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Research Article

Identities Concerning Symmetric f-Biderivations on Lattice

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Abstract: Recently, the concept of lattice derivation has been reintroduced in the examination of a variety of issues. The intention of the current research is to examine the structure of symmetric f-biderivation on lattices and to provide a description of some related properties. Furthermore, symmetric f-derivations are used to characterize the distributive and modular lattices in the domain of f-biderivations and explore the features of kernel.

***Keywords*:** lattice, distributive (joinitive) lattice, symmetric f-biderivation, monotone

MSC: 65L05, 34K06, 34K28

1. Introduction

Since its inception, binary operations have been essential to the study of groups, monoids, semigroups, rings, and other algebraic structures explored in abstract algebra [1]. Lattice theory and its applications now rely heavily on binary operations [2, 3]. Binary operations may be used to explain a variety of concepts and qualities, including the idea of the lattice itself [4]. Furthermore, it is not unexpected that a wide range of theoretical and applied disciplines use binary operations with specific characteristics. For instance, aggregation functions on bounded lattices (as binary operations with certain characteristics) are widely used in many different applied science fields, such as access limitations, information retrieval, cryptanalysis, and information theory. Recently, a lot of research has been done on lattice properties [5]. As an extension of the basic connections between fuzzy sets, they are important for fuzzy set and logic theories [6].

The properties of derivations are an intriguing domain of study within ring theory and other algebraic structures. The concept of derivation was initially introduced primarily in relation to ring structures and has a variety of uses [7, 8]. This idea has been expanded to include lattice structures based on meet and join processes by the author in [9]. Consequently, the operations 'addition' and 'multiplication' undergo a fascinating translation into the domain of lattice theory, being reimagined as the lattice operations $\vee$ and $\wedge$, respectively. This conceptual metamorphosis occurs within the captivating realm of a fictitious perspective applied to lattices, unveiling a new dimension of understanding and potential application. At the same time, BCI algebras, left (right) derivations, f-derivations, (f, g)-derivations, bi-derivations, and n-derivations on lattices were all derived using the concept of ring theory. For some incredible findings, see [10–13] and the references therein.

Definition 1 [2] Let Λ be a non-empty set endowed with the operations $\vee$ and $\wedge$. A lattice $(\Lambda, \wedge, \vee)$ is characterized as a set Λ that satisfies the following properties:

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1. $\kappa \wedge \kappa = \kappa$, $\kappa \vee \kappa = \kappa$;
2. $\kappa \wedge \zeta = \zeta \wedge \kappa$, $\kappa \vee \zeta = \zeta \vee \kappa$;
3. $(\kappa \wedge \zeta) \wedge \gamma = \kappa \wedge (\zeta \wedge \gamma)$, $(\kappa \vee \zeta) \vee \gamma = \kappa \vee (\zeta \vee \gamma)$,
4. $(\kappa \wedge \zeta) \vee \kappa = \kappa$, $(\kappa \vee \zeta) \wedge \kappa = \kappa$ for all $\kappa, \zeta, \gamma \in \Lambda$.

Definition 2 [2] A lattice Λ is characterized as distributive if it adheres to either of the identities (1) or (2)

1. $\kappa \wedge (\zeta \vee \gamma) = (\kappa \wedge \zeta) \vee (\kappa \wedge \gamma)$,
2. $\kappa \vee (\zeta \wedge \gamma) = (\kappa \vee \zeta) \wedge (\kappa \vee \gamma)$.

In the context of any lattice structure, conditions (1) and (2) hold equivalently valid.

Definition 3 [4] A lattice, as referenced in Λ, is considered modular if it satisfies the following identity as specified in

$$\text{If } \kappa \leq \gamma, \text{ then } \kappa \vee (\zeta \wedge \gamma) = (\kappa \vee \zeta) \wedge \gamma. \tag{1}$$

Definition 4 [5] A function $d : \Lambda \longrightarrow \Lambda$ is referred to as a derivation on Λ if it satisfies the following condition for every $\kappa, w \in \Lambda$,

$$d(\kappa \wedge w) = (d(\kappa) \wedge w) \vee (\kappa \wedge d(w)). \tag{2}$$

Example 1 Let $\Lambda = \{0, a, b, c, 1\}$ be a lattice describe as below (Figure 1). It is easily seen that a mapping $d : \Lambda \longrightarrow \Lambda$ will be a derivation.

