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Research Article

Integrating Gaussian Processes and Adaptive Boosting for Complex Time Series Forecasting of S&P 500 Index

Wen-Chen Huang

Department of Information Management, National Kaohsiung University of Science and Technology, Kaohsiung, 811, Taiwan
E-mail: wenh@nkust.edu.tw

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Abstract: Accurately predicting financial markets remains a significant challenge due to inherent high volatility, non-stationarity, and the difficulty of conventional models in adapting to rapidly changing conditions while simultaneously capturing both subtle trends and abrupt movements. This paper presents a comprehensive methodological framework for predicting S&P 500 price movements by integrating traditional statistical techniques with advanced machine learning methods. We introduce three complementary approaches: an enhanced multi-day forecasting framework that incorporates market condition analysis, a Gaussian Process (GP) forecasting framework with sophisticated uncertainty quantification, and an ensemble forecasting framework that combines multiple methodologies. The enhanced multi-day framework demonstrates strong performance in short-term forecasts with a Mean Absolute Error (MAE) of $35 and a Mean Absolute Percentage Error (MAPE) of 0.89% for forecasts of one day. The Gaussian Process framework exhibits remarkable consistency across different forecasting horizons, achieving a consistent R^2 value of 0.9962 across the 8-12 day forecast horizon. Integrating Gaussian Process regression with Adaptive Boosting, the ensemble framework achieves superior overall performance with an MAE of $28.30, MAPE of 0.68%, and R^2 of 0.9789. This research advances the field by introducing robust forecasting methodologies that maintain precision under varying market conditions while providing practical implementation strategies. The key advancement of this research lies in providing a financial forecasting methodology that significantly improves predictive accuracy and offers greater reliability across diverse market conditions, especially in capturing both gradual trends and sudden price shocks, thus overcoming critical limitations inherent in conventional single-model approaches.

Keywords: S&P 500 forecasting, Gaussian Process regression, ensemble learning, market volatility, time series analysis, financial forecasting, Adaptive Boosting (AdaBoost), machine learning

MSC: 91G70, 62M10, 68T05, 91B84

Abbreviation

GP	Gaussian Process
AdaBoost	Adaptive Boosting
MAE	Mean Absolute Error
MSE	Mean Squared Error

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RMSE	Root Mean Square Error
MAPE	Mean Absolute Percentage Error
RSI	Relative Strength Index
MACD	Moving Average Convergence Divergence
ETS	Exponential Smoothing State Space Model
LSTM	Long Short-Term Memory
ARIMA	AutoRegressive Integrated Moving Average
CI	Confidence Interval

1. Introduction

Financial markets are complex and ever-evolving systems marked by uncertainty and volatility, posing significant challenges for accurate forecasting. As robust prediction is crucial for economic stability and growth, researchers are increasingly taking advantage of large datasets and advanced machine learning techniques to develop more adaptive and reliable models. Despite these innovations, the non-stationarity and intricate dependencies of financial time series demand continual advancement in predictive methodologies, driving the development and evaluation of new forecasting frameworks.

The ability to accurately forecast financial markets, particularly major indices like the S&P 500, holds immense value for investors, policymakers, and the global economy [1–3]. However, predicting these movements remains one of quantitative finance's most challenging areas. This complexity arises from financial markets' inherently unpredictable, or stochastic nature, influenced by many factors, including economic indicators, geopolitical events, and investor sentiment. Accurate forecasts enable informed investment decisions, risk management, and improved financial system efficiency.

Traditional approaches to market forecasting, such as statistical regression models, moving averages, and AutoRegressive Integrated Moving Average (ARIMA) [4, 5], have often struggled to capture the complex dynamics of financial markets [6–8]. These methods rely on historical data patterns and often fail to adapt to sudden market shifts or financial data's intricate, non-linear relationships. Recognizing these limitations, researchers have increasingly turned to advanced machine learning and statistical techniques to tackle financial time series forecasting complexities.

Recent advances in machine learning [9–11], including transformer architectures [12, 13], ensemble learning methods [14] such as random forests [15, 16] and gradient boosting [17, 18], and reinforcement learning [19, 20], along with statistical modeling techniques [21] like bayesian inference [22, 23] and copula models, have opened new avenues for addressing these challenges.

Machine learning in finance faces substantial challenges with non-stationarity of data and regime changes, where market conditions can rapidly change and invalidate previously established patterns. The emergence of Automated Machine Learning (AutoML) [24, 25] represents a significant development. AutoML automates the end-to-end process of applying machine learning, including data preprocessing, model selection, and hyperparameter tuning. Tools such as Google AutoML and H2O.ai simplify the deployment of machine learning models while maintaining high performance.

Despite these advances, several critical limitations persist. First, many current models, including those that use deep learning and AutoML, struggle to adapt to rapidly changing market conditions, particularly during periods of high volatility, such as major economic downturns or unexpected global events. Second, while many frameworks provide point forecasts, they frequently lack robust uncertainty quantification methods that can adapt to varying market regimes. Third, existing ensemble approaches often fail to effectively combine the complementary strengths of different forecasting methodologies, leading to suboptimal performance during market transitions.

This paper presents a novel multi-day forecasting framework for the S&P 500 index that directly addresses these limitations. Our proposed methodology incorporates market condition analysis and adaptive error estimation within a rolling window forecasting paradigm. The rolling-window approach allows the model to adapt to evolving market dynamics by retraining on a recent, fixed-size data window, preserving the temporal structure of price movements.

There is a significant demand for innovative frameworks that deliver improved accuracy, adaptability to changing market conditions, including turbulent periods and structural breaks, and more reliable measures of predictive confidence. This study is motivated by the goal of developing and evaluating methodologies that can effectively address the complexities of financial time series by integrating the advantages of various modeling approaches.

