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Research Article

Longitudinal Vibration of Defected Single-Layer Nanoplate Using Nonlocal Elasticity Theory

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Abstract: This article deals with the longitudinal vibrations of a defective single-layer nanoplate. Based on the nonlocal elasticity theory, Kirchhoff plate theory, and von Kármán theory, the longitudinal vibration is investigated. The coupled governing equations for the nonlocal elastic displacement and defect concentration fields are derived by means of Eringen's principle. The analytical presentation of the natural frequency of the defective single-layer nanoplate for longitudinal vibration modes is presented. Moreover, the natural frequencies were determined for various values of the defection and nonlocal elasticity factors. The results reported that an increase in the nonlocal parameter increases the frequencies, while the increase in the defect concentration decreases the natural frequencies of the nanoplate for different values of the half wave number throughout the computation frequencies of the nanoplate. The present work is an attempt to learn more about the effects of the small-scale length and defection factors on the longitudinal vibration behavior of single-layer nanoplates. To the authors' knowledge, these effects have not yet been studied deeply. However, longitudinal vibrations are critical for nanoplate applications because they directly influence the mechanical stability, dynamic response, and functional performance of nanoscale devices. Finally, the longitudinal vibrations determine the natural frequencies and resonant behaviors of nanoplates, affecting their ability to serve as key components in applications such as sensors, actuators, and nano-oscillators.

***Keywords*:** nonlocal elasticity theory, Kirchhoff plate theory, von Kármán theory, Eringen's principle, longitudinal vibrations, defected single-layer nanoplate

MSC: 74H45, 74B20, 74K20

1. Introduction

Nowadays, the field of nanotechnology is seeing fast development in the modern industry. This fact necessitates more research on the nanostructures utilized in drug delivery, energy storage systems, sensors, resonators, Nanoelectromechanical Systems (NEMS), nanooptomechanical structures, and Deoxyribonucleic Acid (DNA) detectors [1–6]. In fact, the classical constitutive principles can correctly anticipate the result if the external characteristic of a continuum is substantially greater than its internal characteristic length. On the other hand, nonlocality is required to account for

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long-range interaction forces since local theories cannot describe the effective mechanical behavior if the external and internal characteristic lengths are comparable. Response functionals [7, 8] characterize constitutive response at a material point of a continuum, which is dependent on the state of all points in nonlocal continuum field theories. The growing interest in miniature electromechanical devices, which have a number of possible uses in nanotechnology, has accelerated the development of mathematical methods capable of capturing size effects in small-scale structures. In this context, the primary goal is to develop computationally efficient and efficient methods for modeling size-dependent behavior and designing small-scale structures by utilizing nonlocal continuum mechanics' unconventional tools instead of atomistic methods, which take a lot of time [9–11]. Eringen [12, 13] created the first theories of nonlocal integral elasticity, in which stress is defined as the convolution integral between the elastic strain field and an appropriate averaging kernel controlled by an internal characteristic length. Eringen's constitutive law has been efficiently adopted to solve vibration and wave propagation problems in nanostructures [14]. Due to its exceptional mechanical, electrical, and thermal properties, single-layer nanoplates are among the key subjects of analysis [15, 16]. Alfwzana et al. [17] investigated the torsional vibration of simply supported nanoplate using nonlocal mathematical model. Selim and El-Safty [18] examined the vibrational analysis of an irregular single-walled carbon nanotube. Olga and Awrejcewicz [19] have reported that the size-effect could not be identified by classical elasticity.

However, in the present analysis, we propose examining the longitudinal vibrations of defected single-layer nanoplate. In fact, recent technological advancement in structures and solids cares more about the design of defect-free structures, where defective structures are characterized by lesser local rigidity and mass in the affected region, necessitating amplified vibrations due to the resonating upper layer [20]-a major drawback in low-frequency and miniature vibrating structures like nanostructures. Besides, defective structures deviate from ideal crystal bodies, affecting the optical behaviour, electrical conductivity, and the mechanical strength, among others-paramount features for nanostructures [21]. Thus, to obtain accurate outcomes in the study, we employed the nonlocal elasticity approach. We assumed the surfaces of the nanoplate are stress-free. In addition, the secular equation that governs the vibrational analysis has been solved in order to determine the frequency equation governing the vibrational analysis. Assuming that the coupling parameter is small, the frequency equations' solution is found and explained [22]. The findings showed that the coupling of elastic displacement and defect-concentration fields influences the longitudinal vibration of nanoplates. Additionally, the manuscript follows the following presentation: Section 2 gives the formulation of the governing equations of motion for the vibrating defective nonlocal single-layer nanoplate. Section 3 provides the precise stress-free boundary conditions. Section 4 determines the resulting solution and frequency equations of the formulated problem. Section 5 gives the numerical results, while section 6 recaps with the concluding remarks.

