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Research Article

# Heterogeneous Belief Asset Pricing from the Perspective of Behavioural Finance: A Partial Differential Game Model with Interaction Terms

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**Abstract:** Asset pricing models have played an important role in the development of modern financial theory. From the Capital Asset Pricing Model (CAPM) to the Arbitrage Pricing Theory (APT), these classical models provide the basic framework for understanding and predicting asset prices. However, most traditional asset pricing models are based on the efficient market hypothesis, which assumes that all investors are rational, have homogeneous expectations, and can process all available information quickly and accurately. Despite their success in some respects, these models have proved inadequate in explaining and predicting market anomalies. For example, market bubbles, excessive volatility, and frequent deviations of asset prices from their fundamental values cannot be adequately explained by traditional rational expectations models. In order to more accurately capture the complex market dynamics, especially given the heterogeneous beliefs and their interactions between investors, this study uses of Partial Differential Equations (PDEs), deeply analyses the mechanism of heterogeneous beliefs influence. This study not only improves our understanding of how financial markets operate, but also provides a solid foundation for the development of effective risk management strategies and policy design. This study contributes to building a more robust and inclusive financial environment to better cope with future challenges and uncertainties.

***Keywords***: behavioural finance, heterogeneous effects, asset pricing

**MSC:** 91G80

## 1. Introduction

Traditional financial theory assumes that all market participants have the same expectations and information processing capabilities, and are perfectly rational. However, in reality, investors have different beliefs, expectations and information acquisition and processing abilities. These differences are called heterogeneous beliefs. The existence of heterogeneous beliefs makes market dynamics more complex and volatile, because each investor not only makes decisions based on his own information, but also is influenced by the behaviour of other investors. This interaction has a profound impact on market prices.


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## 1.1 *Heterogeneous beliefs related research*

Miller was one of the first scholars to explore the impact of heterogeneous beliefs on asset prices, which opened up a new way to study asset pricing issues. He built a static analysis model, discussed the form and the interaction between heterogeneous beliefs, and found that compared with pessimistic investors, optimistic investors' expectations are more likely to reflect in the stock, resulting in asset prices are overvalued. Subsequently, [1] used the divergence of analysts' earnings forecasts to measure the heterogeneous beliefs of investors for the first time, and reached a similar conclusion as Miller. To overcome Miller's static model's inability to link price overvaluation to trading volume, Scheinkman and Xiong et al. proposed a dynamic model based on heterogeneous beliefs. In these models, investors continuously update their expectations based on their understanding of information. Domestic scholars such as [2–4] followed this idea and empirically studied the impact of investors' heterogeneous beliefs on asset pricing by dividing investor heterogeneity.

From the point of view of market microstructure, investors heterogeneous beliefs not only affect the asset pricing, also impact on market liquidity, stability, etc. [5], for example, based on Lucas pure exchange economic model, studies the different types of market investors heterogeneous preferences and beliefs of market equilibrium problem; [6] based on dynamic heterogeneous beliefs asset pricing model, found that even under the rules of the same trade, investors valuing the stock based on different beliefs will lead to market instability. [7] by establishing stage from two to many trading models, the change of investor expectations of Markov chain simulation, reveals the investors heterogeneous beliefs fluctuation degree, the level of risk aversion and the degree of information asymmetry are negatively related with the stock market liquidity. Based on the BH model in the heterogeneous belief pricing theory of investors, [8] designed a model including investor entry and exit mechanism to study the systematic factors and their effects of the stock market volatility, assuming that there are three types of investors in the stock market: fundamental value investors, trend investors and noise traders. It is found that the intensity of conversion between heterogeneous investors is positively correlated with the market volatility, while the changes in the number of investors and previous investment performance have no significant impact on the stock market volatility. Finally, [9–11] discussed theoretically and empirically that investors' optimism and pessimism drive noise traders' buying and selling behaviour and rational traders' arbitrage behaviour, namely, market feedback trading behaviour.

