

#### Research Article

# Research on Pricing and Battery Performance Enhancement Decisions of the New Energy Vehicle Supply Chain Under Different Subsidy Strategies

Yongjian Wang<sup>1\*0</sup>, Zhichao Wang<sup>2</sup>, Zhanjie Wang<sup>3</sup>, Qiang Chen<sup>40</sup>

Received: 6 May 2025; Revised: 9 June 2025; Accepted: 13 June 2025

**Abstract:** Consumers' increasing concern about the battery performance of New Energy Vehicles (NEVs) necessitates effective government subsidies to enhance corresponding investments and production. A subsidy strategy based on battery performance enhancement degree is proposed, and a comparative analysis with the retailer sales subsidy strategy is conducted. Our findings reveal that both subsidy strategies enhance battery performance and market demand, while a sales subsidy strategy potentially lowering the NEV price. The ratio of subsidy coefficients determines the effect of different subsidy strategies on the NEV supply chain's operational decisions. When each subsidy coefficient remains unchanged, an increase in performance sensitivity coefficient or a decrease in enhancement cost coefficient narrows the gap between the equilibrium solutions under the two subsidy strategies. Meantime, it can also further strengthen the advantages of the performance enhancement subsidy in improving member enterprises' profits and consumer surplus. Finally, under identical government subsidy expenditures, the optimal subsidy strategy choice is closely related to the evaluation criteria.

Keywords: government subsidies, new energy vehicles, pricing decisions, battery performance enhancement

MSC: 90B06, 91A80

#### 1. Introduction

Contemporary Mathematics

With the rapid advancement of industrialization and urbanization, the contradiction between human society and natural environment has become increasingly prominent. Transportation energy consumption is particularly a major source of global energy consumption and greenhouse gas emissions [1, 2]. Considering the high energy efficiency and low carbon emission, the NEV industry development is regarded as an effective way to alleviate the above problems [3]. Then, it has become a strategic choice for the high-quality development of the global automobile industry to promote the NEV popularization. Although the NEV market penetration has gradually increased, the battery performance issues (e.g. battery safety and endurance capacity) are still serious. It naturally hinders the purchasing behaviors of potential consumers. According to survey data from iiMedia Research in 2024, the main considerations for Chinese consumers

<sup>&</sup>lt;sup>1</sup>Business School, Xuzhou University of Technology, Xuzhou, 221018, China

<sup>&</sup>lt;sup>2</sup>School of Economics and Management, Shihezi University, Shihezi, 832000, China

<sup>&</sup>lt;sup>3</sup> School of Business Administration, Guizhou University of Finance and Economics, Guiyang, 550025, China

<sup>&</sup>lt;sup>4</sup>Sino-Russian Institute, Jiangsu Normal University, Xuzhou, 221116, China E-mail: wyj19890823@126.com

when purchasing NEVs are endurance capacity (43.6%) and vehicle safety (43.3%). Moreover, the "2024 New Energy Vehicle User Anxiety Insight Report" shows that consumer anxiety about the NEV safety is mainly concentrated on battery safety issues, accounting for nearly 97%. This means that the NEV battery performance enhancement will greatly improve the consumer experience and further promote the popularization and industrial upgrading of NEVs.

However, the NEV battery performance enhancement needs higher costs, which will weaken automakers' investment incentives. To promote the healthy development of the NEV industry, the government has consistently implemented various subsidy strategies. It needs to be clarified that changes in the consumer demand and external situation have also led to a transformation of the government's subsidy strategy. For instance, current government subsidy strategies have been more focused on battery performance enhancement. In October 2024, the Notice on Advancing the 2025 Energy Saving and Emission Reduction Subsidy Budget issued by the Chinese Ministry of Finance publicized the incentives for the demonstration and application of fuel battery vehicles, and the funds amounted to a cumulative total of approximately RMB 1.625 billion. This has not only weakened the negative effect of the complete cancellation of subsidies on enterprises, but also clarified the future development direction of the NEV industry. Data from the China Association of Automobile Manufacturers shows that the sales quantity of NEVs reached 987,000 units in January and February 2025, among which the proportion of vehicles with a range of 1,000 kilometers or more increased from 3.8% in 2024 to 17.4%.

Under different subsidy strategies, the member enterprises in NEV supply chain will optimize optimal operational decisions to obtain higher government subsidies and maximize their profits. For the government, it is an essential issue to choose the appropriate subsidy strategy and analyze its effectiveness. Therefore, it is crucial to explore the pricing and battery performance enhancement decision process in the NEV supply chain based on different government subsidy strategies. This has significant implications for further improving the NEV subsidy mechanism and promoting the NEV industry development.

Based on the above background, the battery performance enhancement subsidy for automakers (i.e., performance enhancement subsidy) and the sales subsidy for retailers (i.e., NEV sales subsidy) strategies are considered in this paper. Through the comparative analysis on the NEV supply chain's optimal operational decisions, total profits and consumer surplus, the following three questions are examined: (1) How do government subsidy strategies affect the member enterprises' operational decisions in the NEV supply chain? (2) Which subsidy strategy is more beneficial to the NEV supply chain, and what are the critical conditions for the preferred results of government subsidy strategies? (3) What are the subsidy effects of different government subsidy strategies under the identical government subsidy and NEV sales subsidy are constructed to analyze the optimal pricing and battery performance enhancement decisions. Then, the effects of key parameters and the critical conditions for the preferred results of different government subsidy strategies are investigated based on the comparative analysis. Meanwhile, given the identical government subsidy expenditures, we further clarified the effectiveness of the above two subsidy strategies.

The contributions of this study are mainly reflected in the following aspects. First, although more and more literature has focused on the NEV battery performance and the positive effect of government subsidy strategies in recent years, few literatures have considered subsidizing automakers based on the NEV battery performance enhancement degree. Second, the pricing and battery performance enhancement decisions of NEV supply chains are investigated under different subsidy strategies (performance enhancement subsidy and NEV sales subsidy). Through rigorous theoretical derivation, the effects of the changes in various parameters on the implementation and preferred results of different subsidy strategies are explored. In the meantime, the subsidy effectiveness and differences of the two government subsidy strategies under the identical government subsidy expenditures are especially clarified.

The remaining work arrangement is as follows: Section 2 describes relevant research. Problem description, notations and basic assumptions are presented in section 3. In section 4, two Stackelberg game models under different subsidy strategies are constructed and solved by backward induction solution. Next, section 5 conducts the comparison and analysis of the equilibrium solutions, and section 6 provides the numerical analysis to verify the main propositions proposed in this paper. Finally, section 7 summarizes the conclusions of this topic and gives the next research directions.

