Contemporary Mathematics
https://ojs.wiserpub.com/index.php/CM/

Research Article

On the Application of Complex Delta Function Leading to New Fractional Calculus Formulae Involving the Generalized Hypergeometric Function and Kinetic Equation

Sara Saud[1], Rekha Srivastava[2], Azza M. Alghamdi[3], Rabab Alharbi[4], Asifa Tassaddiq[1*]

[1] Department of Computer Science, College of Computer and Information Sciences, Majmaah University, Al Majmaah, 11952, Saudi Arabia
[2] Department of Mathematics and Statistics, University of Victoria, Victoria, BC, V8W 3R4, Canada
[3] Department of Mathematics, Faculty of Sciences Al-Baha University, P.O. Box-7738, Alaqiq, Al-Baha, 65799, Saudi Arabia
[4] Department of Mathematics, College of Science, Qassim University, Buraydah, 51452, Saudi Arabia
E-mail: a.tassaddiq@mu.edu.sa

Received: 25 June 2025; **Revised:** 31 October 2025; **Accepted:** 5 November 2025

Abstract: The sun is a vital component of our natural environment, and kinetic equations are important mathematical models that show how quickly a star's chemical composition changes. Taking inspiration from these facts, we develop and solve a novel fractional kinetic equation by calculating the Laplace transform of hypergeometric functions in the complex coefficient parameter. This was a challenging task because the function cannot be integrated concerning the coefficient parameters using classical methods due to the infinite number of singular points of the gamma function involved in it. We achieved it using the distributional representation of the generalized hypergeometric function. Moreover, on the one hand, the role of the delta function is vital to represent the electromotive forces, and on the other, the solution of differential equations of engineering and mathematical physics led to a class of hypergeometric functions. This article is the confluence of both. Therefore, innovative characteristics concerning the Fox-Wright and several related important functions are applied for the simplification of the obtained outcomes. A popular class of fractional transforms involving generalized hypergeometric functions are evaluated using the delta function, and as a distribution, numerous additional features of this function are described.

***Keywords*:** delta function, generalized hypergeometric function, mathematical operators, H-function, kinetic equation

MSC:

1. Introduction

Current hypotheses of gases and astrophysics have greatly enhanced environmental sciences because the role of the sun is crucial in the realm of global warming and a system of differential equations can model the evolution of stars like the sun [1]. Three factors-temperature, pressure, and mass-can be used to characterize stars' internal structure, which is formed completely of gases [2]. In fact, the conversion from the cloud to a star requires a greater gravitational strength as compared with the inside pressure. A protostar is formed and the cloud produces light when fusion of nuclear matter takes

DOI: https://doi.org/10.37256/cm.6620257570

place [1]. Mathematical models to describe the nuclear compositions in these types of stars are used in [1–3]. The basic kinetic equation [2] to study this composition $K(t)$ using the amount of production $P(K)$ and destruction $D(K)$, described as follows:

$$\frac{dK}{dt} = -D(K_t) + P(K_t);\; K_t(t^*) = K(t - t^*),\; t^* > 0. \tag{1}$$

Then ignoring the species disparity and inhomogeneity of $K(t)$, the subsequent equation is formulated, $K_j(t = 0) = K_0$.

$$\frac{dK_j}{dt} = -c_j K_j(t). \tag{2}$$

The following is the result of integrating this equation while ignoring the subscript j,

$$K(t) - K_0 = -d I_{0+}^{-1} K(t). \tag{3}$$

Using the Riemann-Liouville (R-L) fractional integral, subsequent non-integer order kinetic equation can be produced,

$$K(t) - K_0 = -d^{\varepsilon} I_{0+}^{\varepsilon} K(t), \tag{4}$$

where is a constant. The next generalised non-integer kinetic equation [1–4] employing a wide-ranging integrable function $f(t)$ have been studied by various researchers,

$$K(t) - f(t)K_0 = -d^{\varepsilon} I_{0+}^{\varepsilon} K(t). \tag{5}$$

Numerous researchers have made significant contributions to fractional calculus [4]. The literature contains previous studies on a large number of generic families of fractional kinetic equations [5]. Unlike the Hurwitz-Lerch and Mittag-Leffler functions, which have several multi-parameter extensions, Srivastava looked at significantly more general functions in [6, 7]. Moreover, a significant class of general hybrid-type kinetic equations is also considered in the recent research [8, 9]. The link between kinetic equations of fractional order and the theory of continuous-time random walks have recently been discovered, which has led to an increased interest in these equations [10]. The purpose of examining these equations is to identify and then understand certain physical phenomena that are known to control processes such as anomalous propagation, diffusion in porous media, and so forth. According to this review of literature, such an equation with regard to the pth parameter c_p of the generalized hypergeometric function $\Gamma(c_p)\,{}_pF_q\left[\begin{matrix} c_1, \ldots c_p \\ d_1, \ldots d_q \end{matrix}; s\right]$ has not been studied so far. The reason behind it that the function cannot be integrated with respect to the coefficient parameter c_p due to the poles of gamma function. Therefore, main goal of this work is to fill this gap.

Considering the aforementioned, we offer the plan of this study: Basic concepts are given in Section 1.1. New modified representation of the hypergeometric function $\Gamma(c_p)\,{}_pF_q\left[\begin{matrix} c_1, \ldots c_p \\ d_1, \ldots d_q \end{matrix}; s\right]$ as well as the implication to the required kinetic equation is provided in Section 2. Sections 2.1 and 2.2 contain new identities or formulas for fractional calculus employing a hypergeometric function. Section 3 discusses the new distributional representation and its different features. Section 4 discusses further uses of this representation. The last Section 5 concludes this research with suggestions for future directions.

1.1 *Preliminaries*

$\mathbb{R}$ represents the set of real numbers throughout the text. $\mathbb{R}^+$ is a set of positive reals, while $\mathbb{Z}_0$ is a set of positive integers that contains 0. $\Re$ represents real part of a complex number, while $\mathbb{C}$ represents set of such numbers.

For complex numbers $\omega \in \mathbb{C}$ with real part $\Re(\omega) > 0$, the gamma function is defined as [9],

$$\Gamma(\omega) = \int_0^\infty t^{\omega-1} e^{-t} dt. \tag{6}$$

Due to the extensive applications and representations of this well-researched special function, the fundamental Pochhammer symbols $(\omega)_r$ can be defined in terms of the gamma function

$$(\omega)_r = \frac{\Gamma(\omega+r)}{\Gamma(\omega)} = \begin{cases} 1 (r=0), \\ \omega(\omega+1)\ldots(\omega+r-1)(r=n\in\mathbb{N};\ \omega\in\mathbb{C}\backslash\{0\}). \end{cases}$$

The generalized Mittag-Leffler case of three variables $\alpha,\ \beta,\ \gamma \in \mathbb{C}$, and $\Re(\alpha) > 0$ is defined as [11, 12]

$$E^{\gamma}_{\alpha,\ \beta}(\omega) = \sum_{k=0}^{\infty} \frac{(\gamma)_k \omega^k}{k!\Gamma(\alpha k+\beta)}, \tag{7}$$

which is an entire function [13] of type $\sigma = 1$ and order $\varrho = 1/\Re(\alpha)$. However, by taking particular values to parameters $\alpha,\ \beta,\ \gamma$, we can obtain two and one parameter Mittag-Leffler function [11].

By choosing a particular contour which separates the singular points of $\left\{\Gamma\left(1-a_j-A_j\mathfrak{s}\right)\right\}_{j=1}^{n}$ and $\left\{\Gamma\left(b_j+B_j\mathfrak{s}\right)\right\}_{j=1}^{m}$ H-function [14] is defined as follows

$$H^{m,\ n}_{p,\ q}(\omega) = H^{m,\ n}_{p,\ q}\left[\omega \left| \begin{array}{c} (a_i,\ A_i) \\ (b_j,\ B_j) \end{array} \right.\right] = H^{m,\ n}_{p,\ q}\left[\omega \left| \begin{array}{c} (a_1,\ A_1),\ \ldots,\ (a_i,\ A_i) \\ (b_1,\ B_1),\ \ldots,\ (b_j,\ B_j) \end{array} \right.\right]$$

$$= \frac{1}{2\pi i}\int_{\mathscr{L}} \frac{\prod_{j=1}^{m}\Gamma\left(b_j+B_j\mathfrak{s}\right)\prod_{i=1}^{n}\Gamma\left(1-a_j-A_j\mathfrak{s}\right)}{\prod_{j=m+1}^{q}\Gamma\left(1-b_j-B_j\mathfrak{s}\right)\prod_{i=n+1}^{p}\Gamma\left(a_j+A_j\mathfrak{s}\right)}\omega^{-\mathfrak{s}} d\mathfrak{s}, \tag{8}$$

$$\left((1 \leqq m \leqq q;\ 0 \leqq n \leqq p,\ A_i > 0 \wedge B_j > 0,\ a_i \wedge b_j \in \mathbb{C}(i=1,\ \cdots,\ p \wedge j=1,\ \cdots,\ q)\right).$$

Meijer G-function [14] follows from (8) at $A_p = B_q = 1$

$$H^{m,\ n}_{p,\ q}\left[\omega \left| \begin{array}{c} (a_1,\ 1),\ \ldots,\ (a_i,\ 1) \\ (b_1,\ 1),\ \ldots,\ (b_j,\ 1) \end{array} \right.\right] = G^{m,\ n}_{p,\ q}\left[\omega \left| \begin{array}{c} a_1,\ \ldots,\ a_i \\ b_1,\ \ldots,\ b_j \end{array} \right|\right]. \tag{9}$$

Moreover, H-function is related with Fox-Wright function ${}_p\Psi_q$, as follows [14]

$$ {}_p\Psi_q\left[\begin{array}{c}(a_i,\,A_i)\\(b_j,\,B_j)\end{array}\middle|\,\omega\right]=\sum_{m=0}^{\infty}\frac{\prod_{l=1}^{p}\Gamma(a_i+A_i m)}{\prod_{l=1}^{q}\Gamma((b_j+B_j m)}\frac{z^m}{m!}=H^{1,\,p}_{p,\,q+1}\left[-\omega\,\middle|\,\begin{array}{c}(1-a_1,\,A_1),\,\ldots,\,(1-a_i,\,A_i)\\(0,\,1),\,(1-b_1,\,B_1),\,\ldots,\,((1-b_j,\,B_j)\end{array}\right] \tag{10}$$

$$\left(a_i\in\mathbb{R}^+(i=1,\,\ldots p);\; B_j\in\mathbb{R}^+(j=1,\,\ldots q);\; 1+\sum_{i=1}^{q}B_i-\sum_{j=1}^{p}A_j>0\right).$$

The generalized hypergeometric function is defined as [15]

$$\Gamma(c_p)\,{}_pF_q\left[\begin{array}{c}c_1,\,\ldots c_p\\d_1,\,\ldots d_q\end{array};\,s\right]=\sum_{r=0}^{\infty}\frac{(c_1)_r(c_2)_r\ldots(c_p)_r z^r}{(d_1)_r(d_2)_r\ldots(d_q)_r r!}=\sum_{r=0}^{\infty}\frac{z^r\prod_{i=1}^{p}(c_i)_r}{r!\prod_{j=1}^{q}(d_j)_r}, \tag{11}$$

$$\left(c_i;\, d_j\in\mathbb{C};\,\Re(c_i)>0;\, d_j\notin\mathbb{Z}_0^-;\; i=1,\,\ldots.\,p\wedge j=1,\,\ldots.\,q;\; p\leq q+1;\; c_p=v+\imath\theta\right),$$

which has following relationship with Fox-Wright and G-functions

$$ {}_p\Psi_q\left[\begin{array}{c}(a_i,\,1)\\(b_j,\,1)\end{array}\middle|\,\omega\right]={}_pF_q\left[\begin{array}{c}a_i\\b_j\end{array};\,\omega\right]\frac{\Gamma(a_1)\ldots\Gamma(a_i)}{\Gamma(b_1)\ldots\Gamma(b_j)}=G^{1,\,p}_{p,\,q+1}\left[-\omega\,\middle|\,\begin{array}{c}(1-a_1,\,1),\,\ldots,\,(1-a_i,\,1)\\0,\,(1-b_1,\,1),\,\ldots,\,(1-b_j,\,1)\end{array}\right], \tag{12}$$

$\left(a_i>0;\, b_j\notin\mathbb{Z}_0^-\right)$.

