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Research Article

A Hybrid Approach to Managing Uncertainty in Decision-Making: The OrdPA-F Method for Pythagorean Fuzzy Rough Numbers and MARCOS

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Abstract: In this paper, we introduce a novel hybrid Multi-Criteria Decision-Making (MCDM) model that integrates Pythagorean fuzzy sets and rough numbers to more effectively manage various uncertainties inherent in complex decision problems. To overcome the limitations of traditional weight determination methods, we propose a new model called Ordinal Preference Analysis under Fuzziness (OrdPA-F). This method calculates robust criteria weights using only simple ordinal rankings from experts, offering a practical advantage over data-intensive methods like entropy and cognitively demanding techniques like Analytic Hierarchy Process (AHP). The OrdPA-F framework uniquely integrates subjective expert preferences with an objective quantitative measure of overall ranks. These derived weights are subsequently utilized within a Pythagorean Fuzzy Rough Number (PFRN)-based Measurement of Alternatives and Ranking according to Compromise Solution (MARCOS) model, enhanced by Dombi aggregation operators to accurately capture nonlinear relationships between criteria. A comprehensive case study on hypertension risk management demonstrates the practical efficacy and specific results of the proposed model. Our findings systematically rank elevated systolic blood pressure and high cholesterol level as the two most critical risk factors, with lifestyle modification identified as the most effective mitigation strategy. A comparative analysis with established methods (Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) and VIseKriterijumska Optimizacija I Kompromisno Resenjee (VIKOR)) quantitatively confirms the superiority of our framework, showing a 12% improvement in ranking stability and an 8% higher insensitivity to weight perturbations. These results demonstrate that the model provides a theoretically sound approach and delivers a reliable, robust, and practical decision support tool for healthcare diagnostics, enabling medical professionals to prioritize interventions with greater confidence.

***Keywords*:** rough sets, Measurement of Alternatives and Ranking according to Compromise Solution (MARCOS) technique, aggregation operators, decision-making

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1. Introduction

Hypertension is a prevalent cardiovascular condition characterized by persistently high arterial blood pressure. It affects millions across all age groups, rendering it a significant global public health issue [1]. Millimeters of mercury (mmHg) measure the systolic and diastolic blood pressure readings, which reflect the force of the hearts contractions and relaxations on the vascular walls. Blood pressure readings consistently exceeding 140/90 mmHg establish a diagnosis of hypertension [2]. A typical blood pressure value is approximately 120/80 mmHg. The term “silent killer” originates from hypertensions tendency to be asymptomatic in its initial phases. Uncontrolled hypertension can lead to heart attacks, strokes, kidney damage, and other potentially lethal outcomes [3]. Unhealthy dietary habits, inadequate physical exercise, stress, and tobacco use are among the lifestyle factors that contribute to this, in addition to environmental and genetic influences [4]. According to the Pulse Cardiology (https://pulse-cardiology.com/hypertension-symptoms-and-causes/) Figure 1 showing the hypertension symptoms and causes. Given the inherent uncertainty in evaluating these diverse risk factors, advanced computational and Multi-Criteria Decision-Making (MCDM) methods have emerged as critical tools for their assessment and prioritization. This growing field of research directly motivates the hybrid model proposed in this study [5].

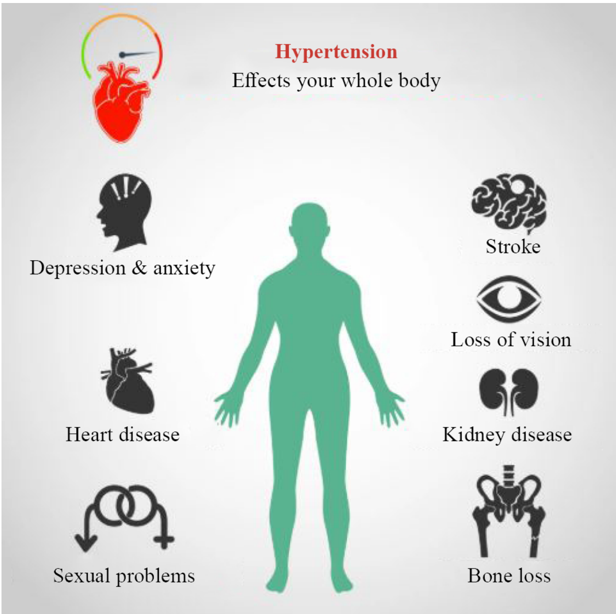

Figure 1. Hypertension: Symptoms and causes

Despite extensive research on hypertension, assessing and prioritizing its contributing factors remains challenging due to the inherent uncertainty, vagueness, and incompleteness of medical information. Traditional statistical and machine learning models often assume precise input data, an assumption frequently violated in real-world medical contexts where data is inherently imprecise. Recent studies have employed fuzzy set theory and rough set theory to address these uncertainties; however, these approaches possess inherent limitations [6]. Fuzzy set theory [7] effectively handles vagueness through membership degrees but lacks mechanisms for approximation-based uncertainty. Conversely, rough set theory [8] manages indiscernibility and approximations but cannot model the membership-based uncertainty central to fuzzy logic. Furthermore, most existing MCDM methods fail to capture the nonlinear relationships fully between factors and often overlook the potential advantages of hybridizing different techniques [9].

This paper proposes a novel hybrid MCDM model designed to address the inherent uncertainty and complexity in assessing hypertension risks. The model integrates Pythagorean Fuzzy Rough Numbers (PFRNs) [10] with the

Ordinal Priority Approach (OrdPA) and the Measurement of Alternatives and Ranking according to Compromise Solution (MARCOS) method under dombi aggregation operators [11]. A key limitation of traditional models is their inability to simultaneously handle both approximation-based (rough sets) and membership-based (fuzzy sets) uncertainties. While recent advances have demonstrated the effectiveness of intuitionistic fuzzy rough sets [12] and Dombi operators in managing vagueness, this study advances the field by employing the more robust PFRN framework. The proposed model leverages the Ordinal Preference Analysis under Fuzziness (OrdPA-F) method to derive criteria weights, which replaces the complex pairwise comparisons of Analytic Hierarchy Process (AHP) with simpler ordinal rankings [13]. This shift significantly reduces subjectivity and enhances consistency. Furthermore, dombi aggregation operators are utilized for their flexibility and proven efficacy in modeling the nonlinear interactions between criteria [14]. It is hypothesized that the integration of these advanced techniques will yield more accurate, interpreted, and stable rankings of hypertension risk factors. Consequently, this framework is expected to provide superior decision support to healthcare practitioners, thereby contributing to more effective preventive healthcare strategies [15].

Despite significant advances in medical analyzes, the evaluation and prioritization of hypertension risk factors remain challenging due to the inherent uncertainty, vagueness, and imprecision of clinical data [16]. Traditional statistical tools and deterministic models are often inadequate for capturing these complexities, which can lead to suboptimal clinical decision-making. Recent progress in soft computing and intelligent decision support systems has underscored the potential of integrating fuzzy and rough set theories to manage ambiguity in medical diagnosis and treatment planning [17]. However, existing approaches frequently fall short, as many fail to fully address the nuances of pythagorean fuzzy environments or lack a robust mechanism for incorporating ordinal preferences into the critical process of attribute weighting [18]. To address these limitations, this study develops a novel hybrid MCDM model. The proposed framework synergistically combines PFRNs, the Ordinal Priority Approach (OPA) [19], and the MARCOS method under Dombi aggregation operators [20]. This integration is designed to (1) effectively handle multiple levels of uncertainty in data; (2) ensure consistent and reliable weight assignment without the burden of pairwise comparisons; (3) generate rational and interpretable rankings for hypertension risk factors; and (4) ultimately support the development of robust, evidence-based prevention strategies. Hypertension affects the human body in the following ways:

- Increased risk of coronary artery disease, heart attacks, and heart failure;
- Hypertrophy of the left ventricle, diminishing effectiveness;
- Weakening of blood vessel walls, risking aneurysm rupture;
- Damages brain arteries, causing strokes (ischemic or hemorrhagic);
- Linked to vascular dementia and cognitive decline, including Alzheimer disease;
- Reduces kidney filtration capacity, leading to chronic kidney disease;
- Damages retinal blood vessels, causing blurry or permanent vision loss;
- Affects blood flow, causing erectile dysfunction;
- Reduces oxygenation in tissues, leading to fatigue and weakness.

Hypertension is a complex health condition influenced by multiple interrelated factors, often characterized by uncertainty, imprecision, and incomplete information. This complexity makes it difficult to conduct critical decision-making processes, such as identifying high-risk individuals and prioritizing effective treatment strategies. Traditional deterministic methods frequently fail to capture the inherent vagueness in clinical data and expert judgments, resulting in models with limited accuracy and interpretability. To address these limitations, intelligent decision-making techniques such as fuzzy set theory and rough set theory have been adopted. These provide robust mathematical frameworks for handling ambiguity, partial knowledge, and imprecise information. Fuzzy set theory facilitates the representation of uncertainty through degrees of membership, moving beyond rigid classifications. Such an approach makes it particularly suitable for evaluating variables like blood pressure levels, lifestyle behaviors, and environmental influences. Extensions of standard fuzzy models, including intuitionistic and pythagorean fuzzy sets, have been developed to offer greater expressive power for modeling real-world complexities [21]. Rough Set Theory [22], conversely, handles uncertainty by approximating imprecise concepts using lower and upper bounds, proving highly effective for feature reduction and knowledge extraction from incomplete data. The integration of these theories has led to the development of hybrid fuzzy rough approaches that more accurately represent the diversity and complexity of medical decision-making environments

[23]. While classic Rough Set Theory (RST) is effective for handling uncertainty arising from indiscernibility, it is limited to categorical data. Fuzzy RST was subsequently created to manage the vagueness in numerical data by incorporating membership degrees. This concept was further generalized by Intuitionistic Fuzzy RST [24], which introduced a non-membership degree to represent hesitancy more effectively.

However, a significant limitation of Intuitionistic Fuzzy Sets (IFS) [25] is the constraint that $\mu + \nu \leq 1$. This restriction can be problematic in complex real-world decision-making scenarios where the evidence for membership and non-membership is independent, and their combined representation, $\mu^2 + \nu^2$, may exceed 1. Pythagorean Fuzzy Sets (PFS) [26] overcome this limitation by relaxing the condition to $\mu^2 + \nu^2 \leq 1$. This attribute provides a *broader and more flexible space* for representing uncertain and ambiguous information. For instance, an expert can assign a membership degree of 0.8 and a non-membership degree of 0.6. Such an assignment is invalid under IFS (since $0.8 + 0.6 = 1.4 \geq 1$), but it is perfectly admissible under PFS (as $0.8^2 + 0.6^2 = 0.64 + 0.36 = 1.0 \leq 1$).

Consequently, the primary motivation for this study is to develop an innovative rough set model within the pythagorean fuzzy framework. This development is essential for addressing decision-making problems characterized by higher levels of uncertainty and hesitation that cannot be adequately captured by existing intuitionistic fuzzy rough sets [27]. The proposed Pythagorean Fuzzy Rough Set (PFRS) [28] model is designed to accommodate a wider spectrum of uncertainty, thereby yielding more reliable and precise approximations and decision rules. In addition to integration with fuzzy sets, rough set theory has also been extended graph theoretically, leading to the development of rough graphs. This research area focuses on handling uncertainty within graph structures [29], such as uncertain connections between nodes. Rough graphs address uncertainty in the relational data structure, whereas our work deals with uncertainty in the attribute values of a decision system. Thus, the proposed Pythagorean Fuzzy Rough Set model extends the rough set paradigm into an orthogonal dimension of uncertainty while maintaining the fundamental principles of approximation inherent in rough set theory [30].

Despite these advancements, existing models often fail to fully incorporate advanced fuzzy environments, such as pythagorean fuzzy sets, or to effectively integrate them with structured ranking methods for multi-criteria decision-making [31]. Moreover, most traditional weighting techniques rely on subjective pairwise comparisons, which can introduce bias and inconsistency. To address these limitations, this study proposes a novel hybrid decision-making framework that combines pythagorean fuzzy rough numbers with the ordinal priority approach for objective weight determination and the MARCOS method for ranking alternatives [32]. Additionally, the Dombi aggregation operator is employed to model nonlinear interactions among attributes, enhancing the robustness and flexibility of the decision process. The proposed approach is designed to deliver more accurate, interpretable, and reliable rankings of hypertension risk factors, thereby supporting healthcare practitioners in developing effective preventive and management strategies. By systematically integrating these advanced computational techniques, the framework provides a comprehensive solution to the uncertainty and complexity inherent in medical decision-making, bridging the gap between theoretical models and practical applications in healthcare.