2. Theoretical formulation of defected single-layer nanoplate

We consider a single-layer nanoplate with nonlocal properties under defect concentration field. The x and y axes are placed along the edges of the nanoplate and the z-axis is normal to the xy-plane as depicted in Figure 1.

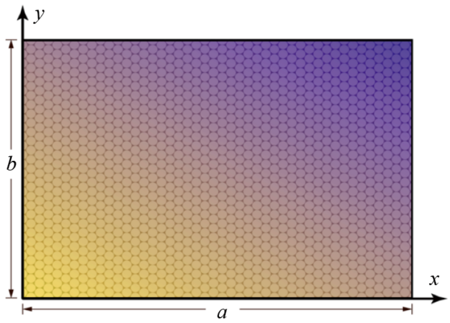

Figure 1. Diagram of single-layer nanoplate

The nanoplate's displacements in the direction of $x,\ y$ and z axes are denoted by $u,\ v$ and w, respectively. The thickness, length, and width of the nanoplate are h, a and b, respectively. In addition, ρ, ϑ and E, represent the nanoplate's mass density, Poisson's ratio, and elastic modulus, respectively.

To investigate the vibrational analysis of nanoplate, the Kirchhoff plate model is used for which the displacement components in x and y directions are given by [23]

$$u = -z\frac{\partial w}{\partial x}, \quad v = -z\frac{\partial w}{\partial y}, \tag{1}$$

where w is the transverse displacement. In addition, the strain components $(\varepsilon_{xx},\ \varepsilon_{yy},\ \varepsilon_{xy})$ in Cartesian coordinate are further expressed as [24]

$$\varepsilon_{xx} = -z\frac{\partial^2 w}{\partial x^2},\ \varepsilon_{yy} = -z\frac{\partial^2 w}{\partial y^2},\ \varepsilon_{xy} = -2z\frac{\partial^2 w}{\partial x \partial y}. \tag{2}$$

Moreover, in the present analysis, the vibration equation of the nanoplate can expressed as

$$\frac{\partial^2 M_{xx}}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_{yy}}{\partial y^2} = \rho h \frac{\partial^2 w}{\partial t^2}, \tag{3}$$

where $M_{xx},\ M_{xy}$, and M_{yy} are the bending moment resultants.

The constitutive relation for the single-layer nanoplate including defect production in the differential form has the following form, in accordance with the nonlocal elasticity theory, which states that the stress at a place is a function of strains at all points in the body [22]

$$\left(1-\mu\nabla^2\right)\sigma_{ij} + \left(1-\mu\nabla^2\right)\delta_i\left(x_i,\ t\right)E\Delta = \sigma_{ij}^*, \tag{4}$$

where σ_{ij}, σ_{ij}^* $(i,\ j = 1 = x,\ 2 = y,\ 3 = z)$ are nonlocal and local stress tensors, respectively, $\mu = (e_0 a)$ is the nonlocal elastic parameter, $\delta_i\left(x_i,\ t\right)$ is the defection coefficient at a point x_i at a time t, E is the Young's modulus, $\Delta = abh$ is the change of the volume of single-layer nanoplate area under defection and ∇^2 is the Laplacian operator, explicitly expressed as follows

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}. \tag{5}$$

Next, the constitutive relation with regard to equation (4) can be written as [25]

$$\left(1-\mu\nabla^2\right)\begin{pmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{pmatrix} + \left(1-\mu\nabla^2\right)\delta_i\left(x_i,\ t\right)E\Delta = \begin{pmatrix}\frac{E}{1-\gamma^2} & 0 & 0\\ 0 & \frac{E}{1-\gamma^2} & 0\\ 0 & 0 & \frac{E}{2(1+\gamma)}\end{pmatrix}\begin{pmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{xy}\end{pmatrix}, \tag{6}$$

where γ is the Poisson ratio of the nanoplate.

