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Research Article

Energy Storage System Selection by Using Complex Intuitionistic Fuzzy Rough MCDM Technique Based on Schweizer-Sklar Operators

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Abstract: Energy Storage System (ESS) is a talented solution to overcome the intermittency (that they do not produce energy all the time) and demand-supply misalliance problems in different renewable energy systems. Selecting the most optimal ESS requires the consideration of different conflicting criteria under uncertainty. This study presents a novel Multi-Criteria Decision-Making (MCDM) framework based on Complex Intuitionistic Fuzzy Rough Sets (CIFRSs) and Schweizer-Sklar aggregation operators to facilitate a more comprehensive and flexible ESS selection process. Specifically, we develop new aggregation operators namely, the Complex Intuitionistic Fuzzy Rough (CIFR) Schweizer-Sklar weighted average and the CIFR Schweizer-Sklar weighted geometric operators to model imprecise, vague, and inconsistent information. CIFR-MCDM methodology captures the intuitionistic, roughness and extra related fuzzy information in one structure. A case study is performed to illustrate the applicability of the suggested method in ranking different ESS alternatives. Comparative analysis with existing approaches confirms the robustness and effectiveness of the proposed framework in handling complex decision environments. The results highlight the potential of the CIFR-MCDM methodology to support informed and reliable ESS selection in renewable energy applications.

***Keywords*:** Energy Storage System (ESS), Complex Intuitionistic Fuzzy Rough-Multi-Criteria Decision-Making (CIFR-MCDM) technique, Schweizer-Sklar t-norm and t-conorm, complex intuitionistic fuzzy rough set

MSC: 03B52, 08A72, 03E72

1. Introduction

Energy Storage System (ESS) is a procedure for storing energy generated ones for use at a later time to reduce the mismatch between energy production and energy demand. ESS improves the dependability and effectiveness of energy systems by balancing supply and demand. Because of pollution, climate change, and the depletion of fossil fuels, the global energy environment is rapidly changing. The growing energy demand has directed to a preference change from fossil fuels to Renewable Energy (RE) sources. RE sources are playing a vital role in this expansion. The portion of electricity generated from renewables, including flywheel energy, compressed air energy, hydropower energy, wind energy, and solar energy, is projected to increase significantly, helping to reduce the carbon intensity of global power production.

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They are becoming more and more important as we change over to more RE sources, such as solar energy and wind energy. These sources of energy are intermittent (that they do not produce energy all the time). For instance, wind energy turbines only generate electricity when the wind is blowing and solar energy only generates electricity during the day. ESSs can help to address this challenge by storing extra energy produced during off-peak times and releasing it back to the grid when demand is high. Now a days, electricity is crucial for supporting human activities, particularly in urban areas. As of 2024, the global generation of electricity continues to grow, driven by increasing energy demands and a strong push towards RE sources. To combat pollution and climate change, this trend reflects a significant change from fossil fuels to cleaner energy options. Data taken from the International Energy Agency (IEA) indicates that electricity demand is growing rapidly, driven by increased electrification in various sectors and the adoption of heat pumps and electric vehicles. RE like solar energy and wind energy, are likely to account for a significant portion of this growth, contributing to the ongoing energy transition away from fossil fuels. Now, according to the IEA, the total global electricity generation in 2024 is expected to reach approximately 29.5 trillion kilowatt-hours, up and around from 23.7 trillion kilowatt-hours in 2014. This growth in energy reflects an average annual increase of around 2.5% over the past decade. Figure 1 shows the increasing rate of global electricity generation over the past decade.

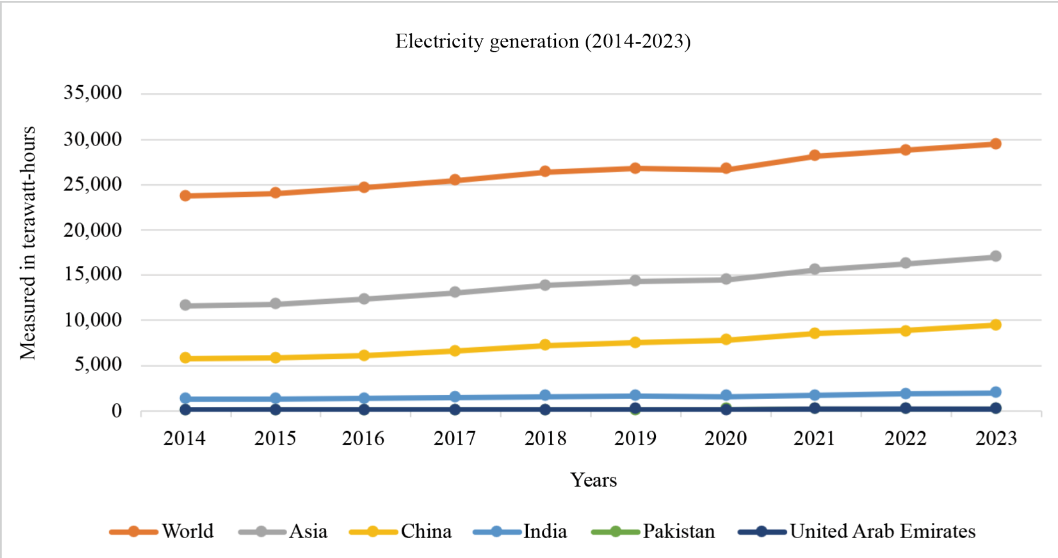

Figure 1. Electricity generation in a decade
Data source: IEA, electricity market report, 2024

Figure 1 describes the trend in global electricity generation over the last ten years. While the detailed data points would be valuable for a more precise analysis. The global and regional trend towards increased electricity generation is not only mirroring rising demand but also a sure sign of escalating pressure on the energy infrastructure. With electricity demand growing especially in the fast-emerging economies of China, India, Pakistan, and the UAE, the necessity for reliable, efficient, and adaptable energy systems grows ever more pressing. These trends further indicate the reliance on variable renewable resources, making strong ESS planning even more imperative.

Table 1 supplements Figure 1 by providing both absolute and relative numbers for the increase in electricity production from 2014 to 2023. These data highlight the regions with the most rapid growth, which are also most likely to be challenged by the most urgent energy balancing and storage needs. This supports the applied importance of creating advanced ESS selection tools, especially for countries undergoing rapid energy transitions.

Table 1. Absolute and relative changes in electricity generation (2014-2023)

Area	2014	2023	Absolute change	Relative change
World	23,749 TWh	29,479 TWh	5,729 TWh	24%
Asia	11,610 TWh	16,216 TWh	4,606 TWh	40%
China	9,459 TWh	13,124 TWh	3,665 TWh	63%
India	1,262 TWh	1,967 TWh	705 TWh	56%
Pakistan	112 TWh	165 TWh	53 TWh	49%
UAE	113 TWh	161 TWh	48 TWh	42%

The world's electricity generation has grown significantly by 24%, showing substantial development in energy production. In Asia, mostly India and China, plays a serious role in this growth, with a noteworthy increase in both absolute and relative terms. Countries like Pakistan and the UAE also show strong growth, demonstrating improvements in energy infrastructure and access. These changes reflect global trends toward increased energy production to meet rising demand, driven by economic growth, urbanization, and industrialization. The significant rise in electricity generation in Asia, particularly in China and India, underscores the region's central role in the global energy landscape. According to IEA predictions, by 2025, half of the world's electricity will come from Asia, with China consuming one-third of all electricity worldwide. Even while the use of clean energy has been increasing throughout Asia, coal still accounts for more than fifty percent of the electricity produced there. No Asian countries rely on RE sources like wind energy, solar energy, or nuclear energy as their primary source of electricity, despite the collective share of these sources doubled in the last decade. RE generation shows a distinct upward trend over the years. Starting from a lower base compared to fossil fuels, renewables increase steadily each year. By 2023, the RE generation will show significant growth, reflecting the global shift towards more sustainable energy sources. Figure 2 shows the trends in electricity generation from four main sources fossil fuels, hydropower, nuclear and renewables over the last decade. Each source is represented by lines in different colors indicating different years.

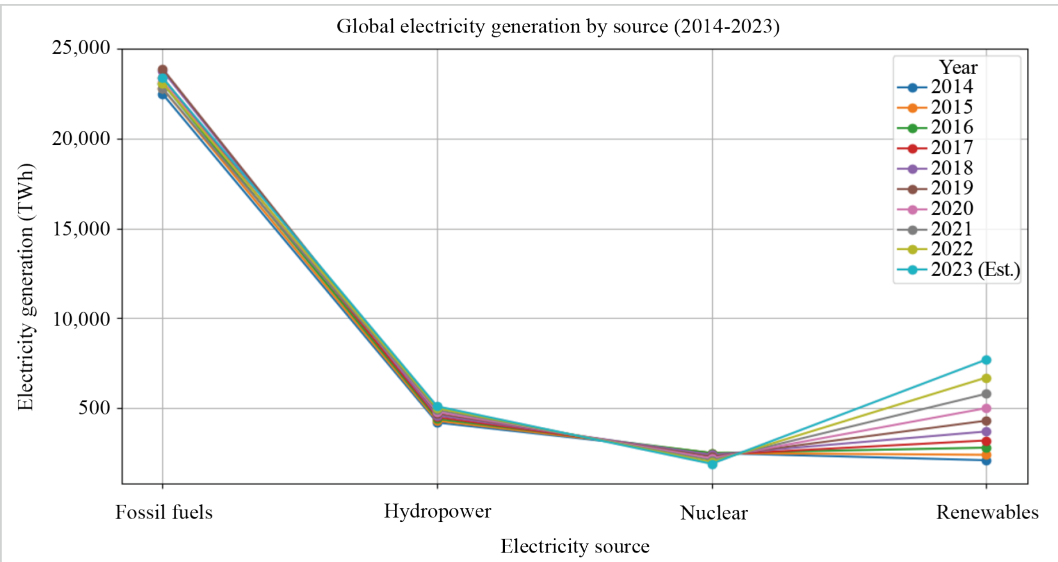

Figure 2. Electricity generation of main sources
Data source: IEA, electricity market report, 2024

Although renewable energy sources are expanding quickly, fossil fuels continue to be the dominant source of electricity. This reflects a global trend towards renewable energy and sustainability, driven by policy changes, technological advancements, and environmental considerations. Many researchers have extensively examined the topic of ESS. The hybrid Multi-Criteria Decision-Making (MCDM) technique is based on the type 2 Fuzzy Sets (FSs) for the selection of ESS and their alternatives given by Ozkan et al. [1]. Colak and Kaya [2] give the MCDM technique for the evaluation of ESS based on the information given in hesitant FS. The selection of renewable ESS by using dual hesitant Pythagorean FS by using the MCDM technique given by Liu and Du [3]. Zhang et al. [4] revised the Multi-Objective Optimization Ratio Analysis (MULTIMOORA) approach for Intuitionistic FSs (IFSs) and proposed a new technique for MCDM for the assessment of ESS technologies. Acar et al. [5] proposed the hesitant fuzzy MCDM technique for sustainable investigation of ESSs. Lu et al. [6] propose a fuzzy group MCDM method for the performance assessment of ESS. Aktas and Kabak [7] develop a group decision making model based on hesitant FS to determine appropriate ESS. Balasundar et al. [8] propose the interval type 2 logic controller-based power-sharing strategy to utilize ESSs in the solar power charging station. Colak and Kaya [9] proposed the sustainable assessment of ESS criteria through an integrated fuzzy based MCDM technique. Shahin et al. [10] introduced an innovative interval-valued circular intuitionistic fuzzy Measurement of Alternatives and Ranking according to Compromise Solution (MARCOS) method to tackle renewable energy source selection. Further, Sahoo et al. [11] emphasized the pivotal role of MCDM techniques in promoting sustainable renewable energy development.

1.1 *Study framework*

This framework is constructed as: in section 1, we address the basic introduction of ESS. In section 2, we have discussed the literature review. In section 3, we revise the basic definitions, operating rules, as well as the idea of Schweizer-Sklar (SS) operational laws for Complex Intuitionistic Fuzzy Rough Numbers (CIFRNs). In section 4, we delivered CIFR Schweizer-Sklar Weighted Average (CIFRSSWA) and CIFR Schweizer-Sklar Weighted Geometric (CIFRSSWG) operators based on CIFRNs and discussed their properties. In section 5, we deliver a CIFR-MCDM technique, a case-study for ESS selection and section 6 is about the comparative study. Section 7 is about the managerial implications of the proposed work. Finally, in section 8 we present the conclusion of this study, capturing the findings and implications.