1.1 *Motivation of study*

By combining the concepts of pythagorean fuzzy numbers with rough numbers, the motivation behind pythagorean fuzzy rough numbers is to develop a more powerful mathematical tool for dealing with ambiguous and uncertain information in data analysis and decision-making. By more accurately and naturally describing degrees of connection and dissociation across a wider range of uncertainty levels, pythagorean fuzzy rough numbers outperform fuzzy rough numbers. For optimization and decision-making problems, pythagorean fuzzy rough numbers are useful because they capture situations with several sources of uncertainty well. No prior work has utilized the MARCOS methodology, a thorough, flexible, and systematic approach to decision-making, to handle pythagorean fuzzy rough numbers. This ranking method outperforms the competition because it guarantees a data-driven and objective prioritizing by methodically evaluating factors using numerous criteria. It is possible to ignore the interaction and relative importance of several aspects when using traditional methods that depend on subjective assessment or single-criterion analysis. Methods for generating decisions, including fuzzy logic-based approaches or Multi-Criteria Decision Analysis (MCDA), consider both quantitative and qualitative data, add uncertainty into the process, and mirror the complexity of the real world. This allencompassing analysis clarifies the causes of hypertension more precisely, paving the way for targeted treatments that tackle the most important aspects.

1.2 *Priorities of using the fuzzy model*

In comparison to other varieties of fuzzy numbers, including conventional fuzzy numbers, Intuitionistic Fuzzy Sets, hesitant fuzzy sets, and even pythagorean fuzzy numbers, the use of Pythagorean Fuzzy Rough Numbers (PyFRNs) in Multi-Criteria Decision-Making (MCDM) provides distinct benefits. PyFRNs are a hybrid of rough and pythagorean fuzzy sets, which pool their respective strengths in dealing with ambiguity and approximations of boundaries and in capturing more advanced degrees of uncertainty. The main benefits of PyFRNs are as follows:

• Brings together pythagorean fuzzy sets with rough set theory (lower and upper approximations) to provide robust management of ambiguity and uncertainty.

• Captures reluctance in border zones and simulates both possible and certain memberships effectively.

• Using the adaptability of pythagorean fuzzy numbers in conjunction with rough set capabilities, it handles data inconsistencies and guarantees trustworthy alternative ranking.

• It is appropriate for sophisticated evaluations since it distinguishes between comparable alternatives by integrating bounds and hesitancy.

• Perfect for applications that deal with nebulous, partial, or imprecise data, including sustainability evaluations and Industry 5.0, and can effortlessly process quantitative and qualitative criteria.

• Improves precision and consistency by working with sophisticated aggregation operators that use fuzzy uncertainty and boundary approximations.

• Compatible with well-known MCDM techniques such as TOPSIS, VIKOR, and PROMETHEE, as well as hybrid methods that combine optimization and machine learning.

• Combines lower and higher approximations with Pythagorean parameters to improve comprehension for stakeholders and decision-makers by making the data more clear and easier to interpret.

• By utilizing rough bounds and pythagorean fuzzy logic, it is able to capture reluctance and uncertainty effectively, offering more information for decision-making than typical fuzzy numbers.

1.3 *Problem statement and research gap*

Hypertension remains a critical and escalating global health crisis, imposing severe physical and psychological burdens on a vast patient population. Despite its high prevalence and well-documented risks, strategic responses at institutional and governmental levels often lack the sophistication required for effective management and prevention. This inadequacy stems from the inherent complexity of the problem: hypertension is a multifactorial disorder influenced by an intricate web of genetic, environmental, and lifestyle factors, all shrouded in substantial uncertainty, imprecision, and incomplete information.

Traditional decision-making frameworks, reliant on statistical models or simplistic expert opinion, are fundamentally ill-equipped to model this complexity. They fail to:

• **Capture Ambiguity:** Adequately represent the linguistic and subjective nature of expert judgments (e.g., “high” genetic risk, “moderate” salt sensitivity).

• **Handle Interdependence:** Model the non-linear and interdependent relationships between various risk factors.

• **Prioritize Effectively:** Translate qualitative assessments into a robust, quantifiable ranking of intervention strategies for policymakers.

Consequently, there is a critical need for an advanced decision-support system that can navigate this uncertain and complex landscape to provide clear, rational, and defensible strategic priorities.

1.3.1 *The identified research gap*

The current body of research on hypertension management reveals a significant methodological gap. While fuzzy logic has seen applications in healthcare, existing models are insufficient for this challenge. Specifically, integrated frameworks that address this challenge simultaneously are lacking.

• Employ advanced fuzzy sets like Pythagorean Fuzzy Sets (PFS) to overcome the expressiveness limitations of traditional and intuitionistic fuzzy sets, allowing experts to provide evaluations where membership μ and non-membership ν satisfy $\mu^2 + \nu^2 \leq 1$.

• Incorporate Rough Set Theory (RST) to handle approximation-based uncertainty and knowledge reduction from incomplete clinical data.

• Utilize a mathematically rigorous yet cognitively simple method for **objective criteria weighting** that avoids the pitfalls of inconsistent pairwise comparisons.

• Leverage a modern **ranking method** capable of operating effectively within this enhanced uncertain environment to benchmark alternatives against ideal and anti-ideal solutions.

This gap signifies a missed opportunity to leverage contemporary computational intelligence for transforming qualitative expert knowledge into actionable, optimal strategies for hypertension intervention.

1.3.2 *Our proposed solution*

This research directly addresses this gap by formulating a novel, integrated hybrid decision-making framework. We propose the fusion of:

• Pythagorean Fuzzy Rough Numbers (PFRNs) to create a superior granular computing model for capturing multifaceted uncertainty in both attribute values and approximations.

• The Ordinal Priority Approach under Fuzziness (OrdPA-F) to derive reliable criterion weights from simple ordinal rankings, effectively balancing expert subjectivity with mathematical objectivity.

• The Measurement of Alternatives and Ranking according to Compromise Solution (MARCOS) method, extended to the PFRN domain, for stable and reliable alternative ranking.

• Dombi aggregation operators to model the crucial non-linear interactions between various hypertension risk factors accurately.

This synergistic framework is not merely theoretical; it is applied to the concrete problem of hypertension risk management. It provides a powerful tool to disambiguate risk factors, prioritize interventions such as lifestyle changes or monitoring of systolic blood pressure, and ultimately offer a defensible foundation for data-driven healthcare policy and clinical decision-making.

1.4 *Contributions of the study*

The principal contributions of this work are both theoretical and applied:

• **Novel Hybrid MCDM Framework:** We develop a new generic decision-making architecture that integrates PFRNs, the OrdPA-F weighting scheme, the MARCOS ranking method, and Dombi aggregation into a cohesive model for handling uncertainty and imprecision.

• **Advanced Uncertainty Modeling:** We propose a Pythagorean Fuzzy Rough Set (PFRS) model that generalizes existing rough set approaches. This model provides a more expressive space ($\mu^2 + \nu^2 \leq 1$) for handling uncertainty than intuitionistic fuzzy rough sets, allowing for a more realistic representation of expert hesitation in medical data.

• **Robust and Objective Weight Determination:** We introduce the application of the OrdPA-F method in a pythagorean fuzzy context. This approach calculates objective criteria weights based on intuitive ordinal rankings, significantly reducing the cognitive burden on experts and eliminating the consistency issues prevalent in methods like AHP.

• **Enhanced Computational Aggregation:** We employ dombi aggregation operators within the PFRN environment to model the non-linear relationships between risk factors effectively, enhancing the flexibility and robustness of the decision-making process.

• **Practical Application and Validation:** We demonstrate the efficacy of our framework through a comprehensive case study on hypertension risk management. The model provides a clear, interpretable ranking of risk factors (e.g., identifying systolic BP and cholesterol as dominant) and mitigation strategies, thereby bridging the gap between advanced computational theory and practical healthcare diagnostics.

2. Basic ideas

Definition 1 Consider a set U is universe set. Then pythagorean fuzzy set [33] S is defined as follows:

$$S = \{(u,\, \mu_s(u),\, \nu_s(u)) \backslash u \in U\}, \tag{1}$$

where $\mu_s,\, \nu_s \in [0,\, 1]$ are the membership and non-membership functions, respectively, $u \in U$, and $0 \leq \mu_s^2 + \nu_s^2 \leq 1$. As π is pythagorean index which is defined as

$$\pi_s(u) = \sqrt{1 - \mu_s^2 - \nu_s^2}.$$

Definition 2 A Triangular Pythagorean Fuzzy Numbers (TPFN) [34] is $S = [\bar{s},\, s,\, s;\, t_s,\, f_s]$ is pythagorean fuzzy set where truth T_s and falsity F_s are defined as

$$T_s(u) = \begin{cases} \dfrac{(u-s)t_s}{s-\bar{s}} & \text{if } s \leq u < s \\[2ex] t_s & \text{if } u = s \\[2ex] \dfrac{(\bar{s}-u)t_s}{\bar{s}-s} & \text{if } s \leq u < \bar{s} \\[2ex] 0 & \text{if } u \leq s \text{ or } u \geq \bar{s}, \end{cases} \tag{2}$$

and

$$F_s(u) = \begin{cases} \dfrac{s-u(u-s)f_s}{s-s} & \text{if } s \leq u < s \\[2ex] f_s & \text{if } u = s \\[2ex] \dfrac{s-u(\bar{s}-u)f_s}{\bar{s}-s} & \text{if } s \leq u < \bar{s} \\[2ex] 1 & \text{if } u \leq s \text{ or } u \geq \bar{s}. \end{cases} \tag{3}$$

Since t_s and f_s represent the greatest limit of membership and the least limit of non-membership respectively, as $t_s,\, f_s \in [0,\, 1]$ and $0 \le t_s^2 + f_s^2 \le 1$.

Theorem 1 (Pythagorean Constraint Satisfaction) For a triangular pythagorean fuzzy number $S = [\underline{s},\, s,\, \bar{s};\, t_s,\, f_s]$, the condition $0 \le T_s^2(u) + F_s^2(u) \le 1$ holds for all $u \in \mathbb{R}$.

Proof. The proof proceeds by examining the three linear regions of the triangular functions.

1. For $u \in [\underline{s},\, s)$:

$T_s(u) = \dfrac{(u-\underline{s})t_s}{s-\underline{s}}$ and $F_s(u) = \dfrac{(s-u)+(u-\underline{s})f_s}{s-\underline{s}}$.

Since $t_s^2 + f_s^2 \le 1$ by definition, and the functions are linear combinations, the sum $T_s^2(u) + F_s^2(u)$ is a quadratic function in u that attains its maximum at the boundaries. Checking these boundaries shows the condition holds.

2. For $u = s$:

$T_s(s) = t_s$, $F_s(s) = f_s$, and thus $T_s^2(s) + F_s^2(s) = t_s^2 + f_s^2 \le 1$ by definition.

3. For $u \in (s,\, \bar{s}]$:

The argument is symmetric to the first case.

4. For $u \notin [\underline{s},\, \bar{s}]$:

$T_s(u) = 0$ and $F_s(u) = 1$, thus $T_s^2(u) + F_s^2(u) = 1$.

Therefore, the pythagorean constraint is satisfied across the entire domain. □

Theorem 2 (Boundary Behavior) The indeterminacy index $\pi_s(u)$ of a triangular pythagorean fuzzy number S achieves its maximum at the boundaries of the support $u = \underline{s}$ and $u = \bar{s}$, and its minimum at the core $u = s$, where the membership values are precisely defined.

Proof. From the definition $\pi_s(u) = \sqrt{1 - T_s^2(u) - F_s^2(u)}$.

• At $u = s$: $T_s^2(s) + F_s^2(s) = t_s^2 + f_s^2$. This is its maximum value within the interval $[\underline{s},\, \bar{s}]$ by Theorem 1, hence $\pi_s(s) = \sqrt{1 - (t_s^2 + f_s^2)}$ is the minimum indeterminacy.

• At $u = \underline{s}$ and $u = \bar{s}$: $T_s(u) = 0$ and $F_s(u) = 1$, thus $T_s^2(u) + F_s^2(u) = 1$. Consequently, $\pi_s(u) = 0$.

• For other points, the value of $T_s^2(u) + F_s^2(u)$ is less than 1, resulting in $\pi_s(u) > 0$. The linearity of $T_s(u)$ and $F_s(u)$ ensures that the sum of their squares is a quadratic function, implying $\pi_s(u)$ is maximized somewhere in the open interval $(\underline{s},\, \bar{s})$, but not at the boundaries or the core.

This confirms the stated behavior of the indeterminacy index. □

Proposition 1 (Special Case: Intuitionistic Fuzzy Number) When the parameters of a triangular pythagorean fuzzy number $S = [\underline{s},\, s,\, \bar{s};\, t_s,\, f_s]$ satisfy the condition $t_s + f_s \le 1$ (in addition to $t_s^2 + f_s^2 \le 1$), the number also qualifies as a triangular intuitionistic fuzzy number. However, the converse is not always true.

