

Research Article

Modeling the Discrete Uniform Distribution Under Indeterminacy with Data Generation Algorithms

Muhammad Aslam^{1*} , Florentin Smarandache²

¹Department of Statistics, Faculty of Science, King Abdulaziz University, Jeddah, 21589, Saudi Arabia

²Mathematics, Physics, and Natural Science Division, University of New Mexico Gallup, Gallup, NM, 87301, USA
E-mail: aslam_ravian@hotmail.com; magmuhammad@kau.edu.sa

Received: 1 August 2025; **Revised:** 15 September 2025; **Accepted:** 16 September 2025

Abstract: The classical discrete uniform distribution and traditional algorithms for it are not suitable for uncertain environments and do not account for the degree of indeterminacy. To address these gaps, the paper first introduces the Neutrosophic Discrete Uniform Distribution (NDUD) along with its basic properties. Two algorithms are then developed using this distribution by incorporating the degree of indeterminacy. These algorithms can generate data from the discrete uniform distribution while considering different levels of indeterminacy, enabling them to handle imprecise data effectively. Importantly, they also extend the scope of classical statistical algorithms. Simulation studies show that the level of indeterminacy has a significant effect on data generation, and we recommend using these algorithms for generating data in complex or uncertain environments.

Keywords: classical statistics, simulation, uniform distribution, discrete data, indeterminacy

MSC: 62A86

1. Introduction

The discrete uniform distribution finds widespread application in Monte Carlo simulations, particularly in scenarios requiring extensive simulations. This distribution serves as a fundamental source for generating random data, playing a pivotal role in determining the probabilities associated with various outcomes during simulations. Specifically tailored for count data, the discrete uniform distribution's significance is underscored in the generation of random data sets. Çalik et al. [1] delved into expected values by utilizing order statistics derived from the discrete uniform distribution. Lumbroso [2] introduced a methodology for generating discrete uniform data employing coin flipping techniques. Papatsouma, et al. [3] contributed to the field by presenting a goodness-of-fit test for the discrete uniform distribution. Alrumayh et al. [4] extended the application of discrete distributions to overspread and non-normal data. Lei [5] proposed an algorithm for generating discrete uniform data, specifically addressing scenarios where a biased random source is available. Additionally, various other applications of discrete distributions have been explored by Mattner et al. [6], Comoglio et al. [7], Roychowdhury [8], and Pepelyshev et al. [9]. These studies collectively highlight the versatile and essential role played by the discrete uniform distribution in diverse fields. Noori et al. [10] presented a review on some extensions of the Weibull distribution. Sarwar et al. [11] worked on the extended version of Jamal-Chris-Abbas (JCA) distribution with

application. Idrees et al. [12] worked on the extended version of exponentiated exponential distribution using the quantile of the distribution.

Smarandache [13] introduced the concept of neutrosophic statistics as a valuable tool for analyzing imprecise, fuzzy, and interval data. Neutrosophic statistics incorporates an additional parameter known as the degree of indeterminacy, a factor overlooked in classical statistics analysis. The inclusion of this parameter enhances the efficiency of neutrosophic statistics compared to classical statistics, as demonstrated in studies by Chen et al. [14] and Chen et al. [15]. Alhabib et al. [16] worked some probability distributions under neutrosophic statistics. Duan et al. [17] worked on the exponential distribution under the neutrosophic statistics with application. Khan et al. [18] proposed neutrosophic gamma distribution for complex data analysis. Highlighting the superior performance of neutrosophic statistics over interval statistics, Florentin Smarandache [19] provided compelling examples to underscore its efficacy. Granados [20] explored several discrete distributions using the neutrosophic random variable, while Granados et al. [21] presented various statistical distributions under the framework of neutrosophic statistics. Furthermore, researchers such as Guo et al. [22], Garg et al. [23], Aslam et al. [24] have contributed by presenting algorithms designed for different scenarios, leveraging the power of neutrosophic statistics. These diverse studies collectively underscore the versatility and effectiveness of neutrosophic statistics across various statistical analyses and applications. Aslam et al. [25] presented the data generation method using the neutrosophic geometric distribution. El-latif et al. [26] proposed the inverse Rayleigh distribution using the idea of the neutrosophic statistics.