## 2. Literature review

The NEV industry development and the corresponding promotion policies have always been the academic concern fields. More literature focuses on the NEV production and pricing decisions [4-6]. When the NEV battery performance (e.g. battery safety and endurance) is concerned by more consumers, it also becomes an influential factor in the NEV market demand. Most of the research on NEV battery focuses on the recycling issues [7, 8]. There is also some literature related to the battery performance enhancement, and more studies pay attention to the endurance capacity. Among them, some literature explores the effect of charging infrastructure construction on the NEV industry development considering consumers' anxiety about the endurance mileage [9-11]. In addition, some studies have pointed out that stronger endurance will improve consumer utility. This will also affect the NEV production pricing and endurance enhancement decisions. For instance, Cheng and Mu [12] assumed that NEV performance (mainly endurance capacity) affects consumers' willingness to pay for Fuel Vehicles (FVs), and formulated a decision model for the joint production of FVs and NEVs under the carbon trading mechanism. Considering the closely relationship between the endurance capacity, environmental performance and consumer utility, Tang et al. [13] comparatively analyzed the effects of independent and cooperative decision-making models on the optimal production and pricing strategies under the dual-credit policy. Also focusing on the effects of FV emissions reduction and NEV endurance capacity on market demand, Lu et al. [14] studied the impact of the dual-credit policy on the automotive industry. In the meantime, it comparatively analyzed the coordinating effects of three types of contracts (namely, cost sharing, revenue sharing, and two-part pricing) on the automotive supply chain. Assuming the NEV demand is highly correlated with sales price, consumer low-carbon preference and endurance capacity concern, Xu et al. [15] showed that higher consumer low-carbon preference and endurance capacity concern will be more favorable to NEV industry development. Ma et al. [16] argued that the dual-credit policy has positive incentives for NEV technological innovations (including endurance enhancement), but information asymmetry will have a negative influence.

Consumers' NEV purchasing behaviors are not only affected by their performance, but also closely related to government strategies. The relevant research mainly involves consumers subsidies [17–19], production-sales subsidies [20, 21] and comparative analyses of the two types of subsidy strategies [22, 23]. However, as the NEV market becomes more mature and consumers are concerned about the endurance capacity, NEV subsidy strategies gradually tend to address the endurance issues. For example, Shi et al. [24] comparatively analyzed the effects of purchasing subsidies and charging infrastructure construction subsidies on the electric vehicle promotion. Moreover, Shao et al. [19] extended the study of Shi et al. [24] to different market structures and pointed out that charging infrastructure construction subsidies are more advantageous in China. Li et al. [3] comparatively analyzed the effects of charging infrastructure construction and charging service fee subsidies, and discussed the government's optimal electric vehicle charging infrastructure subsidy policy choices. Furthermore, focusing on the automaker's technology R&D investment (i.e., endurance capacity enhancement) issue, Zhu et al. [25] proposed a hybrid mechanism model that integrates the advantages of cash subsidy and carbon trading mechanism. Meanwhile, it highlighted the effects of different intervention mechanisms on the NEV market demand, technology R&D investment, total profits and carbon emissions. Chen et al. [26] discussed the influence of consumer subsidies, price subsidies, and endurance subsidies on the NEV production pricing decisions of automakers with different market shares and ranges. However, the corresponding endurance capacity is considered an exogenous known variable.

It can be observed from existing literature that some studies on government subsidies for NEVs have involved in battery endurance, but the corresponding subsidy strategies are mostly based on investment in charging infrastructure, such as [3, 19], etc. It should be noted that Chen et al. [26] considered endurance capacity subsidy, but the subsidy basis is the actual endurance level and the endurance capacity is not modelled as a decision variable. Therefore, to expand the relevant research, this paper considers the battery performance enhancement subsidies given by the government to the automaker, and conducts a comparative analysis with the typical scenario of retailer sales subsidies. Meanwhile, the preferred results and conditions of government subsidy strategies are clarified based on whether the same government expenditure is considered. Finally, scholars have conducted extensive research on NEV subsidies, which provides an

excellent theoretical and methodological foundation for studying pricing and battery performance enhancement decisions in the NEV supply chain under different government subsidy strategies.

# 3. Model description

#### 3.1 Problem description

In this paper, a two-echelon NEV supply chain including an automaker and a retailer is considered, and its structure is shown in Figure 1. The automaker is responsible for producing NEVs and the retailer is responsible for selling NEVs. Specifically, the automaker decides NEV wholesale price and battery performance enhancement degree. The NEVs produced by the automaker are provided to the retailer at a wholesale price  $w_n$  in a "make-to-order" mode. The automaker seeks to attract consumers by enhancing the NEV battery performance (e.g. battery safety and endurance capacity). The corresponding battery performance enhancement degree is expressed as  $\tau_n$ . Additionally, the retailer decides the NEV sales price  $p_n$ , and selling the final product to the consumer based on market demand  $q_n$ . Furthermore, to further promote the NEV industry development and satisfy consumer demand for NEV performance, the government chooses to provide subsidies based on the battery performance enhancement degree by the automaker or the sales quantity of retailers. The corresponding subsidy coefficients are respectively expressed as  $\eta_1$  and  $\eta_2$ . Therefore, driven by both market and policy, the NEV supply chain makes comprehensive decisions on optimal pricing and battery performance enhancement.

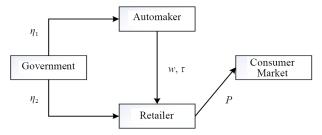


Figure 1. NEV supply chain structure

#### 3.2 Notations

The relevant parameters, decision variable and the corresponding definitions used in this study are shown in Table 1.

Decision variables	Definitions	
$egin{array}{c} w_n \  au_n \ P_n \ q_n \end{array}$	NEV wholesale price Battery performance enhancement degree NEV sales price NEV market demand	
Model parameters	Definitions	
a b	Market Size Consumer price sensitivity coefficient	
λ	Consumer battery performance sensitivity coefficient (hereinafter referred to as "performance sensitivity coefficient")	
β	Battery performance enhancement cost coefficient (hereinafter referred to as "enhancement cost coefficient")	
$\eta_j$	Government subsidy coefficients, $j = 1, 2$ represent performance enhancement subsidy and NEV sales subsidy strategies, respectively	
$egin{array}{c} \pi_m \ \pi_s \ CS \end{array}$	Automaker's profit Retailer's profit Consumer surplus	

Table 1. Notations and definitions

#### 3.3 Basic assumptions

Based on the realistic relevance and theoretical scientificity, the following assumptions are presented:

- (1) NEV market demand is influenced by both sales price and battery performance. Consumers are sensitive to battery performance and prefer NEVs with higher performance and lower sales prices. The market demand function is given as:  $q_n = a bp_n + \lambda(\tau_n + \tau_0)$ . To simplify the analysis processes, it is assumed that  $\tau_0 = 0$ . This will not affect the study results. Thus, the NEV market demand function can be described as  $q_n = a bp_n + \lambda \tau_n$ . Similar assumption can be found in [4, 19].
- (2) The battery performance enhancement cost for automakers can be considered as a one-time investment to be covered by themselves. In general, the greater the battery performance enhancement degree, the higher the corresponding investment cost [16, 27]. Following the law of diminishing returns on investment, the NEV battery performance enhancement cost is assumed to be a quadratic function  $\beta \tau_n^2/2$ .
- (3) It's a Stackelberg game between the automaker and retailer, and the game sequence under different government subsidy strategies is the same. First, the manufacturer acts as a leader and decides the NEV wholesale price and battery performance enhancement degree based on the profit-maximizing objective. Then, the retailer acts as a follower and sets the NEV sales price based on certain automaker decisions and profit-maximizing objective. Thus, the specific decision sequence under different subsidy strategies is shown in Figure 2.