Glue et al. [16] defined the Multiple Erdélyi-Kober (M-E-K) integral operators (see also [17–19]) as follows

$$I^{(\gamma_k),\,(\tau_k)}_{(\alpha_k),\,(\beta_k),\,l}f(z)=\begin{cases}f(z);\quad(\tau_k=0;\;\alpha_k=\beta_k)\\[2ex]\int_0^1 f(z\sigma)H^{l,\,0}_{l,\,l}\left[\sigma\,\middle|\,\begin{array}{c}\left[\gamma_k+\tau_k-\frac{1}{\alpha_k}+1,\,\frac{1}{\alpha_k}\right)_1^l\\\left(\gamma_k-\frac{1}{\beta_k}+1,\,\frac{1}{\beta_k}\right)_1^l\end{array}\right]d\sigma;\;(\sum_k\tau_k>0)\\[2ex]=z^{-1}\int_0^z f(\xi)H^{l,\,0}_{l,\,l}\left[\frac{\xi}{z}\,\middle|\,\begin{array}{c}\left(\gamma_k+\tau_k-\frac{1}{\alpha_k}+1,\,\frac{1}{\alpha_k}\right)_1^l\\\left(\gamma_k-\frac{1}{\beta_k}+1,\,\frac{1}{\beta_k}\right)_1^l\end{array}\right]d\xi;\;(\sum_k\tau_k>0).\end{cases} \tag{13}$$

Here, $\sum_{k=1}^{\infty}\frac{1}{\beta_k}\geq\sum_{k=1}^{\infty}\frac{1}{\alpha_k}$. Order of integration is expressed by $\tau_k{}'s$, $\gamma_k{}'s$, are taken as weights while $\alpha_k's$, $\beta_k's$ are accompanying parameters. Since $H^{m,\,0}_{m,\,m}$ vanishes if $|\sigma|>1$ Thus, in equation (13) the upper limit as infinity becomes worthless. However, [18, 19] also provide a clear definition of the corresponding appropriate R-L type derivative with order $(\tau_k\geq0,\,\ldots,\,\tau_1\geq0)=\tau$.

$$I_{(\alpha_k),\,(\beta_k),\,l}^{(\gamma_k),\,(\tau_k)}[f(z)] := D_\eta I_{(\alpha_k),\,(\beta_k),\,l}^{(\gamma_k+\tau_k),\,(\eta_k-\tau_k)} f(z) = D_\eta \int_0^1 f(z\sigma) H_{l,\,1}^{l,\,0}\left[\sigma \left| \begin{array}{c} \left(\gamma_k+\eta_k+1-\dfrac{1}{\alpha_k},\ \dfrac{1}{\alpha_k}\right)_1^l \\ \left(\gamma_k+1-\dfrac{1}{\beta_k},\ \dfrac{1}{\beta_k}\right)_1^l \end{array}\right.\right] d\sigma, \tag{14}$$

where D_η, is defined as a following polynomial

$$D_\eta = \prod_{r=1}^{m}\prod_{j=1}^{\eta_r} \frac{1}{\beta_r} z\frac{d}{dz} + \gamma_r + j;\ \eta_k = \begin{cases} [\tau_k]+1;\ \tau_k \notin \mathbb{Z} \\ \tau_k;\ \tau_k \in \mathbb{Z} \end{cases} \tag{15}$$

and the corresponding Caputo type multiple Erdélyi-Kober (E-K) derivative operators are stated as [19]

$$^{*}D_{(\alpha_k),\,(\beta_k),\,l}^{(\gamma_k),\,(\tau_k)} f(z) = I_{(\beta_k),\,l}^{(\gamma_k+\tau_k),\,(\eta_k-\tau_k)} D_\eta f(z). \tag{16}$$

Table 1. Popular forms of M-E-K fractional operators [19] when $\alpha_k = \beta_k$

Variations in Equation (11)	Integrand of fractional integral operators [19]
Marichev-Saigo-Maeda (M-S-M) [20–22] ($l = 3;\ \alpha_k = \beta_k = 1$)	$H_{3,\,3}^{3,\,0}\left(\frac{t}{x}\right) = G_{3,\,3}^{3,\,0}\left[\frac{t}{x}\left| \begin{array}{c} \gamma_1'+\gamma_2',\ \tau-\gamma_1,\ \tau-\gamma_2 \\ \gamma_1',\ \gamma_2',\ \tau-\gamma_1-\gamma_2 \end{array}\right.\right]$ $= \frac{x^{-\gamma_1}}{\Gamma(\tau)}(x-t)^{\tau-1} t^{-\gamma_1'} F_3\left(\gamma_1,\ \gamma_1',\ \gamma_2,\ \gamma_2',\ \tau;\ 1-\frac{t}{x};\ 1-\frac{x}{t}\right)$
Saigo [23, 24] ($l = 2;\ \alpha_k = \beta_k = 1$)	$H_{2,\,2}^{2,\,0}\left[\sigma\left| \begin{array}{c} (\gamma_1+\tau_1,\ 1),\ (\gamma_2+\tau_2,\ 1) \\ (\gamma_1,\ 1),\ (\gamma_2,\ 1) \end{array}\right.\right]$ $= G_{2,\,2}^{2,\,0}\left[\sigma\left| \begin{array}{c} \gamma_1+\tau_1,\ \gamma_2+\tau_2 \\ \gamma_1,\ \gamma_2 \end{array}\right.\right]$ $= \frac{\sigma^{\gamma_2}(1-\sigma)^{\tau_1+\tau_2-1}}{\Gamma(\tau_1+\tau_2)}\, {}_2F_1(\tau_2-\gamma_1+\gamma_2,\ \tau;\ \tau_1+\tau_2;\ 1-\sigma)$
Erdélyi-Kober (E-K) [19] ($l = 1;\ \alpha_k = \beta_k = \alpha > 0$)	$\mathrm{H}_{1,\,0}^{1,\,1}\left[\sigma\left| \begin{array}{c} \left(\tau,\ \frac{1}{\alpha}\right) \\ \left(0,\ \frac{1}{\alpha}\right) \end{array}\right.\right] = \alpha\sigma^{\alpha-1} G_{1,\,0}^{1,\,1}\left[\sigma^\alpha\big|_0^\tau\right] = \frac{\alpha\sigma^{\alpha-1}(1-\sigma^\alpha)^{\tau-1}}{\Gamma(\tau)}$
Riemann-Liouville (R-L) [19] ($l = 1 = \alpha_k = \alpha$)	$\mathrm{H}_{1,\,0}^{1,\,1}\left[\sigma\left| \begin{array}{c} (\tau,\ 1) \\ (0,\ 1) \end{array}\right.\right] = G_{1,\,0}^{1,\,1}\left[\frac{t}{x}\Big|_{-}^{\tau}\right] = \frac{(x-t)^{\tau-1}}{\Gamma(\tau)}$

Following the work of [20], fractional calculus operators with the Gauss hypergeometric function kernel were successfully applied in [21, 22] and similarly, the Saigo fractional operators [23, 24] were also used for the significant applications [25, 26].

Delta function is the most popular distibution defined for a suitable function $\wp$ as follows [27, 28]

$$\int_{-\infty}^{\infty} \delta(x-a)\wp(x)dx = \wp(a), \tag{17}$$

and $\delta(ax) = \dfrac{\delta(x)}{|a|}$, $a \neq 0$. Because it cannot be formed from a locally integrable function and instead acts as a continuous linear functional on a set of test functions, this is one of the greatest examples of a singular distribution. The derivatives

$$\left\langle \delta^{(i)}(\omega),\ \wp(\omega) \right\rangle = (-1)^i \wp^{(i)}(0), \tag{18}$$

of delta function behave same like a singular distribution. Delta functions, or singular distributions, are also the Fourier transformations for the frequently used functions, for instance $\sin\omega$, $\cos\omega$, $\sinh\omega$, and $\cosh\omega$ [27]. The exponential function's Fourier transform is also computed in Volume I of [27].

$$F\left[e^{\alpha t};\ \xi\right] = 2\pi\delta(\xi - \iota\alpha), \tag{19}$$

belongs to $\mathscr{Z}'$ therefore, for $\forall g \in \mathscr{Z}'$ [27, 28]

$$g(\omega + a) = \sum_{k=0}^{\infty} \frac{a^k}{k!} g^{(k)}(\omega) \quad (\omega,\ a \in \mathbb{C}), \tag{20}$$

and the subsequent expansion follows which is also computed in Volume I of [27]

$$\delta(\omega + a) = \sum_{k=0}^{\infty} \frac{a^k}{k!} \delta^{(k)}(\omega). \tag{21}$$

Convolution of the delta function with an appropriate function produces

$$\delta(x-a) * g(x) = g(x-a); \quad \delta^{(k)}(x-a) * g(t) = g^{(k)}(x-a). \tag{22}$$

For this research we consider the following representation [29, 30]

$$\Gamma(c_p)\, {}_pF_q \left[\begin{matrix} c_1,\ \ldots c_p \\ d_1,\ \ldots d_q \end{matrix}\ ;\ s \right] = 2\pi \sum_{n,\ r=0}^{\infty} \frac{(-1)^n (s)^r (c_1)_r (c_2)_r \ldots (c_{p-1})_r}{n!r!(d_1)_r (d_2)_r \ldots (d_q)_r} \delta(u - \iota(t+n+r)); \tag{23}$$

$$\left(c_i;\ d_j \in \mathbb{C}; \Re(c_i) > 0;\ d_j \notin \mathbb{Z}_0^-;\ i = 1,\ \ldots.\ p \wedge j = 1,\ \ldots.\ q;\ p \leq q+1;\ c_p = v + \iota\theta\right).$$

We derive numerous new and innovative results using this representation (see [31–35]) for additional such investigations of other special functions. The parameter values, as described in section 2, will be considered normal unless otherwise noted in this article.

2. Distributional representation of the generalized hypergeometric function and generalized kinetic equation

Main results about the generalized hypergeometric function as a complex delta function series are given in this section. This is quite helpful in solving the new integral equation using this function by computing its Laplace transformation in upper parameter c_p. Here and what follows $(c_{i-1})_r$ are the Pochhammer symbols.

Theorem 1 The generalized hypergeometric function has a distributional representation specified as

$$\Gamma(c_p)\,{}_pF_q\left[\begin{array}{c} c_1,\ \ldots c_p \\ d_1,\ \ldots d_q \end{array};\ s\right] = 2\pi\sum_{n,\ r=0}^{\infty}\frac{(-1)^n(s)^r\prod_{i=1}^{p}(c_{i-1})_r}{n!r!\prod_{j=1}^{q}(d_j)_r}\delta(c_p+r+n) \tag{24}$$

$$\left(i=1,\ \ldots.\ p;\ j=1,\ \ldots.\ q;\ p\le q+1;\ c_i;\ d_j\in\mathbb{C};\ \Re(c_i)>0;\ d_j\notin|\ \mathbb{Z}_0^-;\ v+\iota\theta\right).$$

Proof. The modification of (23) that follows is presented as

$$\delta(\theta-\iota(v+r+n))=\delta\left[\frac{1}{\iota}(\iota\theta+(v+r+n))\right]=2\pi|\iota|\delta(v+\iota\theta+n+r)=2\pi\delta(c_p+r+n). \tag{25}$$

This means that by substituting (25) in (23), the specified form (24) can be produced.

Corollary 1 The distributional representation of the generalized hypergeometric function is

$$\Gamma(c_p)\,{}_pF_q\left[\begin{array}{c} c_1,\ \ldots c_p \\ d_1,\ \ldots d_q \end{array};\ s\right] = 2\pi\sum_{n,\ r,\ m=0}^{\infty}\frac{(-1)^n s^r(n+r)^m\prod_{i=1}^{p}(c_{i-1})_r}{n!r!m!\prod_{j=1}^{q}(d_j)_r}\delta^{(m)}(c_p) \tag{26}$$

$$\left(i=1,\ \ldots.\ p;\ j=1,\ \ldots.\ q;\ p\le q+1;\ c_i;\ d_j\in\mathbb{C};\ \Re(c_i)>0;\ d_j\notin\mathbb{Z}_0^-;\ c_p=v+\iota\theta\right).$$

Proof. By using (20) in (24)

$$\delta(c_p+n+r)=\sum_{m=0}^{\infty}\frac{(n+r)^m}{m!}\delta^{(m)}(c_p), \tag{27}$$

the stated form (26) is obtained.

Equations (24) and (26) can be used at s=0 to derive the distributional representation of the gamma function. Thus, for the generalized hypergeometric function with respect to the new representation, it is evident that the concepts related to the Dirac delta function do exist. It provides fresh insights into more recent findings in diverse fields. For instance, applying Laplace transform [23] i-e $L\left\{\delta^{(r)}(\gamma);\ \xi\right\}=\xi^r$, on (26) yields

$$L\left\{\Gamma(c_p)\,{}_pF_q\left[\begin{array}{c} c_1,\ \ldots c_p \\ d_1,\ \ldots d_q \end{array};\ s\right];\ \xi\right\} = 2\pi \sum_{n,\ r,\ m=0}^{\infty} \frac{(-1)^n s^r (n+r)^m \prod_{i=1}^{p} (c_{i-1})_r}{n!r!m! \prod_{j=1}^{q} (d_j)_r} \xi^m. \tag{28}$$

Taking $s = 0$ it provides Equation (48) of [36]

$$L\{\Gamma(c_p);\ \xi\} = 2\pi \exp\left(-e^{\xi}\right),$$

and for a constant γ,

$$L\left(\Gamma(c_p - \sigma)\,{}_pF_q\left[\begin{array}{c} c_1,\ \ldots c_{p-1},\ c_p - \sigma \\ d_1,\ \ldots d_q \end{array};\ s\right];\ \xi\right)$$

$$= 2\pi \sum_{n,\ r,\ m=0}^{\infty} \frac{(-1)^n s^r (n+r)^m \prod_{i=1}^{p} (c_{i-1})_r}{n!r!m! \prod_{j=1}^{q} (d_j)_r} L\left\{\delta^{(r)}(c_p - \sigma);\ \xi\right\} \tag{29}$$

$$= 2\pi \sum_{n,\ r,\ m=0}^{\infty} \frac{(-1)^n s^r (n+r)^m \prod_{i=1}^{p} (c_{i-1})_r}{n!r!m! \prod_{j=1}^{q} (d_j)_r} \xi^r e^{-\xi\sigma}.$$

Moreover, one can compute that

$$L\{\Gamma(c_p - \sigma);\ \xi\} = 2\pi e^{-\xi\sigma} \exp\left(-e^{\xi}\right). \tag{30}$$

Theorem 2 Considering the p-th parameter of the generalized hypergeometric function in a non-integer order kinetic equation

$$K(c_p) - K_0 \Gamma(c_p)\,{}_pF_q\left[\begin{array}{c} c_1,\ c_p \\ d_1,\ d_q \end{array};\ s\right] = -d^{\varepsilon} I_{0+}^{\varepsilon} K(c_p);\ c_p \in \mathbb{R}^+ \wedge d,\ \varepsilon > 0, \tag{31}$$

results in the solution that follows

$$K(c_p) = \frac{2\pi K_0}{c_p} \sum_{n,\ r,\ m=0}^{\infty} \frac{(-1)^n s^r \left(\dfrac{n+r}{c_p}\right)^m \prod_{i=1}^{p} (c_{i-1})_r}{n!r!m! \prod_{j=1}^{q} (d_j)_r} E_{\varepsilon,\ -l}(-d^{\varepsilon} c_p \varepsilon) \tag{32}$$

$$\left(i = 1,\ \ldots.\ p;\ j = 1,\ \ldots.\ q;\ p \le q+1;\ c_i;\ d_j \in \mathbb{C};\ \Re(c_i) > 0;\ d_j \notin \mathbb{Z}_0^-;\ c_p = v + \iota\theta\right)$$