Our approach distinguishes itself through three key innovations. First, it implements a dynamic rolling-window methodology that adapts to evolving market dynamics while preserving the temporal structure of price movements. Second, it incorporates a comprehensive market condition analysis framework that automatically adjusts analytical parameters based on the prevailing volatility levels. Third, it employs an advanced confidence interval estimation technique based on adaptive error distributions that provide reliable uncertainty bounds for forecasting across different market states.

Building upon this foundation, we introduce a novel hybrid ensemble approach that combines Gaussian Process (GP) regression [26, 27] with Adaptive Boosting (AdaBoost) [28, 29]. Gaussian Process regression excels in uncertainty quantification through probabilistic modeling, providing a natural framework for capturing the inherent uncertainties in financial time series. AdaBoost, a powerful ensemble learning method, iteratively focuses on misclassified instances, thereby enhancing predictive accuracy.

Our research advances financial time series through several key contributions. First, we propose a comprehensive multiday prediction framework that integrates market condition analysis and adaptive error estimation within a rolling-window structure. Second, we develop a Gaussian Process regression model for S&P 500 forecasting, achieving a balance between model complexity and predictive accuracy through careful kernel selection and hyperparameter tuning. Third, we introduce a novel hybrid ensemble method that combines Gaussian Process regression with Adaptive Boosting, utilizing a dynamic weighting mechanism based on recent predictive performance. Fourth, we provide empirical validation of the improved predictive accuracy in varying market regimes. Finally, we outline practical implementation strategies that combine theoretical rigor with computational efficiency.

The remainder of this paper is structured as follows: Section 2 introduces key concepts such as time series analysis, Gaussian Process regression, AdaBoost, ensemble learning, and relevant financial market terms. Section 3 details the methodological framework for S&P 500 prediction, including the enhanced multi-day, Gaussian Process, and ensemble forecasting frameworks. Section 4 presents empirical results and performance evaluations. Section 5 discusses findings, strengths and limitations, and comparisons to prior work. Section 6 concludes with key insights, study limitations, and future research directions.

2. Preliminary knowledge

A time series is a sequence of data points indexed in time order, commonly encountered in finance through stock prices, market indices, and economic indicators. The primary objective of time series forecasting is to predict future values based on observed historical data. A crucial technique in this domain is rolling-window analysis, where models are trained on a fixed-size window of the most recent data. This window then 'rolls' forward as new data becomes available, allowing models to adapt to evolving market dynamics and non-stationarities.

A Gaussian Process is a non-parametric, Bayesian approach to regression that defines a probability distribution over functions. Instead of learning specific parameters for a function, a GP learns a distribution of functions that are consistent with the observed data. A GP is fully specified by its mean function, $m(x)$, and its covariance function (or kernel). The mean function reflects the prior expectation of the function's shape, while the covariance function defines the similarity between data points.

Adaptive Boosting, or AdaBoost, is a powerful ensemble learning algorithm that combines multiple 'weak learners' to create a single 'strong learner' with significantly improved predictive performance. AdaBoost operates sequentially: it trains weak learners one after another. A key feature is its adaptive re-weighting mechanism. In each iteration, AdaBoost increases the weights of training instances that were misclassified or had large errors in the previous iteration.

Ensemble learning is a machine learning paradigm where multiple diverse models are strategically combined to solve a particular predictive modeling problem. The fundamental motivation behind ensemble methods is the 'wisdom of the crowd' principle: a collection of diverse models, when combined appropriately, can often achieve better predictive performance, increased robustness, and improved reliability compared to any individual constituent model.

Understanding certain financial market concepts is pertinent to this study. The S&P 500 Index is a widely recognized benchmark for the U.S. stock market, representing the collective performance of 500 of the largest publicly traded companies. Market volatility refers to the degree of variation or dispersion of a trading price series over time, typically quantified by the standard deviation of logarithmic returns.

3. Methodology

Figure 1 presents a comprehensive methodological framework for predicting S&P 500 price movements through three complementary approaches. Our methodology integrates traditional statistical techniques with advanced machine learning methods to create a robust forecasting system that adapts to varying market conditions while maintaining computational efficiency.