1.2 *Motivation and objectives*

The transition toward renewable energy sources has intensified the demand for effective ESSs to ensure grid stability and reliability. Despite significant advancements in energy storage technologies and applications, selecting the appropriate ESS for specific applications remains challenging. Each technology has unique characteristics, making it suitable for certain applications but not others. The need for effective ESSs becomes critical because the demand for renewable energy sources increases. The selection of the most suitable ESS involves multiple criteria, such as cost, capacity, efficiency, lifespan, and environmental impact. Traditional decision-making methods may not adequately capture the complexities and interdependencies among these criteria. As we can see the applications domain of FSs, Fuzzy Rough Sets (FRSs), Intuitionistic Fuzzy Rough Sets (IFRSs) and Cubic Fuzzy Rough Sets (CFRSs) are bound with some conditions and restrictions. All these structures cannot handle the complex intuitionistic information in the form of Upper Approximation (UA) and Lower Approximation (LA). So, to overcome all the problems in the theories mentioned above, the notion of CIFRS is available. CIFRS is the best tool for modeling the information of Membership Grade (MGr) and Non-Membership Grade (N-MGr) with complex information in the form of UA and LA. Additionally, the use of SS t-norm and t-conorm that Schweizer and Sklar [12] formed are noteworthy because they deliver Decision-Making (DM) procedures more flexibility in how they handle uncertainty. SS operations are flexible for aggregating information and are useful for managing vague and awkward data. Considering it as an MCDM problem highlights its intricacy since it entails assessing vendors according to numerous attributes. All this motivates us to propose a CIFR-MCDM method for ESS selection, which comprehensively evaluates important criteria and produces optimal solutions without requiring extensive knowledge from decision-makers.

Numerous Aggregation Operators (AOs) exist in the literature, such as Frank AOs, Dombi AOs, Yager AOs and Schweizer-Sklar AOs, each offering distinct mathematical properties and applicability. The Dombi AOs is a flexible parametric t-norm, t-conorm that uses a parameter $\lambda > 0$ to adjust the sharpness of aggregation. Although considerably flexible, the Dombi AOs expression involves logarithmic and exponential components, making it computationally complex and less straightforward for MCDM purposes. The Frank AO, on the other hand, is another important parametric family that includes a wide variety of t-norms and t-conorms, allowing smooth transitions from logical conjunction to disjunction based on the parameter value. Although Frank AOs and Yager AOs have proven effective in general fuzzy environments, their application within the CIFRS framework remains unexplored. In distinction, the Schweizer-Sklar AOs is based on power means with a parameter ρ is more algebraically tractable, includes several well-known operators as special cases, and has been widely used in MCDM. Its continuity extension and ease of implementation make it particularly suitable for the use with CIFRSs. Therefore, this research applies Schweizer-Sklar operators to construct robust aggregation mechanisms within the CIFR-MCDM framework, enhancing the representation of uncertainty, approximation, and inter-criteria interaction in complex decision-making situations.

Table 2 provides a qualitative comparison between the proposed CIFR-MCDM method and several widely used fuzzy MCDM models. The comparison is based on five critical evaluation dimensions relevant to complex decision-making scenarios such as ESS selection.

Table 2. Comparison of different fuzzy MCDM methods for ESS selection

Methods	Uncertainty handling	Approximation modeling	Aggregation flexibility	Decision consistency	Suitability for complex criteria
FS	Low	No	Basic (e.g., average)	Adequate	Limited
IFS	Medium	No	Basic	Adequate	Adequate
FRS	Medium	Yes	Adequate	Adequate	Adequate
Complex CFS	High	No	Adequate	Adequate	Adequate
CIFRS	Very high	Yes (Upper/Lower)	High	High	Excellent

A. Novel Contributions

Based on this motivation and the requirements needed today. The main contribution of this script are as follows:

• Extension of SS t-norm and t-conorm operations to the CIFRS environment, offering a more adaptable and expressive aggregation process for complex fuzzy data.

• The development of new aggregation operators namely, i.e., the CIFRSSWA and the CIFRSSWG operators. These operators allow for the overall integration of information while allowing varying degrees of uncertainty and interactions between decision criteria.

• The proposed framework is validated through a case study on ESS selection. Comparative analysis demonstrates that the CIFR-MCDM approach provides more consistent, flexible, and accurate decision support than existing fuzzy MCDM techniques, particularly in handling high levels of uncertainty and imprecision.

• The proposed method reduces the cognitive burden on decision-makers by structuring complex evaluations into a systematic process, making it suitable for real-world deployment in energy planning and sustainability assessments.

2. Literature review

Given its capacity to manage ambiguity and uncertainty in criteria and assessments, Fuzzy Set (FS) theory has emerged as a potent tool for MCDM. There are many fuzzy MCDM approaches being developed and explored in research. Different generalized fuzzy structures have utilized the MCDM technique like Sahoo et al. [13] conducted a thorough review of MCDM applications in the energy management problems. Garg and Rani [14] present the application of the

MCDM process for complex intuitionistic FSs. Akram and Ahmad [15] developed the idea of the MCDM technique based on FRSs for the finest water supply strategy. Vojinovic et al. [16] proposed a framework of fuzzy rough MCDM technique for the evaluation of companies engaged in the transport of dangerous goods. Dordevic et al. [17] developed the idea of integrated linear programming fuzzy rough MCDM technique for production optimization. Chen et al. [18] developed the fuzzy rough MCDM technique for the selection of green suppliers in the furniture industry. Xu et al. [19] proposed the idea of evaluating RE sources for employing the hydrogen economy using the fuzzy MCDM technique. Bhowmik et al. [20] developed the concept of the MCDM technique for the evaluation of optimum green energy sources. Wen and Zhang [21] developed the application in the MCDM technique using overlap functions based on FRSs.

The modeling of imprecise and partial information is one of the main research topics in knowledge representation Numerous current methods are built upon extensions of various set theories, including crisp set theory, FS theory, and Rough Set (RS) theory. FS theory, traduced by Zadeh [22] provides a mathematical framework for dealing with uncertainty and gradual membership, making it a fundamental tool in the representation of imprecision. The concept of RS, introduced by Pawlak [23] offers a powerful framework for dealing with vagueness and uncertainty in the data analysis. The focus of rough set theory is on indiscernible items, or those that cannot be discriminated from one another using the information at hand. Knowledge granules are collections of related items based on prior knowledge, and they assist in identifying them. This is helpful in situations when there may be missing details or inadequate information. However, recent researches explore the potential of combining rough sets with other methodologies to address complex problems and improve their performance. It quickly prompted a valid question concerning possible connections between FSs and RSs. Both theories, in essence, address the problem of information granulation. However, while FS theory indicates fuzzy information granulation, RS theory concentrates on crisp information granulation. The core concept of RS theory was that differences in the values of the attributes that define an object's identity lead to its indiscernibility inside an information system. It is evident from this that there is now a significant increase in the similarity and strength of connection between these two ideas. A universal set known as the universe of discourse is defined by two sets denoted as upper and lower approximations in Pawlak's RS.

To address the limitations of RSs, researchers have explored combining them with other methods like Dubois and Prade [24] combined the notion of FS and RS and proposed the new concept of FRS. They combine these two concepts FS and RS and provide a more natural way to handle the ambiguity and uncertainties in the information system. But this notion FRS cannot deal with the intuitionistic data set in the form of upper and lower approximation. To overcome this situation, Cornelis et al. [25] proposed the notion that can handle MGr and N-MGr with roughness. This hybrid model intuitionistic FRS merges RSs and IFS to provide a more flexible framework that allows a more comprehensive assessment of uncertainty by utilizing its components. Its applications span various fields, including pattern recognition, decision support systems and medical diagnosis. Zhang [26] developed an algorithm based on a discernibility matrix to compute all the attributes reduction using intuitionistic FRS. Seikh and Mandal [27] introduced intuitionistic fuzzy Dombi AOs for handling uncertainty in Multi-Attribute Decision-Making (MADM). Their approach provides flexibility through a parameter λ to adjust the sharpness of aggregation. This enhances modeling capabilities under intuitionistic fuzzy environments. Also, Seikh and Mandal [28] proposed Q-rung orthopair fuzzy Frank AOs for MADM problems with unknown attribute weights. This study offers an effective solution for managing higher-order uncertainty using Q-rung orthopair FSs and the Frank operators. Yahya et al. [29] developed Intuitionistic Fuzzy Rough Evaluation based on Distance from Average Solution (IFR-EDAS) method for multi-criteria group decision-making and defined the frank aggregation operators based in Intuitionistic Fuzzy Rough Numbers (IFRNs). Hussain et al. [30] examine the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) approach based on IFR dombi weighted average and IFR dombi weighted geometric AOs. No doubt, FRS and intuitionistic FRS are very beneficial and powerful but they do have certain drawbacks because they do not handle the 2nd dimension data into a single set. To overcome this situation, Yi et al. [31] proposed a novel approach to complex FRS and gave an application in digital marketing. Emam et al. [32] developed artificial intelligence tools in the framework of complex FRS. By combining the ideas of CFS proposed by Tamir et al. [33] and IFS, Ali et al. [34] introduce the idea of CIFS that can effectively handle the information compared to IFS and CFS, allowing new possibilities for research and applications in various fields. CIFS can handle both the complex MGr and N-MGr in one structure, but it cannot handle the rough structure which consists of UA and LA. So, there is a chance of data loss in this structure. When we talk about the concepts of FRS, intuitionistic FRS and complex FRS, we can see that these structures can talk about the roughness in their structures and offer a more balanced approach for representing the information for both vague (fuzzy) and inconsistent or incomplete (rough). But there is no structure that can handle complex MGr and N-MGr with roughness which consists of UA and LA in one structure. To overcome this situation, Mahmood et al. [35] developed a new framework called CIFRS for tackling both 2nd dimensional uncertainties and indiscernibility.

3. Preliminaries

This section contains a basic review of some existing work linked to FSs, FRSs, IFRSs and CIFRSs. All over the article, Q is represented as a universal set.

Definition 1 [22] A FS $\mathbb{A}$ on a universal set Q is presented as

$$\mathbb{A} = \{(c^*, M_{\mathbb{A}}(c^*)) \mid c^* \in Q\},$$

where $M_{\mathbb{A}} : Q \rightarrow [0, 1]$ denotes the truth grade of every element $c^* \in Q$.

Definition 2 [24] For fuzzy approximation space $(Q, \mathbb{R}_r)$ and a FS $\mathbb{A}$ in Q. Then, the UA and LA of $\mathbb{A}$ w.r.t $(Q, \mathbb{R}_r)$ are defined as:

$$\overline{\overline{\mathbb{R}_r}}(\mathbb{A}) = \left\{\left(c^*, M_{\overline{\overline{\mathbb{R}_r}}}(c^*)\right) \mid c^* \in Q\right\},$$

$$\underline{\underline{\mathbb{R}_r}}(\mathbb{A}) = \left\{\left(c^*, M_{\underline{\underline{\mathbb{R}_r}}}(c^*)\right) \mid c^* \in Q\right\},$$

where,

$$M_{\overline{\overline{\mathbb{R}_r}}}(c^*) = \bigvee_{\dot{d}^* \in Q} \left[i(c^*, \dot{d}^*) \wedge m_{\mathbb{A}}(\dot{d}^*)\right],$$

$$M_{\underline{\underline{\mathbb{R}_r}}}(c^*) = \bigwedge_{\dot{d}^* \in Q} \left[(1 - i(c^*, \dot{d}^*)) \vee m_{\mathbb{A}}(\dot{d}^*)\right].$$

Then, the pair $\mathbb{R}_r(\mathbb{A}) = \left(\underline{\underline{\mathbb{R}_r}}(\mathbb{A}), \overline{\overline{\mathbb{R}_r}}(\mathbb{A})\right)$ is termed a FRS.

Definition 3 [36] For an Intuitionistic Fuzzy (IF) approximation space $(Q, \mathbb{R}_r)$ and a FS $\mathbb{A}$ in Q. Then we define the UA and LA of $\mathbb{A}$ w.r.t $(Q, \mathbb{R}_r)$ defined as:

$$\overline{\overline{\mathbb{R}_r}}(\mathbb{A}) = \left\{\left(c^*, M_{\overline{\overline{\mathbb{R}_r}}}(c^*), N_{\overline{\overline{\mathbb{R}_r}}}(c^*)\right) \mid c^* \in Q\right\},$$

$$\underline{\underline{\mathbb{R}_r}}(\mathbb{A}) = \left\{\left(c^*, M_{\underline{\underline{\mathbb{R}_r}}}(c^*), N_{\underline{\underline{\mathbb{R}_r}}}(c^*)\right) \mid c^* \in Q\right\},$$

where,

$$M_{\overline{\overline{\mathbb{R}_r}}}(c^*) = \bigvee_{\dot{d}^* \in Q} \left[i(c^*, \dot{d}^*) \vee m_{\mathbb{A}}(\dot{d}^*)\right],$$

$$M_{\underline{\underline{\mathbb{R}_r}}}(c^*) = \bigwedge_{\dot{d}^* \in Q} \left[i(c^*, \dot{d}^*) \wedge m_{\mathbb{A}}(\dot{d}^*)\right],$$

$$N_{\overline{\overline{\mathbb{R}_r}}}(c^*) = \bigwedge_{\dot{d}^* \in Q} \left[k(c^*, \dot{d}^*) \wedge o_{\mathbb{A}}(\dot{d}^*)\right],$$

$$N_{\underline{\underline{\mathbb{R}_r}}}(c^*) = \bigvee_{\dot{d}^* \in Q} \left[k(c^*, \dot{d}^*) \vee o_{\mathbb{A}}(\dot{d}^*)\right],$$

where, $0 \leq M_{\overline{\overline{\mathbb{R}_r}}}(c^*) + N_{\overline{\overline{\mathbb{R}_r}}}(c^*) \leq 1$, $0 \leq M_{\underline{\underline{\mathbb{R}_r}}}(c^*) + N_{\underline{\underline{\mathbb{R}_r}}}(c^*) \leq 1$. Then, the pair $\mathbb{R}_r(\mathbb{A}) = \left(\underline{\underline{\mathbb{R}_r}}(\mathbb{A}), \overline{\overline{\mathbb{R}_r}}(\mathbb{A})\right) = \left\{\left(c^*, \langle M_{\underline{\underline{\mathbb{R}_r}}}(c^*), N_{\underline{\underline{\mathbb{R}_r}}}(c^*)\rangle, \langle M_{\overline{\overline{\mathbb{R}_r}}}(c^*), N_{\overline{\overline{\mathbb{R}_r}}}(c^*)\rangle\right) \mid c^* \in Q\right\}$ is called the IFRS w.r.t $(Q, \mathbb{R}_r)$.