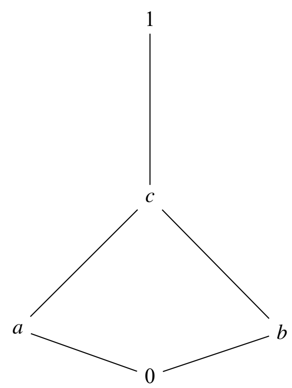

Figure 1. Lattice with 0 (least element) and 1 (greatest element)

Definition 5 [14] A function $d : \Lambda \longrightarrow \Lambda$ is designated as a f-derivation on Λ if it fulfills the following criterion for every $\kappa, w \in \Lambda$,

$$d(\kappa \wedge w) = (d(\kappa) \wedge f(w)) \vee (f(\kappa) \wedge d(w)). \tag{3}$$

Example 2 Let $\Lambda = \{0, a, b, c, 1\}$ be a lattice describe as below. It is easily seen that a mapping $d : \Lambda \longrightarrow \Lambda$ such that $d(a) = a$, $d(b) = b$, $d(c) = c$, $d(c) = 1$, $d(0) = 0$ is not a derivation on Λ. Define a function f such that $f(a) = a$, $f(b) = a$, $f(c) = 1$, $f(1) = 1$, $f(0) = 0$. Then d will be a f-derivation on Λ.

Example 3 Let Λ be a lattice and $k \in \Lambda$. Define a function $d : \Lambda \longrightarrow \Lambda$ by $dw = fw \vee k$ for all $w \in \Lambda$, where $f : \Lambda \longrightarrow \Lambda$ satisfies $f(v \vee u) = fv \vee fu$ for all $u, v \in \Lambda$. Then d is an f-derivation on Λ. Additionally, d will be an isotone derivation, in case f is an increasing function.

Definition 6 [3] A function $\mathscr{D} : \Lambda \times \Lambda \longrightarrow \Lambda$ is called symmetric bi-derivation on Λ, if it is symmetric and satisfy the following condition in each slot for every $\kappa, w, \gamma \in \Lambda$,

$$\mathscr{D}(\kappa \wedge w, \gamma) = (\mathscr{D}(\kappa, \gamma) \wedge w) \vee (\kappa \wedge \mathscr{D}(w, \gamma)). \tag{4}$$

In their ground breaking research, the authors in [15] have ingeniously developed the revolutionary concept of lattice derivation, delving deeply into its fascinating features with keen insight. They meticulously provided a set of stringent equivalent specifications, revealing the conditions under which the derivation emerges as an isotone for a diverse range of lattices, namely, distributive lattices, modular lattices, and those boasting a greatest element. Through their astute exploration, they have utilized isotone derivation as a powerful tool for characterizing both modular and distributive lattices in profound ways. Moreover, they compellingly demonstrated that, within the realm of distributive lattices Λ, the relationship between $D(\Lambda)$ and Λ is not just similar but indeed isomorphic, while the fixed set $Fix_d(\Lambda)$ is unequivocally an ideal of $D(\Lambda)$, contingent upon d being an isotone derivation of lattice Λ. This fascinating discussion genuinely contributes to our understanding of lattice structures by providing broad perspectives and encouraging more research.

Researchers discussed a few relevant features in the work mentioned above on the generalization of the derivations on lattices. Isotone derivation was used to describe distributive and modular lattices. Our study was primarily inspired by the research conducted in [5, 7, 8, 10, 12, 15].

We propose several results on derivations inspired by literature review in order to gain more intriguing properties of the derivations on the lattice. Its significant and distinctive qualities will also be discussed. The study of lattice derivation features is examined in this research from a specifically theoretical perspective.

