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Research Article

Evolutionary Cooperation Dynamics with Mixed Strategy Updating of Combining Super-Rational Imitation and Aspiration Process

Si-Yi Wang[1], Chen-Xu Shi[1], S M Golam Akbar Rabbi[1], Yi Tao[2], Qing-lian Wang[3], Rui-Wu Wang[4*]

[1] School of Modern Posts, Xi'an University of Posts & Telecommunications, Xi'an, China
[2] Institute of Zoology, Chinese Academy of Sciences, Beijing, China
[3] Department of Applied Mathematics, Northwestern Polytechnical University, Xi'an, China
[4] College of Life Sciences, Zhejiang University, Hangzhou, China
E-mail: wangrw@zju.edu.cn

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Abstract: Understanding the mechanisms that generate and maintain cooperation remains a fundamental challenge in evolutionary game theory, with micro-level mechanisms playing a pivotal role in strategy evolution. Hofbauer's theory suggests that imitation effects are more significant than genetic factors in the spread of successful strategies. Building on this framework, previous studies have explored various strategy update mechanisms, though they often focus on single mechanisms. This paper presents a novel mixed strategy update mechanism that integrates super-rational imitation and the aspiration process within a well-mixed finite population. Individuals select between these two strategies with a defined probability. We derived conditions that are conducive to the evolution of cooperation, and apply weak selection approximation to two specific cases: the prisoner's dilemma and the stag hunt games. Theoretical analysis and numerical simulations reveal that cooperation is promoted when cooperators are super-rational, whereas defectors being super-rational impedes cooperation. Moreover, when cooperators exhibit low super-rationality, high aspiration levels support cooperation; conversely, when cooperators display high super-rationality, lower aspiration levels are more beneficial. This study provides important conditions for maintaining cooperation in various evolutionary settings, thereby extending the scope of evolutionary game theory.

***Keywords*:** cooperation dynamics, markov chain, strategy updating, evolutionary games

MSC: 91A22

1. Introduction

Cooperative behavior is observed in various forms both in nature and human society. Cooperation often requires individuals to sacrifice some personal benefits or to avoid over-exploitation of resources, creating a conflict between individual and collective interests-a phenomenon known as the "social dilemma" [1, 2]. The central question of evolutionary game theory is to identify the conditions under which cooperation can emerge and be maintained within populations [3]. In finite populations, micro-level mechanisms, particularly strategy update rules, play a crucial role in the evolution of cooperation [4–11].

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One fundamental question is: what drives the spread of strategies (or behaviors, phenotypes) in a population? Hofbauer and Sigmund proposed that imitation mechanisms may be more influential than genetic factors in the spread of successful strategies [12]. For example, fish schools exhibit coordinated movement through random imitation of neighboring individuals, while the Lappet Butterfly mimics the appearance of leaves to avoid predators [13]. Classic imitation dynamics, rooted in the input-output model [14–17], suggest that the "proportional imitation rule" leads to replicator dynamics, which have proven to be an optimal framework for studying strategy evolution. This approach has been widely applied to explore the evolution of cooperation within imitation dynamics, yielding rich insights into how various imitation styles quantitatively influence cooperative behaviors [18], and how factors like environmental noise affect cooperation in the stochastic imitation dynamic system [19]. h Building on imitation dynamics, scholars have introduced various strategy update mechanisms. Nowak and May integrated "imitating the strategy of the individual with the highest payoffs" into evolutionary dynamics [20]. Szabó and Tökė integrated the imitation process using "Fermi rule", where a randomly selected individual compares the payoff with that of a second randomly selected individual, and the second individual is more likely to be imitated if their payoff is higher [5]. Macy and Flache developed the aspiration process that the idea is derived from the "win-stay, lose-shift" rule and the imitation rule. In the aspiration process, individuals focus more on aspiration levels-or past experiences-rather than direct imitation [21, 22]. For instance, ants use chemical traces to find food, learning from previous experiences rather than imitating others [23]. In this framework, an individual compares its actual payoff with a desired aspiration level and adjusts strategies if the payoff fails to meet expectations. Another extension is super-rational imitation, where individuals imitate the strategies of others based on expected payoffs. If the payoff exceeds expectations, the individual retains its strategy; otherwise, they imitate the successful strategies of others, a principle that is generally accepted across populations [24–27]. For example, highly risk-averse or cautious individuals that rarely switch strategies unless payoff is extremely low in real scenarios.

While cooperation evolution dynamics have been extensively studied under single strategy update mechanisms [8, 28–37], real-world scenarios often involve a mix of strategies. Human behavior is complex, and individuals may adopt different strategy update mechanisms with varying probabilities. Recent work has integrated mixed strategy update rules, such as combining imitation with aspiration, Moran processes with imitation, and payoff-driven updates with conformity-driven ones [38–46]. Studies have shown that combinations of different update rules have distinct effects on cooperation. For instance, increasing the likelihood of using super-rational imitation within a mixed strategy framework tends to promote cooperation more effectively than relying on traditional imitation or super-rational imitation alone [27]. Additionally, these mixed update strategies have been applied beyond cooperation, such as to analyze behaviors like vaccination, driven by imitation and aspiration [47]. These developments not only broaden the scope of evolutionary game theory but also offer new insights into cooperative behavior. However, the combined effects of super-rational imitation and the aspiration process on the evolution of cooperation remain largely unexplored.

In this paper, we introduce a mixed strategy update mechanism that combines super-rational imitation and the aspiration process. Super-rational imitation involves individuals imitating others based on expected payoffs, while the aspiration process centers on individuals' own aspiration levels. In a finite population, we perform theoretical approximations under weak selection and present numerical simulations using the prisoner's dilemma game and the stag hunt game to explore the dynamics of cooperation under varying selection intensities. Our results highlight the conditions that favor the evolution of cooperation, focusing on selection intensity, aspiration levels, and super-rationality. Exploring how strategy update rules dynamically shape cooperation in finite populations may offers insights into resolving social dilemmas through behavioral regulation. These findings offer new perspectives for addressing the social dilemma and advancing the theory of cooperation.