Definition 4 [35] Let $\mathbb{R}_r$ be a CIF relation over Q, and let the pair $(Q, \mathbb{R}_r)$ be an approximation space. Then, for a set $\mathbb{A} = \left\{\left(\dot{d}^*, m_{\mathbb{A}}(\dot{d}^*) + \iota\eta_{\mathbb{A}}(\dot{d}^*), o_{\mathbb{A}}(\dot{d}^*) + \iota p_{\mathbb{A}}(\dot{d}^*)\right) \mid \dot{d}^* \in Q\right\} \in \text{CIFS}(Q)$, we describe the UA and LA of $\mathbb{A}$ w.r.t $(Q, \mathbb{R}_r)$ are defined as:

$$\overline{\overline{\mathbb{R}_r}}(\mathbb{A}) = \left\{\left(c^*, M_{\overline{\overline{\mathbb{R}_r}}}(c^*), N_{\overline{\overline{\mathbb{R}_r}}}(c^*)\right) \mid c^* \in Q\right\},$$

$$\underline{\underline{\mathbb{R}_r}}(\mathbb{A}) = \left\{\left(c^*, M_{\underline{\underline{\mathbb{R}_r}}}(c^*), N_{\underline{\underline{\mathbb{R}_r}}}(c^*)\right) \mid c^* \in Q\right\},$$

where:

$$M_{\overline{\overline{\mathbb{R}_r}}}(c^*) = \bigvee_{\dot{d}^* \in Q} \left[i(c^*, \dot{d}^*) \vee m_{\mathbb{A}}(\dot{d}^*)\right] + \iota \bigvee_{\dot{d}^* \in Q} \left[j(c^*, \dot{d}^*) \vee \eta_{\mathbb{A}}(\dot{d}^*)\right] = m_{\overline{\overline{\mathbb{R}_r}}} + \iota\eta_{\overline{\overline{\mathbb{R}_r}}},$$

$$M_{\underline{\underline{\mathbb{R}_r}}}(c^*) = \bigwedge_{\dot{d}^* \in Q} \left[i(c^*, \dot{d}^*) \wedge m_{\mathbb{A}}(\dot{d}^*)\right] + \iota \bigwedge_{\dot{d}^* \in Q} \left[j(c^*, \dot{d}^*) \wedge \eta_{\mathbb{A}}(\dot{d}^*)\right] = m_{\underline{\underline{\mathbb{R}_r}}} + \iota\eta_{\underline{\underline{\mathbb{R}_r}}},$$

$$N_{\overline{\overline{\mathbb{R}_r}}}(c^*) = \bigwedge_{\dot{d}^* \in Q} \left[k(c^*, \dot{d}^*) \wedge o_{\mathbb{A}}(\dot{d}^*)\right] + \iota \bigwedge_{\dot{d}^* \in Q} \left[l(c^*, \dot{d}^*) \wedge p_{\mathbb{A}}(\dot{d}^*)\right] = o_{\overline{\overline{\mathbb{R}_r}}} + \iota p_{\overline{\overline{\mathbb{R}_r}}},$$

$$N_{\underline{\underline{\mathbb{R}_r}}}(c^*) = \bigvee_{\dot{d}^* \in Q} \left[k(c^*, \dot{d}^*) \vee o_{\mathbb{A}}(\dot{d}^*)\right] + \iota \bigvee_{\dot{d}^* \in Q} \left[l(c^*, \dot{d}^*) \vee p_{\mathbb{A}}(\dot{d}^*)\right] = o_{\underline{\underline{\mathbb{R}_r}}} + \iota p_{\underline{\underline{\mathbb{R}_r}}}.$$

where:

$$0 \leq m_{\overline{\overline{\mathbb{R}_r}}} + o_{\overline{\overline{\mathbb{R}_r}}} \leq 1, \quad 0 \leq m_{\underline{\underline{\mathbb{R}_r}}} + o_{\underline{\underline{\mathbb{R}_r}}} \leq 1,$$

$$0 \leq \eta_{\overline{\overline{\mathbb{R}_r}}} + p_{\overline{\overline{\mathbb{R}_r}}} \leq 1, \quad 0 \leq \eta_{\underline{\underline{\mathbb{R}_r}}} + p_{\underline{\underline{\mathbb{R}_r}}} \leq 1.$$

As $\overline{\overline{\mathbb{R}_r}}(\mathbb{A})$ and $\underline{\underline{\mathbb{R}_r}}(\mathbb{A})$ are CIFRSs. Then the pair

$$\mathbb{R}_r(\mathbb{A}) = \left(\underline{\underline{\mathbb{R}_r}}(\mathbb{A}), \overline{\overline{\mathbb{R}_r}}(\mathbb{A})\right) = \left\{\left(c^*, \langle M_{\underline{\underline{\mathbb{R}_r}}}(c^*), N_{\underline{\underline{\mathbb{R}_r}}}(c^*)\rangle, \langle M_{\overline{\overline{\mathbb{R}_r}}}(c^*), N_{\overline{\overline{\mathbb{R}_r}}}(c^*)\rangle\right) \mid c^* \in Q\right\}$$

is called the CIFRS w.r.t $(Q, \mathbb{R}_r)$. For simplicity, we denote the Complex Intuitionistic Fuzzy Rough Number (CIFRN) as, $\mathbb{R}_r(\mathbb{A}) = \left(\left(m_{\underline{\underline{\mathbb{R}_r}}} + \iota\eta_{\underline{\underline{\mathbb{R}_r}}}, o_{\underline{\underline{\mathbb{R}_r}}} + \iota p_{\underline{\underline{\mathbb{R}_r}}}\right), \left(m_{\overline{\overline{\mathbb{R}_r}}} + \iota\eta_{\overline{\overline{\mathbb{R}_r}}}, o_{\overline{\overline{\mathbb{R}_r}}} + \iota p_{\overline{\overline{\mathbb{R}_r}}}\right)\right)$.

Definition 5 [35] Let two CIFRNs be defined as:

$$\mathbb{R}_r(\mathbb{A}_1) = \left(\left(m_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_1) + \iota\eta_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_1),\ o_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_1) + \iota p_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_1)\right),\ \left(m_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_1) + \iota\eta_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_1),\ o_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_1) + \iota p_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_1)\right)\right),$$

$$\mathbb{R}_r(\mathbb{A}_2) = \left(\left(m_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_2) + \iota\eta_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_2),\ o_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_2) + \iota p_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_2)\right),\ \left(m_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_2) + \iota\eta_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_2),\ o_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_2) + \iota p_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_2)\right)\right).$$

Then:

1. Complement:

$$\mathbb{R}_r(\mathbb{A}_1)^c = \begin{pmatrix} \left(o_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_1) + \iota p_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_1),\ m_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_1) + \iota\eta_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_1)\right), \\ \left(o_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_1) + \iota p_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_1),\ m_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_1) + \iota\eta_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_1)\right) \end{pmatrix}$$

2. Union:

$$\mathbb{R}_r(\mathbb{A}_1) \cup \mathbb{R}_r(\mathbb{A}_2) = \begin{pmatrix} \left(\max\left[m_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_1), m_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_2)\right] + \iota\max\left[\eta_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_1), \eta_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_2)\right],\right. \\ \left.\min\left[o_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_1), o_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_2)\right] + \iota\min\left[p_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_1), p_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_2)\right]\right), \\ \left(\max\left[m_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_1), m_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_2)\right] + \iota\max\left[\eta_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_1), \eta_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_2)\right],\right. \\ \left.\min\left[o_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_1), o_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_2)\right] + \iota\min\left[p_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_1), p_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_2)\right]\right) \end{pmatrix}.$$

3. Intersection:

$$\mathbb{R}_r(\mathbb{A}_1) \cap \mathbb{R}_r(\mathbb{A}_2) = \begin{pmatrix} \Big(\min\Big[m_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_1),\ m_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_2)\Big] + \iota \min\Big[\eta_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_1),\ \eta_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_2)\Big], \\ \max\Big[o_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_1),\ o_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_2)\Big] + \iota \max\Big[p_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_1),\ p_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_2)\Big]\Big), \\ \Big(\min\Big[m_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_1),\ m_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_2)\Big] + \iota \min\Big[\eta_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_1),\ \eta_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_2)\Big], \\ \max\Big[o_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_1),\ o_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_2)\Big] + \iota \max\Big[p_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_1),\ p_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_2)\Big]\Big) \end{pmatrix}.$$

Definition 6 [35] The score function Sc_F of a CIFRN

$$\mathbb{R}_r(\mathbb{A}) = \left(\left(m_{\underline{\underline{\mathbb{R}_r}}} + \iota \eta_{\underline{\underline{\mathbb{R}_r}}},\ o_{\underline{\underline{\mathbb{R}_r}}} + \iota p_{\underline{\underline{\mathbb{R}_r}}}\right),\ \left(m_{\overline{\overline{\mathbb{R}_r}}} + \iota \eta_{\overline{\overline{\mathbb{R}_r}}},\ o_{\overline{\overline{\mathbb{R}_r}}} + \iota p_{\overline{\overline{\mathbb{R}_r}}}\right)\right)$$

is defined as

$$\mathrm{Sc}_F(\mathbb{R}_r(\mathbb{A})) = \frac{1}{8}\left(4 + m_{\overline{\overline{\mathbb{R}_r}}} + \eta_{\overline{\overline{\mathbb{R}_r}}} + m_{\underline{\underline{\mathbb{R}_r}}} + \eta_{\underline{\underline{\mathbb{R}_r}}} - o_{\overline{\overline{\mathbb{R}_r}}} - p_{\overline{\overline{\mathbb{R}_r}}} - o_{\underline{\underline{\mathbb{R}_r}}} - p_{\underline{\underline{\mathbb{R}_r}}}\right), \quad \mathrm{Sc}_F(\mathbb{R}_r(\mathbb{A})) \in [0,\ 1].$$

Definition 7 [35] The accuracy function Acc_F of a CIFRN

$$\mathbb{R}_r(\mathbb{A}) = \left(\left(m_{\underline{\underline{\mathbb{R}_r}}} + \iota \eta_{\underline{\underline{\mathbb{R}_r}}},\ o_{\underline{\underline{\mathbb{R}_r}}} + \iota p_{\underline{\underline{\mathbb{R}_r}}}\right),\ \left(m_{\overline{\overline{\mathbb{R}_r}}} + \iota \eta_{\overline{\overline{\mathbb{R}_r}}},\ o_{\overline{\overline{\mathbb{R}_r}}} + \iota p_{\overline{\overline{\mathbb{R}_r}}}\right)\right)$$

is defined as

$$\mathrm{Acc}_F(\mathbb{R}_r(\mathbb{A})) = \frac{1}{8}\left(m_{\overline{\overline{\mathbb{R}_r}}} + \eta_{\overline{\overline{\mathbb{R}_r}}} + m_{\underline{\underline{\mathbb{R}_r}}} + \eta_{\underline{\underline{\mathbb{R}_r}}} + o_{\overline{\overline{\mathbb{R}_r}}} + p_{\overline{\overline{\mathbb{R}_r}}} + o_{\underline{\underline{\mathbb{R}_r}}} + p_{\underline{\underline{\mathbb{R}_r}}}\right), \quad \mathrm{Acc}_F(\mathbb{R}_r(\mathbb{A})) \in [0,\ 1].$$

From a decision-making point of view, Sc_F measures the quality of a choice by balancing membership and non-membership information. It is an unequivocal measure of what is desired. It helps decision-makers rank alternatives according to how well each meets the criteria needed. The Acc_F, on the other hand, shows how reliable or confident that rating is, offering a view of certainty about information behind it. Both of them help rank preferences and ensure credible information in situations of decision-making uncertainty.

3.1 *Schweizer-Sklar operation*

In this section, we interpreted SS operations known as SS sum and product which are special cases of t-norms and t-conorms.

Definition 8 [12] Suppose r_1 and r_2 be two real numbers with $\varrho \geq 1$. Then, the SS (t-norm and t-conorm) are defined as:

$$r_1 \oplus_{\mathrm{SS}} r_2 = \left(r_1^{\varrho} + r_2^{\varrho} - 1\right)^{\frac{1}{\varrho}}$$

$$r_1 \otimes_{\mathrm{SS}} r_2 = 1 - ((1 - r_1)^{\varrho} + (1 - r_2)^{\varrho} - 1)^{\frac{1}{\varrho}}$$

3.2 *Operations of Schweizer-Sklar on CIFRNs*

In this sub-sequel, we develop the concept of SS operation on CIFRNs.

