Contemporary Mathematics
https://ojs.wiserpub.com/index.php/CM/

Research Article

Hydraulic Analysis of Water Distribution System Under Interval-Valued Fuzzy Set

Zhe Zhao, Zhihao He, Jiazhu Zheng, Yumin Wang*

School of Energy and Environment, Southeast University, Nanjing, China
E-mail: wangyumin@seu.edu.cn

Received: 7 July 2025; **Revised:** 28 August 2025; **Accepted:** 11 September 2025

Abstract: Hydraulic analysis of Water Distribution System (WDS) is usually performed with single uncertainty of pipe roughness coefficient. However, in real world multiple uncertainties exist. In this paper, dual uncertainties of pipe roughness coefficient were considered, which is represented by Interval-Values Fuzzy Set (IVFS). In addition, the fuzziness was assumed to follow trapezoidal fuzzy distribution. The measure of fuzziness and interval of variables based on IVFS were defined. By applying Interval-Values Fuzzy Set-Genetic Algorithm (IVFS-GA) algorithm, pipe flow and nodal pressure under uncertain effects of pipe roughness coefficients were analyzed. The method proposed was applied to three cases to analyze the fuzziness and interval of pipe flow and nodal pressure, which were defined with trapezoidal membership distributions. The fuzziness and interval of pipe flow have no direct relationship, while the fuzziness and interval of nodal pressure have the same trend. In the three WDSs, the intervals of nodal pressures are more significant than pipe flows. The results obtained can help the manager analyze the hydraulic performance of WDS under uncertainty of pipe roughness coefficients.

***Keywords*:** Water Distribution System (WDS), Interval-Values Fuzzy Set (IVFS), trapezoid membership function

MSC: 90B15, 03E72, 90C70

1. Introduction

Water Distribution System (WDS) is one of the critical infrastructures which supply customers with acceptable flow and pressure [1, 2]. Mathematical models of WDS are constructed to represent the real systems and provide accurate estimation of flow and pressure under deterministic information. However, the model input parameters are usually uncertain, which leads to the inaccuracy of model output parameters. Therefore, the uncertainty of flow and pressure should be researched in depth under uncertain conditions [3].

Many methods were proposed to analyze the uncertainty of WDS such as Monte-Carlo simulation, First-Order Reliability Method (FORM), and fuzzy set theory [3–7]. By performing tens of thousands Monte-Carlo simulation, the random distribution of output parameters can be obtained through hydraulic simulation by calling EPANET hydraulic engine. However, Monte-Carlo simulation requires significant computational time [8]. To reduce the computational burden, the FORM based on Taylor series was proposed. However, the FORM is mostly suitable for linear system [9]. To

DOI: https://doi.org/10.37256/cm.6620257746

analyze uncertainty in nonlinear and non-monotonic system, fuzzy set theory was introduced to perform hydraulic analysis and optimization in WDS under uncertainty [9–13]. In fuzzy set theory, the fuzziness of parameters was represented by fuzzy membership functions within the range of [0, 1] [14]. The α-cut set of fuzzy parameters can be obtained for a certain membership degree α. By transferring the membership function of roughness coefficients to α-cut sets, the pipe flow and nodal pressure at each α-cut set can be optimized. As such, the fuzzy membership function of output parameters was estimated and applied to WDS [15]. The wave speeds, pipe friction factors, and nodal demands were supposed to follow triangular fuzzy distribution in transient simulation model to obtain fuzzy nodal pressure by performing optimization process [1, 16]. Instead of triangular fuzzy distribution, trapezoidal fuzzy distribution was considered for pipe roughness coefficients, and fuzziness of nodal pressure and pipe flow was evaluated [17]. By proposing fuzzy set to address uncertainty of nodal demands, the Newton-Raphson iterative process was performed to analyze nodal hydraulic heads, which avoids the optimization approach [18]. Almost all the approaches mentioned above subject to single uncertainty of fuzziness with measure methods of fuzziness proposed. However, in real world, the input parameters exist multiple uncertainties including randomness, intervals, and fuzziness, etc. that cannot be treated as conventional fuzzy sets [19–21]. To deal with uncertainties presented in fuzziness and interval, Interval-Valued Fuzzy Set (IVFS) was proposed and hydraulic analysis with trapezoid fuzzy membership functions was performed. Moreover, the interval of IVFS was measured by defining the fuzzy preference relationship between the lower and upper fuzzy sets. Through the introduction of IVFS, the multiple uncertainties including fuzziness and interval of pipe flow and nodal pressure can be analyzed more completely.

In this paper, firstly, the uncertainty of pipe roughness coefficients was expressed by IVFS with trapezoid membership function distribution. The α-cut sets of pipe roughness coefficient were assumed. Secondly, the Genetic Algorithm (GA) was performed to obtain optimized minimum and maximum pipe flow and nodal pressure at each α-cut set of pipe roughness coefficient based on EPANET-TOOLKIT-MATLAB platform. Thirdly, the method was applied to three WDSs to analyze and compare the uncertainties of pipe flow and nodal pressure. Finally, the conclusion was drawn.

2. Methodology

2.1 *IVFS theory*

2.1.1 *Definition of IVFS*

An interval number $A^{\pm}$ is expressed by Eq. (1) as follows.

$$A^{\pm} = \left\{ x \middle| x^{-} \leq x \leq x^{+},\ x \in R \right\}. \tag{1}$$

The midpoint and radius of interval $A^{\pm}$ are termed as $M(x^{\pm})$ and $R(x^{\pm})$, which can be expressed as $M\left(A^{\pm}\right) = \dfrac{x^{-} + x^{+}}{2}$ and $R\left(A^{\pm}\right) = \dfrac{x^{+} - x^{-}}{2}$, respectively.

A fuzzy set $\tilde{A}$ with trapezoidal distribution is defined as $(x_1,\ x_2,\ x_3,\ x_4)$. The membership function is defined as $\mu_{\tilde{A}}(x)$, which is expressed by Eq. (2) as follows.

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \leq x_1 \\ \dfrac{x - x_1}{x_2 - x_1}, & x_1 \leq x \leq x_2 \\ 1, & x_2 \leq x \leq x_3 \\ \dfrac{x_4 - x}{x_4 - x_3}, & x_3 \leq x \leq x_4 \\ 0, & x \geq x_4 \end{cases} \tag{2}$$

Where x_1 is the minimum possible value, x_2 is the most possible minimum value, x_3 is the most possible maximum value, and x_4 is the maximum possible value.

The greater membership function represents the closer degree the variable belonging to the fuzzy set, and vice versa. The α-cut of fuzzy set $\widetilde{A}$ is expressed as $\left[\underline{x^\alpha},\ \overline{x^\alpha}\right]$, where the elements with the membership degree greater than or equal to α. When $\alpha = 0$, the α-cut set of fuzzy set $\widetilde{A}$ is the interval expressed as $[x_1,\ x_4]$. In case of $x_2 = x_3$, the trapezoidal fuzzy set becomes fuzzy triangular set. The closer distance between x_1 and x_4 represents the less fuzziness. In case of $x_1 = x_2 = x_3 = x_4$, the fuzzy set $\widetilde{A}$ becomes a determined value.

An IVFS is the combination of the interval set and fuzzy set, which can deal with fuzziness and intervals. The IVFS $\widetilde{A^\pm}$ is defined by Eq. (3) as follows.

$$\widetilde{A^\pm} = \left[\widetilde{A^-},\ \widetilde{A^+}\right] = \left[(x_1^-,\ x_2^-,\ x_3^-,\ x_4^-),\ (x_1^+,\ x_2^+,\ x_3^+,\ x_4^+)\right]. \tag{3}$$

Where $\widetilde{A^-}$ is the lower trapezoidal fuzzy number, and $\widetilde{A^+}$ is the upper trapezoidal fuzzy number. The closer distance between x_1^- and x_1^+ or between x_4^- and x_4^+ represents the less intervals.

The membership of $\widetilde{A^-}$ and $\widetilde{A^+}$ are expressed by Eq. (4) and Eq. (5) as follows, respectively.

$$\mu_{\widetilde{A^-}}(x) = \begin{cases} 0, & x \le x_1^- \\ \dfrac{x - x_1^-}{x_2^- - x_1^-}, & x_1^- \le x \le x_2^- \\ 1, & x_2^- \le x \le x_3^- \\ \dfrac{x_4^- - x}{x_4^- - x_3^-}, & x_3^- \le x \le x_4^- \\ 0, & x \ge x_4^- \end{cases} \tag{4}$$

$$\mu_{\widetilde{A^+}}(x) = \begin{cases} 0, & x \le x_1^+ \\ \dfrac{x - x_1^+}{x_2^+ - x_1^+}, & x_1^+ \le x \le x_2^+ \\ 1, & x_2^+ \le x \le x_3^+ \\ \dfrac{x_4^+ - x}{x_4^+ - x_3^+}, & x_3^+ \le x \le x_4^+ \\ 0, & x \ge x_4^+ \end{cases} \tag{5}$$

Where $x_1^- \le x_2^- \le x_3^- \le x_4^-,\ x_1^+ \le x_2^+ \le x_3^+ \le x_4^+$.