2. Main results on symmetric *f*-biderivations

We begin with the following conceptual facts:

Definition 7 [14] A function $\varpi : \Lambda \times \Lambda \longrightarrow \Lambda$ is called symmetric f-biderivation on Λ, if it is symmetric and satisfy the following condition in each slot for every $\kappa, w, \gamma \in \Lambda$,

$$\varpi(\kappa \wedge w, \gamma) = (\varpi(\kappa, \gamma) \wedge f(w)) \vee (f(\kappa) \wedge \varpi(w, \gamma)). \tag{5}$$

Example 4 Let $\Lambda = \{0, 1, 2\}$ be a given lattice described as below:

$$0 \longrightarrow 1 \longrightarrow 2$$

Define $\varpi : \Lambda \times \Lambda \longrightarrow \Lambda$ and $f : \Lambda \longrightarrow \Lambda$ as

$$\varpi(\kappa, \zeta) = \begin{cases} 1, & \text{if } (\kappa, \zeta) = (0, 0) \\ 1, & \text{if } (\kappa, \zeta) = (0, 1) \\ 1, & \text{if } (\kappa, \zeta) = (1, 0) \\ 0, & \text{if } (\kappa, \zeta) = (0, 2) \\ 0, & \text{if } (\kappa, \zeta) = (2, 0) \\ 0, & \text{if } (\kappa, \zeta) = (1, 1) \\ 0, & \text{if } (\kappa, \zeta) = (2, 2) \\ 0, & \text{if } (\kappa, \zeta) = (1, 2) \\ 0, & \text{if } (\kappa, \zeta) = (2, 1) \end{cases}$$

and

$$f(\kappa) = \begin{cases} 1, & \text{if } \kappa = 0 \\ 2, & \text{if } \kappa = 1 \\ 2, & \text{if } \kappa = 2 \end{cases}.$$

We can verify with ease that ϖ will be a f-biderivation on Λ for each $\kappa, \zeta \in \Lambda$.

Next, we will discuss the properties of symmetric f-biderivations as follows:

Theorem 1 If ϖ is a symmetric f-biderivation, then for each $\kappa, \zeta \in \Lambda$ following holds:

(1) $\varpi(\kappa, \kappa) = f(\kappa) \wedge \varpi(\kappa, \kappa)$.

(2) $\varpi(\kappa, \kappa) \leq f(\kappa)$.

Proof.

(1)

$$\begin{aligned} \varpi(\kappa, \kappa) &= \varpi(\kappa \wedge \kappa, \kappa) \\ &= (\varpi(\kappa, \kappa) \wedge f(\kappa)) \vee (f(\kappa) \wedge \varpi(\kappa, \kappa)) \\ &= f(\kappa) \wedge \varpi(\kappa, \kappa) \end{aligned} \tag{6}$$

(2) Direct implication from (1). □

Theorem 2 If ϖ is a symmetric f-biderivation, then $\varpi(\kappa, \kappa) \wedge \varpi(\zeta, \zeta) \leq \varpi(\kappa \wedge \zeta, \kappa \wedge \zeta)$ for each $\kappa, \zeta \in \Lambda$.

Proof.

$$\begin{aligned} \varpi(\kappa \wedge \zeta, \kappa \wedge \zeta) &= (\varpi(\kappa, \kappa \wedge \zeta) \wedge f(\zeta)) \vee (f(\kappa) \wedge \varpi(\zeta, \kappa \wedge \zeta)) \\ &= (((\varpi(\kappa, \kappa) \wedge f(\zeta)) \vee (f(\kappa) \wedge \varpi(\kappa, \zeta))) \wedge f(\zeta)) \end{aligned}$$

$$\vee(f(\kappa)\wedge((\varpi(\zeta,\kappa)\wedge f(\zeta))\vee(f(\kappa)\wedge\varpi(\zeta,\zeta))))$$

$$=(\varpi(\kappa,\kappa)\wedge f(\zeta)\wedge f(\zeta))\vee(f(\kappa)\wedge\varpi(\kappa,\zeta)\wedge f(\zeta))$$

$$\vee(f(\kappa)\wedge\varpi(\zeta,\kappa)\wedge f(\zeta))\vee(f(\kappa)\wedge f(\kappa)\wedge\varpi(\zeta,\zeta))$$

$$=(\varpi(\kappa,\kappa)\wedge f(\zeta))\vee(f(\kappa)\wedge\varpi(\kappa,\zeta)\wedge f(\zeta))$$

$$\vee(f(\kappa)\wedge\varpi(\zeta,\kappa)\wedge f(\zeta))\vee(f(\kappa)\wedge\varpi(\zeta,\zeta))$$