Definition 9 Let $\mathbb{R}_r(\mathbb{A}) = \left(m_{\underline{\underline{\mathbb{R}_r}}} + \iota\eta_{\underline{\underline{\mathbb{R}_r}}},\ o_{\underline{\underline{\mathbb{R}_r}}} + \iota p_{\underline{\underline{\mathbb{R}_r}}},\ m_{\overline{\overline{\mathbb{R}_r}}} + \iota\eta_{\overline{\overline{\mathbb{R}_r}}},\ o_{\overline{\overline{\mathbb{R}_r}}} + \iota p_{\overline{\overline{\mathbb{R}_r}}}\right)$ and $\mathbb{R}_r(\mathbb{A}') = \left(m'_{\underline{\underline{\mathbb{R}_r}}} + \iota\eta'_{\underline{\underline{\mathbb{R}_r}}},\ o'_{\underline{\underline{\mathbb{R}_r}}} + \iota p'_{\underline{\underline{\mathbb{R}_r}}},\ m'_{\overline{\overline{\mathbb{R}_r}}} + \iota\eta'_{\overline{\overline{\mathbb{R}_r}}},\ o'_{\overline{\overline{\mathbb{R}_r}}} + \iota p'_{\overline{\overline{\mathbb{R}_r}}}\right)$ be two CIFRNs. Then:

$$\mathbb{R}_r(\mathbb{A}_1) \oplus \mathbb{R}_r(\mathbb{A}_2) = \begin{pmatrix} (m_{\underline{\underline{\mathbb{R}_r}}} \oplus_{\mathrm{SS}} m'_{\underline{\underline{\mathbb{R}_r}}}) + \iota(\eta_{\underline{\underline{\mathbb{R}_r}}} \oplus_{\mathrm{SS}} \eta'_{\underline{\underline{\mathbb{R}_r}}}), \\ (o_{\underline{\underline{\mathbb{R}_r}}} \oplus_{\mathrm{SS}} o'_{\underline{\underline{\mathbb{R}_r}}}) + \iota(p_{\underline{\underline{\mathbb{R}_r}}} \oplus_{\mathrm{SS}} p'_{\underline{\underline{\mathbb{R}_r}}}); \\ (m_{\overline{\overline{\mathbb{R}_r}}} \oplus_{\mathrm{SS}} m'_{\overline{\overline{\mathbb{R}_r}}}) + \iota(\eta_{\overline{\overline{\mathbb{R}_r}}} \oplus_{\mathrm{SS}} \eta'_{\overline{\overline{\mathbb{R}_r}}}), \\ (o_{\overline{\overline{\mathbb{R}_r}}} \oplus_{\mathrm{SS}} o'_{\overline{\overline{\mathbb{R}_r}}}) + \iota(p_{\overline{\overline{\mathbb{R}_r}}} \oplus_{\mathrm{SS}} p'_{\overline{\overline{\mathbb{R}_r}}}) \end{pmatrix}$$

$$\mathbb{R}_r(\mathbb{A}_1) \otimes \mathbb{R}_r(\mathbb{A}_2) = \begin{pmatrix} (m_{\underline{\underline{\mathbb{R}_r}}} \otimes_{\mathrm{SS}} m'_{\underline{\underline{\mathbb{R}_r}}}) + \iota(\eta_{\underline{\underline{\mathbb{R}_r}}} \otimes_{\mathrm{SS}} \eta'_{\underline{\underline{\mathbb{R}_r}}}), \\ (o_{\underline{\underline{\mathbb{R}_r}}} \otimes_{\mathrm{SS}} o'_{\underline{\underline{\mathbb{R}_r}}}) + \iota(p_{\underline{\underline{\mathbb{R}_r}}} \otimes_{\mathrm{SS}} p'_{\underline{\underline{\mathbb{R}_r}}}); \\ (m_{\overline{\overline{\mathbb{R}_r}}} \otimes_{\mathrm{SS}} m'_{\overline{\overline{\mathbb{R}_r}}}) + \iota(\eta_{\overline{\overline{\mathbb{R}_r}}} \otimes_{\mathrm{SS}} \eta'_{\overline{\overline{\mathbb{R}_r}}}), \\ (o_{\overline{\overline{\mathbb{R}_r}}} \otimes_{\mathrm{SS}} o'_{\overline{\overline{\mathbb{R}_r}}}) + \iota(p_{\overline{\overline{\mathbb{R}_r}}} \otimes_{\mathrm{SS}} p'_{\overline{\overline{\mathbb{R}_r}}}) \end{pmatrix}$$

$$\lambda\mathbb{R}_r(\mathbb{A}_1) = \begin{pmatrix} \oplus_{\mathrm{SS}}(m_{\underline{\underline{\mathbb{R}_r}}}) + \iota \oplus_{\mathrm{SS}} (\eta_{\underline{\underline{\mathbb{R}_r}}}), \\ \otimes_{\mathrm{SS}}(o_{\underline{\underline{\mathbb{R}_r}}}) + \iota \otimes_{\mathrm{SS}} (p_{\underline{\underline{\mathbb{R}_r}}}); \\ \oplus_{\mathrm{SS}}(m_{\overline{\overline{\mathbb{R}_r}}}) + \iota \oplus_{\mathrm{SS}} (\eta_{\overline{\overline{\mathbb{R}_r}}}), \\ \otimes_{\mathrm{SS}}(o_{\overline{\overline{\mathbb{R}_r}}}) + \iota \otimes_{\mathrm{SS}} (p_{\overline{\overline{\mathbb{R}_r}}}) \end{pmatrix}$$

$$(\mathbb{R}_r(\mathbb{A}))^{\lambda} = \begin{pmatrix} \otimes ss(m_{\underline{\underline{\mathbb{R}_r}}}) + \iota \otimes ss\,(\eta_{\underline{\underline{\mathbb{R}_r}}}), \\ \oplus ss(o_{\underline{\underline{\mathbb{R}_r}}}) + \iota \oplus ss\,(p_{\underline{\underline{\mathbb{R}_r}}}); \\ \otimes ss(m_{\overline{\overline{\mathbb{R}_r}}}) + \iota \otimes ss\,(\eta_{\overline{\overline{\mathbb{R}_r}}}), \\ \oplus ss(o_{\overline{\overline{\mathbb{R}_r}}}) + \iota \oplus ss\,(p_{\overline{\overline{\mathbb{R}_r}}}) \end{pmatrix}$$

4. Complex intuitionistic fuzzy rough Schweizer-Sklar average/geometric AOs

In this sequel, we develop the idea of CIFR SS Weighted Averaging (CIFRSSWA), CIFR SS Weighted Geometric (CIFRSSWG) operators and discussed their properties.

Definition 10 Let

$$\mathbb{R}_r\left(\mathbb{A}_J\right) = \left(\underline{\underline{\mathbb{R}_r}}\left(\mathbb{A}_J\right), \overline{\overline{\mathbb{R}_r}}\left(\mathbb{A}_J\right)\right) \quad (J = 1, 2, 3, \ldots, s)$$

be a gathering of CIFRNs defined over Q. Let $\mathbf{w} = (w_1, w_2, w_3, \ldots, w_s)^{\top}$ be the Weight Vector (WV) with $w_J \in [0, 1]$ such that $\sum_{J=1}^{s} w_J = 1$, then a CIFRSSWA operator is defined as

$$\mathrm{CIFRSSWA}(\mathbb{R}_r(\mathbb{A}_1), \mathbb{R}_r(\mathbb{A}_2), \ldots, \mathbb{R}_r(\mathbb{A}_s))$$

$$= \left(\bigoplus_{J=1}^{s} w_J \cdot \underline{\underline{\mathbb{R}_r}}(\mathbb{A}_J), \ \bigoplus_{J=1}^{s} w_J \cdot \overline{\overline{\mathbb{R}_r}}(\mathbb{A}_J)\right)$$

$$= \left(w_1 \cdot \underline{\underline{\mathbb{R}_r}}(\mathbb{A}_1) \oplus w_2 \cdot \underline{\underline{\mathbb{R}_r}}(\mathbb{A}_2) \oplus \cdots \oplus w_s \cdot \underline{\underline{\mathbb{R}_r}}(\mathbb{A}_s), \ w_1 \cdot \overline{\overline{\mathbb{R}_r}}(\mathbb{A}_1) \oplus w_2 \cdot \overline{\overline{\mathbb{R}_r}}(\mathbb{A}_2) \oplus \cdots \oplus w_s \cdot \overline{\overline{\mathbb{R}_r}}(\mathbb{A}_s)\right)$$

Following are the results for the CIFRSSWA operator.

Theorem 1 Using the equation above, we get the CIFRNs and

$$\mathrm{CIFRSSWA}(\mathbb{R}_r(\mathbb{A}_1), \mathbb{R}_r(\mathbb{A}_2), \ldots, \mathbb{R}_r(\mathbb{A}_s))$$

$$= \left(\bigoplus_{J=1}^{s} w_J \cdot \underline{\underline{\mathbb{R}_r}}(\mathbb{A}_J), \ \bigoplus_{J=1}^{s} w_J \cdot \overline{\overline{\mathbb{R}_r}}(\mathbb{A}_J)\right)$$

$$= \left(\begin{pmatrix} 1-\left\{\sum_{\jmath=1}^{s} w_\jmath \left(1-m_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_\jmath)\right)^\rho\right\}^{1/\rho} + \iota\left(1-\left\{\sum_{\jmath=1}^{s} w_\jmath \left(1-\eta_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_\jmath)\right)^\rho\right\}^{1/\rho}\right), \\ \left\{\sum_{\jmath=1}^{s} w_\jmath \left(o_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_\jmath)\right)^\rho\right\}^{1/\rho} + \iota\left\{\sum_{\jmath=1}^{s} w_\jmath \left(p_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_\jmath)\right)^\rho\right\}^{1/\rho} \end{pmatrix}, \begin{pmatrix} 1-\left\{\sum_{\jmath=1}^{s} w_\jmath \left(1-m_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_\jmath)\right)^\rho\right\}^{1/\rho} + \iota\left(1-\left\{\sum_{\jmath=1}^{s} w_\jmath \left(1-\eta_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_\jmath)\right)^\rho\right\}^{1/\rho}\right), \\ \left\{\sum_{\jmath=1}^{s} w_\jmath \left(o_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_\jmath)\right)^\rho\right\}^{1/\rho} + \iota\left\{\sum_{\jmath=1}^{s} w_\jmath \left(p_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_\jmath)\right)^\rho\right\}^{1/\rho} \end{pmatrix} \right)$$

Proof. By using the Mathematical Induction (MI) method, we demonstrate the above equation.
When $s = 1$, then

$$\mathbb{R}_r(\mathbb{A}_1) = \left(\underline{\underline{\mathbb{R}_r}}(\mathbb{A}_1), \overline{\overline{\mathbb{R}_r}}(\mathbb{A}_1)\right) = \left(\begin{pmatrix} 1-\left\{w_1 \left(1-m_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_1)\right)^\rho\right\}^{1/\rho} + \iota\left(1-\left\{w_1 \left(1-\eta_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_1)\right)^\rho\right\}^{1/\rho}\right), \\ \left\{w_1 \left(o_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_1)\right)^\rho\right\}^{1/\rho} + \iota\left\{w_1 \left(p_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_1)\right)^\rho\right\}^{1/\rho} \end{pmatrix}, \begin{pmatrix} 1-\left\{w_1 \left(1-m_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_1)\right)^\rho\right\}^{1/\rho} + \iota\left(1-\left\{w_1 \left(1-\eta_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_1)\right)^\rho\right\}^{1/\rho}\right), \\ \left\{w_1 \left(o_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_1)\right)^\rho\right\}^{1/\rho} + \iota\left\{w_1 \left(p_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_1)\right)^\rho\right\}^{1/\rho} \end{pmatrix} \right)$$