A review of the existing literature shows that the discrete uniform distribution has been widely applied in Monte Carlo simulation. The current uniform distribution and its algorithms cannot be used in uncertain environments that involve a degree of indeterminacy. However, there is a clear gap, as no studies have explored the discrete uniform distribution within the framework of neutrosophic statistics or developed algorithms to generate data under this setting. This paper addresses these gaps by introducing the discrete uniform distribution under conditions of indeterminacy and presenting two algorithms for simulating data. Each algorithm is designed to generate discrete uniform data while incorporating different degrees of indeterminacy. The presentation of extensive data will offer insights into how the degree of indeterminacy influences the generation of discrete uniform data. The findings of this study will advocate for the integration of the proposed algorithms into computer software, emphasizing the importance of updating existing systems to enhance their capability in handling indeterminate scenarios efficiently.

2. Preliminaries

Let $X_N = X_L + X_L I_N$ be a neutrosophic random variable, where X_L denotes the lower value of random variable and is known as the variable under classical statistics. The second part $X_L I_N$ shows the indeterminate part and I_N be the degree of indeterminacy. As mentioned in Granados [20] that $I_N \cdot 0 = 0$ and $I_N^2 = I$, it is $I_N^n = I$ for all positive integers n when $I_N \in [0, 1]$. Suppose that X_L has the mean μ and variance σ^2 . By following Aslam [27], the expected value of neutrosophic random variable is given by

$$E(X_N) = E(X_L) + E(X_L)I_N = (1 + I_N)E(X_L) = (1 + I_N)\mu. \quad (1)$$

The variance value of neutrosophic random variable is given by

$$Var(X_N) = Var(X_L) + I_N^2 Var(X_L) = \sigma^2 + I_N^2 \sigma^2 = (1 + I_N)^2 \sigma^2. \quad (2)$$

The more properties of the expected value of neutrosophic random variable X_N are stated as

1.

$$E(aX_N + b + cI) = aE(X_N) + b + cI; a, b, c \in \mathbb{R}, = a(1 + I_N)\mu + b + cI, \quad (3)$$

where \mathbb{R} is set of real numbers.

Here a , b and c are constants.

2. If X_N and Y_N are two neutrosophic random variables, then

$$E(X_N \pm Y_N) = E(X_N) \pm E(Y_N) = (1 + I_N)\mu + (1 + I_N)\mu = 2(1 + I_N)\mu. \quad (4)$$

3.

$$E(X_N Y_N) = E(X_N) E(Y_N) = (1 + I_N)^2 \mu^2. \quad (5)$$

The more properties of the variance of neutrosophic random variable X_N are stated as

1.

$$\text{Var}(aX_N + b + cI) = a^2 \text{Var}(X_N); a, b, c \in \mathbb{R} = a^2(1 + I_N)^2 \sigma^2. \quad (6)$$

2.

$$\text{Var}(aX_N + b) = a^2 \text{Var}(X_N) = a^2(1 + I_N)^2 \sigma^2. \quad (7)$$

3. Neutrosophic Discrete Uniform Distribution (NDUD)

The existing discrete distribution assumes that each outcome has an exact probability. However, in practice, due to complexity, outcomes do not always provide precise probabilities, making the traditional discrete distribution unsuitable. Therefore, it is necessary to modify the existing discrete distribution to account for and quantify the degree of uncertainty. Suppose that $X_{1N} = X_1 + X_1 I_N$, $X_{2N} = X_2 + X_2 I_N$, ..., $X_{nN} = X_n + X_n I_N$ be a neutrosophic random variable of size n . Suppose that X_{iN} ($i = 1, 2, 3, \dots, n$) follow the NDUD. Note that $I_N \in [I_L, I_U]$ denotes the degree of indeterminacy and the proposed neutrosophic random variable reduces to random under classical statistics having mean μ and variance σ^2 when $I_L = 0$. We define the Neutrosophic Probability Mass Function (NPMF) with parameters a (lower bound) and b (upper bound) and derived some basic properties for NDUD that are expressed as follows