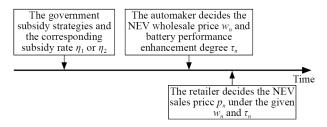


Figure 2. The specific decision sequence

(4) To ensure the non-negativity of the equilibrium solutions under different government subsidy strategies, we set  $4\beta b - \lambda^2 > 0$ .

### 4. Model analysis

# 4.1 Decision models of the NEV supply chain

This section primarily constructs two Stackelberg game models of the NEV supply chain under battery performance enhancement subsidy and NEV sales subsidy strategies. Then, the equilibrium solutions can be obtained by using backward induction, and all proof can be found in Appendix. In the solution process, the retailer first decides on the NEV sales price, and then the automaker simultaneously decides on the NEV wholesale price and battery performance enhancement degree. Finally, this section further analyzes the effects of performance sensitivity coefficient, enhancement cost coefficient and subsidy coefficients.

#### 4.1.1 Performance enhancement subsidy strategy

Under the performance enhancement subsidy strategy, the government can subsidize the automaker based on the battery performance enhancement degree. The corresponding government subsidy expenditure denoted as  $\eta_1 \tau_n$ . Then, the profit functions of the automaker and retailer are shown in Equations (1) and (2). In Equation (1), the first term represents the automaker's NEV wholesale revenue, the second term represents the battery performance enhancement cost, and the third term represents government subsidy revenue. In Equation (2), the first term represents the retailer's NEV sales revenue, and the second term represents the wholesale cost.

$$\pi_m = w_n \left( a - b p_n + \lambda \tau_n \right) - \beta \tau_n^2 / 2 + \eta_1 \tau_n \tag{1}$$

$$\pi_s = p_n \left( a - bp_n + \lambda \tau_n \right) - w_n \left( a - bp_n + \lambda \tau_n \right) \tag{2}$$

#### 4.1.2 NEV sales subsidy strategy

Under the NEV sales subsidy strategy, the government can subsidize the retailer based on the NEV sales quantity. The corresponding government subsidy expenditure denoted as  $\eta_2 q_n$ . Then, the profit functions of the automaker and retailer are shown in Equations (3) and (4). Differing from the above model, the second term in Equation (4) represents the government subsidy revenue obtained by the retailer.

$$\pi_m = w_n \left( a - b p_n + \lambda \tau_n \right) - \beta \tau_n^2 / 2 \tag{3}$$

$$\pi_s = (p_n - w_n)(a - bp_n + \lambda \tau_n) + \eta_2(a - bp_n + \lambda \tau_n) \tag{4}$$

#### 4.2 Equilibrium solution analysis

The above game model can be solved to obtain the equilibrium solutions under different government subsidy strategies, as shown in Lemma 1.

**Lemma 1** Under different government subsidy strategies, the equilibrium solutions for the automaker and retailer, along with their profits and consumer surplus are presented in Table 2.

Variables	Performance enhancement subsidy strategy	NEV sales subsidy strategy
$w_n$	$\frac{2\big(\beta a + \lambda\eta_1\big)}{4\beta b - \lambda^2}$	$\frac{2\beta \left( a+b\eta _{2}\right) }{4\beta b-\lambda ^{2}}$
$ au_n$	$\frac{\lambda a + 4b\eta_1}{4\beta b - \lambda^2}$	$\frac{\lambda\left(a+b\eta_2\right)}{4\beta b-\lambda^2}$
$p_n$	$\frac{3 \left(\beta a + \lambda  \eta_1\right)}{4 \beta b - \lambda^2}$	$\frac{3\beta a + \lambda^2 \eta_2 - \beta b \eta_2}{4\beta b - \lambda^2}$
$q_n$	$\frac{b\big(\beta a + \lambda\eta_1\big)}{4\beta b - \lambda^2}$	$\frac{\beta b \big(a+b\eta_2\big)}{4\beta b-\lambda^2}$
$\pi_m$	$\frac{\beta a^2+2\lambda a\eta_1+4b\eta_1^2}{2(4\beta b-\lambda^2)}$	$\frac{\beta \left(a+b\eta_2\right)^2}{2\left(4\beta b-\lambda^2\right)}$
$\pi_{\scriptscriptstyle S}$	$\frac{b \big(\beta a + \lambda  \eta_1\big)}{4\beta b - \lambda^2}$	$rac{eta^2big(a+b\eta_2ig)^2}{ig(4eta b-\lambda^2ig)^2}$
CS	$rac{big(eta a + \lambda \eta_1ig)^2}{2ig(4eta b - \lambda^2ig)^2}$	$\frac{\beta^2 b \left(a + b \eta_2\right)^2}{2 \left(4\beta b - \lambda^2\right)^2}$

Table 2. Equilibrium solutions under different subsidy strategies

Based on Lemma 1, the effects of performance sensitivity coefficient, enhancement cost coefficient and subsidy coefficients are analyzed, and Propositions 1 and 2 can be obtained as follows.

Proposition 1 (1) 
$$\frac{\partial w_n(\eta_j)}{\partial \lambda} > 0$$
,  $\frac{\partial \tau_n(\eta_j)}{\partial \lambda} > 0$ ,  $\frac{\partial p_n(\eta_j)}{\partial \lambda} > 0$ ,  $\frac{\partial q_n(\eta_j)}{\partial \lambda} > 0$ ,  $\frac{\partial \pi_m(\eta_j)}{\partial \lambda} > 0$ ,  $\frac{\partial \pi_m(\eta_j)}{\partial \lambda} > 0$ ,  $\frac{\partial \pi_m(\eta_j)}{\partial \lambda} > 0$ ,  $\frac{\partial \sigma_m(\eta_j)}{\partial \lambda} > 0$ , where  $j = 1, 2$ .

