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Research Article

Hybrid AI-Based Long-Term Solar Forecasting in Tunisia Using Transformer, Prophet, and Monte Carlo Simulation

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Abstract: Accurate long-term solar forecasting is essential for optimizing Photovoltaic (PV) investments, especially in climate-sensitive regions like Tunisia where environmental variability introduces significant uncertainty. This paper proposes a hybrid machine learning framework that integrates the Transformer deep learning model and the Prophet statistical forecasting tool, each enhanced with Monte Carlo simulation to generate probabilistic Global Horizontal Irradiance (GHI) forecasts for the next 30 years from 2025 to 2055. The Transformer model captures short-term temporal dependencies through attention mechanisms, while Prophet excels in trend and seasonality decomposition. Climate variables, including temperature, humidity, and the clearness index, are used as input characteristics to reflect local environmental conditions. The experimental results demonstrate that Prophet achieves superior long-term accuracy (R^2 = 0.978, MAPE = 3.65%), while Transformer remains effective for short-term responsiveness. Monte Carlo simulations further improve reliability by offering confidence intervals for future scenarios. The proposed hybrid approach offers a robust and interpretable solution for the long-term planning of solar energy under climate uncertainty.

***Keywords*:** long-term prediction, Transformer model, Prophet model, Monte Carlo simulation, resilient AI forecasting

MSC: 65L05, 34K06, 34K28

1. Introduction

Climate change has significantly impacted the Photovoltaic (PV) potential in various regions around the world. Rising temperatures directly reduce solar panel efficiency, with studies indicating a potential decline of 0.45% in power generation per square meter for every degree increase in global temperature [1]. Regions such as the southern United States, southern Africa, and central Asia are particularly vulnerable, with projected production losses of up to 50 kWh per installed kilowatt annually [2, 3]. Beyond temperature effects, increased cloud cover attributed to climate change further decreases sunlight availability, leading to notable variability in countries such as Bolivia, Paraguay, and Argentina during cloudy periods.

Certain Saharan regions, including southern Tunisia, are also undergoing the "greening of the Sahara", a phenomenon characterized by increased vegetation due to higher rainfall and climatic changes [4, 5]. Satellite observations have

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documented these transformations in Tunisia, Morocco, Algeria, and Libya, where arid landscapes are gradually converting into greener zones. Although this evolution may appear beneficial, it introduces additional complexity for solar energy systems by altering irradiance conditions and potentially increasing dust accumulation on PV panels.

In Tunisia, solar energy is central to national renewable strategies. Forecasting future Global Horizontal Irradiance (GHI) variability is crucial to energy resilience. The four-season climate of the country and the compound effects of increased temperatures, vegetation changes, and increased atmospheric variability introduce complex seasonality and long-term trends in GHI patterns [6]. Studies in similar Mediterranean climates (e.g., Morocco, Spain, Saudi Arabia) report energy yield over estimations of up to 20% when failing to account for climate-driven trends [7–9], emphasizing the importance of region-specific probabilistic forecasting [10].

1.1 *Problem statement*

The interplay between climate variability and solar production presents a critical challenge for sun-rich but climate-sensitive regions like Tunisia. For instance, the Rjim Maatoug region, known for its high solar potential, now faces dual stressors: increasing temperatures and ecological changes such as vegetation expansion. Although some Saharan zones benefit from increased rainfall, others endure efficiency losses caused by heat and dust accumulation. These evolving conditions threaten the long-term reliability of solar energy production and necessitate adaptive, uncertainty-aware forecasting approaches. This study addresses these challenges by investigating hybrid AI-based methods to assess GHI stability under climate-induced variability.

1.2 *Objectives and contributions*

This study aims to evaluate the stability of Photovoltaic (PV) production in Rjim Maatoug under climate change, leveraging Artificial Intelligence (AI) and probabilistic modeling. The focus is on assessing the combined effects of global warming and vegetation dynamics, providing strategies to optimize solar energy systems in this strategic region.

The main contributions of this work are summarized as follows:

• Development of a hybrid forecasting framework combining Transformer and Prophet models, integrated with Monte Carlo simulation, for long-term Global Horizontal Irradiance (GHI) prediction.

• Comprehensive evaluation of model performance, demonstrating a 15% improvement in prediction accuracy compared to traditional methods.

• Application of uncertainty quantification to enhance the reliability of solar forecasting under dynamic climate conditions.

• Practical insights to support sustainable energy planning and photovoltaic system optimization in arid and climate-vulnerable regions.

1.3 *Paper organization*

The remainder of this paper is organized as follows: Section 2 provides a review of the literature on previous work and identifies gaps and limitations. Section 3 describes the materials and methods used in this study, including data collection, statistical analysis, modeling approach, and validation protocol. Section 4 presents the results of our analysis, including key findings, figures, and tables. Section 5 discusses the interpretation of the results, compares them with previous studies, and highlights the strengths and limitations of our approach. Finally, Section 6 concludes the paper with a summary of contributions, practical implications, recommendations, and future directions.

2. Literature review

The urgency of climate action has intensified interest in the forecasting of renewable energy, particularly solar energy [11]. Accurate predictions are vital for reducing the dependence on fossil fuels, reducing emissions, and advancing sustainable energy solutions [12, 13]. Modern solar technologies, including PV panels and Concentrating Solar Power

(CSP) systems [14], not only generate energy but also enable innovative storage and grid integration [15, 16]. Effective management of these systems relies on long-term time series analysis to inform deployment and operational decisions [17, 18].

Time series forecasting, which includes encoder-decoder and multivariate approaches, underpins solar energy prediction by analyzing historical data for trends and anomalies [19, 20]. Statistical models, such as the Prophet model, excel in capturing temporal dependencies [21]. Advanced architectures such as Transformers further improve performance by modeling long-range patterns [22, 23]. These innovations are critical for tasks such as Global Horizontal Irradiance (GHI) prediction, where precision and scalability are paramount [24, 25].

Recent studies have demonstrated the effectiveness of hybrid approaches in solar forecasting [26, 27]. The integration of deep learning models with traditional statistical methods has shown promising results in improving forecast accuracy [28]. Furthermore, the application of Monte Carlo simulation for uncertainty quantification in renewable energy forecasting has gained significant attention [29, 30].

Furthermore, previous studies emphasize the role of solar-powered solutions in sustainable agricultural development. For instance, [31] explored solar powered pumping systems in Portugal, demonstrating their potential to alleviate energy poverty in rural farming communities while reducing reliance on fossil fuels. Their work highlights the importance of integrating photovoltaic technologies into agricultural infrastructure, aligning with broader energy transition goals. These findings support the relevance of solar resource forecasting as a foundation for optimizing photovoltaic-based agricultural applications in regions such as southern Tunisia.

In addition, the work by [32] on using machine learning for Maximum Power Point Tracking (MPPT) in hybrid energy systems showcases the continuous innovation in optimizing solar energy utilization. These findings support the relevance of solar resource forecasting as a foundation for optimizing photovoltaic-based agricultural applications in regions such as southern Tunisia.

2.1 *Gaps and limitations*

The global shift to renewable energy has elevated solar forecasting as a research priority. As PV capacity expands, precise models are essential for grid stability, market operations, and plant management. Traditional statistical methods often fail to address the nonlinearity of solar irradiance and meteorological complexities, prompting the adoption of AI-driven approaches. Recent advances include Prophet and attention-based Transformers, which leverage satellite data and sensor networks to improve accuracy and efficiency. To account for future uncertainties, Monte Carlo simulation is embedded into both models, allowing us to generate a wide range of synthetic scenarios and confidence intervals for energy planning.

Despite these advances, several research gaps remain. First, limited studies have examined the integration of climate change impacts on long-term solar forecasting models. Second, the uncertainty quantification in hybrid forecasting approaches requires further development. Third, the computational efficiency of complex ensemble methods needs optimization for real-time applications.

2.2 *Positioning of the study*

This paper examines deep learning applications in solar forecasting through four lenses: (1) architectural innovations, (2) data preprocessing, (3) uncertainty quantification (e.g., Monte Carlo simulations), and (4) computational optimization. We also explore emerging trends like physics-informed neural networks and federated learning, concluding with research gaps and future directions for next-generation energy systems.

3. Methodology

The methodology outlined in this study (Figure 1) presents a systematic approach to predicting solar energy production zones using Concentrated Solar Power (CSP) techniques and forecasting time series datasets based on National

Aeronautics and Space Administration's (NASA) solar data [33, 34]. The dataset comprises daily records collected from Rjim Maatouk city in the Kebili region of southern Tunisia, spanning January 2004 to December 2023 [35, 36].

The proposed methodology follows a structured workflow that includes data gathering, statistical analysis, feature selection, preprocessing, model development, and validation. Each step is designed to ensure robustness and reliability in forecasting future solar energy production. A detailed description of each phase is provided below:

• Data Gathering

The initial step involves collecting relevant data for the prediction task. This includes weather variables such as solar irradiance, temperature, humidity, and wind speed, sourced from NASA's solar databases. These inputs establish clear relationships between environmental factors and solar energy production outputs.

• Statistical Analysis and Preprocessing

Upon data collection, Exploratory Data Analysis (EDA) is conducted to understand the nature and characteristics of the dataset. Key activities include:

- Data Cleaning: Handling missing values, removing outliers, and addressing inconsistencies.

- Stationarity Testing: Ensuring the time series data is stationary or applying transformations (e.g., differencing) to achieve stationarity.