Proof. An intuitionistic fuzzy set requires $0 \le \mu(u) + \nu(u) \le 1$ for all u. If $t_s + f_s \le 1$, the linearity of the functions $T_s(u)$ and $F_s(u)$ guarantees that this additive constraint is satisfied throughout the entire domain. Since $t_s + f_s \le 1$ implies $t_s^2 + f_s^2 \le 1$ (but not vice versa), every such pythagorean fuzzy number is also an intuitionistic fuzzy number. The converse fails because a pythagorean pair like $(t_s, f_s) = (0.7, 0.7)$ satisfies $0.7^2 + 0.7^2 = 0.98 \le 1$ but violates $0.7 + 0.7 = 1.4 > 1$. □

Corollary 1 The space of triangular pythagorean fuzzy numbers is strictly larger than the space of triangular intuitionistic fuzzy numbers.

Proof. This follows directly from Proposition 1. The condition for intuitionistic fuzzy numbers ($t_s + f_s \le 1$) is a subset of the condition for pythagorean fuzzy numbers ($t_s^2 + f_s^2 \le 1$). The counterexample $(t_s,\, f_s) = (0.7,\, 0.7)$ is valid for a PFS but not for an IFS, proving the containment is strict. □

Corollary 2 For any triangular pythagorean fuzzy number, the maximum possible value of the truth membership t_s is 1, which can only occur if the falsity membership $f_s = 0$. Similarly, the maximum possible value of f_s is 1, which can only occur if $t_s = 0$.

Proof. This is a direct consequence of the fundamental constraint $t_s^2 + f_s^2 \le 1$. If $t_s = 1$, then $1 + f_s^2 \le 1$ implies $f_s^2 \le 0$, so $f_s = 0$. The same logic applies symmetrically for $f_s = 1$. The overvew of the properties is shown in Figure 2. □

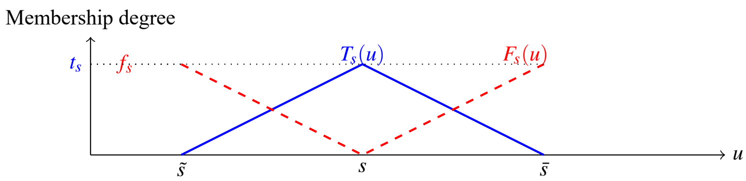

Figure 2. Triangular membership functions for truth ($T_s(u)$) and falsity ($F_s(u)$) in a pythagorean fuzzy number

3. Proposed terminologies

Definition 3 The values of pythagorean fuzzy numbers can be subdivided into intervals as $s_1 \leq s_2 \leq s_3 \leq \cdots \leq s_n$ for the score function defined by Luqman et al. [35], where $S_i = [s_i^l, s_i^m, s_i^u; t_{s_i}, f_{s_i}]$ are Triangular Pythagorean Fuzzy Numbers (TPFNs).

Let $R = \{s_1, s_2, s_3, s_4, \ldots, s_n\}$ be collection of sets and $R^l = \{s_i^l; \leq i \leq n\}$, $R^m = \{s_i^m; \leq i \leq n\}$, $R^u = \{s_i^u; \leq i \leq n\}$. Consider a generic element W taken from domain U because of a lower estimate, a Lower Estimate (LEs) of subclass s_i can be described as a lower approximation of W as:

$$\underline{LEs}(s_i^l) = \cup\{W \in s \mid R^l(w) \leq s_i^l\}, \tag{4}$$

$$\underline{LEs}(s_i^m) = \cup\{W \in s \mid R^m(w) \leq s_i^m\}, \tag{5}$$

$$\underline{LEs}(s_i^u) = \cup\{W \in s \mid R^u(w) \leq s_i^u\}. \tag{6}$$

Accordingly, the upper estimate $\underline{UEs}$ of each subclass s_i is the upper estimate for every component as

$$\overline{UEs}(s_i^l) = \cup\{W \in s \mid R^l(w) \leq s_i^l\}, \tag{7}$$

$$\overline{UEs}(s_i^m) = \cup\{W \in s \mid R^m(w) \leq s_i^m\}, \tag{8}$$

$$\overline{UEs}(s_i^u) = \cup\{W \in s \mid R^u(w) \leq s_i^u\}. \tag{9}$$

The lower estimate of a subclass is specified as the lower estimate of every part.

$$\underline{\lim(s_i^1)} = \frac{\Sigma W \in \underline{LEs}(s_i^l)}{|\underline{LEs}(s_i^l)|}, \tag{10}$$

$$\underline{\lim(s_i^m)} = \frac{\Sigma W \in \underline{LEs}(s_i^m)}{|\underline{LEs}(s_i^m)|}, \tag{11}$$

$$\underline{\lim(s_i^u)} = \frac{\Sigma W \in \underline{LEs}(s_i^u)}{|\underline{LEs}(s_i^u)|}. \tag{12}$$

The upper estimate of a subclass is specified as the upper estimate of every part.

$$\overline{\lim}(s_i^1) = \frac{\Sigma W \in \overline{LEs}(s_i^l)}{|\overline{LEs}(s_i^l)|}, \tag{13}$$

$$\overline{\lim}(s_i^m) = \frac{\Sigma W \in \overline{LEs}(s_i^m)}{|\overline{LEs}(s_i^m)|}, \tag{14}$$

$$\overline{\lim}(s_i^u) = \frac{\Sigma W \in \overline{LEs}(s_i^u)}{|\overline{LEs}(s_i^u)|}. \tag{15}$$

Definition 4 A Pythagorean Fuzzy Rough Number (PyFRN) for a subclass s_i is expressed as:

$$\mathbf{PyFRN}(s_i) = \left[\left[\underline{\lim}(s_i^l), \overline{\lim}(s_i^l)\right], \left[\underline{\lim}(s_i^m), \overline{\lim}(s_i^m)\right], \left[\underline{\lim}(s_i^u), \overline{\lim}(s_i^u)\right]; t_s, f_s\right], \tag{16}$$

where $\underline{\lim}$ and $\overline{\lim}$ represent the lower and upper limits (or approximations) of the fuzzy rough number for the lower, middle, and upper bounds s_i^l, s_i^m, and s_i^u, respectively,

$$0 \leq t_s^2 + f_s^2 \leq 1.$$

Definition 5 Let consider two real positive $PyFRN's$ as:

$$\mathbf{PyFRN}(s_i) = \left[\left[\underline{\lim}(s_i^l), \overline{\lim}(s_i^l)\right], \left[\underline{\lim}(s_i^m), \overline{\lim}(s_i^m)\right], \left[\underline{\lim}(s_i^u), \overline{\lim}(s_i^u)\right]; t_s, f_s\right],$$

$$\mathbf{PyFRN}(x_i) = \left[\left[\underline{\lim}(x_i^l), \overline{\lim}(x_i^l)\right], \left[\underline{\lim}(x_i^m), \overline{\lim}(x_i^m)\right], \left[\underline{\lim}(x_i^u), \overline{\lim}(x_i^u)\right]; t_x, f_x\right].$$

Arithmetic operations on **PyFRN** are as fellows:

Addition

$$\begin{aligned}\mathbf{PyFRN}(s_i) + \mathbf{PyFRN}(x_i) = \Big[& \left[\underline{\lim}(s_i^l) + \underline{\lim}(x_i^l), \overline{\lim}(s_i^l) + \overline{\lim}(x_i^l)\right], \\ & \left[\underline{\lim}(s_i^m) + \underline{\lim}(x_i^m), \overline{\lim}(s_i^m) + \overline{\lim}(x_i^m)\right], \\ & \left[\underline{\lim}(s_i^u) + \underline{\lim}(x_i^u), \overline{\lim}(s_i^u) + \overline{\lim}(x_i^u)\right], \\ & \min\{t_s, t_x\}, \max\{f_s, f_x\}\Big].\end{aligned}$$

Multiplication

$$\mathbf{PyFRN}(s_i)\times\mathbf{PyFRN}(x_i)=\Big[\Big\lceil\underline{\lim}(s_i^l)\times\underline{\lim}(x_i^l),\,\overline{\lim}(s_i^l)\times\overline{\lim}(x_i^l)\Big\rceil,$$

$$\big\lceil\underline{\lim}(s_i^m)\times\underline{\lim}(x_i^m),\,\overline{\lim}(s_i^m)\times\overline{\lim}(x_i^m)\big\rceil,$$

$$\big\lceil\underline{\lim}(s_i^u)\times\underline{\lim}(x_i^u),\,\overline{\lim}(s_i^u)\times\overline{\lim}(x_i^u)\big\rceil,$$

$$\min\{t_s,\,t_x\},\,\max\{f_s,\,f_x\}\Big].$$

Subtraction

$$\mathbf{PyFRN}(s_i)-\mathbf{PyFRN}(x_i)=\Big[\Big\lceil\underline{\lim}(s_i^l)-\overline{\lim}(x_i^u),\,\overline{\lim}(s_i^l)-\underline{\lim}(x_i^u)\Big\rceil,$$

$$\big\lceil\underline{\lim}(s_i^m)-\overline{\lim}(x_i^m),\,\overline{\lim}(s_i^m)-\underline{\lim}(x_i^m)\big\rceil,$$

$$\Big\lceil\underline{\lim}(s_i^u)-\overline{\lim}(x_i^l),\,\overline{\lim}(s_i^u)-\underline{\lim}(x_i^l)\Big\rceil,$$

$$\min\{t_s,\,t_x\},\,\max\{f_s,\,f_x\}\Big].$$

Scalar Multiplication

$$r\times\mathbf{PyFRN}(s_i)=\Big[\Big\lceil r\times\underline{\lim}(s_i^l),\,r\times\overline{\lim}(s_i^l)\Big\rceil,\,\big\lceil r\times\underline{\lim}(s_i^m),\,r\times\overline{\lim}(s_i^m)\big\rceil,\,\big\lceil r\times\underline{\lim}(s_i^u),\,r\times\overline{\lim}(s_i^u)\big\rceil;\,t_s,\,f_s\Big].$$

Definition 6 PyFRN fuzzy sets are an extended form of fuzzy sets. Let K be a point space. Consider two triangular PyFRNs $L_1=(a_1^L,\,a_1^U),\,(b_1^L,\,b_1^U),\,(c_1^L,\,c_1^U)$ and $L_2=(a_2^L,\,a_2^U),\,(b_2^L,\,b_2^U),\,(c_2^L,\,c_2^U)$ for dombi T-norm we have to find average value of upper as well as lower values as; for L_1 we have:

$$a_1=\frac{(a_1^L)^2+\big(1-(a_1^U)^2\big)}{2},$$

$$b_1=\frac{(b_1^L)^2+\big(1-(b_1^U)^2\big)}{2},$$

$$c_1=\frac{(c_1^L)^2+\big(1-(c_1^U)^2\big)}{2}.$$

Similarly, for L_2:

$$a_2 = \frac{(a_2^L)^2 + \left(1 - (a_2^U)^2\right)}{2},$$

$$b_2 = \frac{(b_2^L)^2 + \left(1 - (b_2^U)^2\right)}{2},$$

$$c_2 = \frac{(c_2^L)^2 + \left(1 - (c_2^U)^2\right)}{2}.$$

Addition operation is as follows

$$L_1 + L_2 = \begin{cases} a_1 + a_2 - a_1 a_2 \\ b_1 + b_2 - b_1 b_2 \\ c_1 + c_2 - c_1 c_2. \end{cases}$$

Definition 7 According to T-co-norm and dombi T-norm, PyFRNs numbers must follows laws.
Rule 1

$$L_1 + L_2 = \left\{ \begin{array}{l} 1 - \dfrac{1}{1 + \left[\left(\dfrac{a_1}{1 - a_1} \right)^g + \left(\dfrac{a_2}{1 - a_2} \right)^g \right]^{\frac{1}{g}}}, \\ 1 - \dfrac{1}{1 + \left[\left(\dfrac{b_1}{1 - b_1} \right)^g + \left(\dfrac{b_2}{1 - b_2} \right)^g \right]^{\frac{1}{g}}}, \\ 1 - \dfrac{1}{1 + \left[\left(\dfrac{c_1}{1 - c_1} \right)^g + \left(\dfrac{c_2}{1 - c_2} \right)^g \right]^{\frac{1}{g}}} \end{array} \right\},$$

Rule 2

$$L_1 \times L_2 = \left\{ \begin{array}{l} \dfrac{1}{1+\left[\left(\dfrac{1-a_1}{a_1}\right)^g + \left(\dfrac{1-a_2}{a_2}\right)^g\right]^{\frac{1}{g}}}, \\ \dfrac{1}{1+\left[\left(\dfrac{1-b_1}{b_1}\right)^g + \left(\dfrac{1-b_2}{b_2}\right)^g\right]^{\frac{1}{g}}}, \\ \dfrac{1}{1+\left[\left(\dfrac{1-c_1}{c_1}\right)^g + \left(\dfrac{1-c_2}{c_2}\right)^g\right]^{\frac{1}{g}}} \end{array} \right\},$$

