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Research Article

A Human-Aided Evaluation Based on Distance from Average Solution Method for the Diagnosis of Skin Disease Using T-Spherical Fuzzy Information

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Abstract: Disorders of the skin have been identified as skin diseases. These medical disorders may involve severe skin manifestations, including allergic reactions, frustration, and itching. Numerous skin disorders may be inherited, while other aspects may be caused by lifestyle. To diagnose the various skin disorders based on the symptoms of skin diseases, we introduce the novel idea of Interval-Valued T-Spherical Fuzzy Set (IV-TSFS) that significantly enhances the ability to handle vagueness and unpredictability in the data being gathered. The IV-TSFS takes the concept of T-SFS by incorporating Interval Values (IVs). This innovation greatly improves the capacity to represent and manage uncertainty because they offer a structured and flexible framework that captures real-world ambiguity, vagueness, and unpredictability as compared to other classical fuzzy models. In this article, we construct the extended conventional IV-TSF Evaluation based on Distance from Average Solution (EDAS) approach by using the conventional Evaluation based on Distance from Average Solution (EDAS) method and also identifying a wide range of possibilities and understanding the potential variability in outcomes, which is especially useful in Decision-Making (DM) scenarios. This method provides a balanced view of each alternative's performance, helping decision-makers to rank and select the most suitable option effectively. It is the most powerful way to visualize and compare the performance of various alternatives in a structured and quantitative manner. Firstly, we briefly review the description of T-SFSs and IV-TSFSs and discuss the score function $\dot{S}c_{\curlywedge}(\mathfrak{T})$, accuracy function $\dot{A}c_{\curlywedge}(\mathfrak{T})$, and the basic Operational Laws (OLs) of IV-TSFVs. Next, we explain the extensive interventions of the extended conventional Interval-Valued T-Spherical Fuzzy (IV-TSF) EDAS method to cope with uncertain and unreliable information, which is especially useful in DM scenarios. Finally, a numerical example is provided to effectively diagnose the favorable skin disease based on the symptoms of skin diseases by using the IV-TSF EDAS approach, and several comparative results of our proposed model with other existing Aggregation Operators (AOs) are carried out to demonstrate the invaluable benefits associated with this strategy.

***Keywords*:** Interval-Valued T-Spherical Fuzzy Set (IV-TSFS), intelligent decision-making, Evaluation based on Distance from Average Solution (EDAS) method, skin disorders

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1. Introduction

1.1 *Historical background*

Multiple Attribute Decision Making (MADM) is frequently observed and everyday activities in human life nowadays which usually involves choosing the best option based on several factors. Because of its impressive ability to capture data and unclear information, MADM has been extensively examined and effectively implemented in administration, finance, and other fields of research in recent decades [1–3]. The application of fuzzy set theory to MADM problems has become more and more prevalent in the last few decades mainly because of an unpredictable nature of the available inputs. To deal with such type of issues, Zadeh [4] introduced the concept of Fuzzy Sets (FSs), which has since gained significant attention among researchers. Over the years, numerous conceptual enhancements have been developed to better address uncertainty and imprecision in various applications. However, there are situations, where the traditional FS theory proves to be inadequate. For instance, when someone provides information in the form of Membership Value (MV) and Non-Membership Value (NMV), the FS hypothesis is unable to accommodate it. To overcome these issues, Atanassove [5] developed the conceptual framework of Intuitionistic FS (IFS) by incorporating the concept of NMV into the drawbacks of FS, which states that the sum of MV and NMV $0 \leq \tilde{\varphi}(\eth) + \hat{\mathfrak{u}}(\eth) \leq 1$ are confined to $[0,\ 1]$. IFS is a sophisticated and effective method for handling complicated and unpredictable information in DM scenarios. In certain cases, the concept of IFS is ineffective and unable to accommodate it. To deal with such situations, Yager [6] investigated the Pythagorean FS (PyFS) by modifying the rule of IFS, which states that the sum of the square of MV and NMV $0 \leq \tilde{\varphi}^2(\eth) + \hat{\mathfrak{u}}^2(\eth) \leq 1$ are confined to $[0,\ 1]$. Yager modified the theoretical advancement of PyFS into q-Rung Orthopair FS (q-ROFS), which states that the $\mathfrak{r}$-th power of MV and NMV $0 \leq \tilde{\varphi}^{\mathfrak{r}}(\eth) + \hat{\mathfrak{u}}^{\mathfrak{r}}(\eth) \leq 1$ never exceeds to $[0,\ 1]$ interval. The q-ROFS has gained popularity and heightened interest from numerous scholars due to its organized nature. Although q-ROFS is capable of handling inadequate equivocal evaluation data, it is unable to successfully manage contradicting data in practical situations. To deal with such situations, Cuong et al. [7] invented the theoretical framework of Picture FS (PFS) by incorporating the concept of Abstinence Value (AV) along with MV and NMV, which states that the sum of MV, AV, and NMV $0 \leq \tilde{\varphi}(\eth) + \check{\Upsilon}(\eth) + \hat{\mathfrak{u}}(\eth) \leq 1$ never exceeds to $[0,\ 1]$ interval. In many cases, the concept of PFS is ineffective. To overcome these issues, Mahmood et al. [8] investigated the Spherical FS (SFS) by modifying the rule of PFS, which states that the sum of squares of MV, AV, and NMV $0 \leq \tilde{\varphi}^2(\eth) + \check{\Upsilon}^2(\eth) + \hat{\mathfrak{u}}^2(\eth) \leq 1$ are confined to $[0,\ 1]$. SFS is an improved technique for handling multifaceted and inconsistent data in decision-making situations as compared to PFS. Moreover, Mahmood et al. [8] modified the specifications of SFS to determine the hypothesis of T-Spherical FS (T-SFS), which initiates that the $\mathfrak{r}$-th power of MV, AV, and NMV $0 \leq \tilde{\varphi}^{\mathfrak{r}}(\eth) + \check{\Upsilon}^{\mathfrak{r}}(\eth) + \hat{\mathfrak{u}}^{\mathfrak{r}}(\eth) \leq 1$ are confined to $[0,\ 1]$ interval. T-SFS greatly improves the capacity to represent and manage uncertainty because they offer a structured and flexible framework that captures real-world ambiguity, vagueness, and unpredictability as compared to other classical fuzzy models. T-SFS, and Interval-Valued T-Spherical Fuzzy Set (IV-TSFS) are an extremely successful approaches to dealing with uncertainty and unpredictability in an extensive decision-making scenario and effectively diagnosing skin diseases based on the symptoms of diseases.

Numerous researchers have made important improvements to the establishment of various methods [9–14] for different frameworks since they are crucial in decision-making problems. Singh and Kumar [15] introduced the idea of the Picture Fuzzy Visekriterijumska Optimizacija I Kompromisno Resenje-Technique for Order of Preference by Similarity to Ideal Solution (PF VIKOR-TOPSIS) approach based on information and precision metrics for choosing the best absorption decision-making problems related to the real world. Popovic [16] developed a MADM method that has been a crucial task for both public and private businesses to select a suitable candidate by using the COmbined COmpromise SOlution (CoCoSo) method and also determining the criteria weights. Barukab et al. [17] worked on the development of SFS and also discussed the advantages of the TOPSIS method relying on the Entropy measure for MADM problems. Monika and Sangwan [18] worked on Analytic Hierarchy Process (AHP) and TOPSIS methods by using IV-SFSs to determine the cloud-based computing services and appropriately explain the challenges involving MADM. Hao and MA [19] developed the logical concept of improved executive decision-making scenarios in financial analysis through the use of the PF

MARCOS-based MADM method and also elaborated their ranking results. Wang [20] worked on the development of a VIKOR-based extended TODIM approach, and educational institution's scientific investigations management competence has been evaluated by using IV-PyF frameworks. Anjum et al. [21] explained the theory of the CRiteria Importance Through Intercriteria Correlation-Weighted Aggregated Sum Product ASsessment (CRITIC-WASPAS) model to evaluate the integrated interactive transportation perspectives by using T-SF frameworks for MADM issues. Naz et al. [22] recognized the idealogy for the assessment of sentiment analysis based on the extended-MABAC method under the framework of q-ROFS and appropriately explained the challenges involving MADM. Wang et al. [23] introduced and developed the MADM issues for T-SFSs and also elaborated the advancement of the CoCoSo method depending on the Frank Softmax Aggregation Operators (FSAOs). Gul and Ak [24] introduced a new risk assessment model based on the extended conventional TOPSIS method concerning the failure modes and effect analysis in the concrete manufacturing industry is conducted to demonstrate the innovative model's applicability. Li et al. [25] explained the theory of the enhanced spherical cubic fuzzy WASPAS method for evaluating the performance quality of crowdsourcing logistics processes, which effectively demonstrates the applicability of an innovative model. Radovanovic et al. [26] introduced a new assessment model based on AHP and Grey MARCOS methods under the framework of SFS and appropriately explained the challenges involving MADM. Sangare-Oumar et al. [27] demonstrated the theory of university graduates for MADM problems. The mathematical foundation of the Evaluation based on Distance from Average Solution (EDAS) approach was initially proposed by Keshavarz Ghorabaee et al. [28, 29] as an intriguing MADM technique that is well-known in information identification decision challenges. This approach is highly developed and effective for handling complicated and unpredictable information in DM scenarios. The Table 1 mentioned in section 2 that shows the MADM challenges in the field of scientific research which is helpful for determining the suitable choices based on the EDAS method and also elaborated their concluded remarks.

Table 1. MADM studies in the field of research

Auther's	Criteria used	Aims and Objectives	Method used
Li and Wang [30]	Traffic convenience, product price, protection ability, service quality	Evaluating the service quality of wireless sensor networks.	EDAS and CRITIC method
Ibrahim et al. [31]	Harvesting, processing, refining, composition, stability, antioxidant properties	Evaluating palm oil production in a circular economy.	FWZIC and EDAS method
Xia [32]	Creativity, active learning techniques, communication skills, passion, collaboration	Evaluating the teaching quality.	TODIM and EDAS approach
Jana and Hezam [33]	Better infrastructure, skilled labor, market-based location	Selecting the location of the iron factory and also determine their ranking results.	EDAS approach
Mu [34]	Accounting, interpersonal skills, proficiency, technological skills, ability to work independently	Evaluating the financial management capability that affects the stability of production.	EDAS approach
Wang [35]	Layered and orderly texture, geometric surfaces, flexible modeling mode, semi-mechanized production	Evaluating the quality of ceramic product modeling design.	EDAS and CRITIC method
Kamalakannan et al. [36]	Coughing, shortness of breath, chest pain, weight loss, fatigue, fever, wheezing	To diagnose lung cancer disease and also determine their ranking results.	EDAS method

1.2 *Application of fuzzy set information in skin disease problem*

Several researchers [37, 38] worked on diagnosing different diseases named as Parkinson's disease, viral hepatitis, HIV/AIDS, influenza, Lung cancer, measles, autoimmune diseases, chlamydia, diabetes, skin diseases, and many others. Skin diseases encompass a wide range of conditions caused by various pathogens, each with unique manifestations. Diagnoses and treatment require careful consideration of symptoms, patient history, and often diagnostic sets. Effective management involves identifying the specific pathogens (whether bacterial, viral, fungal, or parasitic) and implementing

an appropriate therapeutic intervention. Early detection and proper care can minimize complications and improve patient outcomes, whether through medication, topical treatments, or lifestyle changes. The application of fuzzy set theory [39–41] in the context of skin disease problems is particularly useful because it helps to deal with the inherent uncertainty and imprecision that are often present in medical data. Skin diseases often have ambiguous or overlapping symptoms, making precise diagnoses and treatment challenging. A fuzzy logic system allows for the handling of vague, subjective information and provides more flexible decision-making processes in medical diagnoses and treatment planning. The Figure 1 mentioned in section 4 that shows different types of skin diseases.

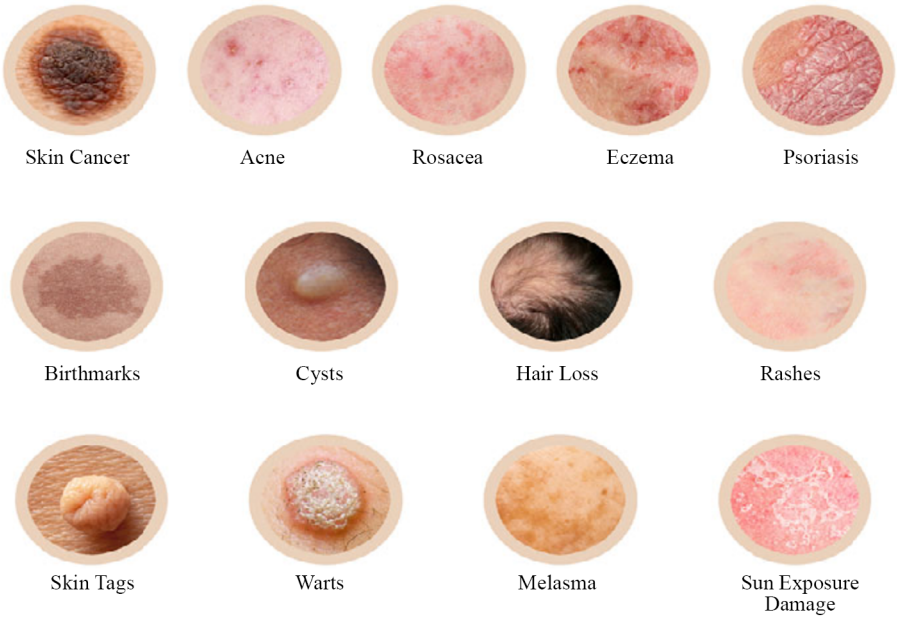

Figure 1. Description of different type of skin diseases (https://www.wfoqfashion.shop/?path=page/ggitem&ggpid=1523921)

1.3 *Motivation and objectives of the research*

The following key points summarize the main contributions of the present study:

• The IV-TSFS is an extremely successful approach to dealing with uncertainty and unpredictability in an extensive decision-making scenario and effectively diagnosing skin diseases based on the symptoms of diseases.

• To explain important amenities, perspectives, necessary steps, correlating approaches, and the historical backdrop of the invented IV-TSF EDAS approach, which may contribute to more comparable and scalable solutions in a wide variety of situations requiring decision-making.

• To effectively diagnose skin diseases based on the symptoms of skin diseases, we have introduced the novel idea of IV-TSFS by using the extended conventional EDAS approach that significantly enhances the ability to handle vagueness and unpredictability in the data being gathered.