Proposition 1 shows that under the above two subsidy strategies, the performance sensitivity coefficient  $\lambda$  and

Proposition 1 shows that under the above two subsidy strategies, the performance sensitivity coefficient  $\lambda$  and enhancement cost coefficient  $\beta$  are critical factors affecting operational decisions of the NEV supply chain. As  $\lambda$  rises or  $\beta$  decreases, the NEV supply chain's operational decisions, member enterprises' profits and consumer surplus all increase accordingly. Therefore, the government and member enterprises should completely consider consumer preferences and investment costs when formulating NEV industry development strategies. For example, they can improve consumer awareness of battery performance through a variety of educational publicity activities to enhance market acceptance of high-performance NEVs. The government can also alleviate automakers' investment pressure by providing financial subsidies, which would drive efficiency improvements across the supply chain and promote high-quality development of the NEV industry. For automakers, they should actively optimize their technology innovation paths to reduce the battery performance enhancement cost and maintain a leading edge in market competition.

Proposition 2 (1) 
$$\frac{\partial w_n(\eta_j)}{\partial \eta_j} > 0$$
,  $\frac{\partial \tau_n(\eta_j)}{\partial \eta_j} > 0$ ,  $\frac{\partial p_n(\eta_1)}{\partial \eta_1} > 0$ ,  $\frac{\partial q_n(\eta_j)}{\partial \eta_j} > 0$ ,  $\frac{\partial \pi_n(\eta_j)}{\partial \eta_j} > 0$ , otherwise,  $\frac{\partial \pi_n(\eta_j)}{\partial \eta_j} < 0$ , where  $j = 1$ ,  $j = 1$ ,

It can be seen from Proposition 2 that the above two subsidy strategies can always increase the NEV wholesale price, battery performance enhancement degree, market demand, member enterprises' profits and consumer surplus. However, they have different effects on NEV sales prices. Specifically, the implement of performance enhancement subsidy strategy would raise the NEV sales price. The main reason is that the automaker is always willing to aggressively invest in battery performance enhancement, which makes the product value increase and indirectly pushes up the final market price. In addition, the effect of the sales subsidy strategy on the NEV sales price mainly depends on the performance sensitivity coefficient  $\lambda$  and the enhancement cost coefficient  $\beta$ . When  $\lambda$  is lower or  $\beta$  is higher, this subsidy strategy will lead to a decrease in the NEV sales price. This is mainly because, only a higher  $\lambda$  or a lower  $\beta$  will encourage the automaker to more actively invest in battery performance enhancement, which in turn increases the product value and price. Otherwise, the lower marginal benefits weaken the automaker's incentives to enhance battery performance. Then, the retailer ultimately chooses to transfer the subsidies to the market to reduce the sales price and expand the sales quantity. Therefore, the government should ensure that the subsidies can be effectively transferred to end consumers when formulating subsidy strategies, and thereby stimulating NEV market demand. Meanwhile, The NEV supply chain should dynamically adjust pricing and battery performance enhancement decisions based on subsidy strategies and parameters such as performance sensitivity and cost coefficients. This would help achieve an overall improvement in market demand, member profits, and consumer surplus.

# 5. Comparison and analysis of results

#### 5.1 Comparative analysis of equilibrium solutions

This section compares the NEV supply chain's operational decisions, members' profits and consumer surplus under different subsidy strategies, and Propositions 3 and 4 can be obtained as follows.

**Proposition 3** If 
$$\eta_1 < \frac{(\lambda^2 - \beta b)\eta_2}{3\lambda}$$
, then  $w_n(\eta_1) < w_n(\eta_2)$ ,  $\tau_n(\eta_1) < \tau_n(\eta_2)$ ,  $p_n(\eta_1) < p_n(\eta_2)$ ,  $q_n(\eta_1) < q_n(\eta_2)$ ; if  $\frac{(\lambda^2 - \beta b)\eta_2}{3\lambda} < \eta_1 < \frac{\lambda \eta_2}{4}$ , then  $w_n(\eta_1) < w_n(\eta_2)$ ,  $\tau_n(\eta_1) < \tau_n(\eta_2)$ ,  $p_n(\eta_1) > p_n(\eta_2)$ ,  $q_n(\eta_1) < q_n(\eta_2)$ ; if  $\frac{\lambda \eta_2}{4} < \eta_1 < \frac{\lambda \eta_2}{3\lambda}$ 

$$\frac{\beta b \eta_2}{\lambda}, \text{ then } w_n(\eta_1) < w_n(\eta_2), \tau_n(\eta_1) > \tau_n(\eta_2), p_n(\eta_1) > p_n(\eta_2), q_n(\eta_1) < q_n(\eta_2); \text{ if } \eta_1 > \frac{\beta b \eta_2}{\lambda}, \text{ then } w_n(\eta_1) > w_n(\eta_2), q_n(\eta_1) > q_n(\eta_1) > q_n(\eta_2).$$

Proposition 3 indicates that as a critical parameter, the ratio of government subsidy coefficients  $(\eta_1/\eta_2)$  determines the effect of different subsidy strategies on the optimal NEV supply chain decisions. When the subsidy intensity of a particular policy is noticeably greater, the corresponding battery performance enhancement and market demand are significantly better. Meanwhile, the increase in the wholesale price caused by subsidies would also raise the NEV sales price. In addition, when each subsidy coefficient is unchanged, an increase in  $\lambda$  or a decrease in  $\beta$  would narrow the gap between the equilibrium solutions of the two subsidy strategies. The reason is that the higher marginal benefits brought by a higher  $\lambda$  or a lower  $\beta$  encourage the automaker to enhance the battery performance and accept a lower wholesale price under the sales subsidy strategy. Accordingly, under the performance enhancement subsidy strategy, the retailer is more likely to reduce the NEV sales price as the market share of high-performance NEVs increases. Therefore, when battery performance concerns are low or performance enhancement costs are high, it is particularly critical for the government to make trade-offs in subsidy intensity of different subsidy strategies. This would help achieve coordinated development and optimal allocation of overall benefits in the NEV supply chain.

**Proposition 4** If 
$$\eta_{1} < \frac{-\lambda a + \sqrt{\lambda^{2}a^{2} + 4\beta b^{2}\eta_{2}(2a + b\eta_{2})}}{4b}$$
, then  $\pi_{m}(\eta_{1}) < \pi_{m}(\eta_{2})$ ,  $\pi_{s}(\eta_{1}) < \pi_{s}(\eta_{2})$ ,  $CS(\eta_{1}) < CS(\eta_{2})$ ; if  $\frac{-\lambda a + \sqrt{\lambda^{2}a^{2} + 4\beta b^{2}\eta_{2}(2a + b\eta_{2})}}{4b} < \eta_{1} < \frac{\beta b\eta_{2}}{\lambda}$ , then  $\pi_{m}(\eta_{1}) > \pi_{m}(\eta_{2})$ ,  $\pi_{s}(\eta_{1}) < \pi_{s}(\eta_{2})$ ,  $CS(\eta_{1}) < CS(\eta_{2})$ ; if  $\eta_{1} > \frac{\beta b\eta_{2}}{\lambda}$ ,  $\pi_{m}(\eta_{1}) > \pi_{m}(\eta_{2})$ ,  $\pi_{s}(\eta_{1}) > \pi_{s}(\eta_{2})$ ,  $CS(\eta_{1}) > CS(\eta_{2})$ .