Hence, for $s = 1$, the equation holds true. Now, for $s = 2$:

$$\begin{aligned} &\mathrm{CIFRSSWA}(\mathbb{R}_r(\mathbb{A}_1), \mathbb{R}_r(\mathbb{A}_2)) \\ &= \left(\bigoplus_{\jmath=1}^{2} w_\jmath \underline{\underline{\mathbb{R}_r}}(\mathbb{A}_\jmath), \bigoplus_{\jmath=1}^{2} w_\jmath \overline{\overline{\mathbb{R}_r}}(\mathbb{A}_\jmath)\right) \\ &= \left(w_1 \underline{\underline{\mathbb{R}_r}}(\mathbb{A}_1) \oplus w_2 \underline{\underline{\mathbb{R}_r}}(\mathbb{A}_2), w_1 \overline{\overline{\mathbb{R}_r}}(\mathbb{A}_1) \oplus w_2 \overline{\overline{\mathbb{R}_r}}(\mathbb{A}_2)\right) \end{aligned}$$

$$
= \left(\begin{array}{l}
\left(\begin{array}{l}
1 - \left\{ w_1 (1 - m_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_1))^{\rho} + w_2 (1 - m_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_2))^{\rho} \right\}^{1/\rho} \\
+ \iota \left(1 - \left\{ w_1 (1 - \eta_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_1))^{\rho} + w_2 (1 - \eta_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_2))^{\rho} \right\}^{1/\rho} \right), \\
\left\{ w_1 (o_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_1))^{\rho} + w_2 (o_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_2))^{\rho} \right\}^{1/\rho} \\
+ \iota \left\{ w_1 (p_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_1))^{\rho} + w_2 (p_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_2))^{\rho} \right\}^{1/\rho}
\end{array} \right), \\
\left(\begin{array}{l}
1 - \left\{ w_1 (1 - m_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_1))^{\rho} + w_2 (1 - m_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_2))^{\rho} \right\}^{1/\rho} \\
+ \iota \left(1 - \left\{ w_1 (1 - \eta_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_1))^{\rho} + w_2 (1 - \eta_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_2))^{\rho} \right\}^{1/\rho} \right), \\
\left\{ w_1 (o_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_1))^{\rho} + w_2 (o_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_2))^{\rho} \right\}^{1/\rho} \\
+ \iota \left\{ w_1 (p_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_1))^{\rho} + w_2 (p_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_2))^{\rho} \right\}^{1/\rho}
\end{array} \right)
\end{array} \right)
$$

$$
= \left(\begin{array}{l}
\left(\begin{array}{l}
1 - \left\{ \sum_{\jmath=1}^{2} w_{\jmath} \left(1 - m_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_{\jmath}) \right)^{\rho} \right\}^{1/\rho} \\
+ \iota \left(1 - \left\{ \sum_{\jmath=1}^{2} w_{\jmath} \left(1 - \eta_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_{\jmath}) \right)^{\rho} \right\}^{1/\rho} \right), \\
\left\{ \sum_{\jmath=1}^{2} w_{\jmath} \left(o_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_{\jmath}) \right)^{\rho} \right\}^{1/\rho} + \iota \left\{ \sum_{\jmath=1}^{2} w_{\jmath} \left(p_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_{\jmath}) \right)^{\rho} \right\}^{1/\rho}
\end{array} \right), \\
\left(\begin{array}{l}
1 - \left\{ \sum_{\jmath=1}^{2} w_{\jmath} \left(1 - m_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_{\jmath}) \right)^{\rho} \right\}^{1/\rho} \\
+ \iota \left(1 - \left\{ \sum_{\jmath=1}^{2} w_{\jmath} \left(1 - \eta_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_{\jmath}) \right)^{\rho} \right\}^{1/\rho} \right), \\
\left\{ \sum_{\jmath=1}^{2} w_{\jmath} \left(o_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_{\jmath}) \right)^{\rho} \right\}^{1/\rho} + \iota \left\{ \sum_{\jmath=1}^{2} w_{\jmath} \left(p_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_{\jmath}) \right)^{\rho} \right\}^{1/\rho}
\end{array} \right)
\end{array} \right)
$$

Hence, for $s = 2$, the expression holds. Now, for $s = k$.

$$
\mathrm{CIFRSSWA}(\mathbb{R}_r(\mathbb{A}_1), \mathbb{R}_r(\mathbb{A}_2), \ldots, \mathbb{R}_r(\mathbb{A}_k))
$$

$$
= \left(\bigoplus_{\jmath=1}^{k} w_{\jmath} \cdot \underline{\underline{\mathbb{R}_r}}(\mathbb{A}_{\jmath}),\ \bigoplus_{\jmath=1}^{k} w_{\jmath} \cdot \overline{\overline{\mathbb{R}_r}}(\mathbb{A}_{\jmath}) \right)
$$

$$
= \left(\begin{pmatrix} 1-\left\{\sum_{\jmath=1}^{k} w_\jmath \left(1-m_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_\jmath)\right)^{\rho}\right\}^{1/\rho} \\ +\iota\left(1-\left\{\sum_{\jmath=1}^{k} w_\jmath \left(1-\eta_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_\jmath)\right)^{\rho}\right\}^{1/\rho}\right), \\ \left\{\sum_{\jmath=1}^{k} w_\jmath \left(o_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_\jmath)\right)^{\rho}\right\}^{1/\rho} + \iota \left\{\sum_{\jmath=1}^{k} w_\jmath \left(p_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_\jmath)\right)^{\rho}\right\}^{1/\rho} \end{pmatrix}, \begin{pmatrix} 1-\left\{\sum_{\jmath=1}^{k} w_\jmath \left(1-m_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_\jmath)\right)^{\rho}\right\}^{1/\rho} \\ +\iota\left(1-\left\{\sum_{\jmath=1}^{k} w_\jmath \left(1-\eta_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_\jmath)\right)^{\rho}\right\}^{1/\rho}\right), \\ \left\{\sum_{\jmath=1}^{k} w_\jmath \left(o_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_\jmath)\right)^{\rho}\right\}^{1/\rho} + \iota \left\{\sum_{\jmath=1}^{k} w_\jmath \left(p_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_\jmath)\right)^{\rho}\right\}^{1/\rho} \end{pmatrix} \right)
$$

Now, for $s = k+1$, we have

$$
\begin{aligned}
&\text{CIFRSSWA}(\mathbb{R}_r(\mathbb{A}_1), \mathbb{R}_r(\mathbb{A}_2), \ldots, \mathbb{R}_r(\mathbb{A}_{k+1})) \\
&= \left(\bigoplus_{\jmath=1}^{k+1} w_\jmath \cdot \underline{\underline{\mathbb{R}_r}}(\mathbb{A}_\jmath),\ \bigoplus_{\jmath=1}^{k+1} w_\jmath \cdot \overline{\overline{\mathbb{R}_r}}(\mathbb{A}_\jmath) \right)
\end{aligned}
$$

$$
= \left(\begin{pmatrix} 1-\left\{\sum_{\jmath=1}^{k+1} w_\jmath \left(1-m_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_\jmath)\right)^{\rho}\right\}^{1/\rho} + \iota\left(1-\left\{\sum_{\jmath=1}^{k+1} w_\jmath \left(1-\eta_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_\jmath)\right)^{\rho}\right\}^{1/\rho}\right), \\ \left\{\sum_{\jmath=1}^{k+1} w_\jmath \left(o_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_\jmath)\right)^{\rho}\right\}^{1/\rho} + \iota \left\{\sum_{\jmath=1}^{k+1} w_\jmath \left(p_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_\jmath)\right)^{\rho}\right\}^{1/\rho} \end{pmatrix}, \begin{pmatrix} 1-\left\{\sum_{\jmath=1}^{k+1} w_\jmath \left(1-m_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_\jmath)\right)^{\rho}\right\}^{1/\rho} + \iota\left(1-\left\{\sum_{\jmath=1}^{k+1} w_\jmath \left(1-\eta_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_\jmath)\right)^{\rho}\right\}^{1/\rho}\right), \\ \left\{\sum_{\jmath=1}^{k+1} w_\jmath \left(o_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_\jmath)\right)^{\rho}\right\}^{1/\rho} + \iota \left\{\sum_{\jmath=1}^{k+1} w_\jmath \left(p_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_\jmath)\right)^{\rho}\right\}^{1/\rho} \end{pmatrix} \right)
$$

Hence, it also holds for $s = k+1$. Thus, by MI, the result holds $\forall\ s \geq 0$. Therefore, from the above theorem, $\underline{\underline{\mathbb{R}_r}}(\mathbb{A}_t)$ and $\overline{\overline{\mathbb{R}_r}}(\mathbb{A}_t)$ are CIFRNs. So, by the definitions, $\bigoplus_{t=1}^{s} w_t \cdot \underline{\underline{\mathbb{R}_r}}(\mathbb{A}_t)$ and $\bigoplus_{t=1}^{s} w_t \cdot \overline{\overline{\mathbb{R}_r}}(\mathbb{A}_t)$ are also CIFRNs. Hence, CIFRSSWA is also a CIFRN.

Properties of associativity and commutativity: For three CIFRNs,

$$
\mathbb{R}_r(\mathbb{A}) = \left(m_{\underline{\underline{\mathbb{R}_r}}} + \iota \eta_{\underline{\underline{\mathbb{R}_r}}},\ o_{\underline{\underline{\mathbb{R}_r}}} + \iota p_{\underline{\underline{\mathbb{R}_r}}},\ m_{\overline{\overline{\mathbb{R}_r}}} + \iota \eta_{\overline{\overline{\mathbb{R}_r}}},\ o_{\overline{\overline{\mathbb{R}_r}}} + \iota p_{\overline{\overline{\mathbb{R}_r}}} \right),
$$

$$\mathbb{R}_r(\mathbb{A}') = \left(m'_{\underline{\underline{\mathbb{R}_r}}} + \iota\eta'_{\underline{\underline{\mathbb{R}_r}}},\ o'_{\underline{\underline{\mathbb{R}_r}}} + \iota p'_{\underline{\underline{\mathbb{R}_r}}},\ m'_{\overline{\overline{\mathbb{R}_r}}} + \iota\eta'_{\overline{\overline{\mathbb{R}_r}}},\ o'_{\overline{\overline{\mathbb{R}_r}}} + \iota p'_{\overline{\overline{\mathbb{R}_r}}}\right),$$

$$\mathbb{R}_r(\mathbb{A}'') = \left(m''_{\underline{\underline{\mathbb{R}_r}}} + \iota\eta''_{\underline{\underline{\mathbb{R}_r}}},\ o''_{\underline{\underline{\mathbb{R}_r}}} + \iota p''_{\underline{\underline{\mathbb{R}_r}}},\ m''_{\overline{\overline{\mathbb{R}_r}}} + \iota\eta''_{\overline{\overline{\mathbb{R}_r}}},\ o''_{\overline{\overline{\mathbb{R}_r}}} + \iota p''_{\overline{\overline{\mathbb{R}_r}}}\right),$$

we get

(1) $\mathbb{R}_r(\mathbb{A}) \oplus_{SS} \mathbb{R}_r(\mathbb{A}') = \mathbb{R}_r(\mathbb{A}') \oplus_{SS} \mathbb{R}_r(\mathbb{A})$,

(2) $(\mathbb{R}_r(\mathbb{A}) \oplus_{SS} \mathbb{R}_r(\mathbb{A}')) \oplus_{SS} \mathbb{R}_r(\mathbb{A}'') = \mathbb{R}_r(\mathbb{A}') \oplus_{SS} (\mathbb{R}_r(\mathbb{A}) \oplus_{SS} \mathbb{R}_r(\mathbb{A}''))$.

Proof. (1) Let

$$\mathbb{R}_r(\mathbb{A}) = \left(m_{\underline{\underline{\mathbb{R}_r}}} + \iota\eta_{\underline{\underline{\mathbb{R}_r}}}, o_{\underline{\underline{\mathbb{R}_r}}} + \iota p_{\underline{\underline{\mathbb{R}_r}}}, m_{\overline{\overline{\mathbb{R}_r}}} + \iota\eta_{\overline{\overline{\mathbb{R}_r}}}, o_{\overline{\overline{\mathbb{R}_r}}} + \iota p_{\overline{\overline{\mathbb{R}_r}}}\right),$$

and

$$\mathbb{R}_r(\mathbb{A}') = \left(m'_{\underline{\underline{\mathbb{R}_r}}} + \iota\eta'_{\underline{\underline{\mathbb{R}_r}}}, o'_{\underline{\underline{\mathbb{R}_r}}} + \iota p'_{\underline{\underline{\mathbb{R}_r}}}, m'_{\overline{\overline{\mathbb{R}_r}}} + \iota\eta'_{\overline{\overline{\mathbb{R}_r}}}, o'_{\overline{\overline{\mathbb{R}_r}}} + \iota p'_{\overline{\overline{\mathbb{R}_r}}}\right),$$

then

$$\mathbb{R}_r(\mathbb{A}) \oplus_{SS} \mathbb{R}_r(\mathbb{A}') = \begin{pmatrix} 1-\left((1-m_{\underline{\underline{\mathbb{R}_r}}})^\rho + (1-m'_{\underline{\underline{\mathbb{R}_r}}})^\rho - 1\right)^{1/\rho} + \iota\left[1-\left((1-\eta_{\underline{\underline{\mathbb{R}_r}}})^\rho + (1-\eta'_{\underline{\underline{\mathbb{R}_r}}})^\rho - 1\right)^{1/\rho}\right], \\ \left((o_{\underline{\underline{\mathbb{R}_r}}})^\rho + (o'_{\underline{\underline{\mathbb{R}_r}}})^\rho - 1\right)^{1/\rho} + \iota\left((p_{\underline{\underline{\mathbb{R}_r}}})^\rho + (p'_{\underline{\underline{\mathbb{R}_r}}})^\rho - 1\right)^{1/\rho}, \\ 1-\left((1-m_{\overline{\overline{\mathbb{R}_r}}})^\rho + (1-m'_{\overline{\overline{\mathbb{R}_r}}})^\rho - 1\right)^{1/\rho} + \iota\left[1-\left((1-\eta_{\overline{\overline{\mathbb{R}_r}}})^\rho + (1-\eta'_{\overline{\overline{\mathbb{R}_r}}})^\rho - 1\right)^{1/\rho}\right], \\ \left((o_{\overline{\overline{\mathbb{R}_r}}})^\rho + (o'_{\overline{\overline{\mathbb{R}_r}}})^\rho - 1\right)^{1/\rho} + \iota\left((p_{\overline{\overline{\mathbb{R}_r}}})^\rho + (p'_{\overline{\overline{\mathbb{R}_r}}})^\rho - 1\right)^{1/\rho} \end{pmatrix}$$