2. Evolutionary game model of mixed strategy update rule

In a well-mixed finite population with N players, each player can choose strategy between Cooperation (C) or Defection (D), and engage in equally possible pairwise games. Self-interactions are excluded. The player who chooses cooperation strategy is a cooperator, and the player who chooses defection strategy is a defector. In paired games, the cooperator gains a interacting with a cooperator. The defector gains d interacting with a defector. If a cooperator meets a defector, the cooperator gains b and the defector gains c [48–50]. Parameters satisfy $a > b,\ c > d$. The payoff matrix is given by

$$\mathbf{P} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \tag{1}$$

For players of strategy type R_i ($R_i = C$ or $R_i = D$), the concept of super-rational aspiration can be defined in equation (2) [24–27].

$$\Pi_{R_i} = (1 - \phi_{R_i})\, \pi_{R_i}^{(max)}. \tag{2}$$

In equation (2), Π_{R_i} is the expected payoff of strategy type R_i, measuring whether players are satisfied with the actual average payoff. ϕ_{R_i} is the super-rationality degree of strategy type R_i, where $0 \le \phi_{R_i} \le 1 - \dfrac{\pi_{R_i}^{(min)}}{\pi_{R_i}^{(max)}}$. $\pi_{R_i}^{(max)}$ $(\pi_{R_i}^{(min)})$ is the maximum (minimum) payoff that an R_i strategy type player possibly gains in payoff matrix (1). Particularly, when $\phi_{R_i} = 0$, players expect to get maximum payoffs (i.e. complete rational), while when $\phi_{R_i} = 1 - \dfrac{\pi_{R_i}^{(min)}}{\pi_{R_i}^{(max)}}$, players expect to get minimum payoffs (i.e. complete super-rational). Players have different degrees of super-rationality at the intermediate parameter range of ϕ_{R_i}. In super-rational imitation mechanism, if the average return is higher than the expected level, the individual is satisfied with the current strategy and will not change the strategy in the next round of the game. If the average return is lower than the expected level, the individual will imitate the strategies of other individuals with a certain probability in the next round of the game [24–27].

In super-rational imitation, if the average payoff is higher than the expected payoff, the player is satisfied with the current strategy and keeps the strategy unchanged in the next round of the game. If the average payoff is lower than the expected payoff, the player imitates other players' strategies with a probability. We use Fermi function as the probability for R_i strategy type player changes into R_j strategy type player ($R_j = C$ or $R_j = D$) [24–27]. Therefore, the probability $W_{R_i \to R_j}$ that an R_i strategy type player transforming into R_j strategy type player is given in equation (3).

$$W_{R_i \to R_j} = \begin{cases} 0 & \text{if} \quad \pi_{R_i} \ge \Pi_{R_i} \\ \dfrac{1}{1 + e^{-\omega(\pi_{R_j} - \pi_{R_i})}} & \text{if} \quad \pi_{R_i} < \Pi_{R_i}. \end{cases} \tag{3}$$

In equation (3), π_{R_i} represents the average payoff of R_i type player and ω represents the selection intensity, see Table 1 for parameters.

Table 1. Main parameters of the model

Parameter	Meaning
N	Number of players in a well-mixed finite population
$C,\ D$	The strategy of Cooperation (C) and Defection (D)
$a,\ b,\ c,\ d$	The payoffs of different paired strategy combinations
P	Payoff matrix
R_i, R_j	The strategy type of players, where $R_i,\ R_j \in \{C,\ D\}$
$\prod_{R_i}$	The expected payoff of strategy type R_i
ϕ_{R_i}	Super-rationality degree of strategy type R_i
$\pi_{R_i}^{(max)}\left(\pi_{R_i}^{(min)}\right)$	The maximum (minimum) payoff that an R_i strategy type player possibly gains in payoff matrix P
$W_{R_i \to R_j}$	The probability of an R_i strategy type player transforming into R_j strategy type player
π_{R_i}	The average payoff of R_i type player
ω	Selection intensity
A_{R_i}	Aspiration level of player R_i
$Z_{R_i \to R_j}$	The probability of an R_i type player changes into a R_j type player
β	The probability to use super-rational imitation
i	Number of cooperators in the population
$\pi_{R_i}(i)$	Average payoff of strategy type R_i
$P_{i,\ j}$	Probability of transitioning from the state i to the state j
$q_t(j)$	The probability that the system in a state j at time step t
ψ_j	The probability of the population being in the state j
$\langle \rho_D \rangle$	The average proportion of strategy type R_i

For aspiration process, R_i type player compares actual average payoff π_{R_i} with aspiration level A_{R_i} [21, 29, 30, 38, 40, 41]. If the actual average payoff is higher than the aspiration level, the original strategy R_i remains unchanged. If the actual average payoff is lower than the aspiration level, the R_i type player imitate a randomly selected player's strategy using probability $Z_{R_i \to R_j}$.

$$Z_{R_i \to R_j} = \frac{1}{1 + e^{-\omega(A_{R_i} - \pi_{R_i})}}. \tag{4}$$

In equation (4), $Z_{R_i \to R_j}$ represents the probability of an R_i type player changes into a R_j type player, A_{R_i} is the aspiration level of player R_i. For the convenience, we assume that both cooperators and defectors have the same aspiration level A_{R_i}, where $\pi_{R_i}^{(min)} \le A_{R_i} \le \pi_{R_i}^{(max)}$. With certain aspiration levels, the larger the average payoff, the less likely a player is to change strategy.

At any time step, an R_i type player is selected to update strategy, with probability β to use super-rational imitation and probability $1 - \beta$ to use aspiration process. When $\beta = 0$, all players use aspiration process. When $\beta = 1$, all players use super-rational imitation. In a certain state with complete super-rational imitation, when players' expected payoffs are low, the payoffs of each player are higher than their expected payoffs. Then, players in the population no longer update strategies. Therefore, we set $\beta \in [0,\ 1)$ in the model. See Figure 1 for the model diagram.

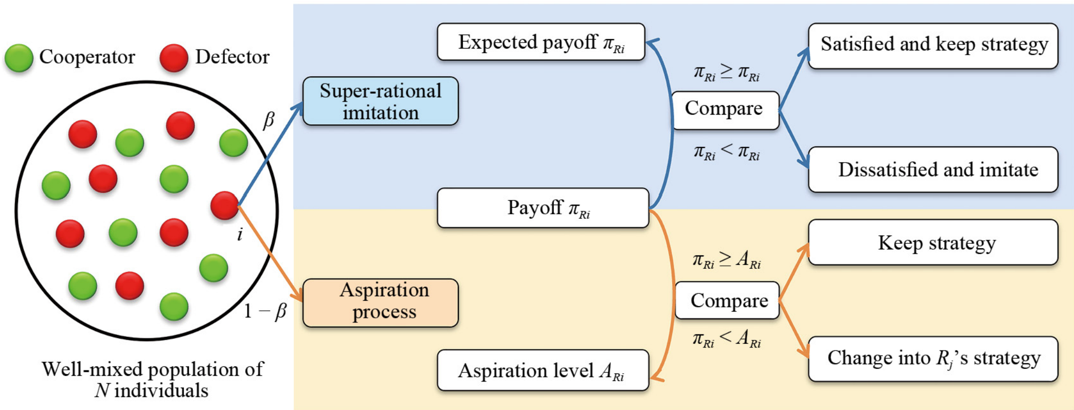

Figure 1. Model diagram

The number of cooperators in the population is i. The average payoffs of the cooperator and the defector are give in equation (5).

$$\begin{cases} \pi_C(i) = \dfrac{(i-1)a+(N-i)b}{N-i} \\ \\ \pi_D(i) = \dfrac{ic+(N-i-1)d}{N-1} \end{cases}, \tag{5}$$

The number of cooperators (i) represents the state of the system, and state variables may remain unchanged, transition to a state $i-1$, or transition to a state $i+1$. The $P_{i,\,j}$ represents the probability of transitioning from state i to state j. Let $P_{i,\,i}$ be the probability of staying in the state i, $P_{i,\,i-1}$ be the probability of transitioning from state i to state $i-1$, and $P_{i,\,i+1}$ be the probability of transitioning from state i to state $i+1$. When $|i-j|>1$, we have $P_{i,\,j}=0$. When $|i-j|\leq 1$, we have equation (6).