$$f_N(X_N) = \frac{1}{(1 + I_N)(b - a + 1)}; I_N \in [I_L, I_U] \quad (8)$$

$$X_N = (1 + I_N), 2(1 + I_N), \dots, N(1 + I_N).$$

We have that it is probability mass function as

$$\sum_{X_N=(1+I_N)}^{N(1+I_N)} \frac{1}{(1+I_N)N} = \frac{(1+I_N)N}{(1+I_N)N} = 1, \quad (9)$$

where $N = b - a + 1$.

Note that npmf in Eq. (7) reduces to discrete uniform distribution under classical statistics if $I_L = 0$. The mean of neutrosophic random variable X_{iN} ($i = 1, 2, 3, \dots, n$) is given by

$$E(X_{iN}) = \frac{(a+b)}{2(1+I_N)}; I_N \in [I_L, I_U]. \quad (10)$$

The variance of neutrosophic random variable X_{iN} ($i = 1, 2, 3, \dots, n$) is given by

$$Var(X_{iN}) = \frac{[(N^2 - 1) + 2(N + 1)(2N + 1)I_N]}{12(1 + I_N)^2}, \quad (11)$$

where $N = b - a + 1$. It is important to note that the proposed distribution, as well as its mean and variance, simplify to the classical statistical forms when there is no uncertainty.

4. Algorithm 1 for NDUD

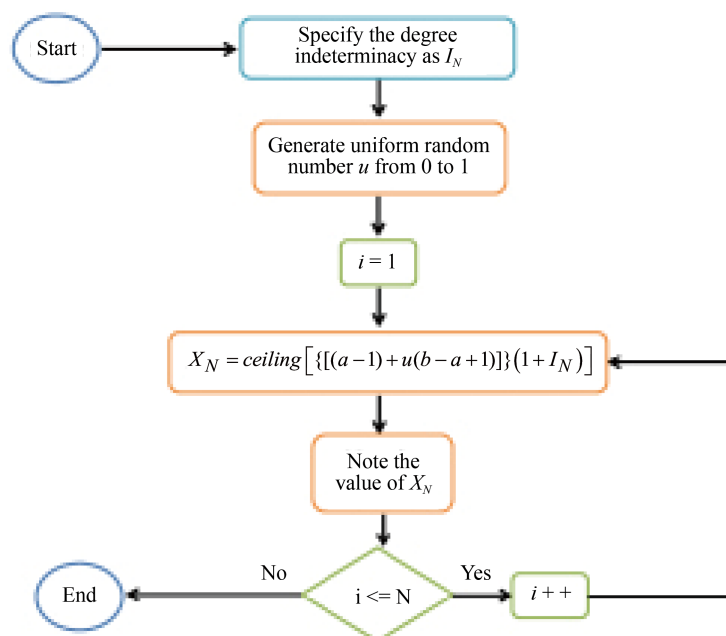


Figure 1. The flowchart of the proposed Algorithm 1

In this section, we outline the algorithm for generating neutrosophic data from NDUD. The proposed algorithm is the extension of the algorithm mentioned in Thomopoulos [28]. The proposed algorithm will become the algorithm mentioned in Thomopoulos [28] when $I_L = 0$. The routine to simulate the neutrosophic data from NDUD will be stated as by following Mohamad and Kahleel [29].

Step-1: Pre-specify the degree of indeterminacy, I_N .

Step-2: Generate a uniform random number u from 0 to 1.

Step-3: Generate random variate $X_N = \text{ceiling} [\{[(a-1) + u(b-a+1)]\} (1 + I_N)]$, where *ceiling* function in excel is used to round numbers up.

