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Research Article

Neutrosophic Binomial Distribution and Algorithm for Managing Uncertainty in Statistical Modeling

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Abstract: This paper presents the neutrosophic binomial distribution, an innovative method for addressing uncertainty in statistical modeling. It explores the properties of neutrosophic random variables and introduces two algorithms that leverage the neutrosophic binomial distribution to generate imprecise data from a conventional binomial distribution. Through extensive simulations with varying parameters, the paper reveals the distinctive characteristics of binomial data generated under uncertainty, in contrast to deterministic settings. The study also highlights practical applications of the proposed neutrosophic binomial distribution. Additionally, it identifies future research directions, including the integration of these algorithms into existing software tools to enhance data generation in uncertain environments.

***Keywords*:** binomial distribution, simulation, data, imprecise data, classical statistics

MSC: 62F99, 60E05, 03E72

1. Introduction

The binomial distribution is utilized in situations where there are two possible outcomes resulting from a random experiment. This distribution assumes that the trials are independent, and the probability of success remains consistent across each trial. It finds numerous applications across various fields. Fischer [1] explored the use of the extended binomial distribution in credit risk models. Chang [2] employed the binomial distribution for testing test questions. Rahman et al. [3] presented their work on the application of an inflated binomial distribution. García-García et al. [4] discussed various applications of the binomial distribution. Short [5] delved into the applications of binomial quantile and proportion bounds in the realm of informatics and engineering. Additional applications can be found in the works of [6, 7].

Smarandache [8] introduced Neutrosophic statistics as a framework applicable for analyzing imprecise data. Serving as an extension of classical statistics, neutrosophic statistics provides a more informative approach, particularly by accommodating indeterminacy in the analysis of imprecise data. Smarandache [9] emphasized the efficacy of neutrosophic statistics in comparison to interval-statistics. Note that both neutrosophic logic and fuzzy systems are employed for handling imprecise data. However, neutrosophic logic is an extension of fuzzy logic. When the indeterminacy in

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neutrosophic logic is set to zero, it simplifies to fuzzy logic. Further details on fuzzy systems can be found in [10–12]. Studies by [13, 14] contributed to the analysis of neutrosophic data, while [15] presented distributions utilizing neutrosophic statistics. Additionally, [16] introduced the negative binomial distribution under indeterminacy. [17] presented neutrosophic statistical distributions along with their applications, and [18] focused on statistical distributions under neutrosophic statistics. AlAita and Aslam [19] worked on the analysis of covariance under neutrosophic statistics. Several works, including those by [20–25], presented algorithms aimed at generating and analyzing neutrosophic data. Collectively, these contributions have significantly expanded the understanding and applications of neutrosophic statistics in dealing with imprecise and indeterminate data. More information can be seen in [26, 27].

The classical binomial distribution under traditional statistics cannot be effectively applied in uncertain environments. To address this issue, the neutrosophic binomial distribution incorporates the degree of indeterminacy. Similar to the traditional binomial distribution, the neutrosophic binomial distribution has numerous applications across various fields where imprecise data can be recorded. The neutrosophic binomial distribution can be particularly useful in practical areas where indeterminacy is prevalent. For instance, in finance and insurance, it can help model uncertain data. In reliability engineering, it can be used to account for uncertain failure rates. In health sciences, it can handle the indeterminacy in patient responses. In environmental sciences, it can be applied to model uncertain weather patterns and climate changes. Additionally, in artificial intelligence and machine learning, the neutrosophic binomial distribution can enhance algorithms to better manage indeterminacy.

While the literature extensively covers applications of the binomial distribution, our exploration reveals a notable gap: the lack of work on algorithms utilizing the neutrosophic binomial distribution. This paper aims to fill this gap by introducing the concept of a binomial distribution based on a newly defined neutrosophic random variable. The main contribution of this paper is to introduce the neutrosophic binomial distribution, detail its properties, develop algorithms for data generation, and conduct an extension study to show the effect of the degree of indeterminacy on data generation from the neutrosophic binomial distribution. We will elucidate the properties of this neutrosophic random variable, outline the construction of the binomial distribution using this variable, and propose two algorithms designed to generate imprecise binomial data incorporating varying degrees of indeterminacy. Additionally, the paper includes a simulation study and comparative analysis, highlighting the impact of indeterminacy on data generation. Through these assessments, we aim to provide insights into the effectiveness and versatility of the proposed algorithms in handling imprecision within the context of the binomial distribution.

2. Neutrosophic random variable

Consider X_L as the lower random variable, also referred to as the determinate random variable, with a mean μ_x and variance σ_x^2. Similarly, suppose Y_L is another lower random variable, known as the determinate random variable, with a mean μ_y and variance σ_y^2. In situations of indeterminacy, the degree of indeterminacy resides in the lower random variable. Consequently, the neutrosophic random variable is defined as $X_N = X_L + X_L I_N$; $I_N \varepsilon [I_L,\ I_U]$; where $I_N \varepsilon [I_L,\ I_U]$ is the degree of indeterminacy, and $Y_N = Y_L + Y_L I_N$; $I_N \varepsilon [I_L,\ I_U]$; where $I_N \varepsilon [I_L,\ I_U]$. In this context, the first part, X_L, represents the random variable under classical statistics, while $X_L I_N$ represents the indeterminate part with I_N denoting indeterminacy. It is noteworthy that the neutrosophic random variable reduces to the random variable under classical statistics when I_L equals zero. Based on this information, the properties of neutrosophic random variables concerning expectations can be explored.