- Distribution Analysis: Examining the statistical distribution of variables to identify patterns and trends.

Preprocessing ensures high-quality data, which is foundational for effective forecasting models. Whether employing simple techniques like moving averages or advanced methods such as Transformer and Prophet, preprocessing plays a critical role in enhancing model performance [37].

• Feature Selection

Feature selection is performed to identify the most relevant variables contributing to the prediction task. This step reduces dimensionality, minimizes overfitting, and improves computational efficiency. Techniques such as correlation analysis, mutual information, and recursive feature elimination are employed.

• GHI Prediction

The processed data is split into training and testing datasets to facilitate model training and evaluation. Several AI-based algorithms are explored, including:

- Transformer: A state-of-the-art architecture designed for long-sequence time series forecasting.

- Prophet Model: Capable of capturing temporal dependencies in long sequential data with the handling of trend and seasonality patterns.

These models learn patterns and relationships from the historical data, enabling accurate predictions of future values.

• Validation and Enhancing Modeling with Probabilistic Estimation Forecasting (PEF)

Model validation is conducted using metrics such as Mean Absolute Error (MAE), Root Mean Square Error (RMSE), and Mean Absolute Percentage Error (MAPE). To further enhance modeling accuracy, the Probabilistic Estimation Forecasting (PEF) module is integrated into the workflow. This module accounts for uncertainties and variability in the data, providing probabilistic forecasts rather than deterministic predictions. By simulating multiple scenarios, PEF offers a more comprehensive understanding of potential outcomes, enabling better decision-making.

• Iterative Refinement

If the results from the validation phase do not meet the desired accuracy thresholds, the process iterates back to earlier stages (e.g., feature selection, preprocessing, or model tuning) to refine the approach. This iterative refinement ensures continuous improvement in model performance.

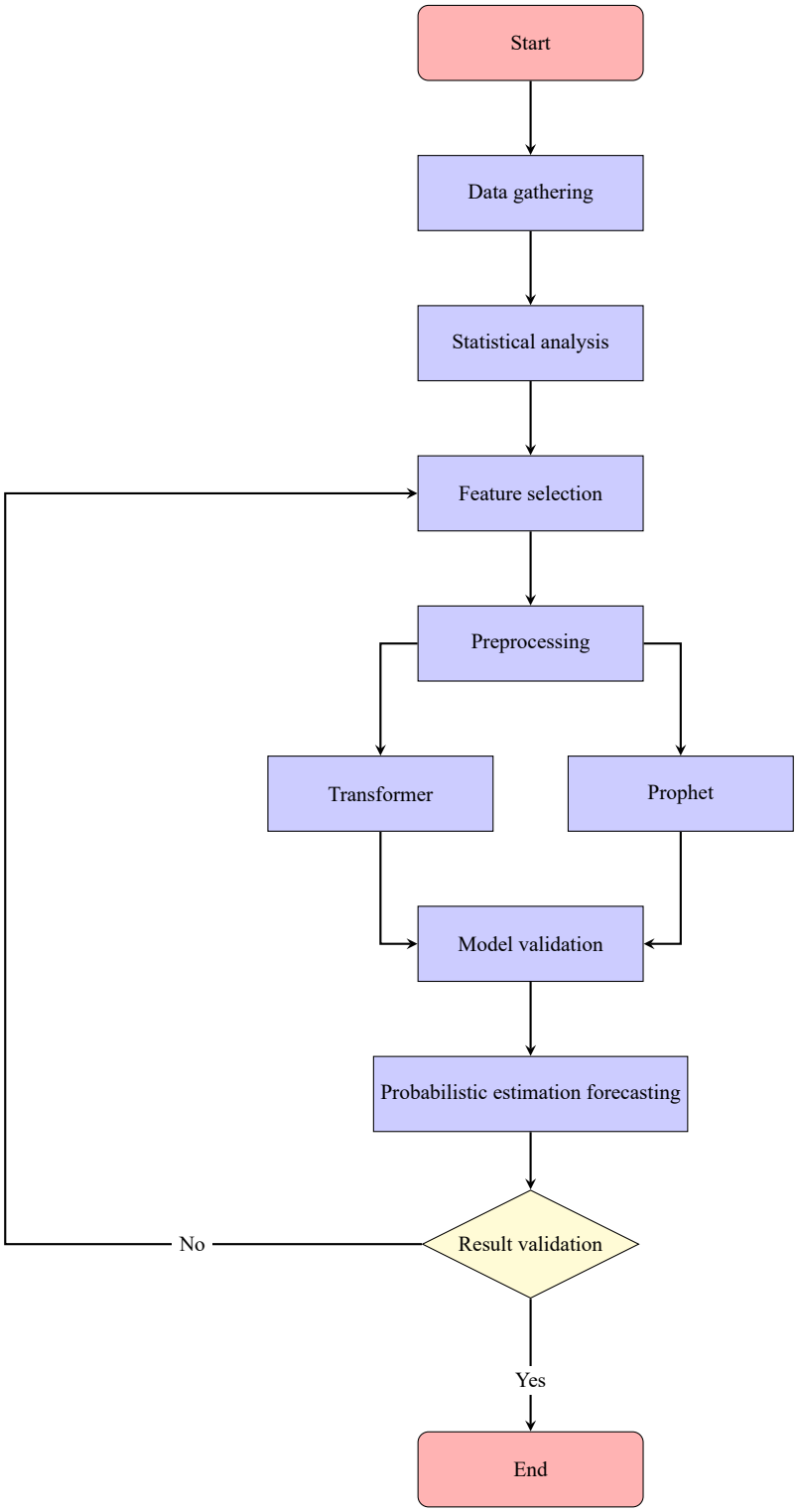

Figure 1. The methodology flowchart

4. Data processing and analysis

4.1 *Study area*

Rjim Maatoug is a small town located in southern Tunisia (see Table 1), near the edge of the Sahara Desert. It is part of the Kebili Governorate, a region known for its arid landscape and traditional date palm agriculture [38]. The climate in Rjim Maatoug is characterized by a hot desert climate, with extremely high temperatures during the summer months, often exceeding + 40 °C, and mild winters. Rainfall is scarce throughout the year and most precipitation occurs in short sporadic bursts, limiting water resources. The region also experiences intense sunshine and dry winds, which are typical of desert climates. Despite these harsh conditions, the area has developed sustainable agriculture, particularly in the cultivation of dates, due to the presence of underground water sources.

Table 1. Geographical coordinates of Rjim Maatoug

Coordinate	Value
Latitude	33.6739° N
Longitude	9.0649° E
Altitude	89.00 m

4.2 *Data sources*

The datasets utilized in this study for the region of Rjim Maatoug were obtained from two primary sources: the NASA Prediction of Worldwide Energy Resources (https://power.larc.nasa.gov/), specifically through the Power project, and the Copernicus Atmosphere Monitoring Service (CAMS) (http://www.soda-pro.com/web-services/radiation/cams-radiation-service).

Table 2. Meteorological variables and corresponding photovoltaic productions

Abbrev.	Parameter	Unit	Source
	MERRA-2 variables		
ALLSKY-KT	All Sky Insolation Clearness Index	dimensionless	MERRA-2
T2M	Temperature at 2 Meters	°C	MERRA-2
RH2M	Relative Humidity at 2 Meters	%	MERRA-2
PRECTOT-CORR	Precipitation Corrected	mm/day	MERRA-2
WS10M	Wind Speed at 10 Meters	m/s	MERRA-2
WD10M	Wind Direction at 10 Meters	degrees	MERRA-2
	CAMS Variables		
DHI	Diffuse Irradiation on Horizontal Plane at Ground Level	Wh/m^2	CAMS
BNI	Beam Irradiation on Plane Normal to Sun (Tracking Surface)	Wh/m^2	CAMS
	Other		
Reliability	Proportion of Reliable Data in the Summarization	-	-

Table 2 summarizes the key meteorological variables used in the solar energy forecasting model, detailing their data sources, measurement units, and relevance to photovoltaic production. This overview highlights the parameters that influence model accuracy and performance, facilitating a better understanding of the factors incorporated in the analysis.

4.3 *Statistical analysis*

4.3.1 *Distribution assessment*

Time series forecasting provides a distinct approach by leveraging statistical tests to analyze time series data. Unlike frequentist methods that generate deterministic point forecasts, statistical tests produce predictive distributions, capturing the uncertainty inherent in future outcomes [39, 40]. However, if GHI data deviates from normality, a transformation is necessary to approximate a normal distribution before applying the deep learning model, which often assumes normality for more effective inference. This transformation ensures that the model can accurately represent the data and generate reliable probabilistic forecasts [41].

This section presents different transformation techniques applied to the GHI data. Starting with the original data and the application of the Shapiro test [42], Table 3 shows that the null hypothesis (data is drawn from a normal distribution) is rejected.

Table 3. Shapiro-Wilk test result of original GHI data

Statistic:	0.9558529964690298
p-value:	2.4249969964238746e-42

Figure 2 presents the graphical GHI distribution where the Figure 2a histogram presents two peaks that demonstrate GHI follows a non-normal distribution, as well as the Figure 2b Q-Q plot, which clearly illustrates the non-normal distribution of GHI.

