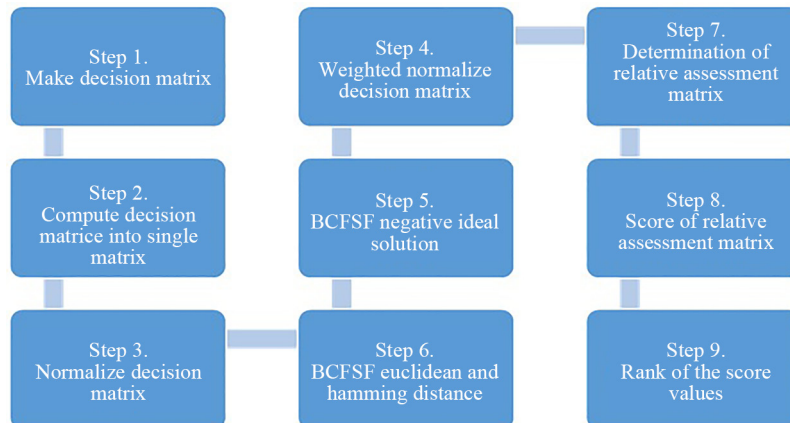


Figure 3. Algorithm of the CODAS method

Step 9 We can rank the options based on the assessment scores' decreasing values. The option that receives the greatest evaluation score is the one that is most preferred. The stepwise processing is also shown in Figure 3.

5. Linguistic term and numerical example

Software engineering is the most influential field in daily life. Let $\dot{u} = \{\dot{u}_1, \dot{u}_2, \dot{u}_3, \dot{u}_4, \dot{u}_5, \dot{u}_6, \dot{u}_7\}$ be set of experts and $v = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ be the set of attributes, where

v_1 = Curious and Open-Minded, v_2 = Self-Motivated and Proactive,

v_3 = Ability to Work Well under Pressure,

v_4 = Good at Prioritizing Tasks, v_5 = Empathetic and Strong Interpersonal Skills

v_6 = Humble and Willing to Learn from others

v_7 = Strong Sense of Responsibility and Ownership.

Suppose $g = \{0.13, 0.14, 0.12, 0.11, 0.15, 0.16, 0.19\}$ and $w = \{0.19, 0.17, 0.13, 0.12, 0.11, 0.10, 0.18\}$ indicate consistently the WVs of the engineers experts \dot{u} and attributes v .

To select for the determination of most quality and quantity work done engineering expert express their assessment in the background of BCFSNs

$$B_{g_{\dot{u}v}} = (Q_{\dot{u}v}, R_{\dot{u}v}) = (\mathcal{E}_{\dot{u}v}^+ + \iota P_{\dot{u}v}^+, \mathcal{E}_{\dot{u}v}^- + \iota P_{\dot{u}v}^-).$$

This data would be accomplished through underneath steps. Firstly, here we make a table for linguistic terms. There is the linguistic term shown in Tables 1-4 respectively and the tabular form of BCFSN data is shown in Tables 5-8 respectively.

Table 1. Linguistic term

Linguistic variables	Alternatives/attributes	v_1	v_2	v_3	v_4	v_5	v_6	v_7
Very Low (VL)	\acute{u}_1	VL	L	ML	M	MH	H	VH
Low (L)	\acute{u}_2	L	ML	M	MH	H	VH	VL
Medium-Low (ML)	\acute{u}_3	ML	M	MH	H	VH	VL	L
Medium (M)	\acute{u}_4	M	MH	H	VH	VL	L	ML
Medium-High (MH)	\acute{u}_5	MH	H	VH	VL	L	ML	M
High (H)	\acute{u}_6	H	VH	VL	L	ML	M	MH
Very High (VH)	\acute{u}_7	VH	VL	L	ML	M	MH	H

Table 2. Linguistic term

Linguistic variables	Alternatives/attributes	v_1	v_2	v_3	v_4	v_5	v_6	v_7
Very Low (VL)	\acute{u}_1	L	ML	M	MH	H	VH	VL
Low (L)	\acute{u}_2	ML	M	MH	H	VH	VL	L
Medium-Low (ML)	\acute{u}_3	M	MH	H	VH	VL	L	ML
Medium (M)	\acute{u}_4	MH	H	VH	VL	L	ML	M
Medium-High (MH)	\acute{u}_5	H	VH	VL	L	ML	M	MH
High (H)	\acute{u}_6	VH	VL	L	ML	M	MH	H
Very High (VH)	\acute{u}_7	VL	L	ML	M	MH	H	VH

Table 3. Linguistic term

Linguistic variables	Alternatives/attributes	v_1	v_2	v_3	v_4	v_5	v_6	v_7
Very Low (VL)	\acute{u}_1	ML	M	MH	H	VH	VL	L
Low (L)	\acute{u}_2	M	MH	H	VH	VL	L	ML
Medium-Low (ML)	\acute{u}_3	MH	H	VH	VL	L	ML	M
Medium (M)	\acute{u}_4	H	VH	VL	L	ML	M	MH
Medium-High (MH)	\acute{u}_5	VH	VL	L	ML	M	MH	H
High (H)	\acute{u}_6	VL	L	ML	M	MH	H	VH
Very High (VH)	\acute{u}_7	L	ML	M	MH	H	VH	VL

Table 4. Linguistic term

Linguistic variables	Alternatives/attributes	v_1	v_2	v_3	v_4	v_5	v_6	v_7
Very Low (VL)	\acute{u}_1	M	MH	H	VH	VL	L	ML
Low (L)	\acute{u}_2	MH	H	VH	VL	L	ML	M
Medium-Low (ML)	\acute{u}_3	H	VH	VL	L	ML	M	MH
Medium (M)	\acute{u}_4	VH	VL	L	ML	M	MH	H
Medium-High (MH)	\acute{u}_5	VL	L	ML	M	MH	H	VH
High (H)	\acute{u}_6	L	ML	M	MH	H	VH	VL
Very High (VH)	\acute{u}_7	ML	M	MH	H	VH	VL	L

[illegible]

Table 6. The tabular form of BCFSN data[illegible]

Table 7. The tabular form of BCFSN data[illegible]

Table 8. The tabular form of BCFSN data[illegible]

Step 2. Here, combine the BCFS matrices presented in Step 1, and obtain a single BCFS decision matrix $M = [B_{g\dot{u}v}]_{p \times q}$ that is given below.