1.

$$E\left(X_L + X_L I_N\right) = E\left(X_L\right) + I_N E\left(X_L\right) = \mu_x + \mu_x I_N \tag{1}$$

2.

$$E\left(X_N+Y_N\right)=E\left(X_N\right)+E\left(Y_N\right)=\left(\mu_x+\mu_x I_N\right)+\left(\mu_y+\mu_y I_N\right) \tag{2}$$

3.

$$E\left(X_N-Y_N\right)=E\left(X_N\right)-E\left(Y_N\right)=\left(\mu_x+\mu_x I_N\right)-\left(\mu_y+\mu_y I_N\right) \tag{3}$$

4. If X_N and Y_N are independent, then,

$$E\left(X_N Y_N\right)=E\left(X_N\right)E\left(Y_N\right)=\left(\mu_x+\mu_x I_N\right)\left(\mu_y+\mu_y I_N\right) \tag{4}$$

5. If a and b are constants, then

$$E\left(a+bX_N\right)=a+bE\left(X_N\right)=a+b\left(\mu_x+\mu_x I_N\right) \tag{5}$$

The properties of neutrosophic random variables based on the variance can be discussed as
1.

$$Var\left(X_L+X_L I_N\right)=Var\left(X_L\right)+I_N^2 Var\left(X_L\right)=\left(1+I_N\right)^2\ \sigma_x^2 \tag{6}$$

2.

$$Var\left(Y_L+Y_L I_N\right)=Var\left(Y_L\right)+I_N^2 Var\left(Y_L\right)=\left(1+I_N\right)^2\ \sigma_y^2 \tag{7}$$

3.

$$Var\left(X_N+Y_N\right)=Var\left(X_N\right)+Var\left(Y_N\right)+2Cov\left(X_N Y_N\right)=\left(1+I_N\right)^2\ \sigma_x^2+\left(1+I_N\right)^2\ \sigma_y^2+\left(E\left(X_N Y_N\right)-\mu_x\mu_y\right) \tag{8}$$

4. If X_N and Y_N are independent, then,

$$Var\left(X_N+Y_N\right)=Var\left(X_N\right)+Var\left(Y_N\right)=\left(1+I_N\right)^2\ \sigma_x^2+\left(1+I_N\right)^2\ \sigma_y^2 \tag{9}$$

5.

$$Var\left(a+bX_N\right)=b^2 Var\left(X_N\right)=b^2\left(1+I_N\right)^2\ \sigma_x^2 \tag{10}$$

Additional information regarding the fundamental operations of neutrosophy is available in the work by [8]. Further insights into the properties of neutrosophic random variables can be found in the study conducted by [17].

3. Neutrosophic binomial distributions

Let X_N be a neutrosophic random variable belonging to the interval $[X_L,\ X_U]$ and following a binomial distribution. The neutrosophic representation of X_N is articulated as $X_N = X_L + X_L I_N;\ I_N \varepsilon [I_L,\ I_U]$, where I_N is a neutrosophic variable within the interval $[I_L,\ I_U]$. It is essential to recognize that the initial component, X_L, signifies the random variable of binomial distribution in classical statistics. The latter part, $X_L I_N$, represents the indeterminate component, and I_N, falling within the interval $[I_L,\ I_U]$, indicates the degree of indeterminacy. Building on this information, the probability function for X_N following a binomial distribution with parameters n and p is defined as follows:

$$f\left(\frac{X_N}{1+I_N}\right) = \left(\begin{array}{c} n \\ \dfrac{X_N}{1+I_N} \end{array}\right) p^{\frac{X_N}{1+I_N}} (1-p)^{n-\frac{X_N}{1+I_N}};\ X_N = 0,\ (1+I_N),\ ..,\ n(1+I_N);\ I_N \in [I_L,\ I_U] \tag{11}$$

We show that it is the probability function as

$$\sum_{X_N=0}^{n(1+I_N)} \left(\begin{array}{c} n \\ \dfrac{X_N}{1+I_N} \end{array}\right) p^{\frac{X_N}{1+I_N}} (1-p)^{n-\frac{X_N}{1+I_N}}$$

$$= \binom{n}{0} p^0 (1-p)^{n-0} + \binom{n}{1} p^1 (1-p)^{n-1} \tag{12}$$

$$+ \binom{n}{2} p^2 (1-p)^{n-2} + \ldots + \binom{n}{n} p^n (1-p)^{n-n} = (p+(1-p))^n = 1$$

The expected value of the neutrosophic random variable X_N is given as by

$$E(X_L + X_L I_N) = E(X_L) + I_N E(X_L) = np(1+I_N);\ I_N \in [I_L,\ I_U] \tag{13}$$

The variance of the neutrosophic random variable X_N is given as by

$$Var(X_L + X_L I_N) = Var(X_L) + I_N^2 Var(X_L) = npq(1+I_N)^2;\ I_N \in [I_L,\ I_U] \tag{14}$$

If I_N is treated as uncertain and lies in the interval, then the mean and variance are given by

$$E(X_N) \in [np(1+I_L),\ np(1+I_U)] \tag{15}$$

$$Var(X_N) \in \left[npq(1+I_L)^2,\ npq(1+I_U)^2\right] \tag{16}$$

It is crucial to emphasize that in classical statistics, the suggested distribution, as well as its expectation and variance, simplifies to the binomial distribution when the lower limit of the neutrosophic variable I, denoted as I_L, equals zero.

4. Algorithm-I

We are about to outline the algorithm for generating imprecise data from the binomial distribution, specifically focusing on scenarios where the parameter n is small. This algorithm is an extension of the one introduced in [28]. The procedure for producing imprecise data from the binomial distribution is delineated as follows:

Step 1: Specify the degree of indeterminacy, denoted as I_N.

Step 2: Set the lower limit, X_L, and compute the neutrosophic random variable, X_N.

Step 3: For each iteration i from 1 to n:

Generate a continuous random variable from the uniform distribution, denoted as u, with a range from 0 to 1.

Step 4: Count the number of instances where u is less than p, where p is the probability threshold derived from u.

Next i

Step 5: Return the computed neutrosophic random variable, X_N.