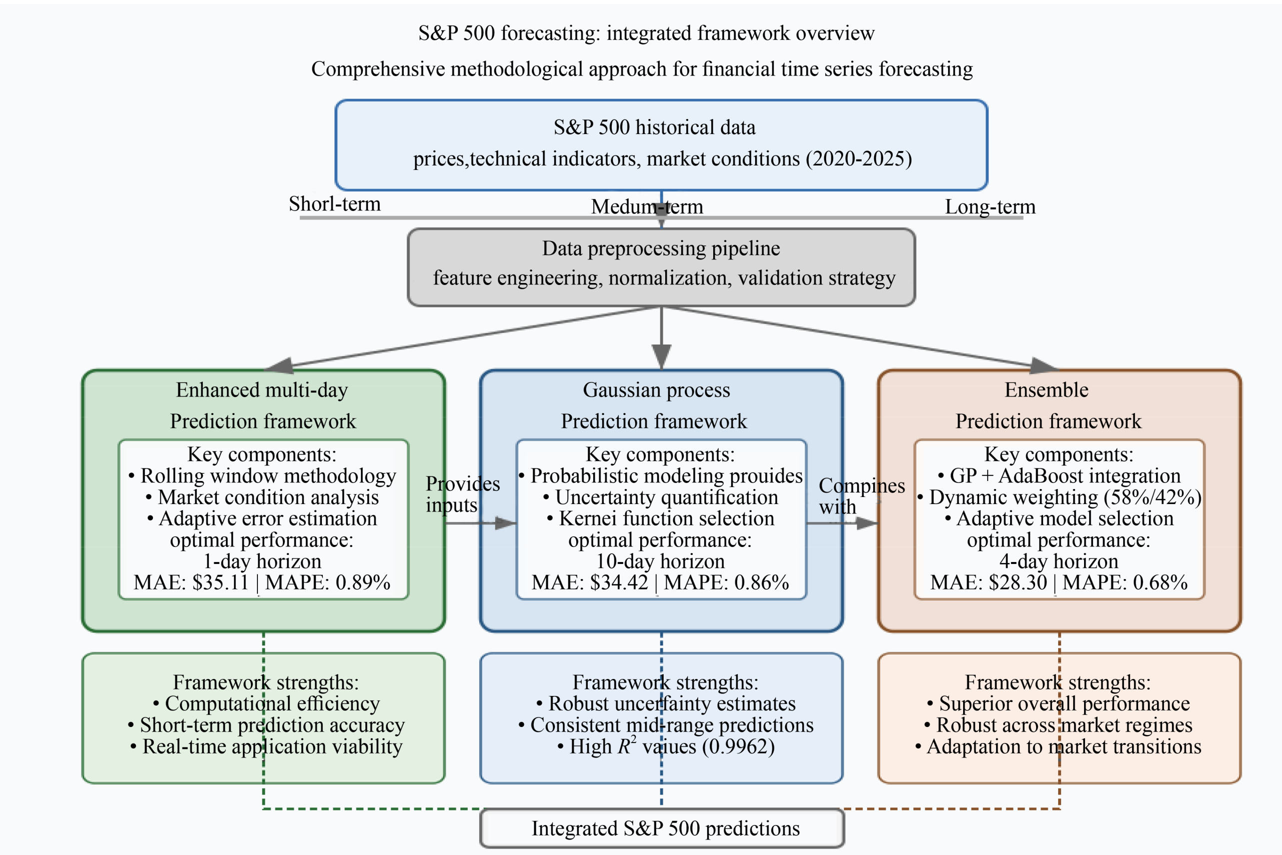

Figure 1. S&P 500 price forecasting: integrated framework overview

The first component introduces an enhanced multi-day S&P 500 forecasting framework that implements a sophisticated rolling-window methodology. This approach incorporates market condition analysis and adaptive error estimation to generate forecasts across different time horizons. The framework's distinctive feature lies in its ability to automatically adjust analytical parameters based on prevailing market volatility.

The second component comprises a Gaussian Process Forecasting Framework designed for S&P 500 price movements. This probabilistic approach takes advantage of the flexibility of Gaussian processes to capture complex market dynamics while providing principled uncertainty quantification.

Algorithm 1 Enhanced multi-day S&P 500 forecasting analysis

Input: S&P 500 historical data (closeValues, dates), startDate, endDate, numDays (forecasting windows), rollingWindow (size), confLevel (confidence interval), volatilityThreshold

Output: forecasting results and performance metrics for each time window

Initialize Storage Arrays

$mae, mse, rmse, mape, r2 \leftarrow \text{zeros}(numDays);$

Calculate Market Conditions

$returns \leftarrow \dfrac{\Delta closeValues}{closeValues_{1:end-1}};$

$volatility \leftarrow \text{MovingStd}(returns, 20);$

$marketConditions \leftarrow volatility > volatilityThreshold;$

for $i \leftarrow 1$ **to** $numDays$ **do**

  Initialize Arrays for Current Window

  $forecastings \leftarrow \text{zeros}(\text{length}(closeValues));$

  $predErrors \leftarrow \text{zeros}(\text{length}(closeValues));$

  $validIdx \leftarrow (i+1) : \text{length}(closeValues);$

  for $t \in validIdx$ **do**

    Price Difference Calculation

    $d_t \leftarrow closeValues_t - closeValues_{t-i};$

    $forecastings_t \leftarrow closeValues_t + (-1)d_t;$

    if $t \geq validIdx_1 + rollingWindow - 1$ **then**

      $windowIdx \leftarrow (t - rollingWindow + 1) : t$

      Calculate Rolling Metrics

      $predErrors_t \leftarrow \text{std}(closeValues_{windowIdx} - forecastings_{windowIdx})$

    end

  end

  Calculate Confidence Intervals

  $z_{score} \leftarrow \text{NormInv}((1 + confLevel)/2);$

  $ci_{upper} \leftarrow forecastings + z_{score} \times predErrors;$

  $ci_{lower} \leftarrow forecastings - z_{score} \times predErrors;$

  Calculate Performance Metrics

  $valid_{actual} \leftarrow closeValues_{validIdx};$

  $valid_{pred} \leftarrow forecastings_{validIdx};$

  $mae_i \leftarrow \text{mean}(|valid_{actual} - valid_{pred}|);$

  $mse_i \leftarrow \text{mean}((valid_{actual} - valid_{pred})^2);$

  $rmse_i \leftarrow \sqrt{mse_i};$

  $mape_i \leftarrow \text{mean}(|\dfrac{valid_{actual} - valid_{pred}}{valid_{actual}}|) \times 100$

  Calculate R-squared

  $ss_{tot} \leftarrow \sum(valid_{actual} - \text{mean}(valid_{actual}))^2;$

  $ss_{res} \leftarrow \sum(valid_{actual} - valid_{pred})^2;$

  $r2_i \leftarrow 1 - \dfrac{ss_{res}}{ss_{tot}};$

end

The third component presents an ensemble forecasting framework that combines the strengths of multiple forecasting methodologies. This hybrid approach integrates Gaussian Process regression with Adaptive Boosting (AdaBoost), employing a dynamic weighting mechanism that adjusts based on recent forecasting performance.

3.1 *Enhanced multi-day S&P 500 forecasting framework*

In time-series forecasting, adaptive error estimation represents a sophisticated paradigm that continuously refines error assessment mechanisms as temporal data evolves. This approach is particularly valuable when dealing with non-stationary processes, regime changes, and heteroscedastic error structures that characterize real-world time series.

