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Research Article

Inventory Optimization in E-Commerce: A Dual-Sourcing Collaborative Replenishment Model Under Demand Uncertainty

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Abstract: Purpose: This study aims to develop a cost-efficient inventory optimization model for e-commerce enterprises facing uncertain demand. The objective is to enhance operational responsiveness and service levels while minimizing total inventory-related costs through a dual-sourcing replenishment strategy that combines lean and flexible suppliers. Design/methodology/approach: A dynamic collaborative replenishment model is formulated using optimization theory and stochastic demand modeling. The model integrates dual sourcing and service-level constraints and is solved through nonlinear programming via a bounded objective method. Matrix Laboratory (MATLAB) is used for implementation and simulation, with a real-world case study from a Chinese e-commerce firm to validate the model. Findings: Simulation results demonstrate that the proposed model significantly reduces total inventory costs-by up to 36%-compared to traditional single-source or fixed-cycle replenishment strategies. The model also improves inventory responsiveness and service reliability, effectively balancing cost control and customer service in volatile demand environments. Practical implications: This model provides a practical decision-support tool for inventory managers in e-commerce. By dynamically allocating orders between lean and flexible suppliers based on real-time demand and service level targets, firms can reduce holding and stockout costs while improving overall supply chain performance, particularly in fast-paced, digitally driven markets.

***Keywords*:** demand forecasting, e-commerce enterprises, inventory control, dynamic collaboration, supply chain management

MSC: 90C15, 90C30

1. Introduction

E-commerce, as a new business model, not only expands the scope of human thinking, communication, and behavior but also simplifies the processes of communication and action, thereby improving the effectiveness and efficiency of business activities. Through e-commerce trading platforms, enterprises establish dynamically transmitted capital flows and information flows, ensuring smooth and efficient logistics. Its development model characterized by low cost, high

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efficiency, networking, digitalization, and globalization, fundamentally addresses the problems of information distortion and delays. E-commerce has grown rapidly and significantly transformed global retail and supply chain dynamics. Statista reported that global e-commerce sales surpassed the USD 6.3 trillion mark in 2024 and are expected to surpass USD 8 trillion by 2027, demonstrating the importance of digital commerce for trade [1]. E-commerce businesses benefit from broad connectivity, reduced administrative costs, and international market exposure; however, they also face complex logistics and operational challenges, most notably, inventory management, which has become a critical factor for competitive advantage, operational flexibility, and customer satisfaction [2]. In contrast to traditional retail, e-commerce supply chains must respond swiftly to volatile consumer demand, short product life cycles, and high service level expectations. Effective inventory control and management in this context help reduce costs, enhance response times, and improve overall supply chain visibilitycite [3, 4].

Conversely, poor inventory management can undermine business performance. Overstocking ties up capital and increases warehousing expenses, while understocking results in missed sales opportunities, delivery delays, and customer dissatisfaction. This is particularly damaging in the e-commerce sector, where stock availability is closely related to customer retention and response time. Studies have also shown that the absence of real-time visibility, fragmented supplier communication, and demand variability are among the main causes of stockout and obsolete inventory [5, 6]. Therefore, innovative approaches to inventory synchronization and demand forecasting are essential to drive growth and enhance efficiency in e-commerce.

To address these pressing challenges, recent studies have proposed coordinated inventory policies involving multiple suppliers and flexible sourcing strategies. One such approach-dynamic collaborative replenishment-enables firms to source inventory jointly from both lean and flexible suppliers. Lean suppliers offer economies of scale and lower unit costs but lack flexibility, whereas flexible suppliers provide greater agility and shorter lead times, albeit at a higher cost [7]. Together, these two types of contracts permit firms to hedge demand risk and trade off inventory costs.

Erkan et al. [8] identified factors influencing electronic procurement, studying the structure of electronic procurement cost and management systems, as well as hindering and driving factors. Giannakis and Louis [9] conducted empirical research on e-commerce and physical industries, highlighting the neglect of small-scale economies. Their analysis of the food industry identified weak infrastructure, lack of technical expertise, and traditional industry resistance as barriers to electronic procurement development. They proposed multi-level, multi-angle solutions rather than unilateral promotion by UN agencies. F. Pan and R. Nagi [10] empirically analyzed the integration of electronic procurement supply chains, finding that larger, more robust companies are more inclined to adopt electronic procurement [3]. Xu et al. [11] examined the use of tracking and delivery processes in Small and Medium-sized Enterprises (SMEs) during electronic procurement, discussing factors influencing electronic procurement in e-commerce and integrating a framework.

Wu and Chuang [12] identified nine electronic procurement studies, constructing third-party commissioned vehicle transportation and scheduling models. Sridhar [13] affirmed the supply chain management model, noting its benefits for overall market performance. Similarly, Swaminathan et al. [14] analyzed issues in traditional supply chains, affirmed the advantages of supply chain management, and advocated for further research. Kaya [15] studied the impact of service levels on single-channel and dual-channel supply chains, finding that single-channel service levels are lower than those in dual channels. Cattani et al. [16] analyzed the effects of different pricing strategies on supply chains, identifying optimal pricing strategies that benefit supply chains and support managerial decision-making.

Chu et al. [17] analyzed channel coordination strategies, finding that they influence manufacturers' and retailers' channel choices and benefit the entire supply chain. Li et al. [18] focused on online direct sales channels, discussing the impact of channel-sharing strategies on manufacturers and retailers in supply chains. Zhang et al. [19] explored factors influencing suppliers' decisions to open offline sales channels, showing that information hiding in online agency models and information sharing in resale models have significant effects.