It is noteworthy that the presented algorithm, referred to as Algorithm-I, is an extension of the algorithm detailed in [28]. Algorithm-I seamlessly reduces to the existing algorithm when the lower limit of indeterminacy, I_L, is set to zero. The procedural steps of Algorithm-I are illustrated in Figure 1.

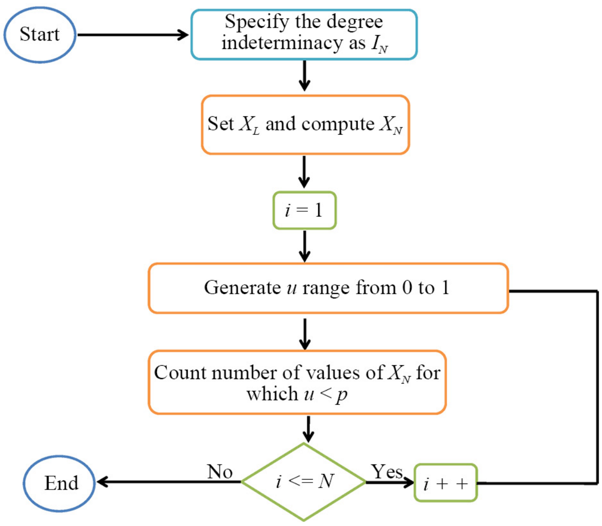

Figure 1. The proposed algorithm-I

5. Algorithm-II

Next, we will formulate Algorithm-II to produce data from the binomial distribution, particularly when n is large and $p \leq 0.5$ with $np > 5$, as outlined in [28]. In such instances, the random variable X_N can be approximated as a neutrosophic normal distribution with a mean of $np(1+I_N)$ and a variance of $npq(1+I_N)^2$. The procedure for generating imprecise data using Algorithm-II from the binomial distribution is presented as follows:

Step 1: Specify the degree of indeterminacy, denoted as I_N.
Step 2: Generate a standard normal variable, z, with a mean of 0 and a variance of 1.
Step 3: For each iteration i from 1 to n:
Step 4: Count the number of values for X_N using the formula

$$X_N = \text{integer}\left[\left((np(1+I_N)) + z\sqrt{npq(1+I_N)^2} + 0.5\right)\right] \tag{17}$$

Step 5: Return the computed neutrosophic random variable, X_N.

It is noteworthy that the presented Algorithm-II serves as an extension of the algorithm outlined in [28]. Algorithm-II seamlessly reverts to the existing algorithm when the lower limit of indeterminacy, I_L, is set to zero. The procedural steps of Algorithm-II are illustrated in Figure 2.

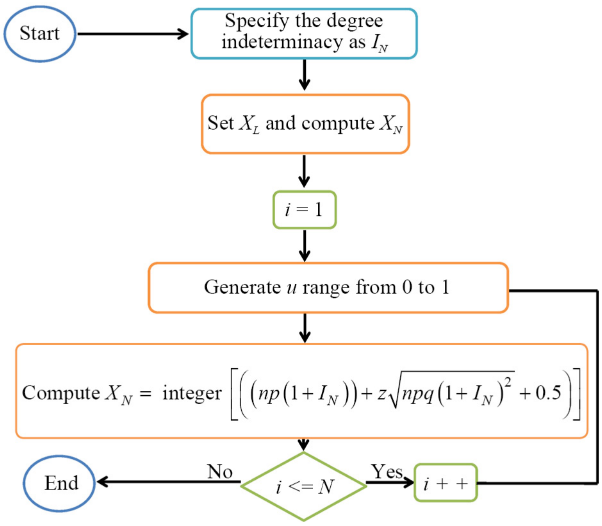

Figure 2. The proposed algorithm-II

6. Simulations studies

This section is dedicated to the simulation studies conducted using both algorithm-I and algorithm-II. Initially, we will delve into the simulation based on algorithm-I, followed by a subsequent discussion on the simulation study based on algorithm-II.

6.1 *Simulation study based on algorithm-I*

Algorithm-I was employed to generate data from the binomial distribution across various combinations of the degree of indeterminacy I_N, sample size (n), and probability (p). The resulting data from algorithm-I has been documented in Tables 1-5. Table 1 displays the binomial distribution for $n = 5$ and $p = 0.30$, while Table 2 presents the distribution for $n = 10$ and $p = 0.30$. Similarly, Table 3 showcases the binomial distribution for $n = 15$ and $p = 0.30$, Table 4 for $n = 20$ and $p = 0.30$, and Table 5 for $n = 20$ and $p = 0.50$. Across Tables 1-5, a consistent trend emerges-an increase in binomial data values as the degree of indeterminacy I_N rises. For instance, in Table 5, with $I_N = 0.1$, the data value is 8, whereas with $I_N = 1$, the data value increases to 14. It is worth noting that, for a fixed value of p and I_N, there is a general upward trend in the data as the sample size increases. Figure 3 visually illustrates the data curves' behavior for $I_N = 0.1$

and $I_N = 0.9$ when $p = 0.30$ and $n = 15$, while Figure 4 demonstrates an increasing trend in data as p rises from 0.30 to 0.50, keeping other parameters constant.