• We have provided a real-world scenario to effectively diagnose favorable skin diseases based on the symptoms of skin diseases by using the IV-TSF EDAS approach.

• We have elaborated several comparative results of our proposed model with other existing AOs carried out to demonstrate the positive effects associated with this strategy.

1.4 *Organization of the suggested work*

The remaining part of this article is particularized as follows: In section 2, we briefly overview the basic concept of T-SFSs, and IV-TSFSs and also discuss the score function $\dot{S}c_{\mathcal{F}}(\mathfrak{T})$, accuracy function $\tilde{A}c_{\mathcal{F}}(\mathfrak{T})$, and the basic Operational

Laws (OLs) of IV-TSFVs. In section 3, we introduced the theoretical foundation of the classical EDAS method by using the IV-TSF framework that is successfully useful in information identification decision challenges. In section 4, we have explained the extensive interventions of the extended conventional IV-TSF EDAS method to cope with uncertain and unreliable information which is especially useful in DM scenarios. Moreover, a numerical illustration is provided to effectively diagnose the favorable skin disease based on the symptoms of skin diseases by using the IV-TSF EDAS approach. In section 5, we investigated several comparative results of our proposed IV-TSF method with other existing AOs to significantly demonstrate the invaluable benefits associated with this strategy. In section 6, we summarized the key points and findings of our proposed method.

2. Preliminaries

In this section, we briefly review the description of T-SFSs, and IV-TSFSs and also discuss the score function $\dot{S}c_{\mathscr{r}}(\mathfrak{F})$, accuracy function $\tilde{A}c_{\mathscr{r}}(\mathfrak{F})$, and the basic Operational Laws (OLs) of IV-TSFVs.

Definition 1 [8] The set $\mathfrak{F} = \left\{\eth,\ \left(\tilde{\omega}_{\mathfrak{F}}(\eth),\ \check{\mathrm{Y}}_{\mathfrak{F}}(\eth),\ \hat{\mathrm{u}}_{\mathfrak{F}}(\eth)\right) \middle| \eth \in \mathscr{G}\right\}$ denoted as a T-SFS where $\mathscr{G}$ is the universe of discourse and $\tilde{\omega}$, $\check{\mathrm{Y}}$, and $\hat{\mathrm{u}}$ representing the MV, AV, and NMV such that $\tilde{\omega}, \check{\mathrm{Y}}, \hat{\mathrm{u}} : \mathscr{G} \to [0,\ 1]$ with the condition $0 \leq \tilde{\omega}^{\mathscr{r}}(\eth) + \check{\mathrm{Y}}^{\mathscr{r}}(\eth) + \hat{\mathrm{u}}^{\mathscr{r}}(\eth) \leq 1$ for $\mathscr{r} \geq 1$ and $\mathfrak{Q}_{\mathfrak{F}}(\eth) = \left(1 - \left(\tilde{\omega}^{\mathscr{r}}(\eth) + \check{\mathrm{Y}}^{\mathscr{r}}(\eth) + \hat{\mathrm{u}}^{\mathscr{r}}(\eth)\right)\right)^{\frac{1}{\mathscr{r}}}$ is called the Refusal Value (RV) of T-SFS and $(\tilde{\omega},\ \check{\mathrm{Y}},\ \hat{\mathrm{u}})$ is known as a T-SFV.

Remark 1 [42] By applying Definition 1, the T-SFS provides the following significant fuzzy sets as listed below:

- q-ROFS for $\check{\mathrm{Y}}_{\mathfrak{F}}(\eth) = 0$.
- S-FS for $\mathscr{r} = 2$.
- PyFS for $\mathscr{r} = 2$ and $\check{\mathrm{Y}}_{\mathfrak{F}}(\eth) = 0$.
- PFS for $\mathscr{r} = 2$.
- IFS for $\mathscr{r} = 1$ and $\check{\mathrm{Y}}_{\mathfrak{F}}(\eth) = 0$.
- FS for $\mathscr{r} = 1$ and $\check{\mathrm{Y}}_{\mathfrak{F}}(\eth) = 0 = \hat{\mathrm{u}}_{\mathfrak{F}}(\eth)$.

Definition 2 [43] The set $\mathfrak{F} = \left\{\left(\eth,\ \left[\tilde{\omega}^{-}_{\mathfrak{F}}(\eth), \tilde{\omega}^{+}_{\mathfrak{F}}(\eth)\right],\ \left[\check{\mathrm{Y}}^{-}_{\mathfrak{F}}(\eth), \check{\mathrm{Y}}^{+}_{\mathfrak{F}}(\eth)\right],\ \left[\hat{\mathrm{u}}^{-}_{\mathfrak{F}}(\eth), \hat{\mathrm{u}}^{+}_{\mathfrak{F}}(\eth)\right]\right) | \eth \in \mathscr{G}\right\}$ denoted as an IV-TSFS where $\mathscr{G}$ is the universe of discourse and $\left[\tilde{\omega}^{-}_{\mathfrak{F}}, \tilde{\omega}^{+}_{\mathfrak{F}}\right],\ \left[\check{\mathrm{Y}}^{-}_{\mathfrak{F}}, \check{\mathrm{Y}}^{+}_{\mathfrak{F}}\right],\ \left[\hat{\mathrm{u}}^{-}_{\mathfrak{F}}, \hat{\mathrm{u}}^{+}_{\mathfrak{F}}\right]$ denoting the lower and upper limits of MV $\tilde{\omega}$, AV $\check{\mathrm{Y}}$, and NMV $\hat{\mathrm{u}}$ that satisfied such conditions $0 \leq \tilde{\omega}^{-}_{\mathfrak{F}}(\eth) \leq \tilde{\omega}^{+}_{\mathfrak{F}}(\eth) \leq 1$, $\check{\mathrm{Y}}^{-}_{\mathfrak{F}}(\eth) \leq \check{\mathrm{Y}}^{+}_{\mathfrak{F}}(\eth) \leq 1$, $\hat{\mathrm{u}}^{-}_{\mathfrak{F}}(\eth) \leq \hat{\mathrm{u}}^{+}_{\mathfrak{F}}(\eth) \leq 1$, and $\left(\tilde{\omega}^{+}_{\mathfrak{F}}(\eth)\right)^{\mathscr{r}} + \left(\check{\mathrm{Y}}^{+}_{\mathfrak{F}}(\eth)\right)^{\mathscr{r}} + \left(\hat{\mathrm{u}}^{+}_{\mathfrak{F}}(\eth)\right)^{\mathscr{r}} \leq 1$, for all $\eth \in \mathscr{G}$ and $\mathfrak{Q}_{\mathfrak{F}}(\eth) = \left[\left(1 - \left(\left(\tilde{\omega}^{-}_{\mathfrak{F}}(\eth)\right)^{\mathscr{r}} + \left(\check{\mathrm{Y}}^{-}_{\mathfrak{F}}(\eth)\right)^{\mathscr{r}} + \left(\hat{\mathrm{u}}^{-}_{\mathfrak{F}}(\eth)\right)^{\mathscr{r}}\right)\right)^{\frac{1}{\mathscr{r}}},\ \left(1 - \left(\left(\tilde{\omega}^{+}_{\mathfrak{F}}(\eth)\right)^{\mathscr{r}} + \left(\check{\mathrm{Y}}^{+}_{\mathfrak{F}}(\eth)\right)^{\mathscr{r}} + \left(\hat{\mathrm{u}}^{-}_{\mathfrak{F}}(\eth)\right)^{\mathscr{r}}\right)\right)^{\frac{1}{\mathscr{r}}}\right]$ is known as RVof IV-TSFS. For an IV-TSFS $\left\{\left(\eth,\ \left[\tilde{\omega}^{-}_{\mathfrak{F}}(\eth), \tilde{\omega}^{+}_{\mathfrak{F}}(\eth)\right],\ \left[\check{\mathrm{Y}}^{-}_{\mathfrak{F}}(\eth), \check{\mathrm{Y}}^{+}_{\mathfrak{F}}(\eth)\right],\ \left[\hat{\mathrm{u}}^{-}_{\mathfrak{F}}(\eth), \hat{\mathrm{u}}^{+}_{\mathfrak{F}}(\eth)\right]\right) | \eth \in \mathscr{G}\right\}$, which is a triple-component $\left(\left[\tilde{\omega}^{-}_{\mathfrak{F}}(\eth), \tilde{\omega}^{+}_{\mathfrak{F}}(\eth)\right],\ \left[\check{\mathrm{Y}}^{-}_{\mathfrak{F}}(\eth), \check{\mathrm{Y}}^{+}_{\mathfrak{F}}(\eth)\right],\ \left[\hat{\mathrm{u}}^{-}_{\mathfrak{F}}(\eth), \hat{\mathrm{u}}^{+}_{\mathfrak{F}}(\eth)\right]\right)$ is said to be an IV-TSFV denoted by $\mathfrak{F} = \left(\left[\tilde{\omega}^{-}_{\mathfrak{F}}, \tilde{\omega}^{+}_{\mathfrak{F}}\right],\ \left[\check{\mathrm{Y}}^{-}_{\mathfrak{F}}, \check{\mathrm{Y}}^{+}_{\mathfrak{F}}\right],\ \left[\hat{\mathrm{u}}^{-}_{\mathfrak{F}}, \hat{\mathrm{u}}^{+}_{\mathfrak{F}}\right]\right)$.

Theorem 1 [42] By applying Definition 2, the IV-TSFS provides the significant fuzzy sets described in the following form:

- TSFS for $\tilde{\omega}^{-}_{\mathfrak{F}}(\eth) = \tilde{\omega}^{+}_{\mathfrak{F}}(\eth),\ \check{\mathrm{Y}}^{-}_{\mathfrak{F}}(\eth) = \check{\mathrm{Y}}^{+}_{\mathfrak{F}}(\eth),\ \hat{\mathrm{u}}^{-}_{\mathfrak{F}}(\eth) = \hat{\mathrm{u}}^{+}_{\mathfrak{F}}(\eth)$.
- IV-SFS for $\mathscr{r} = 2$.
- SFS for $\mathscr{r} = 2$ and $\tilde{\omega}^{-}_{\mathfrak{F}}(\eth) = \tilde{\omega}^{+}_{\mathfrak{F}}(\eth),\ \check{\mathrm{Y}}^{-}_{\mathfrak{F}}(\eth) = \check{\mathrm{Y}}^{+}_{\mathfrak{F}}(\eth),\ \hat{\mathrm{u}}^{-}_{\mathfrak{F}}(\eth) = \hat{\mathrm{u}}^{+}_{\mathfrak{F}}(\eth)$.
- IV-PFS for $\mathscr{r} = 1$.
- PFS for $\mathscr{r} = 1$ and $\tilde{\omega}^{-}_{\mathfrak{F}}(\eth) = \tilde{\omega}^{+}_{\mathfrak{F}}(\eth),\ \check{\mathrm{Y}}^{-}_{\mathfrak{F}}(\eth) = \check{\mathrm{Y}}^{+}_{\mathfrak{F}}(\eth),\ \hat{\mathrm{u}}^{-}_{\mathfrak{F}}(\eth) = \hat{\mathrm{u}}^{+}_{\mathfrak{F}}(\eth)$.
- IV-qROFS $\check{\mathrm{Y}}^{-}_{\mathfrak{F}}(\eth) = \check{\mathrm{Y}}^{+}_{\mathfrak{F}}(\eth) = 0$.
- q-ROFS $\tilde{\omega}^{-}_{\mathfrak{F}}(\eth) = \tilde{\omega}^{+}_{\mathfrak{F}}(\eth),\ \check{\mathrm{Y}}^{-}_{\mathfrak{F}}(\eth) = \check{\mathrm{Y}}^{+}_{\mathfrak{F}}(\eth) = 0,\ \hat{\mathrm{u}}^{-}_{\mathfrak{F}}(\eth) = \hat{\mathrm{u}}^{+}_{\mathfrak{F}}(\eth)$.
- IV-PyFS for $\mathscr{r} = 2$ and $\check{\mathrm{Y}}^{-}_{\mathfrak{F}}(\eth) = \check{\mathrm{Y}}^{+}_{\mathfrak{F}}(\eth) = 0$.
- PyFS for $\mathscr{r} = 2$ and $\tilde{\omega}^{-}_{\mathfrak{F}}(\eth) = \tilde{\omega}^{+}_{\mathfrak{F}}(\eth),\ \check{\mathrm{Y}}^{-}_{\mathfrak{F}}(\eth) = \check{\mathrm{Y}}^{+}_{\mathfrak{F}}(\eth) = 0,\ \hat{\mathrm{u}}^{-}_{\mathfrak{F}}(\eth) = \hat{\mathrm{u}}^{+}_{\mathfrak{F}}(\eth)$.

• IV-IFS for $\mathfrak{r}=1$ and $\check{\Upsilon}_{\mathfrak{F}}^{-}(\eth)=\check{\Upsilon}_{\mathfrak{F}}^{+}(\eth)=0$.
• IFS for $\mathfrak{r}=1$ and $\tilde{\omega}_{\mathfrak{F}}^{-}(\eth)=\tilde{\omega}_{\mathfrak{F}}^{+}(\eth)$, $\check{\Upsilon}_{\mathfrak{F}}^{-}(\eth)=\check{\Upsilon}_{\mathfrak{F}}^{+}(\eth)=0$, $\hat{u}_{\mathfrak{F}}^{-}(\eth)=\hat{u}_{\mathfrak{F}}^{+}(\eth)$.
• IVFS for $\mathfrak{r}=1$ and $\check{\Upsilon}_{\mathfrak{F}}^{-}(\eth)=\check{\Upsilon}_{\mathfrak{F}}^{+}(\eth)=\hat{u}_{\mathfrak{F}}^{-}(\eth)=\hat{u}_{\mathfrak{F}}^{+}(\eth)=0$.
• FS for $\mathfrak{r}=1$ and $\check{\Upsilon}_{\mathfrak{F}}^{-}(\eth)=\check{\Upsilon}_{\mathfrak{F}}^{+}(\eth)=\hat{u}_{\mathfrak{F}}^{-}(\eth)=\hat{u}_{\mathfrak{F}}^{+}(\eth)=0$ and $\tilde{\omega}_{\mathfrak{F}}^{-}(\eth)=\tilde{\omega}_{\mathfrak{F}}^{+}(\eth)$.