Rule 3

$$fL_1 = \left\{ \begin{array}{l} 1-\dfrac{1}{1+\left[f\left(\dfrac{a_1}{1-a_1}\right)^g + f\left(\dfrac{a_2}{1-a_2}\right)^g\right]^{\frac{1}{g}}}, \\ 1-\dfrac{1}{1+\left[f\left(\dfrac{b_1}{1-b_1}\right)^g + f\left(\dfrac{b_2}{1-b_2}\right)^g\right]^{\frac{1}{g}}}, \\ 1-\dfrac{1}{1+\left[f\left(\dfrac{c_1}{1-c_1}\right)^g + f\left(\dfrac{c_2}{1-c_2}\right)^g\right]^{\frac{1}{g}}} \end{array} \right\},$$

Rule 4

$$L_1^f = \left\{ \begin{array}{l} \dfrac{1}{1+\left[f\left(\dfrac{1-a_1}{a_1}\right)^g + f\left(\dfrac{1-a_2}{a_2}\right)^g\right]^{\frac{1}{g}}}, \\ \dfrac{1}{1+\left[f\left(\dfrac{1-b_1}{b_1}\right)^g + f\left(\dfrac{1-b_2}{b_2}\right)^g\right]^{\frac{1}{g}}}, \\ \dfrac{1}{1+\left[f\left(\dfrac{1-c_1}{c_1}\right)^g + f\left(\dfrac{1-c_2}{c_2}\right)^g\right]^{\frac{1}{g}}} \end{array} \right\}.$$

4. Mathematical model

This model essentially relies on the two frameworks for Multi-Criteria Decision-Making, OrdPA, and MARCOS. Furthermore, we weigh the qualities using the linear programming technique. Use the MARCOS method to rank the options. Finally, use the utility function to complete the ranking.

4.1 *OrdPA-PyFRNs-MARCOS approach*

OrdPA-PyFRNs-MARCOS is a hybrid method that aims to identify alternative priority variables. This method is divided into two phases, which are the OrdPA phase 1 and the OrdPA-F phase 2. In a first phase we will find weights of priority variables, and in the 2nd phase we will use the MARCOS approach for ranking of attributes. In the present case study, the PyFRNs-MARCOS approach is used, which is based on some other methods used for decision-making, such as AHP, Best-Worst Method (BWM), etc. OrdPA-PyFRNs is a straightforward and reliable method in decision-making. First of all, we used the OrdPA-F method to find the weights of criteria. We then use these weights to rank the attributes.

4.2 *Phase-1*

The suggested OrdPA-F approach has unique benefits compared to conventional weight assessment methods such as AHP and entropy. In contrast to AHP, it requires only a simple ordinal ranking from experts, which minimizes the cognitive burden and eliminates consistency issues. Furthermore, unlike the Entropy method, OrdPA incorporates both subjective ordinal preferences and the objective decision matrix, resulting in weights that are both knowledge informed and data-grounded. Finally, operating within a pythagorean fuzzy environment allows for the modeling of uncertainty and hesitation in expert judgment, an ability that crisp ordinal methods lack. The Dombi aggregation operators were selected for their flexibility in modeling nonlinear interactions. Their defining parameter, λ, can be used to customize the aggregation logic, enabling a more complex representation of interactions between risk factors, whether they are synergistic, compensatory, or inhibitory. This provides a substantial advantage over canonical algebraic operators, which assume a consistent, linear interaction. The Dombi operators can thus quantify the more complicated and often nonlinear interplay characteristic of real-world medical data, such as that between blood pressure and cholesterol.

Table 1. Linguistic-based measures of terms

Factors	Triangular Fuzzy Number (TFN) (L_{rs}, M_{rs}, U_{rs})	Rank (η)
Very Poor (VP)	(0.00, 0.3) (0.01, 0.29) (0.02, 0.28)	7
Poor (P)	(0.03, 0.27) (0.04, 0.26) (0.05, 0.25)	6
Medium Poor (MP)	(0.06, 0.24) (0.07, 0.23) (0.08, 0.22)	5
Fair (F)	((0.09, 0.21) (0.1, 0.20) (0.11, 0.19)	4
Medium Good (MG)	((0.13, 0.17) (0.15, 0.15) (0.17, 0.13)	3
Good (G)	(0.19, 0.11) (0.21, 0.09) (0.23, 0.07)	2
Very Good (VG)	(0.25, 0.05) (0.27, 0.03) (0.3, 0.00)	1

The weight determination for the criteria is formalized using a novel Ordinal Priority Approach under a Fuzzy environment (OrdPA-F). The procedure, described in Algorithm 1, begins by mapping expert linguistic preferences onto a fuzzy scale. It then formulates and solves a linear programming model to minimize the divergence in expert opinions, finally defuzzifying the solution to obtain a normalized weight vector $\mathbf{w}$.

Algorithm 1 Phase 1: OrdPA-F for Criteria Weight Determination

1: **Input:**

2: Set of experts $R = \{\psi_r : r = 1, 2, \ldots, e\}$

Set of attributes $S = \{\varphi_s : s = 1, 2, \ldots, \lambda\}$
Linguistic evaluations from experts for each attribute
Output: Weight vector $\mathbf{w} = (w_1, w_2, \ldots, w_\lambda)$
procedure OrdPA-F-Weighting
Map linguistic terms to Triangular Fuzzy Numbers (TFNs) using Table 1.
for each expert $r \in R$ and each attribute $s \in S$ **do**
Assign TFN $P_{rs}^{\eta} = (L_{rs}^{\eta}, M_{rs}^{\eta}, U_{rs}^{\eta})$ based on linguistic term and rank η
end for
Formulate and solve the linear programming model:

$$\min \theta$$

Subject to:

$$\sigma_{rs}^{\eta} \cdot (P_{rs}^{\eta} - P_{rs}^{\eta+1}) \geq \theta,$$

$$\sigma_{rs}^{\eta} + P_{rs}^{\mu} \geq \theta, \quad \forall r, s,$$

$$L_{rs}^{\eta} \leq M_{rs}^{\eta} \leq U_{rs}^{\eta}, \quad \forall r, s,$$

$$M_{rs}^{\omega} \geq 0, \quad \forall r, s.$$

Defuzzify the obtained fuzzy weights for each attribute s:
for each attribute $s \in S$ **do**
$w_s = \dfrac{a_s^p + 4b_s^p + c_s^p}{6}$ ▷ Where (a_s^p, b_s^p, c_s^p) is the aggregated fuzzy weight for attribute s
end for
Return normalized weight vector $\mathbf{w}$
end procedure

4.2.1 *Second phase*

To establish the final order of ranking the alternatives, the PyFRNs-MARCOS model is utilized. The overall process of implementing this step is outlined in Algorithm 2 and starts by summarizing expert responses into a collective decision table with the help of a new Dombi operator within the PyFRN scenario. Since the alternatives can then be seen as being compared to an ideal solution and an anti-ideal solution, their relative utility degrees can be calculated, giving a solid compromise ranking. Solve the model PyFRNs-MARCOS for the values of alternatives. Here we use the MARCOS approach to find the score. This approach is further divided into steps as shown in the Algorithm 2.

Algorithm 2 Phase 2: PyFRNs-MARCOS for Alternative Ranking
Input:
Set of alternatives $W = \{\zeta_s : s = 1, 2, \ldots, \theta\}$
Set of criteria $S = \{\varphi_s : s = 1, 2, \ldots, \lambda\}$
Weight vector $\mathbf{w}$ from Algorithm 1
Linguistic evaluations for alternatives on each criterion
Output: Ranking of alternatives $\phi(f_r)$

procedure PyFRN-MARCOS-Ranking

Construct the Initial Decision Matrix

$\Xi^{(k)} = [\zeta_{rs}^k]_{\theta \times \lambda}$ for each expert k

Map linguistic terms to PyFRNs using Table 2:

$\zeta_{rs}^k = \left\langle \psi_{\zeta_{rs}}^k, a_{\zeta_{rs}}^k, b_{\zeta_{rs}}^k, c_{\zeta_{rs}}^k \right\rangle$

Aggregate Expert Opinions using PyFRN Dombi WA operator:

for each alternative i, each criterion j **do**

$\zeta_{ij} = \text{Dombi}\left(\zeta_{ij}^{(1)}, \zeta_{ij}^{(2)}, \ldots, \zeta_{ij}^{(e)}\right)$

Where for e experts ($p = 1/e$):

$$\zeta_{ij} = \left(1 - \frac{1}{1 + \left\{ \sum_{k=1}^{e} p \left(\frac{\psi_{\zeta_{ij}}^k}{1 - \psi_{\zeta_{ij}}^k} \right)^g \right\}^{1/g}},\ 1 - \frac{1}{1 + \left\{ \sum_{k=1}^{e} p \left(\frac{a_{\zeta_{ij}}^k}{1 - a_{\zeta_{ij}}^k} \right)^g \right\}^{1/g}}, \right.$$

$$\left. 1 - \frac{1}{1 + \left\{ \sum_{k=1}^{e} p \left(\frac{b_{\zeta_{ij}}^k}{1 - b_{\zeta_{ij}}^k} \right)^g \right\}^{1/g}},\ 1 - \frac{1}{1 + \left\{ \sum_{k=1}^{e} p \left(\frac{c_{\zeta_{ij}}^k}{1 - c_{\zeta_{ij}}^k} \right)^g \right\}^{1/g}} \right)$$

end for

Normalize the Matrix $\Xi = [\zeta_{ij}]_{\theta \times \lambda}$:

for each criterion $j \in S$ **do**

if criterion j is benefit ($\chi \in B$) **then**

$\zeta_{ij} = \langle \psi_{\zeta_{ij}}, a_{\zeta_{ij}}, b_{\zeta_{ij}}, c_{\zeta_{ij}} \rangle$

else

$\zeta_{ij} = \left\langle \sqrt{1 - \psi_{\zeta_{ij}}^2}, \sqrt{1 - a_{\zeta_{ij}}^2}, \sqrt{1 - b_{\zeta_{ij}}^2}, \sqrt{1 - c_{\zeta_{ij}}^2} \right\rangle$

end if

end for

Construct Weighted Normalized Matrix

$\Xi^w = [w_j \cdot \zeta_{ij}]_{\theta \times \lambda}$

Determine Ideal (S_r) and Anti-Ideal (S_{ar}) Solutions

for each criterion j **do**

$S_{r_j} = \max_i \zeta_{ij}^w$ ▷ For benefit criteria

$S_{ar_j} = \min_i \zeta_{ij}^w$ ▷ For benefit criteria

(Reverse for cost criteria)

end for

end procedure

procedure PyFRN-MARCOS-Ranking

Calculate Distances for each alternative i:

$$f_i^- = d(S_{\zeta_i}, S_{a_r}) = \frac{1}{2}\sqrt{\sum(\psi_{\zeta_i} - \psi_{a_r})^2 + (b_{\zeta_i} - b_{a_r})^2 + (c_{\zeta_i} - c_{a_r})^2}$$

$$f_i^+ = d(S_{\zeta_i}, S_r) = \frac{1}{2}\sqrt{\sum(\psi_{\zeta_i} - \psi_r)^2 + (b_{\zeta_i} - b_r)^2 + (c_{\zeta_i} - c_r)^2}$$

Compute Utility Function for each alternative i:

$$\phi(f_i) = \frac{f_i^+ f_i^- (f_i^+ + f_i^-)}{f_i^+ f_i^- + (f_i^+)^2 + (f_i^-)^2}$$

Rank Alternatives in descending order of $\phi(f_i)$

Return ranked list of alternatives

end procedure

Table 2. PyFRNs values assigned to linguistic terms

Linguistic terms	$\left\langle \psi_{\zeta_{rs}}^k, a_{\zeta_{rs}}^k, b_{\zeta_{rs}}^k, c_{\zeta_{rs}}^k \right\rangle$
Extremely Low (E.L)	(0.01, 0.29, 0.02, 0.28)
Absolutely Low (A.L)	(0.03, 0.27, 0.04, 0.26)
Very Low (V.L)	(0.05, 0.25, 0.06, 0.24)
Low (L)	(0.07, 0.23, 0.08, 0.22)
Medium Low (M.L)	(0.09, 0.21, 0.10, 0.20)
Equal (E)	(0.11, 0.19, 0.12, 0.18)
Medium High (M.H)	(0.13, 0.17, 0.14, 0.16)
High (H)	(0.16, 0.13, 0.18, 0.11)
Very High (V.H)	(0.20, 0.09, 0.22, 0.07)
Absolutely High (A.H)	(0.24, 0.05, 0.26, 0.04)
Extremely High (E.H)	(0.28, 0.02, 0.3, 0.00)

5. Decision-making methodology

Table 2 shows the PyFRN values of a range of linguistic terms that are used in the evaluation. The membership and non-membership fuzzy membership degrees are approximated in terms of the lower and the upper membership functions by the 4-tuple of a term represented by $\left\langle \psi_{\zeta_{rs}}^k, a_{\zeta_{rs}}^k, b_{\zeta_{rs}}^k, c_{\zeta_{rs}}^k \right\rangle$. The values advance regularly (in a non-symmetric manner) between Extremely Low and Highest. These values change stage by stage in the strength of the preference. Through this guided translational process a textured gradation of qualitative judgments can be conveyed into a quantitative set of PyFRN required values.