The governing equations for the nonlinear vibrations of an isotropic single-layer nanoplate can be written as [25],

$$\frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=\rho\frac{\partial^2 u}{\partial t^2}, \tag{7}$$

$$\frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=\rho\frac{\partial^2 u}{\partial t^2}. \tag{8}$$

Then, the resultant in-plane forces and moments may be written as follows [25]

$$N_{xx}=\int_{-h/2}^{h/2}\sigma_{xx}dz, \tag{9}$$

$$N_{xy}=\int_{-h/2}^{h/2}\sigma_{xy}dz, \tag{10}$$

$$N_{yy}=\int_{-h/2}^{h/2}\sigma_{yy}dz, \tag{11}$$

and

$$M_{xx}=\int_{-h/2}^{h/2}z\sigma_{xx}dz, \tag{12}$$

$$M_{xy}=\int_{-h/2}^{h/2}z\sigma_{xy}dz, \tag{13}$$

$$M_{yy}=\int_{-h/2}^{h/2}z\sigma_{yy}dz. \tag{14}$$

In view of equations (1), (6), (9)-(13), the constitutive and bending moment relations can be written in the following forms [26]

$$(1-\mu\nabla^2)\begin{pmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{pmatrix}+\left(1-\mu\nabla^2\right)\delta_i\left(x_i,\ t\right)E\Delta=h\begin{pmatrix}\frac{E}{1-\gamma^2} & 0 & 0\\ 0 & \frac{E}{1-\gamma^2} & 0\\ 0 & 0 & \frac{E}{2(1+\gamma)}\end{pmatrix}\begin{pmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{xy}\end{pmatrix}, \tag{15}$$

$$
(1-\mu\nabla^2)\begin{pmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{pmatrix}+\left(1-\mu\nabla^2\right)\delta_i\left(x_i,\ t\right)E\Delta=\frac{h^{12}}{12}\begin{pmatrix}\dfrac{E}{1-\gamma^2} & 0 & 0\\ 0 & \dfrac{E}{1-\gamma^2} & 0\\ 0 & 0 & \dfrac{E}{2(1+\gamma)}\end{pmatrix}\begin{pmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{xy}\end{pmatrix}. \tag{16}
$$

Taking into account relation (2), the bending moment equation (16) can be written as follows,

$$
\left(1-\mu\nabla^2\right)\left[M_{xx}+\delta_x\left(x_i,\ t\right)E\Delta\right]=-\left(\frac{Eh^3}{12(1-\gamma^2)}\right)\left(\frac{\partial^2 w}{\partial x^2}+\gamma\frac{\partial^2 w}{\partial y^2}\right), \tag{17}
$$

$$
\left(1-\mu\nabla^2\right)\left[M_{yy}+\delta_y\left(x_i,\ t\right)E\Delta\right]=-\left(\frac{Eh^3}{12(1-\gamma^2)}\right)\left(\frac{\partial^2 w}{\partial y^2}+\gamma\frac{\partial^2 w}{\partial x^2}\right), \tag{18}
$$

$$
\left(1-\mu\nabla^2\right)M_{xy}=-\left(\frac{Eh^3}{12(1+\gamma)}\right)\frac{\partial^2 w}{\partial x\partial y}. \tag{19}
$$

Using relations (2) and (6), the stress-strain relations of the nanoplate become,

$$
\sigma_{xx}=\frac{E}{1-\gamma^2}\left(\varepsilon_{xx}+\gamma\varepsilon_{yy}\right)=\frac{E}{1-\gamma^2}\left[\frac{\partial^2 w}{\partial x^2}+\gamma\frac{\partial^2 w}{\partial y^2}\right]-\left(1-\mu\nabla^2\right)\delta_x\left(x_i,\ t\right)E\Delta, \tag{20}
$$

$$
\sigma_{xy}=\frac{E}{2(1+\gamma)}\varepsilon_{xy}=\frac{E}{2(1+\gamma)}\frac{\partial^2 w}{\partial x\partial y}, \tag{21}
$$

$$
\sigma_{yy}=\frac{E}{1-\gamma^2}\left(\varepsilon_{yy}+\gamma\varepsilon_{xx}\right)=\frac{E}{1-\gamma^2}\left[\frac{\partial^2 w}{\partial y^2}+\gamma\frac{\partial^2 w}{\partial x^2}\right]-\left(1-\mu\nabla^2\right)\delta_y\left(x_i,\ t\right)E\Delta. \tag{22}
$$

3. Boundary conditions

The present study is concerning with the analysis of longitudinal vibration of a defected single-layer nanoplate. Assuming the defected nanoplate vibrates exclusively in the longitudinal direction x, then, the boundary conditions at the free surfaces of the single-layer nanoplate will be vanished, namely

$$
\sigma_{yy}=\sigma_{xy}=0,\ \ \delta_y\left(x_i,\ t\right)=0. \tag{23}
$$

Using the boundary conditions (23), the vibration equation of the single-layer nanoplate involving small-scale and defection parameters becomes

$$
\frac{\partial^2 M_{xx}}{\partial x^2}-\rho h\frac{\partial^2 w}{\partial t^2}=0. \tag{24}
$$