Proof. Trivial. □

Definition 3 [43] For any two IV-TSFVs, $\mathfrak{F}_1=\left(\left[\tilde{\omega}_{\mathfrak{F}_1}^{-}(\eth),\tilde{\omega}_{\mathfrak{F}_1}^{+}(\eth)\right],\left[\check{\Upsilon}_{\mathfrak{F}_1}^{-}(\eth),\check{\Upsilon}_{\mathfrak{F}_1}^{+}(\eth)\right],\left[\hat{u}_{\mathfrak{F}_1}^{-}(\eth),\hat{u}_{\mathfrak{F}_1}^{+}(\eth)\right]\right)$ and $\mathfrak{F}_2=\left(\left[\tilde{\omega}_{\mathfrak{F}_2}^{-}(\eth),\tilde{\omega}_{\mathfrak{F}_2}^{+}(\eth)\right],\left[\check{\Upsilon}_{\mathfrak{F}_2}^{-}(\eth),\check{\Upsilon}_{\mathfrak{F}_2}^{+}(\eth)\right],\left[\hat{u}_{\mathfrak{F}_2}^{-}(\eth),\hat{u}_{\mathfrak{F}_2}^{+}(\eth)\right]\right)$ in $\mathcal{G}$. Then the basic operational laws (Ols) over $\mathfrak{F}_1$ and $\mathfrak{F}_2$ are particularized as:

1.

$$\mathfrak{F}_1\bigoplus\mathfrak{F}_2=\left(\begin{array}{l}\left[\left(\left(\tilde{\omega}_{\mathfrak{F}_1}^{-}\right)^{\mathfrak{r}}+\left(\tilde{\omega}_{\mathfrak{F}_2}^{-}\right)^{\mathfrak{r}}-\left(\tilde{\omega}_{\mathfrak{F}_1}^{-}\right)^{\mathfrak{r}}\left(\tilde{\omega}_{\mathfrak{F}_2}^{-}\right)^{\mathfrak{r}}\right)^{\frac{1}{\mathfrak{r}}},\left(\left(\tilde{\omega}_{\mathfrak{F}_1}^{+}\right)^{\mathfrak{r}}+\left(\tilde{\omega}_{\mathfrak{F}_2}^{+}\right)^{\mathfrak{r}}-\left(\tilde{\omega}_{\mathfrak{F}_1}^{+}\right)^{\mathfrak{r}}\left(\tilde{\omega}_{\mathfrak{F}_2}^{+}\right)^{\mathfrak{r}}\right)^{\frac{1}{\mathfrak{r}}}\right],\\ \left[\left(\check{\Upsilon}_{\mathfrak{F}_1}^{-}\check{\Upsilon}_{\mathfrak{F}_2}^{-}\right),\left(\check{\Upsilon}_{\mathfrak{F}_1}^{+}\check{\Upsilon}_{\mathfrak{F}_2}^{+}\right)\right]\\ \left[\left(\hat{u}_{\mathfrak{F}_1}^{-}\hat{u}_{\mathfrak{F}_2}^{-}\right),\left(\hat{u}_{\mathfrak{F}_1}^{+}\hat{u}_{\mathfrak{F}_2}^{+}\right)\right]\end{array}\right),$$

2.

$$\mathfrak{F}_1\bigotimes\mathfrak{F}_2=\left(\begin{array}{l}\left[\left(\tilde{\omega}_{\mathfrak{F}_1}^{-}\tilde{\omega}_{\mathfrak{F}_2}^{-}\right),\left(\tilde{\omega}_{\mathfrak{F}_1}^{+}\tilde{\omega}_{\mathfrak{F}_2}^{+}\right)\right],\\ \left[\left(\left(\check{\Upsilon}_{\mathfrak{F}_1}^{-}\right)^{\mathfrak{r}}+\left(\check{\Upsilon}_{\mathfrak{F}_2}^{-}\right)^{\mathfrak{r}}-\left(\check{\Upsilon}_{\mathfrak{F}_1}^{-}\right)^{\mathfrak{r}}\left(\check{\Upsilon}_{\mathfrak{F}_2}^{-}\right)^{\mathfrak{r}}\right)^{\frac{1}{\mathfrak{r}}},\left(\left(\check{\Upsilon}_{\mathfrak{F}_1}^{+}\right)^{\mathfrak{r}}+\left(\check{\Upsilon}_{\mathfrak{F}_2}^{-}\right)^{\mathfrak{r}}-\left(\check{\Upsilon}_{\mathfrak{F}_1}^{+}\right)^{\mathfrak{r}}\left(\check{\Upsilon}_{\mathfrak{F}_2}^{-}\right)^{\mathfrak{r}}\right)^{\frac{1}{\mathfrak{r}}}\right],\\ \left[\left(\left(\hat{u}_{\mathfrak{F}_1}^{-}\right)^{\mathfrak{r}}+\left(\hat{u}_{\mathfrak{F}_2}^{-}\right)^{\mathfrak{r}}-\left(\hat{u}_{\mathfrak{F}_1}^{-}\right)^{\mathfrak{r}}\left(\hat{u}_{\mathfrak{F}_2}^{-}\right)^{\mathfrak{r}}\right)^{\frac{1}{\mathfrak{r}}},\left(\left(\hat{u}_{\mathfrak{F}_1}^{+}\right)^{\mathfrak{r}}+\left(\hat{u}_{\mathfrak{F}_2}^{+}\right)^{\mathfrak{r}}-\left(\hat{u}_{\mathfrak{F}_1}^{+}\right)^{\mathfrak{r}}\left(\hat{u}_{\mathfrak{F}_2}^{+}\right)^{\mathfrak{r}}\right)^{\frac{1}{\mathfrak{r}}}\right]\end{array}\right),$$

3.

$$\ell\mathfrak{F}_1=\left(\begin{array}{l}\left[\left(1-\left(1-\left(\tilde{\omega}_{\mathfrak{F}_1}^{-}\right)^{\mathfrak{r}}\right)^{\ell}\right)^{\frac{1}{\mathfrak{r}}},\left(1-\left(1-\left(\tilde{\omega}_{\mathfrak{F}_1}^{+}\right)^{\mathfrak{r}}\right)^{\ell}\right)^{\frac{1}{\mathfrak{r}}}\right],\\ \left[\left(\check{\Upsilon}_{\mathfrak{F}_1}^{-}\right)^{\ell},\left(\check{\Upsilon}_{\mathfrak{F}_1}^{+}\right)^{\ell}\right],\left[\left(\hat{u}_{\mathfrak{F}_1}^{-}\right)^{\ell},\left(\hat{u}_{\mathfrak{F}_1}^{+}\right)^{\ell}\right]\end{array}\right),\ \ell>0,$$

4.

$$\mathfrak{F}_1{}^{\ell} = \left(\begin{array}{l} \left[\left(\tilde{\omega}^{-}_{\mathfrak{F}_1} \right)^{\ell}, \left(\tilde{\omega}^{+}_{\mathfrak{F}_1} \right)^{\ell} \right], \left[\left(1 - \left(1 - \left(\breve{\Upsilon}^{-}_{\mathfrak{F}_1} \right)^{\mathfrak{r}} \right)^{\ell} \right)^{\frac{1}{\mathfrak{r}}}, \left(1 - \left(1 - \left(\breve{\Upsilon}^{+}_{\mathfrak{F}_1} \right)^{\mathfrak{r}} \right)^{\ell} \right)^{\frac{1}{\mathfrak{r}}} \right], \\ \left[(1 - \left(1 - \left(\hat{\mathfrak{u}}^{-}_{\mathfrak{F}_1} \right)^{\mathfrak{r}} \right)^{\ell})^{1/\mathfrak{r}}, (1 - \left(1 - \left(\hat{\mathfrak{u}}^{+}_{\mathfrak{F}_1} \right)^{\mathfrak{r}} \right)^{\ell})^{1/\mathfrak{r}} \right] \end{array} \right), \ \ell > 0.$$

Definition 4 [43] For any IV-TSFV, $\mathfrak{F}_1 = \left(\left[\tilde{\omega}^{-}_{\mathfrak{F}_1}(\eth), \tilde{\omega}^{+}_{\mathfrak{F}_1}(\eth) \right], \left[\breve{\Upsilon}^{-}_{\mathfrak{F}_1}(\eth), \breve{\Upsilon}^{+}_{\mathfrak{F}_1}(\eth) \right], \left[\hat{\mathfrak{u}}^{-}_{\mathfrak{F}_1}(\eth), \hat{\mathfrak{u}}^{+}_{\mathfrak{F}_1}(\eth) \right] \right)$ in $\mathcal{G}$, then the score function $\dot{\mathfrak{Sc}}_{\mathcal{r}}(\mathfrak{F}_1)$ is:

$$\dot{\mathfrak{Sc}}\mathrm{r}(\mathfrak{F}_1) = \left(\frac{\left(\tilde{\omega}^{-}_{\mathfrak{F}_1} \right)^{F} \left(1 - \left(\breve{\Upsilon}^{-}_{\mathfrak{F}_1} \right)^{F} - \left(\hat{\mathfrak{u}}^{-}_{\mathfrak{F}_1} \right)^{F} \right) + \left(\tilde{\varphi}^{+}_{\mathfrak{I}_1} \right)^{F} \left(1 - \left(\breve{\Upsilon}^{+}_{\mathfrak{F}_1} \right)^{F} - \left(\grave{\eta}^{+}_{\mathfrak{F}_1} \right)^{F} \right)}{3} \right), \tag{1}$$

Definition 5 [43] For any IV-TSFV, $\mathfrak{F}_1 = \left(\left[\tilde{\omega}^{-}_{\mathfrak{F}_1}(\eth), \tilde{\omega}^{+}_{\mathfrak{F}_1}(\eth) \right], \left[\breve{\Upsilon}^{-}_{\mathfrak{F}_1}(\eth), \breve{\Upsilon}^{+}_{\mathfrak{F}_1}(\eth) \right], \left[\hat{\mathfrak{u}}^{-}_{\mathfrak{F}_1}(\eth), \hat{\mathfrak{u}}^{+}_{\mathfrak{F}_1}(\eth) \right] \right)$ in $\mathcal{G}$, then the accuracy function $\dot{\mathfrak{Sc}}_{\mathcal{r}}(\mathfrak{F}_1)$ is:

$$\tilde{\mathfrak{Acr}}(\mathfrak{F}_1) = \left(\frac{\left(\tilde{\omega}^{-}_{\mathfrak{F}_1} \right)^{F} + \left(\breve{Y}^{-}_{\mathfrak{I}_1} \right)^{F} + \left(\hat{\mathfrak{u}}^{-}_{\mathfrak{F}_1} \right)^{F} + \left(\tilde{\omega}^{+}_{\mathfrak{I}_1} \right)^{F} + \left(\breve{Y}^{+}_{\mathfrak{I}_1} \right)^{F} + \left(\grave{\eta}^{+}_{\mathfrak{F}_1} \right)^{F}}{3} \right), \tag{2}$$

Definition 6 [43] For any two IV-TSFVs, $\mathfrak{F}_1 = \left(\left[\tilde{\omega}^{-}_{\mathfrak{F}_1}(\eth), \tilde{\omega}^{+}_{\mathfrak{F}_1}(\eth) \right], \left[\breve{\Upsilon}^{-}_{\mathfrak{F}_1}(\eth), \breve{\Upsilon}^{+}_{\mathfrak{F}_1}(\eth) \right], \left[\hat{\mathfrak{u}}^{-}_{\mathfrak{F}_1}(\eth), \hat{\mathfrak{u}}^{+}_{\mathfrak{F}_1}(\eth) \right] \right)$ and $\mathfrak{F}_2 = \left(\left[\tilde{\omega}^{-}_{\mathfrak{F}_2}(\eth), \tilde{\omega}^{+}_{\mathfrak{F}_2}(\eth) \right], \left[\breve{\Upsilon}^{-}_{\mathfrak{F}_2}(\eth), \breve{\Upsilon}^{+}_{\mathfrak{F}_2}(\eth) \right], \left[\hat{\mathfrak{u}}^{-}_{\mathfrak{F}_2}(\eth), \hat{\mathfrak{u}}^{+}_{\mathfrak{F}_2}(\eth) \right] \right)$ in $\mathcal{G}$, $\dot{\mathfrak{Sc}}_{\mathcal{r}}(\mathfrak{F}_1)$ and $\dot{\mathfrak{Sc}}_{\mathcal{r}}(\mathfrak{F}_2)$ be the scores of $\mathfrak{F}_1$ and $\mathfrak{F}_2$, respectively, and let $\tilde{\mathfrak{Ac}}_{\mathcal{r}}(\mathfrak{F}_1)$ and $\tilde{\mathfrak{Ac}}_{\mathcal{r}}(\mathfrak{F}_2)$ are the accuracy values of $\mathfrak{F}_1$ and $\mathfrak{F}_2$, respectively, then if $\dot{\mathfrak{Sc}}_{\mathcal{r}}(\mathfrak{F}_1) < \dot{\mathfrak{Sc}}_{\mathcal{r}}(\mathfrak{F}_2)$, then $\mathfrak{F}_1 < \mathfrak{F}_2$; if $\dot{\mathfrak{Sc}}_{\mathcal{r}}(\mathfrak{F}_1) = \dot{\mathfrak{Sc}}_{\mathcal{r}}(\mathfrak{F}_2)$, then (1) if $\tilde{\mathfrak{Ac}}_{\mathcal{r}}(\mathfrak{F}_1) = \tilde{\mathfrak{Ac}}_{\mathcal{r}}(\mathfrak{F}_2)$, then $\mathfrak{F}_1 = \mathfrak{F}_2$; (2) if $\tilde{\mathfrak{Ac}}_{\mathcal{r}}(\mathfrak{F}_1) < \tilde{\mathfrak{Ac}}_{\mathcal{r}}(\mathfrak{F}_2)$, then $\mathfrak{F}_1 < \mathfrak{F}_2$.

3. The EDAS method

The theoretical foundation of the EDAS method was initially introduced by Keshavarz et al. [28, 29] as an intriguing MADM technique that is well-known in the research literature for its successful use in information identification decision challenges. The EDAS approach incorporates the Average Values (AVs) to cope with DM situations to uniquely identify itself. By analyzing the mean value along with positive and negative distance estimations, you can effectively determine how different scenarios or alternatives deviate from the average. This helps in identifying a range of possibilities and understanding the potential variability in outcomes, which is especially useful in DM models. The EDAS method relies on two fundamental components, the first component named the Positive Distance from Average (PDA) measures how much an alternative deviates positively from the average performance across all criteria, and the second component named the Negative Distance from Average (NDA) that captures the negative deviation of an alternative from the average. By analyzing both PDA and NDA, the EDAS method provides a balanced view of each alternative's performance, helping

Decision-Makers (DMrs) to rank and select the most suitable option effectively. It is the most powerful way to visualize and compare the performance of various alternatives in a structured and quantitative manner. Consequently, the higher value of Positive Distance from the Average (PDA) indicates the optimal solution and the lower value of Negative Distance from the Average (NDA) indicates an ideal solution.