The α-cuts of IVFS $\widetilde{A^-}$ and $\widetilde{A^+}$ are expressed by $\left\{\left[\underline{(x^-)^\alpha},\ \overline{(x^+)^\alpha}\right]\right\}$, respectively, which are shown in Figure 1. The closer distance between $\underline{(x^-)}^\alpha$ and $\underline{(x^+)}^\alpha$ as well as between $\overline{(x^-)}^\alpha$ and $\overline{(x^+)}^\alpha$ represents the less intervals. In case of $\underline{(x^-)}^\alpha = \underline{(x^+)}^\alpha$ and $\overline{(x^-)}^\alpha = \overline{(x^+)}^\alpha$ at various α-cuts, the IVFS $\widetilde{A^\pm}$ becomes fuzzy set $\widetilde{A}$, and the interval of $\widetilde{A^\pm}$ vanished.

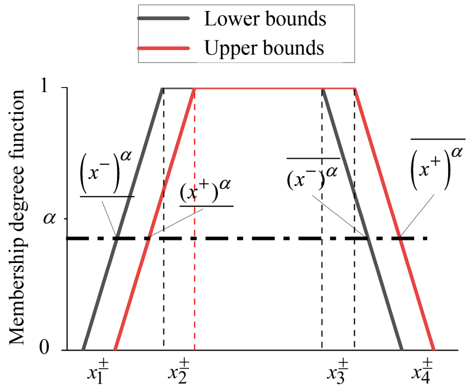


Figure 1. An interval-valued trapezoidal fuzzy number

2.1.2 *Measure of IVFS*

To measure the fuzziness of IVFS, the average dispersion degree $\overline{DD}$ was proposed by Eq. (6) as follows.

$$\overline{DD} = \frac{1}{2}(DD_L + DD_U) = \frac{1}{2}\left(\frac{\sum_{i=1}^{5}\left(x^-_{\max}|_{\alpha_i} - x^-_{\min}|_{\alpha_i}\right)\times 0.2}{\left(x^-_{\max}|_{\alpha_0} + x^-_{\min}|_{\alpha_0}\right)/2} + \frac{\sum_{i=1}^{5}\left(x^+_{\max}|_{\alpha_i} - x^+_{\min}|_{\alpha_i}\right)\times 0.2}{\left(x^+_{\max}|_{\alpha_0} + x^+_{\min}|_{\alpha_0}\right)/2}\right). \tag{6}$$

Where DD_L refers to DD for lower bounds of IVFS variable, DD_U refers to DD for upper bounds of IVFS variable, $x^-_{\max}|_{\alpha_i}$ and $x^-_{\min}|_{\alpha_i}$ refer to maximum and minimum values of lower bounds of IVFS variables corresponding to α_i, which are 0.2, 0.4, 0.6, 0.8, and 1.0, respectively, and $x^+_{\max}|_{\alpha_i}$ and $x^+_{\min}|_{\alpha_i}$ refer to maximum and minimum values of upper bounds of IVFS variables corresponding to α_i, which are 0.2, 0.4, 0.6, 0.8, and 1.0, respectively. The IVFS variable with greater values of DD indicates the IVFS variables are fuzzier.

To measure the interval of IVFS, the fuzzy preference relations is applied and expressed by Eq. (7) as follows.

$$R\left(A^-, A^+\right) = \frac{\overrightarrow{S_L} + \overleftarrow{S_R} + S_O}{\overrightarrow{S_L} + \overleftarrow{S_L} + \overleftarrow{S_R} + \overrightarrow{S_R} + 2S_O}. \tag{7}$$

Where $\overrightarrow{S_L}$ refers to the area that the left part of A^- superior to the left part of A^+, $\overleftarrow{S_R}$ refers to the area that the right part of A^+ superior to the right part of A^-, $\overleftarrow{S_L}$ refers to the area that the right part of A^+ superior to the right part of A^-, $\overrightarrow{S_R}$ refers to the area that the left part of A^- superior to the left part of A^+, and S_O refers to the overlapping part between A^- and A^+. The greater value of $\overrightarrow{S_L}$, $\overleftarrow{S_R}$, $\overleftarrow{S_L}$, $\overrightarrow{S_R}$ corresponds to greater interval between A^- and A^+. When $\overrightarrow{S_L} = \overleftarrow{S_R} = \overleftarrow{S_L} = \overrightarrow{S_R} = 0$, the IVFS $\widetilde{A}^{\pm}$ becomes fuzzy set $\widetilde{A}$ with $R\left(A^-, A^+\right) = 0.5$.

The value of $R\left(A^-, A^+\right)$ ranges between 0 and 1 (Shown in Figure 2). The values of $R\left(A^-, A^+\right)$ means the interval of IVFS $\widetilde{A}^{\pm}$. In case of the value of $R\left(A^-, A^+\right)$ is equal to 0 and 1, the interval of IVFS $\widetilde{A}^{\pm}$ is the biggest, which is shown in Figure 2a and 2b. When the values of $R\left(A^-, A^+\right)$ become closer to 0.5, the interval of IVFS $\widetilde{A}^{\pm}$ becomes the least. The fuzzy preference relations $R\left(A^-, A^+\right)$ can be applied to analyze the interval of pipe flow and nodal pressure, which are expressed as IVFS.

The relationship between A^- and A^+ have several scenarios, which are shown in Figure 2. The figures from Figure 2a to 2j correspond to 10 scenarios between A^- and A^+. In 10 figures, the ranges of $R\left(A^-, A^+\right)$ value are [0, 1], except for Figure 2a and 2b.