The preceding section highlights the advantages of supply chain management models in the e-commerce context, drawing from real-world scenarios to illustrate how online multi-channel sales strategies impact inventory management across various supply chain participants. This paper aims to address a key research gap by developing a dynamic collaborative replenishment model tailored to e-commerce enterprises operating under stochastic demand. Existing studies predominantly focus on static planning horizons, single-supplier systems, or deterministic demand patterns. However, there is a clear need for robust models that can accommodate both demand uncertainty and supplier heterogeneity. To this end, the paper addresses the following research questions:

RQ1: How can e-commerce enterprises optimize replenishment decisions across multiple supplier types under uncertainty?

RQ2: What impact does service level have on total inventory cost within a dual-supplier strategy?

RQ3: Can a simulation-based optimization model provide better decision support compared to fixed-cycle methods?

To address these questions, we propose a novel inventory optimization model that integrates principles from Collaborative Planning, Forecasting, and Replenishment (CPFR) and Joint Inventory Management (JIM). The model employs a dual-objective framework that simultaneously minimizes the expected total inventory costs while ensuring a predefined service level. Stochastic demand is represented using probability distributions (e.g., normal or Poisson), allowing flexibility to adapt to both high and low demand periods. A bounded objective method is applied to transform the bi-objective formulation into a solvable nonlinear program, which is then solved using Matrix Laboratory (MATLAB). The model's novelty lies in its dynamic allocation mechanism between lean and flexible suppliers, as well as its use of real-time inventory updates to enhance replenishment efficiency.

The remainder of the paper is organized as follows: section 2 presents a literature review on supply chain management in e-commerce, focusing on collaborative inventory strategies. Section 3 details the assumptions, mathematical formulation, and solution procedure of the proposed model. In section 4, a case study of a Chinese e-commerce company is introduced, along with numerical simulations and sensitivity analyses to validate the model. Finally, section 5 summarizes the conclusions and outlines future research directions, including extensions to multi-product and multi-tier supply chains.

2. Literature review

The digitalization of commerce and the globalization of markets have revolutionized how supply chains operate, especially within e-commerce enterprises. Traditional supply chains cannot support today's consumer demands for speed, flexibility, and integration. Therefore, there is a pressing need for the evolution of supply chain and inventory management structures in the e-commerce field. This section presents a literature review on the characteristics of e-commerce supply chains, the inventory management requirements in this new environment, and the state-of-the-art collaborative inventory management approaches that deliver responsiveness and efficiency.

2.1 *Supply chain management characteristics of e-commerce enterprises*

E-commerce has brought major changes to how supply chains operate. Unlike traditional supply chains, which were linear and hierarchical, supply chains in the digital era are more networked, responsive, and technology driven. Four main features define Supply Chain Management (SCM) in e-commerce: networking, visibility, flexibility, and agility.

(1) Networking: With the development and application of the Internet, business communication models between enterprises and individuals or between enterprises have shifted from traditional phone calls to online interactions. Supply chain e-commerce network management technologies have further advanced, transforming supply chain models from vertical, linear structures to interconnected networks [20]. Various network information technologies can be applied not only in manufacturing workshops but also extended to upstream and downstream partners, creating value for related enterprises [21].

(2) Visibility: Enterprises in the supply chain leverage the Internet and e-commerce technologies to enhance transparency, achieve full information sharing, and coordinate operational plans across supply chain links at minimal cost [22]. This enables rapid responses to customer demands, effective monitoring of supply chain processes, and synchronized reactions among related enterprises, improving [23].

(3) Flexibility: Flexibility is a new production model proposed to address the drawbacks of mass production, aiming to maximize enterprise benefits. Flexible supply chain management reforms system structures, personnel organization, operational methods, and marketing, using IT or sensing devices to monitor production and inventory in real time [24]. This allows production systems to adapt quickly to market changes, eliminate redundant waste, issue replenishment signals

based on production progress and inventory status, balance production lines and inventory, and elevate inventory levels [25].

(4) Agility: Agility is based on the core competitive capabilities of e-commerce enterprises, seeking available resources with the fastest response. Cloud computing, a new network application model, features ultra-large scale, virtualization, and security, processing millions or billions of data points in seconds [26]. Lean supply chain management requires optimized resource allocation and automatic aggregation of enterprise resources across the industrial chain, along with data integration and sharing, necessitating massive computational, informational, and storage resources. Only through cloud computing can enterprises enhance supply chain responsiveness, reduce management costs, and enable transactions, operations, and queries without regional or time constraints [27].

2.2 *Characteristics of e-commerce inventory management*

Inventory management in the e-commerce environment fundamentally differs from traditional brick-and-mortar retail due to real-time demand fluctuations, limited storage space, and customers' expectations for fast delivery. Informatization, characterized by horizontal integration, accuracy, and real-time capabilities, plays a crucial role in effective inventory control for e-commerce.

2.2.1 *Informatization, horizontal integration, and networking*

Rapid market changes make Information Technology (IT) applications in supply chain systems crucial for understanding user demands and competition. "Horizontal integration" leverages external resources to respond quickly to market demands, with enterprises focusing only on core products and markets while using modern IT to build collaborative partnerships with suppliers and customers [28]. Facing the globalized, competitive buyer's market, fully autonomous, distributed collaboration replaces pyramid-style management structures, enabling agile manufacturing. Systematic and specialized logistics activities directly determine transaction completion and service levels. Scientific methods are used to establish logistics systems for comprehensive, end-to-end management and coordination [29].

