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Research Article

A New Simulation Framework for Analyzing Neutrosophic Data in Experimental Design

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Abstract: A recent simulation-based classical analysis has been developed for interval data. However, a review of the literature indicates that these existing simulations have notable limitations and fail to conform to the neutrosophic statistical framework. In this paper, we propose a novel simulation process designed to analyze neutrosophic data within an appropriate and rigorous neutrosophic framework. We demonstrate that the proposed simulation is more comprehensive and aligns closely with the principles of neutrosophic theory. The results will be obtained through simulation and compared with those of existing methods, with the expectation that the proposed approach provides substantial improvements and is better suited for analyzing neutrosophic data.

***Keywords*:** simulation, interval-data, classical statistics, uncertainty, *p*-value

MSC: 62A86

1. Introduction

Smarandache [1] introduced the neutrosophic theory, which has since attracted the interest of decision-makers across various disciplines, including mathematics, statistics, and artificial intelligence. This theory is grounded in neutrosophic logic—an extension and generalization of fuzzy logic. Later, [2] proposed neutrosophic statistics as an extension of classical statistics, aimed at analyzing imprecise and interval-valued data by incorporating degrees of truth, indeterminacy, and falsity. As a result, neutrosophic statistical analysis is considered more flexible and informative than classical approaches when dealing with uncertainty. It is important to note that classical statistics is applied when data is assumed to be crisp and precise, whereas neutrosophic statistical methods are designed to directly analyze imprecise and uncertain data, which is often encountered in real-world applications. Notably, the results of neutrosophic statistical analysis reduce to those of classical statistics when no uncertainty is present in the observations. Moreover, neutrosophic statistics offer robust and adaptable results that are flexible and well-suited to uncertain environments, often outperforming classical methods in such contexts. For instance, [3, 4] applied neutrosophic statistical techniques to analyze engineering data. [5] further demonstrated, through various numerical examples, that neutrosophic statistical analysis outperforms interval statistics. The application of neutrosophic statistics spans several domains. In distribution theory, contributions can be

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found in the works of [6]. In regression analysis, relevant studies include [7, 8]. Applications in experimental design are presented in [9–13]. More recently, [14] proposed simulation methods for analyzing interval data and conducting analysis of variance to test the null hypothesis that all group means are equal, against the alternative hypothesis that at least one group mean differs from the others. Aslam et al. conducted a critical analysis of the simulation study presented by [15].

Woodall et al. [14] presented a simulation approach for interval data by assuming a uniform distribution over each interval, claiming it to be applicable to neutrosophic statistics. However, the simulation was not developed within the framework of neutrosophic statistics, as it does not account for the fundamental concepts of degrees of truth, indeterminacy, and falsity, which are central to neutrosophic theory. Despite the growing body of literature on neutrosophic statistical analysis, to the best of our knowledge, no study has yet developed a simulation based on a comprehensive neutrosophic statistical framework for testing the null hypothesis that all group means are equal, versus the alternative that at least one group mean differs in the context of analysis of variance. In this paper, we aim to introduce a simulation approach specifically designed for neutrosophic statistical analysis, utilizing data derived from experimental designs. We detail the methodology within the neutrosophic framework and outline the simulation procedure to compute key statistics for experimental designs. The proposed method will be compared with existing simulation approaches. We anticipate that our approach will offer a broader and more insightful analysis within the neutrosophic context. Moreover, the proposed simulation framework is adaptable to any type of experimental design.

2. Methodology

In this part, we introduce the proposed simulation using the neutrosophic statistical framework. As previously noted, neutrosophic analysis relies on the degree of truth, falsity, and indeterminacy. Let L denote the lower bound of the interval data and U denote the upper bound of the neutrosophic data. Also, let X_L is a random variable assumed to follow the uniform distribution on the interval (L, U). The midpoint of the neutrosophic data is expressed as