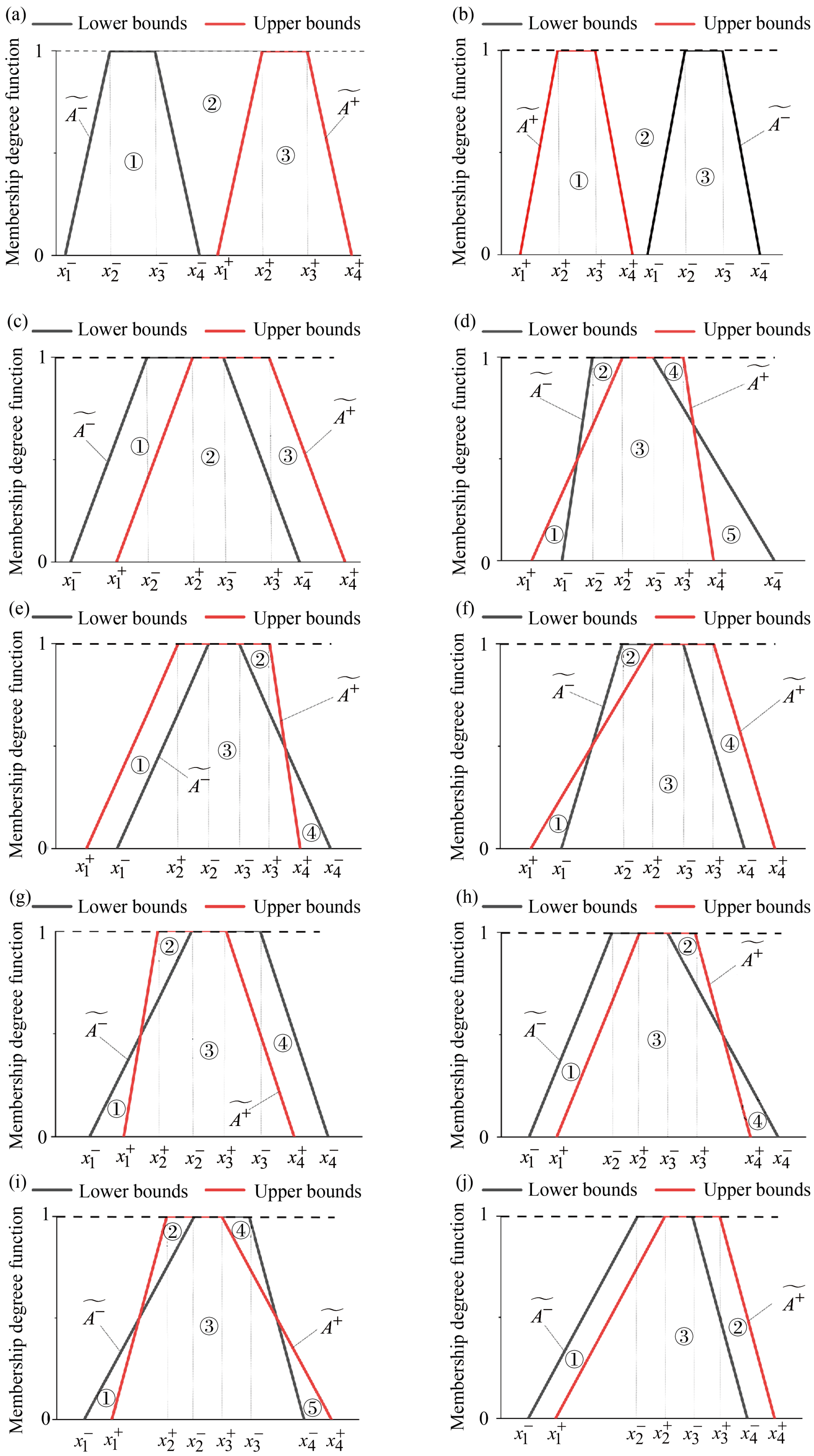


Figure 2. The relationship between A^- and A^+

As such, the variables in Figure 2 have several forms expressed in Table 1.

Table 1. The variables in the relationship between A^- and A^+

Scenario	$\overrightarrow{S_L}$	$\overleftarrow{S_R}$	$\overleftarrow{S_L}$	$\overrightarrow{S_R}$	S_O	$R(A^-, A^+)$
1	① + ②	② + ③	/	/	/	1
2	/	/	① + ②	② + ③	/	0
3	①	③	/	/	②	[0, 1]
4	②	④	①	⑤	③	
5	/	②	①	④	③	
6	②	④	①	/	③	
7	①	/	②	④	③	
8	①	②	/	④	③	
9	①	⑤	②	④	③	
10	①	②	/	/	③	

By applying Eq. (7) and Table 1, the interval of pipe flow and nodal pressure can be analyzed and compared.

2.2 *Hydraulic analysis based on IVFS-GA method*

GA is widely applied to search the best solutions based on selection (reproduction), crossover, and mutation principles. To prevent being trapped into local optimal solutions, GA uses probabilistic search rule instead of deterministic one. The GA process had been described in detail in literatures [11, 17].

The uncertainty of pipe roughness coefficients is described by IVFS (Shown in Section 2.1). By combining the GA mentioned above with IVFS, the membership functions of pipe flows and nodal pressures can be obtained based on EPANET-TOOLKIT-MATLAB platform. The IVFS is introduced to describe pipe flow and nodal pressure under fuzzy and interval uncertainties of pipe roughness coefficients. The roughness coefficients are considered to be fuzzy parameters with trapezoidal distribution, shown in Figure 1. For α-cut levels of 0.0, 0.2, 0.4, 0.6, 0.8, and 1.0, the IVFS of pipe roughness coefficients is $\widetilde{R^{\pm}} = \left[\widetilde{R^-}, \widetilde{R^+}\right] = [(R_1^-, R_2^-, R_3^-, R_4^-), (R_1^+, R_2^+, R_3^+, R_4^+)] = [(110, 118, 122, 130), (115, 123, 127, 135)]$. If $\alpha = 0$, the lower boundary values expressed as R_1^- and R_4^- are assumed to be 110 and 130, respectively, and the upper boundary values expressed as R_1^+ and R_4^+ are assumed to be 115 and 135, respectively. If $\alpha = 1$, the lower boundary values expressed as R_2^- and R_3^- are assumed to be 118 and 122, respectively, and the upper boundary values expressed as R_2^+ and R_3^+ are assumed to be 123 and 127, respectively. As such, for α values of 0.2, 0.4, 0.6, and 0.8, the IVFS of pipe roughness coefficients termed as $\left[\left(\underline{(R^-)^\alpha}, \overline{(R^-)^\alpha}\right), \left(\underline{(R^+)^\alpha}, \overline{(R^+)^\alpha}\right)\right]$ are [(111.6, 128.4), (116.6, 133.4)], [(113.2, 126.8), (118.2, 131.8)], [(114.8, 125.2), (119.8, 130.2)], [(116.4, 123.6), (121.4, 128.6)], respectively. The optimization process by calling GA was performed for lower boundary and upper boundary under each α-cut level of pipe roughness coefficient, respectively. The IVFS of pipe flow and nodal pressure are obtained by performing optimization process expressed as follows.

Objective function:

$$\text{min or max} \quad \left(H_i^{\pm}\right)^{\alpha}; \tag{8}$$

$$\text{min or max} \quad \left(Q_j^{\pm}\right)^{\alpha}. \tag{9}$$

s. t.:

$$\underline{(R^-)}^\alpha \le R \le \overline{(R^-)}^\alpha \text{ for } \widetilde{R^-}; \tag{10}$$

$$\underline{(R^+)}^\alpha \le R \le \overline{(R^+)}^\alpha \text{ for } \widetilde{R^+}; \tag{11}$$

$$\sum (Q_{\text{in}})_i - \sum (Q_{\text{out}})_i = Q_i; \tag{12}$$

$$\sum_l \Delta H = 0. \tag{13}$$

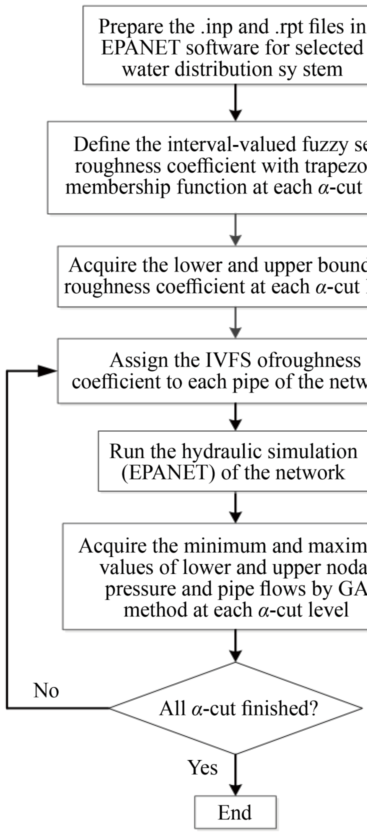


Figure 3. Framework of IVFS-GA method

Where $(H_i^\pm)^\alpha = \left[(H_i^-)^\alpha,\ (H_i^+)^\alpha\right]$ is lower and upper pressures of node i at a certain α-cut level (m), $(Q_j^\pm)^\alpha = \left[(Q_j^-)^\alpha,\ (Q_j^+)^\alpha\right]$ is lower and upper flows of pipe j at a certain α-cut level (L/s or GPM), the minimum and maximum values of $(H_i^-)^\alpha$, $(H_i^+)^\alpha$, $(Q_j^-)^\alpha$, and $(Q_j^+)^\alpha$ are expressed as $\left[\underline{(H_i^-)^\alpha},\ \overline{(H_i^-)^\alpha}\right]$, $\left[\underline{(H_i^+)^\alpha},\ \overline{(H_i^+)^\alpha}\right]$, $\left[\underline{(Q_j^-)^\alpha},\ \overline{(Q_j^-)^\alpha}\right]$, and $\left[\underline{(Q_j^+)^\alpha},\ \overline{(Q_j^+)^\alpha}\right]$, respectively. In Eq. (10) and Eq. (11), $\underline{(R^-)}^\alpha$ and $\overline{(R^-)}^\alpha$ are the α-cuts of lower IVFS pipe roughness coefficient set $\widetilde{R^-}$, $\underline{(R^+)}^\alpha$ and $\overline{(R^+)}^\alpha$ are the α-cuts of upper IVFS pipe oughness coefficient set $\widetilde{R^+}$. In Eq. (12), $(Q_{in})_i$ and $(Q_{out})_i$ are the inflow and outflow of node i, Q_i is the nodal demand at node i. In Eq. (13), $\sum_l \Delta H$ is the algebraic sum of head loss in loop l. The Eqs. (12) and (13) are based on nodal flow continuity and loop pressure loss balance, which can be simulated through EPANET hydraulic engine.