2.2.2 *Accuracy and real-time performance*

E-commerce companies strive to reduce most product inventories to or near zero, saving storage space, lowering management costs, reducing expenses for maintenance, storage, handling, and transportation, minimizing capital tied up in inventory, and avoiding issues like aging, loss, or deterioration [27]. In e-commerce, inventory information is public, and customers demand high accuracy and real-time updates. Minor discrepancies can lead to complaints, damaging corporate image and customer relationships. Thus, high accuracy and real-time inventory information are critical features of e-commerce inventory management [30].

2.3 *Research on e-commerce inventory management strategies*

In the e-commerce environment, various advanced inventory management methods have been proposed to meet supply chain needs. These include multi-echelon inventory management, Vendor-Managed Inventory (VMI), Joint Inventory Management (JIM), and Collaborative Planning, Forecasting, and Replenishment (CPFR).

Vendor-Managed Inventory (VMI): Aims to reduce costs for both suppliers and users, achieving win-win outcomes. Under a mutual agreement, suppliers manage inventory and continuously monitor and adjust the agreement, representing a collaborative policy for ongoing inventory improvement [31].

Collaborative Planning, Forecasting, and Replenishment (CPFR): Focuses on collaboration, planning, forecasting, and replenishment. Upstream and downstream enterprises set common goals, requiring collaborative business strategies, long-term commitments to open communication, and information sharing. Joint plans are made for promotions, product launches, discontinuations, warehouse categorization, and inventory policy changes. Collaborative forecasting reduces inefficiencies, dead stock, and waste, promoting better sales and avoiding resource waste [32–34].

Joint Inventory Management (JIM): A new inventory management model that balances rights and responsibilities between upstream and downstream enterprises, enabling risk-sharing and overcoming traditional inventory management limitations like the bullwhip effect. JIM emphasizes synchronization and consistency across supply chain nodes, with joint participation in inventory planning, ensuring holistic coordination and consistent demand expectations. It fosters mutually beneficial relationships among enterprises in lean supplier alliances, eliminating demand amplification and reflecting new collaborative business relationships [35, 36].

This paper combines two management strategies-CPFR and JIM-using appropriate probability distributions for future demand forecasting and leveraging dynamic collaborative replenishment strategies for lean and flexible supply chains helps e-commerce companies achieve more accurate and lean inventory planning. This cooperation facilitates the design of flexible and effective inventory control systems for rapidly changing digital marketplaces.

3. Dynamic collaborative replenishment optimization model for e-commerce enterprises

3.1 *Principles of dynamic collaborative replenishment for e-commerce inventory control*

In the e-commerce sector, e-commerce firms often select multiple suppliers for sourcing goods to mitigate ordering risks. This study considers a two-tier supply chain comprising an e-commerce firm and two distinct types of suppliers. The e-commerce firm operates on an e-commerce platform, where it posts product information for sale and tracks sales performance. Meanwhile, it faces uncertain market demand. Driven by the principle of maximizing its own economic benefits, the e-commerce firm can replenish its stock from the two suppliers. The synergy between lean and flexible production modes discussed in this paper is from the perspective of the e-commerce firm. It coordinates the resources of the two types of suppliers based on different demand distribution functions while maintaining an appropriate inventory level, thereby meeting market demand at a lower cost.

The two distinct types of suppliers can be understood as two separate firms with lean and flexible production characteristics, respectively, or as two production departments within the same firm that produce the same product but with different production characteristics. The lean supplier is suitable for large-scale production, with a higher production setup cost but a lower variable production cost per unit. In contrast, the flexible supplier is appropriate for flexible small-batch production, with a lower production setup cost but a higher variable production cost per unit. Figure 1 illustrates the dynamic collaborative replenishment structure of the e-commerce firm. Based on Figure 1, it is assumed that there are two types of suppliers: lean suppliers with advantages in large-scale production and flexible suppliers with advantages in flexible production.

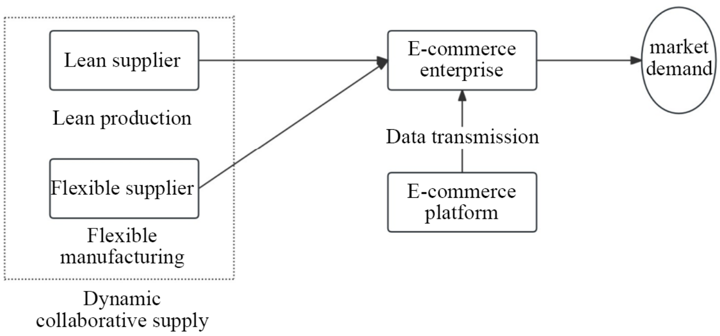

Figure 1. Inventory management replenishment structure diagram of e-commerce enterprises

The entire operation process of the e-commerce firm's supply chain can be briefly described as follows: The entire planning period is divided into several identical cycles, and the e-commerce firm adopts a periodic review policy for replenishment. At the beginning of each cycle, the e-commerce firm checks its inventory level and sales performance on the e-commerce platform and replenishes its stock from both the lean and flexible suppliers to reach the target inventory level. The replenishment strategy is based on minimizing the total replenishment cost under a specified service level, issuing dynamic collaborative replenishment orders to the lean and flexible suppliers, respectively, to coordinate the resource supply of these two types of suppliers. The specific operation process is shown in Figure 2.