$$M = \frac{(L+U)}{2};\ L < U. \tag{1}$$

It is important to note that the degrees of falsity and truth are derived from the bounds and midpoints of the interval data, reflecting a deliberate modeling choice based on the assumption of a uniform distribution over each interval, as indicated in Eq. (1). The midpoint $M = X_{L0}$ of the interval presents the central value in the context of the uniform distribution over each interval. Suppose that δ denotes the amount of fuzziness around each classical data point X_{L0}. The lower and upper bounds of the interval are defined as $L = X_{L0} - \delta$ and $U = X_{L0} + \delta$, respectively. If the value X_L move away from M, the degree of truth decreases. On the other hand, the degree of falsity measures how close the value X_L is to the given lower and upper bounds. It increases as X_L approaches the interval limits and decreases as it moves toward the midpoint. Assume that T_r represents the degree of truth, which is defined as follows

$$T_r = 1 - \left(\frac{|X_L - M|}{(U - L)}\right). \tag{2}$$

Observe that $|X_L - M|$ represents the distance between the value X_L and the midpoint M. Also note that the degree of truth T_r reaches its maximum when $X_L = M$, and T_r decreases as X_L moves away from the midpoint. Note that $T_r = 1$, when $X_L = M$ and $T_r = 0$ when $L = U = X_L = M$ ($\delta = 0$). Next, we define the degree of falsity F_r, which measures how close the value within the interval is to the lower and upper bounds, and is defined as follows

$$F_r = \frac{\min\left(|X_L - L|,\ |X_L - U|\right)}{(U - L)}. \tag{3}$$

Observe that the degree of falsity indicates how distant X_L is from the lower or upper limit of the interval. The value of $F_r = 0$ when X_L lies exactly at the lower or upper bound. The degree of indeterminacy, denoted by I_r, is computed as follows.

$$I_r = \max(0,\ 1 - T_r - F_r). \tag{4}$$

Due to indeterminacy, it may be possible that the sum of three degrees can be greater than one. To make the results similar to the classical statistics if the sum is larger than one, the results can be normalized as follows

$$S_N = T_r + F_r + I_r. \tag{5}$$

Let $T_N = T_r/S_N$, $F_N = F_r/S_N$ and $I_N = I_r/S_N$ be the normalized degree of truth, falsity and indeterminacy. Note that X_L is a random variable, and consequently, all derived quantities such as T_r, F_r, I_r, S_N, T_N, F_N and I_N are also considered random variables. Suppose that X_T denotes the truth value, X_{L0} denotes the mid value and X_F denotes the value associated with falsity. Based on this information, the neutrosophic random variable X_N is defined as follows

$$X_N = T_N X_T + I_N X_{L0} + F_N X_F. \tag{6}$$

The proposed neutrosophic random variable extends the classical random variable by incorporating uncertainty, indeterminacy, and imprecision. It reduces to the classical variable X_T when the data is perfectly certain—i.e., when there is no fuzziness or indeterminacy. This occurs under the conditions $\delta = 0$, $L = U = X_L = X_{L0}$, $T_r = 1$, $F_r = 0$ and $I_r = 0$, which lead to $S_N = 1$, $T_N = 1$, $I_N = 0$ and $F_N = 0$. Under these conditions, the neutrosophic random variable simplifies to $X_N = 1X_T + 0X_{L0} + 0X_F = X_T$. In the case of complete indeterminacy, when, $T_r = 0$, $F_r = 0$ and $I_r = 1$, which lead to $S_N = 1$, $T_N = 0$, $I_N = 1$ and $F_N = 0$. Under these conditions, the neutrosophic random variable simplifies to $X_N = 0X_T + 1X_{L0} + 0X_F = X_{L0}$.

Assume that X_T, X_{L0}, and X_F are mutually independent and follow normal distribution with means μ_T, μ_0 and μ_F, respectively, and variances σ_T^2, σ_0^2 and σ_F^2, respectively. Although T_N, F_N, and I_N are random by nature, they are considered fixed at the time of calculating the expectation and variance. Then, the expected value of the proposed neutrosophic random variable is given by:

$$E\left(X_N\right) = T_N \mu_T + I_N \mu_0 + F_N \mu_F = \mu_N. \tag{7}$$

The variance of the proposed neutrosophic random variable is expressed as

$$\operatorname{Var}\left(X_N\right) = T_N^2 \sigma_T^2 + I_N^2 \sigma_0^2 + F_N^2 \sigma_F^2 = \sigma_N^2. \tag{8}$$