The stress-strain relation of the nanoplate in the longitudinal direction can be written as

$$\sigma_{xx} = \frac{E}{1-\gamma^2}\left[\frac{\partial^2 w}{\partial x^2} + \gamma \frac{\partial^2 w}{\partial y^2}\right] - \left(1-\mu\nabla^2\right)\delta_x(x,\ y,\ t)E\Delta. \tag{25}$$

Therefore, substituting equations (12) and (25) into the vibration equation (24), one gets the following governing equation for motion associated with the defected single-layer elastic nanoplates

$$\frac{\partial^4 w}{\partial x^4} + \mu \frac{\partial^4 \delta_x(x,\ y,\ t)}{\partial x^4} + \gamma \frac{\partial^4 w}{\partial x^2 \partial y^2} - E\Delta \frac{\partial^2 \delta_x(x,\ y,\ t)}{\partial x^2} - \rho\beta \frac{\partial^2 w}{\partial t^2} = 0, \tag{26}$$

where β is expressed explicitly as follows

$$\beta = \frac{2\left(1-\gamma^2\right)}{Eh}. \tag{27}$$

4. Solution method

To solve the vibration equation of the defected single-layer nanoplate (26), we assume the vibrational displacement $w(x,\ y,\ t)$, and the defection coefficient $\delta_x(x,\ y,\ t)$ follow the harmonic solutions of the following form [22]

$$w(x,\ y,\ t) = W(y)e^{i(kx-\omega t)}, \tag{28}$$

$$\delta_x(x,\ y,\ t) = \delta(y)e^{i(kx-\omega t)}, \tag{29}$$

where is the imaginary number, $i=\sqrt{-1}$, $W(y)$ and $\delta(y)$ are function of y only, ω is the angular frequency of the transverse vibration, propagating along the longitudinal x-direction, with the wave number k.

Using solutions (28) and (29) in the vibration equation (26), one gets

$$W(y)k^4 + \mu k^4\delta(y) - \gamma k^2 \frac{d^2 w}{dy^2} - E\Delta k^2\delta(y) + \rho\beta W(y)\omega^2 = 0. \tag{30}$$

The solution of (30) is given as

$$W(y) = \tau e^{iky}, \tag{31}$$

$$\delta(y) = \alpha e^{iky}, \tag{32}$$

where τ is an arbitrary constant and α is the defect concentration parameter.

Substituting equations (31) and (32) into equation (30), one gets

$$\omega = \frac{k}{\sqrt{\rho\beta}}\sqrt{\alpha E\Delta - (1+\alpha\mu+\gamma)k^2}. \tag{33}$$

The natural frequency of the defected single-layer nanoplate can be defined from the well-known relation,

$$f = \frac{\omega}{2\pi} = \frac{k}{\sqrt{2\pi\rho\beta}}\sqrt{\alpha E\Delta - (1+\alpha\mu+\gamma)k^2}. \tag{34}$$

It can be seen from the above equation that, the natural frequency of the single-layer defected nanoplate is influenced by the presence of defection parameter (α). The solution of equation (34) corresponds to defected longitudinal vibrations of the single-layer nanoplate.

5. Numerical results and discussion

The nonlocal elasticity theory, Kirchhoff plate theory and von Kármán theory are proposed as theoretical frameworks for studying longitudinal vibrations of defected single-layer nanoplate.

To verify the accuracy of the suggested formulation, comparisons are made with non-defected nanoplates (α = 0) and local elasticity (μ = 0) cases. Equation (34) has been used to get the natural frequency (Hz) as a function of half wave number k under the effect of the nonlocal parameter (μ) and defection parameter (α). We used the simulation parameters given in Table 1 [27, 28] for numerical calculation to determine the natural frequencies of the defective single-layer nanoplate in order to verify the suggested approach.