Proposition 4 suggests that the ratio of government subsidy coefficients  $(\eta_1/\eta_2)$  is a critical parameter that further determines the effects of subsidy strategies on member enterprises' profits and consumer surplus. Similar to Proposition 3, when the subsidy intensity of a particular policy is noticeably greater, member enterprises' profits and consumer surplus are all correspondingly higher. In addition, when each subsidy coefficient is unchanged, an increase in  $\lambda$  or a decrease in  $\beta$  would strengthen the advantages of the performance enhancement subsidy strategy in improving member enterprises' profits and consumer surplus. The reason is because a higher  $\lambda$  could increase consumers' willingness to pay for high-performance NEVs. Then, the performance enhancement subsidy strategy not only expands R&D investments, but also stimulates market demand and improves the automaker's pricing ability. Meanwhile, a fall in  $\beta$  enables the performance enhancement subsidy strategy to bring higher marginal benefits to the automaker. Subsequently, the lower NEV sales price and higher market demand also contribute to a higher retailer profit and consumer surplus. Therefore, the government should choose specific subsidy strategies based on the NEV industry development stage, consumer demand characteristics and the automaker's technical characteristics, to further improve member enterprises' profits and consumer surplus.

#### 5.2 Analysis of subsidy effects

To further examine the effects of the above two government subsidy strategies, this subsection provides a comparative analysis of the NEV supply chain's operational decisions, member enterprises' profits and consumer surplus based on the identical government subsidy expenditures.

Specifically, the government subsidy expenditures for the above two government subsidy strategies are  $g_{n1} = \frac{(\lambda a + 4b\eta_1)\eta_1}{4\beta b - \lambda^2}$  and  $g_{n2} = \frac{\beta b\eta_2(a + b\eta_2)}{4\beta b - \lambda^2}$ . For a given  $\eta_2$ , the expression of  $\eta_1(\eta_2)$  can be obtained according to  $g_{n1} = g_{n2}$  as follows:

$$\eta_{1}(\eta_{2}) = \frac{-\lambda a + \sqrt{\lambda^{2} a^{2} + 16\beta b^{2} \eta_{2} (a + b \eta_{2})}}{8b}$$
 (5)

It should be noted that the main purpose of government subsidies is to promote NEV production and battery performance enhancement, while member enterprises are primarily concerned with their total profits. Therefore, the

battery performance enhancement degree, market demand, member enterprises' profits and consumer surplus are the main evaluation criteria to comparatively analyze the effects of different government subsidy strategies. Then, we can obtain Proposition 5.

**Proposition 5** Under the identical government subsidy expenditures, (1)  $w_n(\eta_1(\eta_2)) < w_n(\eta_2)$ ,  $\tau_n(\eta_1(\eta_2)) > \tau_n(\eta_2)$ ,  $p_n(\eta_1(\eta_2)) > p_n(\eta_1(\eta_2)) < q_n(\eta_1(\eta_2)) < \tau_m(\eta_1(\eta_2)) < \tau_m(\eta_1(\eta_2$ 

Proposition 5 demonstrates that given the identical government subsidy expenditures, the difference in effectiveness between the two subsidy strategies depends on the evaluation criteria. First, the battery performance enhancement degree is higher under the performance enhancement subsidy strategy. This is conducive to promoting technological upgrading and building long-term competitiveness in the NEV industry. Second, the sales subsidy strategy is more advantageous for improving the NEV market share, member enterprises' profits and consumer surplus. This is mainly because the sales subsidy strategy can directly reduce the NEV sales price, which would lower the purchasing cost for consumers, and thus stimulate the market demand expansion. Additionally, the rising market demand allows the automaker to increase its profit through scale effects even when the wholesale price decreases. Therefore, the government needs to set specific priority targets based on the NEV industry development stage. Meanwhile, it should strive to achieve a balance between efficiency and fairness through the dynamic matching of subsidy strategies and parameter environments.

# 6. Numerical analysis

Contemporary Mathematics

This section aims to verify the main propositions in this paper with numerical simulations. First, the effects of government subsidy strategies on the NEV supply chain's optimal operational decisions are analyzed. Then, we explore the effects of subsidy coefficients ( $\eta_1$ ,  $\eta_2$ ), performance sensitivity coefficient  $\lambda$  and enhancement cost coefficient  $\beta$  on equilibrium solutions and the automaker's profit under the identical government subsidy expenditures. Based on the data provided in Liu et al. [28], and the relevant data of enterprises and dimensionless processing, the parameters are set as follows: a = 100, b = 5,  $\lambda = 3$ ,  $\beta = 2$ .

First, it can be seen from Figure 3 that the NEV battery performance enhancement degree, market demand and wholesale price under each subsidy strategy have been significantly improved with increasing government subsidies. The difference is that under the NEV sales subsidy strategy, the sales price decreases with the increase of the subsidy coefficient because  $\lambda^2 - \beta b < 0$ . However, the increasing subsidy coefficient can always raise the NEV sales price under the performance enhancement subsidy strategy. These are completely consistent with the conclusions of Proposition 2. In addition, the magnitude order of the corresponding equilibrium solutions under the above two subsidy strategies changes when the ratio of the subsidy coefficients  $(\eta_1/\eta_2)$  exceeds a certain critical line. Since  $\lambda^2 - \beta b < 0$ , then  $\eta_1 > \frac{(\lambda^2 - \beta b)\eta_2}{3\lambda}$  is established. Thus, there always exists  $p_n(\eta_1) > p_n(\eta_2)$  as shown in Figure 3c. These are also entirely consistent with the conclusions of Proposition 3. Furthermore, since the validation of the changing process in member enterprises' profits and consumer surplus under different subsidy strategies is similar to the above conditions, it will not be repeated here.