3. The proposed neutrosophic simulation

This section addresses the proposed neutrosophic simulation using the neutrosophic random variable X_N. In this section, we will develop the simulation procedure to carry out the simulation using the information given in the last section. During the simulation, a simple approach will be adopted by assuming a probability distribution over each interval—specifically, a uniform distribution for all intervals. Analysis of Variance (ANOVA) will then be conducted, and the resulting p-values will be recorded to determine whether the differences between the means are statistically significant. The proposed simulation framework is flexible and can be extended to any number of groups and sample sizes for testing the equality of means. For the current study, however, we consider three groups, each with ten observations. The proposed simulation process under the neutrosophic statistical frame work is described in the following steps.

Step 1: Simulate the data $X_L \sim U(L, U)$ within each neutrosophic interval $[L, U]$ by assuming the uniform distribution.

Step 2: Compute the values of T_r, F_r and I_r using Eq. (2), Eq. (3) and Eq. (4), respectively, and then calculate the neutrosophic random variable X_N using Eq. (6).

Step 3: Perform the ANOVA using the F-statistic.

Step 4: Repeat the simulation 5,000 times, computing the F-statistic and the corresponding p-values in each iteration. These values will be stored to determine the lower and upper bounds of the F-statistic and p-values.

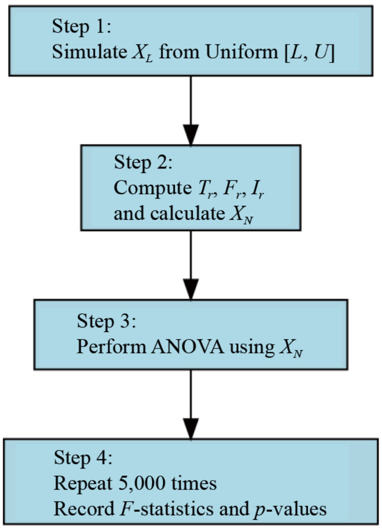

Figure 1. The algorithm for the proposed simulation

Note here that if these steps are repeated for the random variable X_L, the proposed simulation under the neutrosophic statistical framework reduces to the simulation by [14]. The algorithm related to the proposed simulation is shown in Figure 1. Figure 2 presents the values of X_L and X_N for various values of δ. It is evident from the plot in Figure 2 that the difference between these two variables increases as δ increases.