Thus

$$\varpi(\kappa,\kappa)\wedge f(\zeta)\leq\varpi(\kappa\wedge\zeta,\kappa\wedge\zeta) \tag{7}$$

$$\varpi(\zeta,\zeta)\wedge f(\kappa)\leq\varpi(\kappa\wedge\zeta,\kappa\wedge\zeta) \tag{8}$$

Since $\varpi(\kappa,\kappa)\leq f(\kappa)$, we have $\varpi(\kappa,\kappa)\wedge\varpi(\zeta,\zeta)\leq f(\kappa)\wedge\varpi(\zeta,\zeta)$. Hence for each $\kappa,\zeta\in\Lambda$, we obtain

$$\varpi(\kappa,\kappa)\wedge\varpi(\zeta,\zeta)\leq\varpi(\kappa\wedge\zeta,\kappa\wedge\zeta). \tag{9}$$

□

Theorem 3 If ϖ is a symmetric f-biderivation, then $\varpi(\kappa,\zeta)\leq f(\kappa)\wedge f(\zeta)$ for each $\kappa,\zeta\in\Lambda$.
Proof.

$$\begin{aligned}\varpi(\kappa,\zeta)&=\varpi(\kappa,\zeta\wedge\zeta)\\&=(\varpi(\kappa,\zeta)\wedge f(\zeta))\vee(f(\zeta)\wedge\varpi(\kappa,\zeta))\\&=\varpi(\kappa,\zeta)\wedge f(\zeta)\end{aligned} \tag{10}$$

and thus $\varpi(\kappa,\zeta)\leq f(\zeta)$. In addition,

$$\begin{aligned}\varpi(\kappa,\zeta)&=\varpi(\kappa\wedge\kappa,\zeta)\\&=(\varpi(\kappa,\zeta)\wedge f(\kappa))\vee(f(\kappa)\wedge\varpi(\kappa,\zeta))=\varpi(\kappa,\zeta)\wedge f(\kappa)\end{aligned} \tag{11}$$

and so $\varpi(\kappa,\zeta)\leq f(\kappa)$. Therefore, $\varpi(\kappa,\zeta)\leq f(\kappa)\wedge f(\zeta)$ for each $\kappa,\zeta\in\Lambda$. □

Theorem 4 If ϖ is a symmetric f-biderivation, then $\varpi(\kappa\wedge\zeta,\, \kappa\wedge\zeta)\leq f(\kappa)\vee f(\zeta)$ for each $\kappa,\, \zeta\in\Lambda$.
Proof.

$$\begin{aligned}
\varpi(\kappa\wedge\zeta,\, \kappa\wedge\zeta) &= (\varpi(\kappa,\, \kappa\wedge\zeta)\wedge f(\zeta))\vee(f(\kappa)\wedge\varpi(\zeta,\, \kappa\wedge\zeta))\\
&= (((\varpi(\kappa,\, \kappa)\wedge f(\zeta))\vee(f(\kappa)\wedge\varpi(\kappa,\, \zeta)))\wedge f(\zeta))\\
&\quad \vee(f(\kappa)\wedge((\varpi(\zeta,\, \kappa)\wedge f(\zeta))\vee(f(\kappa)\wedge\varpi(\zeta,\, \zeta))))\\
&= (\varpi(\kappa,\, \kappa)\wedge f(\zeta)\wedge f(\zeta))\vee(f(\kappa)\wedge\varpi(\kappa,\, \zeta)\wedge f(\zeta))\\
&\quad \vee(f(\kappa)\wedge\varpi(\zeta,\, \kappa)\wedge f(\zeta))\vee(f(\kappa)\wedge f(\kappa)\wedge\varpi(\zeta,\, \zeta))\\
&= (\varpi(\kappa,\, \kappa)\wedge f(\zeta))\vee(f(\kappa)\wedge\varpi(\kappa,\, \zeta)\wedge f(\zeta))\\
&\quad \vee(f(\kappa)\wedge\varpi(\zeta,\, \kappa)\wedge f(\zeta))\vee(f(\kappa)\wedge\varpi(\zeta,\, \zeta))\\
&= (\varpi(\kappa,\, \kappa)\wedge f(\zeta))\vee\varpi(\kappa,\, \zeta)\vee\varpi(\zeta,\, \kappa)\vee(f(\kappa)\wedge\varpi(\zeta,\, \zeta))
\end{aligned} \tag{12}$$