Table 9. BCFS decision matrix $M = [B_{g\dot{u}v}]_{p \times q}$

$B_{g\dot{u}v}$	v_1	v_2	v_3	v_4	v_5	v_6	v_7
\dot{u}_1	$\begin{pmatrix} 0.40+ \\ \imath 0.36, \\ -0.70- \\ \imath 0.69 \end{pmatrix}$	$\begin{pmatrix} 0.50+ \\ \imath 0.46, \\ -0.60- \\ \imath 0.59 \end{pmatrix}$	$\begin{pmatrix} 0.60+ \\ \imath 0.56, \\ -0.50- \\ \imath 0.49 \end{pmatrix}$	$\begin{pmatrix} 0.70+ \\ \imath 0.66, \\ -0.40- \\ \imath 0.39 \end{pmatrix}$	$\begin{pmatrix} 0.625+ \\ \imath 0.585, \\ -0.475- \\ \imath 0.465 \end{pmatrix}$	$\begin{pmatrix} 0.55+ \\ \imath 0.51, \\ -0.55- \\ \imath 0.54 \end{pmatrix}$	$\begin{pmatrix} 0.475+ \\ \imath 0.435, \\ -0.625- \\ \imath 0.615 \end{pmatrix}$
\dot{u}_2	$\begin{pmatrix} 0.50+ \\ \imath 0.46, \\ -0.60- \\ \imath 0.59 \end{pmatrix}$	$\begin{pmatrix} 0.60+ \\ \imath 0.56, \\ -0.50- \\ \imath 0.49 \end{pmatrix}$	$\begin{pmatrix} 0.70+ \\ \imath 0.66, \\ -0.40- \\ \imath 0.39 \end{pmatrix}$	$\begin{pmatrix} 0.625+ \\ \imath 0.585, \\ -0.475- \\ \imath 0.465 \end{pmatrix}$	$\begin{pmatrix} 0.55+ \\ \imath 0.51, \\ -0.55- \\ \imath 0.54 \end{pmatrix}$	$\begin{pmatrix} 0.475+ \\ \imath 0.435, \\ -0.625- \\ \imath 0.615 \end{pmatrix}$	$\begin{pmatrix} 0.40+ \\ \imath 0.36, \\ -0.70- \\ \imath 0.69 \end{pmatrix}$
\dot{u}_3	$\begin{pmatrix} 0.60+ \\ \imath 0.56, \\ -0.50- \\ \imath 0.49 \end{pmatrix}$	$\begin{pmatrix} 0.70+ \\ \imath 0.66, \\ -0.40- \\ \imath 0.39 \end{pmatrix}$	$\begin{pmatrix} 0.625+ \\ \imath 0.585, \\ -0.475- \\ \imath 0.465 \end{pmatrix}$	$\begin{pmatrix} 0.55+ \\ \imath 0.51, \\ -0.55- \\ \imath 0.54 \end{pmatrix}$	$\begin{pmatrix} 0.475+ \\ \imath 0.435, \\ -0.625- \\ \imath 0.615 \end{pmatrix}$	$\begin{pmatrix} 0.40+ \\ \imath 0.36, \\ -0.70- \\ \imath 0.69 \end{pmatrix}$	$\begin{pmatrix} 0.50+ \\ \imath 0.46, \\ -0.60- \\ \imath 0.59 \end{pmatrix}$
\dot{u}_4	$\begin{pmatrix} 0.70+ \\ \imath 0.66, \\ -0.40- \\ \imath 0.39 \end{pmatrix}$	$\begin{pmatrix} 0.625+ \\ \imath 0.585, \\ -0.475- \\ \imath 0.465 \end{pmatrix}$	$\begin{pmatrix} 0.55+ \\ \imath 0.51, \\ -0.55- \\ \imath 0.54 \end{pmatrix}$	$\begin{pmatrix} 0.475+ \\ \imath 0.435, \\ -0.625- \\ \imath 0.615 \end{pmatrix}$	$\begin{pmatrix} 0.40+ \\ \imath 0.36, \\ -0.70- \\ \imath 0.69 \end{pmatrix}$	$\begin{pmatrix} 0.50+ \\ \imath 0.46, \\ -0.60- \\ \imath 0.59 \end{pmatrix}$	$\begin{pmatrix} 0.60+ \\ \imath 0.56, \\ -0.50- \\ \imath 0.49 \end{pmatrix}$
\dot{u}_5	$\begin{pmatrix} 0.625+ \\ \imath 0.585, \\ -0.475- \\ \imath 0.465 \end{pmatrix}$	$\begin{pmatrix} 0.55+ \\ \imath 0.51, \\ -0.55- \\ \imath 0.54 \end{pmatrix}$	$\begin{pmatrix} 0.475+ \\ \imath 0.435, \\ -0.625- \\ \imath 0.615 \end{pmatrix}$	$\begin{pmatrix} 0.40+ \\ \imath 0.36, \\ -0.70- \\ \imath 0.69 \end{pmatrix}$	$\begin{pmatrix} 0.50+ \\ \imath 0.46, \\ -0.60- \\ \imath 0.59 \end{pmatrix}$	$\begin{pmatrix} 0.60+ \\ \imath 0.56, \\ -0.50- \\ \imath 0.49 \end{pmatrix}$	$\begin{pmatrix} 0.70+ \\ \imath 0.66, \\ -0.40- \\ \imath 0.39 \end{pmatrix}$
\dot{u}_6	$\begin{pmatrix} 0.55+ \\ \imath 0.51, \\ -0.55- \\ \imath 0.54 \end{pmatrix}$	$\begin{pmatrix} 0.475+ \\ \imath 0.435, \\ -0.625- \\ \imath 0.615 \end{pmatrix}$	$\begin{pmatrix} 0.40+ \\ \imath 0.36, \\ -0.70- \\ \imath 0.69 \end{pmatrix}$	$\begin{pmatrix} 0.50+ \\ \imath 0.46, \\ -0.60- \\ \imath 0.59 \end{pmatrix}$	$\begin{pmatrix} 0.60+ \\ \imath 0.56, \\ -0.50- \\ \imath 0.49 \end{pmatrix}$	$\begin{pmatrix} 0.70+ \\ \imath 0.66, \\ -0.40- \\ \imath 0.39 \end{pmatrix}$	$\begin{pmatrix} 0.625+ \\ \imath 0.585, \\ -0.475- \\ \imath 0.465 \end{pmatrix}$
\dot{u}_7	$\begin{pmatrix} 0.475+ \\ \imath 0.435, \\ -0.625- \\ \imath 0.615 \end{pmatrix}$	$\begin{pmatrix} 0.40+ \\ \imath 0.36, \\ -0.70- \\ \imath 0.69 \end{pmatrix}$	$\begin{pmatrix} 0.50+ \\ \imath 0.46, \\ -0.60- \\ \imath 0.59 \end{pmatrix}$	$\begin{pmatrix} 0.60+ \\ \imath 0.56, \\ -0.50- \\ \imath 0.49 \end{pmatrix}$	$\begin{pmatrix} 0.70+ \\ \imath 0.66, \\ -0.40- \\ \imath 0.39 \end{pmatrix}$	$\begin{pmatrix} 0.625+ \\ \imath 0.585, \\ -0.475- \\ \imath 0.465 \end{pmatrix}$	$\begin{pmatrix} 0.55+ \\ \imath 0.51, \\ -0.55- \\ \imath 0.54 \end{pmatrix}$