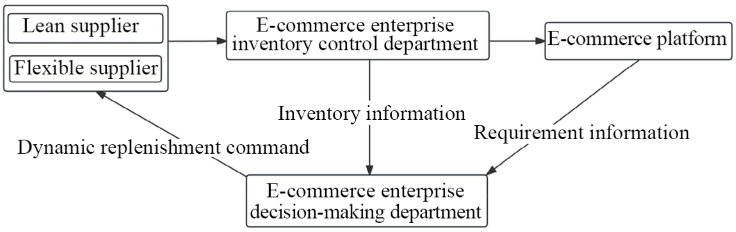

Figure 2. The principle of the supply chain operation process in e-commerce enterprises

3.2 *Parameter definitions and related assumptions*

Assumptions:

(1) Demand Assumption. The demand for a single product of a certain raw material of the e-commerce enterprise is regarded as a random variable. Through the fitting analysis of market demand, it is found to follow a certain probability distribution. For the convenience of model analysis and application, it is assumed that the demand follows a normal distribution.

(2) Replenishment Lead Time Assumption. In the model construction, it is assumed that the replenishment lead times of both the lean supplier and the flexible supplier are non-zero. Moreover, the order lead time of the flexible supplier does not exceed that of the lean supplier.

(3) Inventory and Sales Inspection Assumption. The e-commerce enterprise adopts a periodic inspection method to monitor inventory levels and regularly checks the sales situation on the e-commerce platform. The replenishment strategy is to periodically review and replenish to the target inventory level.

(4) Stockout and Replenishment Assumption. Stockouts are allowed for the e-commerce enterprise, but they incur stockout costs. The stockout products will be replenished with a delay, and if there is excess demand after replenishment, it will be automatically carried over to the next period.

(5) Information Sharing and Delivery Capability Assumption. Information is shared among the e-commerce enterprise, the lean supplier, and the flexible supplier. In each period, the delivery capabilities of all parties are unrestricted, and on-time delivery is guaranteed.

(6) Fixed Ordering Cost Assumption. Each time an order is placed, regardless of the order quantity, both the lean supplier and the flexible supplier must bear a fixed ordering cost.

(7) Delivery Capability Constraint Assumption. The delivery capability of the lean supplier must meet certain constraints and cannot exceed its reserved production capacity. In contrast, the delivery capability of the flexible supplier is unrestricted.

The following symbols are defined:

$$R_i = \begin{cases} \text{Lean supplier} \quad j = 1; \\ \\ \text{Flexible supplier} \quad j = 2. \end{cases}$$

T: Time interval for periodic inventory and sales reviews.

k: Represents the planned quantity for the number of plans for the k-th cycle, assuming the maximum number of plans is K, then $k \in \{1, 2, \cdots, K\}$.

t: Indicates the total planning period duration.

D_t: The sales volume demand of e-commerce enterprises is a random variable. Assuming that it obeys normal distribution and is independent and identically distributed, the mean is μ_t and the variance is σ_t^2, that is $D_t \sim N(\mu_t, \sigma_t)$.

The demand for sales volume of an e-commerce enterprise is considered a random variable, which is assumed to follow a normal distribution. Let μ_t denote the mean and σ_t^2 denote the variance, thus $D_t \sim N(\mu_t, \sigma_t)$.

$f(x)$: The probability distribution function of D_t.

$F(x)$: The cumulative distribution function of D_t.

I_t: The initial inventory of a certain raw material for the e-commerce enterprise at time t.

$I_t{}'$: The ending inventory of a certain raw material for the e-commerce enterprise at time t.

$$\tau_i = \begin{cases} \text{The order lead time of the lean suppliers} \quad j = 1; \\ \\ \text{The order lead time of the flexible suppliers} \quad j = 2. \end{cases}$$

M: Combined ordering lead time for e-commerce companies.

c_i: The fixed cost incurred each time an order is placed with supplier i, with the condition $c_1 > c_2$.

b_i: The unit cost of raw materials ordered each time from supplier i, with the condition $b_1 < b_2$.

H: The inventory holding cost per unit of product required within a unit of time.

C: The stockout cost per unit of product required within a unit of time.

L: Represents the service level, defined as the probability that demand does not exceed the target inventory level during a replenishment cycle, i.e., $P(d_{T+L} \leq U) \geq L$;

$$X_{j,\, t} = \begin{cases} 1 \quad Q_{j,\, t} > 1 \\ & j \in \{1, 2\},\ 1 \leq t \leq N \\ 0 \quad Q_{j,\, t} = 0 \end{cases}$$

Decision variables:

U: Indicates the target inventory level.

$Q_{i,\, t}$: Replenishment quantity of raw material from the supplier $i$in the planning cycle t.

3.3 *Model construction*

The objective function comprises two components: Total procurement costs for raw materials. Service level, defined as the probability of no stockouts.

Under the assumption that sales demand D_t adheres to a normal distribution, the aggregate anticipated demand throughout the planning horizon is deterministic. In alignment with real-world contexts, the possibility of supplier deliveries being backlogged is incorporated. Given that minimizing total costs is tantamount to maximizing the e-commerce enterprise's profit, the primary objective is to reduce total procurement costs.

The service level is defined as the probability that an e-commerce company will not run out of stock. In other words, it refers to the order fulfillment rate for consumers placing orders on the e-commerce platform.

This article limits its range to a fixed interval [60%, 100%]. The service level of an e-commerce enterprise is also the significance level of probability and mathematical statistics α, which can be recorded as L, thus $(1-L)$ representing the probability of out-of-stock occurrence. The significance level αhas a corresponding probability degree Z_α, generally called the critical value. Therefore, the second objective function of e-commerce companies is service level L.