By the previous theorem,

$$\varpi(\kappa,\, \zeta)\vee\varpi(\zeta,\, \kappa)\leq f(\kappa)\vee f(\zeta). \tag{13}$$

Moreover, it is clear that

$$(\varpi(\kappa,\, \kappa)\wedge f(\zeta))\vee(f(\kappa)\wedge\varpi(\zeta,\, \zeta))\leq f(\kappa)\vee f(\zeta). \tag{14}$$

Therefore,

$$\varpi(\kappa\wedge\zeta,\, \kappa\wedge\zeta)\leq f(\kappa)\vee f(\zeta). \tag{15}$$

□

Theorem 5 If ϖ is a monotone symmetric f-biderivation, then for each $\kappa,\, \zeta\in\Lambda$

$$\varpi(\kappa,\, \kappa)=(\varpi(\kappa\vee\zeta,\, \kappa\vee\zeta)\wedge f(\kappa))\vee\varpi(\kappa,\, \kappa). \tag{16}$$

Proof. If $\kappa\leq\zeta$, then $\varpi(\kappa,\, \kappa)\leq\varpi(\zeta,\, \zeta)$ for each $\kappa,\, \zeta\in\Lambda$. We find

$$\varpi(\kappa, \kappa) \le \varpi(\kappa \vee \zeta, \kappa \vee \zeta) \le f(\kappa \vee \zeta). \tag{17}$$

Next consider the case if $\zeta \le \kappa$, then $\varpi(\zeta, \zeta) \le \varpi(\kappa, \kappa)$ and we obtain

$$\varpi(\kappa, \kappa) = \varpi(\kappa \vee \zeta, \kappa \vee \zeta) \le f(\kappa \vee \zeta). \tag{18}$$

Thus, we conclude $\varpi(\kappa, \kappa) \le f(\kappa \vee \zeta)$. Which implies that

$$\begin{aligned}
\varpi(\kappa, \kappa) &= \varpi((\kappa \vee \zeta) \wedge \kappa, (\kappa \vee \zeta) \wedge \kappa) \\
&= (\varpi(\kappa \vee \zeta, (\kappa \vee \zeta) \wedge \kappa) \wedge f(\kappa)) \vee (f(\kappa \vee \zeta) \wedge \varpi(\kappa, (\kappa \vee \zeta) \wedge \kappa)) \\
&= (((\varpi(\kappa \vee \zeta, \kappa \vee \zeta) \wedge f(\kappa)) \vee (\varpi(\kappa \vee \zeta, \kappa) \wedge f(\kappa \vee \zeta))) \wedge f(\kappa)) \\
&\quad \vee (f(\kappa \vee \zeta) \wedge ((\varpi(\kappa, \kappa \vee \zeta) \wedge f(\kappa)) \vee (\varpi(\kappa, \kappa) \wedge f(\kappa \vee \zeta)))) \\
&= ((\varpi(\kappa \vee \zeta, \kappa \vee \zeta) \wedge f(\kappa)) \vee (\varpi(\kappa \vee \zeta, \kappa) \wedge f(\kappa \vee \zeta) \wedge f(\kappa)) \\
&\quad \vee ((f(\kappa \vee \zeta) \wedge \varpi(\kappa, \kappa \vee \zeta) \wedge f(\kappa)) \vee (\varpi(\kappa, \kappa) \wedge f(\kappa \vee \zeta))) \\
&= ((\varpi(\kappa \vee \zeta, \kappa \vee \zeta) \wedge f(\kappa)) \vee (\varpi(\kappa \vee \zeta, \kappa) \wedge f(\kappa \vee \zeta) \wedge f(\kappa)) \\
&\quad \vee (\varpi(\kappa, \kappa) \wedge f(\kappa \vee \zeta))) \\
&= ((\varpi(\kappa \vee \zeta, \kappa) \vee \varpi(\kappa \vee \zeta, \zeta)) \wedge f(\kappa)) \vee (\varpi(\kappa \vee \zeta, \kappa) \wedge f(\kappa \vee \zeta) \wedge f(\kappa)) \\
&\quad \vee (\varpi(\kappa, \kappa) \wedge f(\kappa \vee \zeta)) \\
&= ((\varpi(\kappa, \kappa) \vee \varpi(\zeta, \kappa) \vee \varpi(\zeta, \zeta)) \wedge f(\kappa)) \vee (\varpi(\kappa \vee \zeta, \kappa) \wedge f(\kappa \vee \zeta) \wedge f(\kappa)) \\
&\quad \vee (\varpi(\kappa, \kappa) \wedge f(\kappa \vee \zeta)) \\
&= ((\varpi(\kappa, \kappa) \vee \varpi(\zeta, \kappa) \vee \varpi(\zeta, \zeta)) \wedge f(\kappa)) \vee ((\varpi(\kappa, \kappa) \\
&\quad \vee \varpi(\zeta, \kappa)) \wedge f(k \vee \zeta) \wedge f(\kappa)) \vee \varpi(\kappa, \kappa)
\end{aligned}$$