## 1.2 *Behavioral finance perspective of asset prices abnormal fluctuations*

In 2002, the Nobel Prize in Economics was awarded to Daniel Kahneman, professor of psychology and public affairs at Princeton University. This event marked the beginning of behavioural finance to occupy a place in mainstream economics. Since then, some scholars have begun to combine the research results of cognitive psychology to explore the internal mechanism of abnormal stock market fluctuations from the perspective of behavioural finance. Behavioural finance points out that under uncertainty, investors' judgments and decisions tend not to follow the principle of Bayesian rationality, but influenced by personal behaviour, emotions, habits, and the influence of the cultural background, producing a variety of cognitive biases or psychological preferences. Based on the hypothesis that irrational agent, investors tend to mimic the behaviour of others to make decisions, this leads to price deviate from their fundamental value, causing the stock market bubbles and crashes. Currently, a variety of behavioural finance models study the interaction between investors and its influence on stock market prices, including but not limited to fanaticism and panic model, noise traders, fashion and trend model, the model and the positive feedback traders herding effect model, etc.

(1) Mania and panic model: This model emphasizes the instability of investment behaviour in the short term. Bubbles in the stock market mainly result from the loss of investors' rationality, and even reach the level of "mass hysteria". This phenomenon is closely related to the general irrationality or mass psychology, which is manifested as the accidental deviation of rational behaviour pattern.

(2) Noise Trader model (DSSW model): [12] proposed this model, which revealed that in a limited arbitrage market, unsubstantiated information dissemination will cause investors' emotions to affect each other, causing noise traders to buy and sell the same stock in the same time period, interfering with the normal trading activities of informed traders, resulting in systematic price deviation.

(3) Fads and trends model: This model argues that stock prices are susceptible to pure social dynamics and fashion trends, especially in the absence of mature theories to explain stock values or predict outcomes. Mutual attention among investors can lead to speculative bubbles and eventually a stock market crash.

(4) Positive feedback trader model: also proposed by [13], this model shows that rational investors increase market price volatility by amplifying the positive feedback trading behaviour of other investors.

(5) Herding model: In his 1995 study, Lux constructed a basic model of behaviour imitation and opinion propagation among traders to explain the phenomenon of excessive stock price volatility. In particular, this model takes into account the behaviour of novice investors, who rely more on the behavior of others than fundamental information to form their expectations. Subsequent studies such as [14] further show that the development of financial markets is driven by investor sentiment, while [15, 16] deeply explores the relationship between herding, stock market bubbles and investor interaction.

# 2. Theoretical model

## 2.1 *Model assumptions*

Suppose that there are two tradable assets in an economic system: a risk-free asset (such as cash) and a representative risky asset (such as stocks). The supply of the risk-free asset is perfectly elastic, which means that the quantity demanded by investors has no effect on its price, and its fixed risk-free interest rate is denoted as $Rf$. In contrast, the supply of the risky asset is not perfectly elastic and the supply is limited. The total supply is set as $Q$, and the market price at time $t$ is denoted as $P_t$.

Suppose there are two types of heterogeneous belief investors in the market. Type A customers follow rational expectations but have cognitive biases (such as overconfidence); Type B customers are driven by emotions, and their heterogeneous beliefs are affected by the market sentiment index $S_t$, so the time-to-time change of price $P_t$ at time $t$ is:

$$dP_t = u_t P_t dt + \sigma P_t dW_t \tag{1}$$

Among them, $u_t$ for the instantaneous yield, $\sigma$ for volatility (usually a constant), $W_t$ is a standard Brownian motion. For Type A customers, the return rate is:

$$u_t^a = \theta u_t + (1-\theta)u \tag{2}$$

Where $\theta$ is the coefficient of overconfidence and $\theta > 1$.

For Type B customers, the rate of return is:

$$u_t^b = u_t + \gamma S_t \tag{3}$$

Where $S_t$ is the sentiment factor and $\gamma$ is the sentiment sensitivity coefficient.

## 2.2 *Model construction*

Suppose that the wealth of investor $i$ ($W_i^t$) satisfies:

$$dW_t^i = rW_t^i dt + K_t^i(ut_t^i P_t dt + \sigma P_t dW_t^i) \tag{4}$$

Where $r$ is the risk-free interest rate and $K_t^i$ is the proportion invested.

Using the CRRA risk aversion utility function, we can obtain:

$$U(W_t^i) = \frac{(W_t^i)^{1-\alpha}}{1-\alpha},\ \alpha > 0 \tag{5}$$

Where $\alpha$ denotes risk aversion, where $\alpha$ greater means higher risk aversion.