The specific steps involved in utilizing the classical EDAS method are given as follows:

Step 1: Choose an appropriate criteria that characterizes the available choices.

Step 2: To generate the decision-making matrix $\ddot{\mathsf{E}} = [\ddot{\mathsf{e}}_{\dot{\mathsf{u}}\epsilon}]_{\mathfrak{X}\times\mathfrak{J}}$ of $\dot{\mathsf{u}}^{th}$ alternative $\ddot{\Omega}_{\dot{\mathsf{u}}}$ $(\dot{\mathsf{u}} = 1,\ 2,\ 3,\ \ldots,\ \mathfrak{X})$ according to the ϵ^{th} criterion $\ddot{\mathsf{B}}_{\epsilon}$ $(\epsilon = 1,\ 2,\ 3, \ldots,\ \mathfrak{J})$ that consider the following specifications, as listed below:

$$\ddot{\mathsf{E}} = [\ddot{\mathsf{e}}_{\mathrm{u}\epsilon}]_{\mathfrak{X}\times\mathfrak{J}} = \begin{bmatrix} \ddot{\mathsf{e}}_{11} & \ddot{\mathsf{e}}_{12} & \ldots & \ddot{\mathsf{e}}_{1,\mathfrak{J}} \\ \ddot{\mathsf{e}}_{21} & \ddot{\mathsf{e}}_{22} & \ldots & \ddot{\mathsf{e}}_{2,\mathfrak{J}} \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ \ddot{\mathsf{e}}_{\mathfrak{X}1} & \ddot{\mathsf{e}}_{\mathfrak{X}2} & \ldots & \ddot{\mathsf{e}}_{\mathfrak{X},\mathfrak{J}} \end{bmatrix} \tag{3}$$

$$\gamma = \left[\gamma_1, \gamma_2, \ldots, \gamma_{\mathfrak{J}}\right], \tag{4}$$

for $(\dot{\mathsf{u}} = 1,\ 2,\ 3,\ \ldots,\ \mathfrak{X})$ and $(\epsilon = 1,\ 2,\ 3,\ \ldots,\ \mathfrak{J})$, where γ_{ϵ} is the weight of the criterion ϵ^{th}.

Step 3: To find the average value for each of the given attributes $\ddot{\mathsf{B}}_{\epsilon}$, which can be viewed as follows:

$$\ddot{\mathrm{A}}\mathscr{V}_{\epsilon} = \frac{\sum_{\dot{\mathsf{u}}}^{\mathfrak{X}} \ddot{\mathsf{e}}_{\dot{\mathsf{u}}\epsilon}}{\mathfrak{X}}. \tag{5}$$

Step 4: To develop the positive distance matrix $\rho\text{ÐÂ} = \left(\rho\text{ÐÂ}_{\dot{\mathsf{u}}\epsilon}\right)_{\mathfrak{X}\times\mathfrak{J}}$ and the negative distance matrix $\grave{\mathrm{N}}\text{ÐÂ} = \left(\grave{\mathrm{N}}\text{ÐÂ}_{\dot{\mathsf{u}}\epsilon}\right)_{\mathfrak{X}\times\mathfrak{J}}$ from the average response based on the specific type of attributes $\ddot{\mathsf{B}}_{\epsilon}$, if the ϵ^{th} the attribute is advantageous.

$$\rho\text{ÐÂ}_{\dot{\mathsf{u}}\epsilon} = \frac{\max\left(0,\ \left(\ddot{\mathsf{e}}_{\dot{\mathsf{u}}\epsilon} - \ddot{\mathrm{A}}\mathscr{V}_{\epsilon}\right)\right)}{\ddot{\mathrm{A}}\mathscr{V}_{\epsilon}}, \tag{6}$$

$$\grave{\mathrm{N}}\text{ÐÂ}_{\dot{\mathsf{u}}\epsilon} = \frac{\max\left(0,\ \left(\ddot{\mathrm{A}}\mathscr{V}_{\epsilon} - \ddot{\mathsf{e}}_{\dot{\mathsf{u}}\epsilon}\right)\right)}{\ddot{\mathrm{A}}\mathscr{V}_{\epsilon}}. \tag{7}$$

Step 5: Calculate the weighted sum of $\rho\text{ÐÂ}$ and $\grave{\mathrm{N}}\text{ÐÂ}$ for each alternative $\ddot{\Omega}_{\dot{\mathsf{u}}}$ $(\dot{\mathsf{u}} = 1,\ 2,\ 3,\ \ldots,\ \mathfrak{X})$, as indicated below:

$$\dot{\mathrm{S}}\mathscr{p}_{\dot{\mathsf{u}}} = \sum_{\epsilon=1}^{\mathfrak{J}} \gamma_{\epsilon}\rho\text{ÐÂ}_{\dot{\mathsf{u}}\epsilon}, \tag{8}$$

$$\dot{\mathrm{S}}\dot{\mathrm{N}}_{\dot{\mathsf{u}}} = \sum_{\epsilon=1}^{\mathfrak{J}} \gamma_{\epsilon}\grave{\mathrm{N}}\text{ÐÂ}_{\dot{\mathsf{u}}\epsilon}. \tag{9}$$

Where $\gimel_{\epsilon}$ is the weight of the ϵ^{th} attribute.

Step 6: To normalize the values of $\dot{S}\wp_{\dot{u}}$ and $\dot{S}\dot{N}_{\dot{u}}$ for all alternatives $\ddot{\Omega}_{\dot{u}}$ $(\dot{u} = 1, 2, 3, \ldots, \mathfrak{X})$, as listed below:

$$N\dot{S}\wp_{\dot{u}} = \frac{\dot{S}\wp_{\dot{u}}}{Max_{\dot{u}}\left(\dot{S}\wp_{\dot{u}}\right)}, \tag{10}$$

$$N\dot{S}\dot{N}_{\dot{u}} = 1 - \frac{\dot{S}\dot{N}_{\dot{u}}}{Max_{\dot{u}}\left(\dot{S}\dot{N}_{\dot{u}}\right)}. \tag{11}$$

Step 7: In this step, we have to determine each alternative $\ddot{\Omega}_{\dot{u}}$ appraisal score as follows:

$$\tilde{A}\hat{S}_{\dot{u}} = \frac{1}{2}\left(N\dot{S}\wp_{\dot{u}} + N\dot{S}\dot{N}_{\dot{u}}\right), \tag{12}$$

Step 8: To rank the alternatives $\ddot{\Omega}_{\dot{u}}$ $(\dot{u} = 1, 2, 3, \ldots, \mathfrak{X})$ in descending order based on their evaluation's grading outcomes and then decide which option is most ideal.

4. The MADM issue based on the IV-TSF EDAS method

Consider that $\ddot{\Omega}_{\dot{u}} = (\ddot{\Omega}_1, \ddot{\Omega}_2, \ddot{\Omega}_3 \ldots, \ddot{\Omega}_{\mathfrak{X}})(\dot{u} = 1, 2, 3, \ldots, \mathfrak{X})$ be the finite set of alternatives, and $\ddot{B}_{\epsilon} = (\ddot{B}_1, \ddot{B}_2, \ddot{B}_3 \ldots, \ddot{B}_{\mathfrak{J}})(\epsilon = 1, 2, 3, \ldots, \mathfrak{J})$ be the finite collection of attributes. Now we define the weights of attributes $\gimel_{\epsilon} = \left(\gimel_1, \gimel_2, \ldots, \gimel_{\mathfrak{J}}\right)(\epsilon = 1, 2, 3, \ldots, \mathfrak{J})$, where $\gimel_{\epsilon} \in [0, 1]$, $\sum_{\epsilon=1}^{\mathfrak{J}} \gimel_{\epsilon}$. Assume that $\ddot{H} = \left(\ddot{h}_{\dot{u}\epsilon}\right)_{\mathfrak{X}\times\mathfrak{J}} = \left(\left[\tilde{\omega}^{-}_{\dot{u}\epsilon}, \tilde{\omega}^{+}_{\dot{u}\epsilon}\right], \left[\ddot{Y}^{-}_{\dot{u}\epsilon}, \ddot{Y}^{+}_{\dot{u}\epsilon}\right], \left[\hat{u}^{-}_{\dot{u}\epsilon}, \hat{u}^{+}_{\dot{u}\epsilon}\right]\right)_{\mathfrak{X}\times\mathfrak{J}}$ be the IV-TSF decision matrix, where

$$\check{h}_{\dot{u}\epsilon} = \left(\left[\tilde{\omega}^{-}_{\dot{u}\epsilon}, \tilde{\omega}^{+}_{\dot{u}\epsilon}\right], \left[\ddot{Y}^{-}_{\dot{u}\epsilon}, \ddot{Y}^{+}_{\dot{u}\epsilon}\right], \left[\hat{u}^{-}_{\dot{u}\epsilon}, \hat{u}^{+}_{\dot{u}\epsilon}\right] \right),$$

which is an attribute value $\ddot{B}_{\epsilon}$, given by experts, for the alternative $\ddot{\Omega}_{\dot{u}}$ under $\ddot{B}_{\epsilon}$, $\left[\tilde{\omega}^{-}_{\dot{u}\epsilon}, \tilde{\omega}^{+}_{\dot{u}\epsilon}\right] \subset [0, 1]$, $\left[\ddot{Y}^{-}_{\dot{u}\epsilon}, \ddot{Y}^{+}_{\dot{u}\epsilon}\right] \subset [0, 1]$, $\left[\hat{u}^{-}_{\dot{u}\epsilon}, \hat{u}^{+}_{\dot{u}\epsilon}\right] \subset [0, 1]$, $\left(\tilde{\omega}^{+}_{\dot{u}\epsilon}\right)^{\dagger} + \left(\ddot{Y}^{+}_{\dot{u}\epsilon}\right)^{\dagger} + \left(\hat{u}^{+}_{\dot{u}\epsilon}\right)^{\dagger} \leq 1$, $\dot{u} = 1, 2, 3, \ldots, \mathfrak{X}$, $\epsilon = 1, 2, 3, \ldots, \mathfrak{J}$.

Definition 7 Suppose that $\hat{A} = (\hat{a}_1, \hat{a}_2, \hat{a}_3, \hat{a}_4)$ be the trapezoidal Fuzzy Values (FVs). Based on Equ. (13), the Defuzzified Value (ĎV) of $\hat{A}$ can be found as follows:

$$\tilde{U}(\hat{A}) = \frac{1}{3}\left(\hat{a}_1 + \hat{a}_2 + \hat{a}_3 + \hat{a}_4 - \frac{\hat{a}_3\hat{a}_4 - \hat{a}_1\hat{a}_2}{\hat{a}_3 + \hat{a}_4 - \hat{a}_1 - \hat{a}_2}\right). \tag{13}$$

Definition 8 Suppose that $\hat{A} = (\hat{a}_1, \hat{a}_2, \hat{a}_3, \hat{a}_4)$ be the trapezoidal FVs. The function Φ indicates the largest value between $\hat{A}$ and "0":

$$\Phi(\hat{A}) = \left\{ \begin{array}{l} \hat{A}, \text{ if } \tilde{U}(\hat{A}) > 0 \\ 0, \text{ if } \tilde{U}(\hat{A}) \leq 0 \end{array} \right\}. \tag{14}$$

Definition 9 Suppose that $\dot{\mathrm{N}}_{\check{\mathrm{c}}}=\left(\left[\tilde{\omega}_{\check{\mathrm{c}}}^{-}, \tilde{\omega}_{\check{\mathrm{c}}}^{+}\right], \left[\check{\mathrm{Y}}_{\check{\mathrm{c}}}^{-}, \check{\mathrm{Y}}_{\check{\mathrm{c}}}^{+}\right], \left[\hat{\mathrm{u}}_{\check{\mathrm{c}}}^{-}, \hat{\mathrm{u}}_{\check{\mathrm{c}}}^{+}\right]\right)$ $(\check{\mathrm{c}}=1, 2, 3, \ldots)$ be the collection of IV-TSFVs, and let IV-TSFWA: $ß^{\not{z}} \longrightarrow ß$, if

$$IV-TSFWA_{\psi}\left(\dot{\mathrm{N}}_1, \dot{\mathrm{N}}_2, \dot{\mathrm{N}}_3, \ldots, \dot{\mathrm{N}}_{\not{z}}\right)=\gamma_1\dot{\mathrm{N}}_1\bigoplus\gamma_2\dot{\mathrm{N}}_2\bigoplus, \ldots, \bigoplus\gamma_{\not{z}}\dot{\mathrm{N}}_{\not{z}}. \tag{15}$$

Then, IV-TSFWA is named as an interval-valued T-spherical fuzzy weighted averaging operator, where ß is the collection of all IV-TSFVs, $\psi=(\psi_1, \psi_2, \ldots, \psi_{\not{z}})$ is the weight vectors of the IV-TSFVs $\dot{\mathrm{N}}_{\check{\mathrm{c}}}$ $(\check{\mathrm{c}}=1, 2, 3, \ldots, \not{z})$ and $\psi_{\check{\mathrm{c}}}>0, \sum_{\check{\mathrm{c}}=1}^{\not{z}}\psi_{\check{\mathrm{c}}}$.