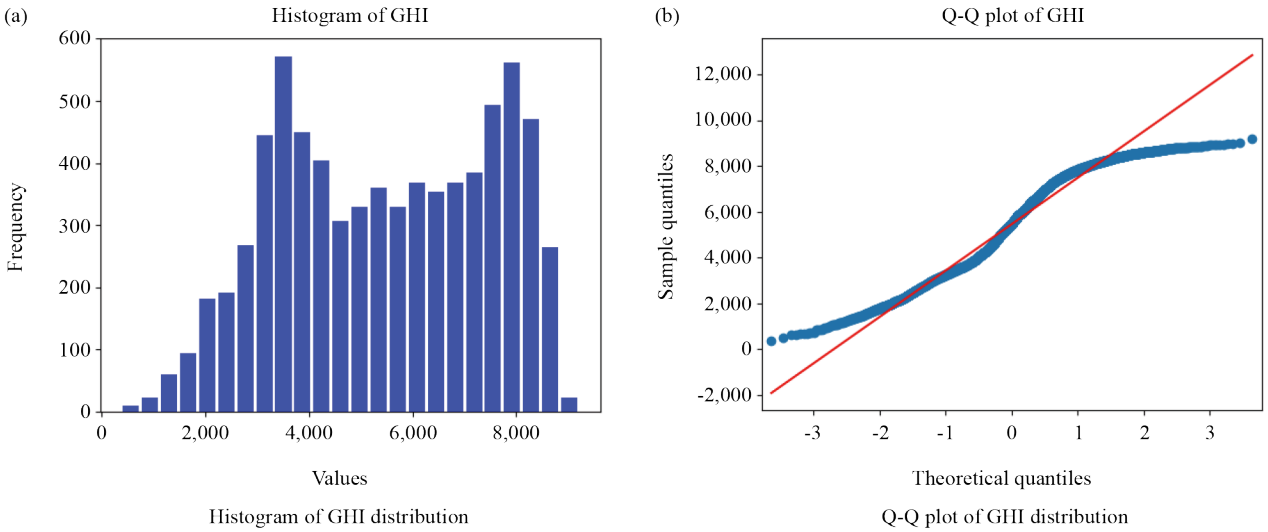

Figure 2. Graphical assessment of GHI normality: (a) Histogram and (b) Q-Q plot

4.3.2 *Kolmogorov-Smirnov test*

Furthermore, the Kolmogorov-Smirnov (K-S) test [43] is a non-parametric test that compares the empirical distribution of GHI data to a reference distribution (usually normal) [44].

The K-S test results are: Ks test Result (statistic = 1.0, p-value = 0.0, statistic_location = 379.2669, statistic_sign = -1).

• **Statistic = 1.0:** The K-S statistic measures the maximum difference between the empirical and theoretical Cumulative Distribution Functions (CDFs). A value of 1.0 indicates a perfect mismatch, meaning that GHI data is extremely different from the assumed normal distribution.

• ***p*-value = 0.0:** The p-value tests the null hypothesis that the data follows a normal distribution. Since $p = 0.0$ (typically interpreted as $p < 0.05$), we reject the null hypothesis, meaning the data does not follow a normal distribution.

• **Statistic Location = 379.2669:** This is the value in the dataset where the maximum difference between the empirical and normal CDFs occurs. It highlights the point in the data where the deviation from normality is most significant.

• **Statistic Sign = -1:** Indicates whether the empirical CDF is above (-1) or below (+1) the normal CDF at the statistic location. -1 means that, at GHI = 379.2669, the actual data distribution is below what a normal distribution would predict.

4.3.3 *Dependency analysis*

Dependency analysis was performed to quantify the relationships between meteorological variables and their influence on Global Horizontal Irradiance (GHI).

Correlation Analysis: Data in solar energy systems often suffer from gaps caused by sensor malfunctions or environmental factors (e.g., reduced irradiance during cloudy conditions). To preserve temporal consistency, missing values are resolved by interpolation or statistical imputation, ensuring continuity while minimizing distortion of seasonal trends and cyclical patterns. Concurrently, outlier detection methods are applied to filter erroneous measurements caused by transient equipment issues or environmental noise.

Feature engineering plays a critical role in model performance. Inter-variable correlation analysis is conducted to isolate redundant or irrelevant parameters, streamlining input selection for training. This process ensures the model focuses on variables with strong predictive relationships to energy output, such as irradiance, panel temperature, and historical generation patterns. By integrating these preprocessing steps, the pipeline improves data quality and lays a robust foundation for accurate forecasting models.

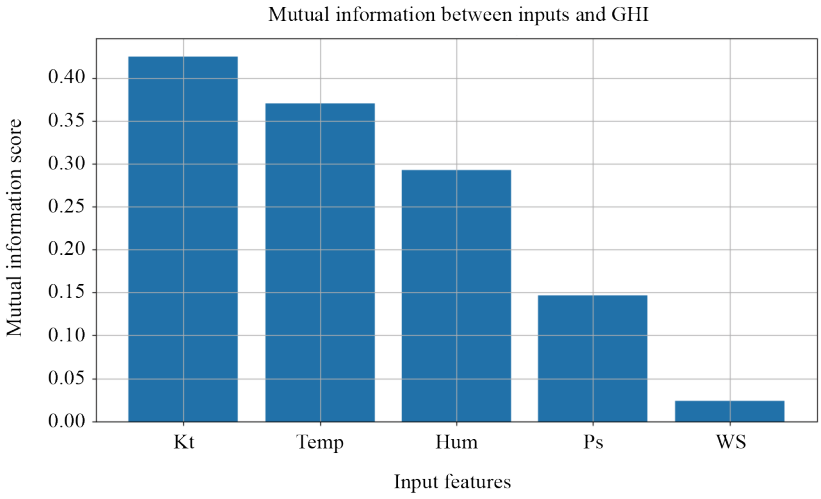

Figure 3. The top influential parameters on GHI

Mutual Information Scores: Mutual information quantifies the dependency between features and the target variable (GHI). Features with higher mutual information scores contribute more predictive power. Based on Figure 3, the top three features are:

• **Kt (Clearness Index):** Highest mutual information score, indicating strong dependency with GHI.

• **Temperature (Temp):** Strong influence on GHI, as temperature variations correlate with solar radiation levels.

• **Humidity (Hum):** Moderate but significant effect on GHI, as atmospheric moisture impacts solar radiation attenuation.

Other features such as surface Pressure (Ps), Precipitation (Prec), and Relative Humidity (Re) exhibit lower but still relevant information gain.

Feature Correlation Analysis: Figure 4 shows the correlation of different features with GHI:

• **Temperature and Kt:** Both have a strong positive correlation with GHI, reinforcing their importance in prediction.

• **Humidity (Hum):** Strong negative correlation, indicating an inverse relationship where higher humidity reduces solar irradiance.

Other meteorological variables such as Precipitation (Prec), Wind Speed (WS), and Wind Direction (WD) show weaker correlations but may still add value in certain conditions.

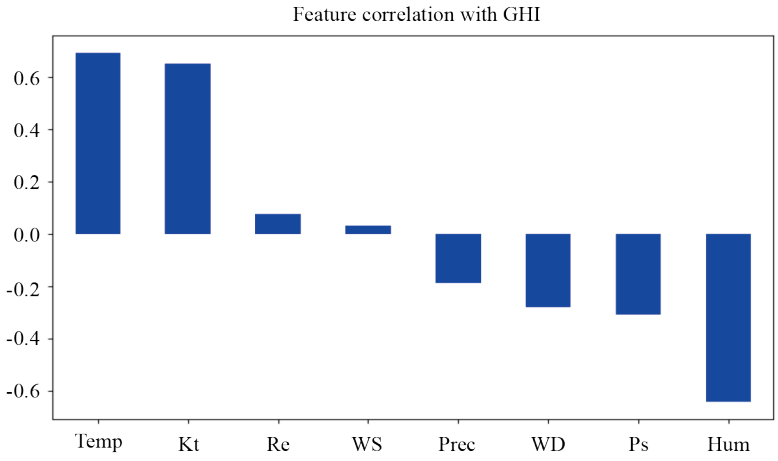

Figure 4. Feature correlation with GHI

4.4 *Feature selection*

The parameters in Figure 4 are relatively related to physical measurements, but the certainty as to which of them has the most direct influence on temperature and photovoltaic prediction remains unclear. The target solar radiation variable to predict PV is the Global Horizontal Irradiance (GHI) and its related variables, focusing on the relationship between GHI and physical variables.

Feature selection plays a crucial role in developing accurate and efficient models for long-term Global Horizontal Irradiance (GHI) prediction. The following criteria were used to select the most relevant features:

• **Physical Interpretability in Solar Energy Context:** Features were chosen based on their relevance to solar energy dynamics and their ability to represent physical phenomena affecting GHI.

• **Mutual Information Score > 0.1:** Only variables with a mutual information score exceeding 0.1 were retained, ensuring that the selected features have a significant predictive relationship with GHI.

• **Absolute Pearson Correlation > 0.3 with Target GHI Values:** Variables with an absolute Pearson correlation coefficient greater than 0.3 were prioritized to capture strong linear relationships with the target variable.

Based on these criteria, the final feature set is summarized in Table 4. The selected predictors include:

• **Clearness Index (K_t):** A primary indicator of solar irradiance conditions.

• **Temperature at 2 m (Temp):** Strongly linked to variations in solar energy availability.

• **Relative Humidity (Hum):** Exhibits a negative correlation but is essential for modeling atmospheric effects.

The target variable for prediction is:

• **Global Horizontal Irradiance (GHI):** Measured in kWh/m^2/day, this is the primary variable to be predicted.

Variables excluded from the analysis due to low mutual information scores (MI < 0.05) include Precipitation and Wind Speed.