Step 3. Here, it has no cost type only benefits type data available. So Table 9 is considered as it is.

Step 4. Weighted normalized BCFS decision matrix by equation (10). For further details, check Tables 10-14.

Table 10. Weighted normalized BCFS decision matrix

$B_{g_{\dot{u}v}}$	v_1	v_2	v_3	v_4	v_5	v_6	v_7
\dot{u}_1	$\begin{pmatrix} 0.07+ \\ \imath 0.06, \\ -0.13- \\ \imath 0.31 \end{pmatrix}$	$\begin{pmatrix} 0.08+ \\ \imath 0.07, \\ -0.10- \\ \imath 0.10 \end{pmatrix}$	$\begin{pmatrix} 0.07+ \\ \imath 0.07, \\ -0.06- \\ \imath 0.06 \end{pmatrix}$	$\begin{pmatrix} 0.08+ \\ \imath 0.07, \\ -0.048- \\ \imath 0.04 \end{pmatrix}$	$\begin{pmatrix} 0.06+ \\ \imath 0.06, \\ -0.05- \\ \imath 0.05 \end{pmatrix}$	$\begin{pmatrix} 0.05+ \\ \imath 0.05, \\ -0.05- \\ \imath 0.05 \end{pmatrix}$	$\begin{pmatrix} 0.08+ \\ \imath 0.07, \\ -0.11- \\ \imath 0.11 \end{pmatrix}$
\dot{u}_2	$\begin{pmatrix} 0.95+ \\ \imath 0.08, \\ -0.11- \\ \imath 0.11 \end{pmatrix}$	$\begin{pmatrix} 0.10+ \\ \imath 0.09, \\ -0.08- \\ \imath 0.08 \end{pmatrix}$	$\begin{pmatrix} 0.09+ \\ \imath 0.08, \\ -0.05- \\ \imath 0.05 \end{pmatrix}$	$\begin{pmatrix} 0.07+ \\ \imath 0.07, \\ -0.05- \\ \imath 0.05 \end{pmatrix}$	$\begin{pmatrix} 0.06+ \\ \imath 0.05, \\ -0.06- \\ \imath 0.05 \end{pmatrix}$	$\begin{pmatrix} 0.04+ \\ \imath 0.04, \\ -0.06- \\ \imath 0.06 \end{pmatrix}$	$\begin{pmatrix} 0.072+ \\ \imath 0.06, \\ -0.12- \\ \imath 0.12 \end{pmatrix}$
\dot{u}_3	$\begin{pmatrix} 0.11+ \\ \imath 0.10, \\ -0.09- \\ \imath 0.09 \end{pmatrix}$	$\begin{pmatrix} 0.11+ \\ \imath 0.11, \\ -0.06- \\ \imath 0.06 \end{pmatrix}$	$\begin{pmatrix} 0.08+ \\ \imath 0.07, \\ -0.06- \\ \imath 0.06 \end{pmatrix}$	$\begin{pmatrix} 0.06+ \\ \imath 0.06, \\ -0.06- \\ \imath 0.06 \end{pmatrix}$	$\begin{pmatrix} 0.05+ \\ \imath 0.04, \\ -0.06- \\ \imath 0.06 \end{pmatrix}$	$\begin{pmatrix} 0.04+ \\ \imath 0.03, \\ -0.07- \\ \imath 0.06 \end{pmatrix}$	$\begin{pmatrix} 0.09+ \\ \imath 0.08, \\ -0.10- \\ \imath 0.10 \end{pmatrix}$
\dot{u}_4	$\begin{pmatrix} 0.13+ \\ \imath 0.12, \\ -0.07- \\ \imath 0.07 \end{pmatrix}$	$\begin{pmatrix} 0.10+ \\ \imath 0.09, \\ -0.08- \\ \imath 0.07 \end{pmatrix}$	$\begin{pmatrix} 0.07+ \\ \imath 0.06, \\ -0.07- \\ \imath 0.07 \end{pmatrix}$	$\begin{pmatrix} 0.05+ \\ \imath 0.05, \\ -0.07- \\ \imath 0.07 \end{pmatrix}$	$\begin{pmatrix} 0.04+ \\ \imath 0.03, \\ -0.07- \\ \imath 0.07 \end{pmatrix}$	$\begin{pmatrix} 0.05+ \\ \imath 0.04, \\ -0.06- \\ \imath 0.05 \end{pmatrix}$	$\begin{pmatrix} 0.10+ \\ \imath 0.10, \\ -0.09- \\ \imath 0.08 \end{pmatrix}$
\dot{u}_5	$\begin{pmatrix} 0.11+ \\ \imath 0.11, \\ -0.09- \\ \imath 0.08 \end{pmatrix}$	$\begin{pmatrix} 0.09+ \\ \imath 0.08, \\ -0.09- \\ \imath 0.09 \end{pmatrix}$	$\begin{pmatrix} 0.06+ \\ \imath 0.05, \\ -0.08- \\ \imath 0.07 \end{pmatrix}$	$\begin{pmatrix} 0.04+ \\ \imath 0.04, \\ -0.08- \\ \imath 0.08 \end{pmatrix}$	$\begin{pmatrix} 0.05+ \\ \imath 0.05, \\ -0.06- \\ \imath 0.06 \end{pmatrix}$	$\begin{pmatrix} 0.06+ \\ \imath 0.05, \\ -0.05- \\ \imath 0.04 \end{pmatrix}$	$\begin{pmatrix} 0.12+ \\ \imath 0.11, \\ -0.07- \\ \imath 0.07 \end{pmatrix}$
\dot{u}_6	$\begin{pmatrix} 0.10+ \\ \imath 0.09, \\ -0.10- \\ \imath 0.10 \end{pmatrix}$	$\begin{pmatrix} 0.08+ \\ \imath 0.07, \\ -0.10- \\ \imath 0.10 \end{pmatrix}$	$\begin{pmatrix} 0.05+ \\ \imath 0.04, \\ -0.91- \\ \imath 0.08 \end{pmatrix}$	$\begin{pmatrix} 0.06+ \\ \imath 0.05, \\ -0.07- \\ \imath 0.07 \end{pmatrix}$	$\begin{pmatrix} 0.06+ \\ \imath 0.06, \\ -0.05- \\ \imath 0.05 \end{pmatrix}$	$\begin{pmatrix} 0.07+ \\ \imath 0.06, \\ -0.04- \\ \imath 0.03 \end{pmatrix}$	$\begin{pmatrix} 0.11+ \\ \imath 0.10, \\ -0.08- \\ \imath 0.08 \end{pmatrix}$
\dot{u}_7	$\begin{pmatrix} 0.09+ \\ \imath 0.08, \\ -0.11- \\ \imath 0.11 \end{pmatrix}$	$\begin{pmatrix} 0.06+ \\ \imath 0.06, \\ -0.11+ \\ \imath 0.11 \end{pmatrix}$	$\begin{pmatrix} 0.06+ \\ \imath 0.05, \\ -0.07- \\ \imath 0.07 \end{pmatrix}$	$\begin{pmatrix} 0.07+ \\ \imath 0.06, \\ -0.06- \\ \imath 0.05 \end{pmatrix}$	$\begin{pmatrix} 0.07+ \\ \imath 0.07, \\ -0.04- \\ \imath 0.04 \end{pmatrix}$	$\begin{pmatrix} 0.06+ \\ \imath 0.05, \\ -0.04- \\ \imath 0.04 \end{pmatrix}$	$\begin{pmatrix} 0.09+ \\ \imath 0.09, \\ -0.09- \\ \imath 0.09 \end{pmatrix}$