The IV-TSFWA operator is an effective tool for integrating IV-TSF data. It can be changed into a more specialized form to accommodate a variety of applications. The IV-TSFWA operator can be modified consistently as follows:

$$\begin{aligned}&\text{IV-TSFWA}_{\psi}(\dot{\mathrm{N}}_1, \dot{\mathrm{N}}_2, \dot{\mathrm{N}}_3, \ldots, \dot{\mathrm{N}}_{\not{z}})\\&=\left(\begin{array}{c}\left[\left(1-\prod_{\check{\mathrm{c}}=1}^{\not{z}}\left(1-\left(\tilde{\omega}_{\check{\mathrm{c}}}^{-}\right)^{\not{r}}\right)^{\psi_{\check{\mathrm{c}}}}\right)^{\frac{1}{\not{r}}}, \left(1-\prod_{\check{\mathrm{c}}=1}^{\not{z}}\left(1-\left(\tilde{\omega}_{\check{\mathrm{c}}}^{+}\right)^{\not{r}}\right)^{\psi_{\check{\mathrm{c}}}}\right)^{\frac{1}{\not{r}}}\right],\\\left[\prod_{\check{\mathrm{c}}=1}^{\not{z}}\left(\check{\mathrm{Y}}_{\check{\mathrm{c}}}^{-}\right)^{\psi_{\check{\mathrm{c}}}}, \prod_{\check{\mathrm{c}}=1}^{\not{z}}\left(\check{\mathrm{Y}}_{\check{\mathrm{c}}}^{+}\right)^{\psi_{\check{\mathrm{c}}}}\right], \left[\prod_{\check{\mathrm{c}}=1}^{\not{z}}\left(\hat{\mathrm{u}}_{\check{\mathrm{c}}}^{-}\right)^{\psi_{\check{\mathrm{c}}}}, \prod_{\check{\mathrm{c}}=1}^{\not{z}}\left(\hat{\mathrm{u}}_{\check{\mathrm{c}}}^{+}\right)^{\psi_{\check{\mathrm{c}}}}\right]\end{array}\right).\end{aligned} \tag{16}$$

Particularly, if $\psi=\left(\frac{1}{\not{z}}, \frac{1}{\not{z}}, \frac{1}{\not{z}}, \ldots, \frac{1}{\not{z}}\right)$, then the IV-TSFWA operator converted to an IV-TSF averaging (IV-TSFA) operator, where

$$\text{IV-TSFA}\left(\dot{\mathrm{N}}_1, \dot{\mathrm{N}}_2, \dot{\mathrm{N}}_3, \ldots, \dot{\mathrm{N}}_{\not{z}}\right)=\frac{1}{\not{z}}\left(\dot{\mathrm{N}}_1\bigoplus\dot{\mathrm{N}}_2\bigoplus, \ldots, \bigoplus\dot{\mathrm{N}}_{\not{z}}\right)$$

$$=\left(\begin{array}{c}\left[\left(1-\prod_{\check{\mathrm{c}}=1}^{\not{z}}\left(1-\left(\tilde{\omega}_{\check{\mathrm{c}}}^{-}\right)^{\not{r}}\right)^{\frac{1}{\not{z}}}\right)^{\frac{1}{\not{r}}}, \left(1-\prod_{\check{\mathrm{c}}=1}^{\not{z}}\left(1-\left(\tilde{\omega}_{\check{\mathrm{c}}}^{+}\right)^{\not{r}}\right)^{\frac{1}{\not{z}}}\right)^{\frac{1}{\not{r}}}\right],\\\left[\prod_{\check{\mathrm{c}}=1}^{\not{z}}\left(\check{\mathrm{Y}}_{\check{\mathrm{c}}}^{-}\right)^{\frac{1}{\not{z}}}, \prod_{\check{\mathrm{c}}=1}^{\not{z}}\left(\check{\mathrm{Y}}_{\check{\mathrm{c}}}^{+}\right)^{\frac{1}{\not{z}}}\right], \left[\prod_{\check{\mathrm{c}}=1}^{\not{z}}\left(\hat{\mathrm{u}}_{\check{\mathrm{c}}}^{-}\right)^{\frac{1}{\not{z}}}, \prod_{\check{\mathrm{c}}=1}^{\not{z}}\left(\hat{\mathrm{u}}_{\check{\mathrm{c}}}^{+}\right)^{\frac{1}{\not{z}}}\right]\end{array}\right). \tag{17}$$

The enhanced conventional EDAS approach based on the IV-TSFSs can be described as follows:

Step 1: Initially, we have to choose an appropriate criteria that characterizes the available choices for specific decision-making issues.

Step 2: To develop the decision-making matrix $\ddot{\mathrm{H}}=\left[\check{\mathrm{h}}_{\ddot{\mathrm{u}}\epsilon}\right]_{\mathfrak{X}\times\not{\mathfrak{z}}}$ of $\ddot{\mathrm{u}}^{th}$ alternative $\ddot{\Omega}_{\ddot{\mathrm{u}}}$ $(\ddot{\mathrm{u}}=1, 2, 3, \ldots, \mathfrak{X})$ according to the ϵ^{th} criterion $\ddot{\mathrm{B}}_{\epsilon}$ $(\epsilon=1, 2, 3, \ldots, \not{\mathfrak{z}})$ that consider the following requirements, as listed below:

$$\ddot{E} = [\check{e}_{\dot{u}\epsilon}]_{\mathfrak{X}\times\mathfrak{J}} = \begin{bmatrix} \check{e}_{11} & \check{e}_{12} & \cdots & \check{e}_{1\mathfrak{J}} \\ \check{e}_{21} & \check{e}_{22} & \cdots & \check{e}_{2\mathfrak{J}} \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ \check{e}_{\mathfrak{X}1} & \check{e}_{\mathfrak{X}2} & \cdots & \check{e}_{\mathfrak{X}\mathfrak{J}} \end{bmatrix},$$

$$= \begin{bmatrix} \begin{pmatrix} [\tilde{\varphi}_{11}^{-}, \tilde{\varphi}_{11}^{+}], [\check{Y}_{11}^{-}, \check{Y}_{11}^{+}] \\ [\hat{u}_{11}^{-}, \hat{u}_{11}^{+}] \end{pmatrix} & \begin{pmatrix} [\tilde{\varphi}_{12}^{-}, \tilde{\varphi}_{12}^{+}], [\check{Y}_{12}^{-}, \check{Y}_{12}^{+}] \\ [\hat{u}_{12}^{-}, \hat{u}_{12}^{+}] \end{pmatrix} & \cdots & \begin{pmatrix} [\tilde{\varphi}_{1\mathfrak{J}}^{-}, \tilde{\varphi}_{1\mathfrak{J}}^{+}], [\check{Y}_{1\mathfrak{J}}^{-}, \check{Y}_{1\mathfrak{J}}^{+}] \\ [\hat{u}_{1\mathfrak{J}}^{-}, \hat{u}_{1\mathfrak{J}}^{+}] \end{pmatrix} \\ \begin{pmatrix} [\tilde{\varphi}_{21}^{-}, \tilde{\varphi}_{21}^{+}], [\check{Y}_{21}^{-}, \check{Y}_{21}^{+}] \\ [\hat{u}_{21}^{-}, \hat{u}_{21}^{+}] \end{pmatrix} & \begin{pmatrix} [\tilde{\varphi}_{22}^{-}, \tilde{\varphi}_{22}^{+}], [\check{Y}_{22}^{-}, \check{Y}_{22}^{+}] \\ [\hat{u}_{22}^{-}, \hat{u}_{22}^{+}] \end{pmatrix} & \cdots & \begin{pmatrix} [\tilde{\varphi}_{2\mathfrak{J}}^{-}, \tilde{\varphi}_{2\mathfrak{J}}^{+}], [\check{Y}_{2\mathfrak{J}}^{-}, \check{Y}_{2\mathfrak{J}}^{+}] \\ [\hat{u}_{2\mathfrak{J}}^{-}, \hat{u}_{2\mathfrak{J}}^{+}] \end{pmatrix} \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ \begin{pmatrix} [\tilde{\varphi}_{\mathfrak{X}1}^{-}, \tilde{\varphi}_{\mathfrak{X}1}^{+}], [\check{Y}_{\mathfrak{X}1}^{-}, \check{Y}_{\mathfrak{X}1}^{+}] \\ [\hat{u}_{\mathfrak{X}1}^{-}, \hat{u}_{\mathfrak{X}1}^{+}] \end{pmatrix} & \begin{pmatrix} [\tilde{\varphi}_{\mathfrak{X}2}^{-}, \tilde{\varphi}_{\mathfrak{X}2}^{+}], [\check{Y}_{\mathfrak{X}2}^{-}, \check{Y}_{\mathfrak{X}2}^{+}] \\ [\hat{u}_{\mathfrak{X}2}^{-}, \hat{u}_{\mathfrak{X}2}^{+}] \end{pmatrix} & \cdots & \begin{pmatrix} [\tilde{\varphi}_{\mathfrak{X}\mathfrak{J}}^{-}, \tilde{\varphi}_{\mathfrak{X}\mathfrak{J}}^{+}], [\check{Y}_{\mathfrak{X}\mathfrak{J}}^{-}, \check{Y}_{\mathfrak{X}\mathfrak{J}}^{+}] \\ [\hat{u}_{\mathfrak{X}\mathfrak{J}}^{-}, \hat{u}_{\mathfrak{X}\mathfrak{J}}^{+}] \end{pmatrix} \end{bmatrix}$$

where

$$\begin{aligned} \gamma_{\epsilon} &= \left(\gamma_1, \gamma_2, \ldots, \gamma_{\mathfrak{J}}\right) \\ &= \left(\left(\left[\left[\tilde{\varphi}_{\gamma 1}^{-}, \tilde{\varphi}_{\gamma 1}^{+}\right], \left[\check{Y}_{\gamma 1}^{-}, \check{Y}_{\gamma 1}^{+}\right], \left[\hat{u}_{\gamma 1}^{-}, \hat{u}_{\gamma 1}^{+}\right]\right]\right), \left(\left[\left[\tilde{\varphi}_{\gamma 2}^{-}, \tilde{\varphi}_{\gamma 2}^{+}\right], \left[\check{Y}_{\gamma 2}^{-}, \check{Y}_{\gamma 2}^{+}\right], \left[\hat{u}_{\gamma 2}^{-}, \hat{u}_{\gamma 2}^{+}\right]\right]\right), \ldots, \right. \\ &\left. \left(\left[\left[\tilde{\varphi}_{\gamma\mathfrak{J}}^{-}, \tilde{\varphi}_{\gamma\mathfrak{J}}^{+}\right], \left[\check{Y}_{\gamma\mathfrak{J}}^{-}, \check{Y}_{\gamma\mathfrak{J}}^{+}\right], \left[\hat{u}_{\gamma\mathfrak{J}}^{-}, \hat{u}_{\gamma\mathfrak{J}}^{+}\right]\right]\right)\right) \end{aligned} \tag{18}$$

and γ_{ϵ} is the weight of criterion ϵ; $(\dot{u} = 1, 2, 3, \ldots, \mathfrak{X})$ and $(\epsilon = 1, 2, 3, \ldots, \mathfrak{J})$.

Step 3: Based on the extended conventional EDAS method, obtaining the average solution for all criteria $\dot{B}_{\epsilon}$ $(\epsilon = 1, 2, 3, \ldots, \mathfrak{J})$ is crucial. This average solution serves as the reference point for calculating PDA and NDA for each alternative $\ddot{O}_{\dot{u}}$ $(\dot{u} = 1, 2, 3, \ldots, \mathfrak{X})$.

$$\widetilde{\ddot{A}\mathscr{V}}_{\epsilon} = \frac{\left(\begin{bmatrix} \left(\sum_{\dot{u}=1}^{\mathfrak{X}} \left(\tilde{\varphi}_{\dot{u}\epsilon}^{-}\right)^{\digamma} - \sum_{\substack{\dot{u}=1 \\ \dot{u} \neq \check{q}}}^{\mathfrak{X}} \left(\tilde{\varphi}_{\dot{u}\epsilon}^{-}\right)^{\digamma} \times \left(\tilde{\varphi}_{\check{q}\epsilon}^{-}\right)^{\digamma} \right)^{\frac{1}{\digamma}}, \left(\sum_{\dot{u}=1}^{\mathfrak{X}} \left(\tilde{\varphi}_{\dot{u}\epsilon}^{+}\right)^{\digamma} - \sum_{\substack{\dot{u}=1 \\ \dot{u} \neq \check{q}}}^{\mathfrak{X}} \left(\tilde{\varphi}_{\dot{u}\epsilon}^{+}\right)^{\digamma} \times \left(\tilde{\varphi}_{\check{q}\epsilon}^{+}\right)^{\digamma} \right)^{\frac{1}{\digamma}} \end{bmatrix}, \right.}{\left. \left[\prod_{\dot{u}=1}^{\mathfrak{X}} \check{Y}_{\dot{u}\epsilon}^{-}, \prod_{\dot{u}=1}^{\mathfrak{X}} \check{Y}_{\dot{u}\epsilon}^{+}\right], \left[\prod_{\dot{u}=1}^{\mathfrak{X}} \hat{u}_{\dot{u}\epsilon}^{-}, \prod_{\dot{u}=1}^{\mathfrak{X}} \hat{u}_{\dot{u}\epsilon}^{+}\right] \right)}}{\mathfrak{X}}, \tag{19}$$

$€ = 1, 2, 3, \ldots, ß$, or

$$\widetilde{Ä\mathscr{V}}_{€} = \left(\begin{array}{l} \left[\left(\left(1 - \Sigma_{ü=1}^{x} \left(\tilde{ω}_{ü€}^{-} \right)^{ŧ} + \Sigma_{\substack{ü=1 \\ ü \neq ą}}^{x} \left(\tilde{ω}_{ü€}^{-} \right)^{ŧ} \times \left(\tilde{ω}_{ą€}^{-} \right)^{ŧ} \right)^{\frac{1}{ŧ}} \right)^{\frac{1}{x}}, \right. \\ \left. \left(\left(1 - \Sigma_{ü=1}^{x} \left(\tilde{ω}_{ü€}^{+} \right)^{ŧ} + \Sigma_{\substack{ü=1 \\ ü \neq ą}}^{x} \left(\tilde{ω}_{ü€}^{+} \right)^{ŧ} \times \left(\tilde{ω}_{ą€}^{+} \right)^{ŧ} \right)^{\frac{1}{ŧ}} \right)^{\frac{1}{x}} \right], \\ \left[\left(\Pi_{ü=1}^{x} \check{Y}_{ü€}^{-} \right)^{1/x}, \left(\Pi_{ü=1}^{x} \check{Y}_{ü€}^{+} \right)^{1/x} \right], \\ \left[\left(\Pi_{ü=1}^{x} \hat{u}_{ü€}^{-} \right)^{1/x}, \left(\Pi_{ü=1}^{x} \hat{u}_{ü€}^{+} \right)^{1/x} \right] \end{array} \right) \tag{20}$$