Given the aforementioned assumptions, it is feasible to construct a dynamic collaborative replenishment optimization model tailored for e-commerce enterprises.

$$Min\ f_1 = \sum_{i=1}^{2}\sum_{t=1}^{N}(c_i * X_{i,\,t} + b_i * Q_{i,\,t} + J_t) t \in \{1,\ 2,\ \cdots,\ N\} \tag{1}$$

$$Max\ f_2 = L \tag{2}$$

Constraints:

$$I_t' = I_t + Q_{1,\,t} - D_t t = (k-1) * T + \tau_1 \tag{3}$$

$$I_t{}' = I_t + Q_{2,\,t} - D_t k = (t-1) * T + \tau_2 \tag{4}$$

$$I_t{}' = I_t - D_t t \neq (k-1) * T + \tau_1 t \neq (k-1) * T + \tau_2 \tag{5}$$

$$I_t{}' = I_t t \in \{1,\ 2,\ \cdots,\ N\} \tag{6}$$

$$Q_{1,\,t} + Q_{2,\,t} = U - I_t t \in \{1,\ 2,\ \cdots,\ N\} \tag{7}$$

$$P\left(d_{T+L} \leq U\right) \geq SL \tag{8}$$

$$X_{i,\,t} \in \{0,\ 1\}\, Q_{1,\,t},\ Q_{2,\,t} \in \{0\} \cup N^+ t \in \{1,\ 2,\ \cdots,\ N\} \tag{9}$$

Where:

Equation (1) denotes the total cost incurred by the e-commerce enterprise, which is composed of three components: the fixed cost associated with placing an order for replenishment, the cost of replenishing a specific raw material product, and the combined cost of inventory management and stockout losses for the e-commerce enterprise.

Equation (2) signifies the service level of the e-commerce enterprise, which is defined as the probability of not experiencing a stockout.

Equation (3) illustrates the change in inventory status following the arrival of raw material products ordered from the lean supplier at a particular time.

Equation (4) depicts the change in inventory status after the receipt of raw material products ordered from the flexible supplier at a certain time.

Equation (5) represents the change in inventory status when the raw material products ordered from the supplier fail to arrive at the scheduled time.

Equation (6) indicates that the initial inventory level of the e-commerce enterprise in period t is equivalent to the final inventory level in period $(t-1)$.

Equation (7) indicates that the quantity of raw material products in period t is equal to the combined total of the dynamic collaborative replenishments from both the lean supplier and the flexible supplier.

Equation (8) stipulates that the service level must be no less than L.

Equation (9) specifies that the decision variables are subject to certain bounds.

During period t, J_t in Equation (1) represents the sum of two types of costs: inventory management costs and stockout costs. This can be specifically denoted by Equation (10).

$$J_t = \begin{cases} H * \dfrac{I'_t + I'_{t+1}}{2}, & I'_t > 0,\ I'_{t+1} > 0 \\ C * \left(-\dfrac{I'_t}{2}\right) + H * \dfrac{I'_{t+1}}{2}, & I'_t \le 0,\ I'_{t+1} \ge 0 \\ C * \left(-\dfrac{I'_t}{2}\right) + H * \dfrac{I'_{t+1}}{2}, & I'_t > 0,\ I'_{t+1} < 0 \\ C * \left(-\dfrac{I'_t + I'_{t+1}}{2}\right), & I'_t \le 0,\ I'_{t+1} \le 0 \end{cases} \tag{10}$$

3.4 *Model solution*

To address the multi-objective function system defined by equations (1) and (2), this study employs the bounded objective method. This approach transforms a multi-objective decision-making model into a single-objective model, which simplifies the solution process. The bound objective method prioritizes one primary objective function while incorporating the other objectives as constraints. This strategy not only yields comprehensive insights into the primary objective function but also enables a sensitivity analysis to be conducted.

By imposing a constraint on the second objective function, specifically equation (2), the value is confined within the range of 60% to 100%. This effectively sets the upper and lower limits for the service level.

$$L \in [60\%,\ 100\%]$$

From Section 3.1, we can see that the order lead time of the lean supplier is τ_1, and the order lead time of the flexible supplier is τ_2, and there are restrictions $\tau_1 > \tau_2$. Hence, the overall order lead time for the dynamic collaborative replenishment of the e-commerce enterprise sourced from these two suppliers can be described by the following relationship:

$$\tau_1 \ge M \ge \tau_2 \tag{11}$$

Within a given period, it is evident from the above that $D_t$represents the stochastic demand for raw material products ordered by the e-commerce enterprise. Here, μ denotes the mean, σ signifies the standard deviation, T indicates the regular inventory check period, and L stands for the lead time for joint orders. Based on equation (11), the expression for the target inventory level U can be derived, along with equation (12).

$$U = \mu * (T + M) + Z * \sigma * (T + M)^{\frac{1}{2}} \tag{12}$$

According to Equations (11) and (12), Equation (13) can be obtained.

$$U \in \left[\mu * (T+\tau_2) + Z * \sigma * (T+\tau_2)^{\frac{1}{2}},\ \mu * (T+\tau_1) + Z * \sigma * (T+\tau_1)^{\frac{1}{2}}\right] \tag{13}$$

The initial value of the target inventory level U, denoted as U_1, can generally be represented as: $U_1 = \mu * (T+\tau_2) + Z * \sigma * (T+\tau_2)^{\frac{1}{2}}$.

Assuming the adjustment parameter is denoted as ω, its value range can be articulated as follows:

$$\omega \in \left[0,\ \mu * (\tau_1 - \tau_2) + Z * \sigma * (T+\tau_1)^{\frac{1}{2}} - Z * \sigma * (T+\tau_2)^{\frac{1}{2}}\right] \tag{14}$$

The target inventory level can be calculated by following these steps, as illustrated in Figure 3.