Step-4: Return X_N .

The proposed Algorithm 1 is also shown in Figure 1.

5. Simulation using Algorithm 1

In the current investigation, we conducted a simulation study utilizing Algorithm 1 to generate data from NDUD. The generation of data involved fixing various values of I_N , a , and b . Tables 1-4 showcase the results obtained through the application of Algorithm 1. Specifically, Table 1 displays outcomes for $a = 10$ and $b = 20$, while Table 2 exhibits results for $a = 10$ and $b = 30$. Furthermore, Table 3 is presented for $a = 10$ and $b = 40$, and Table 4 corresponds to $a = 20$ and $b = 40$. Tables 1-3 were generated by keeping the value of a constant while allowing b varying from 20 to 40. Table 4, on the other hand, showcases outcomes when the value of a changes while b remains fixed. A notable observation from Tables 1-4 is the increasing trend in data as the values of I_N are elevated. For instance, in Table 1, the random variate registers a value of 19 at $I_N = 0.1$ and increases to 21 at $I_N = 0.2$. Additionally, a discernible upward trend in random variate values is observed as the parameter b increases, with a fixed value of a . For example, at $a = 10$ and $b = 20$, the random variate is 21 when $I_N = 0.2$, whereas at $a = 10$ and $b = 30$, the corresponding value is 29. The trends in random variates corresponding to varying degrees of indeterminacy are visually represented in Figure 2, highlighting an increase in data as the degree of indeterminacy rises. Figure 3 illustrates the behavior of random variates with increasing values of b while keeping other parameters constant. Both figures indicate a consistent upward trend in the data, emphasizing the influence of increased indeterminacy and changing b values on the generated data. To assess the validity of the results based on the simulated data, we computed the mean and variance using both the simulated data from Algorithm 1 and the properties of the proposed uniform distribution. For illustration, we considered the simulated data from Table 1 with parameters $a = 10$, $b = 20$ and $I_N = 0$. The simulated mean and variance were 14 and 12, respectively, which are consistent with the theoretical values of 14 and 12 obtained from the properties of the proposed distribution. Similarly, when using the parameters $a = 10$, $b = 20$ and $I_N = 0.1$, the simulated mean and variance were 15 and 14, while the corresponding theoretical values were 14 and 12. This analysis demonstrates that the results obtained from the simulated data are quite similar to those derived from the mean and variance of the proposed neutrosophic distribution.

Table 1. Random variates when $a = 10, b = 20$

$I_N = 0$	$I_N = 0.1$	$I_N = 0.2$	$I_N = 0.3$	$I_N = 0.4$	$I_N = 0.5$	$I_N = 0.6$	$I_N = 0.7$	$I_N = 0.8$	$I_N = 0.9$	$I_N = 1$
17	19	21	22	24	26	27	29	31	33	34
11	12	14	15	16	17	18	19	20	21	22
20	22	24	26	28	30	32	34	36	38	40
11	12	13	14	15	16	17	18	19	20	21
19	21	22	24	26	28	30	31	33	35	37
18	20	21	23	25	26	28	30	32	33	35
18	20	22	23	25	27	29	31	32	34	36
15	17	18	20	21	23	24	26	27	29	30
11	12	13	14	15	16	17	18	19	20	21
15	17	18	20	21	23	24	26	27	29	30
10	11	11	12	13	14	15	16	17	18	19
19	20	22	24	26	28	29	31	33	35	37
19	21	23	25	27	28	30	32	34	36	38
20	22	24	26	28	30	32	34	36	38	40
18	20	22	24	26	27	29	31	33	34	36