Table 1. Random variates from algorithm-I when $n = 5$ and $p = 0.30$

$I_N = 0$	$I_N = 0.1$	$I_N = 0.2$	$I_N = 0.3$	$I_N = 0.4$	$I_N = 0.5$	$I_N = 0.6$	$I_N = 0.7$	$I_N = 0.8$	$I_N = 0.9$	$I_N = 1$
2	3	3	3	3	3	4	4	4	4	4
1	2	2	2	2	2	2	2	2	2	2
1	2	2	2	2	2	2	2	2	2	2
1	2	2	2	2	2	2	2	2	2	2
1	2	2	2	2	2	2	2	2	2	2
1	2	2	2	2	2	2	2	2	2	2
0	0	0	0	0	0	0	0	0	0	0
1	2	2	2	2	2	2	2	2	2	2
2	3	3	3	3	3	4	4	4	4	4
2	3	3	3	3	3	4	4	4	4	4
1	2	2	2	2	2	2	2	2	2	2
2	3	3	3	3	3	4	4	4	4	4
0	0	0	0	0	0	0	0	0	0	0
2	3	3	3	3	3	4	4	4	4	4
1	2	2	2	2	2	2	2	2	2	2

Table 2. Random variates from algorithm-I when $n = 10$ and $p = 0.30$

$I_N = 0$	$I_N = 0.1$	$I_N = 0.2$	$I_N = 0.3$	$I_N = 0.4$	$I_N = 0.5$	$I_N = 0.6$	$I_N = 0.7$	$I_N = 0.8$	$I_N = 0.9$	$I_N = 1$
2	3	3	3	3	3	4	4	4	4	4
5	6	6	7	7	8	8	9	9	10	10
4	5	5	6	6	6	7	7	8	8	8
5	6	6	7	7	8	8	9	9	10	10
4	5	5	6	6	6	7	7	8	8	8
4	5	5	6	6	6	7	7	8	8	8
1	2	2	2	2	2	2	2	2	2	2
3	4	4	4	5	5	5	6	6	6	6
1	2	2	2	2	2	2	2	2	2	2
0	0	0	0	0	0	0	0	0	0	0
4	5	5	6	6	6	7	7	8	8	8
2	3	3	3	3	3	4	4	4	4	4
5	6	6	7	7	8	8	9	9	10	10
2	3	3	3	3	3	4	4	4	4	4
1	2	2	2	2	2	2	2	2	2	2

Table 3. Random variates from algorithm-I when $n = 15$ and $p = 0.30$

$I_N = 0$	$I_N = 0.1$	$I_N = 0.2$	$I_N = 0.3$	$I_N = 0.4$	$I_N = 0.5$	$I_N = 0.6$	$I_N = 0.7$	$I_N = 0.8$	$I_N = 0.9$	$I_N = 1$
4	5	5	6	6	6	7	7	8	8	8
7	8	9	10	10	11	12	12	13	14	14
3	4	4	4	5	5	5	6	6	6	6
7	8	9	10	10	11	12	12	13	14	14
7	8	9	10	10	11	12	12	13	14	14
3	4	4	4	5	5	5	6	6	6	6
3	4	4	4	5	5	5	6	6	6	6
6	7	8	8	9	9	10	11	11	12	12
5	6	6	7	7	8	8	9	9	10	10
6	7	8	8	9	9	10	11	11	12	12
6	7	8	8	9	9	10	11	11	12	12
6	7	8	8	9	9	10	11	11	12	12
5	6	6	7	7	8	8	9	9	10	10
6	7	8	8	9	9	10	11	11	12	12
3	4	4	4	5	5	5	6	6	6	6

Table 4. Random variates from algorithm-I when $n = 20$ and $p = 0.30$

$I_N = 0$	$I_N = 0.1$	$I_N = 0.2$	$I_N = 0.3$	$I_N = 0.4$	$I_N = 0.5$	$I_N = 0.6$	$I_N = 0.7$	$I_N = 0.8$	$I_N = 0.9$	$I_N = 1$
4	5	5	6	6	6	7	7	8	8	8
6	7	8	8	9	9	10	11	11	12	12
3	4	4	4	5	5	5	6	6	6	6
3	4	4	4	5	5	5	6	6	6	6
6	7	8	8	9	9	10	11	11	12	12
3	4	4	4	5	5	5	6	6	6	6
4	5	5	6	6	6	7	7	8	8	8
6	7	8	8	9	9	10	11	11	12	12
4	5	5	6	6	6	7	7	8	8	8
5	6	6	7	7	8	8	9	9	10	10
5	6	6	7	7	8	8	9	9	10	10
4	5	5	6	6	6	7	7	8	8	8
4	5	5	6	6	6	7	7	8	8	8
4	5	5	6	6	6	7	7	8	8	8
4	5	5	6	6	6	7	7	8	8	8

Table 5. Random variates from algorithm-I when $n = 20$ and $p = 0.50$

$I_N = 0$	$I_N = 0.1$	$I_N = 0.2$	$I_N = 0.3$	$I_N = 0.4$	$I_N = 0.5$	$I_N = 0.6$	$I_N = 0.7$	$I_N = 0.8$	$I_N = 0.9$	$I_N = 1$
7	8	9	10	10	11	12	12	13	14	14
8	9	10	11	12	12	13	14	15	16	16
10	11	12	13	14	15	16	17	18	19	20
5	6	6	7	7	8	8	9	9	10	10
6	7	8	8	9	9	10	11	11	12	12
7	8	9	10	10	11	12	12	13	14	14
7	8	9	10	10	11	12	12	13	14	14
7	8	9	10	10	11	12	12	13	14	14
9	10	11	12	13	14	15	16	17	18	18
8	9	10	11	12	12	13	14	15	16	16
10	11	12	13	14	15	16	17	18	19	20
8	9	10	11	12	12	13	14	15	16	16
8	9	10	11	12	12	13	14	15	16	16
7	8	9	10	10	11	12	12	13	14	14
9	10	11	12	13	14	15	16	17	18	18