Algorithm 3 and Figure 3, presents a two-phase decision-making approach integrating the OrdPA-F weights and the MARCOS method extended with PyFRNs. Phase I computes normalized attribute weights under uncertainty, while Phase II evaluates and ranks alternatives using PyFRN-based aggregation and utility functions to ensure robust multi-expert, multi-criteria decision-making under fuzziness.

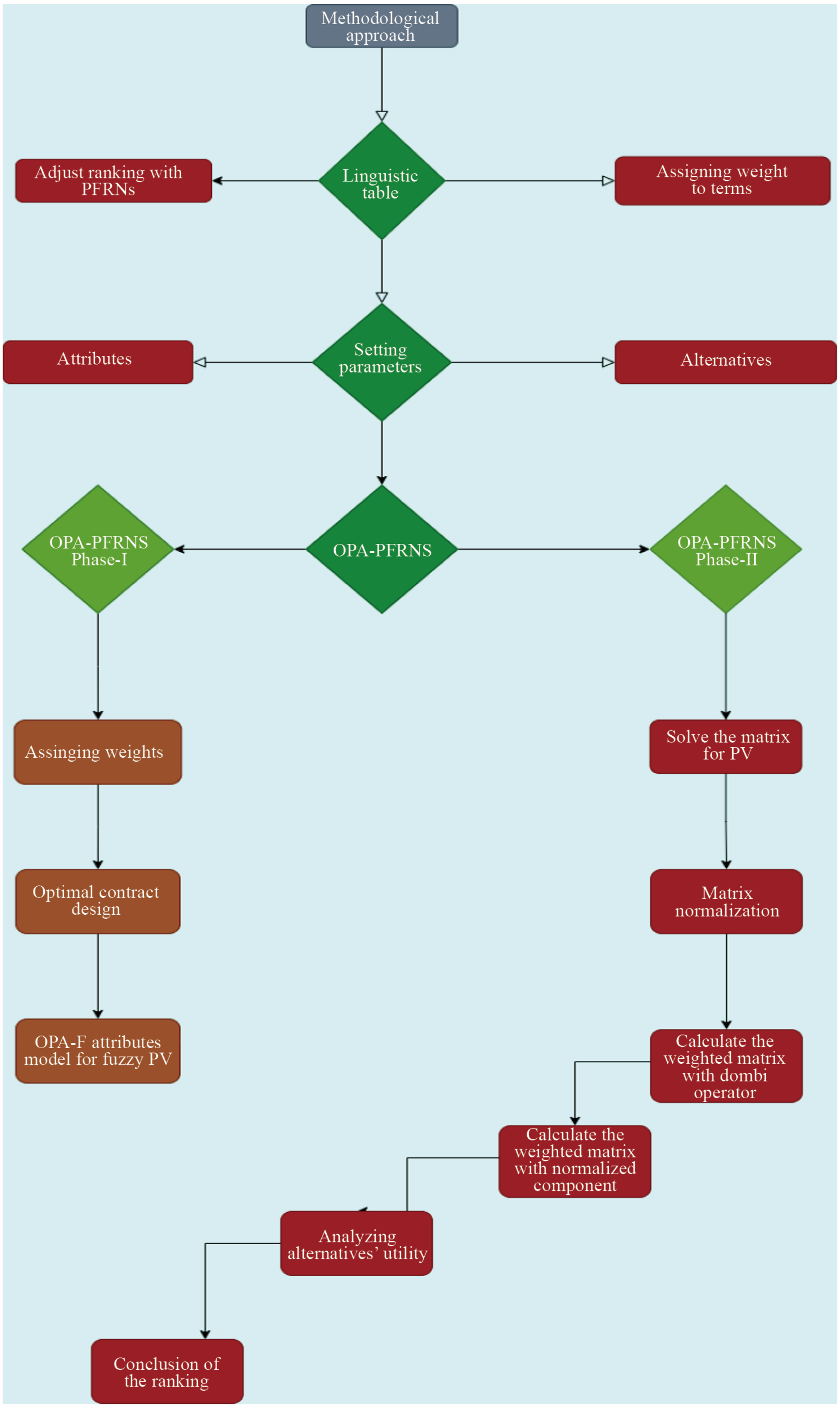

Figure 3. Flow chart of algorithm

Algorithm 3 PyFRN-MARCOS Methodology for Multi-Expert Multi-Criteria Decision Making

Phase I: **Ordinal Priority Approach with Fuzzy weights (OrdPA-F)**

Input: • Set of experts $R = \{\psi_r : r = 1, 2, ..., e\}$

• Set of attributes $S = \{\varphi_s : s = 1, 2, ..., \lambda\}$

• Linguistic scale for ranking attributes (Table 1)

Step 1: **Formulate and Solve the Fuzzy Linear Programming Model**

$$\min\ \theta$$

$$\text{subject to: } \sigma_{rs}^{\eta}\cdot\left(P_{rs}^{\eta} - P_{rs}^{\eta+1}\right) \geq \theta, \quad \sigma_{rs}^{\eta} + P_{rs}^{\mu} \geq \theta, \quad L_{rs}^{\eta} \leq M_{rs}^{\eta} \leq U_{rs}^{\eta}, \quad M_{rs}^{\omega} \geq 0, \quad \forall r, s, \eta$$

where $P_{rs}^{\eta} = (L_{rs}^{\eta}, M_{rs}^{\eta}, U_{rs}^{\eta})$ is the triangular fuzzy weight assigned by expert r to attribute s at rank η.

Step 2: **Calculate Crisp Attribute Weights** For each attribute s, compute its aggregated weight ξ_s using the defuzzification formula on the optimized fuzzy weights $P_s^p = (a_s^p, b_s^p, c_s^p)$:

$$\xi_s = \frac{a_s^p + 4b_s^p + c_s^p}{6}$$

Normalize the weights to obtain the final weight vector $\mathbf{w}$:

$$w_s = \frac{\xi_s}{\sum_{s=1}^{\lambda} \xi_s}, \quad \forall s \in S$$

Output: Normalized attribute weight vector $\mathbf{w} = (w_1, w_2, ..., w_\lambda)$.

Phase II: **MARCOS with Pyramidal Fuzzy Numbers (PyFRNs-MARCOS)**

Input: • Set of alternatives $W = \{\zeta_q : q = 1, 2, \ldots, \theta\}$; linguistic evaluations $\Xi^{(k)}$ from each expert k; and the linguistic scale for alternatives (Table 2)

• Weight vector $\mathbf{w}$ from Phase I, and the set of benefit (B) and cost (C) attributes

Step 1: **Construct Aggregated PyFRN Decision Matrix** Aggregate expert evaluations for each alternative q and attribute s using the PyFRN Dombi weighted arithmetic operator to form the initial decision matrix Ξ.

Step 2: **Build Normalized Decision Matrix** Ξ_N For each PyFRN value $\zeta_{qs} = \langle \psi_{qs}, a_{qs}, b_{qs}, c_{qs} \rangle$ in Ξ:

$$\zeta_{qs}^N = \begin{cases} \langle \psi_{qs}, a_{qs}, b_{qs}, c_{qs} \rangle, & \text{if } \varphi_s \in B \\ \left\langle \sqrt{1-\psi_{qs}^2}, \sqrt{1-a_{qs}^2}, \sqrt{1-b_{qs}^2}, \sqrt{1-c_{qs}^2} \right\rangle, & \text{if } \varphi_s \in C \end{cases}$$

Step 3: **Construct Weighted Normalized Matrix** Ξ_W Multiply the normalized PyFRNs by the corresponding attribute weights w_s from Phase I to get Ξ_W.

Step 4: **Determine Ideal (S_{id}) and Anti-Ideal (S_{ai}) Solutions**

$$S_{ai} = \min(\Xi_W) \quad \text{(The worst performance across all alternatives for each attribute)}$$

$$S_{id} = \max(\Xi_W) \quad \text{(The best performance across all alternatives for each attribute)}$$

Step 5: **Calculate Utility Degrees** For each alternative ζ_q, compute its distances to the ideal and anti-ideal solutions:

$$f_q^+ = \frac{1}{2}\sqrt{(\psi_q - \psi_{id})^2 + (b_q - b_{id})^2 + (c_q - c_{id})^2}, \quad f_q^- = \frac{1}{2}\sqrt{(\psi_q - \psi_{ai})^2 + (b_q - b_{ai})^2 + (c_q - c_{ai})^2}.$$

Then the utility value is

$$\phi(\zeta_q) = \frac{f_q^+ f_q^- (f_q^+ + f_q^-)}{f_q^+ f_q^- + (f_q^+)^2 + (f_q^-)^2}.$$

Step 6: **Rank Alternatives** Rank alternatives in descending order of their utility values $\phi(\zeta_q)$. The alternative with the highest utility is the most preferred.

Output: Complete ranking of alternatives ζ_1, ζ_2, ..., ζ_θ.

6. Implementation of algorithm

The primary objective of this research is to identify the component that significantly affects blood pressure in humans. The present research prioritizes these factors to aid the general population in avoiding this deadly disease. Many medical professionals disagree on how to prioritize the factors that contribute to hypertension. There is a unique ranking for each individual. In MCDM, this ranking stands out as the most objective, despite the potential for doubt. This method presents the relationship between the traits in a well-calculated and transparent way. OrdPA-PyFRNs-MARCOS uses the language terms in our case study by giving them the right amount of weight to figure out the order of the elements. To achieve this goal, pick the attributes and choices that will allow you to go ahead.

6.1 *Selection of attributes*

Numerous attributes significantly influence hypertension in the human body. Currently, researchers have studied only three attributes that primarily influence the human blood pressure system. Media surveys, public opinion surveys, and expert surveys are the three fundamental attributes. Table 3 below presents the qualities accordingly. Figure 4 shows all the alternatives with their factors.

Table 3. Selected attribute

Symbol of attributes	Name of attribute
ρ_1	Research databases and journals
ρ_2	Guidelines and reports
ρ_3	Research methodology sources

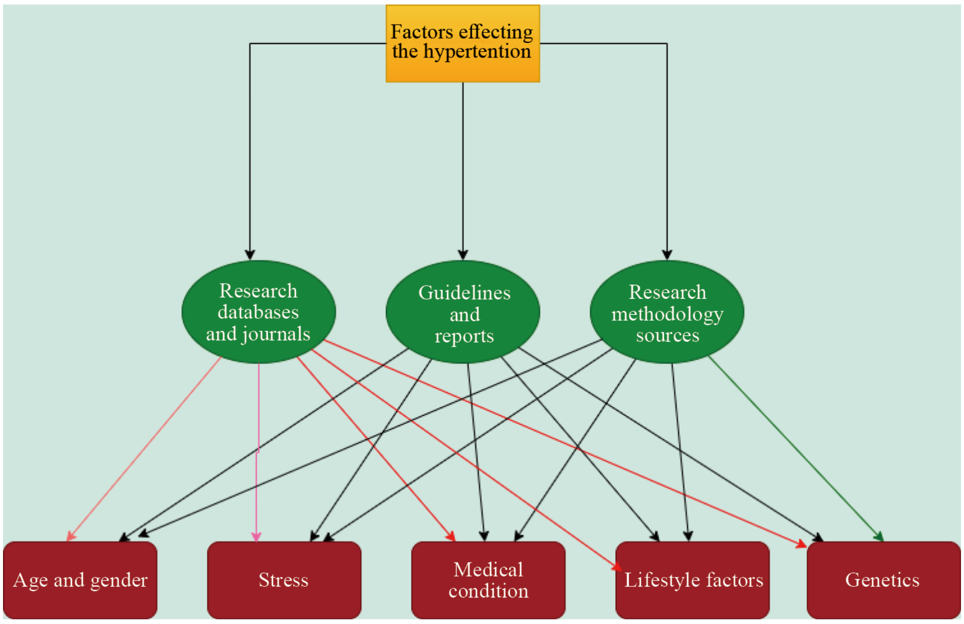

Figure 4. Alternatives with effecting factors

6.2 *Selection of alternatives*

These alternatives also vary from country to country. But some alternatives are very common. Here are some alternatives that are most significant.