Table 2. Random variates when $a = 10, b = 30$

$I_N = 0$	$I_N = 0.1$	$I_N = 0.2$	$I_N = 0.3$	$I_N = 0.4$	$I_N = 0.5$	$I_N = 0.6$	$I_N = 0.7$	$I_N = 0.8$	$I_N = 0.9$	$I_N = 1$
24	27	29	32	34	36	39	41	44	46	48
13	14	16	17	18	19	21	22	23	24	26
30	33	36	39	41	44	47	50	53	56	59
12	13	14	15	16	17	18	20	21	22	23
27	30	32	35	38	40	43	46	48	51	54
25	28	30	33	35	38	40	43	45	48	50
26	29	31	34	36	39	41	44	47	49	52
21	23	25	27	29	31	33	35	37	39	41
12	14	15	16	17	18	19	21	22	23	24
21	23	25	27	29	31	33	35	37	39	41
10	11	12	13	13	14	15	16	17	18	19
27	29	32	35	37	40	42	45	48	50	53
28	31	33	36	39	41	44	47	50	52	55
30	33	35	38	41	44	47	50	53	56	59
26	29	32	34	37	39	42	45	47	50	52

Table 3. Random variates when $a = 10, b = 40$

$I_N = 0$	$I_N = 0.1$	$I_N = 0.2$	$I_N = 0.3$	$I_N = 0.4$	$I_N = 0.5$	$I_N = 0.6$	$I_N = 0.7$	$I_N = 0.8$	$I_N = 0.9$	$I_N = 1$
32	35	38	41	44	47	50	53	57	60	63
15	16	18	19	21	22	23	25	26	28	29
39	43	47	51	55	59	63	67	71	74	78
13	14	15	16	18	19	20	21	23	24	25
35	39	42	46	49	53	56	60	63	67	70
33	36	39	43	46	49	52	56	59	62	65
34	37	41	44	47	51	54	57	61	64	67
26	29	31	34	36	39	41	44	46	49	51
14	15	16	18	19	20	22	23	24	26	27
26	28	31	33	36	38	41	43	46	49	51
10	11	12	13	14	15	16	16	17	18	19
35	38	42	45	49	52	56	59	62	66	69
36	40	44	47	51	54	58	62	65	69	72
39	43	47	51	55	59	62	66	70	74	78
35	38	41	45	48	52	55	58	62	65	69

Table 4. Random variates when $a = 20, b = 40$

$I_N = 0$	$I_N = 0.1$	$I_N = 0.2$	$I_N = 0.3$	$I_N = 0.4$	$I_N = 0.5$	$I_N = 0.6$	$I_N = 0.7$	$I_N = 0.8$	$I_N = 0.9$	$I_N = 1$
34	38	41	45	48	51	55	58	62	65	68
23	25	28	30	32	34	37	39	41	43	46
40	44	48	52	55	59	63	67	71	75	79
22	24	26	28	30	32	34	37	39	41	43
37	41	44	48	52	55	59	63	66	70	74
35	39	42	46	49	53	56	60	63	67	70
36	40	43	47	50	54	57	61	65	68	72
31	34	37	40	43	46	49	52	55	58	61
22	25	27	29	31	33	35	38	40	42	44
31	34	37	40	43	46	49	52	55	58	61
20	22	24	26	27	29	31	33	35	37	39
37	40	44	48	51	55	58	62	66	69	73
38	42	45	49	53	56	60	64	68	71	75
40	44	47	51	55	59	63	67	71	75	79
36	40	44	47	51	54	58	62	65	69	72