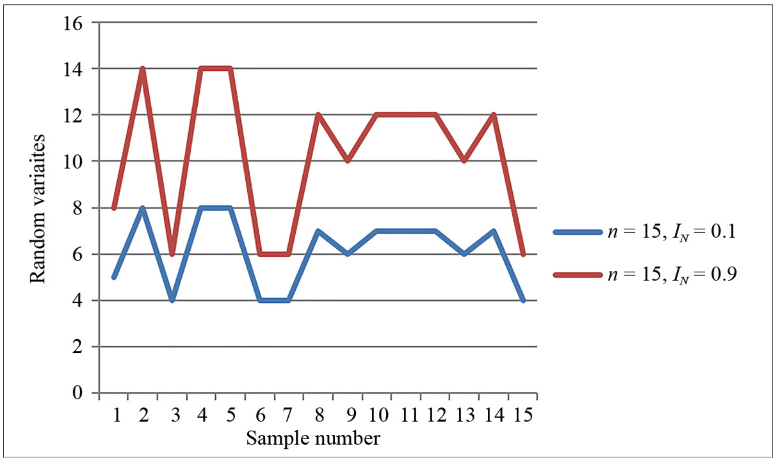

Figure 3. Curves of data when $n = 15$ and $p = 0.30$

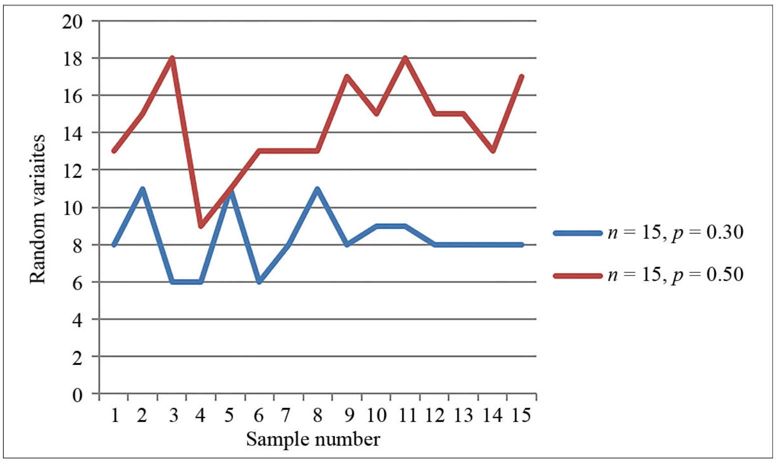

Figure 4. Curves of data when $I_N = 0.8$ and $n = 20$

6.2 *Simulation study based on algorithm-II*

Utilizing algorithm-I, data from the binomial distribution was generated across diverse combinations of the degree of indeterminacy I_N, sample size (n), and probability (p). The outcomes produced by algorithm-I have been documented in Tables 6-9. Table 6 illustrates the binomial distribution for $n = 50$ and $p = 0.40$, while Table 7 depicts the distribution for $n = 50$ and $p = 0.50$. Likewise, Table 8 displays the binomial distribution for $n = 100$ and $p = 0.40$, and Table 9 presents the distribution for $n = 100$ and $p = 0.50$. Across Tables 6-9, a consistent pattern emerges, indicating an increase in binomial data values with rising values of I_N. For instance, in Table 6, with $I_N = 0.1$, the data value is 49, and with $I_N = 1$, the data value rises to 89. The impact of I_N on the generation of binomial data is visually represented in Figure 5. Furthermore, for a fixed value of p and I_N, there is a general upward trend in the data as the sample size increases. Figure 6 illustrates that when $n = 100$ and the value of p increases, there is a concurrent upward trend in the binomial data. Note that we used Excel for the simulation study, and the spreadsheet is available from the author upon reasonable request.

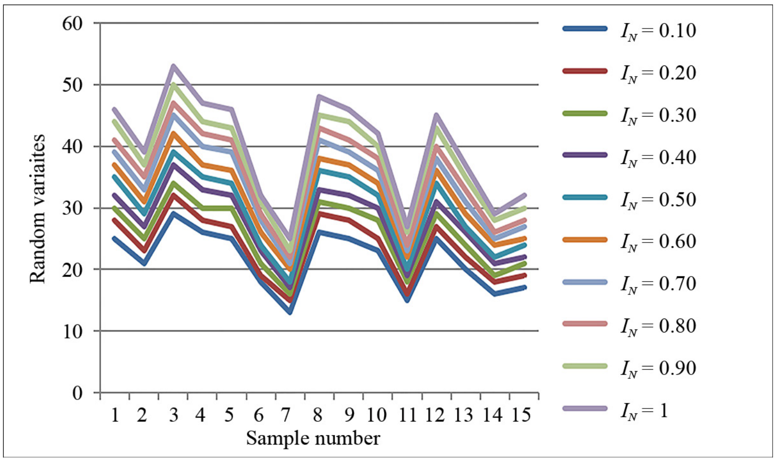

Figure 5. Curves of data when $n = 50$ and $p = 0.40$

Table 6. Random variates from algorithm-II when $n = 50$ and $p = 0.40$

$I_N = 0$	$I_N = 0.1$	$I_N = 0.2$	$I_N = 0.3$	$I_N = 0.4$	$I_N = 0.5$	$I_N = 0.6$	$I_N = 0.7$	$I_N = 0.8$	$I_N = 0.9$	$I_N = 1$
23	25	28	30	32	35	37	39	41	44	46
20	21	23	25	27	29	31	33	35	37	39
26	29	32	34	37	39	42	45	47	50	53
23	26	28	30	33	35	37	40	42	44	47
23	25	27	30	32	34	36	39	41	43	46
16	18	19	21	23	24	26	28	29	31	32
12	13	15	16	17	18	20	21	22	23	25
24	26	29	31	33	36	38	41	43	45	48
23	25	28	30	32	35	37	39	41	44	46
21	23	25	28	30	32	34	36	38	40	42
13	15	16	18	19	20	22	23	24	26	27
22	25	27	29	31	34	36	38	40	43	45
18	20	22	24	26	27	29	31	33	35	37
15	16	18	19	21	22	24	25	26	28	29
16	17	19	21	22	24	25	27	28	30	32