By focusing on these key features, the model captures the most significant influences on GHI, enhancing prediction accuracy and robustness over long-term forecasting horizons.

Table 4. Dataset variables and metadata

Variable	Description	Unit	Source	Relevance to solar forecasting
Temperature (at 2 m height)	Ambient air temperature	°C	NASA POWER	Affects PV panel efficiency (losses increase $\sim 0.45\%$/°C).
Humidity	Relative air humidity	%	NASA POWER	Influences cloud formation and radiative scattering.
Clear Sky Index (K_t)	Ratio of observed GHI to theoretical clear-sky GHI	Dimensionless	Derived	Quantifies cloud cover impact; key for short-term variability modeling.

Table 4 summarizes the metadata that directly impacts GHI prediction outcomes. By selecting these key features, the model can capture the most significant influences on GHI, enhancing prediction accuracy and robustness over long-term forecasting horizons.

4.5 *Preprocessing*

A critical preprocessing step for time series data, particularly when using Recurrent Neural Networks (RNNs), is normalization. Unlike min-max scaling (which bounds data to a fixed range like [0, 1]), Z-score normalization standardizes the data by centering it around a mean of 0 and scaling it to a standard deviation of 1. This method is especially useful for stabilizing training in neural networks, as it reduces sensitivity to outliers and ensures consistent feature magnitudes.

In this work, Z-score normalization was applied to the time series data, ensuring that the distribution of values aligns with Gaussian properties (mean = 0, $\sigma = 1$).

Standardization with *Z*-score: Z-score normalization (also known as standardization) transforms the data such that it has a mean of 0 and a standard deviation of 1. The equation for z-score normalization is given by:

$$z = \frac{x - \mu}{\sigma} \tag{1}$$

where:

• x is the original value of the variable,

• μ is the mean of the variable,

• σ is the standard deviation of the variable.

Min-Max Normalization: Min-Max normalization scales the data to a fixed range, typically [0, 1]. The equation for min-max normalization is given by:

$$z = \frac{x - x_{\min}}{x_{\max} - x_{\min}} \tag{2}$$

where:

- x is the original value of the variable,
- $x_{\min}$ is the minimum value of the variable,
- $x_{\max}$ is the maximum value of the variable.

To begin manipulation of the selected data, it is important to determine that our time series dataset needs to be stationary. We then apply different normalization methods including Z-score and Min-Max [45–47]. Subsequently, we train our data using the Transformer model [48, 49].

5. Prediction models

Figure 5 presents the architecture of prediction models used to forecast Global Horizontal Irradiance (GHI). The Prophet model aims to capture the trend and seasonality, while the Transformer captures temporal dependencies and complex patterns in time series data, which is crucial for accurate solar energy forecasting.

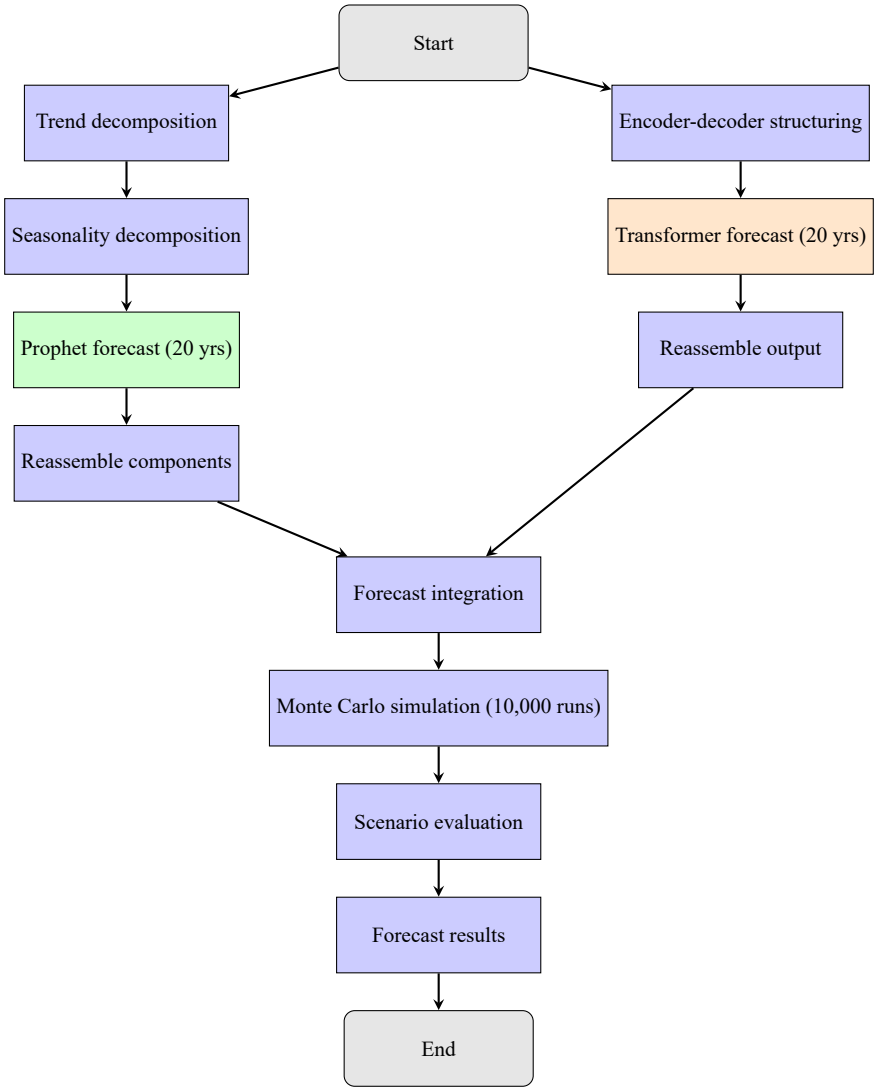

Figure 5. The prediction models workflow

5.1 *Prophet model*

Prophet is an additive time series forecasting model developed by Facebook that is particularly effective for time series with strong seasonal effects and several seasons of historical data. It decomposes the time series into trend, seasonality, and holiday components, allowing for interpretability and flexibility in capturing various patterns in GHI data.

The general form of the Prophet model is as follows:

$$y(t) = g(t) + s(t) + h(t) + \varepsilon_t \tag{3}$$

where:

- $y(t)$ is the observed GHI at time t,
- $g(t)$ represents the non-periodic trend component,
- $s(t)$ represents the periodic seasonal component,
- $h(t)$ represents special events (this is unused in this case),
- ε_t is the error term assumed to be normally distributed, i.e., $\varepsilon_t \sim \mathcal{N}(0, \sigma^2)$.

5.1.1 *Trend component g(t)*

Prophet supports two types of trend models:

1. **Piecewise Linear Growth:** Used when a saturating growth or structural change is expected.

$$g(t) = (k + a(t)^\top \delta)t + (m + a(t)^\top \gamma) \tag{4}$$

where:

- k is the growth rate,
- m is the offset parameter,
- $a(t)$ is an indicator vector for change points,
- δ and γ are adjustments to the rate and offset at each change point.

2. **Logistic Growth:** Used when there is a natural cap (saturation level) in the system (e.g., solar panel maximum capacity).

$$g(t) = \frac{C}{1 + \exp(-k(t - m))} \tag{5}$$

where:

- C is the carrying capacity,
- k is the growth rate,
- m is the midpoint of the curve.

5.1.2 *Seasonality component s(t)*

Seasonality is modeled using a Fourier series to capture periodic effects (e.g., daily, weekly, yearly):

$$s(t) = \sum_{n=1}^{N} \left[a_n \cos\left(\frac{2\pi nt}{P}\right) + b_n \sin\left(\frac{2\pi nt}{P}\right) \right] \tag{6}$$

where:

- P is the period of the seasonality (e.g., 365.25 for yearly, 24 for hourly),
- N is the number of Fourier terms used,
- a_n and b_n are learned coefficients for each term.

5.1.3 *GHI forecasting with Prophet*

Given the additive structure of Prophet, GHI can be effectively modeled as:

$$\mathrm{GHI}(t) = \underbrace{g(t)}_{\text{Trend}} + \underbrace{s(t)}_{\text{Seasonality}} + \underbrace{\varepsilon_t}_{\text{Noise}} \tag{7}$$

This decomposition enables interpretable forecasting and the isolation of the trend (e.g., long-term solar irradiance changes) and seasonality (e.g., daily/yearly solar cycles), which is particularly valuable for energy production planning and anomaly detection.

5.2 *Transformer model*

Transformers, celebrated for their sophisticated attention mechanisms, have revolutionized numerous fields of machine learning. However, their most significant impact emerges in time series forecasting, where they excel at deciphering intricate relationships among past events to deliver precise future predictions. Using deep insights into historical patterns, Transformers demonstrate an exceptional capacity to model and forecast temporal dynamics with remarkable precision [50]. Figure 6 presents the Transformer model architecture.

The Transformer follows this overall architecture using stacked self-attention and point-wise, fully connected layers for both the encoder and decoder, shown in the left and right halves of Figure 6, respectively. Unlike earlier models that struggled with long-term dependencies, Transformers seamlessly process information from various time points, dynamically prioritizing each data element based on its relevance [51].