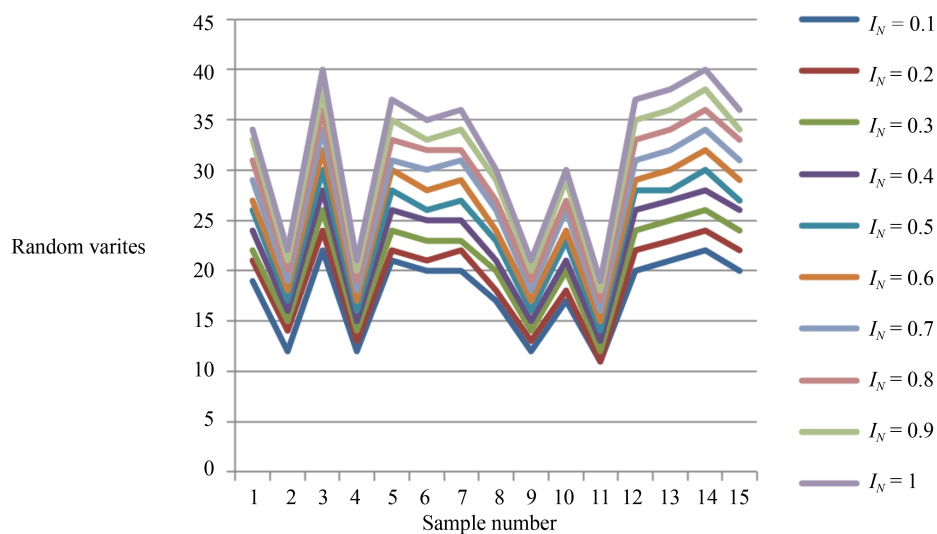


Figure 2. Random variates covers when $a = 10, b = 20$

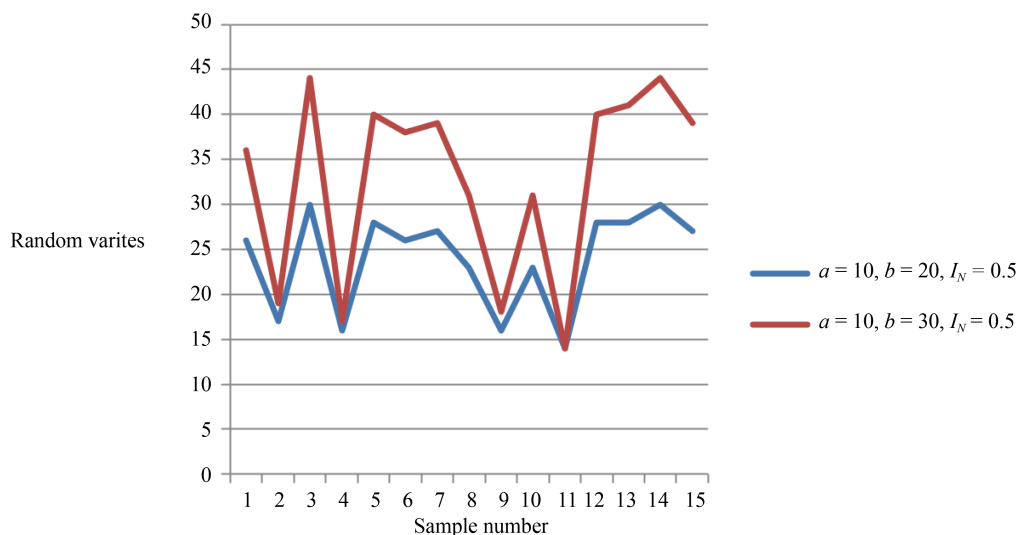


Figure 3. Random variates covers when b changes

6. Algorithm 2 for NDUD

In this section, we will design another algorithm to generate neutrosophic data from NDUD. The proposed algorithm will be developed using inverse probability transformation. The proposed algorithm will be the extension of the algorithm using the classical statistics. The proposed algorithm will become the algorithm under the classical statistics when $I_L = 0$. The routine to simulate the neutrosophic data from NDUD using inverse probability transformation will be stated as

Step-1: Specify the degree of indeterminacy I_N .

Step-2: Generate a uniform random number u from 0 to 1.

Step-3: Generate random variate $X_N = \text{ceiling} [\{[u(b - a + 1)] + 1\} (1 + I_N)]$.

Step-4: Return X_N .

The proposed Algorithm 2 is also shown in Figure 4.