On this basis, the Hamilton-Jacobi-Bellman (HJB) equation is constructed as follows:

$$\left\{ J_t + J_W[rW + K^i(u^i p - rp)] + \frac{1}{2} J_{ww}(K^i \sigma P)^2 \right\} \tag{6}$$

The optimal ratio is solved as follows:

$$K_t^i = \frac{u_t^i - r}{\alpha\sigma^2} \tag{7}$$

On this basis, we calculate the market equilibrium condition. We can set the weights of investors of $a$ and $b$ as $\lambda$ and $1-\lambda$ respectively, and we get the total demand:

$$D_t = \lambda k_t^a + (1-\lambda) k_t^b \tag{8}$$

Considering the strategy interaction effect, we modify the equation:

$$\widetilde{D}_t = D_t + \beta K_t^a K_t^b \tag{9}$$

Where $\beta$ is the coefficient of interaction strength, and we normalize the supply to 1 to obtain:

$$\lambda \frac{u_t^a - r}{\alpha\sigma^2} + (1-\lambda)\frac{u_t^b - r}{\alpha\sigma^2} + \beta \frac{(u_t^a - r)(u_t^b - r)}{\alpha^2\sigma^4} = 1 \tag{10}$$

On this basis, we can set up a partial differential equation simultaneously:

$$\begin{cases} dP_t = u_t P_t dt + \sigma P_t dW_t \\ dS_t = (S - S_t)d_t + \sigma_s dZ_t \\ u_t = F(P_t,\, S_t,\, k_t^a,\, k_t^b) \\ < dW_t,\, dZ_t >= \rho dt \end{cases} \tag{11}$$

The overconfidence $\theta$ of Type A investors and sentiment drive $\gamma S_t$ of Type B investors jointly affect asset pricing, $\beta$ which is used to capture the nonlinear price effect generated by the strategic interaction between the two types of investors for the interaction intensity coefficient $\rho$.

## 3. Equilibrium analysis of the model

The interaction terms between heterogeneous beliefs and investors will have an impact on asset prices and volatility.

### 3.1 *Influence of heterogeneous beliefs*

For Type A investors, they will overestimate the rate of return $u_t^a$, which will lead them to hold more risky assets. When the market equilibrium occurs, the asset price $P_t$ will be pushed higher than the rational expectation level, forming a price bubble. Overconfidence makes Type A overreact to market information. For example, they will amplify fundamental volatility signals, which will lead to higher price fluctuations. The sensitivity $\theta$ of their demand function increases, further exacerbating the volatility.

For Type B investors, sentiment index $S_t$ will affect the expectations of Type B investors by $u_t^b = u_t + \gamma S_t$. Specifically, when $S_t < 0$, Type B investors maintain their optimism, they will continue to increase demand and push up asset prices; however, when $S_t < 0$, Type B investors are pessimistic, they will underestimate the yield and sell assets to drive down the price.

### 3.2 *Influence of interaction term*

The strategic interaction between the two types of investors may produce synergistic or offsetting effects. For example, when the interaction coefficient $\beta > 0$, the demand of the two types of investors is in the same direction, they may chase up and sell off simultaneously, eventually magnifying the total demand. Conversely, when $\beta > 0$, the two types of investors operate in opposite directions, the interaction term will inhibit the total demand, which may show that one side of the bottom fishing and the other side of the stop loss phenomenon. In this case, the equilibrium state will be broken, and there may be a coexistence of bubble state and crash state.

When the difference between the two types of investors' beliefs $u_t^a - u_t^b$ expands, the impact of the interaction term on the aggregate demand increases non-linearly. This can cause the price volatility to exceed the underlying volatility. In this case, the probability of a market crash or boom may be amplified, and the price distribution in this case shows the characteristics of a spike and fat tail.

### 3.3 *Influence under joint action*

Under the positive feedback mechanism, a price increase will lead to an increase in the sentiment index, which will stimulate Type B investors to increase their demand, which will lead to further price increase and continued warming of sentiment, and eventually form a self-reinforcing price bubble. During the bull market, for example, investors tend to show the deviation behaviour such as overconfidence and herd behaviour, these psychological factors can aggravate shares the momentum effect, many see a stock up after many days in a row, other investors may follow suit to buy higher prices. This is a good example of positive feedback.

Under the positive feedback mechanism, prices might affect mood worse, causing investors to sell assets and *b* type, the new to further lead to emotional breakdown prices continue to fall and the market, will eventually cause liquidity spiral effect. The liquidity spiral effect will make the market lose its normal trading function, and the bid-ask spread will expand sharply, or even there will be "no liquidity". This negative feedback mechanism will eventually aggravate the market crisis.

In general, the overconfidence of Type A investors leads to long-term price deviation from fundamentals, while the sentiment fluctuations of Type B investors keep short-term volatility high. The interaction $\beta$ between the two is amplified by the interaction term, so that volatility has a persistent impact on future volatility.