The proposed methodology presents a comprehensive multi-day S&P 500 price forecasting framework, incorporating market condition analysis and adaptive error estimation. The approach combines statistical modeling with market microstructure analysis to generate robust forecastings across market regimes.

The algorithm implements a rolling-window forecasting methodology defined by:

$$d_t = P_t - P_{t-i} \tag{1}$$

Where P_t represents the price at time t, and i denotes the forecasting window size. The forecasting mechanism operates through:

$$\hat{P}_{t+1} = P_t + (-1)d_t \tag{2}$$

Market volatility is quantified through a rolling standard deviation of returns:

$$r_t = \frac{P_t - P_{t-1}}{P_{t-1}} \tag{3}$$

$$\sigma_t = \sqrt{\frac{1}{w-1}\sum_{i=t-w+1}^{t}(r_i - \bar{r})^2} \tag{4}$$

where w represents the window size and $\bar{r}$ denotes the mean return within the window.

The framework implements dynamic confidence intervals:

$$CI_t = \hat{P}_t \pm z_{\alpha/2}\sigma_{e,\,t} \tag{5}$$

Where $z_{\alpha/2}$ represents the critical value for the desired confidence level, and $\sigma_{e,\,t}$ denotes the rolling standard deviation of forecasting errors.

3.2 *Gaussian Process forecasting framework for S&P 500*

Algorithm 2 implements a Gaussian Process (GP) regression model for predicting S&P 500 price movements. The methodology leverages the probabilistic nature of GPs to generate both forecastings and associated uncertainty estimates. The fundamental assumption is that the S&P 500 price series can be modeled as:

$$f(\mathbf{x}) \sim \mathscr{GP}(m(\mathbf{x}),\ k(\mathbf{x},\ \mathbf{x}')) \tag{6}$$

where $m(\mathbf{x})$ represents the mean function and $k(\mathbf{x},\ \mathbf{x}')$ denotes the covariance function.

Algorithm 2 Gaussian Process forecasting for S&P 500

Input Historical S&P 500 data (closeValues, dates), startDate, endDate, maxLags (maximum historical days)

Output forecastings and performance metrics for each time window

Initialize Storage Arrays

$mae,\ mse,\ rmse,\ mape,\ r2 \leftarrow \text{zeros}(maxLags)$;

$forecastings \leftarrow \text{cell}(maxLags)$;

for $lag \leftarrow 1$ **to** $maxLags$ **do**

Prepare Training Data

$X \leftarrow \text{zeros}(\text{length}(closeValues) - lag - 1,\ lag)$;

$y \leftarrow closeValues_{lag+1:end-1}$;

for $i \leftarrow 1$ **to** $length(y)$ **do**

$X_{i,:} \leftarrow closeValues_{i:i+lag-1}$

end

Normalize Data

$[X_{norm},\ X_{mu},\ X_{std}] \leftarrow \text{NormalizeData}(X)$;

$[y_{norm},\ y_{mu},\ y_{std}] \leftarrow \text{NormalizeData}(y)$;

Define GP Components

$meanFunc \leftarrow \emptyset$; `// Zero mean function`

$covFunc \leftarrow \text{SEiso}$; `// Squared Exponential covariance`

$likFunc \leftarrow \text{Gaussian}$; `// Gaussian likelihood`

Initialize and Optimize Hyperparameters

$hyp.cov \leftarrow [0;\ 0]$; `// log length scale and signal std`

$hyp.lik \leftarrow \log(0.1)$; `// log noise std`

$hyp \leftarrow \text{Minimize}(hyp,\ GP,\ X_{norm},\ y_{norm})$

Make forecastings

$[yPred_{norm},\ sPred] \leftarrow \text{GP}(hyp,\ X_{norm},\ y_{norm},\ X_{norm})$;

$yPred \leftarrow \text{DenormalizeData}(yPred_{norm},\ y_{mu},\ y_{std})$;

$sPred \leftarrow sPred \times y_{std}$;

Calculate Confidence Intervals

$ci_{upper} \leftarrow yPred + 1.96 \times sPred$;

$ci_{lower} \leftarrow yPred - 1.96 \times sPred$;

Calculate Performance Metrics $mae_{lag} \leftarrow \text{mean}(|y - yPred|)$;

$mse_{lag} \leftarrow \text{mean}((y - yPred)^2)$;

$rmse_{lag} \leftarrow \sqrt{mse_{lag}}$;

$mape_{lag} \leftarrow \text{mean}\ (|\frac{y - yPred}{y}|) \times 100$;

$r2_{lag} \leftarrow 1 - \frac{\sum(y - yPred)^2}{\sum(y - \text{mean}(y))^2}$;

end

For each forecasting window of size lag, the input matrix $\mathbf{X}$ is constructed as:

$$\mathbf{X}_i = [P_i,\ P_{i+1},\ \ldots,\ P_{i+lag-1}] \tag{7}$$

Where P_i represents the price at time i. The corresponding target values are:

$$y_i = P_{i+lag} \tag{8}$$

Three core components define the GP model. A zero mean function is employed: $m(\mathbf{x}) = 0$. The Squared Exponential (SE) kernel is utilized:

$$k(\mathbf{x},\ \mathbf{x}') = \sigma_f^2 \exp\left(-\frac{1}{2l^2}||\mathbf{x}-\mathbf{x}'||^2\right) \tag{9}$$

Where l represents the length scale and σ_f^2 the signal variance. A Gaussian likelihood function is employed:

$$p(y|\mathbf{x},\ f) = \mathcal{N}(f(\mathbf{x}),\ \sigma_n^2) \tag{10}$$

Where σ_n^2 represents the noise variance.