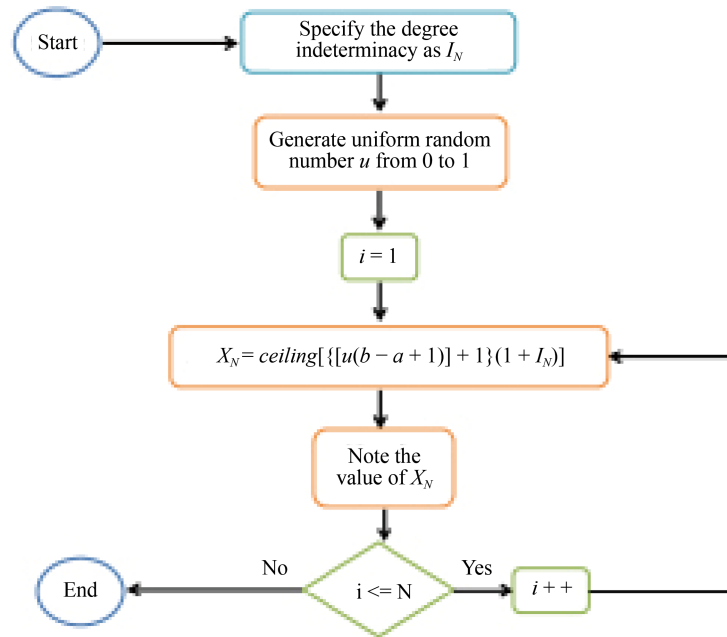


Figure 4. The flowchart of the proposed Algorithm 2

7. Simulation using Algorithm 2

In this ongoing investigation, we executed a simulation study utilizing Algorithm 2 to generate data from NDUD. The data generation process involved maintaining fixed values for I_N , a , and b . Tables 5-8 present the outcomes derived from the application of Algorithm 2. Specifically, Table 5 showcases results for $a=10$ and $b=20$, while Table 6 exhibits outcomes for $a=10$ and $b=30$. Furthermore, Table 7 is presented for $a=10$ and $b=40$, and Table 8 corresponds to $a=20$ and $b=40$. Tables 5-8 were generated by keeping the value of a constant while allowing b to vary from 20 to 40. Table 5, on the other hand, illustrates outcomes when the value of I_N changes while b remains fixed. A notable observation from Tables 5-8 is the consistent upward trend in data as the values of I_N are increased. For example, in Table 5, the random variate registers a value of 10 at $I_N=0.1$ and increases to 14 at $I_N=0.5$. Additionally, a discernible upward trend in random variate values is noted as the parameter b increases, with a fixed value of a . For instance, at $a=10$ and $b=20$, the random variate is 11 when $I_N=0.2$, whereas at $a=10$ and $b=30$, the corresponding value is 20. Visual representations of the trends in random variates corresponding to varying degrees of indeterminacy are illustrated in Figure 5, highlighting an increase in data as the degree of indeterminacy rises. Figure 6 portrays the behavior of random variates with increasing values of b while keeping other parameters constant. Both figures underscore a consistent upward trend in the data, emphasizing the impact of increased indeterminacy and changing b values on the generated data. To assess the validity of the results based on the simulated data, we computed the mean and variance using both the simulated data from Algorithm 2 and the properties of the proposed uniform distribution. For illustration, we considered the simulated data from Table 5 with parameters $a=10$, $b=20$ and $I_N=0$. The simulated mean and variance were 14 and 12, respectively, while the theoretical values from the distribution properties were 7 and 10. Similarly, with parameters $a=10$, $b=20$ and $I_N=0.1$, the simulated mean and variance were 7 and 14, compared to theoretical values of 14 and 12. This analysis indicates that the results obtained from Algorithm 2 are not consistent with the theoretical values. Therefore, data generated using this algorithm should be used with caution, and increasing the sample size may help achieve closer agreement between the simulation results and the theoretical properties of the mean and variance.