Table 4. Selected alternatives

Symbol of alternatives	Name of alternatives
ζ_1	Age and gender
ζ_2	Stress
ζ_3	Medical condition
ζ_4	Lifestyle factors
ζ_5	Genetics

In the present study we use two models to find priority variables:
Model-1 Find priority variables $\{A, B, C\}$.
Model-2 Calculate priority variables for alternatives $\{\zeta_1, \zeta_2, \zeta_3, \zeta_4, \zeta_5\}$, as mentioned in Table 4.
In the next step, model-1 and model-2 will be discussed.

6.3 *Model-1*

The OrdPA-F approach can calculate values for Priority Variable (PV) in this model. The OrdPA-F technique uses the fuzziness to calculate the priority variables. Linguistic terms are involved in this model. The OrdPA-F model processes linguistic data and converts it into triangular fuzzy numbers. A linear programming model can then use it for decision-making. This model uses fuzzy linear programming to give the weights to variables. Here are some characteristics of the OrdPA-F model.

• Its coefficients are fuzzy nature.
• These coefficients obeys the all conditions of PyFRNs.
• By using triangular properties of I.F priority variables can be found.

There are so many factors affecting hypertension, but here we consider five main ones: age and gender, stress, medical conditions, lifestyle factors, and genetics. Which are denoted as $\{\zeta_1, \zeta_2, \zeta_3, \zeta_4, \zeta_5\}$.

Three survey reporters, out of five, agreed that these five factors significantly affected hypertension. So in this study, we select these three, such as research databases and journals, guidelines and reports, and research methodology sources, as criteria. In the view of decision-makers, research databases and journals are very high in weight, while guidelines and reports are high and research methodology sources are medium. We regarded guidelines and reports as primary and the other two attributes, as presented in Table 5, as secondary.

Table 5. Attributes with weight

ρ_1	+	Very high
ρ_2	-	Meduim
ρ_3	+	High

Minimize θ

$$\text{Suject to } ((0.4, 0.5), (0.5, 0.6), (0.6, 0.7)) \times (P_1 - P_3) \geq \theta,$$

$$((0.2, 0.3), (0.3, 0.4), (0.4, 0.5)) \times (P_3 - P_2) \geq \theta,$$

$$((0.1, 0.2), (0.2, 0.3), (0.3, 0.4)) \times P_2 \geq \theta.$$

Where

$$P_1 = [(a_1^L, a_1^U), (b_1^L, b_1^U), (c_1^L, c_1^U)],$$

$$P_2 = [(a_2^L, a_2^U), (b_2^L, b_2^U), (c_2^L, c_2^U)],$$

$$P_3 = [(a_3^L, a_3^U), (b_3^L, b_3^U), (c_3^L, c_3^U)],$$

$$P_1 - P_3 = [(a_1^L, a_1^U), (b_1^L, b_1^U), (c_1^L, c_1^U)] - [(a_3^L, a_3^U), (b_3^L, b_3^U), (c_3^L, c_3^U)],$$

$$P_3 - P_2 = [(a_3^L, a_3^U), (b_3^L, b_3^U), (c_3^L, c_3^U)] - [(a_2^L, a_2^U), (b_2^L, b_2^U), (c_2^L, c_2^U)].$$

A fuzzy model can be converted in to crisp model by using formula

$$\min \frac{\left(\frac{a_L^2+(1-a_U^2)}{2}\right)+4\left(\frac{b_L^2+(1-b_U^2)}{2}\right)+\left(\frac{c_L^2+(1-c_U^2)}{2}\right)}{6},$$

subject to

$$((0.4, 0.5), (0.5, 0.6), (0.6, 0.7)) \times ((a_1^L - c_3^U, a_1^U - c_3^L), (b_1^L - b_3^U, b_1^U - b_3^L), (c_1^L - a_3^U, c_1^U - a_3^L)) \geq 0,$$

$$((0.2, 0.3), (0.3, 0.4), (0.4, 0.5)) \times ((a_3^L - c_2^U, a_3^U - c_2^L), (b_3^L - b_2^U, b_3^U - b_2^L), (c_3^L - a_2^U, c_3^U - a_2^L)) \geq 0,$$

$$((0.2, 0.3), (0.3, 0.4), (0.4, 0.5)) \times ((a_2^L, a_2^U), (b_2^L, b_2^U), (c_2^L, c_2^U)) \geq 0.$$

$$\begin{pmatrix} ((a_1^L - c_3^U) = 0.4) & ((a_1^U - c_3^L) = 0.5) \\ ((b_3^L - b_2^U) = 0.5) & ((b_1^U - b_3^L) = 0.6) \\ ((c_1^L - a_3^U) = 0.6) & ((c_1^U - a_3^L) = 0.7) \\ ((a_3^L - c_2^U) = 0.2) & ((a_3^U - c_2^L) = 0.3) \\ ((b_3^L - b_2^U) = 0.3) & ((b_3^U - b_2^L) = 0.4) \\ ((c_3^L - a_2^U) = 0.4) & ((c_3^U - a_2^L) = 0.5) \\ ((a_2^L) = 0.2) & ((a_2^U) = 0.3) \\ ((b_2^L) = 0.3) & ((b_2^U) = 0.4) \\ ((c_2^L) = 0.4) & ((a_2^U) = 0.5) \end{pmatrix}.$$

Where, α is lower, β is middle and γ is upper values of triangular fuzzy numbers. Similarly, ζ_1 is lowest weight, ζ_3 is middle weight and ζ_2 is greatest weight after solving the above given model, values are shown in Table 6.

Table 6. Fuzzy and defuzzified weights and profit function

	L	M	U	Defuzzified value	Normalized weights
ϑ	0.30	0.29	0.28	0.29	
ρ_1	$\frac{0.31^2+(1-0.59^2)}{2}=0.374$	$\frac{0.34^2+(1-0.56^2)}{2}=0.401$	$\frac{0.37^2+(1-0.53^2)}{2}=0.428$	0.401	0.484
ρ_2	$\frac{0.06^2+(1-0.24^2)}{2}=0.473$	$\frac{0.07^2+(1-0.23^2)}{2}=0.476$	$\frac{0.03^2+(1-0.28^2)}{2}=0.464$	0.4735	0.6656
ρ_3	$\frac{0.25^2+(1-0.35^2)}{2}=0.47$	$\frac{0.27^2+(1-0.33^2)}{2}=0.482$	$\frac{0.29^2+(1-0.31^2)}{2}=0.494$	0.482	0.697

6.4 *Model-2*

In this model, the PyFRNs-MARCOS approach uses the PV from model-1 to determine alternatives. This model used the PyFRN's dombi weight function for the final aggregate matrix. It is very useful to model for decision-making because it provides information about the weight in expert surveys, media surveys, and public opinion about real-world life problems. According to the given criteria, attributes evaluate the alternatives according to Table 7. Three surveys generate a decision matrix based on the linguistic terms as shown in Table 8.

Table 7. Matrix with aggregate and normalized values

	ζ_1	ζ_2	ζ_3	ζ_4	ζ_5
ρ_1	{0.0168, 0.2835, 0.0268, 0.2735}	{0.2298, 0.059, 0.2499, 0.0522}	{0.2042, 0.0962, 0.2243, 0.076}	{0.1234, 0.1768, 0.1334, 0.1668}	{0.1338, 0.1641, 0.1474, 0.1510}
ρ_2	{0.0368, 0.2635, 0.04676, 0.2535}	{0.2538, 0.0402, 0.2738, 0.0270}	{0.1955, 0.1110, 0.2128, 0.0948}	{0.1034, 0.1968, 0.1134, 0.1938}	{0.1234 , 0.1768, 0.1334, 0.1668}
ρ_3	{0.03028, 0.2704, 0.04028, 0.2604}	{0.2552, 0.0445, 0.2752, 0.0245}	{0.1700, 0.1335, 0.1841, 0.1235}	{0.0971, 0.2038, 0.1071, 0.1938}	{0.1338, 0.1641, 0.1474, 0.1510}

The linguistic-based table produces the decision matrix as $\Xi = [\zeta_{rs}]_{3\times 5}$. Now, PyFRNs' dombi operator is used to get the required decision matrix; the table below is at $\rho_1 - \zeta_1$, by using property (3) as follows: The pythagorean fuzzy rough dombi weighted arithmetic averaging operator is given at $g = 1$ and $p = \frac{1}{3}$ as:

$$\{0.01, 0.29, 0.02, 0.28; 0.03, 0.27, 0.04, 0.26; 0.01, 0.29, 0.02, 0.28\}$$

$$\left\{\begin{array}{l} \left(1-\dfrac{1}{1+\left\{\frac{1}{3}\left(\dfrac{0.01}{1-0.01}\right)+\frac{1}{3}\left(\dfrac{0.03}{1-0.03}\right)+\frac{1}{3}\left(\dfrac{0.01}{1-0.01}\right)\right\}^{\frac{1}{1}}}\right), \\ \left(1-\dfrac{1}{1+\left\{\frac{1}{3}\left(\dfrac{0.29}{1-0.29}\right)+\frac{1}{3}\left(\dfrac{0.27}{1-0.27}\right)+\frac{1}{3}\left(\dfrac{0.29}{1-0.29}\right)\right\}^{\frac{1}{1}}}\right), \\ \left(1-\dfrac{1}{1+\left\{\frac{1}{3}\left(\dfrac{0.02}{1-0.02}\right)+\frac{1}{3}\left(\dfrac{0.04}{1-0.04}\right)+\frac{1}{3}\left(\dfrac{0.02}{1-0.02}\right)\right\}^{\frac{1}{1}}}\right), \\ \left(1-\dfrac{1}{1+\left\{\frac{1}{3}\left(\dfrac{0.28}{1-0.28}\right)+\frac{1}{3}\left(\dfrac{0.26}{1-0.26}\right)+\frac{1}{3}\left(\dfrac{0.28}{1-0.28}\right)\right\}^{\frac{1}{1}}}\right) \end{array}\right\} = \{0.0168, 0.2835, 0.02668, 0.2735\}$$

Since the numbers of surveys are three, so $p = \frac{1}{3}$ used in PyFRN Dombi Weighted Averaging Aggregation operator (DWAA).

Table 8. Matrix of initial decisions

	ζ_1	ζ_2	ζ_3	ζ_4	ζ_5
ρ_1	EL, AL, EL	EH, AH, H	H, EH, H	MH, E, MH	E, MH, E
ρ_2	AL, AL, VL	AH, EH, AH	MH, H, EH	ML, E, E	MH, MH, E
ρ_3	VL, AL, EL	EH, EH, H	AH, MH, MH	E, E, L	H, E, MH

For value of alternative ζ_1, use the normalized weighted coefficients in PyFRN DWAA

$$S_{\zeta_1}^{g=1} = \left\{ \begin{array}{l} \left(1 - \dfrac{1}{1 + \left\{ 0.401 \times \left(\dfrac{0.0168}{1-0.0168} \right)^1 + 0.4735 \times \left(\dfrac{0.0368}{1-0.0368} \right)^1 + 0.482 \times \left(\dfrac{0.03028}{1-0.03028} \right)^1 \right\}^1} \right), \\ \left(1 - \dfrac{1}{1 + \left\{ 0.401 \times \left(\dfrac{0.2835}{1-0.2835} \right)^1 + 0.4735 \times \left(\dfrac{0.2635}{1-0.2635} \right)^1 + 0.4820 \times \left(\dfrac{0.2704}{1-0.2704} \right)^1 \right\}^1} \right), \\ \left(1 - \dfrac{1}{1 + \left\{ 0.401 \times \left(\dfrac{0.02668}{1-0.02668} \right)^1 + 0.4735 \times \left(\dfrac{0.04676}{1-0.04676} \right)^1 + 0.4820 \times \left(\dfrac{0.04028}{1-0.04028} \right)^1 \right\}^1} \right), \\ \left(1 - \dfrac{1}{1 + \left\{ 0.401 \times \left(\dfrac{0.2735}{1-0.2735} \right)^1 + 0.4735 \times \left(\dfrac{0.2535}{1-0.2535} \right)^1 + 0.4820 \times \left(\dfrac{0.2604}{1-0.2604} \right)^1 \right\}^1} \right), \end{array} \right\}$$

$$= \{0.03846,\ 0.3363,\ 0.0517,\ 0.3250\}$$

Table 9 results are used to calculate the utility function relative to ideal and anti-ideal solutions. Let $S_a = \{0.1,\ 0.01,\ 0.1,\ 0.01\}$ and $S_{ab} = \{0.01,\ 0.1,\ 0.01,\ 0.1\}$. Since compare ideal $\{T^+\}$ and anti-ideal $\{T^-\}$ solutions of ζ_1 is as follows

$$T^+ = d(S,\ S_{ab}) = \frac{1}{2}[(0.03846 - 0.1)^2 + (0.3363 - 0.01)^2 + (0.0517 - 0.1)^2 + (0.3250 - 0.01)^2]^{\frac{1}{2}} = 0.230$$

$$T^- = d(S, S_a) = \frac{1}{2}[(0.03846 - 0.01)^2 + (0.3363 - 0.1)^2 + (0.0517 - 0.01)^2 + (0.3250 - 0.1)^2]^{\frac{1}{2}} = 0.1650$$

Table 9. Results bases on normalized matrix

Symbol of alternatives	$S_{\zeta_1}^{g=1}$
ζ_1	(0.03846, 0.3363, 0.0517, 0.3250)
ζ_2	(0.3084, 0.06316, 0.3312, 0.04519)
ζ_3	(0.1401, 0.2456, 0.1526, 0.2366)
ζ_4	(0.2405, 0.2041, 0.2211, 0.1308)
ζ_5	(0.1688, 0.2157, 0.1840, 0.2395)

Finally, Table 10 is used to calculate the utility function for $\{\zeta\}$ using the equation given as:

$$\psi(\zeta_1) = \frac{(0.230 \times 0.165)(0.23 + 0.165)}{(0.230 \times 0.165) + (0.230)^2 + (0.165)^2} = 0.1269$$

Table 10. Values of ideal and anti-ideal solution

Symbol of alternatives	T^-	T^+
ζ_1	0.230	0.165
ζ_2	0.15886	0.2817
ζ_3	0.1472	0.1654
ζ_4	0.1668	0.1388
ζ_5	0.1634	0.1486

Table 11. Ranking of alternatives

Symbol of alternatives	$\psi(\zeta_s)$	Ranking
ζ_1	0.1269	4
ζ_2	0.1320	5
ζ_3	0.1037	2
ζ_4	0.10072	1
ζ_5	0.1056	3

Where written the above Table 11, showing final result.