Proof. First, let's apply the Laplace transform (see [1–3]) to (31)

$$L\{K(c_p);\ \xi\} - K_0 L\left\{\Gamma(c_p)\,{}_pF_q\left[\begin{array}{c} c_1,\ \dots c_p \\ d_1,\ \dots d_q \end{array};s\right];\ \xi\right\} = L\left\{-d^{\varepsilon} I_{0+}^{\varepsilon} K(c_p) : \xi\right\}, \tag{33}$$

wherever

$$K(s) = L[K(t):\xi] = \int_0^\infty e^{-\xi t} K(t)dt,\ \Re(\xi) > 0 \tag{34}$$

$$L\left\{I_{0+}^{\varepsilon} K(c_p);\ \xi\right\} = \xi^{-\varepsilon} K(\xi). \tag{35}$$

Then, by employing (24) in (33), we get

$$K(\xi) = 2\pi K_0 \sum_{n,\ r,\ m=0}^{\infty} \frac{(-1)^n s^r (n+r)^m \prod_{i=1}^{p} (c_{i-1})_r}{n!r!m! \prod_{j=1}^{q} (d_j)_r} \xi^m - \left(\frac{\xi}{d}\right)^{-\varepsilon} K(\xi), \tag{36}$$

and then expresses equation (36) above as follows

$$K(\xi)\left[1 + \left(\frac{\xi}{d}\right)^{-\varepsilon}\right] = 2\pi K_0 \sum_{n,\ r,\ m=0}^{\infty} \frac{(-1)^n s^r (n+r)^m \prod_{i=1}^{p} (c_{i-1})_r}{n!r!m! \prod_{j=1}^{q} (d_j)_r} \xi^m. \tag{37}$$

After doing a quick calculation, the outcome can be ascertained as

$$K(\xi) = 2\pi K_0 \sum_{n,\ r,\ m=0}^{\infty} \frac{(-1)^n s^r (n+r)^m \prod_{i=1}^{p} (c_{i-1})_r}{n!r!m! \prod_{j=1}^{q} (d_j)_r} \xi^m \sum_{l=0}^{\infty} \left[-\left(\frac{\xi}{d}\right)^{-\varepsilon}\right]^l. \tag{38}$$

Furthermore, let $\varepsilon m - p > 0$; $\varepsilon > 0$; $L^{-1}\{\omega^{-\varepsilon};\ c_p\} = \dfrac{(c_p)^{\varepsilon-1}}{\Gamma(\varepsilon)}$; we compute L^{-1} of (38) given by

$$K(c_p) = 2\pi K_0 \sum_{n,\ r,\ m=0}^{\infty} \frac{(-1)^n s^r (n+r)^m \prod_{i=1}^{p} (c_{i-1})_r}{n!r!m! \prod_{j=1}^{q} (d_j)_r} c_p^{-m-1} \times \sum_{l=0}^{\infty} \frac{\left(-d^{\varepsilon} c_p^{\varepsilon}\right)^l}{\Gamma(\varepsilon l - m)}. \tag{39}$$

At last, we can obtain (32) by applying (7) in (39).

Remark 1 Response rate or the solution $K(c_p)$ is a function of the fractional parameter ε, and it is noteworthy that the solution approach is traditional [1–3]. Usually, $K(c_p)$ is expressed in closed form using the Mittag-Leffler function, which can be observed in (39). This leads to a well-defined and finite sum over the coefficients in (32).

$$\sum_{n,r=0}^{\infty} \frac{(-1)^n \left(\dfrac{n+r}{c_p}\right)^p \prod_{i=1}^{p} (c_{i-1})_r}{n!r! \prod_{j=1}^{q} (d_j)_r} = \exp\left(-e^{1/c_p}\right) {}_{p-1}\Psi_q \left[\begin{array}{c} (c_{i-1},\ 1) \\ (d_j,\ 1) \end{array} \middle|\, e^{\frac{1}{c_p}}\right]. \tag{40}$$

In the same way, $\lim_{c_p \to \infty}$ leads to the following

$$\lim_{c_p \to \infty} \exp\left(-e^{1/c_p}\right) {}_{p-1}\Psi_q \left[\begin{array}{c} (c_{i-1},\ 1) \\ (d_j,\ 1) \end{array} \middle| e^{\frac{1}{c_p}} \right] = \exp(-1)_{p-1}\Psi_q \left[\begin{array}{c} (c_{i-1},\ 1) \\ (d_j,\ 1) \end{array} \middle| 1 \right]. \tag{41}$$

2.1 *M-E-K fractional integral operators and the generalized hypergeometric function*

Lemma 1 By means of the Laplace transform of generalized hypergeometric function, prove the following identity

$$\sum_{n,\ r=0}^{\infty} {}_0\Psi_0 \left[\begin{smallmatrix} - \\ - \end{smallmatrix} \middle| (n+r)\xi \right] \frac{(-1)^n s^r}{n!r!m!} \frac{\prod_{i=1}^{p} (c_{i-1})_r}{\prod_{j=1}^{q} (d_j)_r} = \exp\left(-e^{\xi}\right) {}_{p-1}\Psi_q \left[\begin{array}{c} (c_{p-1},\ 1) \\ (d_q,\ 1) \end{array} \middle| s\, e^{\xi} \right]. \tag{42}$$

Proof. Equation (28) gives us the following result.

$$\begin{aligned} L\left(\Gamma(c_p)\, {}_pF_q \left[\begin{array}{c} c_1,\ \ldots c_p \\ d_1,\ \ldots d_q \end{array} ;\ s \right];\ \xi \right) &= 2\pi \sum_{n,\ r,\ m=0}^{\infty} \frac{(-1)^n s^r (n+r)^m \prod_{i=1}^{p} (c_{i-1})_r}{n!r!m! \prod_{j=1}^{q} (d_j)_r} \xi^m \\ &= 2\pi \sum_{n,\ r=0}^{\infty} {}_0\Psi_0 \left[\begin{smallmatrix} - \\ - \end{smallmatrix} \middle| (n+r)\xi \right] \frac{(-1)^n s^r \prod_{i=1}^{p} (c_{i-1})_r}{n!r! \prod_{j=1}^{q} (d_j)_r}, \end{aligned} \tag{43}$$

then

$$\begin{aligned} L(\Gamma(c_p))\, {}_pF_q \left[\begin{array}{c} c_1,\ \ldots c_p \\ d_1,\ \ldots d_q \end{array} ;\ s \right];\ \xi &= 2\pi \sum_{n,\ r,\ m=0}^{\infty} \frac{(-1)^n s^r (n+r)^m \Pi_{i=1}^{p} (c_{i-1})_r}{n!r!m! \Pi_{j=1}^{q} (d_j)_r} \xi^m \\ &= \sum_{n,\ r=0}^{\infty} \frac{\left(-e^{\xi}\right)^n}{n!} {}_{p-1}\Psi_q \left[\begin{array}{c} (c_{p-1},\ 1) \\ (d_q,\ 1) \end{array} \middle| s\, e^{\xi} \right] \\ &= \exp\left(-e^{\xi}\right) {}_{p-1}\Psi_q \left[\begin{array}{c} (c_{p-1},\ 1) \\ (d_q,\ 1) \end{array} \middle| s\, e^{\xi} \right]. \end{aligned} \tag{44}$$

Therefore, from both of the above Equations (43-44), the required result is established.

Remark 2 Note that the following general conclusion can be drawn from (42):

$$\sum_{n,\ r=0}^{\infty} \frac{(-1)^n s^r}{n!r!m!} \frac{\prod_{i=1}^{p} (c_{i-1})_r}{\prod_{j=1}^{q} (d_j)_r} {}_p\Psi_q \left[\begin{array}{c} (a_i,\ A_i) \\ (b_j,\ B_j) \end{array} \middle| (n+r)\xi \right] = \exp\left(-e^{\xi}\right) {}_p\Psi_{q+1} \left[\begin{array}{cc} (c_{i-1},\ 1) & (a_i,\ A_i) \\ (d_j,\ 1) & (b_j,\ B_j) \end{array} \middle| s\, e^{\xi} \right]. \tag{45}$$

Theorem 3 M-E-K fractional transform including the generalized hypergeometric function is

$$I^{(\gamma_k),\,(\tau_k)}_{(\alpha_k),\,(\beta_k),\,l}\left(\xi^{\chi-1}L\left(\Gamma(c_p)\,{}_pF_q\left[\begin{matrix} c_1,\ \ldots c_p \\ d_1,\ \ldots d_q \end{matrix}\ ;\ s\right];\ \xi\right)\right)$$

$$=2\pi\xi^{\chi-1}\exp\left(-e^{\xi}\right){}_{l+p-1}\Psi_{l+q}\left[\begin{matrix} (c_{i-1},\ 1) & \left(\gamma_k+1+\dfrac{\chi-1}{\beta_k},\ \dfrac{1}{\beta_k}\right)_1^l \\ (d_j,\ 1) & \left(\gamma_k+\tau_k+1+\dfrac{\chi-1}{\alpha_k},\ \dfrac{1}{\alpha_k}\right)_1^l \end{matrix}\ \middle|\ s\,e^{\xi}\right] \tag{46}$$

$$([-\beta_k(1+\gamma_k)]<p;\ \delta_k\geq 0;\ k=1,\ \ldots,\ l;\ i=1,\ \ldots\ p;\ j=1,\ \ldots.\ q).$$

Proof. Consider the following:

$$\begin{aligned} &I^{(\gamma_k),\,(\tau_k)}_{(\alpha_k),\,(\beta_k),\,l}\left(\xi^{\chi-1}L\left(\Gamma(c_p)\,{}_pF_q\left[\begin{matrix} c_1,\ \ldots c_p \\ d_1,\ \ldots d_q \end{matrix}\ ;\ s\right];\ \xi\right)\right) \\ &=I^{(\gamma_k),(\tau_k)}_{(\alpha_k),\,(\beta_k),\,l}\left(\xi^{\chi-1}2\pi\sum_{n,\ r,\ m=0}^{\infty}\frac{(-1)^n s^r (n+r)^m \prod_{i=1}^{p}(c_{i-1})_r}{n!r!m!\prod_{j=1}^{q}(d_j)_r}\xi^m\right), \end{aligned} \tag{47}$$

then due to the uniform convergence of the integral, we exchange the role of sum and integration to obtain

$$\begin{aligned} &I^{(\gamma_k),\,(\tau_k)}_{(\alpha_k),\,(\beta_k),\,l}\left(\xi^{\chi-1}L\left(\Gamma(s)_pF_q\left[\begin{matrix} c_1,\ \ldots c_p \\ d_1,\ \ldots d_q \end{matrix}\ ;\ s\right];\ \xi\right)\right) \\ &=2\pi\sum_{n,\ r,\ m=0}^{\infty}\frac{(-1)^n s^r (n+r)^m}{n!r!m!}\frac{\prod_{i=1}^{p}(c_{i-1})_r}{\prod_{j=1}^{q}(d_j)_r}I^{(\gamma_k),\,(\tau_k)}_{(\alpha_k),\,(\beta_k),\,l}\left(\xi^{\chi-1}\xi^m\right), \end{aligned} \tag{48}$$

and the following action of multiple E-K operators [16, 17] is a key step of this proof

$$I^{(\gamma_k),\,(\tau_k)}_{(\alpha_k),\,(\beta_k),\,l}(z^c)=\prod_{k=1}^{l}\frac{\Gamma\left(\gamma_k+1+\dfrac{c}{\beta_k}\right)}{\Gamma\left(\gamma_k+\tau_k+1+\dfrac{c}{\alpha_k}\right)}z^c;\ q\left(k=1,\ \ldots,\ l\wedge c>[-\beta_k(1+\gamma_k)];\ \tau_k\geq 0\right), \tag{49}$$

which gives

$$I^{(\gamma_k),\,(\tau_k)}_{(\alpha_k),\,(\beta_k),\,l}\left(\xi^{\chi-1}L\left(\Gamma(c_p)\,{}_pF_q\left[\begin{matrix}c_1,\,\ldots c_p\\ d_1,\,\ldots d_q\end{matrix}\,;\,s\right];\,\xi\right)\right)$$

$$=2\pi\sum_{n,\,r,\,m=0}^{\infty}\frac{(-1)^n s^r (n+r)^m\prod_{i=1}^{p}(c_{i-1})_r}{n!r!m!\prod_{j=1}^{q}(d_j)_r}\prod_{k=1}^{l}\frac{\Gamma\left(\gamma_k+1+\dfrac{\chi+p-1}{\beta_k}\right)}{\Gamma\left(\gamma_k+\tau_k+1+\dfrac{\chi+p-1}{\alpha_k}\right)}\xi^{m+\chi-1}, \tag{50}$$

and then applying Equation (10) to Equation (50) yields the following outcome

$$I^{(\gamma_k),\,(\tau_k)}_{(\alpha_k),\,(\beta_k),\,l}\left(\xi^{\chi-1}\left(\Gamma(c_p)\,{}_pF_q\left[\begin{matrix}c_1,\,\ldots c_p\\ d_1,\,\ldots d_q\end{matrix}\,;\,s\right];\,\xi\right)\right)$$

$$=2\pi\xi^{\chi-1}\sum_{n,\,r,\,m=0}^{\infty}\frac{(-1)^n s^r\prod_{i=1}^{p}(c_{i-1})_r}{n!r!m!\prod_{j=1}^{q}(d_j)_r}\,{}_l\Psi_l\left[\begin{matrix}\left(\gamma_k+1+\dfrac{\chi-1}{\beta_k},\,\dfrac{1}{\beta_k}\right)_1^l\\ \left(\gamma_k+\tau_k+1+\dfrac{\chi-1}{\alpha_k},\,\dfrac{1}{\alpha_k}\right)_1^l\end{matrix}\,\middle|\,(n+r)\xi\right] \tag{51}$$

$$([-\beta_k(1+\gamma_k)]<p;\ \tau_k\geq 0;\ k=1,\,\ldots,\,l).$$

Applying Lemma 1 thus produces the correct simplified form.