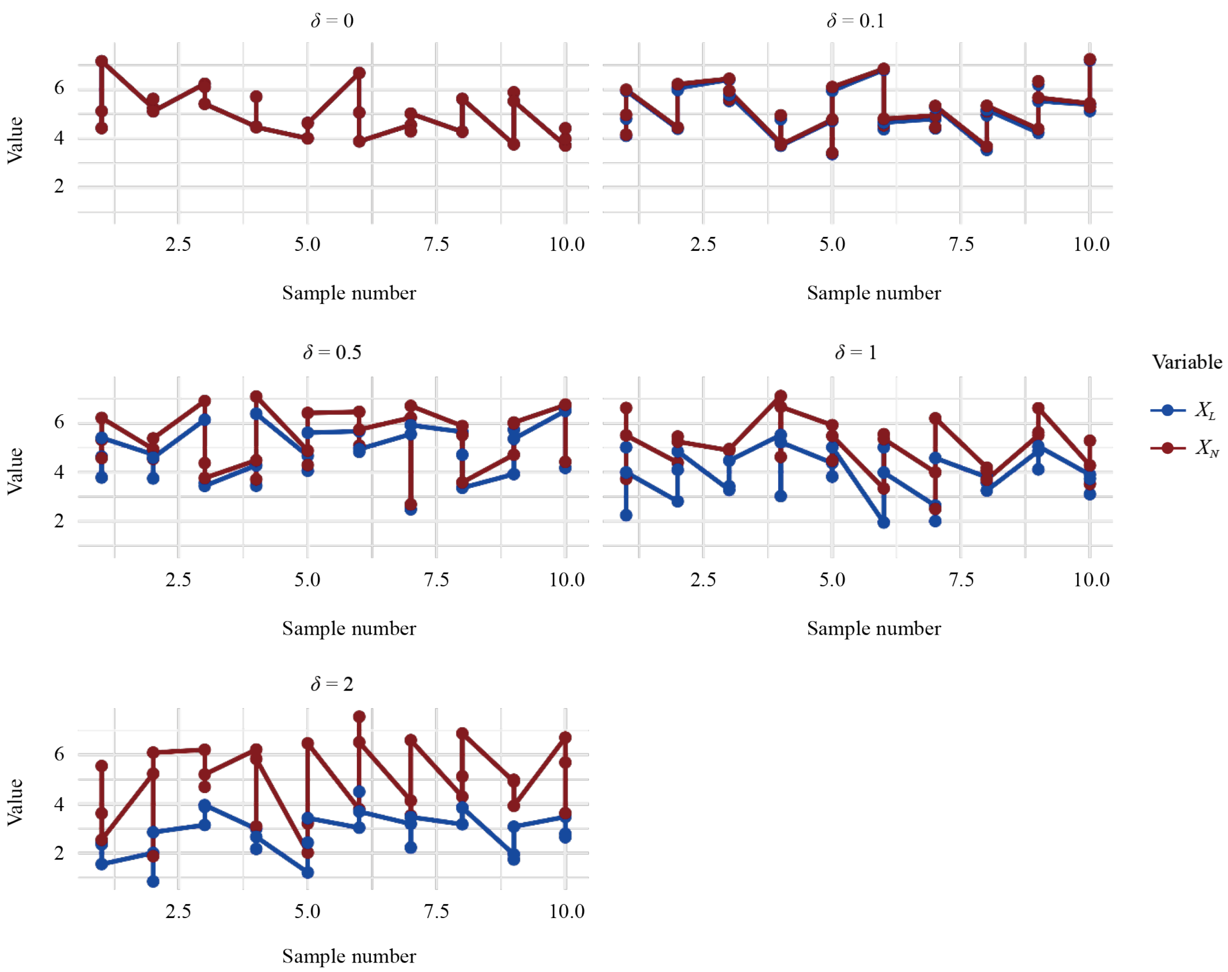

Figure 2. Plot of X_L and X_N for various values of δ

4. Discussions on the results

In this section, we discussed the results obtained from the proposed simulation under the neutrosophic statistical framework for various values of δ in terms of the sum of squares, the p-values, and the power of the test, respectively.

4.1 *Discussion based on sum of square*

We also compared the results from the proposed simulation under the neutrosophic statistical framework with those from the simulation by [14], focusing first on the sum of squares. The Total Sum of Squares (TSS), Between-group Sum of Squares (BSS), and Error Sum of Squares (ESS) obtained using the proposed simulation when $\delta = 0.5$, along with the corresponding values from the simulation by [14] when $\delta = 0$, are shown in Figure 3. Figure 3 presents the distributions of TSS, ESS, and BSS based on 5,000 simulation runs. It is evident from the figure that the distributions of the sums of squares obtained from the proposed neutrosophic simulation differ from those of the existing simulation. Specifically, the proposed simulation results in a lower ESS distribution and a higher BSS distribution compared to the existing method. These differences highlight the impact of incorporating the three neutrosophic components—degrees of indeterminacy, truth, and falsity—particularly at $\delta = 0.5$, as seen in the distinct behavior of the distributions in Figure 3.