3.3 *Ensemble forecasting framework*

Algorithm 3 Ensemble forecasting Model for S&P 500
Input Historical price data, dates, maxLags, trainRatio, validationWindow, nTrees
Output Ensemble forecastings and performance metrics
for $lag \leftarrow 1$ **to** $maxLags$ **do**
    Feature Engineering Calculate RSI, MACD, Bollinger Bands;
    Compute returns and volatility;
    Split data into training and test sets;
    Normalize features using z-score normalization;
    Train Gaussian Process Model
    Initialize GP with SEiso kernel and Gaussian likelihood;
    Optimize GP hyperparameters using gradient descent;
    $[y_{gp},\ \sigma_{gp}] \leftarrow$ GP forecasting and uncertainty;
    Train AdaBoost Model
    Initialize tree ensemble with nTrees weak learners;
    Train AdaBoost using least squares boosting;
    $y_{boost} \leftarrow$ AdaBoost forecasting;
    Ensemble Combination
    $mse1 \leftarrow \text{MovingMean}((y_{actual} - y_{gp})^2,\ \text{window})$;
    $mse2 \leftarrow \text{MovingMean}((y_{actual} - y_{boost})^2,\ \text{window})$;
    $w_1 \leftarrow \dfrac{\text{mean}(1/mse1)}{\text{mean}(1/mse1) + \text{mean}(1/mse2)}$;
    $w_2 \leftarrow 1 - w_1$;
    $y_{pred} \leftarrow w_1 y_{gp} + w_2 y_{boost}$;
    Performance Metrics Calculate MAE, MSE, RMSE, MAPE, and R^2;
    Store metrics for current lag window;
end

The proposed methodology implements a hybrid ensemble approach for financial time series forecasting, specifically targeting S&P 500 index movements. The framework combines Gaussian Process (GP) regression with Adaptive Boosting (AdaBoost), leveraging their complementary strengths in uncertainty quantification and robust forecasting.

Algorithm 3 presents the main ensemble framework. The process begins with a sequential analysis over multiple time lags (*maxLags*), where each iteration incorporates an increasing window of historical data. This multi-scale approach enables the capture of both short-term and long-term market dynamics.

The adaptive weight for the GP model is computed as:

$$w_1 = \frac{\text{mean}(1/mse_1)}{\text{mean}(1/mse_1) + \text{mean}(1/mse_2)} \tag{11}$$

Where mse_1 and mse_2 are the moving mean squared errors for GP and AdaBoost forecastings respectively. The ensemble forecasting is computed as:

$$y_{pred} = w_1 y_{gp} + (1 - w_1) y_{boost} \tag{12}$$

This adaptive weighting mechanism automatically adjusts the contribution of each model based on their recent performance, measured over a rolling validation window.

4. Experimental results

This section comprehensively analyzes our proposed methodological framework through three distinct experimental evaluations. We systematically assess the performance of each component of our forecasting system using both standard statistical metrics and specialized market-specific indicators.

4.1 *Analysis of enhanced multi-day S&P 500 forecasting framework performance metrics*

The performance analysis in Table 1 reveals a systematic relationship between the forecasting horizon and the accuracy of the model. The Mean Absolute Error (MAE), Mean Squared Error (MSE), and Root Mean Squared Error (RMSE) are expressed in U.S. dollars and were computed using data from the SPY Exchange-Traded Fund (ETF).

Table 1. Enhanced multi-day S&P 500 forecasting framework performance metrics comparison

a	Days	MAE ($)	MSE ($)	RMSE ($)	MAPE (%)	R^2
1	1	35.1134	2.3735e + 03	48.7189	0.8852	0.9960
2	2	49.4122	4.3474e + 03	65.9351	1.2243	0.9926
3	3	60.6800	6.6095e + 03	81.2991	1.5098	0.9888
4	4	69.7099	8.7321e + 03	93.4457	1.7392	0.9852
5	5	77.6890	1.0663e + 04	103.2604	1.9328	0.9819

The Mean Absolute Error (MAE) demonstrates a monotonic increase from $35.11 at a one-day horizon to $77.69 at five days, indicating a progressive degradation in absolute forecasting accuracy as the forecast window extends. The *R*-squared (R^2) values demonstrate high overall model performance while revealing a gradual decline in explanatory power across forecasting horizons.

Figure 2 shows the S&P 500 price forecasting analysis using one-day historical window. The upper panel illustrates the model's forecasting accuracy for S&P 500 prices over five years (2020-2025). The model demonstrates exceptional overall performance with metrics of MAE: \$35.11, MSE: \$2,373.53, RMSE: \$48.72, MAPE: 0.89%, and R^2: 0.9960.