Table 7. Random variates from algorithm-II when $n = 50$ and $p = 0.50$

$I_N = 0$	$I_N = 0.1$	$I_N = 0.2$	$I_N = 0.3$	$I_N = 0.4$	$I_N = 0.5$	$I_N = 0.6$	$I_N = 0.7$	$I_N = 0.8$	$I_N = 0.9$	$I_N = 1$
28	31	34	36	39	42	45	48	51	53	56
24	27	29	32	34	37	39	42	44	47	49
31	35	38	41	44	47	50	53	56	60	63
28	31	34	37	40	43	46	48	51	54	57
28	31	33	36	39	42	45	47	50	53	56
21	23	25	27	30	32	34	36	38	40	42
17	19	21	22	24	26	27	29	31	33	34
29	32	35	38	41	43	46	49	52	55	58
28	31	34	36	39	42	45	48	51	53	56
26	29	31	34	37	39	42	45	47	50	52
18	20	22	24	26	28	29	31	33	35	37
27	30	33	36	38	41	44	47	49	52	55
23	26	28	30	33	35	37	40	42	44	46
20	22	24	25	27	29	31	33	35	37	39
21	23	25	27	29	31	33	35	37	39	41

Table 8. Random variates from algorithm-II when $n = 100$ and $p = 0.40$

$I_N = 0$	$I_N = 0.1$	$I_N = 0.2$	$I_N = 0.3$	$I_N = 0.4$	$I_N = 0.5$	$I_N = 0.6$	$I_N = 0.7$	$I_N = 0.8$	$I_N = 0.9$	$I_N = 1$
44	49	53	58	62	66	71	75	80	84	89
39	43	47	51	55	59	63	67	71	75	79
49	54	59	63	68	73	78	83	88	93	98
45	49	54	58	63	67	72	76	81	85	90
44	48	53	57	62	66	70	75	79	84	88
35	38	42	45	48	52	55	59	62	66	69
29	32	35	38	41	44	47	49	52	55	58
46	50	55	59	64	68	73	77	82	86	91
44	49	53	58	62	66	71	75	80	84	89
42	46	50	54	58	63	67	71	75	79	83
31	34	37	40	43	46	49	52	55	58	62
43	48	52	56	61	65	69	74	78	82	87
38	41	45	49	53	56	60	64	68	71	75
33	36	39	42	46	49	52	55	59	62	65
34	37	41	44	48	51	54	58	61	65	68

Table 9. Random variates from algorithm-II when $n = 100$ and $p = 0.50$

$I_N = 0$	$I_N = 0.1$	$I_N = 0.2$	$I_N = 0.3$	$I_N = 0.4$	$I_N = 0.5$	$I_N = 0.6$	$I_N = 0.7$	$I_N = 0.8$	$I_N = 0.9$	$I_N = 1$
54	60	65	71	76	82	87	92	98	103	109
49	54	59	64	69	74	79	84	89	94	99
59	65	71	77	83	89	94	100	106	112	118
55	60	66	71	77	82	88	93	99	104	110
54	59	65	70	76	81	86	92	97	103	108
45	49	53	58	62	67	71	76	80	85	89
39	43	47	50	54	58	62	66	70	74	78
56	61	67	72	78	83	89	95	100	106	111
54	60	65	71	76	82	87	92	98	103	109
52	57	62	67	72	78	83	88	93	98	103
41	45	49	53	57	61	65	69	73	77	81
53	59	64	70	75	80	86	91	96	102	107
48	52	57	62	67	71	76	81	86	90	95
42	47	51	55	59	64	68	72	76	80	85
44	48	53	57	61	66	70	75	79	83	88

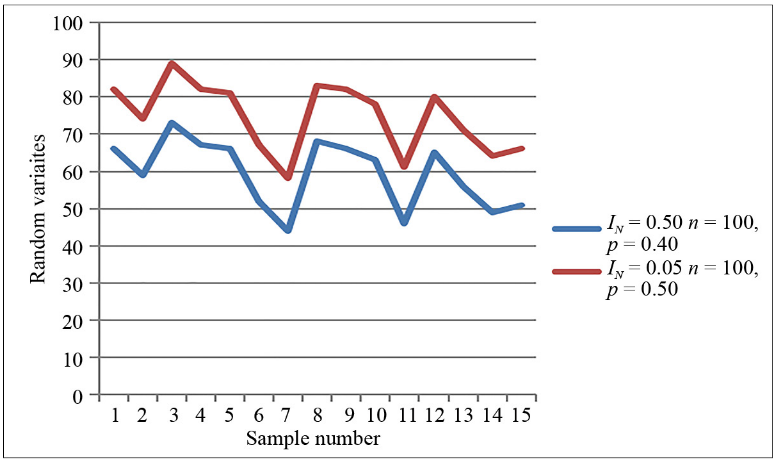

Figure 6. Curves of data when p varies

7. Comparative studies

In this section, we will compare the binomial data generated from the existing algorithm as mentioned in [28]. Note that in our study, the random variable X_L denotes the classical statistics. In our proposed algorithms, when $I_L = 0$, these are reduced to the algorithms under classical statistics as mentioned in [28]. We will first compare the results from algorithm-I and then we will compare the results from the second algorithm.

7.1 *Comparison based on algorithm-I*

As previously mentioned, the algorithm-I proposed here is an extension of the existing algorithm within the framework of classical statistics. Binomial data is derived from the current algorithm and presented in the initial column of Tables 1-5. Upon inspecting Tables 1-5, a distinction is evident between the binomial data generated by the existing algorithm under classical statistics and the data generated under various levels of indeterminacy. Notably, a discernible upward trend in the data is observed as I_N progresses from 0 to 1. For instance, when I_L is set at 0, the data in Table 5 registers as 7, while with I_L at 0.2, the corresponding data in Table 5 increases to 9. The augmentation in the values of I_U correlates with an escalating trend in the data. Figure 7, based on the information in Table 5, provides a comparative analysis of the data derived from the proposed algorithm-I and the existing algorithm under classical statistics. From Figure 7, it is apparent that the existing algorithm yields smaller data compared to the proposed algorithm-I. The lower curve, representing classical statistics data, is contrasted with two other curves denoting data when I_U is set at 0.5 and I_U at 1. The analysis and Figure 7 clearly show an upward trend in the variates as the degree of indeterminacy increases, emphasizing its impact on the data generated from the neutrosophic binomial distribution.