Corollary 2 For $(i=1,\,\ldots.\,p;\ j=1,\,\ldots.\,q;\ p\leq q+1;\ c_i\wedge d_j\in\mathbb{C};\ \Re(c_i)>0;\ d_j\notin\mathbb{Z}_0^-)$ M-S-M fractional integral operator including the generalized hypergeometric function is

$$I_{0+}^{\gamma_1,\,\gamma_1',\,\gamma_2,\,\gamma_2',\,\tau}\left(\xi^{x-1}L\left\{\Gamma(c_p)\,{}_pF_q\left[\begin{matrix}c_1,\,\ldots c_p\\ d_1,\,\ldots d_q\end{matrix}\,;\,s\right];\,\xi\right\}\right)=2\pi\xi^{\tau+\chi-\gamma_1-\gamma_1'-1}\exp\left(-e^{\xi}\right)$$

$${}_{p+2}\Psi_{q+3}\left[\begin{matrix}(c_{i-1},\,1) & (\chi,\,1) & (\chi+\tau-\gamma_1-\gamma_1'-\gamma_2,\,1) & (\chi+\gamma_2'-\gamma_1',\,1)\\ (d_j,\,1) & (\chi+\gamma_2',\,1) & (\chi+\tau-\gamma_1-\gamma_1',\,1) & (\chi+\tau-\gamma_1'-\gamma_2,\,1)\end{matrix}\,\middle|\,s\,e^{\xi}\right].$$

Proof. This can be achieved by using $(\alpha_k=\beta_k)$ in (46) and then using the case $l=3$ from Table 1.

Corollary 3 For $(i=1,\,\ldots.\,p;\ j=1,\,\ldots.\,q;\ p\leq q+1;\ c_i\wedge d_j\in\mathbb{C};\ \Re(c_i)>0;\ d_j\notin\mathbb{Z}_0^-)$ Saigo fractional integral operator including the generalized hypergeometric function is

$$I_{0+}^{\gamma_1,\,\gamma_2,\,\tau}\left(\xi^{\chi-1}L\left\{\Gamma(c_p)\,{}_pF_q\left[\begin{matrix}c_1,\,\ldots c_p\\ d_1,\,\ldots d_q\end{matrix}\,;\,s\right];\,\xi\right\}\right)=2\pi\xi^{\tau+\chi-\gamma_1-\gamma_1-1}\exp\left(-e^{\xi}\right)$$

$${}_{p+2}\Psi_{q+3}\left[\begin{matrix}(c_{i-1},\,1) & (\chi,\,1) & (\chi+\gamma_2-\gamma_1,\,1)\\ (d_j,\,1) & (\chi-\gamma_2,\,1) & (\chi+\tau+\gamma_2,\,1)\end{matrix}\,\middle|\,s\,e^{\xi}\right].$$

Proof. This can be achieved by using $(\alpha_k = \beta_k)$ in (46) and then using the case $l = 2$ from Table 1 .

Corollary 4 For $(i = 1, \ldots. p;\ j = 1, \ldots. q;\ p \leq q+1;\ c_i \wedge d_j \in \mathbb{C};\ \Re(c_i) > 0;\ d_j \notin \mathbb{Z}_0^-)$ Erdélyi−Kober fractional integral operator including the generalized hypergeometric function is

$$I_{0+}^{\gamma,\,\tau}\left(\xi^{\chi-1}L\left\{\Gamma(c_p)\,{}_pF_q\left[\begin{matrix} c_1,\ldots c_p \\ d_1,\ldots d_q\end{matrix};s\right];\xi\right\}\right) = 2\pi\xi^{\chi-1}\exp\left(-e^{\xi}\right){}_p\Psi_{q+1}\left[\begin{matrix}(c_{i-1},\,1) & (\chi+\gamma,\,1)\\ (d_j,\,1) & (\chi+\gamma+\tau,\,1)\end{matrix}\,\middle|\, s\,e^{\xi}\right].$$

Proof. This can be achieved by using $(\alpha_k = \beta_k)$ in (46) and then using the case $l = 1$ from Table 1 .

Corollary 5 For $(i = 1, \ldots. p;\ j = 1, \ldots. q;\ p \leq q+1;\ c_i \wedge d_j \in \mathbb{C};\ \Re(c_i) > 0;\ d_j \notin \mathbb{Z}_0^-)$ R-L fractional integral operator including the generalized hypergeometric function is

$$I_{0+}^{\tau}\left(\xi^{\chi-1}L\left\{\Gamma(c_p)\,{}_pF_q\left[\begin{matrix} c_1,\ldots c_p \\ d_1,\ldots d_q\end{matrix};s\right];\xi\right\}\right) = 2\pi\xi^{\chi-1}\exp\left(-e^{\xi}\right){}_p\Psi_{q+1}\left[\begin{matrix}(c_{i-1},\,1) & (\chi,\,1)\\ (d_j,\,1) & (\tau+\chi,\,1)\end{matrix}\,\middle|\, s\,e^{\xi}\right].$$

Proof. This can be achieved by using $(\alpha_k = \beta_k)$ in (46) and then using the case $l = 1$ from Table 1 . Similarly, the corresponding left side formulae for $\left(i = 1, \ldots. p;\ j = 1, \ldots. q;\ p \leq q+1;\ c_i \wedge d_j \in \mathbb{C};\ \Re(c_i) > 0;\ d_j \notin \mathbb{Z}_0^-\right)$ are listed as follows:

$$I_{0-}^{\gamma_1,\,\gamma_1',\,\gamma_2,\,\gamma_2',\,\tau}\left(\xi^{\chi-1}\left\{\Gamma(c_p)\,{}_pF_q\left[\begin{matrix} c_1,\ldots c_p \\ d_1,\ldots d_q\end{matrix};s\right];\frac{1}{\xi}\right\}\right) = 2\pi\xi^{\tau+\chi-\gamma_1-\gamma_1'-1}\exp\left(-e^{\xi}\right)$$

$${}_{p+2}\Psi_{q+3}\left[\begin{matrix}(c_{i-1},\,1) & (1-\chi-\tau+\gamma_1+\gamma_1',\,1) & (1-\chi+\gamma_1+\gamma_2'-\tau,\,1) & (1-\chi-\gamma_1,\,1)\\ (d_j,\,1) & (1-\chi,\,1) & (1-\chi+\gamma_1+\gamma_1'+\gamma_2+\gamma_2'-\tau,\,1) & (1-\chi+\gamma_1-\gamma_2,\,1)\end{matrix}\,\middle|\, s\,e^{\frac{1}{\xi}}\right]$$

$$I_{-}^{\gamma_1,\,\gamma_2,\,\tau}\left(\xi^{\chi-1}L\left\{{}_pF_q\left[\begin{matrix} c_1,\ldots c_p \\ d_1,\ldots d_q\end{matrix};s\right];\frac{1}{\xi}\right\}\right)$$

$$=2\pi\xi^{\chi-\gamma_1-1}\exp\left(-e^{\xi}\right){}_{p+1}\Psi_{q+2}\left[\begin{matrix}(c_{i-1},\,1) & (\gamma_1-\chi+1,\,1) & (\gamma_2-\chi+1,\,1)\\ ((d_j,\,1) & (1-\chi,\,1) & ((\gamma_1+\gamma_2+\tau-\chi+1,\,1)\end{matrix}\,\middle|\, s\,e^{\frac{1}{\xi}}\right]$$

$$I_{0-}^{\gamma,\,\tau}\left(\xi^{\chi-1}L\left\{\Gamma(c_p)\,{}_pF_q\left[\begin{matrix} c_1,\ldots c_p \\ d_1,\ldots d_q\end{matrix};s\right];\frac{1}{\xi}\right\}\right)$$

$$= 2\pi\xi^{\chi-1}\exp\left(-e^{\xi}\right){}_p\Psi_{q+1}\left[\begin{matrix}(c_{i-1},\,1) & (1-\chi+\gamma,\,-1)\\ (d_j,\,1) & (\tau-\chi+1+\gamma,-1)\end{matrix}\,\middle|\, se^{\frac{1}{\xi}}\right]$$

$$I_{-}^{\tau}\left(\xi^{\chi-1}L\left\{\Gamma(c_p)\,{}_pF_q\left[\begin{matrix} c_1,\ldots c_p \\ d_1,\ldots d_q\end{matrix};s\right];\frac{1}{\xi}\right\}\right) = 2\pi\xi^{\chi+\tau-1}\exp\left(-e^{\xi}\right){}_p\Psi_{q+1}\left[\begin{matrix}(c_{i-1},\,1) & (1-\tau-\chi-1)\\ (d_j,\,1) & (1-\chi,\,-1)\end{matrix}\,\middle|\, s\,e^{\frac{1}{\xi}}\right].$$

2.2 *Multiple E-K derivatives including the generalized hypergeometric function*

Using the method from Theorem 1 and the distributional representation of the generalized hypergeometric function, we may deduce the new derivative formulae involving a generalized hypergeometric function. Here, we deduce them directly while altering the overall outcome using Theorem 4 of [19], which is

$$D^{(\gamma_k),\,(\tau_k)}_{(\alpha_k),\,(\beta_k),\,l}\left\{z^c\,{}_p\Psi_q\left[\begin{matrix}(a_i,\alpha_i)_1^p\\(b_j,\beta_j)_1^q\end{matrix};\lambda z^\mu\right]\right\}=z^c\left[{}_{p+l}\Psi_{q+l}\left|\begin{matrix}(a_i,\,\alpha_i)_1^p,\left(\gamma_k+\tau_k+1+\dfrac{c}{\alpha_k},\,\dfrac{1}{\alpha_k}\right)_1^l\\(b_j,\,\beta_j)_1^q,\left(\gamma_k+1+\dfrac{c}{\beta_k},\,\dfrac{1}{\beta_k}\right)_1^l\end{matrix}\right.;\lambda z^\mu\right]\right\},\tag{52}$$

$$(\mu>0;\;\Re(\tau_i)>0,\;\Re(\gamma_i)>-1;\;\lambda\neq 0;\;|\lambda z^\mu|<1\text{ when }q+1=p).$$

M-E-K fractional derivatives containing generalized hypergeometric function are computed using first (52) on (28) and then using (42)

$$\left(D^{(\gamma_k),\,(\tau_k)}_{(\alpha_k),\,(\beta_k),\,l}\xi^{\chi-1}L\left\{\Gamma(c_p)\,{}_pF_q\left[\begin{matrix}c_1,\,\ldots c_p\\d_1,\,\ldots d_q\end{matrix};s\right];\xi\right\}\right)$$

$$=2\pi\xi^{\chi-1}\exp\left(-e^{\xi}\right){}_m\Psi_{m+1}\left[\begin{matrix}(c_{i-1},\,1)_{i=1}^p & \left(\gamma_k+\tau_k+1+\dfrac{c}{\alpha_k},\,\dfrac{1}{\alpha_k}\right)_1^l\\(d_j,\,1)_{j=1}^q & \left(\gamma_k+1+\dfrac{c}{\beta_k},\dfrac{1}{\beta_k}\right)_1^l\end{matrix}\,\middle|\,s\,e^{\xi}\right]\tag{53}$$

$$\left(\mu>0;\;\Re(\tau_i)>0,\;\Re(\gamma_i)>-1;\;\lambda\neq 0;\;|\lambda z^\mu|<1\text{ when }q+1=p;\;p\leq q+1;\;c_i;\;d_j\in\mathbb{C};\;\Re(c_i)>0;\;d_j\notin\mathbb{Z}_0^-\right).$$

Hence, the following special cases of the result (53) for $l=3$ (M-S-M derivative); $l=2$ (Saigo fractional derivative); $l=1$ (E-K and R-L fractional derivatives) are listed as follows:

$$D_{0+}^{\gamma_1,\,\gamma_1',\,\gamma_2,\,\gamma_2',\,\tau}\xi^{\chi-1}L\left\{\Gamma(c_p)\,{}_pF_q\left[\begin{matrix}c_1,\,\ldots c_p\\d_1,\,\ldots d_q\end{matrix};s\right];\xi\right\}=2\pi\xi^{\tau+\chi-\gamma_1-\gamma_1'-1}\exp\left(-e^{\xi}\right)$$

$${}_{p+2}\Psi_{q+3}\left[\begin{matrix}(c_{i-1},\,1) & (\chi,\,1) & (\chi-\gamma_2+\gamma_1,\,1) & (\chi+\gamma_1+\gamma_1'+\gamma_2'-\tau,\,1)\\(d_j,\,1) & (\chi-\gamma_2,\,1) & (\chi-\tau+\gamma_1++\gamma_2',\,1) & (\chi-\tau+\gamma_1'+\gamma_1,\,1)\end{matrix}\,\middle|\,s\,e^{\xi}\right];$$

$$D_{0+}^{\gamma_1,\,\gamma_2,\,\tau}\xi^{\chi-1}L\left\{\Gamma(c_p)\,{}_pF_q\left[\begin{matrix}c_1,\,\ldots c_p\\d_1,\,\ldots d_q\end{matrix};s\right];\xi\right\}=2\pi\xi^{\chi-\gamma_1-1}\exp\left(-e^{\xi}\right)$$

$${}_{p+1}\Psi_{q+2}\left[\begin{matrix}(c_{i-1},\,1) & (\chi,\,1) & (\chi+\tau+\gamma_2+\gamma_1,\,1)\\(d_j,\,1) & (\chi+\gamma_2,\,1) & (\chi+\tau,\,1)\end{matrix}\,\middle|\,s\,e^{\xi}\right];$$

$$D_{0+}^{\gamma,\,\tau}\xi^{\chi-1}L\left\{\Gamma(c_p)\,{}_pF_q\left[\begin{array}{c}c_1,\,\dots c_p\\ d_1,\,\dots d_q\end{array};\,s\right];\,\xi\right\}=2\pi\xi^{\chi-1}\exp\left(-e^{\xi}\right){}_p\Psi_{q+1}\left[\begin{array}{cc}(c_{i-1},\,1) & (\gamma+\tau+\chi,\,1)\\ (d_j,\,1) & (\gamma+\chi,\,1)\end{array}\middle|\,s\,e^{\xi}\right];$$

$$D_{0+}^{\tau}\xi^{\chi-1}L\left\{\Gamma(c_p)\,{}_pF_q\left[\begin{array}{c}c_1,\,\dots c_p\\ d_1,\,\dots d_q\end{array};\,s\right];\,\xi\right\}=2\pi\xi^{\chi-1}\exp\left(-e^{\xi}\right){}_p\Psi_{q+1}\left[\begin{array}{cc}(c_{i-1},\,1) & (\chi,\,1)\\ (d_j,\,1) & (\chi-\tau,\,1)\end{array}\middle|\,s\,e^{\xi}\right],$$

and the corresponding left hand formulae are given as:

$$D_{-}^{\gamma_1,\,\gamma_1',\,\gamma_2,\,\gamma_2',\,\tau}\xi^{\chi-1}L\left\{\Gamma(c_p)\,{}_pF_q\left[\begin{array}{c}c_1,\,\dots c_p\\ d_1,\,\dots d_q\end{array};\,s\right];\,\frac{1}{\xi}\right\}=2\pi\xi^{\tau+\chi-\gamma_1-\gamma_1'-1}\exp\left(-e^{\xi}\right)$$

$${}_{p+2}\Psi_{q+3}\left[\begin{array}{cccc}(c_{i-1},\,1) & (1-\chi+\gamma_2',\,1) & (1-\gamma_1'-\chi-\gamma_2+\tau_1,\,1) & (1-\chi-\gamma_1-\gamma_1'+\tau,\,1)\\ (d_j,\,1) & (1-\chi,\,1) & (1-\chi-\gamma_1'+\gamma_2',\,1) & (1-\chi+\delta-\gamma_1'-\gamma_1-\gamma_2,\,1)\end{array}\middle|\,s\,e^{\frac{1}{\xi}}\right];$$

$$D_{-}^{\gamma_1,\,\gamma_2,\,\tau}\xi^{\chi-1}L\left\{\Gamma(c_p)\,{}_pF_q\left[\begin{array}{c}c_1,\,\dots c_p\\ d_1,\,\dots d_q\end{array};\,s\right];\,\frac{1}{\xi}\right\}=2\pi\xi^{\chi-\gamma_1-1}\exp\left(-e^{\xi}\right)$$

$${}_{p+1}\Psi_{q+2}\left[\begin{array}{ccc}(c_{i-1},\,1) & (1-\chi-\gamma_2,\,1) & (1-\chi+\tau+\gamma_1,\,1)\\ (d_j,\,1) & (1-\chi+\tau-\gamma_2,\,1) & (1-\chi,\,1)\end{array}\middle|\,s\,e^{\frac{1}{\xi}}\right];$$

$$D_{-}^{\gamma,\,\tau}\xi^{\chi-1}L\left\{\Gamma(c_p)\,{}_pF_q\left[\begin{array}{c}c_1,\,\dots c_p\\ d_1,\,\dots d_q\end{array};\,s\right];\,\frac{1}{\xi}\right\}=2\pi\xi^{\chi-1}\exp\left(-e^{\xi}\right){}_p\Psi_{q+1}\left[\begin{array}{cc}(c_{i-1},\,1) & (1-\chi+\gamma+\tau,\,1)\\ (d_j,\,1) & (1-\chi+\gamma,\,1)\end{array}\middle|\,s\,e^{\frac{1}{\xi}}\right];$$

$$D_{-}^{\tau}\xi^{\chi-1}L\left\{\Gamma(c_p)\,{}_pF_q\left[\begin{array}{c}c_1,\,\dots c_p\\ d_1,\,\dots d_q\end{array};\,s\right];\,\frac{1}{\xi}\right\}=2\pi\xi^{\chi-1}\exp\left(-e^{\omega}\right){}_p\Psi_{q+1}\left[\begin{array}{cc}(c_{i-1},\,1) & (\tau-\chi+1,\,1)\\ (d_j,\,1) & (1-\chi,\,1)\end{array}\middle|\,s\,e^{\frac{1}{\xi}}\right].$$

3. Behaviour of distributional representation of the generalized hypergeometric function

It is important whether the delta function used to build the new series representation of the generalized hypergeometric function is accurately described in terms of the distributional idea. The results may hold over complex domain due to the fact stated at page 200 of [28],

$$\langle f(\omega),\,\chi(\omega)\rangle=\langle f(t+\iota u),\,\chi(t+\iota u)\rangle=\langle f(t),\,\chi(t+\iota u-\iota u)\rangle=\langle f(t),\,\chi(t)\rangle.$$

Moreover, a class of test functions closed under the Fourier transform contains infinitely differentiable and fastly decreasing functions is $\mathscr{S}$ (space $\mathscr{S}'$ is its dual which contains functions of slow growth). Essentially, the Fourier transforms of dual space $\mathscr{D}'$ ($\mathscr{D}$ is the space of test functions with compact support) do not belong to $\mathscr{D}'$ but they belong to a different space

$\mathscr{Z}'$ that is a space of complex functions. We remark that the Fourier transforms of are the elements of $\mathscr{D}$, which are entire functions and do not vanish but only on a specific interval $\omega_1 < t < \omega_2$, that yields the subsequent enclosure

$$\mathscr{Z} \subset \mathscr{S} \subset \mathscr{S}' \subset \mathscr{Z}' \wedge \mathscr{Z} \cap \mathscr{D} \equiv 0 \wedge \mathscr{D} \subset \mathscr{S} \subset \mathscr{S}' \subset \mathscr{D}',$$

and

$$|\omega^{\sigma} \wp(\omega)| \leq A_{\sigma} e^{\eta|\Im(\omega)|}; \quad (\forall \wp \in \mathscr{Z};\ \sigma \in \mathbb{Z}_0).$$

It involves the imaginary part u of ω and the constants η and A_{σ}, which are determined by $\wp$. Hence, we prove the following theorem.

Theorem 4 Generalized hypergeometric function $\Gamma(c_p)\, {}_pF_q \left[\begin{array}{c} c_1, \ldots c_p \\ d_1, \ldots d_q \end{array} ; s \right]$ is a singular distribution (generalized function) over $\mathscr{Z}$.

Proof. For $\wp_1(c_p),\ \wp_2(c_p) \in \mathscr{Z} \wedge C_1,\ C_2 \in \mathbb{C}$, we take the subsequent combination

$$\begin{aligned} &\left\langle \Gamma(c_p)\, {}_pF_q \left[\begin{array}{c} c_1, \ldots c_p \\ d_1, \ldots d_q \end{array} ; s \right], C_1 \wp_1(c_p) + C_2 \wp_2(c_p) \right\rangle \\ &= \left\langle 2\pi \sum_{n,\, r=0}^{\infty} \frac{(-1)^n (s)^r \prod_{i=1}^{p} (c_{i-1})_r}{n! r! \prod_{j=1}^{q} (d_j)_r} \delta(c_p + n + r),\ C_1 \wp_1(c_p) + C_2 \wp_2(c_p) \right\rangle. \end{aligned} \tag{54}$$

$$\begin{aligned} &\Rightarrow \left\langle \Gamma(c_p)\, {}_pF_q \left[\begin{array}{c} c_1, \ldots c_p \\ d_1, \ldots d_q \end{array} ; s \right], C_1 \wp_1(c_p) + C_2 \wp_2(c_p) \right\rangle \\ &= C_1 \left\langle \Gamma(c_p)\, {}_pF_q \left[\begin{array}{c} c_1, \ldots c_p \\ d_1, \ldots d_q \end{array} ; s \right], \wp_1(c_p) \right\rangle + C_2 \left\langle {}_pF_q \left[\begin{array}{c} c_1, \ldots c_p \\ d_1, \ldots d_q \end{array} ; s \right], \wp_2(c_p) \right\rangle. \end{aligned} \tag{55}$$

Then, we choose an arbitrary sequence. $\{\wp_\ell\}_{\ell=1}^{\ell=\infty} \to 0$ using $\{\langle \delta(c_p + n + r)),\ \wp_\ell \rangle\}_{\ell=1}^{\ell=\infty} \to 0$.

$$\begin{aligned} &\Rightarrow \left\{ \left\langle \Gamma(c_p)\, {}_pF_q \left[\begin{array}{c} c_1, \ldots c_p \\ d_1, \ldots d_q \end{array} ; s \right], \wp_\ell(c_p) \right\rangle \right\}_{\ell=1}^{\ell=\infty} \\ &= 2\pi \sum_{n,\, r=0}^{\infty} \frac{(-1)^n (s)^r \prod_{i=1}^{p} (c_{i-1})_r}{n! r! \prod_{j=1}^{q} (d_j)_r} \{\langle \delta(c_p + n + r)),\ \wp_\ell(c_p) \}_{\ell=1}^{\ell=\infty} \to 0. \end{aligned} \tag{56}$$

In order to examine how new representations converge, take into account the following

$$\left\langle \Gamma(c_p)\, {}_pF_q\left[\begin{array}{c} c_1,\ \ldots c_p \\ d_1,\ \ldots d_q \end{array} ;\, s\right],\ \wp(c_p)\right\rangle = 2\pi \sum_{n,\ r=0}^{\infty} \frac{(-1)^n (s)^r \prod_{i=1}^{p} (c_{i-1})_r}{n!r! \prod_{j=1}^{q} (d_j)_r} \left\langle \delta\left(c_p + n + r\right)\right),\ \wp(c_p);\ (\forall \wp(c_p) \in \mathscr{Z})$$

$$= 2\pi \sum_{n,\ r=0}^{\infty} \frac{(-1)^n (s)^r \prod_{i=1}^{p} (c_{i-1})_r}{n!r! \prod_{j=1}^{q} (d_j)_r} \wp(-(n+r)), \tag{57}$$

wherever

$$\text{sum over the coefficients} = \sum_{n,\ r=0}^{\infty} \frac{(-1)^n (s)^r \prod_{i=1}^{p} (c_{i-1})_r}{n!r! \prod_{j=1}^{q} (d_j)_r} = e^{-1}\, {}_{p-1}\Psi_q \left[\begin{array}{c} (c_{i-1},\ 1)_{i=1}^{p} \\ (d_j,\ 1)_{j=1}^{q} \end{array} \middle| s \right]. \tag{58}$$

Consequently, Equation (57) displays that $\left\langle \Gamma(c_p)\, {}_pF_q\left((c)_p;\ (d)_q;\ s\right),\ \wp(c_p)\right\rangle;\ \forall \wp(z) \in \mathscr{Z}$ is the inner product of the two types of functions that increase slowly and diminish swiftly, and it is convergent. It's also corroborated by the Abel theorem. Consequently, the behavior of a generalized hypergeometric function is similar to a distribution over $\mathscr{Z}$.

To help our grasp the previous topic, consider the example that follows.

Example 1 For $\wp(c_p) = t^{c_p \xi}$ $(\xi > 0;\ c_p \in \mathbb{C})$, consider the following

$$\left\langle \Gamma(c_p)\, {}_pF_q\left[\begin{array}{c} c_1,\ \ldots c_p \\ d_1,\ \ldots d_q \end{array} ;\, s\right],\ \wp(\gamma)\right\rangle = \left\langle \Gamma(c_p)\, {}_pF_q\left[\begin{array}{c} c_1,\ \ldots c_p \\ d_1,\ \ldots d_q \end{array} ;\, s\right],\ t^{c_p \xi}\right\rangle$$

$$= 2\pi \sum_{n,\ r=0}^{\infty} \frac{(-1)^n (s)^r \prod_{i=1}^{p} (c_{i-1})_r}{n!r! \prod_{j=1}^{q} (d_j)_r} t^{-n\xi - r\xi} \tag{59}$$

$$= 2\pi \exp\left(-t^{-\xi}\right)\, {}_{p-1}\Psi_q \left[\begin{array}{c} (c_{i-1},\ 1) \\ (d_j,\ 1) \end{array} \middle| s\, t^{-\xi} \right].$$

For $s = 0$, it leads to

$$\int_{-\infty}^{+\infty} t^{c_p \xi} \Gamma(c_p)\, dc_p = 2\pi \sum_{n=0}^{\infty} \frac{\left(-t^{-\xi}\right)^n}{n!} = \exp\left(-t^{-\xi}\right). \tag{60}$$

These findings offer fresh insights into the possibility of additional findings of this kind. For instance, based on $\tau = e^{-1}$ in Equation (60), one can obtain the Laplace transform of $\Gamma(c_p)\, {}_pF_q\left[\begin{array}{c} c_1,\ \ldots c_p \\ d_1,\ \ldots d_q \end{array}\right]$.

3.1 *Validation of the distributional representation of generalized hypergeometric function using classical Fourier transform*

Verifying the stability of the new identities attained by innovative representation is the main objective of this part. A generalized hypergeometric function's Fourier transform representation is obtained by taking $u_i = v_i = 1$ in Equation (2.1) of [30]

$$\Gamma(c_p)\,{}_pF_q\left[\begin{array}{c} c_1,\ \ldots c_p \\ d_1,\ \ldots d_q \end{array};\ s\right] = \sqrt{2\pi}\mathscr{F}\left[e^{\Re(c_p)x}\exp\left(-e^x\right){}_{p-1}F_q\left[\begin{array}{c} c_1,\ \ldots c_{p-1} \\ d_1,\ \ldots d_q \end{array};\ se^x\right];\ \xi\right]. \tag{61}$$

Since the duality condition is preserved by the Fourier transform, any function $u(t)$

$$\mathscr{F}[\sqrt{2\pi}\mathscr{F}[u(t);\ s];\ \xi] = 2\pi u(-\xi). \tag{62}$$

Equation (61) yields the following result when this characteristic is applied.

$$\begin{aligned}\mathscr{F}\left\{\Gamma(c_p)\,{}_pF_q\left[\begin{array}{c} c_1,\ \ldots c_p \\ d_1,\ \ldots d_q \end{array};\ s\right];\ \xi\right\} &= \mathscr{F}\left[\sqrt{2\pi}\mathscr{F}\left[e^{\Re(c_p)x}\exp\left(-e^x\right)_{p-1}F_q\left[\begin{array}{c} c_1,\ \ldots c_{p-1} \\ d_1,\ \ldots d_q \end{array};\ se^x\right]\right];\ \xi\right] \\ &= f(-\xi) = 2\pi\exp\left(-e^{-\xi}\right)e^{-\Re(c_p)\xi}{}_{p-1}F_q\left[\begin{array}{c} c_1,\ \ldots c_{p-1} \\ d_1,\ \ldots d_q \end{array};\ se^{-\xi}\right].\end{aligned} \tag{63}$$

For $c_p = v + \iota\theta$, the matching form of the above identity is provided as

$$\begin{aligned}&\int_{-\infty}^{+\infty} e^{i\theta\xi}\Gamma(v+\iota\theta)_pF_q\left[\begin{array}{c} c_1,\ \ldots c_{p-1},\ v+\iota\theta \\ d_1,\ \ldots\ldots d_q \end{array};\ s\right]d\theta \\ &= 2\pi e^{-\Re(c_p)\xi}\exp\left(-e^{-\xi}\right){}_{p-1}F_q\left[\begin{array}{c} c_1,\ \ldots c_{p-1} \\ d_1,\ \ldots d_q \end{array};\ se^{-\xi}\right].\end{aligned} \tag{64}$$

By substituting $\tau = e$; $\gamma = v + \iota\theta$, this is the particular form of (57). These specifics show th representation agree with those obtained using more conventional techniques. Furthermor can be obtained by assuming $\xi = 0$ in (64):

$$\int_{-\infty}^{+\infty}\Gamma(v+\iota\theta)_pF_q\left[\begin{array}{c} c_1,\ \ldots c_{p-1},\ v+\iota\theta \\ d_1,\ \ldots\ldots d_q \end{array};\ s\right]d\theta = 2\pi e^{-1}{}_{p-1}F_q\left[\begin{array}{c} c_1,\ \ldots c_{p-1} \\ d_1,\ \ldots d_q \end{array};\ se^{-\xi}\right]. \tag{65}$$

3.2 *Distributional (generalized) properties of the generalized hypergeometric function*

New distribution features, based on the ideas and methodology presented in chapter 7 of [28], are offered here for a generalized hypergeometric function as a consequence of its new relation with delta function [28, 29].