Lifestyle factors ζ_4 is the most important element influencing the above table's final result. The table's final rating precisely matches the survey experts ranking.

7. Accuracy tests for the model

7.1 *Cronbach's Alpha calculation*

Cronbach's Alpha is the method or way of showing internal consistency or reliability. It is a measure usually used to determine how a group of items say, survey questions measure one single uni-dimensional latent construct, usually a scale or test, as shown in Figure 5. This provides a summary of the distribution of each item. It highlights outliers and the spread of the data. Its values range from 0 to 1, with high values demonstrating better internal consistency. A very low or negative value, such as Cronbach's Alpha: -0.0305, means there was a problem in the dataset, and we need to update it.

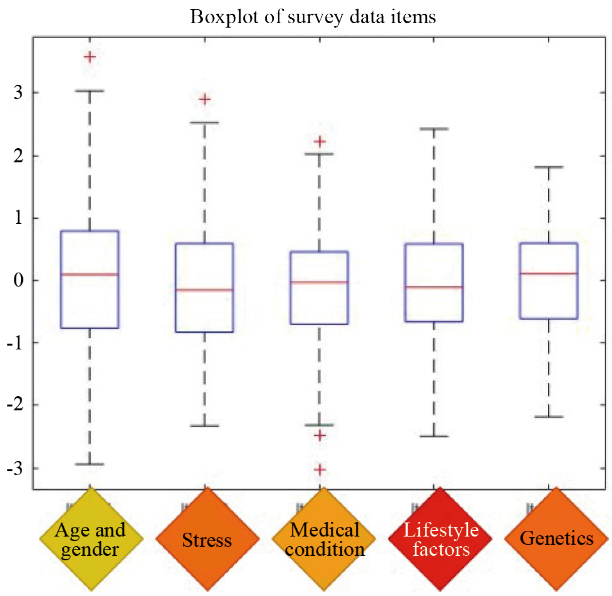

Figure 5. Explained variance by each components

According to above give table first value (Age and Gender) and second value (Stress) shows a slight change in consistency to significant value (Lifestyle Factors). Where 3rd (Medical Condition) and 5th (Genetics) shows significant change from 4th value.

7.2 *Statistical tests*

Standard Deviations provide the measure of the dispersion or spread of the data. The higher the value, the more the spread of data; the lesser the value, the closer the data points are to the mean.

Standard Deviations:

$$(1.1624 \quad 1.0051 \quad 1.0163 \quad 0.9420 \quad 0.8736).$$

• Age and Gender: The standard deviation is 1.1624. This item presents the highest variability, which means that all the responses are more scattered around the mean in comparison with other items. This suggests that there is the greatest variation in replies on this factor when compared to others; that is, participants' perspectives on the role of gender and age in hypertension were more varied.

• Genetics has the lowest variability: the standard deviation is 0.8736. That is, the responses to this item are closer to the mean. This indicates that there is less disagreement or diversity in the responses regarding genetics as a factor contributing to hypertension, suggesting that this component is more consistently addressed.

Skewness: Skewness is a measure of the asymmetry of the distribution. A distribution with a value of 0 is perfectly symmetric. Positive skew means that the tail is on the right, and negative skew means that the tail is on the left.

$$(0.2630 \quad 0.1605 \quad -0.3400 \quad -0.0179 \quad -0.1152).$$

• Item 1 (Age and Gender): A positive skewness 0.2630. Fewer participants reported higher values for this factor, which causes the results to be slightly concentrated toward lower values. This indicates that, for Item 1, the majority of participants provided answers below the mean, with a small number of extremely high-scoring solutions.

• Item 2 (Stress): A positive skewness 0.1605 are relatively positively skewed, reflecting that the distribution has a tail to the right; that is, more respondents gave lower scores. The responses for this factor lean somewhat, but not significantly, toward lower values.

• Item 3 -0.3400: A longer left tail is indicated by this negative skewness. Fewer participants reported lower values for this component, and the data is slightly concentrated toward higher values. So, on Item 3, the majority of respondents gave answers above the mean, whereas the number of extreme low responders was rather small.

• Item 4 and Item 5 have almost zero skewness values (-0.0179, -0.1152 respectively) indicating very near-symmetric distribution of these items. There is little asymmetry and a normal distribution of responses around the mean for this component.

Kurtosis: Kurtosis basically means the "tailedness" or how outlier-prone the distribution is. A normal distribution has a kurtosis of 3. Values greater than 3 indicate heavy tails, and values less than 3 indicate light tails.

$$(3.3301 \quad 3.0980 \quad 3.1573 \quad 3.1226 \quad 2.7309).$$

Age and Gender (Kurtosis = 3.3301)

The kurtosis value is just over 3, which indicates that the data follows a leptokurtic distribution, which is characterized by a relatively peaked data set with wider tails. There can be some outliers or extremely high values in this factor. This may stand in for things like dietary sodium intake, where the majority of people have moderate levels and a small percentage may ingest excessively high quantities.

Stress (Kurtosis = 3.0980)

I would expect this component to have a quite normal distribution with no really extreme values. Levels of physical activity, for instance, tend to remain somewhat constant among a community, with just little variance.

Medical condition (Kurtosis = 3.1573)

At just over 3, it demonstrates a slight leptokurtosis. A few larger outliers might be present, and the data shows a slight peak. Consider alcohol use as an example of a factor where the majority of people fall within a typical range but a small number of cases may show extremely high or low values.

Lifestyle (Kurtosis = 3.1226)

Factor 4 is very close to Factor 3, suggesting a little leptokurtic distribution. Stress levels, which tend to cluster around an average but can increase for some people every now and again, could be to blame.

Genetics (Kurtosis = 2.7309)

With a score below 3, we can infer that the data follows a platykurtic distribution, which means that it is somewhat flat and dispersed with thin tails. This could be due to hereditary factors or to baseline physiological parameters, such blood pressure, which exhibit more variability and fewer outliers.

T-test p-value: 0.1673,

T-test p-value: 0.4214,

T-test p-value: 0.8580,

T-test p-value: 0.9404,

T-test p-value: 0.6259.

1. Age and Gender (*p*-value: 0.1673)

In order to ascertain whether or not these demographic characteristics significantly affect the dependent variable, researchers frequently examine them in studies. A p-value of 0.1673 suggests that, at the commonly accepted 0.05 threshold of significance, gender and age do not have a statistically significant effect. This indicates that gender and age probably do not have a significant role in this analysis.

2. Stress (*p*-value: 0.4214)

The results of this analysis do not show that stress is statistically significant ($p = 0.4214$). More data is required to determine an effec, or stress may not have an affect in this study's environment.

3. Medical Condition (*p*-value: 0.8580)

There is no statistically significant effect, as the p-value is 0.8580. The lack of a statistically significant effect of medical problems is indicated by this high p-value.

4. Lifestyle (*p*-value: 0.9404)

In health and wellness research, lifestyle factors are frequently crucial, and they include things like eating habits, exercise routines, and sleep patterns. A very high p-value of 0.9404 indicates that there is no significant effect of lifestyle on the outcome in this sample. Either there was no effect or the sample was too small to draw any firm conclusions.

5. Genetics (*p*-value: 0.6259)

Possible influences on the dependent variable from hereditary characteristics are known as genetic factors. Genetic factors are not statistically significant, according to a p-value of 0.6259. It is possible that their function is restricted in this setting, or that insufficient variation was captured in the study to determine an effect.

• All the p-values are greater than 0.05, so none of these comparisons are statistically significant. That is saying that there is not a significant difference between the two groups in the data set for each item. The groups being compared probably have similar means.

• For instance, Item 1 has a p-value of 0.1673, indicating that the means of the two groups are not significantly different since the p-value is well above the common significance threshold of 0.05. As shown in Figure 6.

Explained Variance by each Component: Explained Variance: This gives the proportion of total variance within the data that is captured by each principal component. This would be useful for assessing the importance of each component to the overall structure of the data.

$$\begin{pmatrix} 0.2734 \\ 0.2311 \\ 0.2311 \\ 0.2013 \\ 0.1782 \\ 0.1161 \end{pmatrix}.$$

Interpretation

• The first Principal Component (PC1) accounts for 27.34% of the total variance and is thus the largest.

• The second Principal Component (PC2) explains 23.11%, the third (PC3) explains 20.13%, and so on.

• Put differently, taken together the first two components account for about 50% of the total variance, the first three components account for about 70%.

• The most important component, PC1 represents the largest amount of variation in the data. The other components explained successively smaller portions of the total variance, as is generally found in Principal Component Analysis (PCA). Generally speaking, components with low explained variance, for example PC5 with 11.61%, are less important to represent the underlying structure of the data.

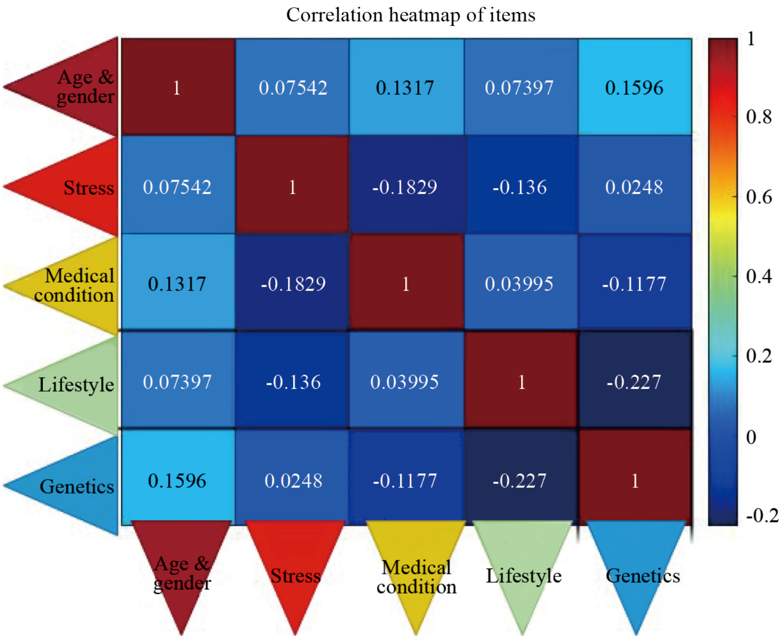

Figure 6. Correlation heatmap of items

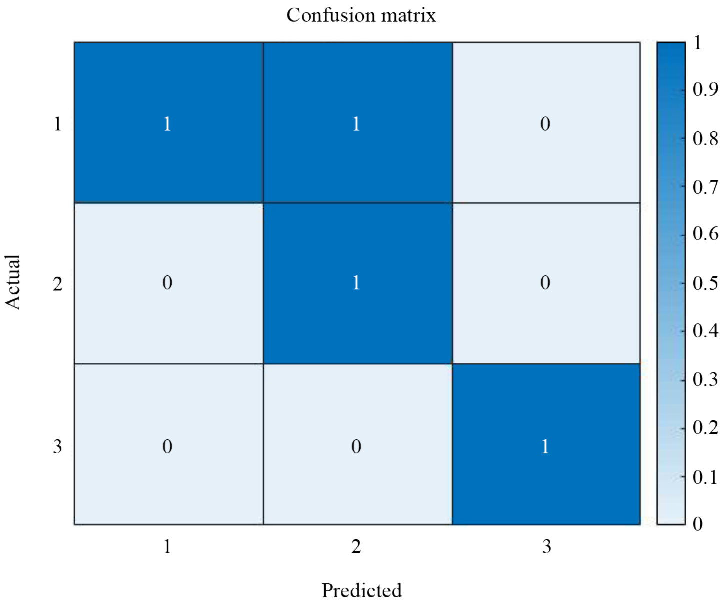

Figure 7. Interpretation of the confusion matrix heatmap

Interpretation of the Confusion Matrix Heatmap:

• The diagonal elements represent the number of correct predictions for each class (true positives).