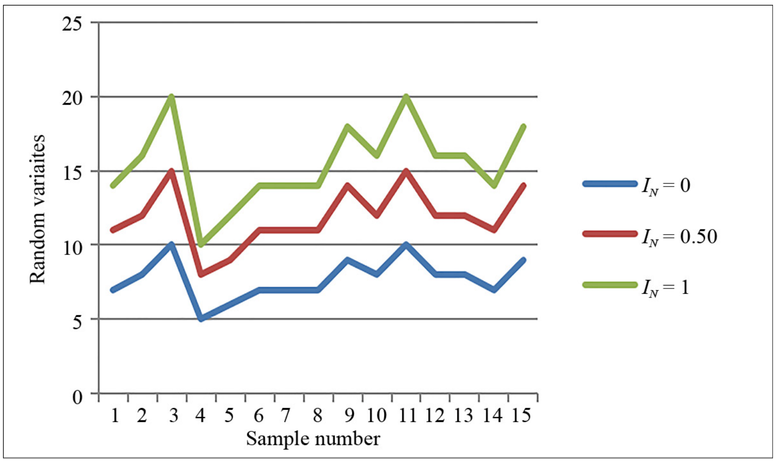

Figure 7. Comparison between the existing and the proposed algorithm-I

7.2 *Comparison based on algorithm-II*

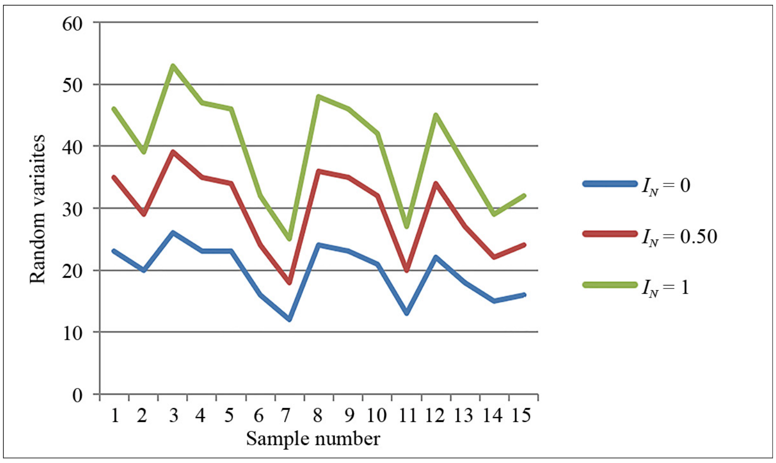

Figure 8. Comparison between the existing and the proposed algorithm-II

As previously mentioned, the proposed algorithm-II serves as an extension of the existing algorithm within the framework of classical statistics. Binomial data is generated using the existing algorithm and is then organized in the initial column of Tables 6-9. Upon scrutiny of Tables 6-9, a clear distinction is observable between the binomial data produced by the existing algorithm under classical statistics and the data generated under varying levels of indeterminacy. Significantly, a discernible upward trend in the data is evident as I_N progresses from 0 to 1. To illustrate, when I_L is set at 0, the data in Table 6 records as 23, while with I_L at 0.2, the corresponding data in Table 6 rises to 28. This trend continues with the increase in the values of I_N. Figure 8, derived from the data presented in Table 6, facilitates a comparative analysis of the data obtained from the proposed algorithm-II and the existing algorithm under classical statistics. Figure 8 distinctly reveals that the existing algorithm generates data smaller than the proposed algorithm-II. The lower curve, representing classical statistics data, is juxtaposed with two other curves denoting data when I_U is set at 0.5 and I_U at 1. Through this analysis, it becomes apparent that the degree of indeterminacy significantly influences the generation of binomial data.

8. Sample moments vs. theoretical moments

In this section, we compare the results obtained from sample moments with the theoretical moments. For this purpose, we calculate the mean and variance from the simulated data and from the properties of the proposed neutrosophic binomial distribution. We consider $n = 100$, $p = 0.5$, and $I_N = 0$ and $I_N = 0.1$. First, under classical statistics ($I_N = 0$), the sample mean and variance are 49.55 and 38.09, respectively, while the theoretical mean and variance from the binomial distribution are 50 and 25, respectively. Next, under neutrosophic statistics ($I_N = 0.1$), the sample mean and variance are 54.6 and 46.25, respectively, whereas the theoretical mean and variance from the neutrosophic binomial distribution are 55 and 30.25, respectively. Figure 9 shows the Quantile-Quantile (QQ) plot, from which it is evident that the simulated data reasonably follows the neutrosophic binomial distribution. From this comparison, it is observed that the sample means under both classical and neutrosophic statistics are very close to their respective theoretical means. However, the sample variances are higher than the corresponding theoretical variances, which are likely due to the relatively small sample size and could be improved by increasing the number of simulated observations.