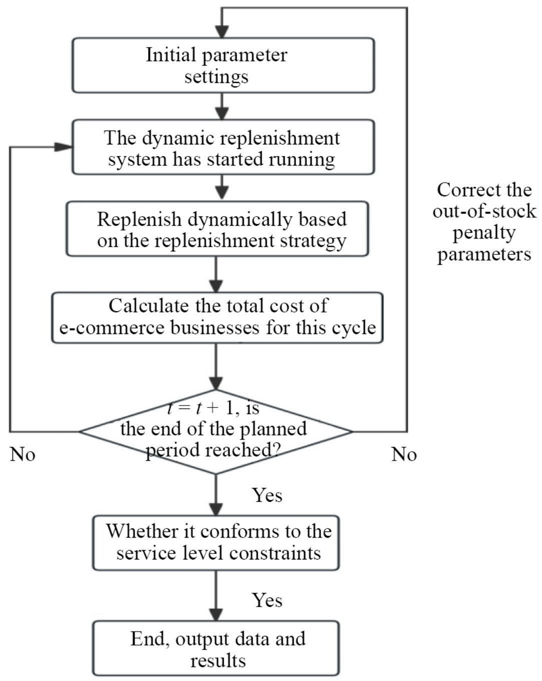

Figure 3. Dynamic collaborative replenishment flow chart for e-commerce enterprises

Step 1: Reasonably set the initial target inventory level U_1 and the stockout penalty parameter δ. If the service level fails to meet the specified constraints within a planning period, then the value of δ should be increased. This adjustment dynamically alters the order ratios from preferred and flexible suppliers to ensure the service level complies with the given constraints. Calculate and record the minimum cost corresponding to U_1 at this point.

Step 2: Reasonably adjust the parameter value ω: $U = U_1 + \omega$. Go to step 1 to calculate U the minimum cost corresponding to this time.

Step 3: The e-commerce enterprise should identify the U value that corresponds to the minimum cost from the set of all considered U values, based on the lowest cost associated with each.

The dynamic collaborative replenishment strategy implemented by e-commerce enterprises through the e-commerce platform can be outlined as follows: At the commencement of each cycle, the e-commerce enterprise ascertains the dynamic collaborative order quantity for raw material products from two suppliers, relying on the current inventory status and a rational forecast of sales for the forthcoming cycle. The objective of this dynamic collaborative replenishment strategy is to optimize and minimize the total cost incurred by the e-commerce enterprise for the next cycle.

$$Min\ \lambda = (e_1 * X_{1,\,t} + e_2 * X_{2,\,t} + b_1 * Q_{1,\,t} + b_2 * Q_{2,\,t}) + J(\tau_2) + J(\tau_1 - \tau_2) + J(T - \tau_1) \tag{15}$$

In the objective function, as given by Equation (15), $J(\tau_2)$ represents the total cost of inventory management and stockout for the e-commerce enterprise from time t to $t + \tau_2$. The function $J(\tau_1 - \tau_2)$ denotes the total cost of inventory management and stockout for the e-commerce enterprise from time $t + \tau_2$ to $t + \tau_1$. Meanwhile, $J(\tau_2)$ also signifies the total cost of inventory management and stockout for the e-commerce enterprise from time $t + \tau_1$ to $t + T$.

The objective function, specifically Equation (15), is a nonlinear programming function. Within each ordering cycle, the order quantities of a particular raw material product placed by the e-commerce enterprise to both the lean and flexible suppliers are fixed, which can be regarded as a resource allocation problem. This nonlinear programming function can be programmed and solved using MATLAB software. Once parameters such as inventory levels, fixed ordering costs, variable production costs, and service levels are defined, the model can calculate the replenishment quantities for each cycle. Upon obtaining the replenishment instructions for each cycle, the relevant values can be substituted into Equation (1) or Equation (15) to compute the total cost incurred by the e-commerce enterprise over the entire planning horizon.

4. Numerical analyses

4.1 *Problem description*

A Chinese e-commerce company based in Yiwu, specializing in small household items, was established in 2003. Over the past decade, it has grown from a small-scale workshop into a mid-sized enterprise with an annual turnover of USD 100 million. The company operates directly managed stores in Qingdao, Nanjing, and Hangzhou, and maintains two small manufacturing facilities in Yiwu and Xuzhou. It handles all processes from product design to marketing independently, characterizing it as a typical enterprise that both manufactures and sells its own products. In the past three years, its average annual sales have reached USD 100 million, positioning it as a leader in the home goods sector.

In terms of marketing, the company runs a flagship online store on Tmall and also engages in wholesale of small commodities through Alibaba, distributing finished products to retailers, which is the primary source of its annual sales revenue.

The main product categories of the company include storage boxes, water cups, and hand warmers. These three categories have accounted for over 90% of total annual sales in the past three years. The rising popularity of rechargeable hand warmers and innovative storage boxes, coupled with the company's focus on quality management, has significantly boosted its market share.

The company's core products are mature items with stable demand. However, as a home goods enterprise heavily reliant on online store sales, its customer base is widely dispersed. Factors such as store reputation and customer reviews greatly influence sales volume, leading to significant fluctuations in product demand.

Due to the lack of a systematic and scientific approach to demand forecasting, and simply relying on past annual sales data provided by the e-commerce platform to calculate averages, the company often faces issues of overstocking or understocking, resulting in substantial annual losses. Therefore, enhancing the accuracy of demand forecasting is essential for effective inventory management.