$$\begin{cases} P_{i,\,i+1} = \beta\,\dfrac{i}{N}\cdot\dfrac{N-i}{N}\,W_{D\to C}(i)+(1-\beta)\,\dfrac{N-i}{N}\,Z_{D\to C}(i) \\ \\ P_{i,\,i-1} = \beta\,\dfrac{i}{N}\cdot\dfrac{N-i}{N}\,W_{C\to D}(i)+(1-\beta)\,\dfrac{i}{N}\,Z_{C\to D}(i) \\ \\ P_{i,\,i} = 1-P_{i,\,i+1}-P_{i,\,i-1} \end{cases}. \tag{6}$$

Particularly, for super-rational imitation, when $\phi_C = 0$ $(\phi_D = 0)$, cooperators (defectors) pursue the possibly maximum payoff (i.e. completely rational). When $\phi_C = \phi_D = 0$, players only update their strategies based on the payoff difference, pursuing strategies with highest payoffs. In this case, super-rational imitation transforms into an imitation process. The mixed strategy updating combining super-rational imitation and aspiration process transforms into the mixed strategy updating combining imitation and super-rational imitation. This situation is presented in previous studies [24–27].

When $\phi_{R_i} = 1-\dfrac{\pi_{R_i}^{(min)}}{\pi_{R_i}^{(max)}}$, players are in a state of completely super-rational, and satisfied with their payoffs at any time. If players use super-rational imitation, they no longer update their strategies, and the system state is stalled. However, they may still use aspiration process. In this case, the evolution of the system is completely determined by aspiration process.

The addition of aspiration process allows the system to escape from stagnation. Since each player has the possibility to use aspiration process. Therefore, there is always a non-zero probability for players to change strategies. Therefore, the mixed strategy update rule does not have an absorption state, but rather a stationary distribution [51, 52].

Let $q_t(j)$ be the probability that the system in a state j at time step t, where $j = 1, \ldots, N$. We can obtain the main equation of $q_t(j)$ in equation (7).

$$q_{t+1}(j) = q_t(j-1)P_{j-1,\ j} + q_t(j+1)P_{j+1,\ j} + q_t(j)(1 - P_{j,\ j+1} - P_{j,\ j-1}). \tag{7}$$

The stationary distribution characterizes the steady-state characteristics of Markov chains after long-term operation. Based on the properties of Markov chains, there exists a unique stationary distribution $\psi = (\psi_0,\ \psi_1,\ \psi_2,\ \cdots,\ \psi_N)$, where $\psi_j (j = 1, \ldots, N)$ represents the probability of the population being in the state j. We have $\lim_{t\to\infty} q_t(j) = \psi_j$, and ψ_j satisfies the equilibrium equation

$$\psi_j = \psi_{j-1}P_{j-1,\ j} + \psi_{j+1}P_{j+1,\ j} + \psi_j(1 - P_{j,\ j+1} - P_{j,\ j-1}) \tag{8}$$

and meet the detailed balance conditions [53]

$$\begin{aligned} \psi_{j-1}P_{j-1,\ j} &= \psi_j P_{j,\ j-1}, \\ \psi_{j+1}P_{j+1,\ j} &= \psi_j P_{j,\ j+1}. \end{aligned} \tag{9}$$

Since $\sum_{j=0}^{N} \psi_j = 1$ and $\psi_j = \dfrac{P_{j-1,\ j}}{P_{j,\ j-1}}\ \psi_{j-1}$, we have the following equations.

$$\begin{aligned} \psi_j &= \frac{P_{j-1,\ j}}{P_{j,\ j-1}}\ \psi_{j-1} \\ &= \frac{P_{0,\ 1}}{P_{j,\ j-1}} \prod_{i=1}^{j-1} \frac{P_{i,\ i+1}}{P_{i,\ i-1}}\ \psi_0\ , \quad \text{for } j = 1,\ 2,\ \cdots,\ N \\ \Rightarrow \sum_{j=0}^{N} \psi_j &= \psi_0 \left[1 + \sum_{j=1}^{N} \frac{P_{0,\ 1}}{P_{j,\ j-1}} \prod_{i=1}^{j-1} \frac{P_{i,\ i+1}}{P_{i,\ i-1}} \right]\ . \end{aligned} \tag{10}$$

Thus, ψ_j can be normalized as

$$\psi_j = \begin{cases} \dfrac{1}{1+\sum_{j=1}^{N} \dfrac{P_{0,\,1}}{P_{j,\,j-1}} \prod_{i=1}^{k-1} \dfrac{P_{i,\,i+1}}{P_{i,\,i-1}}} & \text{for } j=0 \\[2ex] \dfrac{\dfrac{P_{0,\,1}}{P_{j,\,j-1}} \prod_{i=1}^{j-1} \dfrac{P_{i,\,i+1}}{P_{i,\,i-1}}}{1+\sum_{j=1}^{N} \dfrac{P_{0,\,1}}{P_{j,\,j-1}} \prod_{i=1}^{k-1} \dfrac{P_{i,\,i+1}}{P_{i,\,i-1}}} & \text{for } j=1,\, 2,\, \cdots,\, N. \end{cases} \tag{11}$$

The average proportion of cooperators and defectors are expressed as

$$\begin{cases} \langle \rho_C \rangle = \sum_{j=0}^{N} \dfrac{j}{N}\, \psi_j \\[2ex] \langle \rho_D \rangle = 1 - \langle \rho_C \rangle \end{cases} . \tag{12}$$

2.1 *Evolutionary cooperation dynamics of mixed strategy updating under weak selection*

For systems without absorption states, we use the average proportion of cooperators to measure whether it is conducive to cooperation. We mainly focus on whether the average proportion of cooperators is higher than that of defectors (i.e. whether the average proportion of cooperators is higher than $\frac{1}{2}$). For the mixed strategy updating, when players in the population use super-rational imitation, two special cases are considered. The case one is that cooperators have complete super-rationality and defectors have no super-rationality. The case two is that cooperators have no super-rationality and defectors have complete super-rationality. Then we discuss the evolutionary cooperation dynamics in two special cases under weak selection ($\omega \to 0$).