$$= ((\varpi(\kappa, \kappa) \vee \varpi(\zeta, \kappa) \vee \varpi(\zeta, \zeta)) \wedge f(\kappa)) \vee (\varpi(\kappa, \kappa) \wedge f(\kappa \vee \zeta) \wedge f(\kappa))$$

$$\vee (\varpi(\zeta, \kappa) \wedge f(\kappa \vee \zeta) \wedge f(\kappa)) \vee \varpi(\kappa, \kappa)$$

$$= ((\varpi(\kappa, \kappa) \vee \varpi(\zeta, \kappa) \vee \varpi(\zeta, \zeta)) \wedge f(\kappa)) \vee \varpi(\kappa, \kappa)$$

$$\vee (\varpi(\zeta, \kappa) \wedge f(\kappa)) \vee \varpi(\kappa, \kappa))$$

$$=((\varpi(\kappa, \kappa) \vee \varpi(\zeta, \kappa) \vee \varpi(\zeta, \zeta)) \wedge f(\kappa)) \vee \varpi(\kappa, \kappa)$$

$$= (\varpi(\kappa \vee \zeta, \kappa \vee \zeta) \wedge f(\kappa)) \vee \varpi(\kappa, \kappa).$$

This completes the proof. □

Example 5 We observe the following by putting values in Example 4 to validate the conclusion of Theorem 5. Consider for each $\kappa, \zeta \in \Lambda$

$$\varpi(\kappa, \kappa) = (\varpi(\kappa \vee \zeta, \kappa \vee \zeta) \wedge f(\kappa)) \vee \varpi(\kappa, \kappa). \tag{19}$$

If $\kappa = 0$, $\zeta = 1$, then

$$[(\varpi(0 \vee 1, 0 \vee 1) \wedge f(0)) \vee \varpi(0, 0)] = [(\varpi(1, 1) \wedge f(0)) \vee \varpi(0, 0)] = (0 \wedge 1) \vee 1 = 0 \vee 1 = \varpi(0, 0). \tag{20}$$

If $\kappa = 1$, $\zeta = 0$, then

$$[(\varpi(1 \vee 0, 1 \vee 0) \wedge f(1)) \vee \varpi(1, 1)] = [(\varpi(1, 1) \wedge 2) \vee \varpi(1, 1)] = (0 \wedge 2) \vee 0 = 0 \vee 0 = \varpi(1, 1). \tag{21}$$

If $\kappa = 0$, $\zeta = 2$, then

$$[(\varpi(0 \vee 2, 0 \vee 2) \wedge f(0)) \vee \varpi(0, 0)] = [(\varpi(2, 2) \wedge f(0)) \vee \varpi(0, 0)] = (0 \wedge 1) \vee 1 = 0 \vee 1 = \varpi(0, 0). \tag{22}$$

If $\kappa = 1$, $\zeta = 2$, then

$$[(\varpi(1 \vee 2, 1 \vee 2) \wedge f(1)) \vee \varpi(1, 1)] = [(\varpi(2, 2) \wedge 2) \vee \varpi(1, 1)] = (0 \wedge 2) \vee 0 = 0 \vee 0 = \varpi(2, 2). \tag{23}$$

If $\kappa = 2$, $\zeta = 0$, then

$$[(\varpi(2\vee 0,\, 2\vee 0)\wedge f(2))\vee \varpi(2,\, 2)] = [(\varpi(2,\, 2)\wedge 2)\vee \varpi(2,\, 2)] = (0\wedge 2)\vee 0 = 0\vee 0 = \varpi(2,\, 2). \tag{24}$$