Taking the sales of water cups as an example, the sales data over the past six weeks are presented in Table 1.

Table 1. Sales report of water cups of e-commerce enterprises in the last six weeks

Week	1	2	3	4	5	6
Sales	701	699	702	700	719	666

From Table 1, it can be observed that the weekly sales volume is approximately 700 units, with an average daily sales volume of about 100 units. Assuming that each water cup requires one unit of raw materials such as glass, and the daily demand follows a normal distribution $D_t \sim N(100,\ 50)$, the annual total planning period for the e-commerce enterprise is set at $t = 360$ days, with a regular cycle inspection length $T = 9$ days. The lead time for orders from the preferred supplier, the first type of supplier, is set at $\tau_1 = 4$ days, and for the flexible supplier, the second type of supplier, it is set at $\tau_2 = 2$ days. The setup costs are set at $e_1 = 900$, $e_2 = 90$, the unit purchase costs are $b_1 = 10$, $b_2 = 13$, the unit holding cost is set at $C = 2$, the unit holding damage cost $H = 0.9$, and the service level is set at 95%. Time is calculated in one-day units. Generate the daily demand sales volume for water cups in MATLAB software. The daily demand should not be less than zero; if a day's generated data is less than zero in the simulation, correct it to the average of 100.

4.2 *Model calculation results*

E-commerce enterprises order raw materials for water cups based on the annual sales figures provided by the e-commerce platform from the previous year. They adopt a monthly ordering method, meaning that a fixed monthly order quantity is determined at the beginning of each year, and an order is placed once a month. Assuming a 30-day month, they need to order $100 \times 30 = 3,000$ units per month in one go. Assuming that these quantities are all purchased from the preferred supplier, each cost can be calculated accordingly.

Fixed fee per order: $900 \times 12 = 10,800$.

Inventory storage fee: $\dfrac{3,000}{2} \times 0.9 \times 360 = 486,000$.

Ordering cost: $3,000 \times 12 \times 10 = 360,000$.

Total cost: $10,800 + 486,000 + 360,000 = 856,800$.

After generating the simulated daily demand data using MATLAB, the dynamic collaborative replenishment model and method proposed in this paper were applied for computation. The simulation data utilized in this study were derived from 100 simulation runs, and the set of simulated market demand values that resulted in a median total replenishment cost-under identical parameter settings-was selected. The initial inventory level was set at 1,200 units. The simulation outcomes are depicted in Figure 4, which presents both the market demand and the dynamic collaborative replenishment curve for the e-commerce enterprise.

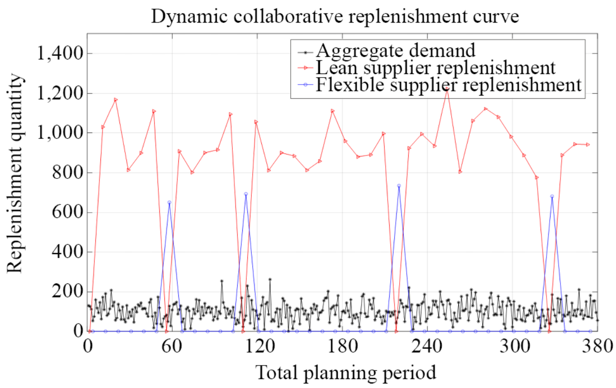

Figure 4. E-commerce enterprise demand and dynamic collaborative replenishment curve

For comparative purposes, the dynamic collaborative replenishment method presented in this paper is juxtaposed against three alternative methods: the monthly replenishment method, the replenishment method utilizing solely the lean supplier, and the replenishment method employing solely the flexible supplier. The total procurement costs associated with these four approaches are detailed in Table 2.

Table 2. Comparison of total replenishment costs of four methods for e-commerce companies

Replenishment method	Total cost	Total cost increase
Collaborative replenishment	629,460	-
Lean supplier replenishment	639,740	10,280
Flexible suppliers replenishment	645,530	16,070
Replenish goods every month	856,800	227,340

As seen in Table 2, the monthly replenishment method results in the highest cost at 856,800, while the dynamic collaborative replenishment method developed in this study incurs the lowest cost at 629,460-a reduction of 227,340, representing a 36% decrease. Among the total costs calculated under the traditional monthly replenishment method, inventory management costs alone account for 323,760, indicating that a large portion of the enterprise's working capital is tied up in inventory. Such high inventory management costs should be of significant concern to the e-commerce enterprise.

As depicted in Table 2, the monthly replenishment method yields the highest cost at 856,800, whereas the dynamic collaborative replenishment method developed in this study incurs the lowest cost at 629,460-a reduction of 227,340, which represents a 36% decrease. Among the total costs calculated under the traditional monthly replenishment method, inventory management costs alone amount to 323,760, indicating that a substantial portion of the enterprise's working capital is immobilized in inventory. Such elevated inventory management costs should be a matter of considerable concern for the e-commerce enterprise.

4.3 *Sensitivity analysis*

In order to fully analyze the collaborative replenishment strategy of e-commerce enterprises, the following sensitivity analysis is conducted on three model parameters: service level L, target inventory level U_1 and variance σ^2. Table 3 provides a comparison of the total collaborative replenishment costs for the e-commerce enterprise under various initial inventory levels. The figures presented in Table 3 are averages derived from 100 simulation runs. As per Table 3, there is a downward trend in the total cost of dynamic collaborative replenishment as the initial inventory level gradually increases from 600 to 1,200 units. This suggests that the initial inventory level selected in the dynamic collaborative replenishment model is appropriate for this e-commerce enterprise. Table 4 illustrates the one-to-one correspondence between critical values and service levels. Each critical value is associated with a particular service level. Often referred to as the safety factor, the critical value can typically be determined using the standard normal distribution table.