Case 1 When cooperators have complete super-rationality and defectors have no super-rationality, the transition probability is expressed in equation (13).

$$P_{i,\,i+1} = \begin{cases} \beta\, \dfrac{i}{N} \cdot \dfrac{N-i}{N} \cdot \dfrac{1}{1+e^{-\omega(\pi_C(i)-\pi_D(i))}} + (1-\beta)\dfrac{N-i}{N} \cdot \dfrac{1}{1+e^{-\omega(A-\pi_D(i))}} & \text{for } i < N-1 \\[2ex] (1-\beta)\dfrac{N-i}{N} \cdot \dfrac{1}{1+e^{-\omega(A-\pi_D(i))}} & \text{for } i \geq N-1 \end{cases} ,$$

$$P_{i,\,i-1} = (1-\beta)\, \frac{i}{N} \cdot \frac{1}{1+e^{-\omega(A-\pi_C(i))}} . \tag{13}$$

Based on equation (11), when $j < N-j$, we have $\dfrac{\psi_{N-j}}{\psi_j} = \prod_{i=j}^{N-j-1} \dfrac{P_{i,\,i+1}}{P_{i+1,\,i}}$. Since $N-j-1 < N-1$, we have equation (14).

$$\frac{P_{N-j-1,\ N-j}}{P_{j+1,\ j}} = \frac{\beta}{1-\beta} \cdot \frac{N-j-1}{N} \cdot \frac{1+e^{-\omega(A-\pi_C(j+1))}}{1+e^{-\omega(\pi_C(N-j-1)-\pi_D(N-j-1))}}$$

$$+ \frac{1+e^{-\omega(A-\pi_C(j+1))}}{1+e^{-\omega(A-\pi_D(N-j-1))}}. \tag{14}$$

Under weak selection ($\omega \to 0$), the first-order Taylor expansion of $\frac{P_{N-j-1,\ N-j}}{P_{j+1,\ j}}$ on ω is

$$\frac{P_{N-j-1,\ N-j}}{P_{j+1,\ j}} \approx 1 + \frac{\beta}{1-\beta} \cdot \frac{N-j-1}{N} + D_1\omega. \tag{15}$$

In equation (15), D_1 is expressed as

$$D_1 = \frac{\beta}{1-\beta} \cdot \frac{N-j-1}{N} \cdot \frac{\pi_C(j+1)+\pi_C(N-j-1)-\pi_D(N-j-1)-A}{2}$$

$$+ \frac{\pi_C(j+1)-\pi_D(N-j-1)}{2}\ . \tag{16}$$

When $\beta \in (0,\ 1)$, we have the inequality

$$1 + \frac{\beta}{1-\beta} \cdot \frac{N-j-1}{N} > 1. \tag{17}$$

Since $\omega \to 0$, we have the inequality

$$\frac{\psi_{N-j}}{\psi_j} = \prod_{i=j}^{N-j-1} \frac{P_{i,\ i+1}}{P_{i+1,\ i}} > 1\ . \tag{18}$$

If N is an even number, we have

$$\langle\rho_C\rangle - \langle\rho_D\rangle = \sum_{j=0}^{\frac{N}{2}-1} \left(\frac{j}{N} - \frac{N-j}{N}\right)\psi_j + \sum_{j=\frac{N}{2}+1}^{N} \left(\frac{j}{N} - \frac{N-j}{N}\right)\psi_j$$

$$= \sum_{j=0}^{\frac{N}{2}-1} \frac{2j-N}{N}\psi_j - \sum_{j=0}^{\frac{N}{2}-1} \frac{2j-N}{N}\psi_{N-j} = \sum_{j=0}^{\frac{N}{2}-1} \frac{2j-N}{N}\left(\psi_j - \psi_{N-j}\right)\ . \tag{19}$$

Based on equation (18), we have $\langle\rho_C\rangle > \langle\rho_D\rangle$. Since $\langle\rho_C\rangle + \langle\rho_D\rangle = 1$, we have $\langle\rho_C\rangle > 1/2$. Similarly, if N is an odd number, we have $\langle\rho_C\rangle > 1/2$. Therefore, the average proportion of cooperators is higher than that of defectors in case one under weak selection.

Case 2 When cooperators have no super-rationality and defectors have complete super-rationality, the transition probability is

$$P_{i,\ i+1} = (1-\beta)\ \frac{N-i}{N} \cdot \frac{1}{1+e^{-\omega(A-\pi_D(i))}}\ ,$$

$$P_{i,\ i-1} = \begin{cases} \beta\ \frac{i}{N} \cdot \frac{N-i}{N} \cdot \frac{1}{1+e^{\pi_D(i)-\pi_C(i))}} + (1-\beta)\ \frac{1}{1+e^{-\omega(A-\pi_C(i))}} & \text{for } i < N \\ (1-\beta)\ \frac{1}{1+e^{-\omega(A-\pi_C(i))}} & \text{for } i = N \end{cases}. \tag{20}$$

Under weak selection ($\omega \to 0$), the first-order Taylor expansion of $\frac{P_{N-j-1,\ N-j}}{P_{j+1,\ j}}$ on ω is

$$\frac{P_{N-j-1,\ N-j}}{P_{j+1,\ j}} \approx \frac{(1-\beta)\frac{j+1}{N}}{\beta\frac{j+1}{N} \cdot \frac{N-j-1}{N} + (1-\beta)\frac{j+1}{N}} + D_2\omega, \tag{21}$$

In equation (21), D_2 is expressed as

$$D_2 = \frac{1}{8}(1-\beta)\left(\frac{j+1}{N}\right)^2 \left[\frac{N-j-1}{N}\beta A - \left(1 - \frac{j+1}{N}\beta\right)\pi_D(N-j-1)\right.$$

$$\left. - \frac{N-j-1}{N}\beta\left(\pi_D(j+1) - \pi_C(j+1)\right) + (1-\beta)\pi_C(j+1)\right]$$

$$\times \left[\beta\frac{j+1}{N} \cdot \frac{N-j-1}{N} + (1-\beta)\frac{j+1}{N}\right]^{-2}. \tag{22}$$

When $\beta \in (0,\ 1)$, we have the inequality

$$\frac{(1-\beta)\dfrac{j+1}{N}}{\beta\dfrac{j+1}{N}\cdot\dfrac{N-j-1}{N}+(1-\beta)\dfrac{j+1}{N}}<1$$

$$\Rightarrow \frac{P_{N-j-1,\ N-j}}{P_{j+1,\ j}}<1$$

$$\Rightarrow \frac{\psi_{N-j}}{\psi_i}=\prod_{i=1}^{N-j-1}\frac{P_{i,\ i+1}}{P_{i+1,\ i}}<1. \tag{23}$$

Based on equation (23), we have $\langle\rho_C\rangle<1/2$.

Therefore, the average proportion of cooperators is lower than that of defectors in case two under weak selection.

Through the above deduction process, we have obtained the evolutionary cooperation dynamics in two special cases. For two special cases with $\beta\in(0,\ 1)$, we conducted weak selection approximation and found that when cooperators have complete super-rationality and defectors have no super-rationality, the average proportion of cooperators is higher than $1/2$. When cooperators have no super-rationality and defectors have complete super-rationality, the average proportion of cooperators is lower than $1/2$. The results indicate that under the approximation of weak selection for two special cases, cooperators with super-rationality are advantageous for cooperation, while defectors with super-rationality inhibit cooperation. The game type and expected payoff do not affect the result of whether the average proportion of cooperators is higher than that of defectors.