By calling EPANET toolkit functions in MATLAB environment, the hydraulic simulation was performed by EPANET simulation, and the optimization model expressed in Eq. (8)-(13) was performed by GA. By combining the optimization results of the minimum and maximum values for lower and upper boundaries of $(Q_j^-)^\alpha$ and $(Q_j^+)^\alpha$ under each α-cut level, the IVFS of pipe flow with trapezoidal distribution membership function can be obtained. Similarly, by combing the optimization results of the minimum and maximum values for lower and upper boundaries of $(H_i^-)^\alpha$ and $(H_i^+)^\alpha$ under each α-cut level, the IVFS of nodal pressure with trapezoidal distribution membership function can also be obtained. As such, the IVFS of nodal pressure and pipe flow can be plotted and analyzed.

The framework of IVFS-GA method was described in Figure 3. The detailed process is described as follows.

1. Select a network and prepare the .inp and .rpt files of the network with hydraulic information of pipes, nodes, reservoirs, and tanks.

2. Compile the GA code in MATLAB environment to call EPANET toolkit program.

3. Define the values of $\left[\left(\underline{(R^-)^\alpha}, \overline{(R^-)^\alpha}\right), \left(\underline{(R^+)^\alpha}, \overline{(R^+)^\alpha}\right)\right]$ at various α-cut levels for pipe roughness coefficient.

4. Perform GA to acquire the minimum and maximum values of $(Q_j^-)^\alpha$, $(Q_j^+)^\alpha$, $(H_i^-)^\alpha$, and $(H_i^+)^\alpha$ at each α-cut level.

5. Plot the IVFS of pipe flows and nodal pressures with trapezoidal distribution membership function.

6. Analyze the fuzziness and interval of node and pipe in the network.

2.3 *Case study*

Case 1 The network in Case 1 has a source, three loops, 32 nodes, and 34 pipes, which is shown in Figure 4 [22]. Node 1 is a reservoir with a hydraulic pressure of 100 m, and the elevations for all the nodes are assumed to be zero. The detailed nodal demands and pipe information can be found in reference [17].

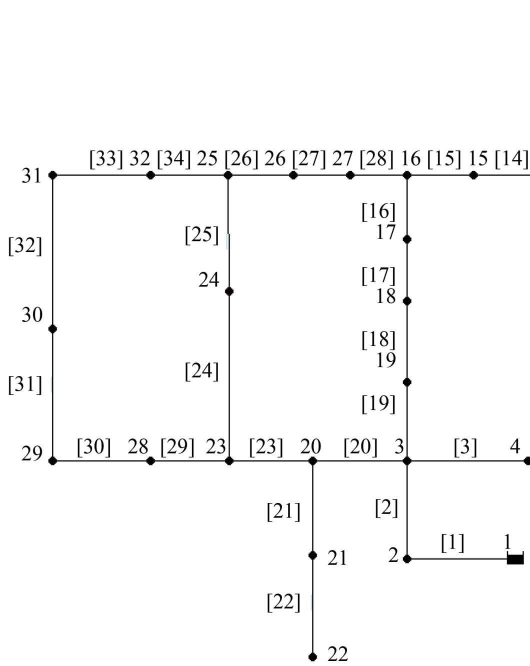


Figure 4. Layout of Case 1

Case 2 The network in Case 2 has three source nodes (node 10, 65, and 165), 19 nodes, and 40 pipes, which is shown in Figure 5. The details can be found in reference [17].

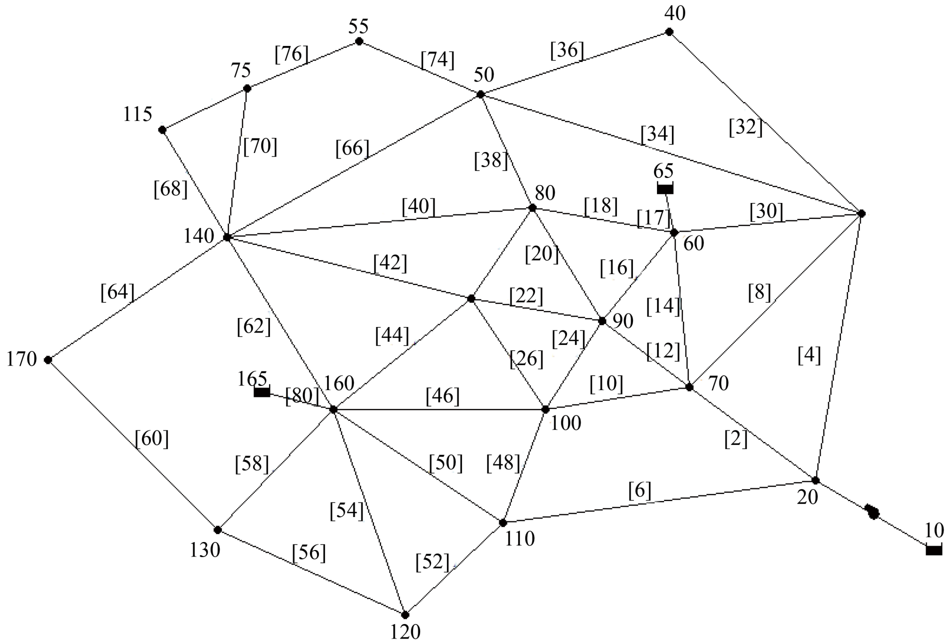


Figure 5. Layout of Case 2

Case 3 The network in Case 3 has one source node, a pump station, 34 nodes, one tank, and 40 pipes (Shown in Figure 6). The details can be found in literatures [23, 24].

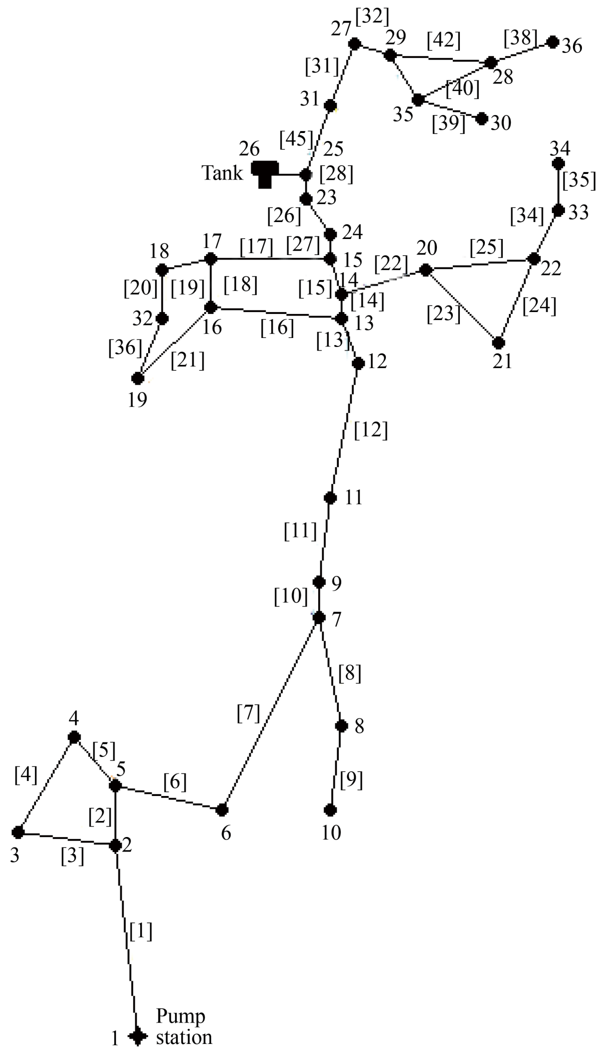


Figure 6. Layout of Case 3

3. Results and discussions

The proposed method was applied to three WDSs of Case 1, 2, and 3. Among them, Case 1 and 2 were WDSs applied to compare the method in this paper with other literatures, and Case 3 is WDSs applied to illustrate the application of the method proposed in this paper. The fuzziness of pipe flows and nodal pressures were measured by DD expressed in Eq. (6), and the interval of pipe flows and nodal pressures were measured by $R(A^-, A^+)$ expressed in Eq. (7). The effects of uncertain roughness coefficient on the pipe flow and nodal pressure were compared.

Case 1 By applying IVFS-GA method to Case 1, the minimum and maximum values of $(Q_j^-)^\alpha$, $(Q_j^+)^\alpha$, $(H_i^-)^\alpha$, and $(H_i^+)^\alpha$ under the IVFS of pipe roughness coefficient $\left[\left(\underline{(R^-)^\alpha}, \overline{(R^-)^\alpha}\right), \left(\underline{(R^+)^\alpha}, \overline{(R^+)^\alpha}\right)\right]$ at various α-cut levels were acquired. The IVFS of pipe flows and nodal pressures with trapezoidal distribution membership function can be obtained and shown in Figure 7.