# 4. Numerical simulation analysis

In order to analyse the influence mechanism of heterogeneous beliefs on asset prices and volatility, we assume the following parameters: $r = 5\%$, $\sigma = 20\%$, $\alpha = 2$, $\lambda = 60\%$, $\theta = 1.2$, $\gamma = 0.5$, $\beta = 0.3$, $u = 10\%$, $S_t = 0.2$(bull market)$/-0.2$(bear market).

In the bull market scenario, $S_t = 0.2$, $u_t = 8\%$, the return rate $u_t$ is lower than the long-run equilibrium return rate, which reflects that the sentiment drive is overvalued in the short term, asset prices rise, the underlying volatility $\sigma = 20\%$, but after introducing the interaction term, the additional volatility $\sigma = 30\%$. In the bear market scenario, $S_t = -0.2$, $u_t = -2\%$ shows that the return rate is underestimated under pessimistic sentiment, and the asset price falls, and the additional volatility is amplified to $\sigma = 34\%$ by the interaction term.

Therefore, we can conclude that overconfidence of Type A investors drives up demand in bull markets, but intensifies selling in bear markets. The sentiment sensitivity of Type B investors leads to a high dependence of returns on the sentiment index. In the bull market, the interaction term increases the volatility by 50% (20% → 30%). In a bear market, it increases by 70% (20% → 34%).

# 5. Policy recommendations

## 5.1 *Build a dynamic regulatory system*

### 5.1.1 *Establish a dynamic monitoring and correction mechanism for investor behaviour*

(1) Establish a behavioural finance database

Regulators should build a high-frequency behavioural data platform for stock exchanges and securities firms to track the characteristics of investors' trading behaviour, such as turnover rate, position concentration, stop loss frequency and other important indicators, combined with heterogeneous belief parameters in the model, such as overconfidence coefficient $\theta$, sentiment sensitivity $\gamma$, and build a real-time monitoring index system.

(2) A hierarchical warning system for cognitive bias should be implemented

When the $\theta$ value of a certain group of investors is continuously higher than the threshold (such as $\theta > 1.5$), or the volatility of the sentiment index exceeds the historical extreme value, the regulatory alert will be automatically triggered. For example, in 2021, US stock market retail group event, the monitoring of $\theta$ value of Robinhood user group can identify irrational trading risks in advance.

(3) Develop intelligent systems for investor education

The system analyses individual trading records using machine learning and pushes customized risk tips. For example, the psychological test of overconfidence is pushed for investors who frequently chase up and kill down, and behaviour correction is carried out based on their trading loss cases.

### 5.1.2 *Counter-cyclical adjustment mechanism of leverage tools*

(1) Introducing nonlinear margin rules

Flexible margin requirements will be implemented for asset classes where momentum trading is concentrated, such as industry ETFs and cryptocurrencies. For example, when the 20-day volatility of an ETF exceeds 40 per cent, its margin ratio will be raised in a step from 50 per cent to 70 per cent.

(2) Stress tests will be normalized

In order to effectively deal with the chain reaction brought by the liquidity spiral, banks, funds and securities firms are required to conduct regular stress tests simultaneously, and the frequency of tests should be at least quarterly or semi-annual. Special attention should be paid to the possible liquidity evaporation risk of momentum trading concentrated assets to ensure that potential hidden dangers can be found in time. Once a capital gap is found in the stress test, the relevant institutions should make up for it by increasing capital or reducing leverage within 90 days. At the same time, the parameters under stress scenarios should be incorporated into the daily risk management index system to enhance the adaptability to the complex market environment.

## 5.2 *Improve the design of market microstructure*

### 5.2.1 *Establish trading mechanism circuit breaker system*

(1) Implement volatility layered circuit breaker

On the basis of the existing price circuit breaker, a volatility circuit breaker mechanism will be added. When the effective volatility $\sigma$ calculated by the model exceeds twice the base value (e.g., from 20% to 40%), procedural trading is suspended for 10 minutes; when it exceeds 3 times, algorithmic trading is prohibited for the whole day.

(2) Establish liquidity adjustment funds

To cope with liquidity risks in market fluctuations, special funds can be set up by central banks or exchanges to automatically intervene to control price spreads and provide market-making support when necessary. The mechanism is that when the bid-ask spread on the order book exceeds three times its daily average, the fund will inject liquidity by submitting limit orders to stop the negative feedback loop between price and liquidity caused by widening spreads. The intervention is aimed at stabilising market prices, ensuring smooth trading and preventing the spread of a liquidity crisis.