To further verify the theoretical results, we fixed some parameters and conducted numerical simulations. Let $a=3,\ b=1,\ c=4,\ d=2$ in payoff matrix (1), the game is the prisoner's dilemma game ($c>a>d>b$). The super-rationality degree of cooperators (defectors) can be obtained from the payoff matrix (1), which are $\phi_C\in\left[0,\ \frac{2}{3}\right]$ and $\phi_D\in\left[0,\ \frac{1}{2}\right]$ respectively. Figure 2 shows the average proportion of cooperators with probability β in two special cases of the mixed strategy updating. The green curve represents the case one, while the red curve represents the case two. For each set of parameters, the simulation is repeated for 100 times.

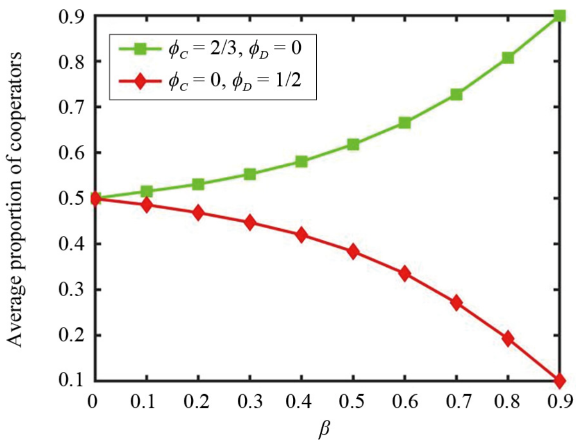

Figure 2. Fraction of cooperators under weak selection, where $N=100$, $\omega=0.001$, $A_{R_i}=4$, $a=3,\ b=1,\ c=4,\ d=2$

The results indicate that when cooperators have super-rationality, the higher the probability β of using super-rational imitation, the more beneficial it is for cooperation. When defectors have super-rationality, the higher the probability β of using super-rational imitation, the less conducive it is to cooperation. That is to say, cooperation is promoted when cooperators are super-rational, while cooperation is suppressed when defectors are super-rational.

To further analyze the impact on cooperation with different degrees of super-rationality for cooperators and defectors, we presented the average proportion of cooperators with different values of ϕ_C-ϕ_D under weak selection. The black line represents the average proportion of cooperators when players only use the aspiration process ($\beta = 0$). Figure 3a, Figure 3b, and Figure 3c represent different frequencies of using super-rational imitation, and $\beta = 0.1$, 0.5, and 0.9, respectively.

Figure 3 illustrates that when the super-rationality degree of the defector is $\phi_D = 0$, the average proportion of cooperators is higher than the case where the super-rationality degree of the defector is not zero, and also higher than the average proportion of cooperators when the player only uses the aspiration process in updating. Moreover, the average proportion of cooperators increases with the increase of their super-rationality. When the defector is completely super-rational $\left(\phi_D = \frac{1}{2}\right)$, the average proportion of cooperators is always lower than the situation that the player only uses the aspiration process. The numerical results indicate that for the prisoner's dilemma game under weak selection, the mixed strategy updating promotes cooperation when cooperators are super-rational, inhibit cooperation when defectors are super-rational. The inhibitory effect on cooperation reaches its maximum when defectors are in a completely super-rational state. As the probability of using super-rational imitation increases, the promoting effect of super-rational cooperators increases, and the inhibitory effect of super-rational defectors increases.

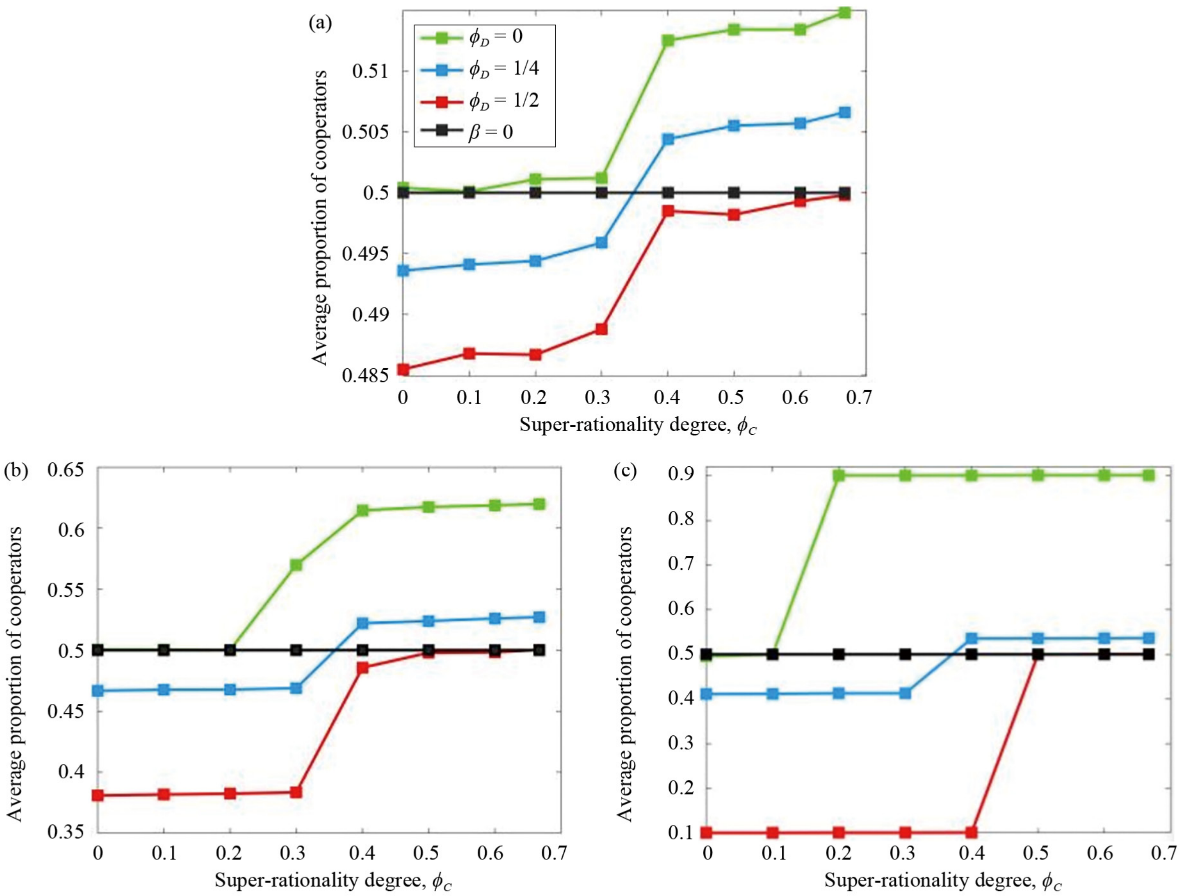

Figure 3. Average proportion of cooperators with different values of ϕ_C-ϕ_D. In panel (a), (b) and (c), $\beta = 0.1$, 0.5 and 0.9, respectively. The common parameters are $\omega = 0.001$, $A_{R_i} = 4$, $a = 3$, $b = 1$, $c = 4$, $d = 2$

2.2 *Evolutionary cooperation dynamics under arbitrary selection intensities*

To explore the average proportion of cooperators under arbitrary selection intensity, a color-coded proportion of cooperators in the prisoner's dilemma game with parameter space ω-A_{R_i} is given in Figure 4. Figure 4a depicts case one and Figure 4b depicts case two. In Figure 4a, the average proportion of cooperators decreases with increasing A_{R_i} and ω. In Figure 4b, the average proportion of cooperators increases with the increase of A_{R_i} and decreases with the increase of ω. When the selection intensity is low ($\omega \leq 0.02$), the impact of aspiration level (A_{R_i}) on the average proportion of cooperators is small, but as the selection intensity increase, the impact of aspiration level (A_{R_i}) increases. The results indicate that low selection intensity is beneficial for cooperation, and the super-rationality degree of cooperators promotes cooperation at low aspiration levels (A_{R_i}), while the super-rationality of defectors promotes cooperation at high aspiration levels (A_{R_i}).