Table 1. Simulation parameters for the defective single-layer nanoplate [27, 28]

E	γ	ρ	h	$a = b$
1.06 TPa	0.25	2,250 kg/m^3	0.34 nm	10 nm

Figure 2 shows the effect of nonlocal parameters ($\mu = 1.0,\ 2.0,\ 3.0$ nm) on the natural frequencies of the single-layer nanoplate versus half wave numbers (***k***), at different defection parameter ($\alpha = -5,\ -10,\ -15,\ -20$). It is observed that the natural frequency of the considered nanoplate vary depending on both nonlocal elasticity and defection parameters. It can be seen that, the increase of nonlocal parameter reduces the value of the natural frequency, as be shown in Figures 2 and 3. Thus, a mechanism on the natural frequencies variation is due to coupling of defect concentration field and the nonlocal elasticity of the nanoplates. The effects of defection parameter ($\alpha = -5,\ -10,\ -15,\ -20$) on the natural frequency of the nanoplate at constant values of the nonlocal parameter $\mu = 1.0$ nm is shown in Figure 4. From the curves in this figure, it is can be seen that, the natural frequency has a maxima value at ($\alpha = -15$).

It is clear that both the nonlocal elasticity and the defection parameters affect the natural frequency curves of the nanoplate. Additionally, it can be observed that, at small values of wave number ($\boldsymbol{k} < 2$), the effects of nonlocal and defection factors are minor, and the results display nearly identical values of the natural frequencies of the nanoplate under consideration. The discrepancies, however, become apparent when the value of (***k***) exceeds 2. Figures 2 and 3 show that the natural frequencies of the nanoplate rise as the nonlocal parameter (μ) increases. The effects of the defection parameter ($\alpha = -5,\ -10,\ -15,\ -20$) on the defected nanoplate's natural frequency are examined in Figure 2. Figure 4 shows the natural frequencies at constant nonlocal parameter value ($\mu = 1.0$) for various defection parameter values ($\alpha = -5,\ -10,\ -15,\ -20$)-a paramount factor for controlling defect concentration and mechanical properties of structures in defect engineering; besides, defects and material imperfections are purposely introduced in solids to attain desired structures with preferred characteristics and optimized performance [29, 30]. Finally, it can be observed that the natural frequency of the single-layered nanoplates varies depending on the defection parameter change.