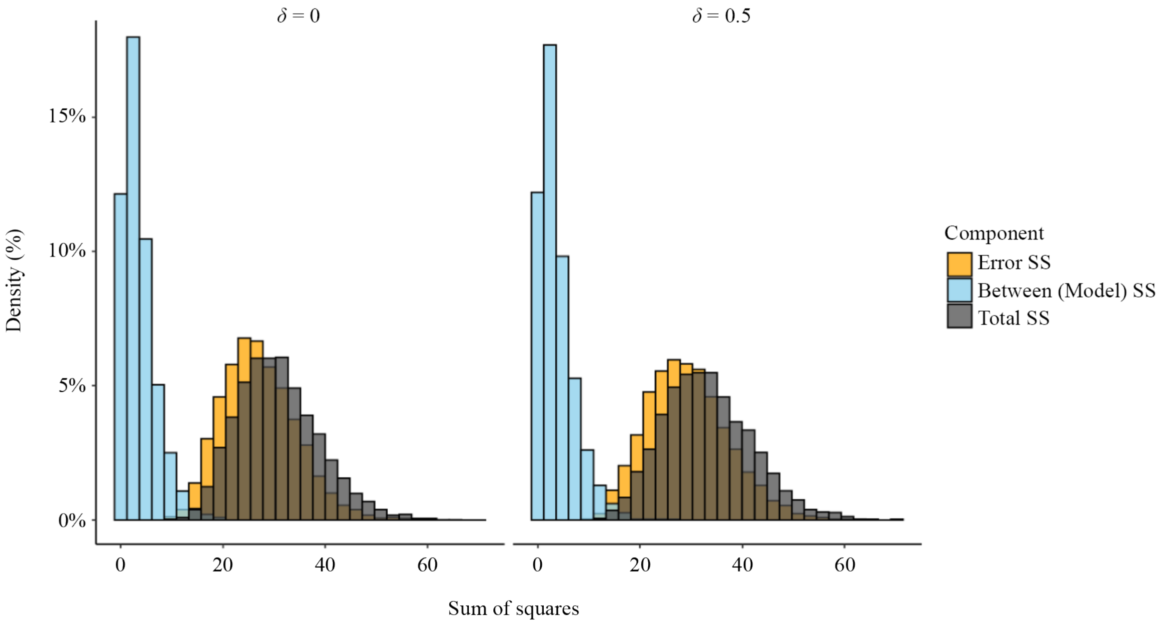

Figure 3. The sum of squares from two simulations

4.2 *Discussion based on p-values*

We compared the *p*-values obtained from the proposed simulation under the neutrosophic statistical framework with those from the simulation by [14]. Using both the proposed and existing simulations, *p*-values were computed for various values of δ. These computed *p*-values are presented in Figure 4. As shown in Figure 4, there were differences in the *p*-values between the two simulations across different δ values. The figure clearly illustrates that ignoring the degrees of indeterminacy, truth, and falsity may lead to misleading conclusions when using *p*-values for hypothesis testing. Consequently, for more accurate and reliable decision-making, it is recommended that decision-makers adopt the proposed simulation within the neutrosophic framework. This approach offers greater flexibility and precision in statistical inference, especially in contexts where uncertainty and imprecision are inherent in the data.

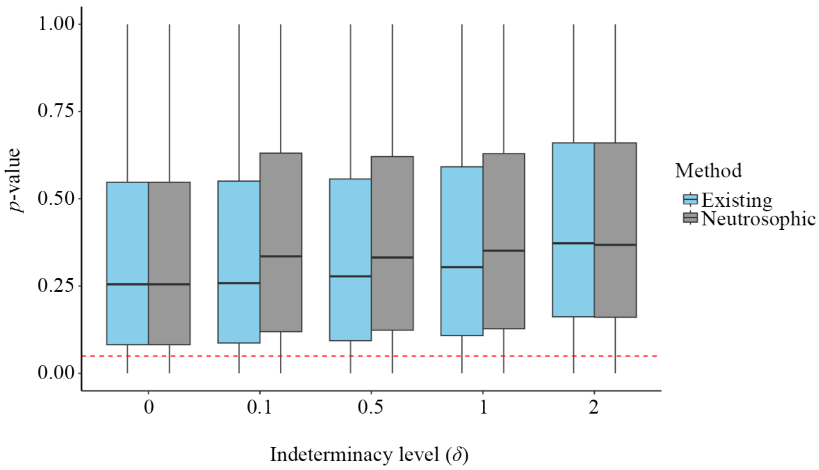

Figure 4. The *p*-values from two simulations

4.3 *Discussion based on the power of the test*

We discussed the results obtained from the power of the test using both the proposed simulation under the neutrosophic framework and the existing simulation by [14]. The power of the test plays a critical role in the analysis of variance, as it reflects the ability to detect true effects and informs sound decision-making in statistical inference. Simulation results for the power of the test across various values of δ were presented in Figure 5. To validate our approach, we included results for $\delta = 0$, which confirmed that both simulations produced identical outcomes, aligning with the results reported by [14]. However, for other values of δ, the power of the test obtained from the proposed simulation differed from that of the existing method. Figure 5 clearly illustrates that the proposed simulation offers a distinct advantage, as it incorporates the degrees of indeterminacy, truth, and falsity—factors not considered by the traditional simulation. In summary, the proposed simulation under the neutrosophic framework provides a more flexible and realistic approach for evaluating the power of the test, sum of squares, and p-values, especially in contexts involving uncertainty.