Theorem 5 For any test function $\wp(z) \in \mathscr{Z}$, the generalized hypergeometric function has the following properties as a generalized function (distribution), where γ is an arbitrary real or complex constants.

P-1) Any distribution g being an element of the dual space $\mathscr{Z}'$ and a generalized hypergeometric function together have the following combined effect:

$$\left\langle \Gamma(c_p)\, {}_pF_q \left[\begin{array}{c} c_1, \ldots c_p \\ d_1, \ldots d_q \end{array} ; s \right] + g,\ \wp(c_p) \right\rangle = \left\langle \Gamma(c_p)\, {}_pF_q \left[\begin{array}{c} c_1, \ldots c_p \\ d_1, \ldots d_q \end{array} ; s \right],\ \wp(c_p) \right\rangle + \langle g,\ \wp(c_p) \rangle .$$

P-2) The following is obtained by multiplying the generalized hypergeometric function by an arbitrary constant, γ

$$\left\langle \gamma\, \Gamma(c_p)\, {}_pF_q \left[\begin{array}{c} c_1, \ldots c_p \\ d_1, \ldots d_q \end{array} ; s \right],\ \wp(c_p) \right\rangle = \left\langle \Gamma(c_p)\, {}_pF_q \left[\begin{array}{c} c_1, \ldots c_p \\ d_1, \ldots d_q \end{array} ; s \right],\ \gamma\, \wp(c_p) \right\rangle .$$

P-3) Shifting property of the generalized hypergeometric function using any complex constant γ

$$\left\langle \Gamma(c_p - \gamma)\, {}_pF_q \left[\begin{array}{c} c_1, \ldots c_{p-1},\ c_p - \gamma \\ d_1, \ldots d_q \end{array} ; s \right],\ \wp(c_p) \right\rangle = \left\langle \Gamma(c_p)\, {}_pF_q \left[\begin{array}{c} c_1, \ldots c_p \\ d_1, \ldots d_q \end{array} ; s \right],\ \wp(c_p + \gamma) \right\rangle .$$

P-4) Generalized hypergeometric function is transposed as

$$\left\langle \Gamma(-c_p)\, {}_pF_q \left[\begin{array}{c} c_1, \ldots c_{p-1},\ -c_p \\ d_1, \ldots d_q \end{array} ; s \right],\ \wp(c_p) \right\rangle = \left\langle \Gamma(c_p)\, {}_pF_q \left[\begin{array}{c} c_1, \ldots c_p \\ d_1, \ldots d_q \end{array} ; s \right],\ \wp(-c_p) \right\rangle .$$

P-5) A positive constant γ multiplied by the independent variable c_p

$$\left\langle \Gamma(c_p \gamma)\, {}_pF_q \left[\begin{array}{c} c_1, \ldots c_{p-1},\ c_p \gamma \\ d_1, \ldots d_q \end{array} ; s \right],\ \wp(c_p) \right\rangle = \left\langle \Gamma(c_p)\, {}_pF_q \left[\begin{array}{c} c_1, \ldots c_p \\ d_1, \ldots d_q \end{array} ; s \right],\ \frac{1}{\gamma} \wp\left(\frac{c_p}{\gamma} \right) \right\rangle .$$

P-6) Differentiating a generalized hypergeometric function as a distribution

$$\left\langle \frac{d^m}{d{c_p}^m} \left(\Gamma(c_p)\, {}_pF_q \left[\begin{array}{c} c_1, \ldots c_p \\ d_1, \ldots d_q \end{array} ; s \right] \right),\ \wp(c_p) \right\rangle = 2\pi \sum_{n,\ r=0}^{\infty} \frac{(-1)^n (s)^r \prod_{i=1}^{p} (c_{i-1})_r}{n! r! \prod_{j=1}^{q} (d_j)_r} (-1)^m \wp^m(-n-r).$$

P-7) The distributional Fourier transform of a generalized hypergeometric function

$$\left\langle \mathscr{F} \left[\Gamma(c_p)\, {}_pF_q \left[\begin{array}{c} c_1, \ldots c_p \\ d_1, \ldots d_q \end{array} ; s \right] \right],\ \wp(c_p) \right\rangle = \left\langle \Gamma(c_p)\, {}_pF_q \left[\begin{array}{c} c_1, \ldots c_p \\ d_1, \ldots d_q \end{array} ; s \right],\ \mathscr{F}[\wp](c_p) \right\rangle .$$

P-8) Dual characteristic of Fourier transforms for the generalized hypergeometric function

$$\left\langle \mathscr{F}\left[\Gamma(c_p)\,{}_pF_q\left[\begin{matrix} c_1,\,\ldots c_p \\ d_1,\,\ldots d_q\end{matrix}\;;\,s\right]\right],\,\mathscr{F}\left[\wp(c_p)\right]\right\rangle = 2\pi\left\langle \Gamma(c_p)\,{}_pF_q\left[\begin{matrix} c_1,\,\ldots c_p \\ d_1,\,\ldots d_q\end{matrix}\;;\,s\right],\,\wp(-c_p)\right\rangle.$$

P-9) Parseval's characteristic of Fourier transform for the generalized hypergeometric function

$$\left\langle \mathscr{F}\left[\Gamma(c_p)\,{}_pF_q\left[\begin{matrix} c_1,\,\ldots c_p \\ d_1,\,\ldots d_q\end{matrix}\;;\,s\right]\right],\,\overline{\mathscr{F}\left[\wp(c_p)\right]}\right\rangle = \overline{\left\langle \mathscr{F}\left[\Gamma(c_p)\,{}_pF_q\left[\begin{matrix} c_1,\,\ldots c_p \\ d_1,\,\ldots d_q\end{matrix}\;;\,s\right]\right]},\,\mathscr{F}\left[\wp(c_p)\right]\right\rangle$$

$$=2\pi\left\langle \Gamma(c_p)\,{}_pF_q\left((c)_p;\,(d)_q;\,s\right),\,\overline{\left[\wp(\Re(c_p))\right]}\right\rangle.$$

P-10) The generalized hypergeometric function and differentiation property of the Fourier transform

$$\left\langle \mathscr{F}\left[\frac{d^m}{d{c_p}^m}\left(\Gamma(c_p)\,{}_pF_q\left[\begin{matrix} c_1,\,\ldots c_p \\ d_1,\,\ldots d_q\end{matrix}\;;\,s\right]\right),\,\wp(c_p)\right]\right\rangle = \left\langle (-\iota t)^m\Gamma(c_p)\,{}_pF_q\left[\begin{matrix} c_1,\,\ldots c_p \\ d_1,\,\ldots d_q\end{matrix}\;;\,s\right],\,\mathscr{F}[\wp](c_p)\right\rangle.$$

P-11) Generalized hypergeometric function s' Taylor series

$$\left\langle \Gamma(c_p+\gamma)\,{}_pF_q\left[\begin{matrix} c_1,\,\ldots c_{p-1},\,c_p+\gamma \\ d_1,\,\ldots d_q\end{matrix}\;;\,s\right],\,\wp(c_p)\right\rangle = \left\langle \sum_{n=0}^{\infty}\frac{(c_1)^n}{n!}\frac{d^n}{dc_p^n}\left(\Gamma(c_p)\,{}_pF_q\left[\begin{matrix} c_1,\,\ldots c_p \\ d_1,\,\ldots d_q\end{matrix}\;;\,s\right]\right),\,\mathscr{F}[\wp](c_p)\right\rangle.$$

P-12) Generalized hypergeometric function has the property of convolution

$$\Gamma(c_p)\,{}_pF_q\left[\begin{matrix} c_1,\,\ldots c_p \\ d_1,\,\ldots d_q\end{matrix}\;;\,s\right]*f(c_p) = 2\pi\sum_{n,\,r,\,m=0}^{\infty}\frac{(-1)^n s^r (n+r)^m\prod_{i=1}^{p}(c_{i-1})_r}{n!r!m!\prod_{j=1}^{q}(d_j)_r}\frac{d^m}{dc_p^m}\left(f(c_p)\right).$$

P-13) Suppose f is a distribution with bounded support, then we have the following identity

$$\mathscr{F}\left[e^{-\Re(c_p)\xi}\exp\left(-e^{-\xi}\right){}_{p-1}F_q\left((c)_{p-1};\,(d)_q;\,se^{-\xi}\right)*f(s)\right] = \mathscr{F}\left[f(z);\,c_p\right]\Gamma(c_p)\,{}_pF_q\left[\begin{matrix} c_1,\,\ldots c_p \\ d_1,\,\ldots d_q\end{matrix}\;;\,s\right].$$

Proof. Theorem 4's methodology and the characteristics of the delta function can be utilized to accomplish P-1)-P-5). Similarly, Equation (18) is used to show the property P-6)

$$\left\langle \frac{d^m}{dc_p^m}\left(\Gamma(c_p)\,{}_pF_q\left[\begin{matrix} c_1,\,\ldots c_p \\ d_1,\,\ldots d_q\end{matrix}\;;\,s\right]\right),\,\wp(c_p)\right\rangle = \sum_{n,\,r=0}^{\infty}\frac{(-1)^n(s)^r\prod_{i=1}^{p}(c_{i-1})_r}{n!r!\prod_{j=1}^{q}(d_j)_r}(-1)^m\wp^m(-n-r).$$

According to Theorem 4, the above is a finite sum of functions that are of slow growth and fast decay. Alternatively, the Fourier transform of delta function features can be used to demonstrate outcomes P-7)-P-8). Consequently, result P-8) is confirmed by the following:

$$\left\langle \mathscr{F}\left[\Gamma(c_p)\,{}_pF_q\left[\begin{array}{c} c_1,\,\ldots c_p \\ d_1,\,\ldots d_q \end{array};\,s\right]\right],\,\wp(c_p)\right\rangle = 2\pi\sum_{n,\,r=0}^{\infty}\frac{(-1)^n(s)^r\prod_{i=1}^{p}(c_{i-1})_r}{n!r!\prod_{j=1}^{q}(d_j)_r}\left\langle \mathscr{F}\left[\delta\left(c_p+n+r\right)\right],\,\wp(c_p)\right\rangle$$

$$= 2\pi\sum_{n,\,r=0}^{\infty}\frac{(-1)^n(s)^r\prod_{i=1}^{p}(c_{i-1})_r}{n!r!\prod_{j=1}^{q}(d_j)_r}\left\langle \delta\left(c_p+n+r\right),\,\mathscr{F}\left[\wp(c_p)\right]\right\rangle$$

$$= \left\langle \Gamma(c_p)\,{}_pF_q\left[\begin{array}{c} c_1,\,\ldots c_p \\ d_1,\,\ldots d_q \end{array};\,s\right],\,\mathscr{F}\left[\wp(c_p)\right]\right\rangle.$$

Similarly, Parseval's identity P-9) for the Fourier transform is determined and provided as

$$\left\langle \mathscr{F}\left[\Gamma(c_p)\,{}_pF_q\left((c)_p;\,(d)_q;\,s\right)\right],\,\overline{\mathscr{F}[\wp(\gamma)]}\right\rangle = \left\langle \overline{\mathscr{F}\left[\Gamma(c_p)\,{}_pF_q\left((c)_p;\,(d)_q;\,s\right)\right]},\,\mathscr{F}\left[\wp(c_p)\right]\right\rangle$$

$$= 2\pi\left\langle \left[\Gamma(c_p)\,{}_pF_q\left[\begin{array}{c} c_1,\,\ldots c_p \\ d_1,\,\ldots d_q \end{array};\,s\right]\right],\,\overline{\left[\Re(c_p)\right]}\right\rangle.$$