Step 4: The accuracy values of IV-TSFVs must be determined by using the accuracy function's mathematical formulation, $Ŧ = \left(\left[\tilde{ω}_{ü€}^{-}, \tilde{ω}_{ü€}^{+} \right], \left[\check{Y}_{ü€}^{-}, \check{Y}_{ü€}^{+} \right], \left[\hat{u}_{ü€}^{-}, \hat{u}_{ü€}^{+} \right] \right)$, where $(ü = 1, 2, 3, \ldots, x)$ and $(€ = 1, 2, 3, \ldots, ß)$ and also calculate the accuracy values of $\widetilde{Ä\mathscr{V}}_{€}$. To defuzzify $Ŧ$, the scoring function $ˀAcr(Ŧ)$ is suggested as follows:

$$ˀAcr(Ŧ) = \left(\frac{\left(\tilde{ω}_{ü€}^{-} \right)^{ŧ} + \left(\check{Y}_{ü€}^{-} \right)^{ŧ} + \left(\hat{u}_{ü€}^{-} \right)^{ŧ} + \left(\tilde{ω}_{ü€}^{+} \right)^{ŧ} + \left(\check{Y}_{ü€}^{+} \right)^{ŧ} + \left(ή_{ü€}^{+} \right)^{ŧ}}{3} \right), \tag{21}$$

Step 5: In this step, we have to develop the positive distance matrix $ƿĐÂ = \left(ƿĐÂ_{ü€} \right)_{x \times ß}$ and the negative distance matrix $ǸĐÂ = \left(ǸĐÂ_{ü€} \right)_{x \times ß}$ from the average response based on the particular type of attributes $Ꞵ_{€}$ as follows:

$$\widetilde{ƿĐÂ}_{ü€} = \frac{\max \left(0, \left(ˀAcr \left(\left[\tilde{ω}_{ü€}^{-}, \tilde{ω}_{ü€}^{+} \right], \left[\check{Y}_{ü€}^{-}, \check{Y}_{ü€}^{+} \right], \left[\hat{u}_{ü€}^{-}, \hat{u}_{ü€}^{+} \right] \right) \right) - ˀAcr \left(\widetilde{Ä\mathscr{V}}_{€} \right) \right)}{\tilde{U} \left(\widetilde{Ä\mathscr{V}}_{€} \right)}, \tag{22}$$

$$\widetilde{ǸĐÂ}_{ü€} = \frac{\max \left(0, ˀAcr \left(\widetilde{Ä\mathscr{V}}_{€} \right) - \left(ˀAcr \left(\left[\tilde{ω}_{ü€}^{-}, \tilde{ω}_{ü€}^{+} \right], \left[\check{Y}_{ü€}^{-}, \check{Y}_{ü€}^{+} \right], \left[\hat{u}_{ü€}^{-}, \hat{u}_{ü€}^{+} \right] \right) \right) \right)}{\tilde{U} \left(\widetilde{Ä\mathscr{V}}_{€} \right)}. \tag{23}$$

According to the above equations, $\widetilde{ƿĐÂ}_{ü€}$ indicates the positive distance of $ü^{th}$ possibility from the average response in terms of $€^{th}$ criterion and $\widetilde{ǸĐÂ}_{ü€}$ indicates the negative distance of $ü^{th}$ possibility from the average response in terms of $€^{th}$ criterion.

Step 6: To determine the weighted sum of positive distance $\widetilde{\rho Đ}$ and negative distance $\widetilde{ṄĐ}$ for each alternative $\ddot{Q}_{ù}$ $(ù = 1, 2, 3, \ldots, x)$, as given below:

$$\widetilde{Ṣ\mathcal{P}_{ù}} = \sum_{€=1}^{ß} ɣ_{€} \times \widetilde{\rho Đ}\hat{A}_{ù€}, \tag{24}$$

and

$$\widetilde{ṢṆ_{ù}} = \sum_{€=1}^{ß} ɣ_{€} \times \widetilde{ṄĐ}\hat{A}_{ù€}, \tag{25}$$

where $(ù = 1, 2, 3, \ldots, x)$ and $(€ = 1, 2, 3, \ldots, ß)$.

Step 7: Obtaining the predetermined values of $\widetilde{Ṣ\mathcal{P}_{ù}}$ and $\widetilde{ṢṆ_{ù}}$ for every potential choice. The maximum of $\widetilde{Ṣ\mathcal{P}_{ù}}$ and $\widetilde{ṢṆ_{ù}}$ needs to be ascertained using a defuzzification procedure. The IV-TSFVs yield T-spherical $\widetilde{\mathcal{N}Ṣ\mathcal{P}_{ù}^{t-sph}}$ and $\widetilde{\mathcal{N}ṢṄ_{ù}^{t-sph}}$ acquired in the following manners.

$$\widetilde{\mathcal{N}Ṣ\mathcal{P}_{ù}^{t-sph}} = \frac{\widetilde{Ṣ\mathcal{P}_{ù}}}{Max_{ù}\left(\widetilde{Ṣ\mathcal{P}_{ù}}\right)}, \tag{26}$$

$$\widetilde{\mathcal{N}Ṣ\mathcal{P}_{ù}^{t-sph}} = \frac{\left(\sum_{€=1}^{ß} ɣ_{€} \times \widetilde{\rho Đ}\hat{A}_{ù€}\right)}{Max_{ù}\left(\sum_{€=1}^{ß} ɣ_{€} \times \widetilde{\rho Đ}\hat{A}_{ù€}\right)}, \tag{27}$$

$(ù = 1, 2, 3, \ldots, x)$, and

$$\widetilde{\mathcal{N}ṢṄ_{ù}^{t-sph}} = 1 - \frac{\widetilde{ŚṄ_{î}}}{Max_{î}\left(\widetilde{ŚṄ_{î}}\right)}, \tag{28}$$

$$\widetilde{NṢṄ_{ù}^{t-sph}} = 1 - \frac{\sum_{€=1}^{ß} ɣ_{€} \times \widetilde{ṄĐ}\hat{A}_{ù€}}{Max_{î}\left(\sum_{€=1}^{ß} ɣ_{€} \times \widetilde{ṄĐ}\hat{A}_{ù€}\right)}, \tag{29}$$

$(ù = 1, 2, 3, \ldots, x)$.

Step 7: To calculate each alternative's T-spherical appraisal $(\tilde{A}Ś^{T-sph})$ score, which appears to be as follows:

$$\bar{A}Ṣ^{T-sph} = \frac{1}{2}\left(\widetilde{\mathcal{N}Ṣ\mathcal{P}_{ù}^{t-sph}} + \widetilde{\mathcal{N}ṢṄ_{ù}^{t-sph}}\right), \tag{30}$$

where $0 \leq \bar{A}\dot{S}^{T-sph} \leq 1$.

Step 8: In the final step, we have to rank the alternatives $\ddot{Q}_{\ddot{u}}$ $(\ddot{u} = 1, 2, 3, \ldots, \ddot{x})$ in descending order based on their evaluation's Appraisal Scoring $(\tilde{A}\acute{S}^{T-sph})$ outcomes. The choice that achieves the highest rating is considered the best one.

4.1 *Numerical example*

In this section, we establish an application for determining the diagnoses of skin diseases based on the symptoms of skin diseases by applying the extended conventional EDAS approach under the environment of IV-TSFVs.

Pathology is the specific field of medicine that investigates the causes, procedures, and effects of diseases. It entails identifying the disease's etiology, understanding its pathophysiology, identifying the architectural changes linked to the disease process (morphological alterations), and determining the functional effects of those alterations. Determining the right course of treatment requires precisely identifying the root cause of the disease. Various harmful diseases that occur in the human body are named as Parkinson's disease, viral hepatitis, Human Immunodeficiency Virus/Acquired Immunodeficiency Syndrome (HIV/AIDS), influenza, Lung cancer, measles, autoimmune diseases, chlamydia, diabetes, skin diseases, and many others. In real-world scenarios, we will discuss various types of skin disorders that are harmful to the human body system. Irritation, inflammatory processes, scratching, and other skin abnormalities can be caused by these conditions. Certain skin disorders may be inherited, while other aspects may be caused by lifestyle. Pharmaceuticals, moisturizers, antibiotics, and lifestyle modifications can all be used to treat skin diseases. There are numerous types of skin disorders and aside from knowing that they all negatively affect the skin, the majority of them are entirely unrelated. Skin problems can be classified as being caused by infection (antibiotic-resistant bacteria, viral-based or fungal infections), allergic reactions, inflammatory reactions, and many others. There are a few prevalent human disease types are given below:

Bacterial infection:

• **Cellulitis:** Pathogenic bacteria such as streptococcus or staphylococcus that enter through an injury or scrape are usually the cause of cellulitis. It can cause pain, swelling, and redness in the affected area, and it occasionally needs to be treated with antibiotics.

• **Impetigo:** usually appears as reddish eruptions and is extremely contagious and irritating. Compared to youngsters, it is more frequently observed in children and infants. It can show up anywhere on the body, although it is most prevalent around the edges of the nose and mouth. It usually needs to be treated with antibiotics.

These infections can be seen in Figure 2 below.

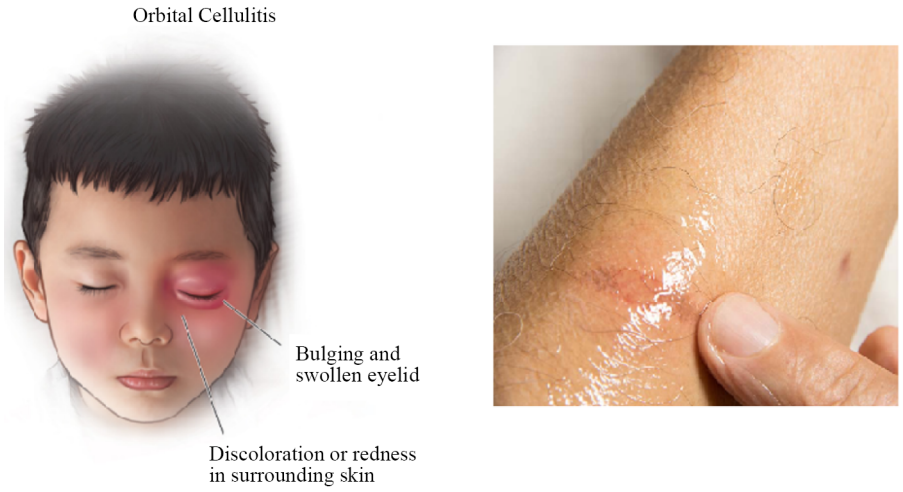

Figure 2. Descritption to Bacterial infection of types "Cellulitis" & "Impetigo" (https://www.wfoqfashion.shop/?path=page/ggitem&ggpid=1523921) (https://www.pearlchemistgroup.co.uk/blog/post/impetigo-vs-other-skin-infections-how-to-tell-the-difference)

Viral infection:

• **Warts** are often harmless growths caused by viruses and typically go away without any medical intervention, but it take some time to disappear. Furthermore, a variety of therapies are accessible, such as exfoliating medication, and laser treatments.

• **Chickenpox:** A widespread infectious disease, chickenpox is characterized by red, ranging itchy patches. Although it can affect persons of any age group, it is most prevalent in children. Fever and headaches are other possible side effects. Several topical remedies can reduce symptoms.

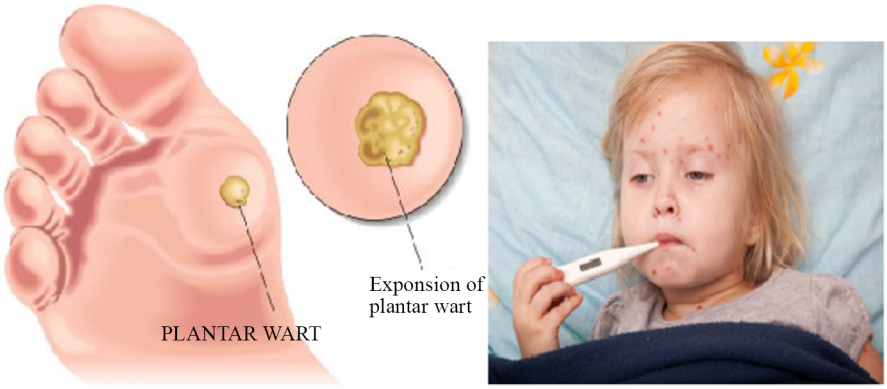

Figure 3. Descritption to Viral infection of types "Warts" & "Chicken" (https://footandanklespec.com/plantar-warts/https://www.pearlchemistgroup.co.uk/blog/post/impetigo-vs-other-skin-infections-how-to-tell-the-difference)

An example of the viral infections can be seen in Figure 3 below.

Fungal infection:

• **Ringworm:** It includes an athlete's foot. In general, it manifested as an irritating, red, and bumpy patch. It is susceptible to treatment with antifungal solutions or moisturizers.

• **Yeast infection:** The candidiasis fungus normally occurs in modest quantities on the body, that represents the root cause of yeast infection. Infections happen when yeast accumulates and grows excessively. It is susceptible to treatment with antifungal solutions or moisturizers.

These two types of infections can be observed in Figure 4 below.

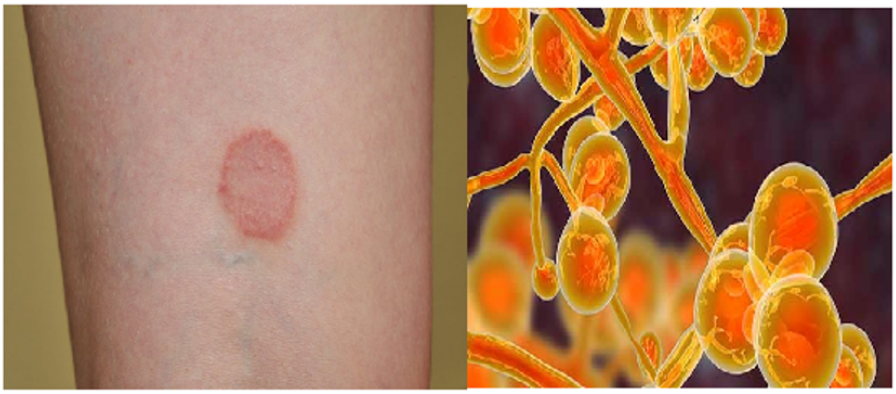

Figure 4. Descritption to Viral infection of types "Ringworm" & "Yeast infection" (https://www.usdermatologypartners.com/services/general-dermatology/ringworm-treatment) (https://www.hopkinsmedicine.org/health/conditions-and-diseases/candidiasis-yeast-infection)

Allergic reactions:

• **Eczema:** Atopic eczema, or atopic dermatitis, is often linked to a person's hypersensitivity to allergens like pollen, dust mites, or certain foods. Though eczema itself is not an allergy, these triggers can certainly worsen the condition. It makes the skin painful, cracked, damaged, and irritated. Topical steroid creams are frequently used to reduce symptoms of atopic eczema, although avoiding triggers can help to reduce confrontations.