Step 5. Evaluate BCFS negative ideal solution by equations (11) and (12) respectively:

Table 11. BCFS negative ideal solution

v_1	v_2	v_3	v_4	v_5	v_6	v_7
$\begin{pmatrix} 0.076+ \\ \imath 0.0684, \\ -0.133- \\ \imath 0.1311 \end{pmatrix}$	$\begin{pmatrix} 0.068+ \\ \imath 0.0612, \\ -0.119- \\ \imath 0.1173 \end{pmatrix}$	$\begin{pmatrix} 0.052+ \\ \imath 0.0468, \\ -0.091- \\ \imath 0.0897 \end{pmatrix}$	$\begin{pmatrix} 0.048+ \\ \imath 0.0432, \\ -0.084- \\ \imath 0.0828 \end{pmatrix}$	$\begin{pmatrix} 0.044+ \\ \imath 0.0396, \\ -0.077- \\ \imath 0.0759 \end{pmatrix}$	$\begin{pmatrix} 0.040+ \\ \imath 0.0360, \\ -0.070- \\ \imath 0.0690 \end{pmatrix}$	$\begin{pmatrix} 0.072+ \\ \imath 0.0648, \\ -0.126- \\ \imath 0.1242 \end{pmatrix}$

Table 12. BCFSF Euclidean Distance ($BCFSFED_{\dot{u}}$)

0.340939	0.339180	0.337028	0.336059	0.336272	0.336401	0.339787
0.342475	0.340739	0.338758	0.339062	0.339275	0.339398	0.343182
0.336376	0.334892	0.334363	0.334529	0.334617	0.334630	0.336979
0.333031	0.333091	0.332345	0.332429	0.332443	0.331580	0.333601
0.343538	0.343797	0.343114	0.343318	0.342189	0.341184	0.342382
0.347015	0.347237	0.346399	0.345145	0.344024	0.343039	0.345755
0.355958	0.356242	0.353383	0.352020	0.350819	0.351253	0.354454

Step 6. Calculate the BCFSF Euclidean Distance ($BCFSFED_{\dot{u}}$) and BCFSF Hamming Distance ($BCFSFHD_{\dot{u}}$) of the alternatives from the BCFS negative ideal solution by using Equations (13) and (14), respectively.