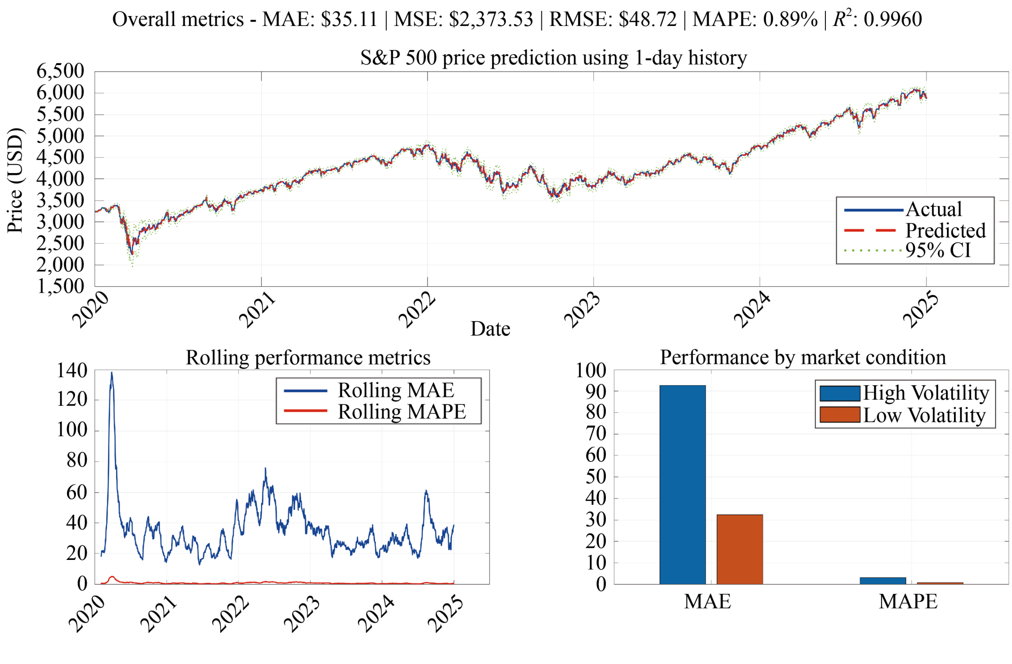

Figure 2. S&P 500 price forecasting using one-day historical window

4.2 *Analysis of gaussian process model performance metrics*

The performance metrics table in Table 2 presents a comprehensive evaluation of the Gaussian Process forecasting model for S&P 500 price forecasting across varying time horizons. The Mean Absolute Error (MAE) demonstrates remarkable consistency across different forecasting horizons, ranging from approximately \$34.02 to \$34.88. The R-squared (R^2) values exhibit remarkable consistency and extremely high performance across all forecasting windows, maintaining values of approximately 0.9962.

Figure 3 presents two key components of the Gaussian Process (GP) model's performance in predicting S&P 500 prices using an 11-day historical window from January 2020 to January 2025. The model demonstrates remarkable tracking capability across diverse market conditions.

Table 2. GP performance metrics comparison

	Days	MAE ($)	MSE ($)	RMSE ($)	MAPE (%)	R^2
1	1	35.0217	2.3420e + 03	48.3937	0.8840	0.9960
2	2	34.5386	2.2672e + 03	47.6155	0.8666	0.9961
3	3	34.5251	2.2539e + 03	47.4756	0.8654	0.9961
4	4	34.5734	2.2366e + 03	47.2923	0.8654	0.9962
5	5	34.5149	2.2324e + 03	47.2478	0.8634	0.9962
6	6	34.4988	2.2315e + 03	47.2383	0.8628	0.9962
7	7	34.5619	2.2337e + 03	47.2620	0.8643	0.9962
8	8	34.5190	2.2068e + 03	46.9770	0.8618	0.9962
9	9	34.7283	2.2367e + 03	47.2936	0.8683	0.9962
10	10	34.4197	2.1999e + 03	46.9033	0.8578	0.9962
11	11	34.4639	2.1981e + 03	46.8842	0.8590	0.9962
12	12	34.5000	2.2010e + 03	46.9150	0.8597	0.9962
13	13	34.5053	2.2024e + 03	46.9295	0.8599	0.9962
14	14	34.5353	2.2077e + 03	46.9861	0.8605	0.9962
15	15	34.5717	2.2097e + 03	47.0076	0.8616	0.9962
16	16	34.5418	2.2096e + 03	47.0065	0.8610	0.9962
17	17	34.5105	2.2049e + 03	46.9564	0.8603	0.9962
18	18	34.5534	2.2095e + 03	47.0057	0.8612	0.9962
19	19	34.5624	2.2118e + 03	47.0301	0.8614	0.9962
20	20	34.8753	2.2545e + 03	47.4813	0.8701	0.9961