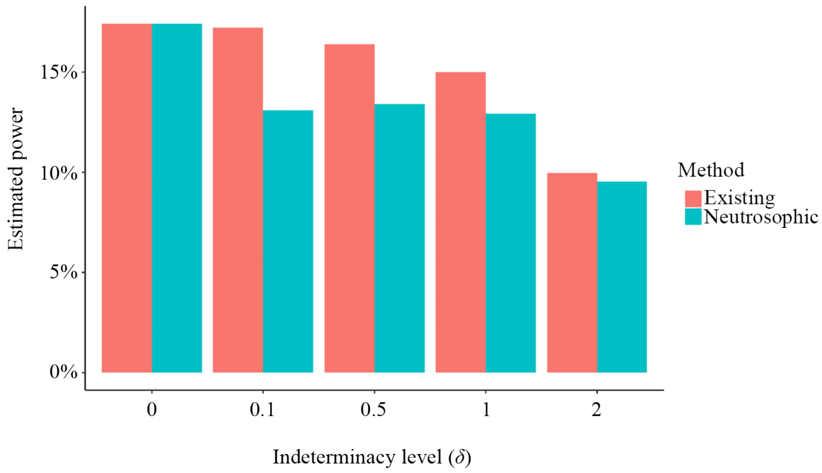

Figure 5. Power of the test from two simulations

5. Application

In this section, we demonstrate the application of the proposed methodology using real data obtained from [16]. The lower and upper bounds of each interval are presented in Table 1. According to [16] "A food company wished to test four different package designs for a new product. Ten stores, with approximately equal sales volumes, are selected as the experimental units. Package designs 1 and 4 are assigned to three stores each and package designs 2 and 3 are assigned to two stores each". Our objective is to test whether the mean sales are the same across all four package designs at a significance level of $\alpha = 0.05$. The hypotheses are stated as follows:

$$H_{0N}: \mu_{N1} = \mu_{N2} = \mu_{N3} = \mu_{N4} \text{ Vs. } H_{1N}: \text{ not all } \mu_{Ni} \text{ are equal}$$

Note that the hypothesis is used to evaluate whether the neutrosophic means across different groups are equal, based on the corresponding group means from classical statistics. We conducted the analysis of variance using both the classical method proposed by [14] and the newly proposed neutrosophic method. The corresponding p-values obtained from both approaches are summarized in Table 2. As shown, the two methods yield different results: the proposed method consistently produces smaller p-values compared to the classical approach. Specifically, Table 2 indicates that the p-value interval for the classical method is (0.00006, 0.7992), whereas the proposed method yields an interval of (0.00009, 0.58288). Comparing these intervals with the significance level $\alpha = 0.05$, we observe that α lies within both ranges. Therefore, the evidence is insufficient to make a definitive conclusion regarding the rejection or acceptance of the null hypothesis H_{0N}. This situation highlights the relevance and strength of the neutrosophic approach. Unlike the classical method, the proposed method incorporates degrees of truth (T), falsity (F), and indeterminacy (I) in the decision-making process. According to [2], when the significance level falls within the p-value interval, the neutrosophic framework becomes particularly useful in quantifying the uncertainty around the decision. It allows us to assess how strongly the data supports the rejection (T) or acceptance (F) of H_{0N}, while also capturing the level of indeterminacy (I) present in the evidence. If, $\min\{\text{neutrosophic } p\text{-value}\} < \alpha < \max\{\text{neutrosophic } p\text{-value}\}$, in such cases, the rejection or acceptance of the null hypothesis becomes indeterminate. Therefore, the probability of rejecting H_{0N} at the significance level α is given by:

$$T = \frac{\alpha - \min\{\text{neutrosophic } p\text{-value}\}}{\max\{\text{neutrosophic } p\text{-value}\} - \min\{\text{neutrosophic } p\text{-value}\}} \tag{9}$$