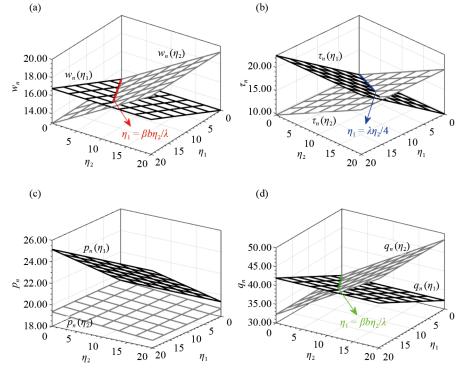


Figure 3. The effects of government subsidies on the equilibrium solutions

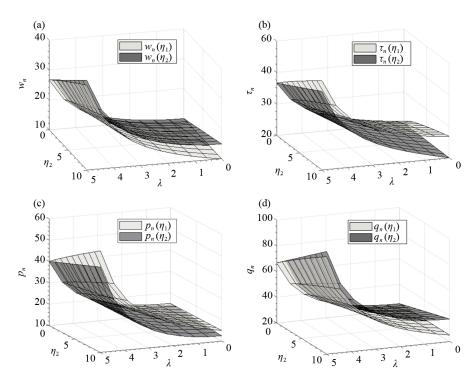


Figure 4. Effect of subsidy coefficients and  $\lambda$  on equilibrium solutions under identical government subsidy expenditures

Figure 4 shows the effects of performance enhancement subsidy and sales subsidy strategies on NEV supply chain's optimal operational decisions considering the identical government subsidy expenditures. Specifically, the performance enhancement subsidy is more conducive to enhancing the NEV battery performance. The sales subsidy is more conducive to lowering NEV sales price and increasing market demand, which is completely consistent with the conclusion of Proposition 5. Moreover, the rising performance sensitivity coefficient  $\lambda$  can increase the NEV supply chain's decision variable values. Meanwhile, it narrows the gap between the equilibrium solutions of the two subsidy strategies. These are completely consistent with the conclusions of Propositions 1 and 3. However, the gap between the equilibrium solutions becomes more noticeable as the subsidy intensity increases. This not only assists the government in formulating more precise subsidy strategies according to the industry development targets, but also helps achieve a balance between short-term market promotion and long-term industrial upgrading.

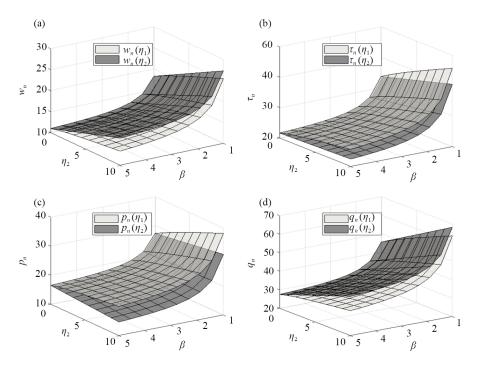


Figure 5. Effect of subsidy coefficients and  $\beta$  on equilibrium solutions under identical government subsidy expenditures

Figure 5 again verifies the conclusion of Proposition 5, it will not be repeated here. Moreover, the increasing government subsidies further widened the gap between the equilibrium solutions under the two subsidy strategies, which is consistent with the conclusion in Figure 4. Furthermore, a decreasing enhancement cost coefficient  $\beta$  can increase the NEV supply chain's decision variable values. Meantime, the gap between the equilibrium solutions becomes narrower as  $\beta$  decreases. This is consistent with the conclusions of Propositions 1 and 3.

Combined with the conclusions of Figures 3 and 4, it can also be concluded that the increasing  $\lambda$  or the decreasing  $\beta$  can help reduce uncertainty in subsidy strategy selection for the government. Consequently, it is beneficial for improving the stability and predictability of the government's subsidy effect.

Since the comparison results of the retailer's profit and consumer surplus are similar to the market demand, we just further show the effects of different subsidy strategies on the automaker's profit. It can be seen from Figure 6 that the NEV sales subsidy strategy brings a higher profit to the automaker. This is consistent with the conclusion of Proposition 5. That is, there always exists  $\pi_m(\eta_1(\eta_2)) < \pi_m(\eta_2)$  under the identical government subsidy expenditures. Moreover, the growing government subsides further extends the advantage of the NEV sales subsidy strategy in improving the automaker's profit. Furthermore, an increasing  $\lambda$  or a decreasing  $\beta$  brings the automaker a higher profit under the two

subsidy strategies. Meantime, it narrows the gap between the automaker's profits, which is consistent with the conclusion of Proposition 4. This also means that a higher  $\lambda$  or a lower  $\beta$  is more beneficial to the performance enhancement subsidy strategy in promoting the automaker's total profit.

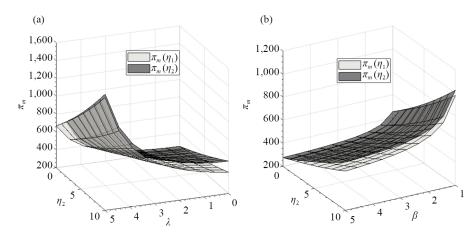


Figure 6. Effects of subsidy coefficients with  $\lambda$  and  $\beta$  on the automaker's profit under identical government subsidy expenditures

# 7. Conclusions

# 7.1 Concluding remarks

As the NEV industry matures, its battery performance has increasingly become a central concern for consumers. However, the high costs associated with battery performance enhancement have weakened automakers' investment incentives. This necessitates the continuous implementation and adjustment of government subsidy strategies. Therefore, to analyze the implementation effects of different government subsidy strategies, this paper investigates the effects of performance enhancement subsidy and sales subsidy strategies on the NEV supply chain's operational decisions, member enterprises' profits and consumer surplus. Then, this paper clarifies the effects of critical parameters on the subsidy effectiveness, and the critical conditions under which a certain strategy is preferred through theoretical and numerical analysis. Meanwhile, the specific effectiveness of the above two subsidy strategies are further comparatively analyzed under the identical government subsidy expenditures.

The main conclusions proposed in this paper are as follows. First, both battery performance enhancement subsidy and sales subsidy strategies can improve the NEV battery performance, market demand and benefits for all participants in the supply chain. Among these, the sales subsidy may reduce the final sales price, while the performance enhancement subsidy focuses more on technological research and development. Second, the ratio of subsidy coefficients  $\eta_1/\eta_2$ , consumer performance sensitivity  $\lambda$  and performance enhancement cost coefficients  $\beta$  jointly affect the effectiveness of different subsidy strategies. A higher  $\lambda$  or a lower  $\beta$  would narrow the gap in effectiveness between above two strategies, while a increasing subsidy intensity would widen the gap in effectiveness. Third, under the identical government subsidy expenditures, the performance enhancement subsidy is preferred for the purpose of NEV technological upgrading. The sales subsidy strategy is more beneficial to the NEV market expansion and benefits for all participants in the supply chain.

#### 7.2 Managerial insights

Based on the above conclusions, this paper provides managerial insights for the government in choosing appropriate subsidy strategies and for NEV supply chain member enterprises in making optimal operational decisions.