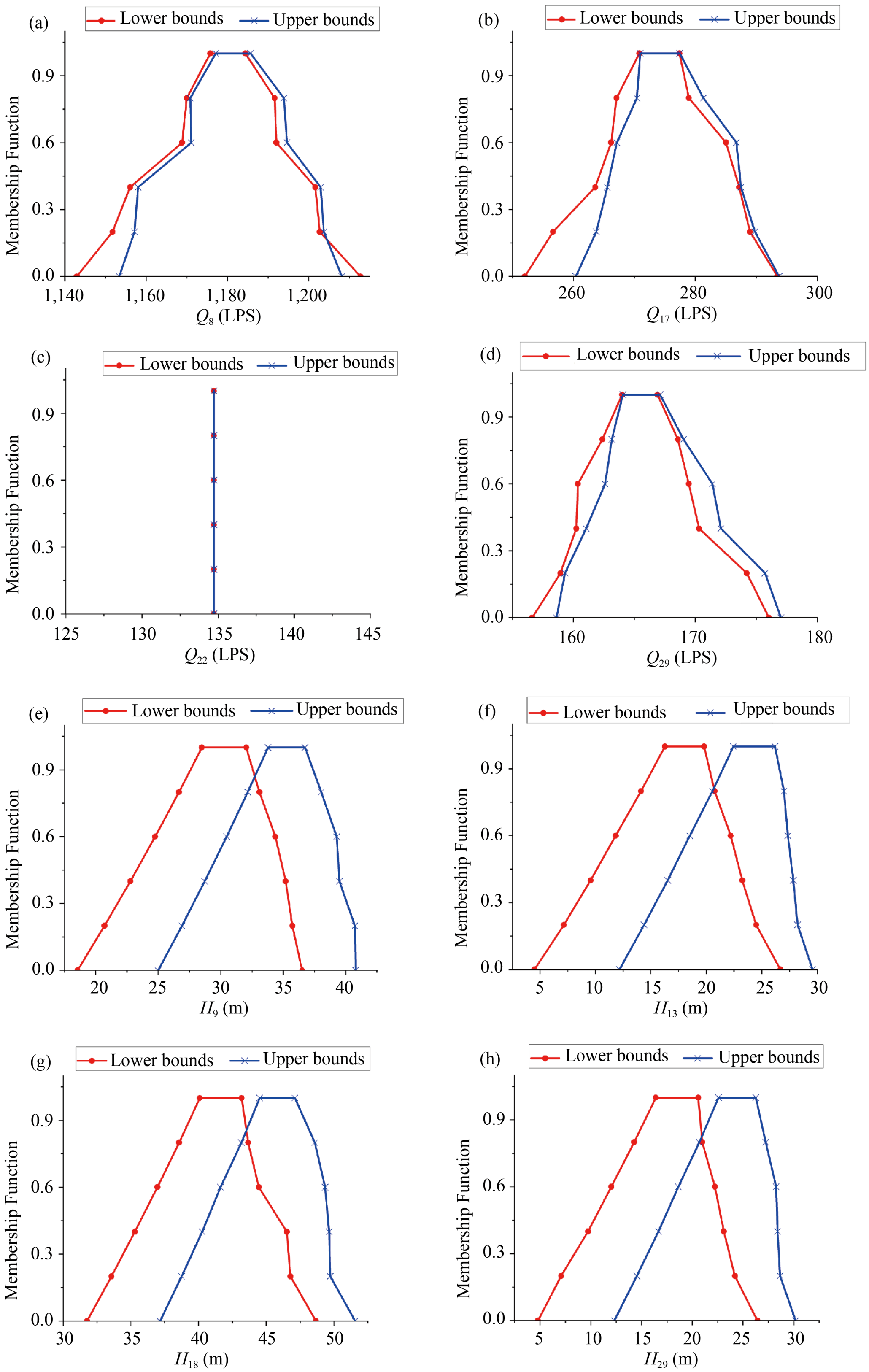


Figure 7. Membership function of IVFS for pipe flow and nodal pressure in Case 1. (a) Flow in pipe 8; (b) Flow in pipe 17; (c) Flow in pipe 22; (d) Flow in pipe 29; (e) Pressure at node 9; (f) Pressure at node 13; (g) Pressure at node 18; (h) Pressure at node 29

The results obtained for flows of pipe 8, pipe 17, pipe 22, and pipe 29 and pressures of node 9, node 13, node 18, and node 29 were similar with the conclusion in the literature [15]. By applying Eq. (6) and Eq. (7) to the membership function of IVFS for pipe flow and nodal pressure in Figure 7, the fuzziness and interval of pipe flow and nodal pressure can be measured (Shown in Figure 8 and Table 2).

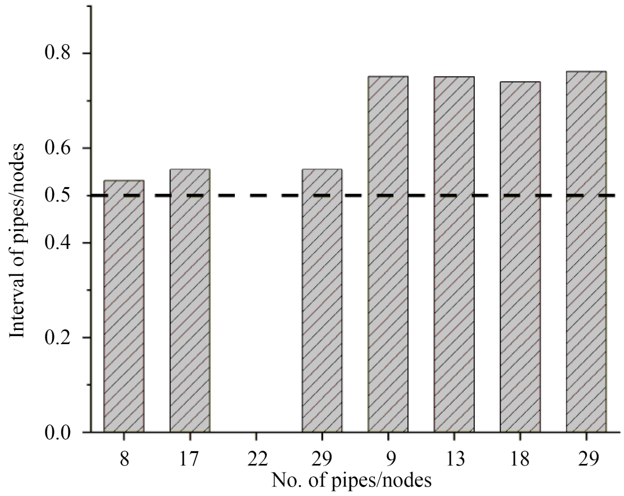


Figure 8. Interval of pipes/nodes in Case 1

Table 2. Fuzziness and interval of pipes/nodes in Case 1

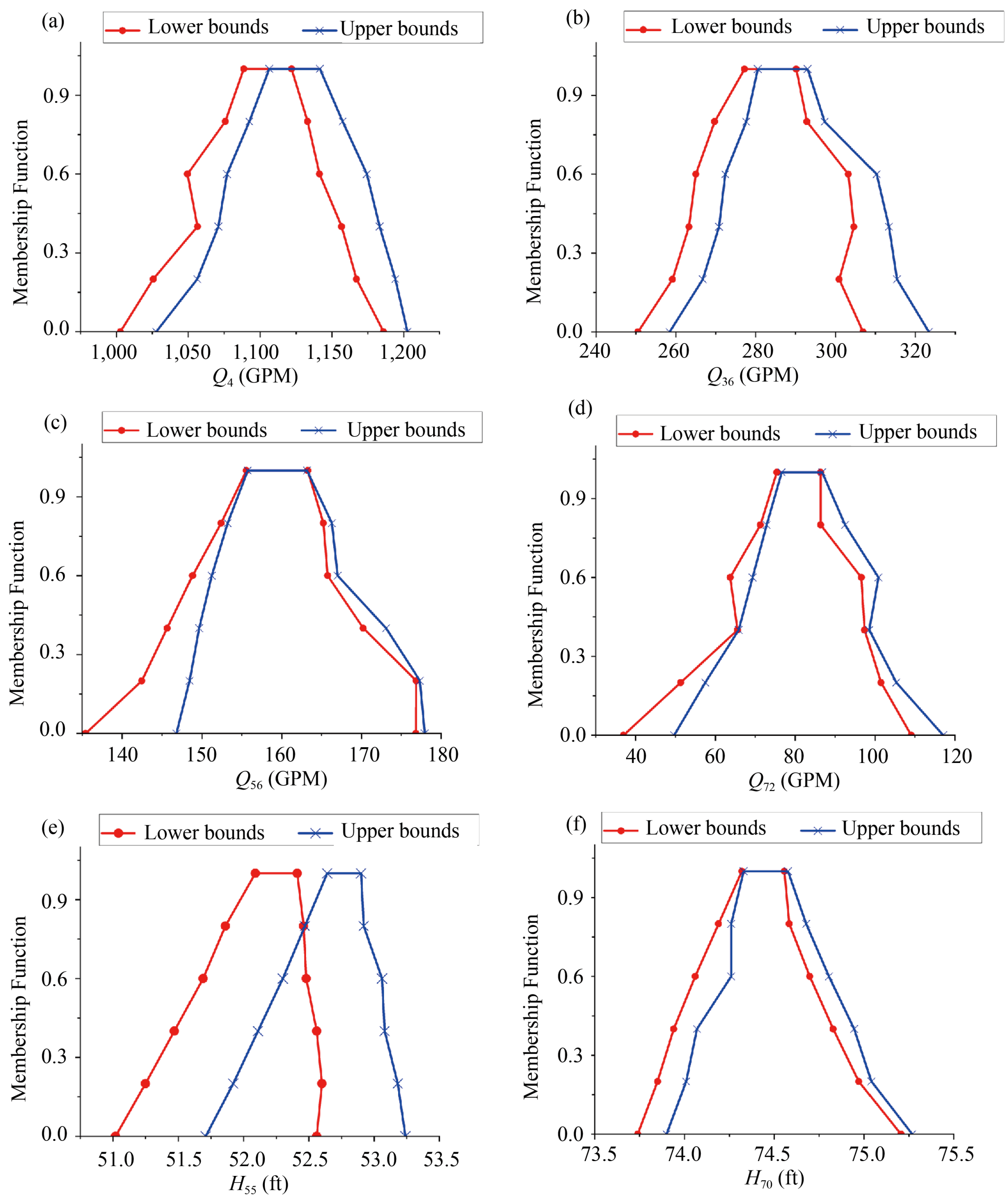
Pipes/Nodes	Fuzziness	Interval	Pipes/Nodes	Fuzziness	Interval
Pipe 8	0.03	0.53	Pipe 17	0.07	0.55
Pipe 22	0.00	0.00	Pipe 29	0.06	0.55
Node 9	0.30	0.75	Node 13	0.54	0.75
Node 18	0.18	0.74	Node 29	0.55	0.76