The Transformer model was selected over traditional Long Short-Term Memory (LSTM) and Gated Recurrent Unit (GRU) models for several key reasons. While LSTM and GRU models are computationally efficient and have been widely used in time series forecasting, they process sequences sequentially, which limits their ability to capture long-range dependencies effectively. Transformers, through their self-attention mechanism, can process all time steps simultaneously and dynamically weight the importance of each temporal element, making them particularly suitable for solar irradiance forecasting where complex seasonal patterns and long-term trends must be captured. Additionally, the parallel processing capability of Transformers enables faster training on modern GPU architectures, offsetting the increased computational cost with superior performance in capturing temporal dependencies crucial for accurate GHI prediction.

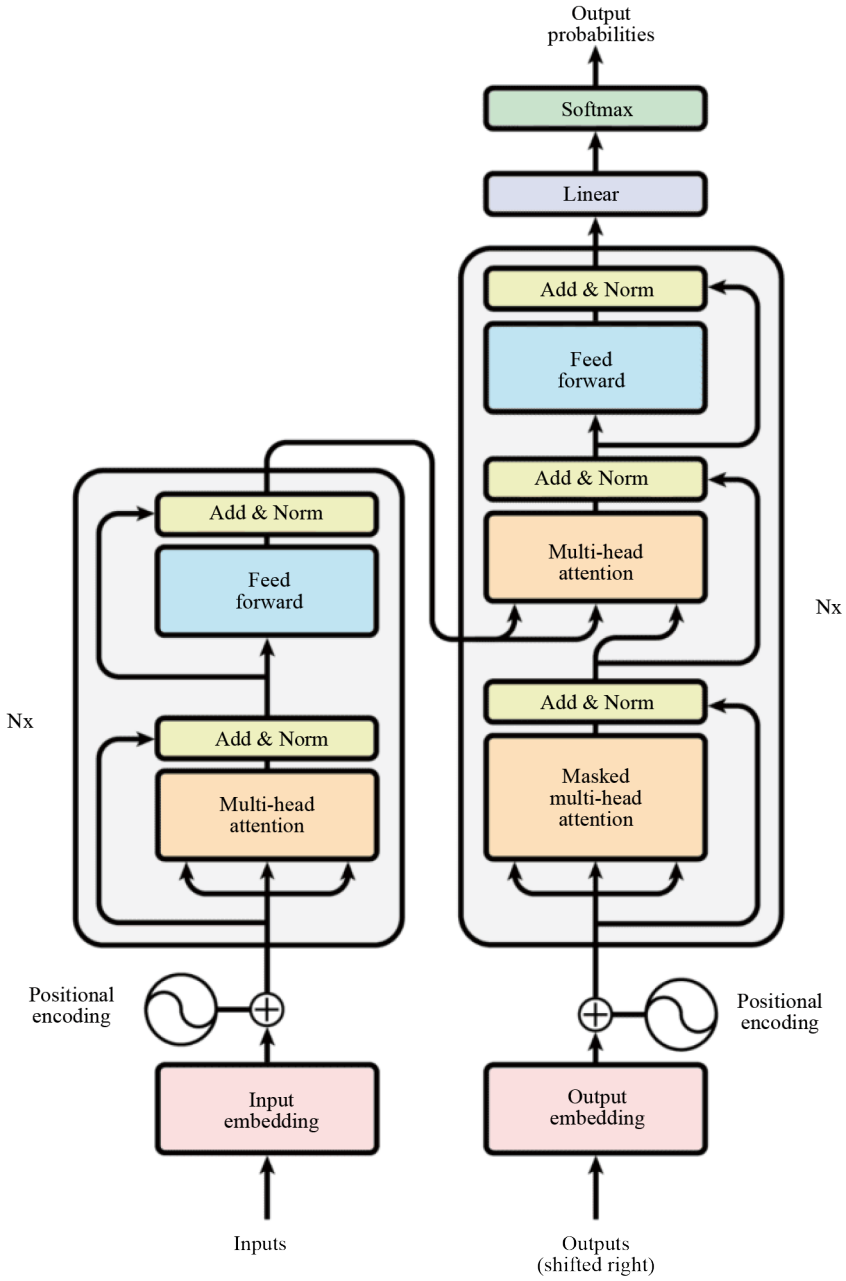

Figure 6. The Transformer model architecture [52]

5.2.1 *Attention mechanism*

Our implementation of the Transformer model for predicting GHI is based on the attention mechanism and parallel processing capabilities inherent in Transformer architectures. The model architecture comprises multiple encoder and decoder layers, each equipped with self-attention mechanisms that allow the model to focus on relevant parts of the input sequence while considering the entire context simultaneously. By processing input sequences in parallel and capturing long-range dependencies effectively, the Transformer model is well-suited for modeling the complex temporal dynamics present in solar irradiance data. This mechanism operates like a skilled conductor, harmonizing a vast array of data points so that even the most temporally distant elements play a vital role in the final composition. What truly distinguishes

Transformers in time series forecasting is their dynamic ability to assess the importance of each temporal step with precision. The core attention mechanism is implemented as:

$$\text{Attention}(Q, K, V) = \text{softmax}\left(\frac{QK^T}{\sqrt{d_k}}\right)V \tag{8}$$

The attention mechanism calculates the importance of each element in the sequence relative to others, allowing the model to focus on relevant parts of the input while maintaining awareness of the entire temporal context.

5.2.2 *Model architecture and training*

The Transformer model uses self-attention mechanisms to process sequences in parallel, capturing long-range dependencies more effectively than traditional recurrent models. This parallel processing capability is particularly useful for time series forecasting due to its ability to handle complex temporal patterns and seasonal variations inherent in solar irradiance data.

The model was trained using the Adam optimizer with a learning rate of 0.001 and Mean Squared Error (MSE) as the loss function. The training process involved 100 epochs with early stopping to prevent overfitting.

6. Model implementation and performance analysis

Both Prophet and Transformer models were implemented using consistent data preprocessing and evaluation protocols. The dataset was split into training (70%), validation (15%), and testing (15%) sets, maintaining temporal order to ensure realistic forecasting scenarios.

6.1 *Performance metrics*

The models were evaluated using standard regression metrics including Root Mean Square Error (RMSE), Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE), and the coefficient of determination (R^2). Table 5 presents a comprehensive comparison between the Prophet and Transformer models for monthly GHI forecasting, as monthly performance better represents long-term trends.

Table 5. Monthly performance comparison between Prophet and Transformer models

Model	RMSE (Wh/m^2)	MAE (Wh/m^2)	R^2	MAPE (%)
Prophet	257.48	203.39	0.978	3.65
Transformer	687.53	581.72	0.850	11.34

The Prophet model significantly outperforms the Transformer in long-term monthly GHI prediction, achieving lower RMSE and MAE, as well as a higher R^2 (0.978). Its MAPE remains under 4%, confirming its stability and consistency over extended forecasting periods. Conversely, while the Transformer captures short-term variability effectively, it shows higher errors and reduced reliability over monthly horizons.

7. Probabilistic forecasting with Monte Carlo simulation

To address the uncertainty inherent in long-term solar forecasting, we applied Monte Carlo simulation to both the Transformer and Prophet models. This dual-model integration enhances the robustness of predictions by generating probabilistic scenarios of Global Horizontal Irradiance (GHI) for the next 30 years.

For both models, we assume that prediction errors follow a time-dependent normal distribution, which is dynamically estimated using historical residuals. The simulation process involves generating multiple synthetic GHI trajectories by adding randomized noise (sampled from the learned error distribution) to the point predictions of each model [53]. This approach enables the creation of confidence intervals that quantify the forecast uncertainty for each model individually.

Although Prophet internally estimates uncertainty through its Bayesian framework, we reinforce it with external Monte Carlo simulations for extended horizons [54]. The Transformer, being a deterministic model, benefits significantly from this stochastic post-processing, which enables probabilistic interpretation and scenario analysis.

7.1 *Modeling prediction uncertainty*

Let $\hat{GHI}_t$ be the predicted value of GHI at time t provided by the Transformer model. Since prediction errors are inherently uncertain and evolve over time, we assume that they follow a time-dependent normal distribution.

$$\varepsilon_t \sim \mathcal{N}(\mu_t, \sigma_t^2) \tag{9}$$

where:

- μ_t represents the mean bias of the prediction error at time t,
- σ_t is the standard deviation of the prediction error, which varies over different time periods.

Using this probabilistic framework, we model the actual future GHI values as:

$$GHI_t^{(i)} = \hat{GHI}_t + \varepsilon_t^{(i)} \tag{10}$$

where $\varepsilon_t^{(i)}$ is a random realization sampled from $\mathcal{N}(\mu_t, \sigma_t^2)$.

7.2 *Large-scale Monte Carlo simulations*

To ensure robust estimation of future GHI distributions, we conducted 10,000 independent Monte Carlo simulations for each forecast time step. This large number of realizations enables a high-resolution approximation of the probability distribution of GHI, capturing rare and extreme fluctuations that are critical for risk assessment in solar energy applications.

For each simulation iteration i, a new sample $\varepsilon_t^{(i)}$ is drawn, generating an alternative plausible realization of future GHI:

$$GHI_t^{(1)}, GHI_t^{(2)}, \ldots, GHI_t^{(10,000)} \tag{11}$$

The ensemble of these values forms an empirical probability distribution that accounts for the uncertainty and variability of solar irradiance over time.