Table 13. BCFSF Hamming Distance ($BCFSFHD_{\dot{u}}$) values of alternatives

0.025852	0.023237	0.017859	0.016530	0.015135	0.013748	0.024569
0.026282	0.023621	0.018143	0.016725	0.015315	0.013912	0.024833
0.025677	0.023064	0.017633	0.016260	0.014894	0.013534	0.024265
0.025412	0.022707	0.017372	0.016024	0.014681	0.013379	0.024014
0.026756	0.023899	0.018284	0.016861	0.015503	0.014131	0.025426
0.027038	0.024156	0.018490	0.017124	0.015744	0.014349	0.025700
0.028046	0.025058	0.019286	0.017863	0.016424	0.014909	0.026669

Table 14. Total of $BCFSFED_{\dot{u}}$ and $BCFSFHD_{\dot{u}}$ for each alternative

Alternatives	Total of $BCFSFED_{\dot{u}}$	Total of $BCFSFHD_{\dot{u}}$
\dot{u}_1	2.365667	0.136931
\dot{u}_2	2.382890	0.138831
\dot{u}_3	2.346386	0.135327
\dot{u}_4	2.328518	0.133588
\dot{u}_5	2.399523	0.140858
\dot{u}_6	2.418615	0.142600
\dot{u}_7	2.474129	0.148255

Step 7. Determine Relative Assessment (RA) matrix by equations (15) and (16) is shown in Table 15:

Table 15. Relative Assessment (RA) matrix

0	-0.017190	0.019311	0.037273	-0.033720	-0.052650	-0.107230
0.017256	0	0.036632	0.054657	-0.016600	-0.035590	-0.090380
-0.019250	-0.036380	0	0.017899	-0.052840	-0.071700	-0.126090
-0.037020	-0.054090	-0.017840	0	-0.070490	-0.089280	-0.143480
0.033989	0.016667	0.053431	0.071521	0	-0.019060	-0.074050
0.053249	0.035860	0.072754	0.090909	0.019125	0	-0.055200
0.109691	0.092100	0.129395	0.147747	0.075159	0.055828	0

Step 8. Compute the Assessment Score ($AS_{\hat{u}_i}$) of each alternative given below in Table 16.

Table 16. Assessment Score ($AS_{\hat{u}_i}$)

Alternative	$AS_{\hat{u}_i}$
\hat{u}_1	-0.15421
\hat{u}_2	-0.03403
\hat{u}_3	-0.28836
\hat{u}_4	-0.41220
\hat{u}_5	0.082493
\hat{u}_6	0.216696
\hat{u}_7	0.609919

Step 9. We can rank the options based on the assessment scores. Here, the best engineer is \hat{u}_7 .

$$\hat{u}_7 > \hat{u}_6 > \hat{u}_5 > \hat{u}_2 > \hat{u}_1 > \hat{u}_3 > \hat{u}_4$$

The ranking of alternatives is shown in a graph in Figure 4.

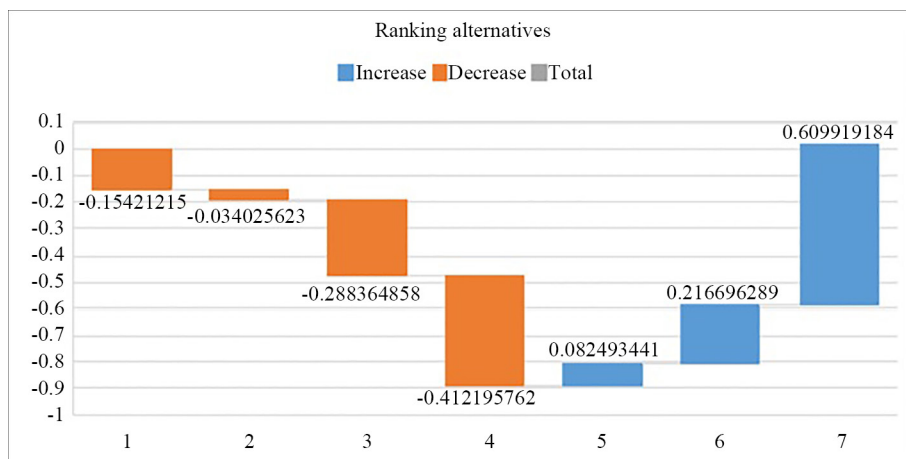


Figure 4. Ranking alternatives

6. Comparative study

Many researchers worked on ambiguity and impression dilemmas. Here, we highlight some prevailing notions for comparison with our proposed work. Zhang [19] initiated the BFS. Mahmood and Rehman [21] demonstrated BCFS. Jana et al. [32] interpreted a robust AO for the MCDM method with a BFS environment. Ahmmad [27] initiated the classification of renewable energy trends by utilizing the novel entropy measures under the environment of q-rung Orthopair FSSs and Thirunavukarasu et al. [28] demonstrated the theory of CFSS and its applications. Our proposed work consists of BCFSFAO with the CODAS method. Our work contains two-dimensional MG and positive and negative aspects with parameterization. Now, Zhang [19] demonstrated BFS that does not cover our data due to a deficiency of two-dimensional MG with parameterization. After this, Mahmood and Rehman [21] demonstrated BCFS that does not

handle our data due to a deficiency of parameterization. Moreover, Jana et al. [32] interpreted a robust AO for the MCDM method with a BFS environment that does not tackle our data due to a lack of two-dimensional MG. Furthermore, Ahmmad [27] initiated the classification of renewable energy trends by utilizing the novel entropy measures under the environment of q -rung Orthopair FSSs, which does not tackle our data due to the deficiency of two-dimensional MG and positive and negative aspects. Additionally, Thirunavukarasu et al. [28] demonstrated the theory of CFSS and its applications. This does not tackle our data due to the lack of bipolarity (positive and negative aspects). From this comparison, our proposed work is more supremacy and dominant from the above-prevailing notions. The comparison Table 17 is given below:

Table 17. Comparison of techniques based on BCFSFAAO

Techniques	Score values	Ranking
Zhang [19]	×	×
Mahmood and Rehman [21]	×	×
Jana et al. [32]	×	×
Ahmad [27]	×	×
Thirunavukarasu et al. [28]	×	×
BCFSFAAO	$v_1 = -0.15421, v_2 = -0.03403,$ $v_3 = -0.28836, v_4 = -0.4122,$ $v_5 = 0.082493, v_6 = 0.216696,$ $v_7 = 0.609919$	$v_7 > v_6 > v_5 > v_2 > v_1 > v_3 > v_4$

The comparison table by a graph in Figure 5.