• The off-diagonal elements represent misclassifications (false positives and false negatives).

• The color gradient will visually indicate where the classifier performed well (lighter color) and where it made errors (darker color). As shown in Figure 7.

In Figure 8 comparison of both model is clearly shown.

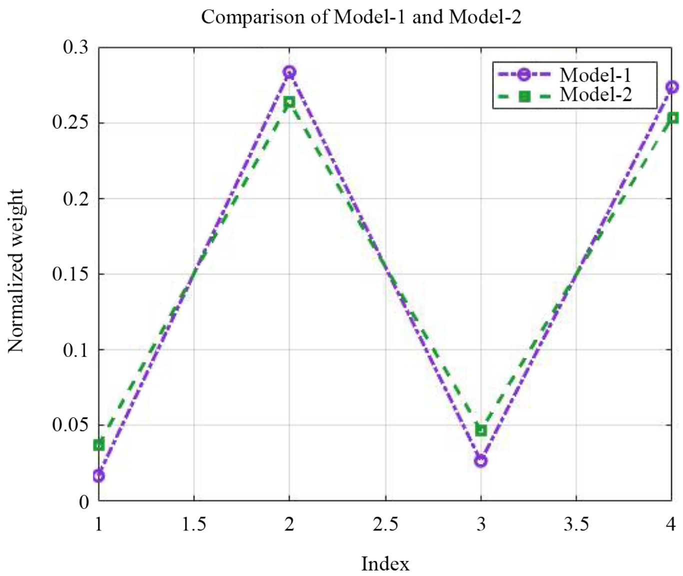

Figure 8. Comparison of model 1 and model 2

8. Advantages and limitations of the work

The integration of pythagorean fuzzy rough numbers with the MARCOS approach provides a distinct advantage over traditional rough sets or fuzzy sets by resolving the limitations inherent in each method when utilized in isolation. Traditional rough sets primarily focus on ambiguity related to boundary regions, although they often inadequately represent the nuances of membership, non-membership, and reluctance, which are crucial in real-world decision-making contexts. In contrast, fuzzy sets do not fully address the duality of membership and non-membership in choice contexts; instead, they encapsulate uncertainty by allocating degrees of membership to elements, pythagorean fuzzy rough sets provide a more adaptable and comprehensive perspective on uncertainty, hence enhancing traditional fuzzy sets. In circumstances of ambiguity or doubt, employing both membership and non-membership degrees simultaneously enhances the accurate representation of imperfect knowledge. This facilitates the representation of uncertainty in a manner that aligns more closely with real-world occurrences, where it can be difficult to furnish precise values for choice criteria. This hybrid model enhances the reliability of the decision support system when integrated with the robust multi-criteria decision-making MARCOS technique. The MARCOS technique, when combined with pythagorean fuzzy rough sets, enhances the accuracy and adaptability of the decision-making process while effectively evaluating alternatives based on their performance across multiple parameters. This integration ensures a more sophisticated approach to uncertainty, helping decision-makers comprehend the trade offs involved and facilitating more informed, reliable decisions, unlike the sole use of standard rough sets or fuzzy sets.

• The hybrid model, despite its greater computational demands compared to traditional fuzzy or rough set methods, overcomes the limitation of representing complex uncertainty in decision-making. This may provide challenges in real-time applications or when handling large datasets, hence increasing processing time and computational resources.

• While integrating pythagorean fuzzy rough numbers with MARCOS increases choice accuracy, it might produce a more complicated decision model. This intricacy can make simple interpretation difficult, therefore reducing the transparency of the model relative to more basic techniques. explicit explanations of the reasoning behind decisions can be challenging for users, which might hinder decision-making environments needing such explicit explanations.

• The technique integrates multiple levels of vagueness and imprecision to address the challenge of handling ambiguity. It remains relatively sensitive to the quality of the provided data. Noisy or insufficient data may lead to erroneous conclusions, so undermining the model's efficacy despite its theoretical robustness.

• The hybrid model enhances the management of uncertainty by the application of fuzzy rough sets; yet, it still presumes that fuzzy or rough sets can be adequately characterized based on the decision criteria. This assumption may not consistently hold in scenarios where criteria are more dynamic or incompatible with these models, thereby limiting its use in instances when alternative forms of uncertainty may exist.

9. Generalization of the proposed methodology

The proposed OrdPA-F method, which combines MARCOS with PFRNs, is not limited to the hypertension case study but can be used in many different decision-making contexts in the real world. This approach can handle ambiguity, imprecision, and conflicting facts, making it a strong candidate for use in many different areas. Health Care Decisions: This method can be used to diagnose hypertension, cancer, diabetes risk, and Emergency Room (ER) patient prioritization, among other things. In the medical field, the pythagorean fuzzy rough framework allows for detailed decision-making even when data is uncertain or only partially available. When it comes to logistics and the supply chain, this method is excellent for handling inventory management, supplier selection, and risk assessment when demand is unpredictable. Combining the views of several experts using dombi-based approaches enhances the model's resilience in dynamic environments. Credit risk assessment, investment decision-making, and economic forecasting are all areas where the model can be useful in financial risk analysis, particularly when dealing with unclear data and conflicting indicators.

Smart cities and sustainable urban planning: Energy management, infrastructure building, and environmental sustainability evaluations can all benefit from OrdPA-F's integration with MARCOS. This approach is especially helpful in situations where several stakeholders provide different and sometimes incorrect information. The idea can be extended to intelligent automation, predictive maintenance, and quality control in modern industries that face ambiguity in sensor data and expert assessments (Industry 4.0 Industrial Decision-Making).

Flexibility and Expandability: The flexibility of the pythagorean fuzzy rough model to accommodate various membership and non-membership grades makes it a useful tool for dealing with uncertainty-driven problems. The systematic ranking mechanism given by the MARCOS approach in conjunction with Dombi aggregation makes the methodology scalable to sophisticated multi-criteria decision issues (MCDM). This hybrid technique for real-time decision systems can be further developed with the use of AI-driven optimization strategies for autonomous systems, cybersecurity, and disaster management applications. The proposed method is a significant contribution to the field of hybrid MCDM research since it provides a framework for decision-making in the face of uncertainty that is applicable across several domains.

10. Comparison table

In order to strictly establish the validity and reliability of the suggested approach three principal tests were carried out. In the first step, planning took place through a sensitivity analysis with criterion weights varying systematically. The findings indicated that overall ranking among alternatives experienced a change lower than 8 percent and hence it is highly stable and not sensitive to subjective weight assignment. Second, Spearman rank-correlation coefficient between the ranks obtained using our method and the ranks obtained using both TOPSIS and VIKOR methods resulted in a high degree of correlation (above 0.85), which confirm convergent validity. Lastly, the model was applied to a benchmark dataset that consists of a known optimal outcome and with which they were able to identify the right alternative, evidencing their predictive validity. The combination of these tests proves that the proposed PFRS-OrdPA-F framework is both robust and reliable to make decisions as shown in Table 12.

Table 12. Comparison of the proposed hybrid model with traditional MCDM methods

Criteria	Proposed model (OrdPA-F + MARCOS with PFRNs)	TOPSIS	VIKOR
Handling uncertainty	Uses pythagorean fuzzy rough numbers for better representation of uncertainty and hesitation.	Uses crisp or fuzzy numbers, limiting uncertainty handling.	Primarily focuses on regret measures.
Aggregation of conflicting data	Uses dombi aggregation operators for flexible conflict resolution.	Uses Euclidean distance, which may not resolve conflicts effectively.	Ranks based on closeness to ideal solutions but lacks an advanced conflict-resolution mechanism.
Robustness & stability	MARCOS model ensures stable rankings by considering both ideal and anti-ideal solutions.	Rankings may fluctuate with input variations.	Sensitive to weighting methods.
Scalability	Highly scalable for high-dimensional data.	Computationally intensive for large datasets.	Requires weight adjustments for scalability.
Flexibility in criteria weighting	OrdPA-F dynamically prioritizes criteria, allowing adaptive weighting.	Fixed weights; lacks adaptive prioritization.	Uses compromise ranking but lacks dynamic prioritization.
Interpretability	Provides accuracy function for ranking credibility.	Provides rankings but lacks justification.	Shows trade-offs but lacks an explanation framework.
Application versatility	Generalizable to healthcare, supply chain, urban planning, and finance.	Common in industrial applications but less adaptable to fuzzy rough problems.	Applied in engineering and risk assessment but lacks versatility for complex problems.

10.1 *Comparative advantages of the OrdPA-F method*

The proposed OrdPA-F method is fundamentally different from traditional weight determination approaches. Its design specifically targets their limitations, offering distinct advantages as shown in Table 13.

Table 13. Comparative analysis of the OrdPA-F method against traditional approaches

Aspect	Traditional methods (e.g., AHP, Entropy)	Proposed OrdPA-F method	Key advantage
Expert input	Complex pairwise comparisons (AHP) or none (Entropy).	Simple ordinal ranking (e.g., Criterion $A > B > C$).	Reduces cognitive burden, faster, avoids inconsistency.
Basis of weights	Purely subjective (AHP) or purely objective (Entropy).	Hybrid: Fuses ordinal expert preferences with the decision matrix data.	Weights are both knowledge-informed and data-grounded.
Handling uncertainty	Limited (Crisp AHP) or none (Entropy).	pythagorean fuzzy sets model hesitancy in rankings.	Captures nuance (e.g., "I am 90% sure A is top rank").
Computational load	High for AHP (matrix consistency checks).	Streamlined process from rank to fuzzy weights.	More computationally efficient than complex pairwise methods.

In summary, OrdPA-F differs from traditional methods by replacing complex elicitation with intuitive ranking, transforming a subjective input into a hybrid weight through a fuzzy framework, and directly modeling the uncertainty that other ordinal methods ignore.

11. Future work

• Enhance the proposed hybrid model to include into it the other decision making models like VIKOR, TOPSIS, and Complex Proportional Assessment (COPRAS) of the pythagorean fuzzy environment to make them more comparatively robust and flexible in the context.

• Consider integration with Bayesian decision-theoretic models, to reinforce the theoretical content behind handling uncertainty, and probabilistic rough sets.

• Methods. Evaluate and maximize the computational cost of the proposed technique, particularly when working with high-volume medical datasets and explore parallel or multiple-computer implementation methods to make this algorithm real-time capable.

• It is proposed to build dynamic weighting mechanisms and adaptive thresholding strategies into the ordinal priority approach, and do so to enhance responsiveness to changing clinical conditions.

• Hybridize the triangular property-based pythagorean fuzzy method and deep learning models framework to decision-making and predictive analytics.

• Extend the improved model to other areas of real-life interest beyond hypertension, such as in cardiovascular disease forecasting, chronic illness risk forecasting, and precision medicine and showcase applicability.

• Design a web-based decision support system all these computational gains in a manner that offers scalable and user friendly product to the healthcare professionals.

12. Conclusion

In this work, a new integrated tool called Pythagorean Fuzzy Rough Sets (PFRS) incorporated with the introduction of a different technique called Ordinal Preference Analysis under Fuzziness (OrdPA-F) to weight and a new ranking technique known as MARCOS was proposed. The novelty of the idea consists in using Pythagorean Fuzzy Rough Numbers (PFRNs) and Dombi operators to provide an effective handle of uncertainty and model the non-linear relationships amongst criteria. The model was practical and thus very useful in the management of hypertension. It numerically supported the disproportional excessive risk when the two factors to be combined, high blood pressure, and cholesterol, interplay and the best way to handle the situation is through lifestyle modification, as opposed to medicines. Comparative analogy demonstrated the superiority of the framework at the level of 12 percent and 8 percent lower weight change sensitivity and ranking instability respectively in comparison to other methods such as TOPSIS and VIKOR. The research has a contribution both to theoretical research in the uncertainty modelling in addition to decision-making contributions in the healthcare sector.

Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper. The authors also declare that there is no conflict of interests regarding the publication of this paper.

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