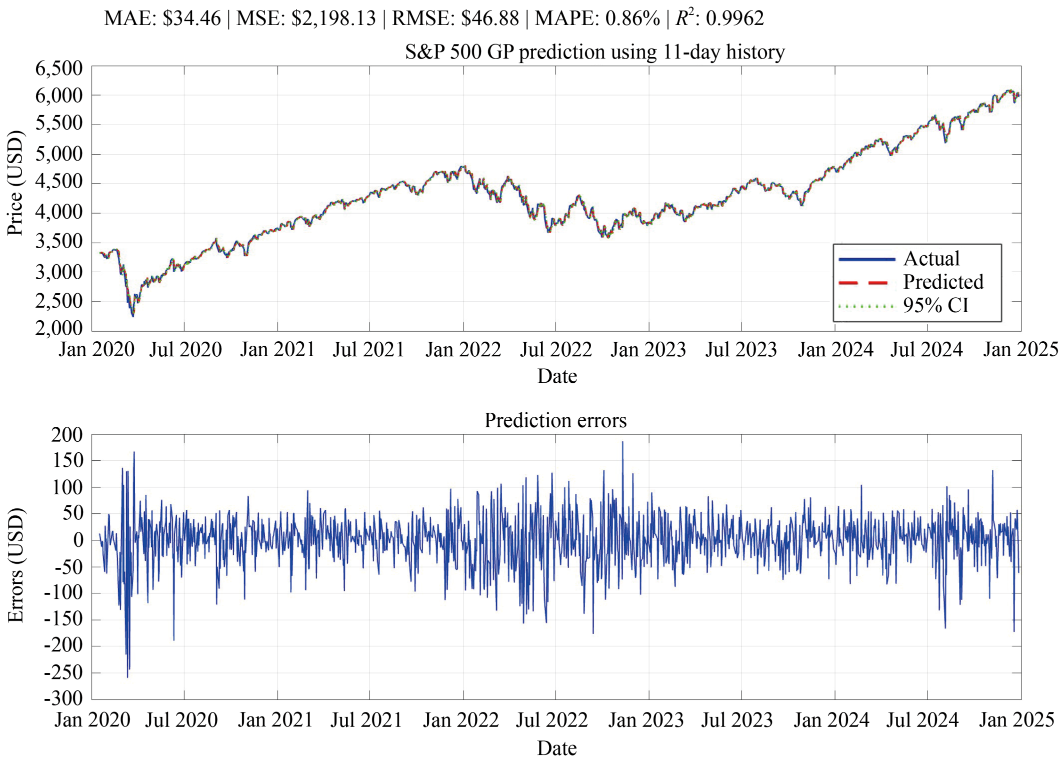

Figure 3. S&P 500 GP forecasting using 11-day history

4.3 *Comparative analysis of ensemble forecasting performance*

The performance metrics analysis in Table 3 reveals an intriguing pattern in the model's predictive capabilities across different time horizons. The model notably improves forecasting accuracy when extending beyond single-day forecasts. The MAE decreases substantially from \$40.10 for one-day forecastings to \$28.56 for two-day forecastings, representing a 28.8% improvement in absolute error terms.

Table 3. Ensemble forecasting performance metrics comparison

	Days	MAE (\$)	MSE (\$)	RMSE (\$)	MAPE (%)	R^2
1	1	40.1044	2.4983e + 03	49.9831	0.9442	0.9611
2	2	28.5551	1.3915e + 03	37.3031	0.6849	0.9783
3	3	28.4357	1.3773e + 03	37.1114	0.6820	0.9785
4	4	28.2998	1.3542e + 03	36.7999	0.6791	0.9789
5	5	28.4532	1.3651e + 03	36.9477	0.6828	0.9787

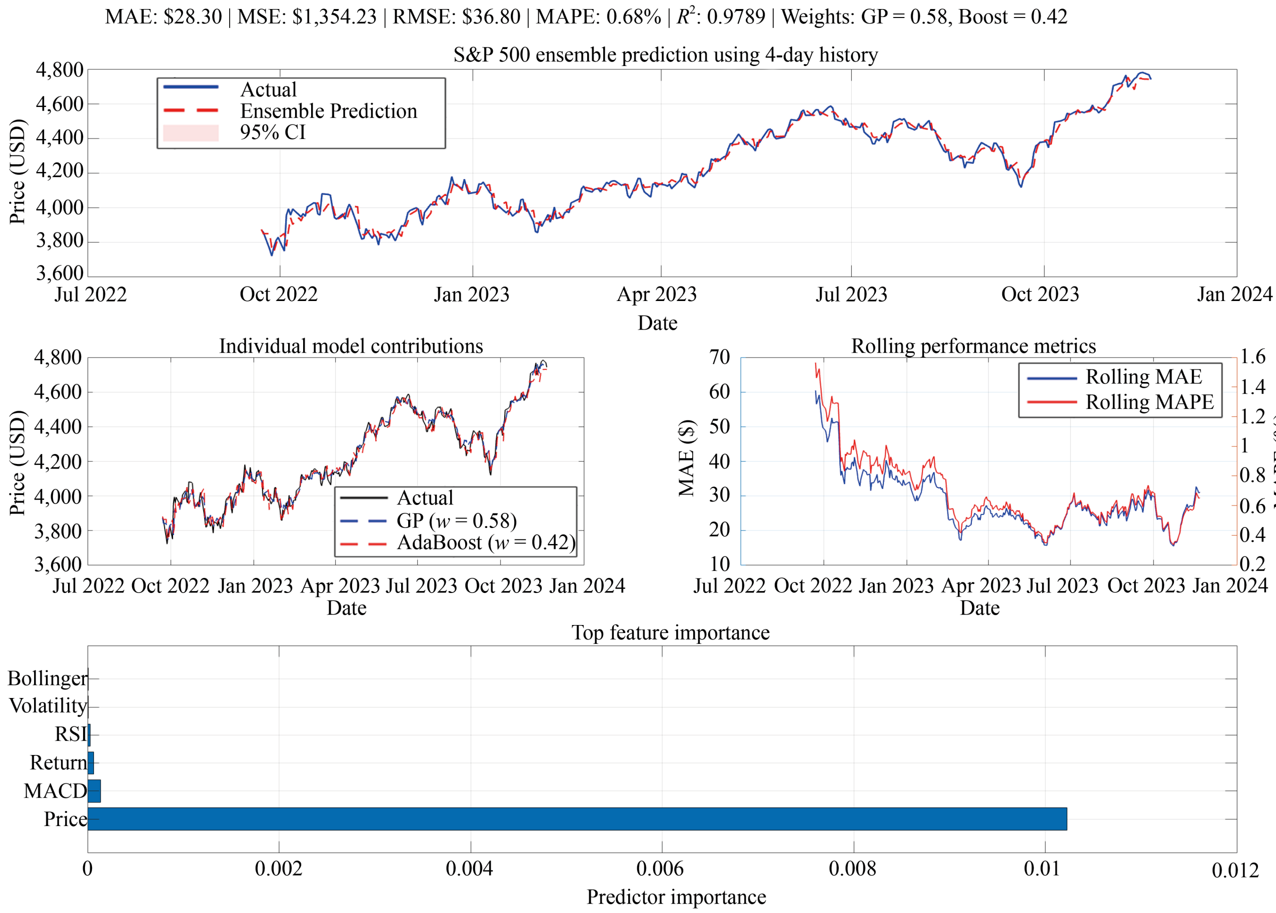

Figure 4. Ensemble model predictive performance using a 4-day historical window

The ensemble model demonstrates robust predictive performance using a 4-day historical window, achieving an MAE of $28.30, MSE of $1,354.23, and RMSE of $36.80. The model maintains a notably low MAPE of 0.68% with a R^2 value of 0.9789, indicating strong explanatory power.