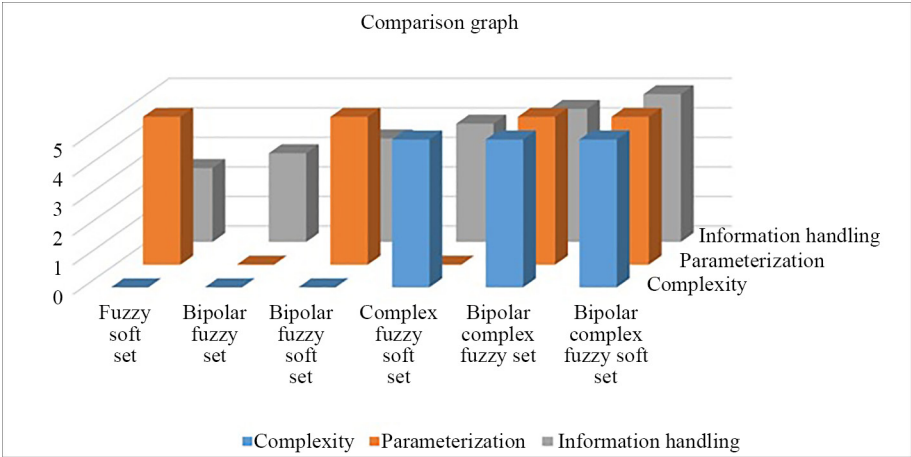


Figure 5. Comparison graph

7. Sensitivity analysis

To check the limitations and robustness of the proposed method, a sensitivity analysis was conducted by varying the threshold parameter Θ . Initially, the threshold value was set to $\Theta = 0.02$, resulting in the following ranking of alternatives:

$$\acute{u}_7 > \acute{u}_6 > \acute{u}_5 > \acute{u}_2 > \acute{u}_1 > \acute{u}_3 > \acute{u}_4.$$

When the threshold parameter was increased to $\Theta \geq 2.5$, the ranking changed to:

$$\dot{u}_7 > \dot{u}_6 > \dot{u}_5 > \dot{u}_4 > \dot{u}_3 > \dot{u}_2 > \dot{u}_1.$$

It is evident that while the top-ranked alternative \dot{u}_7 remains consistent across different values of Θ , some fluctuations are observed in the rankings of the lower alternatives. This indicates that although the proposed method is stable in identifying the best alternative, it is somewhat sensitive to changes in the threshold parameter when evaluating lower-ranked options. The sensitivity analysis is shown in Table 18.

Table 18. Sensitivity analysis

Parameter	Ranking	Changes
$\Theta < 2.5$	$\dot{u}_7 > \dot{u}_6 > \dot{u}_5 > \dot{u}_2 > \dot{u}_1 > \dot{u}_3 > \dot{u}_4$	$\dot{u}_2 > \dot{u}_1 > \dot{u}_3 > \dot{u}_4$
$\Theta \geq 2.5$	$\dot{u}_7 > \dot{u}_6 > \dot{u}_5 > \dot{u}_4 > \dot{u}_3 > \dot{u}_2 > \dot{u}_1$	$\dot{u}_4 > \dot{u}_3 > \dot{u}_2 > \dot{u}_1$

8. Conclusion

A highly used approach to managing complex DM problems with two-dimensional parameters to consider both positive and negative elements and essential vagueness is the BCFSS framework. It efficiently deals with uncertainty and fuses different opinions to enable a better study. BCFSFAO framework was developed to extend further BCFSS capability by presenting an innovative AO known as Fuzzy Aggregation Operator (FAO). Bipolar Complex Fuzzy Soft Weighted Arithmetic Aggregation Operator (BCFSFWAAO) and Bipolar Complex Fuzzy Soft Weighted Geometric Aggregation Operator (BCFASFWGAO) are the two crucial components of the BCFSFAO framework. Additionally, the framework now includes the CODAS approach, which offers a sound solution for numerical problems and facilitates sound DM. CpE, the science and engineering of designing, constructing, operating, and supporting the hardware and software basics of modern computer systems and computer-controlled devices, is the discipline to which the framework has been applied. Software Engineering (SE), its concentration on the development, evaluation, and maintenance of software programs, has been the focus in CpE. The CODAS method has also been applied to analyze a numerical case in SE and has effectively generated insightful results. The excellence and effectiveness of the BCFSFAO framework have then been assessed by comparing its results with those of other methods. For the comparison part, a chart that graphically presents the results has been constructed to enhance the understanding of the data. Upon analysis, the options were listed in decreasing order, and \dot{u}_7 was found to be the best answer both qualitatively and quantitatively. Figure 6 gives a representation of this finding. Looking ahead, we are committed to extending this research into multi-criteria decision-making [35, 36] and exploring applications of Aczel-Alsina aggregation operators [37–39] to further refine and expand the framework’s capabilities.

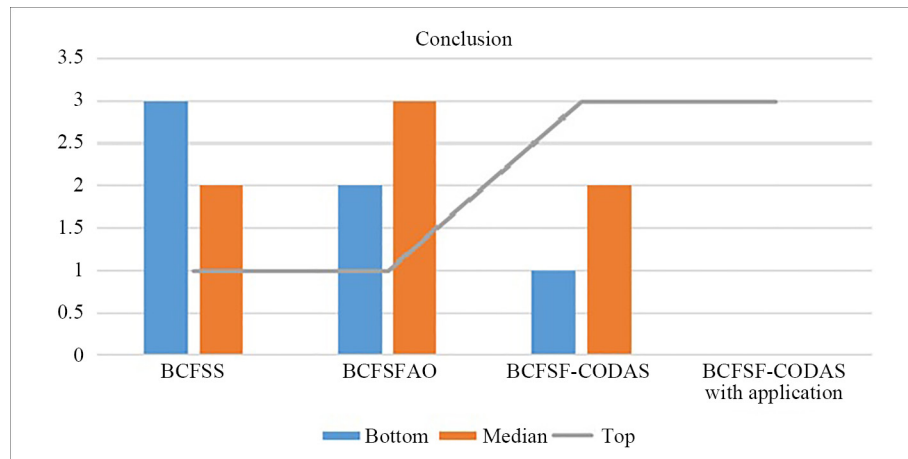


Figure 6. Conclusion

Data availability

The data generated and analyzed for this article is contained in the manuscript and anyone can use it by just citing the article.