$$= \begin{pmatrix} 1-\left((1-m'_{\underline{\underline{\mathbb{R}_r}}})^\rho + (1-m_{\underline{\underline{\mathbb{R}_r}}})^\rho - 1\right)^{1/\rho} + \iota\left[1-\left((1-\eta'_{\underline{\underline{\mathbb{R}_r}}})^\rho + (1-\eta_{\underline{\underline{\mathbb{R}_r}}})^\rho - 1\right)^{1/\rho}\right], \\ \left((o'_{\underline{\underline{\mathbb{R}_r}}})^\rho + (o_{\underline{\underline{\mathbb{R}_r}}})^\rho - 1\right)^{1/\rho} + \iota\left((p'_{\underline{\underline{\mathbb{R}_r}}})^\rho + (p_{\underline{\underline{\mathbb{R}_r}}})^\rho - 1\right)^{1/\rho}, \\ 1-\left((1-m'_{\overline{\overline{\mathbb{R}_r}}})^\rho + (1-m_{\overline{\overline{\mathbb{R}_r}}})^\rho - 1\right)^{1/\rho} + \iota\left[1-\left((1-\eta'_{\overline{\overline{\mathbb{R}_r}}})^\rho + (1-\eta_{\overline{\overline{\mathbb{R}_r}}})^\rho - 1\right)^{1/\rho}\right], \\ \left((o'_{\overline{\overline{\mathbb{R}_r}}})^\rho + (o_{\overline{\overline{\mathbb{R}_r}}})^\rho - 1\right)^{1/\rho} + \iota\left((p'_{\overline{\overline{\mathbb{R}_r}}})^\rho + (p_{\overline{\overline{\mathbb{R}_r}}})^\rho - 1\right)^{1/\rho} \end{pmatrix}$$

$$= \mathbb{R}_r(\mathbb{A}') \oplus_{SS} \mathbb{R}_r(\mathbb{A})$$

So, $\mathbb{R}_r(\mathbb{A}) \oplus_{SS} \mathbb{R}_r(\mathbb{A}') = \mathbb{R}_r(\mathbb{A}') \oplus_{SS} \mathbb{R}_r(\mathbb{A})$. It is commutative.

Proof. (2) It is obvious.

These algebraic properties ensure that the aggregation of CIFRNs is order-independent and grouping-invariant, which are essential features for consistent MCDM.

Remarks CIFRSSWA also meets the requirements for boundedness, monotonicity and idempotency.

Definition 11 Suppose $\mathbb{R}_r(\mathbb{A}_\jmath) = \left(\underline{\underline{\mathbb{R}_r}}(\mathbb{A}_\jmath), \overline{\overline{\mathbb{R}_r}}(\mathbb{A}_\jmath)\right)$, $\jmath = 1, 2, \ldots, s$, be a gathering of CIFRNs, and $w = (w_1, w_2, \ldots, w_s)^\top$ be the WV with $w_\jmath \in [0, 1]$ such that $\sum_{\jmath=1}^{s} w_\jmath = 1$, then a CIFRSSWG operator is defined as

$$\mathrm{CIFRSSWG}\left(\mathbb{R}_r(\mathbb{A}_1), \mathbb{R}_r(\mathbb{A}_2), \ldots, \mathbb{R}_r(\mathbb{A}_s)\right)$$

$$= \left(\bigotimes_{\jmath=1}^{s} w_\jmath \underline{\underline{\mathbb{R}_r}}(\mathbb{A}_\jmath), \bigotimes_{\jmath=1}^{s} w_\jmath \overline{\overline{\mathbb{R}_r}}(\mathbb{A}_\jmath)\right)$$

$$= \left(\left(w_1 \underline{\underline{\mathbb{R}_r}}(\mathbb{A}_1) \otimes w_2 \underline{\underline{\mathbb{R}_r}}(\mathbb{A}_2) \otimes \cdots \otimes w_s \underline{\underline{\mathbb{R}_r}}(\mathbb{A}_s)\right), \left(w_1 \overline{\overline{\mathbb{R}_r}}(\mathbb{A}_1) \otimes w_2 \overline{\overline{\mathbb{R}_r}}(\mathbb{A}_2) \otimes \cdots \otimes w_s \overline{\overline{\mathbb{R}_r}}(\mathbb{A}_s)\right)\right)$$

The following are the outcomes for the CIFRSSWG operator.

Theorem 2 By using the equation above, we get the CIFRNs and

$$\mathrm{CIFRSSWA}(\mathbb{R}_r(\mathbb{A}_1), \mathbb{R}_r(\mathbb{A}_2), \ldots, \mathbb{R}_r(\mathbb{A}_s))$$

$$= \left(\bigotimes_{\jmath=1}^{s} w_\jmath \cdot \underline{\underline{\mathbb{R}_r}}(\mathbb{A}_\jmath), \bigotimes_{\jmath=1}^{s} w_\jmath \cdot \overline{\overline{\mathbb{R}_r}}(\mathbb{A}_\jmath)\right)$$

$$= \begin{pmatrix} \left(\left\{\sum_{\jmath=1}^{s} w_\jmath \left(m_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_\jmath)\right)^\rho\right\}^{1/\rho} + \iota \left(\left\{\sum_{\jmath=1}^{s} w_\jmath \left(\eta_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_\jmath)\right)^\rho\right\}^{1/\rho}\right)\right), \\ 1 - \left\{\sum_{\jmath=1}^{s} w_\jmath \left(1 - o_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_\jmath)\right)^\rho\right\}^{1/\rho} + \iota \left(1 - \left\{\sum_{\jmath=1}^{s} w_\jmath \left(1 - p_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_\jmath)\right)^\rho\right\}^{1/\rho}\right), \\ \left(\left\{\sum_{\jmath=1}^{s} w_\jmath \left(m_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_\jmath)\right)^\rho\right\}^{1/\rho} + \iota \left(\left\{\sum_{\jmath=1}^{s} w_\jmath \left(\eta_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_\jmath)\right)^\rho\right\}^{1/\rho}\right)\right), \\ 1 - \left\{\sum_{\jmath=1}^{s} w_\jmath \left(1 - o_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_\jmath)\right)^\rho\right\}^{1/\rho} + \iota \left(1 - \left\{\sum_{\jmath=1}^{s} w_\jmath \left(1 - p_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_\jmath)\right)^\rho\right\}^{1/\rho}\right) \end{pmatrix}$$

Proof. Similar to Theorem 1.

Example 1 Let

$$\mathbb{R}_r(\mathbb{A}_1) = \left\{ \begin{array}{c} (0.10 + \iota 0.40,\ 0.30 + \iota 0.50) \\ (0.20 + \iota 0.30,\ 0.40 + \iota 0.60) \end{array} \right\},$$

$$\mathbb{R}_r(\mathbb{A}_2) = \left\{ \begin{array}{c} (0.50 + \imath 0.30,\ 0.30 + \imath 0.60) \\ (0.20 + \imath 0.40,\ 0.30 + \imath 0.40) \end{array} \right\},$$

$$\mathbb{R}_r(\mathbb{A}_3) = \left\{ \begin{array}{c} (0.3 + \imath 0.3,\ 0.1 + \imath 0.4) \\ (0.5 + \imath 0.7,\ 0.3 + \imath 0.1) \end{array} \right\}$$

and

$$\mathbb{R}_r(\mathbb{A}_4) = \left\{ \begin{array}{c} (0.4 + \imath 0.6,\ 0.3 + \imath 0.1) \\ (0.2 + \imath 0.5,\ 0.3 + \imath 0.4) \end{array} \right\}$$

be four CIFRNs and the parameter is $\varrho = 2$. Let the WV be $\boldsymbol{\theta} = (0.31,\ 0.14,\ 0.34,\ 0.21)^{\top}$. Now we use CIFRSSWG operator get the aggregated result as follows:

$$\mathrm{CIFRSSWA}(\mathbb{R}_r(\mathbb{A}_1), \mathbb{R}_r(\mathbb{A}_2), \mathbb{R}_r(\mathbb{A}_3), \mathbb{R}_r(\mathbb{A}_4))$$

$$= \left(\begin{array}{c} \left(\left\{ \sum_{J=1}^{4} w_J \left(m_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_J) \right)^2 \right\}^{1/2} + \imath \left(\left\{ \sum_{J=1}^{4} w_J \left(\eta_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_J) \right)^2 \right\}^{1/2} \right) \right), \\ 1 - \left\{ \sum_{J=1}^{4} w_J \left(1 - o_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_J) \right)^2 \right\}^{1/2} + \imath \left(1 - \left\{ \sum_{J=1}^{4} w_J \left(1 - p_{\underline{\underline{\mathbb{R}_r}}}(\mathbb{A}_J) \right)^2 \right\}^{1/2} \right), \\ \left(\left\{ \sum_{J=1}^{4} w_J \left(m_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_J) \right)^2 \right\}^{1/2} + \imath \left(\left\{ \sum_{J=1}^{4} w_J \left(\eta_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_J) \right)^2 \right\}^{1/2} \right) \right), \\ 1 - \left\{ \sum_{J=1}^{4} w_J \left(1 - o_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_J) \right)^2 \right\}^{1/2} + \imath \left(1 - \left\{ \sum_{J=1}^{4} w_J \left(1 - p_{\overline{\overline{\mathbb{R}_r}}}(\mathbb{A}_J) \right)^2 \right\}^{1/2} \right) \end{array} \right)$$

$$
= \left\{ \begin{pmatrix} \left(\left(0.3 \times (0.1)^4 + 0.4 \times (0.5)^4 + 0.2 \times (0.3)^4 + 0.1 \times (0.4)^4\right)^{\frac{1}{4}} \right) \\ + \imath \left(\left(0.3 \times (0.4)^4 + 0.4 \times (0.3)^4 + 0.2 \times (0.3)^4 + 0.1 \times (0.6)^4\right)^{\frac{1}{4}} \right) \\ \left(1 - \left(0.3 \times (1-0.3)^4 + 0.4 \times (1-0.3)^4 + 0.2 \times (1-0.1)^4 + 0.1 \times (1-0.3)^4\right)^{\frac{1}{4}} \right) \\ + \imath \left(1 - \left(0.3 \times (1-0.3)^4 + 0.4 \times (1-0.3)^4 + 0.2 \times (1-0.1)^4 + 0.1 \times (1-0.3)^4\right)^{\frac{1}{4}} \right), \\ \left(\left(0.3 \times (0.2)^4 + 0.4 \times (0.2)^4 + 0.2 \times (0.5)^4 + 0.1 \times (0.2)^4\right)^{\frac{1}{4}} \right) \\ + \imath \left(\left(0.3 \times (0.3)^4 + 0.4 \times (0.4)^4 + 0.2 \times (0.7)^4 + 0.1 \times (0.5)^4\right)^{\frac{1}{4}} \right) \\ \left(1 - \left(0.3 \times (1-0.4)^4 + 0.4 \times (1-0.3)^4 + 0.2 \times (1-0.3)^4 + 0.1 \times (1-0.3)^4\right)^{\frac{1}{4}} \right) \\ + \imath \left(1 - \left(0.3 \times (1-0.6)^4 + 0.4 \times (1-0.4)^4 + 0.2 \times (1-0.1)^4 + 0.1 \times (1-0.4)^4\right)^{\frac{1}{4}} \right) \end{pmatrix} \right\}
$$

$$
= \{(0.0685 + \imath 0.069), (0.723 + \imath 0.854), (0.041 + \imath 0.107), (0.7745 + \imath 0.805)\}
$$

Remark CIFRSSWG also meets the requirements for boundedness, monotonicity and idempotency.

5. CIFR-MCDM method

Let's suppose $Y_s = \{Y_1, Y_2, \ldots, Y_m\}$ be a set of alternatives, $\mathcal{C}_r = \{\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_n\}$ be a set of criteria, and $\mathbf{w} = (w_1, w_2, \ldots, w_n)^T$ be the weights of criteria with $w_r \in [0, 1]$, $r = 1, 2, 3, \ldots, n$, and $\sum_{r=1}^{n} w_r = 1$.

Moreover, based on the discovered criteria, the specialist will calculate the evaluated values of the alternatives. These evaluated values are CIFRNs; that is,

$$
\mathbf{M} = (\mathbb{M}_{sr})_{m \times n} = \left(\left(\underline{\underline{M}}_{sr}, \underline{\underline{N}}_{sr} \right), \left(\overline{\overline{M}}_{sr}, \overline{\overline{N}}_{sr} \right) \right)_{m \times n} = \left(\left(\underline{\underline{\mu}}_{sr} + \imath\, \underline{\underline{\eta}}_{sr}, \underline{\underline{\delta}}_{sr} + \imath\, \underline{\underline{\tau}}_{sr} \right), \left(\overline{\overline{\mu}}_{sr} + \imath\, \overline{\overline{\eta}}_{sr}, \overline{\overline{\delta}}_{sr} + \imath\, \overline{\overline{\tau}}_{sr} \right) \right)_{m \times n}
$$

which will form a CIFR decision matrix. To handle the MCDM problem, we interpret the algorithm described below.