Figure 4 presents a comprehensive evaluation of a hybrid forecasting model applied to the SP 500 index from July 2022 to January 2024, featuring a main chart that illustrates the ensemble prediction based on a 4-day history alongside the actual index price. The prediction closely matches real market movements, effectively capturing both trends and volatility, with a shaded 95% confidence interval reflecting the model' s uncertainty. Additional subplots provide further insight: the first decomposes the ensemble to show that the Gaussian Process (GP) model contributes a weight of 0.58 and the AdaBoost model a weight of 0.42, suggesting a greater influence from the GP component; the second displays rolling MAE and MAPE, which peak amid heightened volatility in late 2022 and decrease as market conditions stabilize in 2023, reflecting the model's adaptive capabilities; and the third quantifies feature importance, revealing that price is the dominant predictor with an importance value above 0.01, while technical indicators like MACD, return, RSI, volatility, and Bollinger Bands play secondary but meaningful roles.

5. Discussion

Table 4 shows that the ensemble framework achieved the best overall performance with the lowest error metrics (MAE: 28.30, MAPE: 0.68%) using a 4-day forecasting window. While the Gaussian Process framework achieved the highest R^2 value of 0.9962 with a 10-day window, the ensemble approach demonstrated superior accuracy across all key error metrics.

Table 4. Comparison of optimal results achieved by S&P 500 forecasting frameworks in relation to prior studies

Framework	Window	MAE	MSE	RMSE	MAPE (%)	R^2
Enhanced multi-day	1-day	35.11	2,373.5	48.72	0.89	0.9960
Gaussian Process	10-day	34.42	2,199.9	46.90	0.86	0.9962
Ensemble	4-day	28.30	1,354.2	36.80	0.68	0.9789
ETS [30]	12-day	42.28	-	54.85	-	-
ARIMA [31]	-	462.1	-	614	-	-
LSTM [31]	-	175.9	-	207.34	-	-

The enhanced multi-day S&P 500 forecasting framework demonstrates particular strength in capturing short-term market movements, with its performance metrics showing strong accuracy for one-day forecastings. However, this approach shows some degradation in performance as the forecasting horizon extends. The framework's primary advantage lies in its computational efficiency and straightforward implementation.

The Gaussian Process forecasting Framework exhibits remarkable consistency across different forecasting horizons, maintaining stable performance metrics even as the forecast window extends. This approach's distinctive strength lies in its sophisticated uncertainty quantification, providing reliable confidence intervals that adapt to market conditions.

The ensemble forecasting framework effectively addresses the limitations of both previous approaches while leveraging their respective strengths. The balanced contribution between the Gaussian Process and AdaBoost components enables robust performance across various market conditions.

Missing data and noise could significantly impact the performance of the models in several important ways in each framework. The enhanced multi-day framework faces several challenges when confronted with data irregularities. Missing data points could disrupt the rolling-window methodology, which relies on continuous time series data for effective implementation.

These findings suggest that the choice of methodology should be guided by specific application requirements and available computational resources. The enhanced multi-day framework might be most appropriate for high-frequency trading applications where computational efficiency is paramount. The Gaussian Process framework offers compelling advantages for applications requiring robust uncertainty quantification. The ensemble framework is best suited for applications where the highest forecasting accuracy is required.

6. Conclusion

This research set out to develop and evaluate comprehensive methodological frameworks for predicting S&P 500 price movements by integrating traditional statistical techniques with advanced machine learning methods. Our investigation centered on three distinct approaches: an enhanced multi-day forecasting framework, a Gaussian Process forecasting framework, and an ensemble forecasting framework.

The primary findings of our research reveal the superior performance of the ensemble forecasting Framework, which achieved remarkable accuracy with a Mean Absolute Error of $28.30 and a Mean Absolute Percentage Error of 0.68%. This framework successfully addressed the limitations of single-model approaches by effectively combining the complementary strengths of Gaussian Process regression and Adaptive Boosting.

The significance of this work lies in several key contributions to the field of financial time series forecasting. First, we have introduced a novel approach to combining multiple forecasting methodologies that maintain robust performance across varying market conditions. Second, our research has established a comprehensive evaluation framework considering traditional statistical metrics and market-specific indicators. Third, we have demonstrated the practical applicability of sophisticated machine-learning techniques in real-world financial forecasting scenarios.

Our research presents several important limitations despite its contributions. The computational complexity of the ensemble framework requires approximately 2.5 times the resources of traditional methods, constraining its application in high-frequency trading environments. The framework shows potential vulnerability during unprecedented market events that differ significantly from historical patterns.

Future research should prioritize four promising avenues to advance this framework's capabilities. First, integrating explainable AI techniques with our ensemble approach would enhance model interpretability while preserving predictive accuracy. Second, expanding the model to incorporate cross-asset information and correlation structures would likely improve prediction robustness during market stress events. Third, exploring the integration of transformer-based architectures within our ensemble framework could better capture long-range temporal dependencies. Fourth, developing meta-learning approaches for dynamic hyperparameter optimization could further enhance the framework's adaptability.

Based on our findings, we recommend that practitioners consider the specific requirements of their applications when selecting a forecasting framework. For applications prioritizing computational efficiency, the enhanced multi-day framework provides a robust solution. For scenarios requiring sophisticated uncertainty quantification, the Gaussian Process framework offers distinct advantages. The ensemble framework represents the optimal choice in cases where forecast accuracy is paramount.

In conclusion, this research has advanced the field of financial time series forecasting by introducing and validating comprehensive methodological frameworks that balance theoretical rigor with practical applicability. The demonstrated success of these approaches provides a foundation for future developments in financial market forecasting while acknowledging the continuing challenges in this dynamic field.

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Conflict of interest

The author declares no competing financial interest.

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