By applying triangular membership function, the fuzziness of pipe 17 is the most significant among selected pipes of 8, 17, 22, and 29 [15]. However, by applying trapezoidal membership function, the fuzziness of pipe flows measured by dispersion degree $\left(\overline{DD}\right)$ follow the order of pipe 17 (0.07) > pipe 29 (0.06) > pipe 8 (0.03) > pipe 22 (0.00), which is the same with the results obtained in reference [17]. Although the fuzziness obtained between triangular membership function and trapezoidal membership function is not completely the same, they express the same trend. The results indicated that in Case 1, the fuzziness of pipe 17 and pipe 29 are more significant than other selected pipes. The interval of pipes decreased in the order of pipe 17 (0.55) = pipe 29 (0.55) = pipe 8 (0.53) > pipe 22 (0.00). The reason is that the distance between pipe 17 (0.55) and 0.50 is the same with the distance between pipe 8 (0.53) and 0.50, which indicated that the interval of pipe 17 and pipe 8 is the same. The condition is also suitable for pipe 29. The results showed that in Case 1, the interval of pipe 17, pipe 29, and pipe 8 are the same, which indicted the effects of uncertain roughness coefficients on interval of pipe flow are the same. In addition, pipe 22 is not affected by uncertain roughness coefficients including fuzziness as well as interval.

By applying triangular membership function, the fuzziness of node 29 is the most significant among selected nodes of 9, 13, 18, and 29 [15]. By applying trapezoidal membership function, the fuzziness of nodal pressure measured by $\left(\overline{DD}\right)$ follow the order of node 29 (0.55) > node 13 (0.54) > node 9 (0.30) > node 18 (0.18). The results indicated that in Case 1, the fuzziness of node 29 and node 13 are more significant than other selected nodes. The interval of nodal pressure decreased in the order of node 29 (0.76) > node 9 (0.75) = node 13 (0.75) > node 18 (0.74). The results showed that in Case 1, the intervals of node 29, node 9, node 13, and node 18 are almost the same, which also indicated the effects of uncertain roughness coefficients on interval of nodal pressure are almost the same. The results indicated that in Case 1, the effects of uncertain roughness coefficients on fuzziness and interval of pipe flows and nodal pressure are almost the same.

In addition, the effects of uncertain pipe roughness coefficient on nodal pressure are greater than the effects on pipe flows.

Case 2 By applying IVFS-GA method to Case 2, The IVFS of pipe flows and nodal pressures with trapezoidal distribution membership function can be obtained and showed in Figure 9.



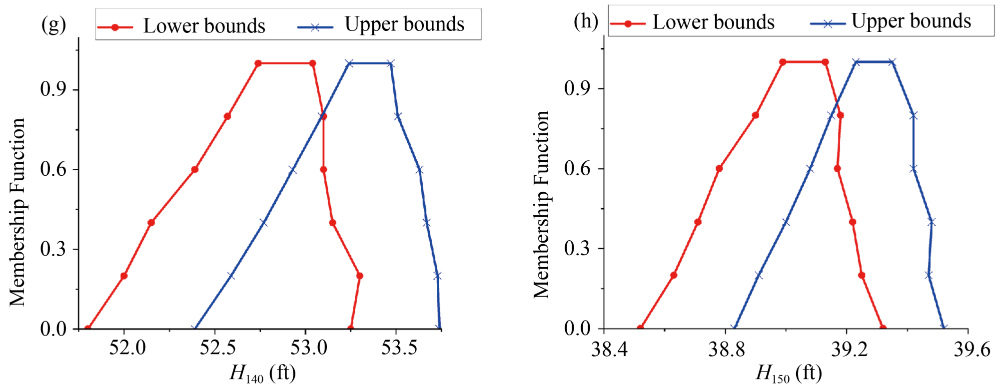


Figure 9. Membership function of IVFS for pipe flow and nodal pressure in Case 2. (a) Flow in pipe 4; (b) Flow in pipe 36; (c) Flow in pipe 56; (d) Flow in pipe 72; (e) Pressure at node 55; (f) Pressure at node 70; (g) Pressure at node 140; (h) Pressure at node 150

The results obtained for flows of pipe 4, pipe 36, pipe 56, and pipe 72 and pressures of node 55, node 70, node 140, and node 150 are similar to the conclusion in the literature [15]. The fuzziness and interval of pipe flow and nodal pressure can be obtained by the same method measured as Case 1 (Shown inFigure 10 and Table 3).