5.1 *Algorithm*

To solve the MCDM dilemma, we establish the algorithm in the framework of CIFRSs by using CIFRSSWA and CIFRSSWG operators.

Stage 1: Aggregate the CIFR decision-matrix by using the CIFRSSWA operator and CIFRSSWG operators.

Stage 2: Attain the score values of the aggregated results and then the accuracy values if the score values of two different aggregated outcomes are the same.

Stage 3: In this stage, use the score values to rank the alternatives and attain the finest alternative.

Stage 4: End.

5.2 *Case study*

A rapidly growing coastal city wishes to enhance the exploitation of RE sources particularly wind and solar energy. However, these sources are not steady; hence, the proper energy storage system should be included to ensure a steady power supply. For this purpose, the energy department of the city is supposed to identify the right energy storage system for the city. These are the choices that the decision-makers have to make and they are cost, time, effects on the environment, and scalability. Thus, they have been left with the following four strategies.

Y_1: Lithium-ion Batteries: These are batteries that can be charged again and the electrolyte found in the battery is lithium ions. These are majorly used in portable devices and electric cars and are gradually extending to large storage uses.

Y_2: Pumped Hydro Storage: This technique involves the storing of energy in a water reservoir by pumping water from one level to the other especially when there is excess electricity. When there is energy demand the water can flow back to the lower reservoir through turbines and hence generate energy.

Y_3: Compressed Air Energy Storage: This entails the use of compressed air energy storage to enhance the effectiveness of the compressors used in energy storage. This technology uses the excess electricity to compress air and store it in the underground cavern or tanks as the case may be. The compressed air is then stored and when electricity is needed the compressed air is released and heated then passes through a turbine.

Y_4: Flywheel Energy Storage: This system has the form of the stored energy the rotational kinetic energy of the system. A flywheel stores a large amount of energy and it is maintained charged to the degree that it can deliver the energy when required.

Which will be assessed by the following four criteria.

$\mathcal{C}_1$: Cost Efficiency: This criterion includes the costs that are required at the start of the project and the depreciable costs for each asset for the useful life of the asset. This is an important factor in its economics and some of the parameters are the cost per kWh for storage and the life of the system.

$\mathcal{C}_2$: Energy Density: This refers to how much energy could be put in per volume or mass and the higher the energy density, the more compressed the solutions are that can be offered especially in urban centers where space is often a huge issue.

$\mathcal{C}_3$: Environmental Impact: This criterion evaluates the sustainability index of each of the alternatives from the initial conceptualization and through the use of the product as well as its disposal. They include extraction of materials, manufacturing and use emissions and disposal or recovery at the end of the product's life.

$\mathcal{C}_4$: Operational Flexibility: This defines the capacity of the storage system in responding to the supply and demand of energy in the most effective manner. These are such factors as response time, cycling efficiency which is defined as the number of cycles in charging and discharging the system and the ability to provide services to the grid.

This MCDM problem entails the evaluation of these alternatives concerning the outlined criteria to determine the right energy storage system for the city's RE project. The decision will determine the future of energy in the city, its environmental plan and all the economic development. The weights for each criteria are (0.3, 0.4, 0.2, 0.1) which is devised by a decision maker. The assessment values of these energy storage systems are interpreted in Table 3, provided by the decision maker.

Table 3. Complex intuitionistic fuzzy rough numbers

	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$
Y_1	$\begin{pmatrix}(0.1+\imath 0.4,\ 0.3+\imath 0.5)\\(0.2+\imath 0.3,\ 0.4+\imath 0.6)\end{pmatrix}$	$\begin{pmatrix}(0.5+\imath 0.3,\ 0.3+\imath 0.6)\\(0.2+\imath 0.4,\ 0.3+\imath 0.4)\end{pmatrix}$	$\begin{pmatrix}(0.3+\imath 0.3,\ 0.1+\imath 0.4)\\(0.5+\imath 0.7,\ 0.3+\imath 0.1)\end{pmatrix}$	$\begin{pmatrix}(0.4+\imath 0.6,\ 0.3+\imath 0.1)\\(0.2+\imath 0.5,\ 0.3+\imath 0.4)\end{pmatrix}$
Y_2	$\begin{pmatrix}(0.2+\imath 0.5,\ 0.5+\imath 0.2)\\(0.3+\imath 0.2,\ 0.4+\imath 0.2)\end{pmatrix}$	$\begin{pmatrix}(0.2+\imath 0.3,\ 0.3+\imath 0.5)\\(0.3+\imath 0.8,\ 0.3+\imath 0.2)\end{pmatrix}$	$\begin{pmatrix}(0.4+\imath 0.6,\ 0.4+\imath 0.2)\\(0.5+\imath 0.4,\ 0.2+\imath 0.4)\end{pmatrix}$	$\begin{pmatrix}(0.6+\imath 0.3,\ 0.3+\imath 0.2)\\(0.2+\imath 0.6,\ 0.7+\imath 0.3)\end{pmatrix}$
Y_3	$\begin{pmatrix}(0.6+\imath 0.4,\ 0.2+\imath 0.5)\\(0.5+\imath 0.6,\ 0.3+\imath 0.1)\end{pmatrix}$	$\begin{pmatrix}(0.4+\imath 0.3,\ 0.2+\imath 0.7)\\(0.5+\imath 0.1,\ 0.1+\imath 0.7)\end{pmatrix}$	$\begin{pmatrix}(0.4+\imath 0.2,\ 0.5+\imath 0.5)\\(0.3+\imath 0.8,\ 0.4+\imath 0.2)\end{pmatrix}$	$\begin{pmatrix}(0.7+\imath 0.4,\ 0.3+\imath 0.4)\\(0.5+\imath 0.4,\ 0.5+\imath 0.3)\end{pmatrix}$
Y_4	$\begin{pmatrix}(0.7+\imath 0.3,\ 0.2+\imath 0.6)\\(0.2+\imath 0.3,\ 0.8+\imath 0.5)\end{pmatrix}$	$\begin{pmatrix}(0.4+\imath 0.6,\ 0.6+\imath 0.4)\\(0.2+\imath 0.7,\ 0.5+\imath 0.3)\end{pmatrix}$	$\begin{pmatrix}(0.4+\imath 0.5,\ 0.3+\imath 0.4)\\(0.2+\imath 0.4,\ 0.3+\imath 0.5)\end{pmatrix}$	$\begin{pmatrix}(0.3+\imath 0.7,\ 0.3+\imath 0.2)\\(0.7+\imath 0.2,\ 0.3+\imath 0.5)\end{pmatrix}$

Stage 1: Using the CIFRSSWA operators to determine the values, $Y_s = \{Y_1, Y_2, Y_3, Y_4\}$

$$Y_1 = \begin{pmatrix}(0.7615+\imath 0.791),\ (0.037+\imath 0.126),\\ \\(0.719+\imath 0.833),\ (0.0555+\imath 0.095)\end{pmatrix}$$

$$Y_2 = \begin{pmatrix}(0.732+\imath 0.824),\ (0.076+\imath 0.062),\\ \\(0.7715+\imath 0.852),\ (0.0705+\imath 0.0345)\end{pmatrix}$$

$$Y_3 = \begin{pmatrix}(0.8635+\imath 0.766),\ (0.0435+\imath 0.1685),\\ \\(0.851+\imath 0.792),\ (0.044+\imath 0.108)\end{pmatrix}$$

$$Y_4 = \begin{pmatrix}(0.7075+\imath 0.8405),\ (0.0915+\imath 0.104),\\ \\(0.4992+\imath 0.0611),\ (0.1595+\imath 0.093)\end{pmatrix}$$

Stage 2: The obtained score values of $\mathrm{Sc}_F(Y_1)$, $\mathrm{Sc}_F(Y_2)$, $\mathrm{Sc}_F(Y_3)$, and $\mathrm{Sc}_F(Y_4)$ are:

$$\mathrm{Sc}_F(Y_1) = 0.8488, \quad \mathrm{Sc}_F(Y_2) = 0.8670, \quad \mathrm{Sc}_F(Y_3) = 0.8645, \quad \mathrm{Sc}_F(Y_4) = 0.8523$$

Stage 3: Rank the values Y_1, Y_2, Y_3, and Y_4 based on the above score values of the overall CIFRNs:

$$Y_2 > Y_3 > Y_4 > Y_1$$

Y_2 is selected as the best alternative.

Stage 4: End.

The graphical representation of ESS alternatives based on CIFRSSWA operators is discussed in Figure 3.

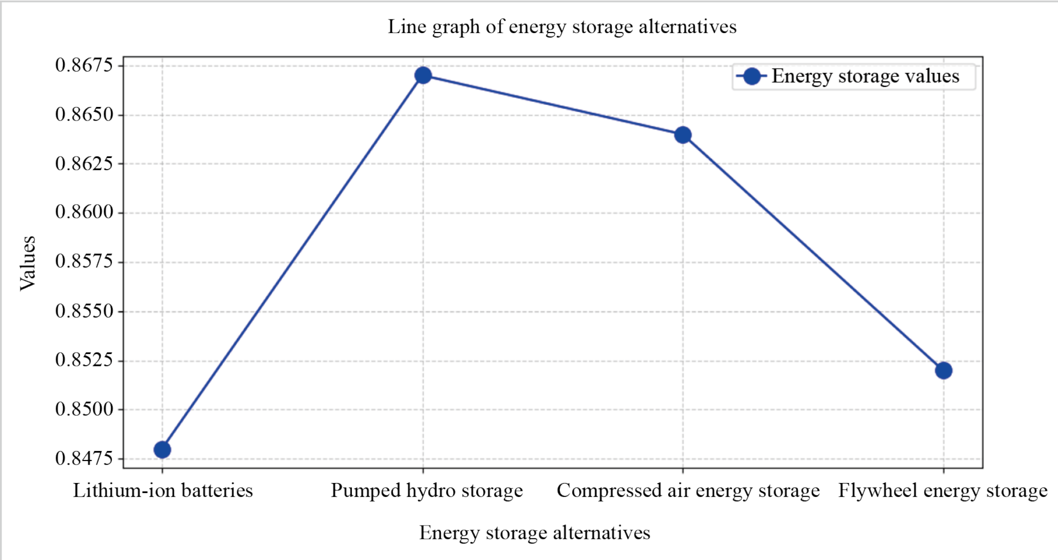

Figure 3. Ranking of ESS alternatives using CIFRSSWA

Now, we use CIFRSSWG operator then the result of the above dilemma is as follows:

Stage 1: Using the CIFRSSWG operators to determine the values, $Y_s = \{Y_1, Y_2, \ldots, Y_m\}$:

$$Y_1 = \begin{pmatrix} (0.0685 + \imath 0.069), (0.723 + \imath 0.854), \\ (0.041 + \imath 0.107), (0.7745 + \imath 0.805) \end{pmatrix}$$

$$Y_2 = \begin{pmatrix} (0.048 + \imath 0.096), (0.804 + \imath 0.758), \\ (0.0585 + \imath 0.168), (0.7795 + \imath 0.7155) \end{pmatrix}$$

$$Y_3 = \begin{pmatrix} (0.1265 + \imath 0.054), (0.7265 + \imath 0.9015), \\ (0.109 + \imath 0.128), (0.716 + \imath 0.772) \end{pmatrix}$$

$$Y_4 = \begin{pmatrix} (0.126 + \imath 0.135), (0.7985 + \imath 0.836), \\ (0.0425 + \imath 0.1295), (0.8705 + \imath 0.827) \end{pmatrix}$$

Stage 2: The obtained score values of $\mathrm{Sc}_F(Y_i)$ are:

$$\mathrm{Sc}_F(Y_1) = 0.1411, \quad \mathrm{Sc}_F(Y_2) = 0.1641, \quad \mathrm{Sc}_F(Y_3) = 0.1626, \quad \mathrm{Sc}_F(Y_4) = 0.1376$$

Stage 3: Rank the alternatives based on their score values:

$$Y_2 > Y_3 > Y_1 > Y_4$$

Y_2 is selected as the best alternative.

Stage 4: End.

The graphical representation of ESS alternatives based on CIFRSSWG operators is discussed in Figure 4.