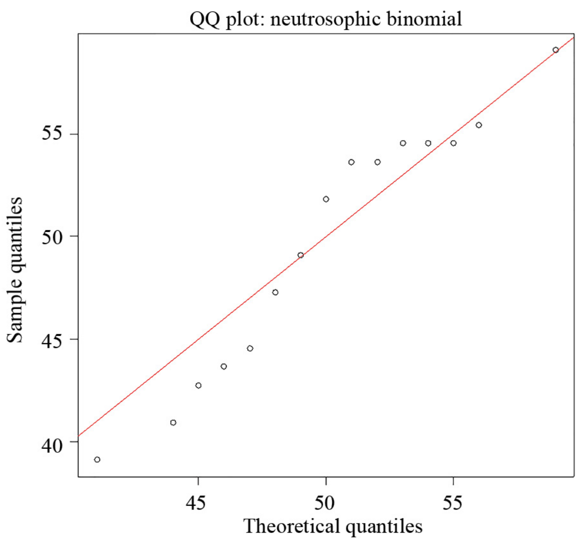

Figure 9. QQ plot of neutrosophic binomial distribution

8.1 *Comparative in confidence intervals*

Now, we present the comparative analysis in terms of 95% Confidence Intervals (CI) using both the simulated data and the theoretical results. For the simulated data under classical statistics ($I_N = 0$), the 95% CI calculated using the t-value 2.145 (with 14 degrees of freedom and $\alpha = 0.05$) is 46.24 to 53.01. Using the theoretical results and the z-value

1.96, the corresponding 95% CI is 47.47 to 52.53. For the simulated data under neutrosophic statistics ($I_N = 0.1$), the 95% CI using the t-value 2.145 is 50.83 to 58.36, while the CI based on the theoretical properties and the z-value 1.96 is 52.21 to 57.78. The summary of these results is presented in Table 10. This comparison indicates that the confidence limits obtained from the simulated data, using both classical and neutrosophic algorithms are reasonably close to those derived from the theoretical properties, demonstrating good agreement between the simulation and the model.

Table 10. Comparison of 95% CI for classical statistics and neutrosophic statistics

Framework	I_N	Method	Test statistic	95% CI
Classical statistics	0	simulation	$t = 2.145$	46.24-53.01
Classical statistics	0	theoretical	$z = 1.96$	47.47-52.53
Neutrosophic statistics	0.1	simulation	$t = 2.145$	50.83-58.36
Neutrosophic statistics	0.1	theoretical	$z = 1.96$	52.21-57.78

9. Updating software

The simulation and comparative studies indicate notable distinctions between the data produced by the proposed algorithms and that generated by the established algorithms referenced in [28]. The research highlights the significant impact of indeterminacy on data generation in uncertain conditions. Upon thorough exploration of the literature and to the best of our knowledge, no software currently exists for generating imprecise data from the binomial distribution. Our findings suggest the need for a revision of existing algorithms responsible for generating binomial data. It is imperative to update these algorithms to facilitate the generation of imprecise data, taking into account the degree of indeterminacy. Generating imprecise data within an indeterminate environment can enhance the accuracy of decision-making. Relying on traditional algorithms based on classical statistics may lead decision-makers astray when faced with indeterminacy in the data. Consequently, we recommend extending the algorithms from the binomial distribution to the neutrosophic binomial distribution under uncertainty, allowing the effect of the degree of indeterminacy on the generated data to be observed.

10. Limitations and potential applications

The binomial distribution finds application across various fields. In acceptance sampling plans, it calculates the probability of lot acceptance. For time-truncated life tests within control charts, the binomial distribution assesses the average run length. Additionally, risk management incorporates the binomial distribution. Within an uncertain environment, the neutrosophic binomial distribution is applicable, as highlighted in studies by [29–31]. However, the proposed distribution and associated algorithms have limitations. They are not applicable in the absence of indeterminacy in the data. When all observations are 100% precise (i.e., $I_L = 0$), the proposed algorithms revert to classical statistical methods. Furthermore, these algorithms are specifically designed for generating imprecise data from the binomial distribution and cannot be used for other purposes.

11. Application

In this section, we will discuss the application of the proposed binomial distribution under neutrosophic statistics, following [17]. Suppose X_L represents the number of questions answered correctly when $p = 0.30$ and $n = 20$. From Table 4, we consider the minimum passing grade to be 6 when $I_N = 0.2$. What is the probability of passing the exam when $I_N = 0.2$?

The probability of passing the exams using the binomial distribution when $I_N = 0.20$ is calculated as follows:

$$P(X_N \geq 6) = P(X_L(1 + I_N) \geq 6) = P\left(X_L \geq \frac{6}{(1+0.20)}\right)$$

$$P(X_L \geq 5) = \sum_{X_L=5}^{n} \binom{n}{X_L} p^{X_L}(1-p)^{n-X_L}$$

$$P(X_L \geq 5) = \sum_{X_L=5}^{20} \binom{20}{X_L} (0.30)^{X_L}(1-0.30)^{20-X_L} = 0.7624$$

The probability of the student passing the exams is 0.7624 under a degree of indeterminacy of 0.20.

12. Concluding remarks

This paper introduced the concept of the neutrosophic binomial distribution and explored several properties of neutrosophic random variables. Two algorithms are proposed, leveraging the neutrosophic binomial distribution, to generate imprecise data from the binomial distribution. Extensive simulations were conducted with various parameters using both algorithms, revealing distinctions between data generated under indeterminacy and that produced in a certain environment. The study's key conclusion is that binomial data generated under indeterminacy differs significantly from data generated in a certain environment. As a result, the paper recommends the application of the proposed algorithms specifically in scenarios where indeterminacy is present. Additionally, suggesting the potential updating of existing software to incorporate the proposed algorithms is identified as a promising avenue for future research. Furthermore, the paper encourages the extension of the neutrosophic concept to other statistical distributions, presenting an intriguing area for future research endeavors. This expansion could offer valuable insights into handling indeterminacy across a broader spectrum of statistical applications. Future research may focus on the proposed algorithms for the Jamal-Chris-Abbas (JCA) distribution [32], the sum of weighted gamma distribution [33], and the generalized Kavya & Ma-noharan (KM) distribution [34]. When the degree of indeterminacy is unknown, its estimation through a suitable distribution can be considered in future research. The proposed algorithms based on the beta-binomial distribution offer potential for further extension in future research.

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Confilict of interest

The authors declare no competing financial interest.

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