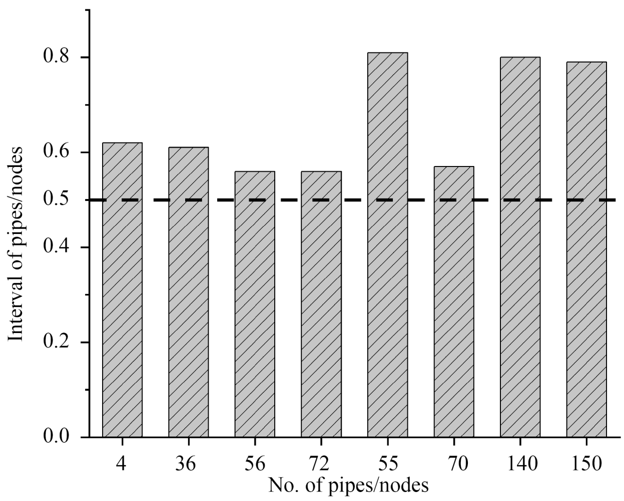


Figure 10. Interval of pipes/nodes in Case 2

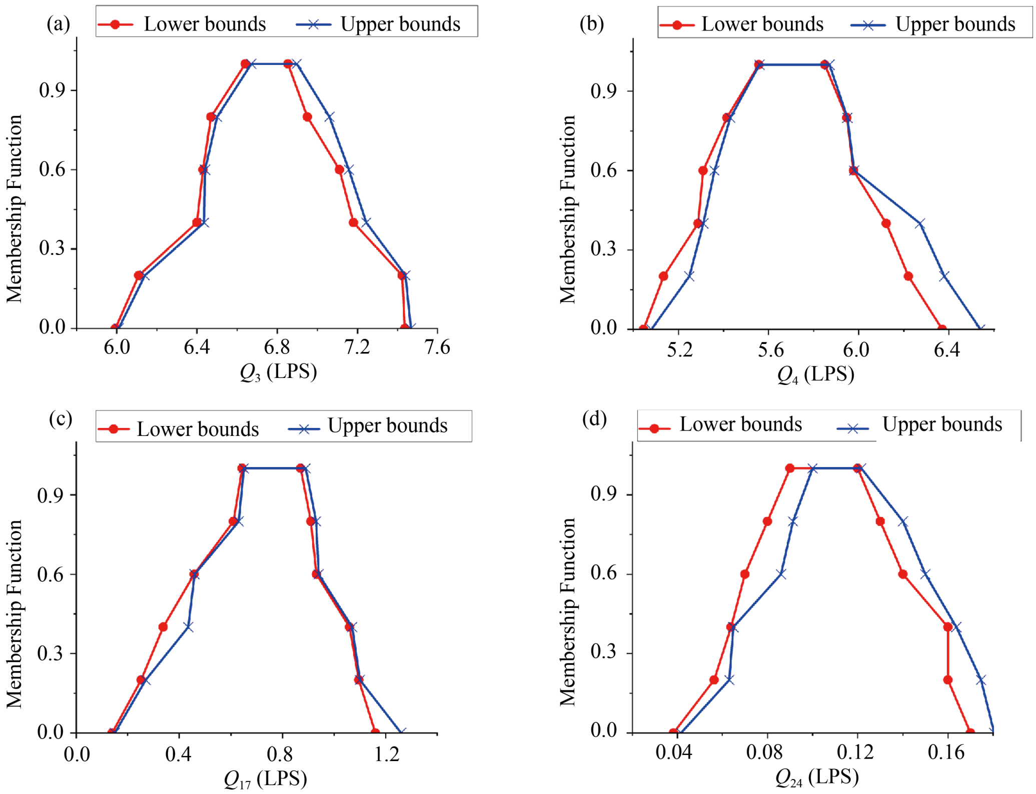
Table 3. Fuzziness and interval of pipes/nodes in Case 2

Pipes/Nodes	Fuzziness	Interval	Pipes/Nodes	Fuzziness	Interval
Pipe 4	0.08	0.62	Pipe 36	0.11	0.61
Pipe 56	0.12	0.56	Pipe 72	0.36	0.56
Node 55	0.02	0.81	Node 70	0.01	0.57
Node 140	0.02	0.80	Node 150	0.01	0.79

By applying trapezoidal membership function, the fuzziness of pipe flows measured by $\overline{DD}$ decreased in the order of pipe 72 (0.36) > pipe 56 (0.12) > pipe 36 (0.11) > pipe 4 (0.08). The results indicated that in Case 2, the fuzziness of pipe 72 is the most significant among selected pipes. The interval of pipes decreased in the order of pipe 4 (0.62) > pipe 36 (0.61) > pipe 56 (0.56) = pipe 72 (0.56). The results showed that in Case 2, the intervals of pipe 4 and pipe 36 are more significant than pipe 56 and pipe 72. The results indicated that different from Case 1, the effects of uncertain roughness coefficients on interval of pipe flow are not the same. The results also indicated that in Case 2, the effects of uncertain roughness coefficients on fuzziness and interval of pipe flows have no direct relationships.

The fuzziness of nodal pressure measured by $\overline{DD}$ decreased in the order of node 55 (0.02) = node 140 (0.02) > node 150 (0.01) = node 70 (0.01). The results indicated that in Case 2, the fuzziness of node 55, node 140, node 150, and node 70 are almost the same. The interval of nodal pressure decreased in the order of node 55 (0.81) > node 140 (0.80) > node 150 (0.79) > node 70 (0.57). The results showed that in Case 2, the intervals of node 55, node 140, and node 150 are almost the same, which are greater than the interval of node 70. The results also indicated that the effects of uncertain roughness coefficients on fuzziness and interval of nodal pressure are the same. Similar with Case 1, generally the effects of uncertain pipe roughness coefficient on nodal pressure are greater than the effects on pipe flows in Case 2.

Case 3 By applying IVFS-GA method to Case 3, The IVFS of pipe flows and nodal pressures with trapezoidal distribution membership function can be obtained. The results obtained for flows of pipe 3, pipe 4, pipe 17, and pipe 24 and pressures of node 5, node 7, node 14, and node 29 are shown in Figure 11. Similar with Case 2, the fuzziness and interval of pipe flow and nodal pressure can be obtained (Shown in Figure 12 and Table 4).



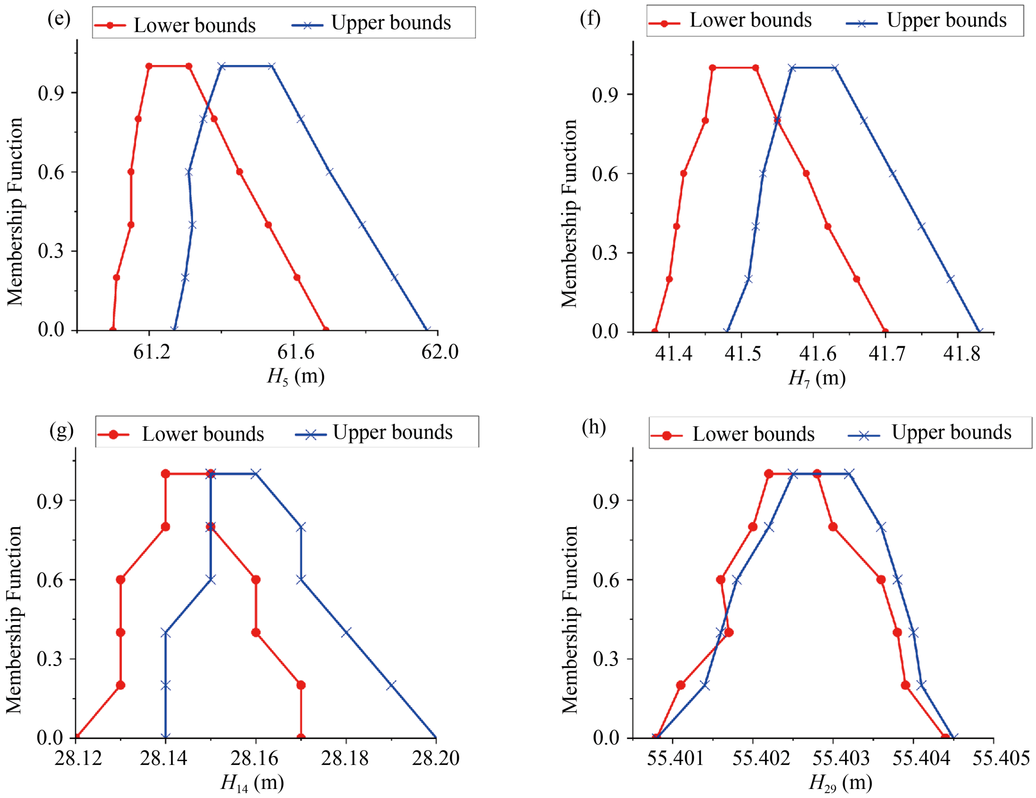


Figure 11. Membership function of IVFS for pipe flow and nodal pressure in Case 3. (a) Flow in pipe 3; (b) Flow in pipe 4; (c) Flow in pipe 17; (d) Flow in pipe 24; (e) Pressure at node 5; (f) Pressure at node 7; (g) Pressure at node 14; (h) Pressure at node 29

By applying trapezoidal membership function, the fuzziness of pipes flow measured by $\overline{DD}$ decreased in the order of pipe 17 (0.75) > pipe 24 (0.65) > pipe 4 (0.12) > pipe 3 (0.11). The results indicated that in Case 3, the fuzziness of pipe 17 is the most significant among selected pipes. The interval of pipes decreased in the order of pipe 24 (0.55) > pipe 4 (0.54) > pipe 3 (0.52) = pipe 17 (0.52). The results showed that in Case 3, the intervals of pipe 24, pipe 4, pipe 3, and pipe 17 are almost the same. The results indicated that similar with Case 1, the effects of uncertain roughness coefficients on interval of pipe flow are almost the same. The results indicated that in Case 3, the fuzziness and interval of IVFS for pipe flows have no direct relationships.

The fuzziness of nodal pressure measured by $\overline{DD}$ decreased in the order of node 5 (0.50) > node 7 (0.40) > node 14 (0.10) > node 29 (0.00). The results indicated that in Case 3, the fuzziness of node 5 is the most significant among the selected nodes. The interval of nodal pressure decreased in the order of node 7 (0.78) > node 5 (0.77) > node 14 (0.75) > node 29 (0.55). The results showed that in Case 3, the intervals of node 7, node 5, and node 14 are almost the same, which are greater than the interval of node 29. The results indicated that in Case 3, the fuzziness and the interval of IVFS for nodal pressure are almost the same. The results also indicated that generally the effects of uncertain pipe roughness coefficient on nodal pressure are greater than the effects on pipe flows in Case 3.