An example of the Allergic infections are portrayed in Figure 5 below.

Figure 5. Descritption to allergic reaction of type "Eczema" (https://www.medicalnewstoday.com/articles/322484#types)

To diagnose the most favorable skin diseases or skin disorders $\ddot{\Omega}_{\ddot{u}}$ $(\ddot{u} = 1, 2, 3, 4)$ based on the symptoms of skin diseases $\ddot{B}_{\epsilon}$ $(\epsilon = 1, 2, 3, 4)$ under the framework of IV-TSFVs. The following description is listed below:

$\ddot{B}_1$: represents skin discoloration, which means any skin modification that differs from the original complexion which includes birthmarks, problems with skin pigmentation, tumors on the skin, and many others.

$\ddot{B}_2$: tells about skin dryness, dehydration, and lack of natural oils can disrupt the skin's barrier, leading to skin dryness. Moisturizing regularly and using gentle skincare products can help to maintain your skin.

$\ddot{B}_3$: describes the life-threatening skin rashes that are rare, but require immediate medical attention which includes Pemphigus Vulgaris (PV), Stevens-Johnson Syndrome (SJS), Toxic Epidermal Necrolysis (TEN), and many others.

$\ddot{B}_4$: tells about open sores which are also called ulcers**,** that can happen for a variety of reasons, including inadequate circulation of blood, trauma, or pressure.

Table 2. Decision matrix $\ddot{H}$ of IV-TSF information

	$\ddot{B}_1$	$\ddot{B}_2$	$\ddot{B}_3$	$\ddot{B}_4$
$\ddot{\Omega}_1$	[0.015, 0.356] [0.056, 0.328] [0.217, 0.267]	[0.267, 0.309] [0.165, 0.276] [0.317, 0.456]	[0.198, 0.317] [0.287, 0.317] [0.165, 0.276]	[0.217, 0.326] [0.219, 0.389] [0.327, 0.387]
$\ddot{\Omega}_2$	[0.156, 0.327] [0.165, 0.217] [0.254, 0.376]	[0.136, 0.378] [0.333, 0.409] [0.276, 0.467]	[0.015, 0.237] [0.247, 0.365] [0.254, 0.376]	[0.157, 0.276] [0.416, 0.537] [0.317, 0.465]
$\ddot{\Omega}_3$	[0.167, 0.217] [0.016, 0.376] [0.201, 0.317]	[0.187, 0.318] [0.028, 0.426] [0.316, 0.467]	[0.145, 0.287] [0.067, 0.367] [0.376, 0.425]	[0.013, 0.156] [0.054, 0.267] [0.256, 0.346]
$\ddot{\Omega}_4$	[0.156, 0.217] [0.265, 0.365] [0.158, 0.465]	[0.136, 0.365] [0.365, 0.389] [0.127, 0.476]	[0.198, 0.345] [0.265, 0.364] [0.067, 0.256]	[0.187, 0.376] [0.243, 0.317] [0.034, 0.342]

To define the suitable weight vectors $\gamma_\epsilon = (0.17, 0.28, 0.33, 0.22)^T$ provided by the decision-makers that is used for determining the diagnosis of skin diseases Ω_υ $(\upsilon = 1, 2, 3, 4)$ based on the symptoms of skin diseases β_ϵ $(\epsilon = 1, 2, 3, 4)$. The following Table 2 shows the IV-TSF decision matrix $H = \left(h_{\upsilon\epsilon}\right)_{4\times4}$ as given below:

4.2 *MADM approach based on our proposed IV-TSF method*

To develop a sophisticated algorithm by utilizing the IV-TSF EDAS method combined with the corresponding weight vectors given as: $\gamma_\epsilon = (0.17, 0.28, 0.33, 0.22)^T$ where $\gamma_\epsilon \in [0, 1]$, $\sum_{\epsilon=1}^{s} \gamma_\epsilon$. The desired information based on the IV-TSFVs is presented in Table 2. The detailed analysis and structured approach must yield some insightful results.

Step 1: Firstly, we have to determine the average response of IV-TSFVs concerning the attributed values β_ϵ $(\epsilon = 1, 2, 3, 4)$. The average response serves as the reference point for calculating PDA and NDA for each alternative Ω_υ $(\upsilon = 1, 2, 3, \ldots, \mathfrak{X})$ as follows:

$$\widetilde{A\mathscr{V}}_{\epsilon_1} = (0.0091, 0.0334, 0.1818, 0.3275, 0.2565, 0.3465),$$

$$\widetilde{A\mathscr{V}}_{\epsilon_2} = (0.0025, 0.0295, 0.2903, 0.3820, 0.2753, 0.4210),$$

$$\widetilde{A\mathscr{V}}_{\epsilon_3} = (0.0035, 0.0171, 0.0413, 0.3590, 0.2873, 0.3888),$$

$$\widetilde{A\mathscr{V}}_{\epsilon_4} = (0.0051, 0.0362, 0.2845, 0.3588, 0.0965, 0.3848).$$

Step 2: We have to determine the accuracy function $Acr(\mathfrak{F})$ of IV-TSFVs to diagnose skin diseases based on the symptoms of skin diseases β_ϵ $(\epsilon = 1, 2, 3, 4)$ by using Definition 5. The following Table 3 shows some pertinent results as given below:

Table 3. shows the accuracy values $Acr(\mathfrak{F})$ to effectively diagnose the skin diseases

	β_1	β_2	β_3	β_4
Ω_1	0.0366	0.0410	0.0360	0.0619
Ω_2	0.0669	0.0949	0.0831	0.0895
Ω_3	0.0402	0.0489	0.0688	0.0442
Ω_4	0.0691	0.1280	0.0271	0.0486

Step 3: To determine the accuracy values of $\widetilde{A\mathscr{V}}$ associated with the attributed values β_ϵ $(\epsilon = 1, 2, 3, 4)$ based on the Definition 5. The following suitable results are described as follows:

$$Acr\left(\widetilde{A\mathscr{V}}_{\epsilon_1}\right) = 0.0332,\ Acr\left(\widetilde{A\mathscr{V}}_{\epsilon_2}\right) = 0.0586,\ Acr\left(\widetilde{A\mathscr{V}}_{\epsilon_3}\right) = 0.0429,\ Acr\left(\widetilde{A\mathscr{V}}_{\epsilon_4}\right) = 0.0424. \tag{31}$$

Step 4: We have to compute the positive distance PDA and the negative distance matrix NDA from the average response based on the particular type of attributes β_ϵ as follows:

$$\widetilde{\rho\text{Đ}\hat{\text{A}}}_{\epsilon_1} = (0.0647,\ 0.6414,\ 0.1332,\ 0.6823),$$

$$\widetilde{\rho\text{Đ}\hat{\text{A}}}_{\epsilon_2} = (0.0000,\ 0.6920,\ 0.0000,\ 1.3228),$$

$$\widetilde{\rho\text{Đ}\hat{\text{A}}}_{\epsilon_3} = (0.0000,\ 0.7655,\ 0.4921,\ 0.0000),$$

$$\widetilde{\rho\text{Đ}\hat{\text{A}}}_{\epsilon_4} = (0.3721,\ 0.8975,\ 0.0358,\ 0.1196),$$

$$\widetilde{\text{Ǹ}\text{Đ}\hat{\text{A}}}_{\epsilon_1} = (0.0000,\ 0.0000,\ 0.0000,\ 0.0000),$$

$$\widetilde{\text{Ǹ}\text{Đ}\hat{\text{A}}}_{\epsilon_2} = (0.3343,\ 0.0000,\ 0.1849,\ 0.0000),$$

$$\widetilde{\text{Ǹ}\text{Đ}\hat{\text{A}}}_{\epsilon_3} = (0.1319,\ 0.0000,\ 0.0000,\ 0.3021),$$

$$\widetilde{\text{Ǹ}\text{Đ}\hat{\text{A}}}_{\epsilon_4} = (0.0000,\ 0.0000,\ 0.0000,\ 0.0000).$$

Step 5: After that, we have to determine the weighted sum of positive distance $\widetilde{\rho\text{Đ}\hat{\text{A}}}_{\epsilon}$ and negative distance $\widetilde{\text{Ǹ}\text{Đ}\hat{\text{A}}}_{\epsilon}$ to diagnose the skin diseases $\text{Ǫ}_{\grave{\text{u}}}$ $(\grave{\text{u}} = 1,\ 2,\ 3,\ 4)$ based on the IV-TSFVs are described as follows:

$$\begin{aligned} &\widetilde{\text{Ş}\rho_1} = 0.3846,\ \widetilde{\text{Ş}\rho_2} = 0.4848,\ \widetilde{\text{Ş}\rho_3} = 0.3767,\ \widetilde{\text{Ş}\rho_4} = 0.3527, \\ &\widetilde{\text{Ş}\text{Ņ}_1} = 0.0000,\ \widetilde{\text{Ş}\text{Ņ}_2} = 0.1178,\ \widetilde{\text{Ş}\text{Ņ}_3} = 0.0889,\ \widetilde{\text{Ş}\text{Ņ}_4} = 0.0000. \end{aligned} \tag{32}$$

Step 6: To obtain the predetermined values of $\widetilde{\text{Ş}\rho_{\grave{\text{u}}}}$ and $\widetilde{\text{Ş}\text{Ņ}_{\grave{\text{u}}}}$ for every potential choice. The maximum of $\widetilde{\text{Ş}\rho_{\grave{\text{u}}}}$ and $\widetilde{\text{Ş}\text{Ņ}_{\grave{\text{u}}}}$ needs to be ascertained using a defuzzification procedure. The IV-TSFVs yield T-spherical $\mathcal{N}\widetilde{\text{Ş}\rho_{\grave{\text{u}}}^{t-sph}}$ and $\mathcal{N}\widetilde{\text{Ş}\text{Ǹ}_{\grave{\text{u}}}^{t-sph}}$ acquired as follows:

$$\begin{aligned} &\mathcal{N}\widetilde{\text{Ş}\rho_1^{t-sph}} = 0.7934,\ \mathcal{N}\widetilde{\text{Ş}\rho_2^{t-sph}} = 1.0000,\ \mathcal{N}\widetilde{\text{Ş}\rho_3^{t-sph}} = 0.7772,\ \mathcal{N}\widetilde{\text{Ş}\rho_4^{t-sph}} = 0.7275, \\ &\mathcal{N}\widetilde{\text{Ş}\text{Ǹ}_1^{t-sph}} = 1.0000,\ \mathcal{N}\widetilde{\text{Ş}\text{Ǹ}_2^{t-sph}} = 0.0000,\ \mathcal{N}\widetilde{\text{Ş}\text{Ǹ}_3^{t-sph}} = 0.2457,\ \mathcal{N}\widetilde{\text{Ş}\text{Ǹ}_4^{t-sph}} = 1.0000. \end{aligned} \tag{33}$$

Step 7: To determine the IV-T-spherical appraisal $(\tilde{\text{Ă}}\acute{\text{S}}^{T-sph})$ score to effectively diagnose the favorable skin diseases, which appears to be as follows:

$$\tilde{A}\acute{S}_1^{T-sph} = 0.8967,\ \tilde{A}\acute{S}_2^{T-sph} = 0.5000,\ \tilde{A}\acute{S}_3^{T-sph} = 0.5114,\ \tilde{A}\acute{S}_4^{T-sph} = 0.8638. \tag{34}$$

Step 8: In the final step, we have to diagnose the favorable skin diseases (alternatives) $\ddot{\Omega}_{\grave{u}}$ $(\grave{u} = 1,\ 2,\ 3,\ 4)$ in descending order based on their evaluation's appraisal scoring $(\tilde{A}\acute{S}^{T-sph})$ consequences. By utilizing the given weights $\gamma_{\epsilon} = (0.17,\ 0.28,\ 0.33,\ 0.22)^T$ to identify the favourable skin diseases based on the symptoms of skin diseases under the framework of IV-TSF EDAS approach. The desired ranking results are given as $\left(\tilde{A}\acute{S}_1^{T-sph} > \tilde{A}\acute{S}_4^{T-sph} > \tilde{A}\acute{S}_3^{T-sph} > \tilde{A}\acute{S}_2^{T-sph}\right) : \ddot{\Omega}_1 > \ddot{\Omega}_4 > \ddot{\Omega}_3 > \ddot{\Omega}_2$. According to the above discussions, we see that $\ddot{\Omega}_1$ is a favorable skin disease to effectively diagnose by using IV-TSF information.

The visual illustration of skin diseases (alternatives) $\ddot{\Omega}_{\grave{u}}$ $(\grave{u} = 1,\ 2,\ 3,\ 4)$ based on the symptoms of skin diseases related to the IV-TSF EDAS method is shown in Figure 6.

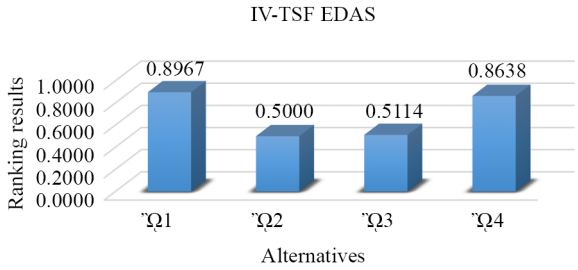

Figure 6. shows the visual illustration of skin diseases $\ddot{\Omega}_{\grave{u}}$ $(\grave{u} = 1,\ 2,\ 3,\ 4)$

4.3 *Sensitivity analysis*

To demonstrate the usefulness and the efficiency of the criteria weights $\gamma_{\epsilon} = \left(\gamma_{\epsilon_1},\ \gamma_{\epsilon_2},\ \gamma_{\epsilon_3},\ \gamma_{\epsilon_4}\right)$, we have to diagnose the favorable skin diseases $\ddot{\Omega}_{\grave{u}}$ $(\grave{u} = 1,\ 2,\ 3,\ 4)$ based on the symptoms of skin diseases $\ddot{B}_{\epsilon}$ $(\epsilon = 1,\ 2,\ 3,\ 4)$ by using the IV-TSF EDAS method. Now we take different criteria weights γ_{ϵ}, and determine the ranking results to precisely identify the root cause of the skin diseases by using the IV-TSF EDAS method which are shown in the following Table 4 and Figure 7. If we take the criteria weights as $\gamma_{\epsilon} = (0.31,\ 0.42,\ 0.11,\ 0.16)$, then the ranking results of the skin diseases $\ddot{\Omega}_{\grave{u}}$ based on their evaluation rating findings are similar as $\ddot{\Omega}_4 > \ddot{\Omega}_1 > \ddot{\Omega}_3 > \ddot{\Omega}_2$. We notice that $\ddot{\Omega}_4$ is the favorable skin disease. The remaining all other criteria weights $\left(\gamma_{\epsilon_1},\ \gamma_{\epsilon_2},\ \gamma_{\epsilon_3},\ \gamma_{\epsilon_4}\right)$ have the same level of ranking results based on their evaluation's rating findings are similar as $\ddot{\Omega}_1 > \ddot{\Omega}_4 > \ddot{\Omega}_2 > \ddot{\Omega}_3$. It means that $\ddot{\Omega}_1$ is a favorable skin disease to effectively diagnose by using IV-TSF information.