Table 3. Analysis of the sensitivity of collaborative replenishment of e-commerce enterprises under different initial values

Initial value	600	700	800	900	1,000	1,100	1,200	1,250
Collaborative replenishment costs	638,520	635,460	633,580	632,540	631,220	629,980	629,460	631,150

Table 4. Correspondence between the service level and the critical value

Service level (%)	96	95	90	80	60
Critical value	1.75	1.65	1.29	0.85	0.26

According to Table 5, as the service level of the e-commerce enterprise decreases from 96% to 80%, the total cost also exhibits a gradual downward trend, indicating that higher service levels are correlated with higher safety stock levels. When the service level further decreases from 80% to 60%, the shortage cost increases significantly, leading to a gradual increase in the total cost. It is generally recommended that the service level should not be lower than 90%. In this study, the dynamic collaborative replenishment model is set to a service level of 95%.

Table 5. Sensitivity analysis of total cost of dynamic collaborative replenishment for e-commerce enterprises under different service levels (Note: The values in parentheses indicate the additional costs incurred compared to collaborative replenishment)

Service level (%)	Collaborative replenishment	Lean supplier replenishment	Flexible suppliers replenishment
96	634,540	644,990 (10,450)	650,990 (16,450)
95	629,460	639,740 (10,280)	665,530 (16,070)
90	601,760	611,880 (10,120)	617,720 (15,960)
80	588,757	599,737 (10,980)	605,407 (16,650)
60	591,880	603,130 (11,250)	609,110 (17,230)

Table 6 shows a comparison of total dynamic collaborative replenishment costs under different variances. Calculate the values for σ equal to 30, 40, 50, 60, 70, and 80, respectively. The values in Table 6 are also the average of 100 calculations. The analysis results indicate that as σ increases from 30, the total cost of e-commerce enterprises shows a gradually increasing trend.

Table 6. Sensitivity analysis of dynamic collaborative replenishment costs of e-commerce companies under different variances

Variance	30	40	50	60	70	80
Collaborative replenishment costs	600,460	60,838	629,460	649,940	664,210	689,690

The smaller of σ is, the smaller the fluctuation in daily demand of e-commerce enterprises, and the daily demand is relatively stable. The larger σ is, the greater the fluctuation in daily demand of e-commerce enterprises, and the difference in daily demand is significant. This aligns with the actual operational context of e-commerce enterprises. During peak sales seasons such as Singles' Day (also known as Double Eleven), the demand value typically surges, leading to substantial and variable daily demand, which in turn causes an increase in total costs during these periods

5. Conclusions and prospects

5.1 *Conclusions*

E-commerce has become a vital driver of global trade, offering enterprises expanded market reach and competitive advantage. However, it also presents new complexities for traditional supply chains and inventory management practices, especially under conditions of uncertain demand. This study has developed a dynamic collaborative replenishment optimization model specifically designed for e-commerce enterprises. The model integrates lean and flexible sourcing strategies within a bi-objective decision-making framework that aims to minimize total inventory costs while ensuring a predefined service level.

By incorporating elements from Collaborative Planning, Forecasting, and Replenishment (CPFR) and Joint Inventory Management (JIM), and by applying probabilistic demand modeling, this approach enables more accurate and agile inventory decisions. A bounded objective method was employed to transform the dual-objective model into a solvable nonlinear programming problem, and the model was implemented using MATLAB. Numerical simulations and a real-world case study demonstrated that the proposed strategy significantly outperforms conventional replenishment methods in terms of cost efficiency and service reliability. Thus, the findings highlight the practical value of dynamic supplier collaboration in e-commerce environments. The model provides a robust decision-support tool for inventory managers seeking to balance cost control with responsiveness, particularly in digitally-enabled and demand-volatile markets.

5.2 *Future research directions*

Although this study provides significant insights into dynamic collaborative replenishment, there are several areas that require further exploration:

(1) The current model is centered on a simplified two-tier supply chain that includes a single e-commerce enterprise, one lean supplier, and one flexible supplier, and it only considers one product. In reality, supply chains often encompass multiple products and span multiple tiers. Future research should expand the model to address multi-product, multi-tier supply chains to more accurately mirror real-world complexities.

(2) The study employs MATLAB programming to solve the bi-objective decision model. However, in more intricate situations involving multiple products and multi-tier supply chains, the use of intelligent optimization algorithms (such as genetic algorithms, particle swarm optimization, etc.) will be necessary to improve computational efficiency and decrease solution times. Further investigation into these algorithms represents a promising avenue for future work.

(3) The research is based on a specific product from a Chinese e-commerce enterprise. Considering the highly variable nature of demand across different industries, future studies could carry out empirical research on products from various sectors to validate and adapt the model for a range of market conditions.

Author contributions

Conceptualization, Xiang Ziquan and Muhammad Hamza Naseem; Funding acquisition, Xiang Ziquan; Methodology, Xiang Ziquan and Muhammad Hamza Naseem; Resources, Xiang Ziquan; Software, Xiang Ziquan and Fatima Sayeeda Ahmad; Validation, Xiang Ziquan and Muhammad Hamza Naseem; Writing-original draft, Xiang Ziquan and Muhammad Hamza Naseem; Writing-review & editing, Muhammad Hamza Naseem and Fatima Sayeeda Ahmad.

Conflict of interest

The authors declare no conflicts of interest.

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