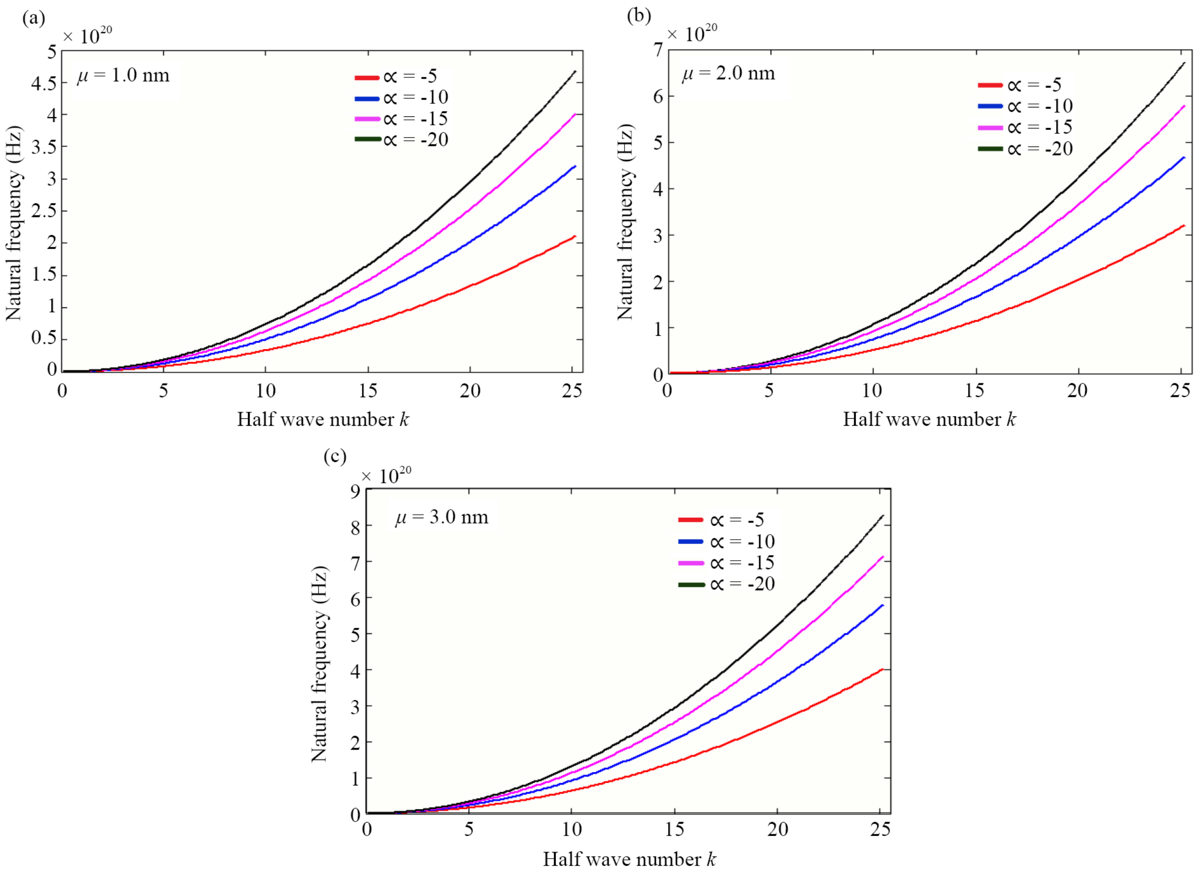

Figure 2. Changes in the nonlocal nanoplate's natural frequency in relation to half wave number ($\boldsymbol{k}$) with varying defection parameter values ($\alpha = -5, -10, -15, -20$) at $\mu = 1.0, 2.0, 3.0$ nm

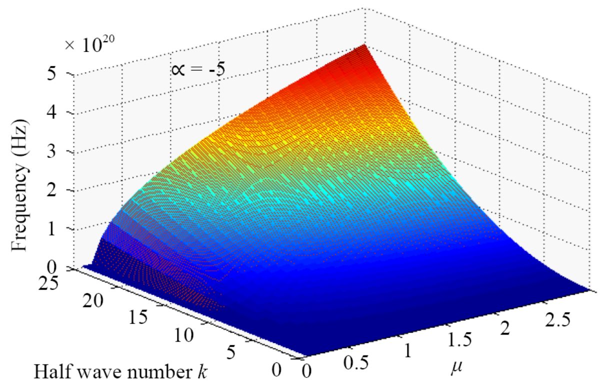

Figure 3. Changes in the nonlocal nanoplate's natural frequency in relation to half wave number ($\boldsymbol{k}$) with varying nonlocal elasticity parameter values ($\mu = 0.0, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0$ nm)

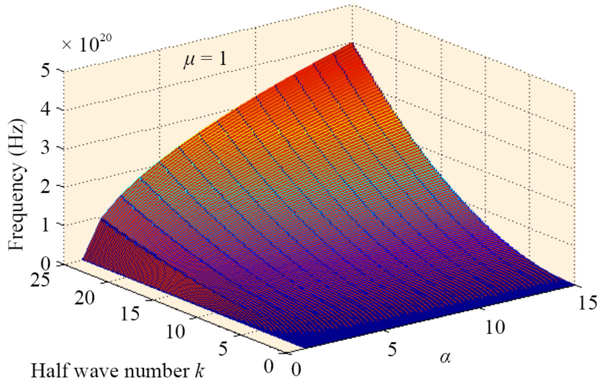

Figure 4. Changes in the nonlocal nanoplate's natural frequency in relation to half wave number ($\boldsymbol{k}$) with varying defection parameter values ($\alpha = -5, -10, -15, -20$) at $\mu = 1.0$ nm

6. Conclusions

The present analysis examines the longitudinal vibration of the nonlocal defective single-layer nanoplate based on nonlocal elasticity theory under the effects of defection and nonlocality parameters.

The key conclusions of the current study are summarized as follows:

• New governing and natural frequency equations were developed after it was determined to address the theoretical model of the longitudinal vibration of a nonlocal defective single-layer nanoplate.

• For a range of defection and nonlocal elasticity factor values, the natural frequencies were calculated.

• The presence of defection and nonlocal elasticity has a considerable impact on the natural frequencies of single-layer nanoplates.

• According to the results, across a range of half wave number ($\boldsymbol{k}$) values across the nanoplate's computation frequencies, increasing the nonlocal parameter raises the frequencies, while raising the defection parameter (α) lowers the nanoplate's natural frequencies.

• The goal of this effort is to gain a better understanding of how the vibration behavior of single-layer nanoplates is affected by defection variables and small-scale length.

• Since single-layer nanoplates are the most prevalent structural element of nanocomposite materials, the results obtained may be useful in the design of nano oscillators and nanodevices.

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Data availability

Data sharing is not applicable to this article as no data sets were generated during the current study.

Ethical approval

This article does not contain any studies with human participants performed by the authors.

Conflicts of interest

The authors declare that they have no conflicts of interest to report regarding the present study.

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