For the government, first, subsidy strategies and the ratio of coefficients should be dynamically adjusted considering the NEV industry development stage. During the industrial technology growth period, the performance enhancement subsidy coefficient can be improved to directly incentivize automakers to invest more in battery technology research and development. The sales subsidy coefficient can be moderately improved to rapidly stimulate consumer demand by lowering the NEV final price. Second, the government should precisely match subsidy strategies according to the NEV industry development targets. If technological innovation is the primary target, a performance enhancement subsidy should be prioritized to drive automakers to break through battery technology bottlenecks (such as high energy density and fast charging technology) and build long-term industry barriers. If market expansion is the short-term target, a sales subsidy can better reduce the NEV final price and stimulate demand growth. Concurrently, the construction of charging infrastructure can be coordinated to alleviate consumer endurance anxiety and amplify the synergistic effects of subsidy policies. Third, it is necessary to strengthen consumers' performance sensitivity and reduce performance enhancement costs through policy instruments. For instance, the government can enhance consumer awareness of core performance characteristics such as battery safety and endurance through science popularization campaigns and safety standard certification. Alternatively, it could provide R&D expense deductions and technical sharing platform construction support to automakers to reduce the marginal cost of battery performance enhancement.

For the NEV supply chain member enterprises, automakers can proactively optimize their R&D strategies to reduce the battery performance enhancement costs through large-scale production and supply chain vertical integration. Additionally, it is essential to optimize pricing and R&D investment decisions through dynamic matching subsidy strategies. For example, under performance enhancement subsidies, automakers can moderately increase wholesale prices and performance enhancement degree to capture the market with high-value-added products. Retailers should instead concentrate on scenario-based promotion of high-range, high-safety vehicles (such as long-distance travel and household use scenarios) to leverage product performance advantages for premium pricing. Simultaneously, they can improve consumer surplus through membership services and extended warranty policies to strengthen their high-end market share.

#### 7.3 Future research

The limitations of this study are mainly reflected in the following aspects. This paper focuses primarily on a two-echelon NEV supply chain that includes an automaker and a retailer. Future research could incorporate more participants (such as battery suppliers and recyclers) to make the study issues more realistic. In addition, the effects of subsidy combination strategies can be considered to explore more targeted government subsidy schemes. Last but not least, the evaluation criteria such as environmental and social welfare indicators can be integrated and analyzed.

# Acknowledgement

The authors thank the editor and the anonymous referees for their constructive comments. This work is supported by the Ministry of Education Humanities and Social Science Youth Fund under grant number 22YJC630148, the National Natural Science Fund of China under grant number 72162006, and the Social Science Fund of Jiangsu Province under grant number 20EYD002.

# Data availability statement

The assignment of model parameters has been given in the text, no other data needs to be shared.

#### **Conflict of interest**

The authors declare no competing financial interest.

# References

- [1] Zhang X, Bai X. Incentive policies from 2006 to 2016 and new energy vehicle adoption in 2010-2020 in China. *Renewable and Sustainable Energy Reviews*. 2017; 70: 24-43.
- [2] Xu X, Wei Z, Ji Q, Wang C, Gao G. Global renewable energy development: Influencing factors, trend predictions and countermeasures. *Resources Policy*. 2019; 63: 101470.
- [3] Li J, Jiang M, Li G. Does the new energy vehicles subsidy policy decrease the carbon emissions of the urban transport industry? Evidence from Chinese cities in Yangtze River Delta. *Energy*. 2024; 298: 131322.
- [4] Lou G, Ma H, Fan T, Zhang Y, Chen L. Impact of the dual-credit policy on improvements in fuel economy and the production of internal combustion engine vehicles. *Resources, Conservation and Recycling*. 2020; 156: 104712.
- [5] He H, Li S, Wang S, Chen Z, Zhang J, Zhao J, et al. Electrification decisions of traditional automakers under the dual-credit policy regime. *Transportation Research Part D: Transport and Environment*. 2021; 98: 102956.
- [6] Pu J, Chun W, Wang Z, Chen W. Operation strategy for new energy vehicle enterprises based on dual credit policy. *Journal of Industrial and Management Optimization*. 2023; 19(8): 5724-5748.
- [7] Zhang Z, Guo M, Yang W. Analysis of NEV power battery recycling under different government reward-penalty mechanisms. *Sustainability*. 2022; 14(17): 10538.
- [8] Yi Y, Fu A, Li Y, Zhang A. Battery recycling and coordination in information leakage prevention under blockchain technology in a new energy vehicles supply chain. *Energy Economics*. 2024; 139: 107862.
- [9] Lim M, Mak H, Rong Y. Toward mass adoption of electric vehicles: Impact of the range and resale anxieties. *Manufacturing & Service Operations Management*. 2015; 17(1): 101-119.
- [10] Neaimeh M, Salisbury SD, Hill GA, Blythe PT, Scoffield DR, Scoffield DR. Analysing the usage and evidencing the importance of fast chargers for the adoption of battery electric vehicles. *Energy Policy*. 2017; 108: 474-486.
- [11] Yu JJ, Tang CS, Li MK, Shen ZJM. Coordinating installation of electric vehicle charging stations between governments and automakers. *Production and Operations Management*. 2022; 31(2): 681-696.
- [12] Cheng Y, Mu D. Joint production strategies of new energy vehicle with carbon price disruption. *Journal of Systems Engineering*. 2018; 33(6): 780-792.
- [13] Tang J, Yang F, Xu J. A production and pricing research on two-echelon automotive supply chains considering consumer preference under dual-credit regulation. *Industrial Engineering and Management*. 2021; 26(1): 121-129.
- [14] Lu C, Wang Q, Chen Q. Automobile supply chain coordination considering auto price, emission reduction and the mileage range in one charge under the "double points" policy. *Systems Engineering-Theory & Practice*. 2021; 41(10): 2595-2608.
- [15] Xu Y, Ma X, Zhou G. Coordination of automobile supply chain considering relative endurance level under the dual-credit policy. *Sustainability*. 2022; 14(21): 13704.
- [16] Ma M, Meng W, Huang B, Li Y. The influence of dual credit policy on new energy vehicle technology innovation under demand forecast information asymmetry. *Energy*. 2023; 271: 127106.
- [17] Zhang J, Huang J. Vehicle product-line strategy under government subsidy programs for electric/hybrid vehicles. *Transportation Research Part E: Logistics and Transportation Review.* 2021; 146: 102221.
- [18] Cheng F, Chen T, Chen Q. Cost-reducing strategy or emission-reducing strategy? The choice of low-carbon decisions under price threshold subsidy. *Transportation Research Part E: Logistics and Transportation Review.* 2022; 157: 102560.
- [19] Shao J, Jiang C, Cui Y, Tang Y. A game-theoretic model to compare charging infrastructure subsidy and electric vehicle subsidy policies. *Transportation Research Part A: Policy and Practice*. 2023; 176: 103799.
- [20] Wee S, Coffman M, La Croix SL. Do electric vehicle incentives matter? Evidence from the 50 U.S. states. *Research Policy*. 2018; 47(9): 1601-1610.
- [21] Yao X, Shao Z, Wang Z, Zhu Z, Chen Z, Wu Q. Policy incentives and market mechanisms dual-driven framework for new energy vehicles promotion. *Energy Policy*. 2025; 199: 114530.
- [22] Gu H, Liu Z, Qing Q. Optimal electric vehicle production strategy under subsidy and battery recycling. *Energy Policy*. 2017; 109: 579-589.
- [23] Yang D, Qiu L, Yan J, Chen Z, Jiang M. The government regulation and market behavior of the new energy automotive industry. *Journal of Cleaner Production*. 2019; 210: 1281-1288.
- [24] Shi L, Sethi S, Çakanyldirim M. Promoting electric vehicles: Reducing charging inconvenience and price via station and consumer subsidies. *Production and Operations Management*. 2022; 31(12): 4333-4350.