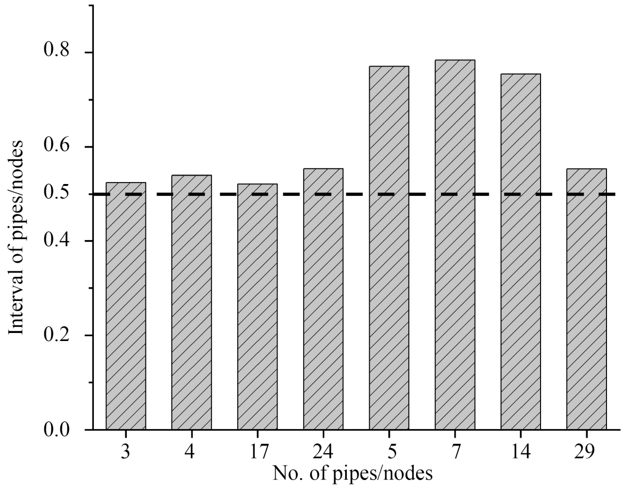


Figure 12. Interval of pipes/nodes in Case 3

Table 4. Interval of pipes/nodes in Case 3

Pipes/Nodes	Fuzziness	Interval	Pipes/Nodes	Fuzziness	Interval
Pipe 3	0.11	0.52	Pipe 4	0.12	0.54
Pipe 17	0.75	0.52	Pipe 24	0.65	0.55
Node 5	0.50	0.77	Node 7	0.40	0.78
Node 14	0.10	0.75	Node 29	0.00	0.55

4. Conclusion

In this paper, the IVFS of pipe roughness coefficient is proposed to analyze the effects of multiple uncertainties of pipe roughness coefficient on pipe flows and nodal pressure. The proposed method was performed based on IVFS-GA algorithm. Based on the interval values of α-cut sets defined for lower and upper pipe roughness coefficients, the minimum and maximum values of pipe flows and nodal pressures were obtained. Accordingly, the IVFS of pipe flows and nodal pressures were obtained, and the fuzziness and interval of pipe flows and nodal pressures were determined and compared among pipes and nodes. The method was applied to three networks. The results indicated that the fuzziness and interval of pipe flows have no direct relationships, while the fuzziness and interval of nodal pressure have close relationship. The results also indicated that the uncertainty of pipe roughness coefficients has more significant effects on interval of nodal pressure than pipe flows, which may due to the fact that the hydraulic relation between nodal pressure and pipe roughness coefficient is closer than the hydraulic relation between pipe flow and pipe roughness coefficient. Moreover, the effects of uncertainty for pipe roughness coefficients on flow among pipes and pressure among nodes should be researched in depth, which may have relationships with flow, nodal demand, topology of pipes and nodes, etc. In this paper, only the effects of fuzziness and interval of pipe roughness coefficients on pipe flows and nodal pressure were analyzed. The effects of the other parameters of hydraulic and water quality in WDS should also be researched in future research. In addition, the effect of uncertain parameters on water quality is an important issue, which have close relationship with human health and should be paid more attention. The results obtained can give managers more message under complex uncertain conditions.

Data availability statement

All data, models, and code generated or used during the study appear in the submitted article.

Acknowledgments

This work was funded by NationalSocial ScienceFund of China (Grant No. 24BJY139). This work was also funded by Research and Demonstration of Key Technologies for Water Quality Safety Assurance in Danjiangkou Reservoir Area and Upstream Watershed (2024YFC3211100). This work is also funded by Key Lab of Functional Polymers for Sustainability of Jiangsu, School of Energy and Environment. Southeast University. We also acknowledge U. S. Environmental Protection Agency for supplying EPANET software freely downloaded on website.

Conflict of interest

The author declares no competing financial interest.

References

[1] Sabzkouhi AM, Haghighi A. Uncertainty analysis of transient flow in water distribution networks. *Water Resources Management*. 2018; 32: 3853-3870.

[2] Hou B, Zhou B, Wu S. Optimal design of water distribution network considering the uncertainty and correlation of nodal demands. *Water Resources Management*. 2025. Available from: https://doi.org/10.21203/rs.3.rs-4544931/v1.

[3] Kang DS, Pasha MFK, Lansey K. Approximate methods for uncertainty analysis of water distribution systems. *Urban Water Journal*. 2009; 6(3): 233-249.

[4] Bargiela A, Hainsworth GD. Pressure and low uncertainty in water systems. *Journal of Water Resources Planning and Management*. 1989; 115(2): 212-229.

[5] Duan H-F, Tung Y-K, Ghidaoui MS. Probabilistic analysis of transient design for water supply systems. *Journal of Water Resources Planning and Management*. 2010; 136(6): 678-687.

[6] Ghelichi Z, Tajik J, Pishvaee MS. A novel robust optimization approach for an integrated municipal water distribution system design under uncertainty: a case study of Mashhad. *Computers & Chemical Engineering*. 2018; 110: 13-34.

[7] Revelli R, Ridolfi L. Fuzzy approach for analysis of pipe networks. *Journal of Hydraulic Engineering*. 2002; 128(1): 93-101.

[8] Abhijith GR, Ostfeld A. Assessing uncertainties in mechanistic modeling of quality fluctuations in drinking water distribution systems. *Journal of Environmental Engineering*. 2024; 150(1): 04023091.

[9] Hwang H, Lansey K, Jung D. Accuracy of first-order second-moment approximation for uncertainty analysis of water distribution systems. *Journal of Water Resources Planning and Management*. 2018; 144(2): 04017087.

[10] Gupta R, Bhave PR. Fuzzy parameters in pipe network analysis. *Civil Engineering and Environmental Systems*. 2007; 24(1): 33-54.

[11] Sabzkouhi AM, Haghighi A. Uncertainty analysis of pipe-network hydraulics using a many-objective particle swarm optimization. *Journal of Hydraulic Engineering*. 2016; 142(9): 04016030.

[12] Salehi S, Fontana ME, Tscheikner-Gratl F, Herrera M, Sadiq R, Mian HR. A fuzzy group decision-making model for water distribution network rehabilitation. *Urban Water Journal*. 2024; 21(3): 364-379.

[13] Yuan L, Zhou Z, He W, Wu X, Degefu DM, Cheng J, Chai L, Ramsey TS. A fuzzy logic approach within the DPSIR framework to address the inherent uncertainty and complexity of water security assessments. *Ecological Indicators*. 2025; 170: 112984.

[14] Garg M, Kumar S. Water distribution system selection with modified fuzzy VIKOR approach based on novel knowledge and accuracy measure. *Evolutionary Intelligence*. 2025; 18(4): 77.

[15] Sivakumar P, Prasad RK, Chandramouli S. Uncertainty analysis of looped water distribution networks using linked EPANET-GA method. *Water Resources Management*. 2016; 30(1): 331-358.

[16] Haghighi A, Keramat A. A fuzzy approach for considering uncertainty in transient analysis of pipe networks. *Journal of Hydroinformatics*. 2012; 14(4): 1024-1035.

[17] Wang Y, Zhu G. Analysis of water distribution system under uncertainty based on genetic algorithm and trapezoid fuzzy membership. *Journal of Pipeline Systems Engineering and Practice*. 2021; 12(4): 04021043.

[18] Spiliotis M, Tsakiris G. Water distribution network analysis under fuzzy demands. *Civil Engineering and Environmental Systems*. 2012; 29(2): 107-122.

[19] Li MW, Li YP, Huang GH. An interval-fuzzy two-stage stochastic programming model for planning carbon dioxide trading under uncertainty. *Energy*. 2011; 36(9): 5677-5689.

[20] Li YP, Huang GH, Nie SL, Huang YF. IFTSIP: interval fuzzy two-stage stochastic mixed-integer linear programming: a case study for environmental management and planning. *Civil Engineering and Environmental Systems*. 2006; 23(2): 73-99.

[21] Lu HW, Huang GH, He L. Development of an interval-valued fuzzy linear-programming method based on infinite α-cuts for water resources management. *Environmental Modelling & Software*. 2010; 25(3): 354-361.

[22] Suribabu CR. Resilience-based optimal design of water distribution network. *Applied Water Science*. 2017; 7(7): 4055-4066.

[23] Boccelli DL, Tryby ME, Uber JG, Rossman LA, Zierolf ML, Polycarpou MM. Optimal scheduling of booster disinfection in water distribution systems. *Journal of Water Resources Planning and Management*. 1998; 124(2): 99-111.

[24] Köker E, Altan-Sakarya AB. Chance constrained optimization of booster chlorination in water distribution networks. *CLEAN-Soil, Air, Water*. 2015; 43(5): 717-723.