It is possible to demonstrate property P-10) by taking into account the following

$$\left\langle \mathscr{F}\left[\frac{d}{dc_p}\left(\Gamma(c_p)\,{}_pF_q\left[\begin{array}{c} c_1,\,\ldots c_p \\ d_1,\,\ldots d_q \end{array};\,s\right]\right)\right],\,\wp(c_p)\right\rangle$$

$$= 2\pi\sum_{n,\,r=0}^{\infty}\frac{(-1)^n(s)^r\prod_{i=1}^{p}(c_{i-1})_r}{n!r!\prod_{j=1}^{q}(d_j)_r}\left\langle \mathscr{F}\left[\delta^{(1)}\left(c_p+n+r\right)\right],\,\wp(c_p)\right\rangle;$$

$$\left\langle \mathscr{F}\left[\frac{d}{dc_p}\left(\Gamma(c_p)\,{}_pF_q\left[\begin{array}{c} c_1,\,\ldots c_p \\ d_1,\,\ldots d_q \end{array};\,s\right]\right)\right],\,\wp(c_p)\right\rangle$$

$$= 2\pi\sum_{n,\,r=0}^{\infty}\frac{(-1)^n(s)^r\prod_{i=1}^{p}(c_{i-1})_r}{n!r!\prod_{j=1}^{q}(d_j)_r}\left\langle \mathscr{F}\left[\delta\left(c_p+n+r\right)\right],\,\wp^{(1)}(c_p)\right\rangle;$$

$$\left\langle \mathscr{F}\left[\frac{d}{dc_p}\left(\Gamma(c_p)\,{}_pF_q\left[\begin{array}{c} c_1,\,\ldots c_p \\ d_1,\,\ldots d_q \end{array};\,s\right]\right)\right],\,\wp(c_p)\right\rangle$$

$$= 2\pi\sum_{n,\,r=0}^{\infty}\frac{(-1)^n(s)^r\prod_{i=1}^{p}(c_{i-1})_r}{n!r!\prod_{j=1}^{q}(d_j)_r}\left\langle \delta\left(c_p+n+r\right),\mathscr{F}\left[\wp^{(1)}(c_p)\right]\right\rangle;$$

$$\left\langle \Gamma(c_p)\,{}_pF_q\left[\begin{array}{c} c_1,\ \ldots c_p \\ d_1,\ \ldots d_q \end{array};\ s\right],\ \mathscr{F}\left[\wp^{(1)}(c_p)\right]\right\rangle = \left\langle \Gamma(c_p)\,{}_pF_q\left((c)_p;\ (d)_q;\ s\right),\ (-\imath t)\wp(c_p)\,e^{-ic_pt}\right\rangle;$$

$$\left\langle \mathscr{F}\left[\frac{d}{dc_p}\left(\Gamma(c_p)\,{}_pF_q\left[\begin{array}{c} c_1,\ \ldots c_p \\ d_1,\ \ldots d_q \end{array};\ s\right)\right],\ \wp(c_p)\right\rangle = \left\langle (-\imath t)\mathscr{F}\left[\Gamma(c_p)\,{}_pF_q\left[\begin{array}{c} c_1,\ \ldots c_p \\ d_1,\ \ldots d_q \end{array};\ s\right]\right],\ \wp(c_p)\right\rangle;$$

and so forth, it results

$$\left\langle \mathscr{F}\left[\frac{d^m}{dc_p{}^m}\left(\Gamma(c_p)\,{}_pF_q\left[\begin{array}{c} c_1,\ \ldots c_p \\ d_1,\ \ldots d_q \end{array};\ s\right]\right)\right],\ \wp(c_p)\right\rangle = \left\langle (-\imath t)^m\mathscr{F}\left[\Gamma(c_p)\,{}_pF_q\left[\begin{array}{c} c_1,\ \ldots c_p \\ d_1,\ \ldots d_q \end{array};\ s\right]\right],\ \wp(c_p)\right\rangle.$$

Equation (17) makes it possible to demonstrate the outcome number P-11) as follows:

$$\left\langle \Gamma(c_p+\gamma)\,{}_pF_q\left[\begin{array}{c} c_1,\ \ldots c_{p-1},\ c_p+\gamma \\ d_1,\ \ldots d_q \end{array};\ s\right],\ \wp(c_p)\right\rangle$$

$$=2\pi\sum_{n,\ r=0}^{\infty}\frac{(-1)^n(s)^r\prod_{i=1}^{p}(c_{i-1})_r}{n!r!\prod_{j=1}^{q}(d_j)_r}\left\langle\delta(c_p+n+r+\gamma),\ \wp(c_p)\right\rangle$$

$$=2\pi\sum_{n,\ r=0}^{\infty}\frac{(-1)^n(s)^r\prod_{i=1}^{p}(c_{i-1})_r}{n!r!\prod_{j=1}^{q}(d_j)_r}\left\langle\delta(c_p+n+r),\ \wp(c_p-\gamma)\right\rangle$$

$$=\lim_{v\to\infty}\left\langle 2\pi\sum_{n,\ r=0}^{\infty}\frac{(-1)^n(s)^r\prod_{i=1}^{p}(c_{i-1})_r}{n!r!\prod_{j=1}^{q}(d_j)_r}\delta(c_p+n+r),\sum_{m=0}^{v}\frac{(-\gamma)^n}{n!}\wp^{(m)}(c_p)\right\rangle$$

$$=\lim_{v\to\infty}\left\langle\sum_{m=0}^{v}\frac{(\gamma)^m}{m!}\frac{d^m}{dc_p{}^m}\left(\Gamma(c_p)\,{}_pF_q\left[\begin{array}{c} c_1,\ \ldots c_p \\ d_1,\ \ldots d_q \end{array};\ s\right]\right),\ \wp(c_p)\right\rangle,$$

as required. Next, result P-11) can be demonstrated using Equation (22), which is further explained by the example that follows [28].

Example 2 Consider $f(c_p)=\exp(ac_p)$ then

$$\Gamma(c_p)\,{}_pF_q\left[\begin{array}{c} c_1,\ \ldots c_p \\ d_1,\ \ldots d_q \end{array};\ s\right]*\exp(ac_p)=2\pi\sum_{n,\ r,\ m=0}^{\infty}\frac{(-1)^ns^r(n+r)^m\prod_{i=1}^{p}(c_{i-1})_r}{n!r!m!\prod_{j=1}^{q}(d_j)_r}\delta^{(m)}(c_p)*\exp(ac_p);\ a>0$$

$$\Gamma(c_p)\,{}_pF_q\left[\begin{array}{c} c_1,\ \ldots c_p \\ d_1,\ \ldots d_q \end{array};\ s\right]*\exp(ac_p)=2\pi\sum_{n,\ r,\ m=0}^{\infty}\frac{(-1)^ns^r(n+r)^m\prod_{i=1}^{p}(c_{i-1})_r}{n!r!m!\prod_{j=1}^{q}(d_j)_r}\frac{d^m}{dc_p^m}\left(\exp(ac_p)\right)$$

$$\Gamma(c_p)\, _pF_q\left[\begin{array}{c} c_1,\ \dots c_p \\ d_1,\ \dots d_q \end{array};\ s\right]*\exp(ac_p)=2\pi\sum_{n,\ r,\ m=0}^{\infty}\frac{(-1)^n s^r\,(ac_p(n+r))^m\prod_{i=1}^{p}(c_{i-1})_r}{n!r!m!\prod_{j=1}^{q}(d_j)_r}$$

$$=2\pi\exp(-ac_p)\, _pF_q\left[\begin{array}{c} c_1,\ \dots c_{p-1} \\ d_1,\ \dots d_q \end{array};\ se^{ac_p}\right].$$

The following identities can also be further computed using the definitions of $\sinh ac_p$ and $\cosh ac_p$

$$\Gamma(c_p)\, _pF_q\left[\begin{array}{c} c_1,\ \dots c_p \\ d_1,\ \dots d_q \end{array};\ s\right]*\sinh ac_p=2\pi\sum_{n,\ r,\ m=0}^{\infty}\frac{(-1)^n s^r(n+r)^m\prod_{i=1}^{p}(c_{i-1})_r}{n!r!m!\prod_{j=1}^{q}(d_j)_r}\frac{d^m}{dc_p^m}(\sinh ac_p)$$

$$\Gamma(c_p)\, _pF_q\left[\begin{array}{c} c_1,\ \dots c_p \\ d_1,\ \dots d_q \end{array};\ s\right]*\cosh ac_p=2\pi\sum_{n,\ r,\ m=0}^{\infty}\frac{(-1)^n s^r(n+r)^m\prod_{i=1}^{p}(c_{i-1})_r}{n!r!m!\prod_{j=1}^{q}(d_j)_r}\frac{d^m}{dc_p^m}(\cosh ac_p).$$

Since Fourier ($\mathscr{F}$) and inverse Fourier ($\mathscr{F}^{-1}$) act like continuous linear functionals from $\mathscr{D}' \to \mathscr{Z}'$ therefore, next result P - 12) is demonstrated [23]. Thus, in light of Equation (60), $\left[\exp\left(-e^{-\xi}\right)e^{-\Re(c_p)\xi}\, _{p-1}F_q\left[\begin{array}{c} c_1,\ \dots c_{p-1} \\ d_1,\ \dots d_q \end{array};\ se^{-\xi}\right]\right] \in \mathscr{D}'$. The proof of property P-13) is therefore completed in the light of Theorem 7.9.1 as given and proved in [23]. This will be further demonstrated with the aid of the example that follows.

Example 3 Analyze the following distribution $f(c_p)$ with bounded support

$$f(c_p)=\begin{cases} 1 & |c_p|<1 \\ 0 & |c_p|\geq 1 \end{cases}.$$

Therefore, with the previously stated data, we arrive at

$$\mathscr{F}\left[f(c_p)*2\pi e^{-\Re(c_p)\xi}\exp\left(-e^{-\xi}\right)\, _{p-1}F_q\left[\begin{array}{c} c_1,\ \dots c_{p-1} \\ d_1,\ \dots d_q \end{array};\ se^{-\xi}\right]\right]$$

$$=\mathscr{F}[f(c_p)]\,\mathscr{F}\left[2\pi e^{-\Re(c_p)\xi}\exp\left(-e^{-\xi}\right)\, _{p-1}F_q\left[\begin{array}{c} c_1,\ \dots c_{p-1} \\ d_1,\ \dots d_q \end{array};\ se^{-\xi}\right]\right]$$

$$=\frac{\sin\xi}{\xi}\Gamma(c_p)\, _pF_q\left[\begin{array}{c} c_1,\ \dots c_p \\ d_1,\ \dots d_q \end{array};\ s\right].$$

Utilizing the new representation, the obtained result is novel and useful.

4. Further applications and discussion

The above explanation focuses on the convergent behavior of $\Gamma(c_p)\,{}_pF_q\left[\begin{array}{c} c_1,\ \dots c_p \\ d_1,\ \dots d_q \end{array};\, s\right]$ and the presence of delta function ensures this fact for a wider range of functions. This topic is further discussed in this conversation. The Dirac delta function transfers all functions to their zero value in a linear fashion. Therefore, (18) can be used to obtain the subsequent results for a real t:

$$\left\langle \Gamma(c_p)\,{}_pF_q\left[\begin{array}{c} c_1,\ \dots c_p \\ d_1,\ \dots d_q \end{array};\, s\right],\ \wp(c_p)\right\rangle$$

$$=2\pi \sum_{n,\ r,\ m=0}^{\infty} \frac{(-1)^n s^r (n+r)^m \prod_{i=1}^{p}(c_{i-1})_r}{n!r!m!\prod_{j=1}^{q}(d_j)_r} \left\langle \delta^{(m)}(c_p),\ \wp(c_p)\right\rangle \tag{66}$$

$$=2\pi \sum_{n,\ r,\ m=0}^{\infty} \frac{(-1)^n s^r (n+r)^m \prod_{i=1}^{p}(c_{i-1})_r}{n!r!m!\prod_{j=1}^{q}(d_j)_r} (-1)^m \wp^{(m)}(0).$$

Example 4 Consider $\wp(c_p) = e^{ac_p}$ then $\wp^{(p)}(0) = a^p$

$$\left\langle \Gamma(c_p)\,{}_pF_q\left[\begin{array}{c} c_1,\ \dots c_p \\ d_1,\ \dots d_q \end{array};\, s\right],\ e^{ac_p}\right\rangle$$

$$=2\pi \sum_{n,\ r,\ m=0}^{\infty} \frac{(-1)^n s^r (n+r)^m \prod_{i=1}^{p}(c_{i-1})_r}{n!r!m!\prod_{j=1}^{q}(d_j)_r} (-1)^m a^m$$

$$=2\pi \sum_{n,\ r=0}^{\infty} \frac{(-1)^n (s)^r \prod_{i=1}^{p}(c_{i-1})_r}{n!r!\prod_{j=1}^{q}(d_j)_r} e^{-an-ar} \tag{67}$$

$$=2\pi \exp\left(-e^{-a}\right)\,{}_{p-1}F_q\left[\begin{array}{c} c_1,\ \dots c_{p-1} \\ d_1,\ \dots d_q \end{array};\, se^{-a}\right].$$

Example 5 Consider $\wp(c_p) = \sin ac_p$, then $\wp^{(m)}(0) = (-a)^{2m+1}$; $\wp^{(m)}(0) = 0$; $m = 0,\ 2\ ,4,\ \dots$

$$\left\langle \Gamma(c_p)\,{}_pF_q\left[\begin{array}{c} c_1,\ \dots c_p \\ d_1,\ \dots d_q \end{array};\, s\right],\ \wp(c_p)\right\rangle$$

$$=2\pi \sum_{n,\ r,\ m=0}^{\infty} \frac{(-1)^n s^r (n+r)^{2m+1} \Pi_{i=1}^{p}(c_{i-1})_r}{n!r!(2m+1)!\Pi_{j=1}^{q}(d_j)_r} (-a)^{2m+1}$$

$$=2\pi\sum_{n,\ r=0}^{\infty}\frac{(-1)^n(s)^r\Pi_{i=1}^{p}(c_{i-1})_r}{n!r!\Pi_{j=1}^{q}(d_j)_r}\,\mathrm{sina}(-n-r)$$

$$=\Im\left(2\pi\sum_{n,\ r=0}^{\infty}\frac{(-1)^n(s)^r\Pi_{i=1}^{p}(c_{i-1})_r}{n!r!\Pi_{j=1}^{q}(d_j)_r}e^{\imath(a(-r-n))}\right)$$

$$=\Im\left(2\pi\exp\left(-e^{-\imath a}\right)_{p-1}F_q\left[\begin{array}{c}c_1,\ \ldots c_{p-1}\\ d_1,\ \ldots d_q\end{array};\ se^{-\imath a}\right]\right). \tag{68}$$

Likewise, if $\wp(c_p)=\mathrm{cosac}_p$ then $\wp^{(m)}(0)=(-a)^{2m}$; $\wp^{(m)}(0)=0$; $m=1,\ 3,\ 5,\ \ldots$

$$\left\langle\Gamma(c_p)\,_pF_q\left[\begin{array}{c}c_1,\ \ldots c_p\\ d_1,\ \ldots d_q\end{array};\ s\right],\ \mathrm{cosac}\,c_p\right\rangle=\Re\left(2\pi\exp\left(-e^{-\imath a}\right)_{p-1}F_q\left[\begin{array}{c}c_1,\ \ldots c_{p-1}\\ d_1,\ \ldots d_q\end{array};\ se^{-\imath a}\right]\right). \tag{69}$$

Hence, the distributional representation is capable of producing more creative outcomes in several ways. A fractional kinetic Equation was previously solved in Section 2. Take note that when evaluating the following findings regarding the products of a broad class of special functions, (42) and (46) are taken into account.