- [25] Zhu X, Chiong R, Wang M, Liu K, Ren M. Is carbon regulation better than cash subsidy? The case of new energy vehicles. *Transportation Research Part A: Policy and Practice*. 2021; 146: 170-192.
- [26] Chen Y, Zhang Y, Zhang S. The optimal design of differentiated subsidy policies for new energy vehicle firms by considering the difference in market share and endurance mileage. *International Journal of Industrial Engineering Computations*. 2024; 15(2): 427-442.
- [27] Wang Y, Zhang X, Cheng TCE, Wu TH. Choice of the co-opetition model for a new energy vehicle supply chain under government subsidies. *Transportation Research Part E: Logistics and Transportation Review.* 2023; 179: 103326.
- [28] Liu C, Liu J, Shao LL, Zhang Y. Contract design to incentive supplier innovation under dual-credit policy. *Chinese Journal of Management*. 2022; 19(6): 928-937.

# **Appendix**

**Proof of Lemma 1** Since different game models are solved in the same way, only the proof procedure for the performance enhancement subsidy case is given here. Specifically, the model is solved using backward induction based on the above decision sequence.

First, let  $\frac{\partial \pi_s(\eta_2)}{\partial p_n} = 0$ , and we have  $p_n = \frac{a + \lambda \tau_n + bw_n}{2b}$ . Substituting  $p_n = \frac{a + \lambda \tau_n + bw_n}{2b}$  into the automaker's profit function  $\pi_m(\eta_2)$  and further solving for  $\frac{\partial \pi_m(\eta_2)}{\partial w_n} = 0$  and  $\frac{\partial \pi_m(\eta_2)}{\partial \tau_n} = 0$ , we can obtain  $w_n = \frac{a + \lambda \tau_n}{2b}$  and  $\tau_n = \frac{\lambda w_n + 2\eta_1}{2\beta}$ . Joining the above equations, we can obtain the equilibrium solutions and the corresponding profits and consumer surplus under the performance enhancement subsidy strategy.

**Proof of Proposition 5** Assuming  $w_n(\eta_1(\eta_2)) = \frac{2(\beta a + \lambda \eta_1(\eta_2))}{4\beta b - \lambda^2} > w_n(\eta_2) = \frac{2\beta(a + b\eta_2)}{4\beta b - \lambda^2}$ , we have  $16\beta b^3 \eta_2^2 (\lambda^2 - 4\beta b) > 0$ . This contradicts the previous setting  $4\beta b - \lambda^2 > 0$ . Thus, the above assumption is not valid. Ultimately, we can obtain  $w_n(\eta_1(\eta_2)) < w_n(\eta_2)$ . Based on the above proof process, it can also be obtained that  $q_n(\eta_1(\eta_2)) < q_n(\eta_2)$ ,  $\pi_s(\eta_1(\eta_2)) < \pi_s(\eta_2)$ ,  $CS(\eta_1(\eta_2)) < CS(\eta_2)$ .

 $\pi_s(\eta_1(\eta_2)) < \pi_s(\eta_2), CS(\eta_1(\eta_2)) < CS(\eta_2).$  Assuming  $\tau_n(\eta_1(\eta_2)) = \frac{\lambda a + 4b\eta_1(\eta_2)}{4\beta b - \lambda^2} < \tau_n(\eta_2) = \frac{\lambda (a + b\eta_2)}{4\beta b - \lambda^2}$ , we have  $b\eta_2(a + b\eta_2)(4\beta b - \lambda^2) < 0$ . This contradicts the previous setting  $4\beta b - \lambda^2 > 0$ . Thus, the above assumption is not valid. Ultimately, we can obtain  $\tau_n(\eta_1(\eta_2)) > \tau_n(\eta_2)$ .

According the expressions of  $p_n(\eta_1(\eta_2))$ ,  $p_n(\eta_2)$ , it can also be obtained that  $p_n(\eta_1(\eta_2)) > p_n(\eta_2)$  when  $\lambda^2 < \beta b$ . If  $\lambda^2 > \beta b$ , we have  $b\eta_2(3\lambda^2a + 4\lambda^2b\eta_2 - \beta b^2\eta_2)(4\beta b - \lambda^2) > 0$  through assuming  $p_n(\eta_1(\eta_2)) = \frac{3(\beta a + \lambda \eta_1(\eta_2))}{4\beta b - \lambda^2} > 0$ 

 $p_n(\eta_2) = \frac{3\beta a + \lambda^2 \eta_2 - \beta b \eta_2}{4\beta b - \lambda^2}.$  Since  $\lambda^2 > \beta b$  and  $4\beta b - \lambda^2 > 0$ , there always exists  $p_n(\eta_1(\eta_2)) > p_n(\eta_2)$ . Ultimately, we can obtain  $p_n(\eta_1(\eta_2)) > p_n(\eta_2)$ .

Finally, assuming  $\pi_m(\eta_1(\eta_2)) > \mu_n(\eta_2)$ .

Finally, assuming  $\pi_m(\eta_1(\eta_2)) = \frac{\beta a^2 + 2\lambda a \eta_1(\eta_2) + 4b \eta_1^2(\eta_2)}{2(4\beta b - \lambda^2)} > \pi_m(\eta_2) = \frac{\beta (a + b \eta_2)^2}{2(4\beta b - \lambda^2)}$ , we have  $16\beta a^2 b^3 \eta_2^2 (\lambda^2 - 4\beta b) > 0$ . This contradicts the previous setting  $4\beta b - \lambda^2 > 0$ . Thus, the above assumption is not valid. Ultimately, we can obtain  $\pi_m(\eta_1(\eta_2)) < \pi_m(\eta_2)$ .

To sum up, proposition 5 is proved.