The visual illustration of skin diseases (alternatives) $\ddot{\Omega}_{\grave{u}}$ $(\grave{u} = 1,\ 2,\ 3,\ 4)$ based on the symptoms of skin diseases $\ddot{B}_{\epsilon}$ $(\epsilon = 1,\ 2,\ 3,\ 4)$ by using different criteria weights $\gamma_{\epsilon} = \left(\gamma_{\epsilon_1},\ \gamma_{\epsilon_2},\ \gamma_{\epsilon_3},\ \gamma_{\epsilon_4}\right)$, associated with the IV-TSF EDAS method as shown in Figure 7.

Table 4. exhibits the variation of criteria weights γ_{ϵ} by using the IV-TSF EDAS method

Criteria weights	Ranking and ordering results
$\gamma_{\epsilon_1}=0.04, \gamma_{\epsilon_2}=0.26, \gamma_{\epsilon_3}=0.32, \gamma_{\epsilon_4}=0.38$	$\ddot{\Omega}_1 > \ddot{\Omega}_4 > \ddot{\Omega}_2 > \ddot{\Omega}_3$ $\left(\tilde{A}\acute{S}_1^{T-sph}=0.8452, \tilde{A}\acute{S}_4^{T-sph}=0.7235, \tilde{A}\acute{S}_2^{T-sph}=0.6979, \tilde{A}\acute{S}_3^{T-sph}=0.2612\right)$
$\gamma_{\epsilon_1}=0.22, \gamma_{\epsilon_2}=0.17, \gamma_{\epsilon_3}=0.32, \gamma_{\epsilon_4}=0.29$	$\ddot{\Omega}_1 > \ddot{\Omega}_4 > \ddot{\Omega}_2 > \ddot{\Omega}_3$ $\left(\tilde{A}\acute{S}_1^{T-sph}=0.8623, \tilde{A}\acute{S}_4^{T-sph}=0.7799, \tilde{A}\acute{S}_2^{T-sph}=0.5000, \tilde{A}\acute{S}_3^{T-sph}=0.3475\right)$
$\gamma_{\epsilon_1}=0.21, \gamma_{\epsilon_2}=0.35, \gamma_{\epsilon_3}=0.02, \gamma_{\epsilon_4}=0.42$	$\ddot{\Omega}_1 > \ddot{\Omega}_4 > \ddot{\Omega}_2 > \ddot{\Omega}_3$ $\left(\tilde{A}\acute{S}_1^{T-sph}=0.8305, \tilde{A}\acute{S}_4^{T-sph}=0.7778, \tilde{A}\acute{S}_2^{T-sph}=0.7610, \tilde{A}\acute{S}_3^{T-sph}=0.1741\right)$
$\gamma_{\epsilon_1}=0.31, \gamma_{\epsilon_2}=0.42, \gamma_{\epsilon_3}=0.11, \gamma_{\epsilon_4}=0.16$	$\ddot{\Omega}_4 > \ddot{\Omega}_1 > \ddot{\Omega}_3 > \ddot{\Omega}_2$ $\left(\tilde{A}\acute{S}_4^{T-sph}=1.0000, \tilde{A}\acute{S}_1^{T-sph}=0.9009, \tilde{A}\acute{S}_3^{T-sph}=0.5046, \tilde{A}\acute{S}_2^{T-sph}=0.4873\right)$
$\gamma_{\epsilon_1}=0.18, \gamma_{\epsilon_2}=0.22, \gamma_{\epsilon_3}=0.38, \gamma_{\epsilon_4}=0.22$	$\ddot{\Omega}_1 > \ddot{\Omega}_4 > \ddot{\Omega}_3 > \ddot{\Omega}_2$ $\left(\tilde{A}\acute{S}_1^{T-sph}=0.8987, \tilde{A}\acute{S}_4^{T-sph}=0.8433, \tilde{A}\acute{S}_3^{T-sph}=0.5551, \tilde{A}\acute{S}_2^{T-sph}=0.5000\right)$
$\gamma_{\epsilon_1}=0.12, \gamma_{\epsilon_2}=0.05, \gamma_{\epsilon_3}=0.26, \gamma_{\epsilon_4}=0.57$	$\ddot{\Omega}_1 > \ddot{\Omega}_2 > \ddot{\Omega}_4 > \ddot{\Omega}_3$ $\left(\tilde{A}\acute{S}_1^{T-sph}=0.7938, \tilde{A}\acute{S}_2^{T-sph}=0.7655, \tilde{A}\acute{S}_4^{T-sph}=0.6059, \tilde{A}\acute{S}_3^{T-sph}=0.1054\right)$
$\gamma_{\epsilon_1}=0.21, \gamma_{\epsilon_2}=0.13, \gamma_{\epsilon_3}=0.34, \gamma_{\epsilon_4}=0.32$	$\ddot{\Omega}_1 > \ddot{\Omega}_4 > \ddot{\Omega}_2 > \ddot{\Omega}_3$ $\left(\tilde{A}\acute{S}_1^{T-sph}=0.8513, \tilde{A}\acute{S}_4^{T-sph}=0.7389, \tilde{A}\acute{S}_2^{T-sph}=0.5000, \tilde{A}\acute{S}_3^{T-sph}=0.2926s\right)$

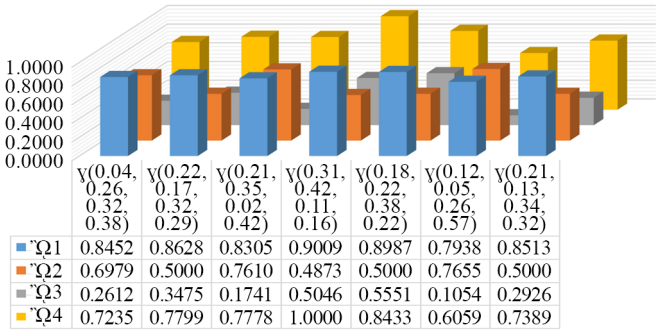

Figure 7. exhibits the variation of criteria weights γ_{ϵ} by using the IV-TSF EDAS method

5. Comparative study

To assess the potential advantages of the recommended approach and also exemplify its usefulness and effectiveness in real-world scenarios, we accomplish a comparative study between the recently developed approach and the other widely recognized strategies that are applied in the form of IV-TSFVs, q-ROFVs, and PFVs. The extensive ranking outcomes of our recently developed IV-TSF EDAS approach with other conventional approaches such as IV-TSF Weighted Averaging (IV-TSFWA) operator, IV-TSFW Geometric (IV-TSFWG) operator initially developed by Ullah et al. [43], IV-TSF Dombi WA (IV-TSFDWA) operator, IV-TSF Dombi WG (IV-TSFDWG) operator originally invented by Ullah et al. [44], q-ROF Einstein Prioritized WA (q-ROFEPWA) operator, q-ROF Einstien Prioritized WG (q-ROFEPWG) operator firstly introduced by Riaz et al. [45], PF Interactional Partitioned Heronian Mean WA (PF-IPHMWA) operator, and PF Interactional Partitioned Heronian Mean WG (PF-IPHMWG) operator presented by Lin et al. [46]. Numerous other established AOs such as q-ROFEPWA operator, q-ROFEPWG operator, PF-IPHMWA operator, and PF-IPHMWG operator cannot deal with MADM concerns under the prerequisites of IV-TSFVs listed in Table 5.

Table 5. Ranking outcomes to effectively diagnose the skin diseases

AOs	Ranking and ordering consequences
IV-TSF EDAS	$\ddot{\Omega}_1 > \ddot{\Omega}_4 > \ddot{\Omega}_3 > \ddot{\Omega}_2$
IV-TSFWA	$\ddot{\Omega}_2 > \ddot{\Omega}_3 > \ddot{\Omega}_4 > \ddot{\Omega}_1$
IV-TSFWG	$\ddot{\Omega}_2 > \ddot{\Omega}_3 > \ddot{\Omega}_1 > \ddot{\Omega}_4$
IV-TSFDWA	$\ddot{\Omega}_2 > \ddot{\Omega}_1 > \ddot{\Omega}_3 > \ddot{\Omega}_4$
IV-TSFDWG	$\ddot{\Omega}_2 > \ddot{\Omega}_3 > \ddot{\Omega}_1 > \ddot{\Omega}_4$
q-ROFEPWA	Not applicable
q-ROFEPWG	Not applicable
PF-IPHMWA	Not applicable
PF-IPHMWG	Not applicable

We see that our recently developed IV-TSF EDAS approach has different ranking results as compared to other established AOs such as IV-TSFWA operator, IV-TSFWG operator, IV-TSFDWA operator, and IV-TSFDWG operator. The extensive ranking results of the IV-TSF EDAS method are utilized effectively to diagnose favorable skin diseases which are given as $\ddot{\Omega}_1 > \ddot{\Omega}_4 > \ddot{\Omega}_3 > \ddot{\Omega}_2$. It means that $\ddot{\Omega}_1$ is the favorable skin disease based on the symptoms of skin diseases $\ddot{B}_\epsilon$ $(\epsilon = 1, 2, 3, 4)$. Various other recognized AOs such as IV-TSFWA operator, IV-TSFWG operator, IV-TSFDWA operator, and IV-TSFDWG operator have the same level of comprehensive ranking findings to effectively diagnose the favorable skin diseases which are described as $\ddot{\Omega}_2 > \ddot{\Omega}_3 > \ddot{\Omega}_1 > \ddot{\Omega}_4$. It means that the $\ddot{\Omega}_2$ is the favourable skin disease according to these types of conventional AOs which are shown in Table 5 and Figure 8. The EDAS method provides a balanced view of each alternative's performance, helping Decision-Makers (DMrs) to rank and select the most suitable option effectively. It is the most powerful way to visualize and compare the performance of various alternatives in a structured and quantitative manner. It was developed to improve upon existing MADM techniques like TOPSIS, VIKOR, IV-TSFWA, IV-TSFWG, IV-TSFDWA, and, IV-TSFDWG operators by providing a more intuitive and balanced approach to decision-making. The IV-TSF EDAS approach offers more precise and optimal solutions as compared to conventional AOs, especially in uncertain, fuzzy, and real-world decision-making environments. The distinctions, deficiencies, and unexplored aspects associated with mental abilities can be considerably enhanced by our proposed IV-TSF EDAS approach, relying on the IV-TSF environment. As a result, our suggested methodology seems more advantageous and comprehensible than other popular methods.

The following Figure 8 shows the visual illustration of the comparative ranking results of our suggested IV-TSF EDAS method with other established AOs to effectively diagnose the favorable skin diseases (alternatives) $\ddot{\Omega}_{\ddot{u}}$ $(\ddot{u} = 1, 2, 3, 4)$ based on the symptoms of skin diseases $\ddot{B}_\epsilon$ $(\epsilon = 1, 2, 3, 4)$ under the framework of IV-TSFVs.

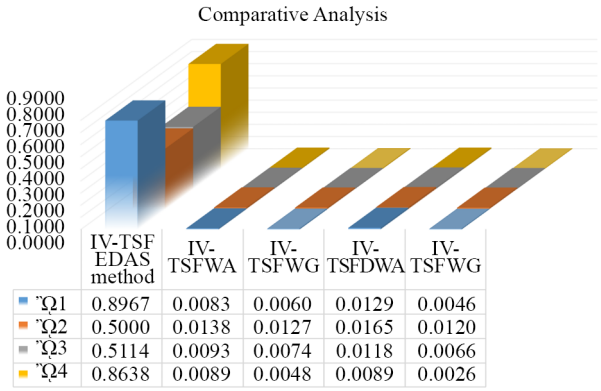

Figure 8. exhibits the extensive ranking results to effectively diagnose skin diseases $\ddot{\Omega}_{\ddot{u}}$ $(\ddot{u} = 1, 2, 3, 4)$

6. Conclusion

Various harmful diseases occur in the human body which are named including Parkinson's disease, viral hepatitis, HIV/AIDS, influenza, Lung cancer, measles, autoimmune diseases, chlamydia, diabetes, skin diseases, and many others. Determining the right course of treatment requires precisely identifying the root cause of these diseases. To effectively diagnose skin diseases based on the symptoms of skin diseases, we have introduced the novel idea of IV-TSFS, which relies on the basic concept of T-SFS by introducing the interval values. The developmental approach significantly enhances the ability to handle vagueness and predictability in the data being gathered. In this paper, we have developed an innovative idea of an extended classical EDAS (Evaluation based on distance from average solution) approach under the framework of IV-TSFS to cope with erratic data that is used in the various components of the classical EDAS approach. An effective extended conventional EDAS technique is being used for identifying the treatment of skin diseases under the framework of IV-TSFS and effectively determining their ranking results. We have also examined important amenities, perspectives, necessary steps, correlating approaches, and the historical backdrop of the invented IV-TSF EDAS approach, which may contribute to more comparable and scalable solutions in a wide variety of situations requiring decision-making. Furthermore, a numerical illustration gives the potential to effectively diagnose favorable skin diseases based on the symptoms of skin diseases by using the IV-TSF EDAS method and also determine their comprehensive comparative results.

In future research, we aim to develop the concept of the IV-TSF EDAS method in a wide range of other disciplines of science such as exemplary employee selection [47], optimal selection of an electric power wheelchair [48], assessment of Chinese international education in Higher Education Institutions (HEI) [49], FP-Intuitionistic Multi Fuzzy N-soft (FP-IMFNS) set [50], induced FP-Hesitant N-Soft (IFP-HN-S) set [50], IFP-Intuitionistic Multi Fuzzy N-Soft (IFP-IMFN-S) set [51], Induced Ifp-Hesitant N-Soft (IIFP-HN-S) set [51], anthropogenic influences on urban river water quality [52], bibliometric analysis of the MADM problem [53], and many other disciplines of science.

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Data availability

Data sharing does not apply to this article as no datasets were generated or analyzed during the current study.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

Conflict of interest

The authors declare that they have no competing interests.

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