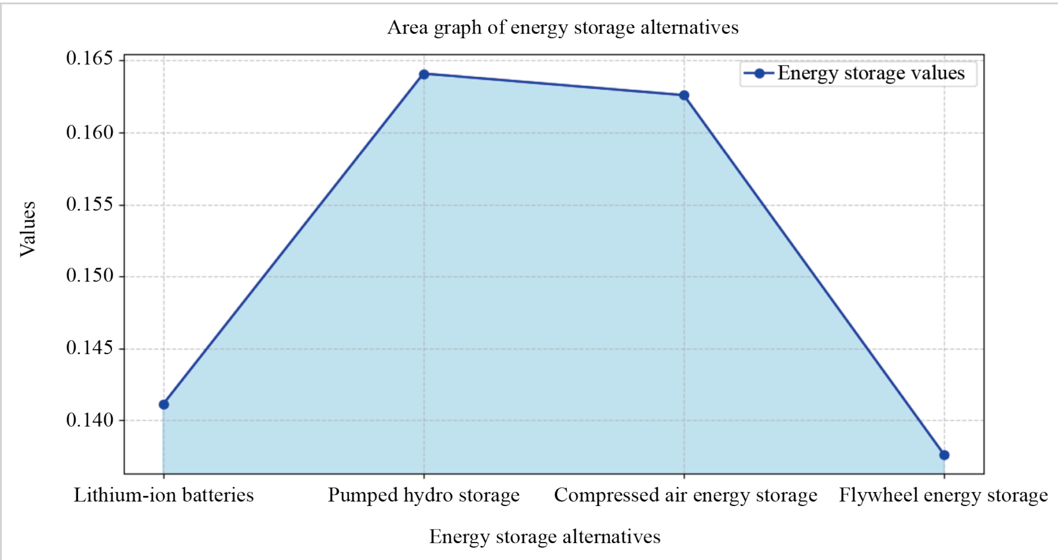

Figure 4. Ranking of ESS alternatives using CIFRSSWG

5.3 *Sensitivity analysis*

Sensitivity analysis is essential in decision-making, especially in ranking of energy storage systems. In this study, we conduct a sensitivity analysis using the Schweizer-Sklar AOs to examine the stability and robustness of the proposed CIFR-MCDM method. By systematically varying the weights assigned to decision criteria, we assess how changes in input preferences influence the ranking of energy storage alternatives. This analysis provides deeper insights into the resilience of the proposed model under different weighting scenarios and further reinforces the validity and reliability of our approach in complex decision environments.

We considered six different scenarios by adjusting the weights of our four criteria's (cost efficiency, energy density, environmental impact and operational flexibility). Different weights for each criterion are:

$$\omega_1 = (0.25,\ 0.45,\ 0.20,\ 0.10),\ \omega_2 = (0.30,\ 0.30,\ 0.30,\ 0.10),\ \omega_3 = (0.20,\ 0.40,\ 0.30,\ 0.10),$$

$$\omega_4 = (0.35,\ 0.35,\ 0.20,\ 0.10),\ \omega_5 = (0.25,\ 0.35,\ 0.25,\ 0.15),\ \omega_6 = (0.30,\ 0.40,\ 0.20,\ 0.10).$$

The assessment values of energy storage systems are are taken from Table 3. The aggregated values of CIFRSSWA and CIFRSSWG are in Table 4 and Table 5.

Table 4. Outcomes for CIFRSSWA operators according to changes in weight

Variations of weights	$Sc_F(Y_1)$	$Sc_F(Y_2)$	$Sc_F(Y_3)$	$Sc_F(Y_4)$
ω_1	0.85113	0.86825	0.8585	0.85516
ω_2	0.85419	0.87081	0.868	0.85181
ω_3	0.85869	0.87319	0.85788	0.85738
ω_4	0.84663	0.86588	0.86863	0.84959
ω_5	0.861219	0.873781	0.869188	0.855344
ω_6	0.84888	0.86706	0.86356	0.85238

Table 5. Outcomes for CIFRSSWG operators according to changes in weight

Variations of weights	$Sc_F(Y_1)$	$Sc_F(Y_2)$	$Sc_F(Y_3)$	$Sc_F(Y_4)$
ω_1	0.14263	0.1655	0.159	0.13922
ω_2	0.14956	0.16544	0.1645	0.13694
ω_3	0.15256	0.16806	0.15713	0.14013
ω_4	0.13963	0.16288	0.16638	0.13603
ω_5	0.158156	0.166844	0.163313	0.140281
ω_6	0.14113	0.16419	0.16269	0.13763

Tables 6 and 7 discuss how all the alternatives are ranked.

Table 6. Alternative rankings using CIFRSSWA aggregation operators

CIFRSSWA operators	Ranking
ω_1	$Y_2 > Y_3 > Y_4 > Y_1$
ω_2	$Y_2 > Y_3 > Y_1 > Y_4$
ω_3	$Y_2 > Y_1 > Y_3 > Y_4$
ω_4	$Y_3 > Y_2 > Y_4 > Y_1$
ω_5	$Y_2 > Y_3 > Y_1 > Y_4$
ω_6	$Y_2 > Y_3 > Y_4 > Y_1$

Sensitivity analysis shows that the ranking of ESS remains relatively stable across different weight criteria. From the Table 4 and Table 5, we note that, the score values for each alternative show minimal fluctuations, representing the robustness of the proposed CIFRSSWA and CIFRSSWG operators. This consistency across scenarios confirms the reliability of the model in handling uncertainty and changing decision-maker preferences.

Table 7. Alternative rankings using CIFRSSWG aggregation operators

CIFRSSWG operators	Ranking
ω_1	$Y_2 > Y_3 > Y_1 > Y_4$
ω_2	$Y_2 > Y_3 > Y_1 > Y_4$
ω_3	$Y_2 > Y_3 > Y_1 > Y_4$
ω_4	$Y_3 > Y_2 > Y_1 > Y_4$
ω_5	$Y_2 > Y_3 > Y_1 > Y_4$
ω_6	$Y_2 > Y_3 > Y_1 > Y_4$

6. Comparative analysis

In this study, we conduct a comparative evaluation to demonstrate the advantages of the proposed CIFR-MCDM method by juxtaposing it with other well-established FS based approaches. These include models grounded in complex intuitionistic fuzzy sets, fuzzy rough sets, and intuitionistic fuzzy rough sets each connected to different types of aggregation operators. For ease of comparison, we selected representative studies that utilize a variety of fuzzy-based aggregation operators within Multi-Criteria Decision-Making (MCDM) frameworks. The outcomes of these methods are compared against the results produced by our proposed CIFR-MCDM framework. The theories considered for comparison include: Garg and Rani [14], Chen et al. [18], Yahya et al. [29], Hussain et al. [30], Emam et al. [32], and Seikh and Chatterjee [37]. Table 8 provides a summary of ranking and performance results produced by these models. When applied to choose situation with high uncertainty and connected criteria, like ESS selection, this comparison clearly shows the suggested CIFR-MCDM technique's better flexibility, approximation capability, and consistency.

Table 8. Comparative analysis between proposed and existing methods

Methods	Score values of alternatives	Rankings
Garg and Rani [14]	Nil	Nil
Chen et al. [18]	Nil	Nil
Yahya et al. [29]	Nil	Nil
Hussain et al. [30]	Nil	Nil
Emam et al. [32]	Nil	Nil
Seikh and Chatterjee [37]	Nil	Nil
CIFRSSWA (proposed work)	$\mathrm{Sc}_F(Y_1) = 0.8488$, $\mathrm{Sc}_F(Y_2) = 0.8670$, $\mathrm{Sc}_F(Y_3) = 0.8645$, $\mathrm{Sc}_F(Y_4) = 0.8523$	$Y_2 > Y_3 > Y_4 > Y_1$
CIFRSSWG (proposed work)	$\mathrm{Sc}_F(Y_1) = 0.1411$, $\mathrm{Sc}_F(Y_2) = 0.1641$, $\mathrm{Sc}_F(Y_3) = 0.1626$, $\mathrm{Sc}_F(Y_4) = 0.1376$	$Y_2 > Y_3 > Y_1 > Y_4$

• Garg and Rani [14] suggested strong averaging/geometric AOs in complex IFSs and applied them to MCDM problems. Their model, however, does not include rough approximations, limiting its usability to handle boundary-based uncertainty. Our CIFR-MCDM model, however, integrates intuitionistic and rough set theory in a complex setting, yielding a more descriptive fuzzification of fuzzy data.

• Chen et al. [18] introduced a fuzzy-rough MCDM approach to green supplier selection with eco-friendly materials. Although their approach is a rough set-based approach, it is restricted by the traditional fuzzy logic and does not have phase-handling capability of complicated FSs. Our contribution is built on this by adding complex intuitionistic fuzzy rough logic, enabling more ambiguity and vagueness modeling in multi-criteria assessment.

• Yahya et al. [29] utilized an intuitionistic fuzzy rough Evaluation based on Distance from Average Solution (EDAS) approach with Frank aggregation operators. Their work combines intuitionistic fuzzy rough sets but avoids the complicated domain. However, our CIFR-MCDM model includes more complex expert opinions using complex-valued membership functions, therefore having a higher expressiveness in real uncertain situations.

• Hussain et al. [30] presented intuitionistic fuzzy rough Dombi AOs using a TOPSIS approach. Although they use Dombi AOs for soft aggregation but do not address complex numbers in their approach, restricting cyclic or multidimensional uncertainty modeling. This work builds upon that by using Schweizer-Sklar operators in a CIFR framework, with enhanced flexibility and generality.

• Emam et al. [32] constructed sophisticated fuzzy rough Frank AOs and applied them to describe AI tools in civil engineering. The paper employs Frank operators but not intuitionistic fuzzy logic, which our suggested CIFR-MCDM approach accomplishes. The additional intuitionistic element supports decision credibility since it includes hesitation as well as rough set approximations, which is better applicable to uncertain MCDM contexts such as ESS selection.

• Seikh and Chatterjee [37] proposed an interval-valued Fermatean FS-based Step-wise Weight Assessment Ratio Analysis-Additive Ratio Assessment (SWARA-ARAS) model for renewable energy selection. Though Fermatean sets possess more uncertainty representation capability, their model only uses real-valued data and does not include rough approximations. In contrast, our CIFR-MCDM approach combines both roughness and intuitionistic complex-valued information and thus has more potential in managing multi-layered uncertainty in ESS-related decisions.

7. Managerial implications

The CIFR-MCDM model suggested in this paper carries significant practical benefits for decision-makers and entrepreneurs who are involved in renewable energy planning, infrastructure development, and sustainability initiatives. Importantly, the CIFR setting allows organizations to replicate actual decision-making contexts with abundant conflicting, incomplete, or imprecise information, which is actually very common in energy projects.

Managers can utilize the set-up to:

• Rank ESS solutions against strategic objectives like cost reductions, environmental regulation, or long-term availability.

• Improve management of conflicting criteria by incorporating uncertainties, indecisiveness, and boundary-focused approximations into the evaluation system.

• Reduce reliance on personal judgments by enabling data-driven decision-making through a structured aggregation approach, thus increasing transparency and strengthening stakeholder trust.

• Apply sensitivity analysis for dynamically changing decisions, studying the effect of changes in strategic priorities (e.g., cost vs. sustainability emphasis) on the selection of the best ESS.

• Compare with existing techniques, showing that the proposed CIFR-MCDM model exhibits better and more robust results under uncertainty than traditional fuzzy MCDM models.

Finally, this framework promotes stronger and more resilient decision-making so that organizations select energy storage systems that meet both technical requirements and long-term business objectives as well as sustainability objectives.

8. Conclusion

A significant drawback of the MCDM methods is incapable of handling roughness, membership grade and non-membership grade with extra fuzzy information (known as 2nd dimension) that is usually linked to the criteria. The limitations of traditional MCDM methods pose a significant challenge, as they increase the risk of data loss. Ignoring the roughness, membership and non-membership aspects of attributes and the 2nd dimension information that should be considered makes it difficult to make accurate and dependable decisions when selecting the optimal ESS. Failing to account for these critical factors can lead to less optimal outcomes, as important facts and insights might be overlooked or missed during the decision-making process. Despite significant advancements in ESS technologies, selecting the finest ESS remains challenging. Therefore, for this reason, this manuscript developed a new framework CIFRS to cover all the limitations that exist in the existing theories and use an MCDM technique in the environment of CIFRS to produce a CIFR-MCDM technique. This technique covers the complex plane for both MGr and N-MGr along with the LA and UA. Additionally, we have developed the basic operations and properties of the developed CIFRS. Furthermore, we have covered SS operational laws based on the developed notion of CIFRS and we have discussed aggregation operators like CIFRSSWA and CIFRSSWG aggregation operator and we discuss a case study of energy storage system selection. In order to illustrate the significant benefits, we also have a comparison analysis of the initiated theory.

8.1 *Limitation*

Although the theory of CIFRSs is a powerful tool for modeling uncertainty, vagueness and boundary based data, it still has some limitations. The CIFR-MCDM framework is specifically designed to work with complex intuitionistic fuzzy data in the form of LA and UA. However, in more advanced scenarios, where information is provided in the form of complex pythagorean FSs, complex picture FSs or bipolar complex FSs, the existing fails to handle such type of information due to the incompatibility in structure and semantic interpretation. These limitations open several potential avenues for future work.

8.2 *Future work*

Our approach could be extended to various real-life decision-making problems, such as the electronic waste management and electric vehicle adoption problem by Sheikh and Chatterjee [38, 39]. Our long-term goal is to develop this hypothesis further within the context of complex picture fuzzy environment [40], ELimination Et Choice Translating Reality (I) (ELECTRE-I) method [41, 42], bipolar complex fuzzy soft sets [43], Aczel Alsina Heronian mean operators [44] and frank power aggregation operators [45].

Data availability

The data generated for this article is entirely included in the manuscript and anyone can use it by just citing this article.

Ethics declaration statement

The authors state that this is their original work and it is neither submitted nor under consideration in any other journal simultaneously.

Human and animal participants

This article does not contain any studies with human participants or animals performed by any of the authors.

Conflict of interest

The authors declare no competing financial interest.

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