Ethics declaration statement

The authors state that this is their original work and it is neither submitted nor under consideration in any other journal simultaneously.

Human and animal participants

This article does not contain any studies with human participants or animals performed by any of the authors.

Conflict of interest

The authors state that they have no conflicts of interest for the publication of this work.

References

- [1] Mahmood T, Rehman UU, Jaleel A, Ahmmad J, Chinram R. Bipolar complex fuzzy soft sets and their applications in decision-making. *Mathematics*. 2022; 10(7): 1048. Available from: <https://doi.org/10.3390/math10071048>.
- [2] Mahmood T, Jaleel A, Rehman UU. Pattern recognition and medical diagnosis based on trigonometric similarity measures for bipolar complex fuzzy soft sets. *Soft Computing*. 2023; 27(16): 11125-11154. Available from: <https://doi.org/10.1007/s00500-023-08176-y>.
- [3] Mahmood T, Jaleel A, Ur Rehman U. Determination of the most influential robot in the medical field by utilizing the bipolar complex fuzzy soft aggregation operators. *Expert Systems with Applications*. 2024; 251: 123878. Available from: <https://doi.org/10.1016/j.eswa.2024.123878>.

- [4] Jaleel A. WASPAS technique utilized for agricultural robotics system based on Dombi aggregation operators under bipolar complex fuzzy soft information. *Journal of Innovative Research in Mathematical and Computational Sciences*. 2022; 1(2): 67-95.
- [5] Jaleel A, Mahmood T, Albaity M. Analysis and applications of bipolar complex fuzzy soft power dombi aggregation operators for robot selection in artificial intelligence. *IEEE Access*. 2024; 12: 32218-32237. Available from: <https://doi.org/10.1109/ACCESS.2024.3368502>.
- [6] Jaleel A, Mahmood T, Emam W, Yin S. Interval-valued bipolar complex fuzzy soft sets and their applications in decision-making. *Scientific Reports*. 2024; 14(1): 11589. Available from: <https://doi.org/10.1038/s41598-024-58792-3>.
- [7] Ghorabae MK, Amiri M, Zavadskas EK, Hooshmand R, Antuchevicienė J. Fuzzy extension of the CODAS method for multi-criteria market segment evaluation. *Journal of Business Economics and Management*. 2017; 18(1): 1-19. Available from: <https://doi.org/10.3846/16111699.2016.1278559>.
- [8] Simic V, Karagoz S, Deveci M, Aydin N. Picture fuzzy extension of the CODAS method for multi-criteria vehicle shredding facility location. *Expert Systems with Applications*. 2021; 175: 114644. Available from: <https://doi.org/10.1016/j.eswa.2021.114644>.
- [9] Bolturk E. Pythagorean fuzzy CODAS and its application to supplier selection in a manufacturing firm. *Journal of Enterprise Information Management*. 2018; 31(4): 550-564. Available from: <https://doi.org/10.1108/JEIM-01-2018-0020>.
- [10] Karagoz S, Deveci M, Simic V, Aydin N, Bolukbas U. A novel intuitionistic fuzzy MCDM-based CODAS approach for locating an authorized dismantling center: A case study of Istanbul. *Waste Management and Research*. 2020; 38(6): 660-672. Available from: <https://doi.org/10.1177/0734242X19899729>.
- [11] Peng X, Garg H. Algorithms for interval-valued fuzzy soft sets in emergency decision-making based on WDBA and CODAS with new information measure. *Computers and Industrial Engineering*. 2018; 119: 439-452. Available from: <https://doi.org/10.1016/j.cie.2018.04.001>.
- [12] Zadeh LA. Fuzzy sets. *Information and Control*. 1965; 8(3): 338-353. Available from: [https://doi.org/10.1016/S0019-9958\(65\)90241-X](https://doi.org/10.1016/S0019-9958(65)90241-X).
- [13] Ramot D, Milo R, Friedman M, Kandel A. Complex fuzzy sets. *IEEE Transactions on Fuzzy Systems*. 2002; 10(2): 171-186. Available from: <https://doi.org/10.1109/91.995119>.
- [14] Ur Rehman U. Selection of database management system by using multi-attribute decision-making approach based on probability complex fuzzy aggregation operators. *Journal of Innovative Research in Mathematical and Computational Sciences*. 2023; 2(1): 1-16.
- [15] Hussain A, Latif S, Ullah K. A novel approach of picture fuzzy sets with unknown degree of weights based on schweizer-sklar aggregation operators. *Journal of Innovative Research in Mathematical and Computational Sciences*. 2022; 1(2): 18-39.
- [16] Atanassov KT. *On Intuitionistic Fuzzy Sets Theory*. Heidelberg: Springer; 2012. Available from: <https://doi.org/10.1007/978-3-642-29127-2>.
- [17] Garg H, Rani D. Novel aggregation operators and ranking method for complex intuitionistic fuzzy sets and their applications to the decision-making process. *Artificial Intelligence Review*. 2020; 53: 3595-3620. Available from: <https://doi.org/10.1007/s10462-019-09772-x>.
- [18] Ali Z. Decision-making techniques based on complex intuitionistic fuzzy power interaction AOs and their applications. *Journal of Innovative Research in Mathematical and Computational Sciences*. 2022; 1(1): 107-125.
- [19] Zhang WR. Bipolar fuzzy sets and relations: a computational framework for cognitive modeling and multi-agent decision analysis. In: *NAFIPS/IFIS/NASA '94. Proceedings of the First International Joint Conference of The North American Fuzzy Information Processing Society Biannual Conference. The Industrial Fuzzy Control and Intelligence*. San Antonio, TX, USA: IEEE; 1994. p.305-309. Available from: <https://doi.org/10.1109/IJCF.1994.375115>.
- [20] Jana C, Pal M, Wang JQ. Bipolar fuzzy Dombi aggregation operators and their application in the multiple-attribute decision-making process. *Journal of Ambient Intelligence and Humanized Computing*. 2019; 10: 3533-3549. Available from: <https://doi.org/10.1007/s12652-018-1076-9>.
- [21] Mahmood T, Ur Rehman U. A novel approach towards bipolar complex fuzzy sets and their applications in generalized similarity measures. *International Journal of Intelligent Systems*. 2022; 37(1): 535-567. Available from: <https://doi.org/10.1002/int.22639>.