$$\int_0^1\xi^{\chi-1}\exp\left(-e^{\xi}\right)_{p-1}F_q\left[\begin{array}{c}c_1,\ \ldots c_{p-1}\\ d_1,\ \ldots d_q\end{array};\ s\,e^{\xi}\right]H_{l,\ l}^{l,\ 0}\left[\xi\left|\begin{array}{c}\left(\gamma_k+1+\dfrac{\chi-1}{\beta_k},\ \dfrac{1}{\beta_k}\right)_1^l\\ \left(\gamma_k+\tau_k+1+\dfrac{\chi-1}{\alpha_k},\ \dfrac{1}{\alpha_k}\right)_1^l\end{array}\right.\right]d\xi$$

$$=2\pi\xi^{\chi-1}\exp\left(-e^{\xi}\right)_{p+m-1}\Psi_{q+m+1}\left[s\,e^{\xi}\left|\begin{array}{cc}(c_{i-1},\ 1) & \left(\gamma_k+1+\dfrac{\chi-1}{\beta_k},\ \dfrac{1}{\beta_k}\right)_1^l\\ (d_j,\ 1) & \left(\gamma_k+\tau_k+1+\dfrac{\chi-1}{\alpha_k},\ \dfrac{1}{\alpha_k}\right)_1^l\end{array}\right.\right]. \tag{70}$$

Consequently, we also obtain

$$\int_0^1\Gamma(\xi c_p)\,_pF_q\left[\begin{array}{c}c_1,\ \ldots c_{p-1},\ \xi c_p\\ d_1,\ \ldots d_q\end{array};\ s\right]H_{l,\ l}^{l,\ 0}\left[c_p\left|\begin{array}{c}\left(\gamma_k+1+\dfrac{\chi-1}{\beta_k},\ \dfrac{1}{\beta_k}\right)_1^l\\ \left(\gamma_k+\tau_k+1+\dfrac{\chi-1}{\alpha_k},\ \dfrac{1}{\alpha_k}\right)_1^l\end{array}\right.\right]dc_p$$

$$=2\pi\sum_{n,\ r=0}^{\infty}\frac{(-1)^n(s)^r\prod_{i=1}^{p}(c_{i-1})_r}{n!r!\prod_{j=1}^{q}(d_j)_r}\int_0^1\delta(c_p\xi+n+r)H_{l,\ l}^{l,\ 0}\left[c_p\left|\begin{array}{c}\left(\gamma_k+1+\dfrac{\chi-1}{\beta_k},\ \dfrac{1}{\beta_k}\right)_1^l\\ \left(\gamma_k+\tau_k+1+\dfrac{\chi-1}{\alpha_k},\ \dfrac{1}{\alpha_k}\right)_1^l\end{array}\right.\right]dc_p$$

$$=2\pi\xi^{-1}\sum_{n,\ r=0}^{\infty}\frac{(-1)^n(s)^r\prod_{i=1}^{p}(c_{i-1})_r}{n!r!\prod_{j=1}^{q}(d_j)_r}H_{l,\ l}^{l,\ 0}\left[-\frac{n+r}{\xi}\middle|\begin{array}{c}\left(\gamma_k+1+\frac{\chi-1}{\beta_k},\ \frac{1}{\beta_k}\right)_1^l\\\left(\gamma_k+\tau_k+1+\frac{\chi-1}{\alpha_k},\ \frac{1}{\alpha_k}\right)_1^l\end{array}\right]$$

$$=2\pi\xi^{-1}\exp\left(-e^{\xi}\right)H_{l+p-1,\ l+q}^{l,0}\left[s\,e^{1/\xi}\middle|\begin{array}{cc}(c_{i-1},\ 1) & \left(\gamma_k+1+\frac{\chi-1}{\beta_k},\ \frac{1}{\beta_k}\right)_1^l\\(d_j,\ 1) & \left(\gamma_k+\tau_k+1+\frac{\chi-1}{\alpha_k},\ \frac{1}{\alpha_k}\right)_1^l\end{array}\right], \tag{71}$$

and

$$\int_0^1\Gamma(\xi c_p)\,{}_pF_q\left[\begin{array}{c}c_1,\ \ldots c_{p-1},\ \xi c_p\\d_1,\ \ldots d_q\end{array};s\right]G_{m,\ m}^{m,\ 0}\left[c_p\middle|\begin{array}{c}(\gamma_k+\tau_k)_1^m\\(\gamma_k)_1^m\end{array}\right]dc_p$$

$$=2\pi\sum_{n,\ r=0}^{\infty}\frac{(-1)^n(s)^r\prod_{i=1}^{p}(c_{i-1})_r}{n!r!\prod_{j=1}^{q}(d_j)_r}\int_0^1\delta(c_p\xi+n+r)\,G_{m,\ m}^{m,\ 0}\left[c_p\middle|\begin{array}{cc}- & (\gamma_k+\tau_k)_1^m\\- & (\gamma_k)_1^m\end{array}\right]dc_p$$

$$=2\pi\xi^{-1}\sum_{n,\ r=0}^{\infty}\frac{(-1)^n(s)^r\prod_{i=1}^{p}(c_{i-1})_r}{n!r!\prod_{j=1}^{q}(d_j)_r}G_{m,\ m}^{m,\ 0}\left[-\frac{n+r}{\xi}\middle|\begin{array}{c}(\gamma_k+\tau_k)_1^m\\(\gamma_k)_1^m\end{array}\right] \tag{72}$$

$$=2\pi\xi^{-1}\exp\left(-e^{\xi}\right)G_{m+p-1,\ m+q}^{m,\ 0}\left[s\,e^{\xi}\middle|\begin{array}{cc}((c)_{p-1},\ 1) & (\gamma_k+\tau_k)_1^m\\(d_q,\ 1) & (\gamma_k)_1^m\end{array}\right].$$

5. Conclusion and future directions

The new fractional transformations of a generalized hypergeometric function have been computed by applying multiple E-K operators form fractional calculus. Consequently, as special examples for the many other well-known fractional transforms, equivalent new images are created. More general kinetic Equation in parameter c_p is designed and solved using the distributional representation, which is also applied to examine the Laplace transformation of the generalized hypergeometric function $\Gamma(c_p)\,{}_pF_q\left[\begin{array}{c}c_1,\ \ldots c_p\\d_1,\ \ldots d_q\end{array};s\right]$. Particular examples involving the original Mittag-Leffler function are given as corollaries. The purpose of this work was made possible in large part by a recently derived formulation of the generalized hypergeometric function as well as the associated Laplace transform. We can therefore draw the conclusion that this finding is important for extending the use of the generalized hypergeometric function [37, 38] beyond its original context [39].

Author contributions

Each author equally contributed to writing and finalizing the article. All authors have read and agreed to the published version of the manuscript.

Data availability statement

The original contributions presented in this study are included in the article material. Further inquiries can be directed to the corresponding author, Asifa Tassaddiq, at the email address a.tassaddiq@mu.edu.sa.

Acknowledgement

The author extends the appreciation to the Deanship of Postgraduate Studies and Scientific Research at Majmaah University for funding this research work through the project number (PGR-2025-2134).

Conflict of interest

The authors declare that they have no competing interests.

References

[1] Haubold HJ, Mathai AM. The fractional kinetic equation and thermonuclear functions. *Astrophysics and Space Science*. 2000; 273(1): 53-63.

[2] Saxena RK, Mathai AM, Haubold HJ. Unified fractional kinetic equations and a fractional diffusion equation. *Astrophysics and Space Science*. 2004; 290: 299-310.

[3] Mathai AM, Haubold HJ. *Erdélyi-Kober Fractional Calculus: From a Statistical Perspective, Inspired by Solar Neutrino Physics*. Singapore: Springer Briefs in Mathematical Physics; 2018.

[4] Srivastava HM. Some parametric and argument variations of the operators of fractional calculus and related special functions and integral transformations. *Journal of Nonlinear and Convex Analysis*. 2021; 22(8): 1501-1520.

[5] Srivastava HM, Tomovski Z. Fractional calculus with an integral operator containing generalized Mittag-Leffler function in the kernel. *Applied Mathematics and Computation*. 2009; 211(1): 198-210.

[6] Srivastava HM. A survey of some recent developments on higher transcendental functions of analytic number theory and applied mathematics. *Symmetry*. 2021; 13(12): 2294.

[7] Srivastava HM. An introductory overview of fractional-calculus operators based upon the Fox-Wright and related higher transcendental functions. *Journal of Advanced Engineering and Computation*. 2021; 5: 135-166.

[8] Srivastava HM. A general family of hybrid-type fractional-order kinetic equations involving the Liouville-Caputo fractional derivative. *Montes Taurus Journal of Pure and Applied Mathematics*. 2024; 6(3): 48-61.

[9] Srivastava HM. Some general fractional-order kinetic equations associated with the Riemann-Liouville fractional derivative. *Journal of the Indian Academy of Mathematics*. 2023; 45(2): 55-76.

[10] Hilfer R, Anton L. Fractional master equations and fractal time random walks. *Physical Review E*. 1995; 51: R848-R851.

[11] Mittag-Leffler MG. About the new function $E_{\alpha}(x)$. *French Academy of Sciences*. 1903; 137: 554-558.

[12] Prabhakar TR. A singular integral equation with a generalized Mittag-Leffler function in the kernel. *Yokohama Mathematical Journal*. 1971; 19(1): 7-15.

[13] Gorenflo R, Kilbas AA, Mainardi F, Rogosin S. *Mittag-Leffler Functions: Theory and Applications*. Berlin: Springer; 2014.

[14] Kilbas AA. *H-Transforms: Theory and Applications*. Florida: CRC Press; 2004.

[15] Lebedev NN. *Special Functions and Their Applications*. Englewood Cliffs, NJ: Prentice Hall; 1965.

[16] Galu'e L, Kiryakova VS, Kalla SL. Solution of dual integral equations by fractional calculus. *Mathematics Balkanica (New Series)*. 1993; 7: 53-72.

[17] Galu'e L, Kalla SL, Srivastava HM. Further results on an H-function generalized fractional calculus. *Journal of Fractional Calculus*. 1993; 4: 89-102.

[18] Srivastava HM, Saxena RK. Operators of fractional integration and their applications. *Applied Mathematics and Computation*. 2001; 118: 1-52.

[19] Kiryakova V. Unified approach to fractional calculus images of special functions-A survey. *Mathematics*. 2020; 8(12): 2260.

[20] Marichev OI. Volterra equation of Mellin convolutional type with a Horn function in the kernel. *Proceedings of the Academy of Sciences of the BSSR, Series of Physical and Mathematical Sciences*. 1974; 1: 128-129.

[21] Srivastava HM, AbuJarad ESA, Jarad F, Srivastava G, AbuJarad MHA. The Marichev-Saigo-Maeda fractional-calculus operators involving the (p, q)-extended Bessel and Bessel-Wright functions. *Fractal and Fractional*. 2021; 5(4): 210.

[22] Srivastava HM, Karlsson PW. *Multiple Gaussian Hypergeometric Series*. Toronto: Halsted Press; 1985.

[23] Saigo M, Maeda N. More generalization of fractional calculus. In: *Transform Methods & Special Functions, Varna'96*. Singapore: Science Culture Technology Publishing; 1998. p.386-400.

[24] Saigo M. A remark on integral operators involving the Gauss hypergeometric functions. *Mathematical Reports of the College of General Education, Kyushu University*. 1978; 11(2): 135-143.

[25] Srivastava HM, Saigo M. Multiplication of fractional calculus operators and boundary value problems involving the Euler-Darboux equation. *Journal of Mathematical Analysis and Applications*. 1987; 121: 325-369.

[26] Raina RK, Srivastava HM, Kilbas AA, Saigo M. Solvability of some Abel-type integral equations involving the Gauss hypergeometric function as kernels in the spaces of summable functions. *ANZIAM Journal*. 2001; 43: 291-320.

[27] Gel'fand IM, Shilov GE. *Generalized Functions: Properties and Operations*. New York: Academic Press; 1969.

[28] Zamanian AH. *Distribution Theory and Transform Analysis*. New York: Dover Publications; 1987.

[29] Al-Lail MH, Qadir A. Fourier transform representation of the generalized hypergeometric functions with applications to the confluent and Gauss hypergeometric functions. *Applied Mathematics and Computation*. 2015; 263: 392-397.

[30] Pal A, Jana RK, Shukla AK. Some integral representations of the $pR_q(\alpha, \beta; z)$ function. *International Journal of Applied and Computational Mathematics*. 2020; 6: 72.

[31] Chaudhry MA, Qadir A. Fourier transform and distributional representation of Gamma function leading to some new identities. *International Journal of Mathematics and Mathematical Sciences*. 2004; 39: 2091-2096.

[32] Tassaddiq A, Srivastava R. New results involving the generalized Krätzel function with application to the fractional kinetic equations. *Mathematics*. 2023; 11(4): 1060.

[33] Tassaddiq A, Srivastava R. New results involving Riemann zeta function using its distributional representation. *Fractal and Fractional*. 2022; 6(5): 254.

[34] Tassaddiq A, Srivastava R, Kasmani RM, Almutairi DK. Distributional representation of a special Fox-Wright function with an application. *Mathematics*. 2023; 11(15): 3372.

[35] Tassaddiq A, Cattani C. Fractional distributional representation of gamma function and the generalized kinetic equation. *Alexandria Engineering Journal*. 2023; 82: 577-586.

[36] Tassaddiq A. A new representation of the extended k-gamma function with applications. *Mathematical Methods in the Applied Sciences*. 2021; 44(14): 11174-11195.

[37] Koornwinder TH. Fractional integral and generalized Stieltjes transforms for hypergeometric functions as transmutation operators. *Symmetry, Integrability and Geometry: Methods and Applications*. 2015; 11: 074.

[38] Zaidi A, Almuthaybiri S. Explicit evaluations of subfamilies of the hypergeometric function ${}_3F_2(1)$ along with specific fractional integrals. *AIMS Mathematics*. 2025; 10: 5731-5761.

[39] El-Nabulsi RA. Modelling of KdV-soliton through fractional action and emergence of lump waves. *Qualitative Theory of Dynamical Systems*. 2024; 23(Suppl 1): 298.