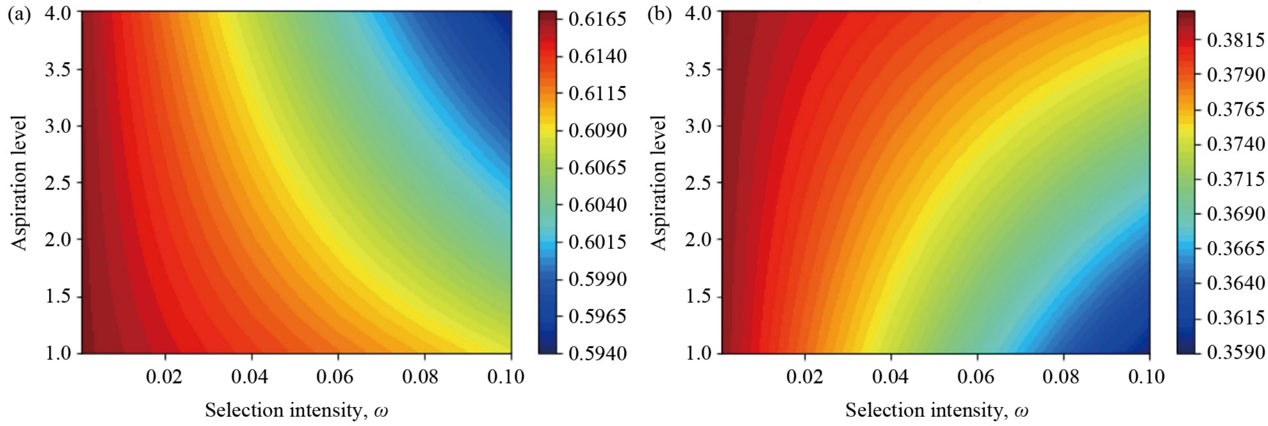

Figure 4. Color-coded average proportion of cooperators in the parameter region ω-A_{R_i}. In panel (a), $\phi_C = 0.67$, $\phi_D = 0$. In panel (b), $\phi_C = 0$, $\phi_D = 0.5$. The common parameters are $a = 3$, $b = 1$, $c = 4$, $d = 2$, $\beta = 0.5$. The color bar represents the cooperation level

Figure 5 applies stag hunt game to test the robustness of the results in Figure 4. Figure 5a depicts case one and Figure 5b depicts case two. In Figure 5a, the average proportion of cooperators decreases with increasing A_{R_i} and increases with increasing selection intensity ω. In Figure 5b, the average proportion of cooperators increases with the increase of A_{R_i}. When the aspiration level $A_{R_i} > 3$, the average proportion of cooperators increases with the increase of selection intensity. When the aspiration level $A_{R_i} \leq 3$, the average proportion of cooperators decreases with the increase of selection intensity. When the selection intensity is low ($\omega \leq 0.01$), the aspiration level A_{R_i} has a relatively small impact on the average proportion of cooperators, but as the selection intensity increases, the influence of A_{R_i} increases. The results indicate that when cooperators are super-rational, high selection intensity and low aspiration level A_{R_i} are beneficial for cooperation. When defectors are super-rational, high aspiration level A_{R_i} and high selection intensity are conducive to cooperation. When the selection intensity is low ($A_{R_i} \leq 3$), low selection intensity and high aspiration level A_{R_i} are conducive to cooperation.

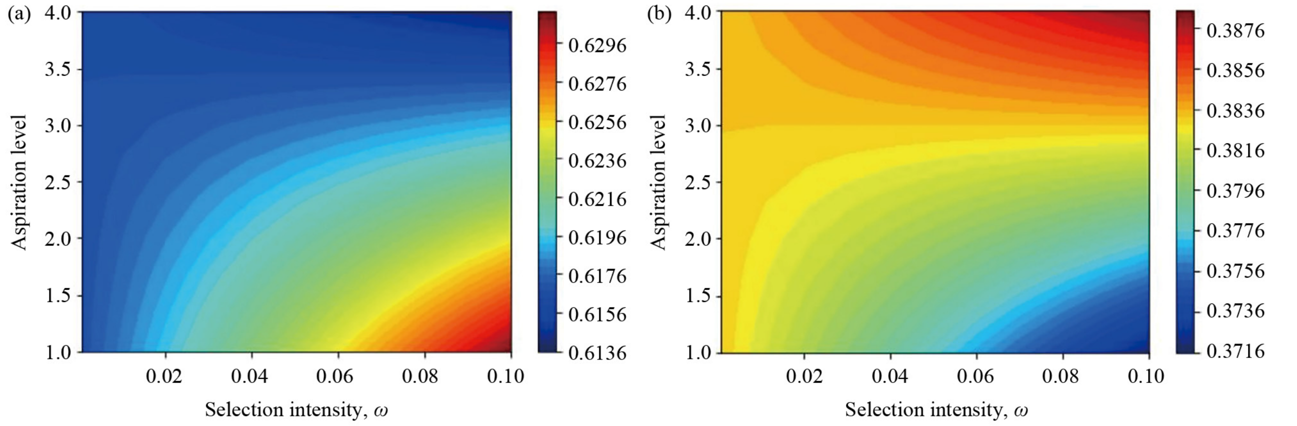

Figure 5. Color-coded average proportion of cooperators in the parameter region ω-A_{R_i}. In panel (a), $\phi_C = 0.75$, $\phi_D = 0$. In panel (b), $\phi_C = 0$, $\phi_D = 0.33$. The common parameters are $a = 4$, $b = 1$, $c = 3$, $d = 2$, $\beta = 0.5$. The color bar represents the cooperation level

Comparing Figure 4 and Figure 5, we found the impact of aspiration level A_{R_i} on the average proportion of cooperators is different in two cases, which is related to players' super-rationality degrees. To further study the impact of super-rationality on the evolution of cooperation, we present color-coded average proportion of cooperators in the prisoner's dilemma game and the stag hunt game in the parameter space A_{R_i}-ϕ_C in Figure 6 and Figure 7, respectively.