### 5.2.2 *Intelligent upgrading of information disclosure system*

(1) The public disclosure system of sentiment index will be implemented

Referring to the construction method $S_t$ in the model, the market sentiment index is published by an independent third party, including multiple data sources such as social media sentiment analysis and option skewness index. Fund managers are required to disclose the regular reports the exposure of its portfolio to $S_t$.

(2) Establish a tiered disclosure system for information shocks

A "cooling-off period" disclosure rule should be implemented for information that may lead to heterogeneous belief differentiation, such as Federal Reserve policy signals, industry disruptive technological breakthroughs, and China-US trade war. For example, the market is forced to close for 30 minutes after major policy announcements for investors to digest the information.

(3) Develop a trading strategy filing system

Institutional investors are required to report the $\beta$ value parameters (interaction strength) of their strategies, and additional capital requirements are imposed on high-risk strategies with $\beta > 0.4$ to prevent multiple equilibrium risks in the model.

## 5.3 *Supporting safeguard measures for policy implementation*

### 5.3.1 *Improve relevant laws and regulations*

The article of "nonlinear market fluctuation" is added to the Securities Law to clarify the special disposal power of the regulatory authorities when $\sigma$ exceeds the threshold. The Bankruptcy Law is amended to add a "special disposal procedure for fintech enterprises" to prevent systemic risks caused by the collapse of algorithmic trading systems.

### 5.3.2 *Build a team of professional regulatory talents*

The Conduct Finance Regulatory Authority shall be set up in the central bank and major regulatory institutions, equipped with professional teams with both mathematical modelling ability and market experience. The Bank has jointly trained "financial engineering + cognitive psychology" talents with top universities to develop a new generation of regulatory technology tools.

### 5.3.3 *Improve financial infrastructure*

We will invest in building a quantum-secure financial communication network to prevent an instantaneous liquidity crisis caused by HFT systems being hacked. Building disaster recovery centres for distributed trading in major financial centres to ensure the continuous operation of core market functions under extreme circumstances.

## 6. Conclusion and outlook

By integrating heterogeneous beliefs and strategy interactions into a unified analytical framework, this study provides a new methodological tool for the development of behavioural finance theory. Theoretically, the model achieves three innovative breakthroughs: firstly, the interaction term is introduced to capture the nonlinear game effects among different agents, thus surpassing the linear superposition assumption in the traditional rational expectations model; secondly, a dynamic feedback mechanism is constructed to reflect the changes of sentiment index, which successfully integrates quantifiable emotional factors from psychology into the asset pricing process. Finally, we use partial differential game method to describe the coevolution path among price, belief and liquidity in a continuous-time framework. In practice, the model not only provides a micro foundation for explaining market anomalies (such as momentum effect and volatility clustering), but also provides quantitative support for regulators to identify systemic risks (such as liquidity spirals) and design countercyclical regulatory tools (such as dynamic margin rules). This enables us to better understand market dynamics, and provides a scientific basis for formulating effective policy measures, which helps to improve market stability and efficiency. In this way, this study not only advances the academic understanding of the complexity of financial markets, but also provides valuable guidance for practical operations. It is recommended to systematically deal with the risk of market instability caused by heterogeneous beliefs and interaction terms.

However, this study still has some limitations. On the one hand, this study only distinguishes rational investors from emotional investors, and ignores the multi-level cognitive differences existing in the real market. Further research can introduce continuous belief distribution function or agent-based modelling. On the other hand, this study focuses on single asset pricing. In the future, the multi-asset framework should be extended to study the contagion path of heterogeneous beliefs among stock, bond and derivative markets and the subsequent systemic risk.

In general, this study represents a shift in finance from the traditional hypothesis of "rational man" to a more realistic research paradigm of "real man". Through a rigorous mathematical framework, this model shows how irrational behaviour reshapes macro market dynamics through micro-level interactions among individuals, providing a new perspective for understanding the complexity of financial systems. With the rapid progress of computing technology and the deep integration of interdisciplinary methods, this model has the potential to achieve accurate prediction of financial risks and proactive governance of market failures. In doing so, the dialogue between theory and practice, and between science and policy, will continue to deepen, promoting the development of a more resilient and inclusive modern financial system.

## Conflict of interest

The authors declare no conflict of interest.

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