- [22] Naeem M, Mahmood T, Ur Rehman U, Mehmood F. Classification of renewable energy and its sources with a decision-making approach based on bipolar complex fuzzy frank power aggregation operators. *Energy Strategy Reviews*. 2023; 49: 101162. Available from: <https://doi.org/10.1016/j.esr.2023.101162>.
- [23] Molodtsov D. Soft set theory-first results. *Computers & Mathematics with Applications*. 1999; 37(4-5): 19-31. Available from: [https://doi.org/10.1016/S0898-1221\(99\)00056-5](https://doi.org/10.1016/S0898-1221(99)00056-5).
- [24] Maji PK, Biswas R, Roy AR. Soft set theory. *Computers & Mathematics with Applications*. 2003; 45(4-5): 555-562. Available from: [https://doi.org/10.1016/S0898-1221\(03\)00016-6](https://doi.org/10.1016/S0898-1221(03)00016-6).
- [25] Ali MI, Feng F, Liu X, Min WK, Shabir M. On some new operations in soft set theory. *Computers & Mathematics with Applications*. 2009; 57(9): 1547-1553. Available from: <https://doi.org/10.1016/j.camwa.2008.11.009>.
- [26] Cagman N, Enginoglu S, Citak F. Fuzzy soft set theory and its applications. *Iranian Journal of Fuzzy Systems*. 2011; 8(3): 137-147.
- [27] Ahmad J. Classification of renewable energy trends by utilizing the novel entropy measures under the environment of q -rung orthopair fuzzy soft sets. *Journal of Innovative Research in Mathematical and Computational Sciences*. 2023; 2(2): 1-17. Available from: <https://doi.org/10.62270/jirmcs.v2i2.19>.
- [28] Thirunavukarasu P, Suresh R, Ashokkumar V. Theory of complex fuzzy soft set and its applications. *International Journal of Innovative Research in Science and Technology*. 2017; 3(10): 13-18.
- [29] Maji PK. More on intuitionistic fuzzy soft sets. In: Sakai H, Chakraborty MK, Hassanien AE, Ślęzak D, Zhu W. (eds.) *Rough Sets, Fuzzy Sets, Data Mining and Granular Computing*. Heidelberg: Springer; 2009. p.231-240. Available from: https://doi.org/10.1007/978-3-642-10646-0_28.
- [30] Kumar T, Bajaj RK. On complex intuitionistic fuzzy soft sets with distance measures and entropies. *Journal of Mathematics*. 2014; 2014(1): 972198. Available from: <https://doi.org/10.1155/2014/972198>.
- [31] Riaz M, Tehrim ST. Bipolar fuzzy soft mappings with application to bipolar disorders. *International Journal of Biomathematics*. 2019; 12(7): 1950080. Available from: <https://doi.org/10.1142/S1793524519500803>.
- [32] Jana C, Pal M, Wang J. A robust aggregation operator for multi-criteria decision-making method with bipolar fuzzy soft environment. *Iranian Journal of Fuzzy Systems*. 2019; 16(6): 1-16.
- [33] Jana C, Pal M, Wang JQ. Bipolar fuzzy Dombi prioritized aggregation operators in multiple attribute decision-making. *Soft Computing*. 2020; 24: 3631-3646. Available from: <https://doi.org/10.1007/s00500-019-04130-z>.
- [34] Jana C, Pal M. Application of bipolar intuitionistic fuzzy soft sets in decision-making problems. *International Journal of Fuzzy System Applications*. 2018; 7(3): 32-55. Available from: <https://doi.org/10.4018/IJFSA.2018070103>.
- [35] Kousar S, Kausar N. Multi-criteria decision-making for sustainable agritourism: An integrated fuzzy-rough approach. *Spectrum of Operational Research*. 2025; 2(1): 134-150. Available from: <https://doi.org/10.31181/sor21202515>.
- [36] Kumar R, Pamucar D. A comprehensive and systematic review of multi-criteria decision-making (MCDM) methods to solve decision-making problems: Two decades from 2004 to 2024. *Spectrum of Decision Making and Applications*. 2025; 2(1): 178-197. Available from: <https://doi.org/10.31181/sdmap21202524>.
- [37] Hussain A, Ullah K, Pamucar D, Haleemzai I, Tatić D. Assessment of solar panel using multiattribute decision-making approach based on intuitionistic fuzzy aczel alsina heronian mean operator. *International Journal of Intelligent Systems*. 2023; 2023(1): 6268613. Available from: <https://doi.org/10.1155/2023/6268613>.
- [38] Jabeen K, Ullah K, Akram M, Haleemzai I. Interval-valued picture fuzzy Aczel-Alsina aggregation operators and their application by using the multiattribute decision-making problem. *Journal of Mathematics*. 2023; 2023(1): 1707867. Available from: <https://doi.org/10.1155/2023/1707867>.
- [39] Imran R, Ullah K, Ali Z, Akram M. A multi-criteria group decision-making approach for robot selection using interval-valued intuitionistic fuzzy information and aczel-alsina bonferroni means. *Spectrum of Decision Making and Applications*. 2024; 1(1): 1-32. Available from: <https://doi.org/10.31181/sdmap1120241>.