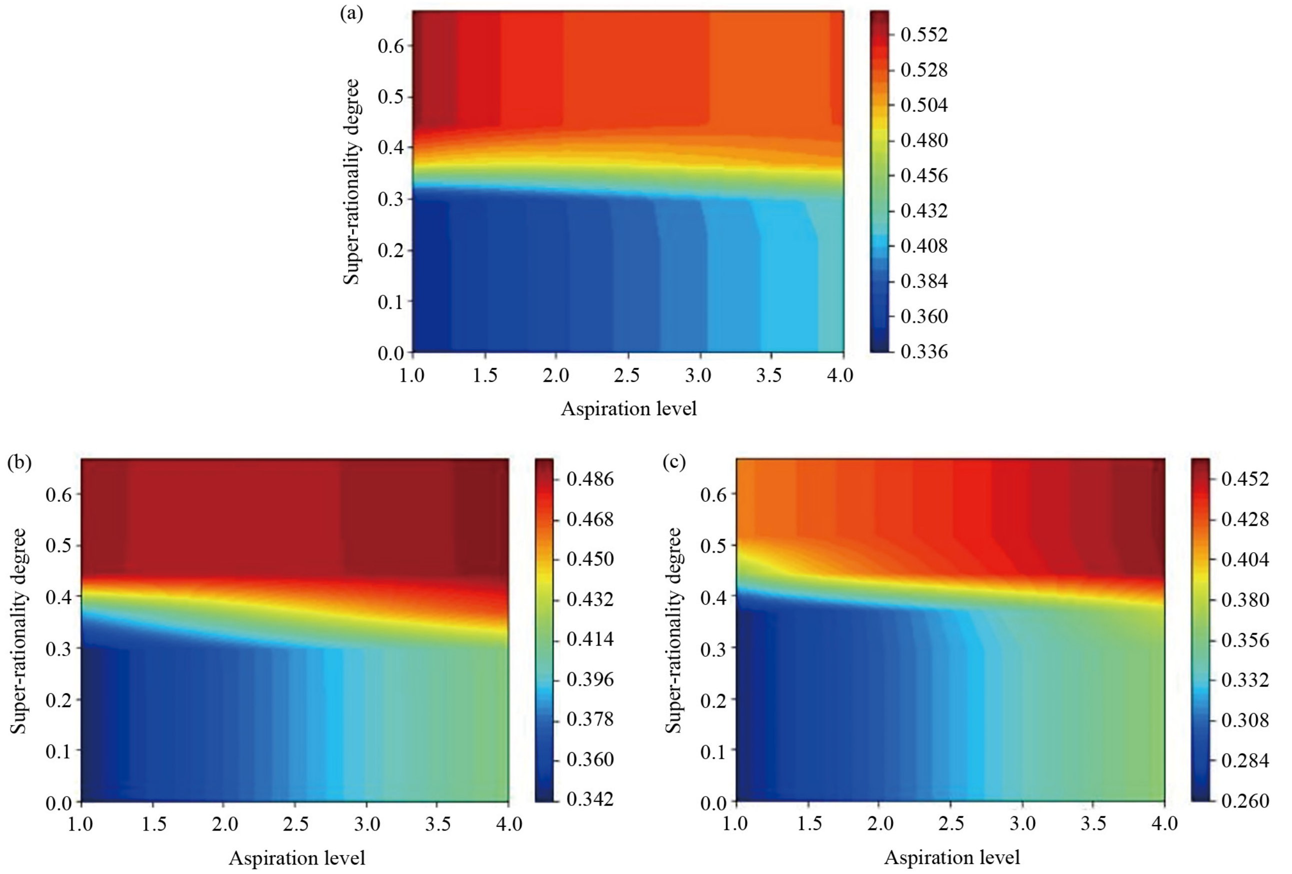

Figure 6. Color-coded average proportion of cooperators in the parameter region A_{R_i}-ϕ_C. In panel (a), (b) and (c), ϕ_D=0, 0.25 and 0.5, respectively. The common parameters are $a = 3$, $b = 1$, $c = 4$, $d = 2$, $\beta = 0.5$, $\omega = 0.5$

In Figure 6a, defector have no super-rationality ($\phi_D = 0$). When the super-rationality degree of the cooperator is $\phi_C > 0.37$, the average proportion of cooperators decreases as the aspiration level A_{R_i} increases. When the super-rationality degree of the cooperator is $\phi_C \leq 0.37$, the average proportion of cooperators increases as the aspiration level A_{R_i} increases. In Figure 6b, the super-rationality degree of the defector is $A^D = 0.25$. When the super-rationality degree of the cooperator is $\phi_C > 0.37$ and the aspiration level is $A_{R_i} \leq 2.286$, the average proportion of cooperators decreases as the aspiration level A_{R_i} increases, while when the aspiration level is $A_{R_i} > 2.286$, the average proportion of cooperators increases as the aspiration level A_{R_i} increases. When the super-rationality degree of cooperators is low, the proportion of cooperators increases with the aspiration level A_{R_i} increases. In Figure 6c, the super-rationality degree of the defector is $\phi_D = 0.5$ (i.e. completely super-rational), the average proportion of cooperators increases as the aspiration level A_{R_i} increases.

The results indicate that when the cooperator is completely super-rational and the defector have no super-rationality, the average proportion of cooperators is the highest when the aspiration level A_{R_i} is the lowest. With the increase of aspiration level A_{R_i}, the average proportion of cooperators only increases when the cooperator's super-rationality degree is small or the defector's super-rationality degree is not zero.