If $\kappa = 2$, $\zeta = 1$, then

$$[(\varpi(2\vee 1,\, 2\vee 1)\wedge f(2))\vee \varpi(2,\, 2)] = [(\varpi(2,\, 2)\wedge 2)\vee \varpi(2,\, 2)] = (0\wedge 2)\vee 0 = 0\vee 0 = \varpi(2,\, 2). \tag{25}$$

Our next attempt is to discuss the kernel. Consider the kernel of a symmetric f-biderivation ϖ, which is defined as a set

$$\mathfrak{K}_r(\varpi) = \{k\in\Lambda \mid \varpi(k,\, 0) = 0\}. \tag{26}$$

Proposition 1 Given a lattice Λ having least element 0. Then $k\wedge w\in\mathfrak{K}_r(\varpi)$ for every $k,\, w\in\mathfrak{K}_r(\varpi)$.

Proof. We are given that $k,\, w\in\Lambda$. By definition of $\mathfrak{K}_r(\varpi)$, we obtain $\varpi(k,\, 0) = 0$, $\varpi(w,\, 0) = 0$. Assume that

$$\begin{aligned}\varpi(k\wedge w,\, 0) &= (\varpi(k,\, 0)\wedge f(w))\vee(f(k)\wedge\varpi(w,\, 0))\\ &= (0\wedge f(w))\vee(f(k)\wedge 0)\\ &= 0\vee 0\\ &= 0.\end{aligned} \tag{27}$$

Hence $k\wedge w\in\mathfrak{K}_r(\varpi)$ for every $k,\, w\in\mathfrak{K}_r(\varpi)$. □

Proposition 2 Given a monotone symmetric f-biderivation on a lattice Λ having least element 0. If $k\leq w$ and $w\in\mathfrak{K}_r(\varpi)$, then $k\in\mathfrak{K}_r(\varpi)$.

Proof. We are given that $w\in\Lambda$, by definition of $\mathfrak{K}_r(\varpi)$, we obtain $\varpi(w,\, 0) = 0$. Assume that $k\leq w$, it follows that

$$\varpi(k,\, 0)\leq\varpi(w,\, 0) = 0, \quad \text{that is,} \quad \varpi(k,\, 0) = 0. \tag{28}$$

Consider

$$\begin{aligned}\varpi(k,\, 0) &= \varpi(k\wedge w,\, 0)\\ &= (\varpi(k,\, 0)\wedge f(w))\vee(f(k)\wedge\varpi(w,\, 0))\end{aligned}$$

$$= (0 \wedge f(w)) \vee (f(k) \wedge 0)$$

$$= 0 \vee 0$$

$$= 0. \tag{29}$$

We conclude $k \in \mathfrak{K}_r(\varpi)$ as $\varpi(k, 0) = 0$. □

If the f-biderivation is not monotone, then the preceding proposition need not be true. We present the following counterexample to demonstrate this.

Example 6 Consider Λ and ϖ as described in Example 4. Note that $\varpi(2, 0) = 0$ and $\varpi(1, 0) = 0$ which implies that $2 \in \mathfrak{K}_r(\varpi)$ and $1 \notin \mathfrak{K}_r(\varpi)$. But $1 \leq 2$. Thus the previous proposition is not true as ϖ is not monotone.

3. Conclusion

The key features and significant properties discussed in this research. From a strictly theoretical perspective, the method of examining the characteristics of lattice derivations is taken into consideration. Our theoretical investigations should ideally continue and conclude the ones that is modify by many researchers [6, 13, 14] and establish the interesting features of this subject. However, the concept raises striking questions for further study as an application in allied fields such as computer sciences, combinatorics, discrete mathematics, and extended algebraic structure.

The partial order is a key feature in the structure of lattices. Lattices and partial orders are main ideas in discrete mathematics and are frequently utilized in computer science, particularly in database theory, data structures, and computation theory. Lattices and partial orders are essential to many applications in computer science, engineering, and other fields. These structures develop much interest among researchers, making it possible for future work to efficiently organize, compare, and optimize data, offering crucial tools for solving challenging issues across limited a range of fields.

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Conflict of interest

The authors declare no competing financial interest.

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