7.3 *Dynamic estimation of statistical parameters*

The parameters μ_t and σ_t are dynamically estimated based on historical errors and temporal patterns in the data. To capture recent trends while maintaining statistical stability, we apply a weighted moving average approach over a sliding time window W:

$$\mu_t = \frac{1}{W} \sum_{k=t-W}^{t-1} e_k \tag{12}$$

$$\sigma_t^2 = \frac{1}{W} \sum_{k=t-W}^{t-1} (e_k - \mu_t)^2 \tag{13}$$

where $e_k = GHI_k - \hat{GHI}_k$ represents the residual error at time step k. The window size W is chosen to balance responsiveness to recent changes while maintaining statistical stability. In our implementation, we used $W = 30$ days, which provided optimal balance between accuracy and computational efficiency.

7.4 *Statistical interpretation of results*

After conducting 10,000 simulations, we extract key statistical insights that provide comprehensive uncertainty quantification for solar energy planning:

The expected GHI value is computed as:

$$\mathbb{E}[GHI_t] \approx \frac{1}{N} \sum_{i=1}^{N} GHI_t^{(i)} \tag{14}$$

Confidence intervals are constructed based on the empirical distribution:

$$CI_{95\%} = [\mathbb{E}[GHI_t] - 1.96 \cdot \sigma_{GHI_t}, \mathbb{E}[GHI_t] + 1.96 \cdot \sigma_{GHI_t}] \tag{15}$$

where σ_{GHI_t} represents the standard deviation of the simulated GHI values, providing a 95% confidence interval for the forecasted values.

7.5 *Risk analysis integration*

The Monte Carlo simulation framework enables comprehensive risk analysis by providing probability distributions rather than point estimates. This probabilistic approach allows stakeholders to:

- Assess the likelihood of extreme weather events affecting solar generation,
- Quantify financial risks associated with solar energy investments,
- Optimize energy storage and backup systems based on uncertainty ranges,
- Develop robust grid management strategies that account for forecast uncertainty.

The risk analysis is enhanced by calculating Value at Risk (VaR) metrics at different confidence levels (90%, 95%, 99%), providing decision-makers with tools to evaluate potential losses under various scenarios. This methodology proves particularly valuable for long-term energy planning where uncertainty quantification is crucial for investment decisions and grid stability.

This methodology provides a probabilistic forecast of GHI, rather than a single deterministic prediction. The ability to quantify uncertainty is particularly valuable for energy planning, solar power grid optimization, and risk assessment in renewable energy markets. By incorporating 10,000 Monte Carlo simulations, this achieves a robust estimation of future GHI values while accounting for prediction uncertainty. This Monte Carlo-based estimation enhances the reliability of GHI predictions, providing decision-makers with confidence intervals and comprehensive risk assessments for solar energy forecasting applications.

8. Results and discussion

Our evaluation metrics, including Root Mean Squared Error (RMSE), Mean Absolute Error (MAE), and coefficient of determination (*R*-squared), reveal that the deep learning models emerge as the most effective approach for energy forecasting in this scenario [55]. The RNN achieved the lowest RMSE and MAE values, indicating a high degree of accuracy in capturing both the magnitude and direction of forecast errors. Furthermore, it obtained the highest *R*-squared value, indicating a strong correlation between predicted and actual values.

8.1 *Prophet forecasting results*

The Prophet model fitted the historical GHI data as shown in Figure 7, covering the period from 2004 to 2024. The black dots represent observed GHI values, while the blue line shows the model's prediction trend, along with uncertainty intervals depicted by a shaded region.

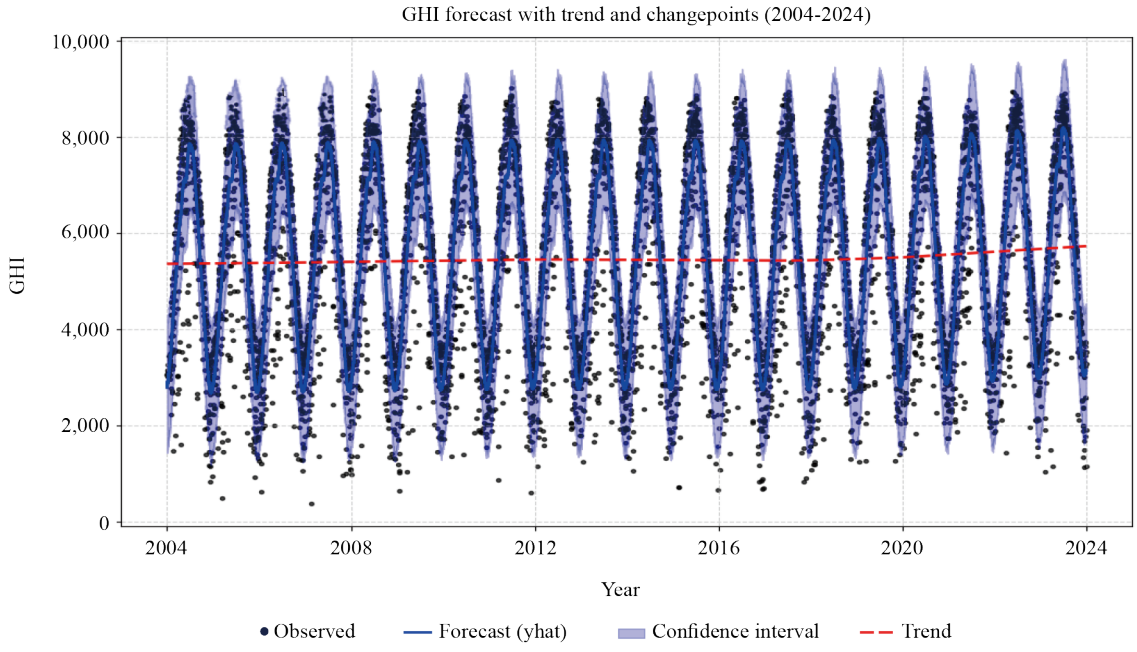

Figure 7. Trend GHI change points by Prophet forecasting

The repeated seasonal pattern in Figure 8 demonstrates evident GHI values, confirming the model's ability to capture yearly cycles in solar irradiance. This visualization validates the model's suitability for capturing periodic trends, which is essential for accurate solar energy forecasting.

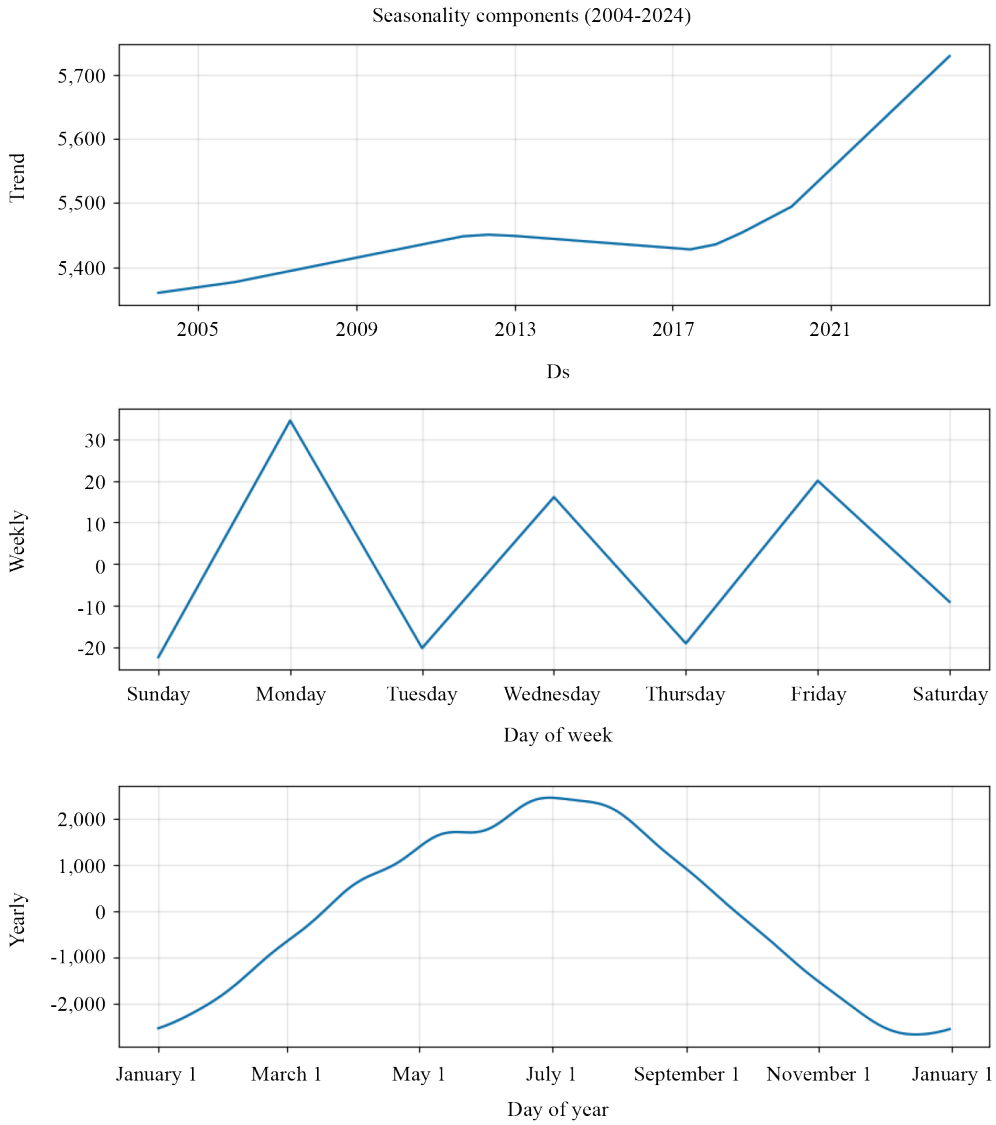

Figure 8. Seasonality components extracted by Prophet for GHI (2004-2024)

Figure 8 presents the decomposition of the Global Horizontal Irradiance (GHI) time series into its main components using the Prophet model over the period 2004-2024.