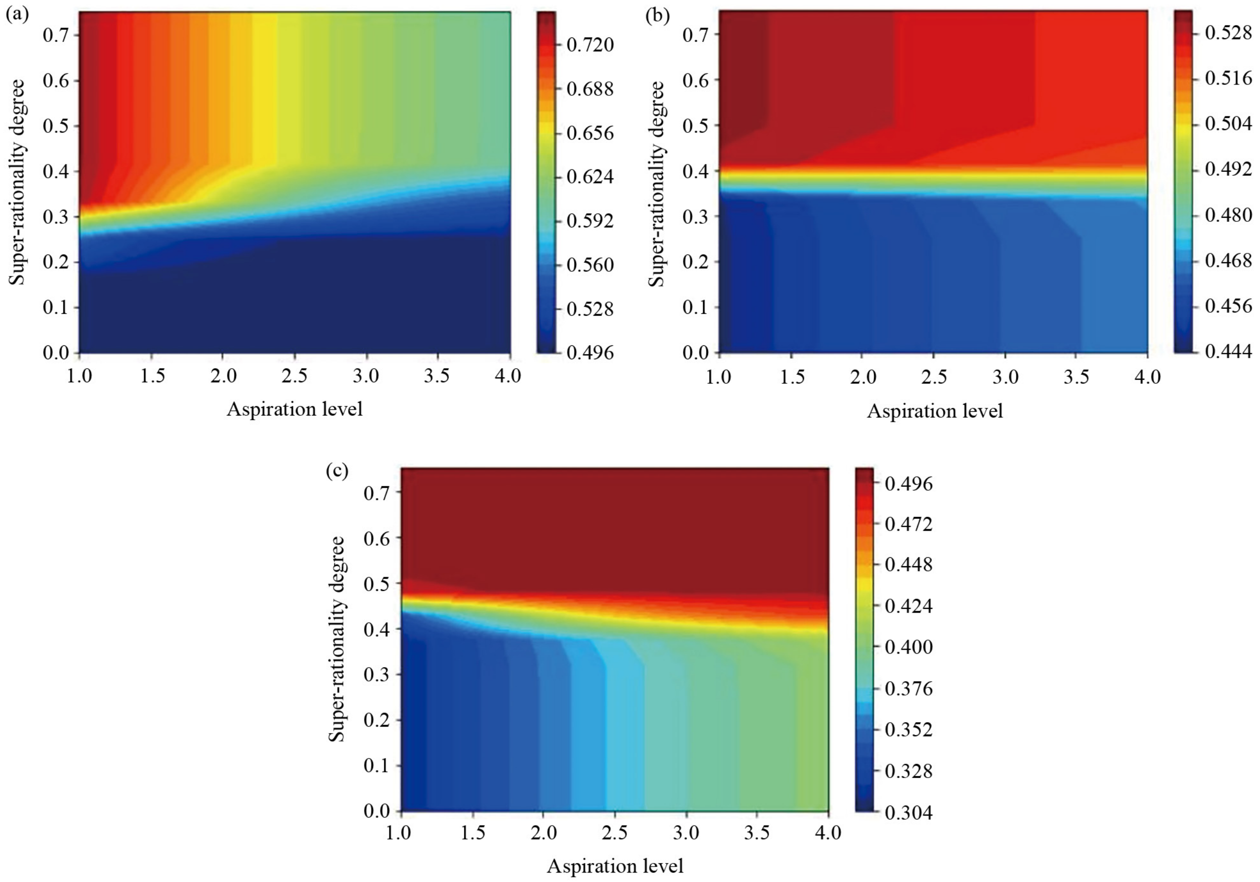

Figure 7. Color-coded average proportion of cooperators in the parameter region A_{R_i}-ϕ_C. In panel (a), (b) and (c), $\phi_D = 0,\ 0.17$ and 0.33, respectively. The common parameters are $a = 4,\ b = 1,\ c = 3,\ d = 2,\ \beta = 0.5,\ \omega = 0.5$

Figure 7 applies stag hunt game to test the robustness of the results in Figure 6, and provides color-coded average proportion of cooperator in the parameter space A_{R_i}-ϕ_C. In Figure 7a, defectors do not possess super-rationality ($\phi_D = 0$). It shows that when the super-rationality degree of cooperators is higher than 0.2, the average proportion of cooperators decreases with the increase of aspiration level A_{R_i}. When the super-rationality degree of cooperators is lower than 0.2, average proportion of cooperators fluctuates a little, and the impact of aspiration level A_{R_i} on the average proportion of cooperators is minimal. In Figure 7b, the super-rationality degree of defectors is $\phi_D = 0.17$. It shows that when the cooperator's degree of super-rational is higher than 0.4, the average proportion of cooperators decreases with the increase of aspiration level A_{R_i}. When the cooperator's degree of super-rationality is lower than 0.35, the average proportion of cooperators increases with the increase of aspiration level A_{R_i}. In Figure 7c, the super-rationality degree of defectors is $\phi_D = 0.33$ (i.e. completely super-rational). It shows that when the cooperator's degree of super-rationality is higher than 0.5, the fluctuation of the average proportion of cooperators is small, and the influence of aspiration level A_{R_i} on the average proportion of cooperators is small. When the cooperator's super-rationality degree is lower than 0.4, the average proportion of cooperators increases with the increase of aspiration level A_{R_i}.

The results indicate that when the cooperator is completely super-rational and the defector does not possess super-rationality, the lower the aspiration level A_{R_i}, the more favorable it is for cooperation. The average proportion of cooperators will only increase with the increase of aspiration level A_{R_i} when the cooperator's degree of super-rationality is small and the defector's degree of super-rationality is not zero. Figure 6 and Figure 7 present the parameter ranges of the super-rationality degree and aspiration level A_{R_i} that are conducive to the evolution of cooperation in the mixed strategy updating.

3. Conclusion

This paper is based on a symmetric 2×2 game in a well-mixed finite population, introducing a mixed strategy updating of combining super-rational imitation and aspiration process. We studied the cooperation evolution, and provided more favorable conditions for the evolution of cooperation. In the mixed update rule, individuals no longer use only one update rule, but rather use the super-rational imitation and aspiration process with a certain probability. The expected payoff, as a criterion for whether an individual imitates other individuals' strategies, reflects the super-rationality degree of the individual. The lower the degree of super-rationality, the higher the expected payoff, and the individual is more likely to imitate strategies with higher payoffs. The aspiration level is an inherent property of an individual, which can be understood as the standard of their self-demand. The higher the aspiration level, the higher the individual's self-demand and the higher the payoffs they intend to achieve.In the aspiration process, individuals have aspiration levels, and in the super-rational imitation, individuals with different strategies have different degrees of super-rationality, which determines their different expected payoffs.

This paper mainly focuses on the impact of the individual's super-rationality degree and aspiration level on the evolution of cooperation. Firstly, we considered the evolutionary cooperation dynamics in two special cases under weak selection, namely, cooperators have complete super-rationality and defectors have no super-rationality, and cooperators have no super-rationality and defectors have complete super-rationality. The results of theoretical and numerical analysis indicate that in the mixed strategy update mechanism under weak selection, cooperation is promoted when cooperators have super-rationality, and cooperation is inhibited when defectors have super-rationality. Next, we studied the evolutionary cooperation dynamics under arbitrary selection intensity. The results show that the influence of the individual's aspiration level on the system's evolution outcome increases with the increase of selection intensity. For the prisoner's dilemma game and the stag hunt game, the higher the aspiration level, the more favorable it is for cooperation when the cooperator's super-rationality degree is small or the defector's super-rationality degree is not zero. In both types of games, the average proportion of cooperators is the highest when the cooperator is completely super-rational but the defector does not possess super-rationality and has the lowest aspiration level. The numerical results provide favorable conditions for the evolution of cooperation under different selection intensities, aspiration levels, and super-rationality degrees in a mixed strategy update mechanism.

This study advances evolutionary game theory by demonstrating that real populations employ hybrid behavioral update mechanisms (beyond singular strategies like pure imitation) and introducing the novel concept of super-rational imitation, thereby enhancing behavioral realism. Yet, there is still room for improvement. In future works, a natural extension would be to let individuals (or the population) adapt β or A through experience, or to let individuals differ in their aspiration levels. This would speak to whether the system endogenously selects for more or less super-rational behaviors in cooperators versus defectors. In addition, exploring how super-rational imitation interacts with quantum enhanced or mutation-driven dynamics could yield deeper insights into the behavioral mechanisms that promote collective action [54]. Future work can continue to consider the evolutionary cooperation dynamics under other mixed strategy update mechanisms, such as mutation, super-rationality, Moran process, aspiration process, etc., thus to theoretically expand evolutionary game theory.

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Conflict of interest

The authors declare no competing financial interest.

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