Thus, the probability of not rejecting the H_0 at α is given by

$$F = \frac{\max\{\text{neutrosophic } p\text{-value}\} - \alpha}{\max\{\text{neutrosophic } p\text{-value}\} - \min\{\text{neutrosophic } p\text{-value}\}} \tag{10}$$

The degree of indeterminacy in the p-values is calculated by

$$I = \left(\frac{\text{Upper } p\text{-value } - \text{ Lower } p\text{-value}}{\text{Upper value}}\right) \tag{11}$$

For the given p-values (0.00009, 0.58288), the values of T, F and I at $\alpha = 0.05$ are computed as follows

$$T = \frac{0.05 - 0.00009}{0.58288 - 0.00009} = \frac{0.04991}{0.58279} = 0.0856$$

$$F = \frac{0.58288 - 0.05}{0.58288 - 0.00009} = \frac{0.5329}{0.58279} = 0.9143$$

$$I = \left(\frac{0.58288 - 0.00009}{0.58288}\right) = \frac{0.58279}{0.58288} = 0.9998$$

It is important to note that when $I > 0$, the sum of the degrees of truth and falsity satisfies $T + F + I > 1$. To align this with the probabilistic framework, we normalize T, F and I. Let T', F' and I' represent the normalized values of T, F and I, respectively, defined as follows:

$$\text{Total} = 0.0856 + 0.9143 + 0.9998 = 1.9997$$

We normalize each component, so that their sum is 1 as follows

$$T' = \frac{0.0856}{1.9997} = 0.0428,\ F' = \frac{0.9143}{1.9997} = 0.4571 \text{ and } I' = \frac{0.9998}{1.9997} = 0.4999$$

Based on the three neutrosophic components, the interpretation of H_0 is as follows: there is a 4.28% degree of belief (truth) that H_{0N} should be rejected, a 45.71% degree of belief (falsity) that it should not be rejected, and a 49.99% degree of indeterminacy. This exceptionally high level of indeterminacy reflects substantial ambiguity and overlapping evidence in the data, indicating that a clear decision—whether to reject or retain the null hypothesis—cannot be made with confidence. When the p-value interval includes the significance level α, as observed in this case, the outcome remains inconclusive. Consequently, greater caution is advised, and further data collection or investigation may be necessary to reach a more definitive conclusion.

Table 1. The neutrosophic data for volumes

Package design (i)	Store j or observations j		
1	[9, 13]	[14, 18]	-
2	[11, 19]	[10, 21]	[11, 15]
3	[15, 21]	[14, 20]	[17, 23]
4	[15, 23]	[21, 27]	-

Table 2. The p-values results

Statistics	The existing method	The proposed method
Min	0.00006	0.00009
Q1	0.03516	0.03432
Median	0.08665	0.08145
Mean	0.12402	0.11527
Q3	0.17838	0.1743
Max	0.7992	0.58288
Power ($p < 0.05$)	0.3351	0.3412

6. Concluding remarks

A simulation-based classical analysis had recently been developed for interval data. However, a review of the literature indicated that these existing simulations had notable limitations and did not conform to the neutrosophic statistical framework. In this paper, we propose a novel simulation process tailored to analyze neutrosophic data within a rigorous and appropriate neutrosophic framework. We demonstrated that the proposed simulation was more comprehensive and aligned more closely with the principles of neutrosophic theory. The results—including the sum of squares, p-values, and the power of the test—revealed significant differences between the proposed simulation and the existing method by [14]. The simulation presented by [14, 17], merely referencing neutrosophic concepts in the title of their paper, does not incorporate any actual neutrosophic analysis. This may be seen as a misleading attempt that could confuse researchers working in the field of neutrosophic statistics. Based on these findings, the proposed simulation is recommended for generating neutrosophic data across various fields such as health, industry, reliability, artificial intelligence, and other domains where imprecise data are expected. In neutrosophic analysis, the width of intervals and associated statistical measures depend on the distribution of the degree of indeterminacy. Therefore, investigating the influence of indeterminacy and the potential misuse of the uniform distribution—particularly in relation to information loss, cost implications, and interval width—represents an important direction for future research in the analysis of interval-valued data.

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Conflict of interest

The authors declare no competing financial interest.

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