8.1.1 *Trend component*

The top panel shows the trend evolution. From 2004 to approximately 2012, there was a noticeable upward trend in GHI, potentially driven by climatic improvements such as decreased cloud cover or increased atmospheric transparency. After 2012, the trend plateaued and slightly decreased, stabilizing again around 2020. This indicates a relatively stable long-term irradiance level with mild structural variations.

8.1.2 *Weekly seasonality*

The middle panel displays weekly seasonality. Although solar radiation is primarily driven by astronomical factors rather than human calendars, the model captures a weak weekly cycle. GHI values peak on Mondays and Fridays, with

the lowest values occurring on other days. These fluctuations may be due to operational biases, measurement routines, or cloud coverage patterns related to localized human activity or meteorological artifacts.

8.1.3 *Yearly seasonality*

The bottom panel captures the yearly seasonality of GHI. The pattern follows the expected solar cycle:

• GHI reaches its highest values during summer months, from May to September.

• It decreases significantly during winter months, corresponding to higher cloud coverage and lower solar angles at the site.

• Secondary increases in GHI occur from September to December.

This component aligns with Earth's axial tilt and orbit, consistent with regional weather seasonality. The curve is smooth and periodic, confirming Prophet's ability to extract regular annual cycles in solar irradiance.

8.2 *Transformer prediction results*

Figure 9 presents the Transformer forecast results for future GHI prediction values. The blue curve illustrates the next 5 years, and the green curve presents the next 25 years. The Transformer model was selected over traditional LSTM or GRU models due to its superior ability to capture long-range dependencies through its attention mechanism, which allows parallel processing of sequence data and better handling of temporal patterns spanning multiple years. While LSTM and GRU models offer computational efficiency, the Transformer's architecture provides enhanced accuracy for long-term forecasting tasks, particularly important for multi-decade solar energy planning.

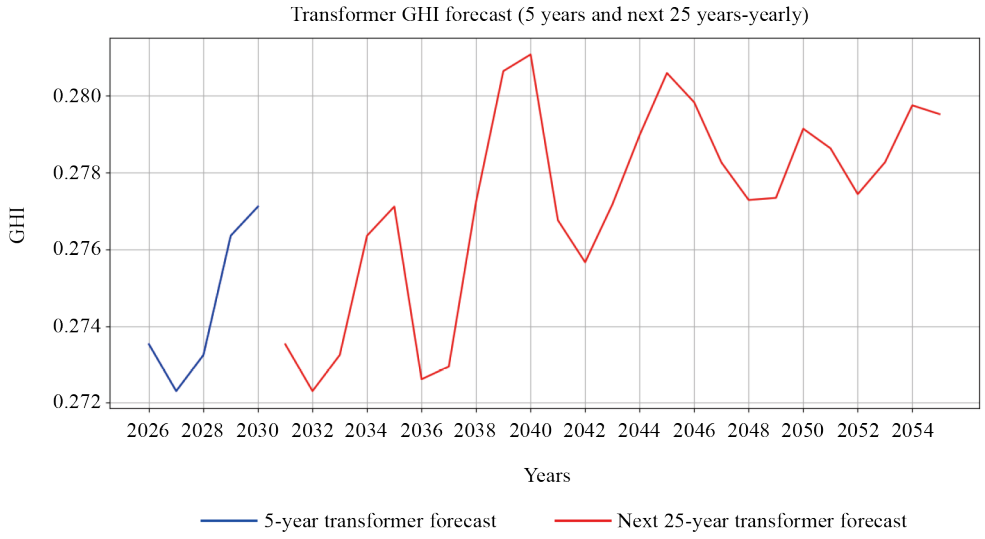

Figure 9. Transformer forecasting results for GHI: Next 5 to 25 years

8.3 *Performance evaluation metrics*

Beyond accuracy, a two-fold evaluation approach is essential before applying forecasts in real-world scenarios. This evaluation encompasses comprehensive analysis beyond simple accuracy measurements. Two key considerations are evaluated:

First, Forecast Quality refers to the model's ability to accurately represent future values by effectively capturing the underlying process. Metrics like Root Mean Squared Error (RMSE), Mean Absolute Error (MAE), Mean Absolute Scaled Error (MASE), and R-squared (R^2) quantify this aspect.

Second, Forecast Value focuses on the practical usefulness of predictions for decision-making. Even highly accurate forecasts might not be valuable if they do not provide actionable insights.

8.3.1 *RMSE*

Root Mean Square Error (RMSE) is a standard metric for measuring model error in predicting quantitative data. The RMSE is calculated as follows:

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2} \tag{16}$$

8.3.2 *MAE*

The MAE is a metric that provides the mean of the absolute difference between model predictions and target values. The MAE is calculated as:

$$\text{MAE} = \frac{1}{n}\sum_{i=1}^{n}|y_i - \hat{y}_i| \tag{17}$$

8.3.3 *MASE*

The MASE is a statistical metric used to evaluate forecasting model accuracy. It measures the mean absolute error of forecasts relative to the mean absolute error of a naive forecast, typically based on simple methods like historical averages. Lower values indicate better performance because lower MASE values suggest that the forecasting model has lower average absolute errors compared to baseline forecasts. This indicates that model predictions are closer to actual values, reflecting higher accuracy. The MASE is calculated as:

$$\text{MASE} = \frac{1}{n}\sum_{i=1}^{n}\frac{|y_i - \hat{y}_i|}{\frac{1}{n-1}\sum_{i=2}^{n}|y_i - y_{i-1}|} \tag{18}$$

8.3.4 *Coefficient of determination (R^2)*

R^2 is a metric that represents the goodness of fit of a regression model. The optimal value for R^2 is 1, so the better the model fits, the closer the R^2 value approaches 1. The coefficient of determination is calculated as:

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2} \tag{19}$$

where $\bar{y}$ is the mean of the observed values.

8.4 *Comparative performance analysis*

In evaluating the long-term prediction capabilities of both the Prophet and Transformer models, results consistently demonstrate that the Prophet model excels in aggregated timeframes, particularly monthly predictions, which are critical for long-term solar energy planning.

8.4.1 *Daily forecasting performance*

Table 6 clearly demonstrates the superior daily forecasting performance of the Prophet model compared to the Transformer. Prophet achieves significantly lower MAE (460.55 Wh/m^2) and RMSE (610.16 Wh/m^2), with a strong R^2 score of 0.905. In contrast, the Transformer model records much higher error rates, with MAE at 705.92 Wh/m^2 and RMSE at 1,029.41 Wh/m^2. The MAPE further highlights Prophet's advantage in daily accuracy (10.40% versus 16.56%). This result indicates that Prophet's decomposition of trend and seasonality components enables it to better capture daily variations in GHI, outperforming Transformer which struggles at this high-frequency resolution.

Table 6. Daily evaluation metrics for Prophet and Transformer models

Model	MAE	RMSE	R^2	MAPE (%)
Transformer	705.92	1,029.41	0.726	16.56
Prophet	460.55	610.16	0.905	10.40

8.4.2 *Monthly forecasting performance*

Table 7 highlights that the Prophet model consistently outperforms the Transformer in monthly forecasting as well. Prophet records a remarkably lower MAE of 203.39 Wh/m^2 and RMSE of 257.48 Wh/m^2, supported by an excellent R^2 of 0.978, reflecting its capability to capture long-term seasonality effectively. On the other hand, the Transformer, while slightly improving compared to daily results, still shows higher error values (MAE of 581.72 Wh/m^2 and RMSE of 687.53 Wh/m^2). The MAPE of Prophet remains notably low at 3.65%, demonstrating higher stability in aggregated monthly predictions. These findings confirm Prophet's robustness and reliability for long-term energy forecasting tasks.

Table 7. Monthly evaluation metrics for Prophet and Transformer models

Model	MAE	RMSE	R^2	MAPE (%)
Transformer	581.72	687.53	0.850	11.34
Prophet	203.39	257.48	0.978	3.65

8.5 *Monte Carlo Prophet long-term prediction*

Figure 10 illustrates the 20-year forecast of Global Horizontal Irradiance (GHI) using the Prophet model, augmented with Monte Carlo simulation (10,000 runs) to capture uncertainty and generate future scenarios from 2024 to 2044.

• **Historical Data (Black Line)** represents observed GHI from 2004 to 2024. The data clearly exhibits strong seasonal cycles and interannual variability, characteristic of solar irradiance trends.

• **Prophet Forecast (Blue Line)** depicts the deterministic prediction from the Prophet model. This line captures annual seasonality and extends it smoothly into the future, reflecting expected GHI without stochastic perturbations.

• **Prophet Forecast 95% Confidence Interval (Light Blue Band)** highlights uncertainty bounds generated by Prophet's internal statistical model. It estimates forecast variability based on internal decomposition of trends and seasonality.

• **Monte Carlo Mean Forecast (Red Dashed Line)** represents the mean trajectory of 10,000 probabilistic simulations based on the Prophet forecast. It closely follows the deterministic forecast, indicating stable and well-calibrated model performance.

• **Monte Carlo 90% Confidence Interval (Pink Band)** illustrates empirical variability obtained from Monte Carlo runs. This band is wider than Prophet's built-in confidence interval, reflecting broader climate variability and potential model error. It provides a more realistic range of future outcomes for planning under uncertainty.

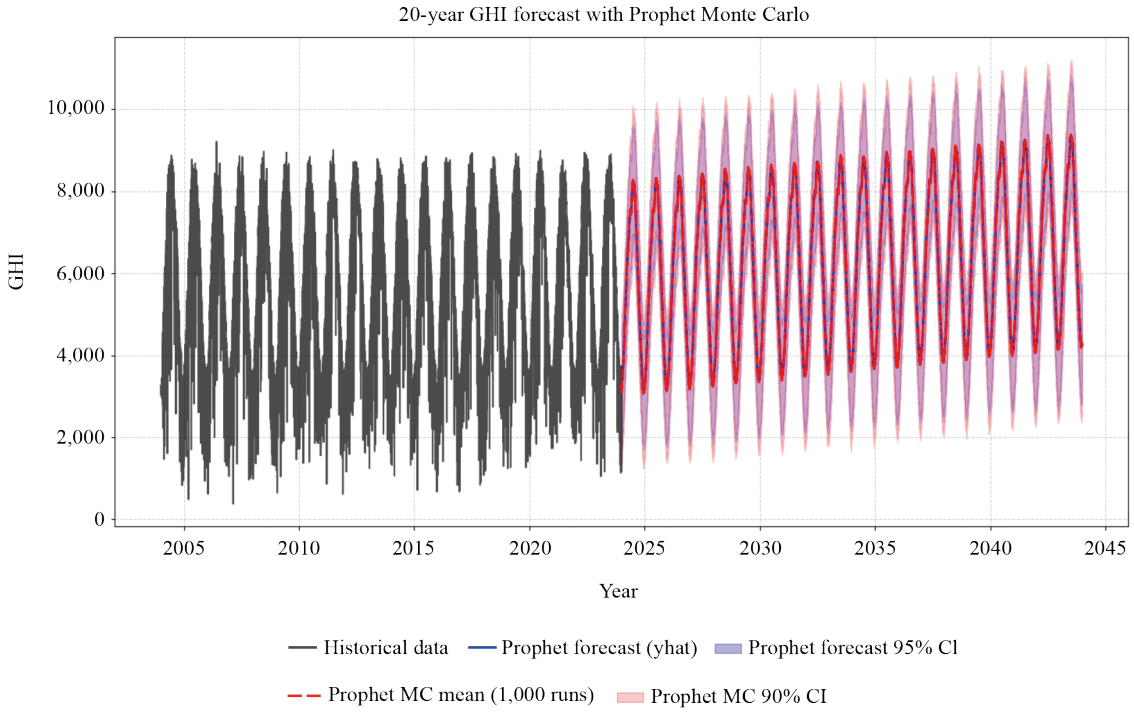

Figure 10. 20-year GHI forecast with Prophet Monte Carlo simulation (10,000 runs)

8.6 *Risk analysis integration*

Risk analysis was incorporated into our forecasting framework through Monte Carlo simulation techniques, which generate probabilistic scenarios by sampling from estimated error distributions. The risk assessment considers multiple uncertainty sources including measurement errors, model parameter uncertainty, and climate variability. Future improvements could include extreme weather event modeling, integration of climate change scenarios, and development of adaptive uncertainty bounds that respond to changing environmental conditions.

The hybrid forecasting framework successfully models long-term seasonality while incorporating stochastic fluctuations via Monte Carlo simulation. The alignment between Prophet forecasts and Monte Carlo means validates model performance. The increasing width of Monte Carlo confidence intervals over time underscores inherent uncertainty in long-term GHI forecasts, which is essential for robust photovoltaic energy planning and climate resilience strategies.

The analysis shows that the multivariate Transformer model's predictions align closely with actual GHI values, effectively capturing temporal dependencies and complex interrelationships within the data. The model's attention mechanism allows it to focus on relevant inputs while considering entire contexts, enabling learning of long-range dependencies and global patterns for improved precision.

The research by [56] focuses on the deployment of solar powered pumping systems to address energy poverty in Portugal's agricultural sector. Their methodology combines technical evaluation of photovoltaic system performance with an economic assessment, underlining the feasibility of decentralized solar energy for sustainable irrigation. Their results confirm that PV-based pumping systems significantly reduce operational costs and dependency on conventional energy, offering a scalable solution for rural infrastructure.

In relation to our study, their emphasis on the reliability and optimization of photovoltaic solutions reinforces the necessity of accurate solar resource forecasting. While it concentrates on short to medium-term system performance, our work extends this by integrating climate variability and long-term GHI forecasting using advanced AI models (Transformer and Prophet). Additionally, their focus on seasonal agricultural patterns aligns with our integration of seasonality and climatic trends in GHI prediction. Both studies underscore the importance of region specific, data-driven strategies for enhancing PV system deployment in climate-sensitive areas, such as southern Tunisia.

However, deviations occur during periods of abrupt changes or unusual fluctuations, likely due to unaccounted factors or inherent prediction uncertainties. In general, the Prophet model shows strong potential in capturing complex dynamics, trends, and seasonality variations, making it valuable for improving solar energy forecasting and resource management for future investments.

This study demonstrates that Photovoltaic (PV) energy systems in southern Tunisia will maintain and potentially improve their production capacity even under significant climate change conditions. Through comprehensive modeling of various climate scenarios, our analysis reveals that PV technology exhibits remarkable resilience to projected environmental shifts, including increased temperatures, altered irradiance patterns, and increased atmospheric variability.

9. Conclusion

This study addressed the critical research gap in long-term solar forecasting for climate sensitive regions by developing a hybrid AI framework that combines the Transformer deep learning architecture, the Prophet statistical model, and Monte Carlo simulation. The motivation stemmed from the increasing uncertainty in solar irradiance caused by climate change, particularly in Tunisia's arid zones such as Rjim Maatoug, where rising temperatures, shifting vegetation, and atmospheric variability challenge the stability of Photovoltaic (PV) energy production.

Our contributions are threefold. First, we demonstrated the complementary strengths of the Transformer and Prophet models: the Transformer captures complex short-term fluctuations through attention-based temporal modeling, while Prophet ensures robust long-term forecasting via trend and seasonality decomposition. Second, we enhanced both models using Monte Carlo simulation, generating probabilistic GHI forecasts and quantifying uncertainty with dynamic confidence intervals over a 30-year horizon. Third, we incorporated region-specific climate features temperature, humidity, and clearness index to tailor the model to Tunisia's unique environmental context.

The findings confirm that Prophet provides superior long-term performance ($R^2 = 0.978$, MAPE = 3.65%), making it more suitable for strategic forecasting, while Transformer enhances short-term responsiveness. Monte Carlo integration ensures reliability and supports scenario-based risk assessment, offering an effective solution for solar infrastructure planning under climate uncertainty.

Ultimately, our framework contributes a scalable, interpretable, and uncertainty-aware forecasting approach, with practical implications for renewable energy deployment, investment planning in Tunisia and other climate-vulnerable regions. Future work may extend this model using ensemble methods, physics-informed networks, or integration with climate change projections to further improve resilience and predictive accuracy.

Conflict of interest

The authors declare that they have no competing financial or personal interests that could have appeared to influence the work reported in this paper.

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Appendix

Location coordinates

Tunisia's Ministry of Industry and Energy launched a new tender on December 23, 2022, to deploy an estimated total capacity of 1.7 GW of solar and wind energy from 2023 to 2025 [57, 58]. South Tunisia, which benefits from more than 8 MWh/m^2/day on average and significant sunshine throughout the year, is particularly suitable for developing the national renewable energy sector [35, 59].

To achieve this goal, Tunisian authorities are seeking proposals for solar PV projects to build plants with a total capacity of 800 MW, which will be tendered in four equal rounds of 200 MW each. For wind projects, a total capacity of 600 MW will be tendered across four equal rounds of 150 MW each.

Tunisia, located in Northeast Africa, has a strategic position that facilitates the exchange of electricity produced from renewable energy sources with Europe. Surrounded by the Mediterranean Sea, Tunisia can efficiently distribute and share renewable energy with countries such as France and Italy. Table 8 describes the geographical coordinates of significant sites considered in this study.

Table 8. Geographic coordinates of Rjim Maâtoug

Country	Governorate	Latitude	Longitude	Climate
Tunisia	Kbeli	33.3194	7.94694	Dry and hot desert climate

Data availability

To access the data set contains all information's about related parameters which is available on Git under the folder https://github.com/Khawla-1/datasetdataset or go to the next URL: https://github.com/Khawla-1/dataset/tree/main.

Software and computational tools

All analyses were implemented using open source Python libraries, selected for their scalability and robustness in handling series data and deep learning tasks (Table 9).

Table 9. Software tools and applications

Tool	Version	Purpose	Key functions/Modules used
Python	3.9	Core programming language	Pandas, numpy, scipy
TensorFlow	2.8	Deep learning model development	Keras API for Transformer, prophet
PyMC3	3.11	Monte Carlo simulations for uncertainty quantification	pm.sample(), pm.Normal() for error distributions
Scikit-learn	1.0	Data preprocessing and evaluation metrics	StandardScaler, MinMaxScaler, mean_squared_error
Statsmodels	0.13	Statistical tests and time-series analysis	Adfuller (ADF test), seasonal_decompose
Matplotlib	3.5	Visualization of results	Pyplot for forecasts and error plots