Table 5. Random variates when $a = 10, b = 20$

$I_N = 0$	$I_N = 0.1$	$I_N = 0.2$	$I_N = 0.3$	$I_N = 0.4$	$I_N = 0.5$	$I_N = 0.6$	$I_N = 0.7$	$I_N = 0.8$	$I_N = 0.9$	$I_N = 1$
9	10	11	12	13	14	15	16	16	17	18
3	4	4	4	5	5	5	5	6	6	6
12	13	14	16	17	18	19	20	21	23	24
3	3	3	3	4	4	4	4	4	5	5
11	12	13	14	15	16	17	18	19	20	21
10	11	12	13	14	14	15	16	17	18	19
10	11	12	13	14	15	16	17	18	19	20
7	8	9	9	10	11	11	12	13	14	14
3	3	3	4	4	4	4	5	5	5	5
7	8	9	9	10	11	11	12	13	13	14
2	2	2	2	2	2	2	2	3	3	3
11	12	13	14	15	16	17	18	19	20	21
11	12	13	14	15	16	17	18	20	21	22
12	13	14	16	17	18	19	20	21	22	24
10	11	12	13	14	15	16	17	18	19	20

Table 6. Random variates when $a = 10, b = 30$

$I_N = 0$	$I_N = 0.1$	$I_N = 0.2$	$I_N = 0.3$	$I_N = 0.4$	$I_N = 0.5$	$I_N = 0.6$	$I_N = 0.7$	$I_N = 0.8$	$I_N = 0.9$	$I_N = 1$
16	18	20	21	23	24	26	28	29	31	32
5	6	6	6	7	7	8	8	9	9	10
22	24	26	28	30	32	35	37	39	41	43
4	4	4	5	5	5	6	6	6	7	7
19	21	23	25	26	28	30	32	34	36	38
17	19	21	22	24	26	27	29	31	32	34
18	20	22	23	25	27	29	30	32	34	36
13	14	15	16	18	19	20	21	22	24	25
4	5	5	6	6	6	7	7	7	8	8
13	14	15	16	17	19	20	21	22	23	25
2	2	2	2	2	2	3	3	3	3	3
19	21	22	24	26	28	30	31	33	35	37
20	22	24	26	27	29	31	33	35	37	39
22	24	26	28	30	32	34	36	39	41	43
18	20	22	24	26	27	29	31	33	35	36

Table 7. Random variates when $a = 10, b = 40$

$I_N = 0$	$I_N = 0.1$	$I_N = 0.2$	$I_N = 0.3$	$I_N = 0.4$	$I_N = 0.5$	$I_N = 0.6$	$I_N = 0.7$	$I_N = 0.8$	$I_N = 0.9$	$I_N = 1$
24	26	28	31	33	35	38	40	42	44	47
7	7	8	9	9	10	11	11	12	12	13
31	34	38	41	44	47	50	53	56	59	62
5	5	6	6	7	7	7	8	8	9	9
27	30	33	36	38	41	44	46	49	52	54
25	27	30	32	35	37	39	42	44	47	49
26	29	31	34	36	39	41	44	46	49	51
18	20	21	23	25	27	28	30	32	34	35
6	6	7	7	8	8	9	9	10	10	11
18	20	21	23	25	26	28	30	32	33	35
2	2	2	2	2	3	3	3	3	3	3
27	30	32	35	37	40	43	45	48	51	53
28	31	34	37	40	42	45	48	51	54	56
31	34	37	40	43	47	50	53	56	59	62
27	29	32	34	37	40	42	45	47	50	53

Table 8. Random variates when $a = 20, b = 40$

$I_N = 0$	$I_N = 0.1$	$I_N = 0.2$	$I_N = 0.3$	$I_N = 0.4$	$I_N = 0.5$	$I_N = 0.6$	$I_N = 0.7$	$I_N = 0.8$	$I_N = 0.9$	$I_N = 1$
16	18	20	21	23	24	26	28	29	31	32
5	6	6	6	7	7	8	8	9	9	10
22	24	26	28	30	32	35	37	39	41	43
4	4	4	5	5	5	6	6	6	7	7
19	21	23	25	26	28	30	32	34	36	38
17	19	21	22	24	26	27	29	31	32	34
18	20	22	23	25	27	29	30	32	34	36
13	14	15	16	18	19	20	21	22	24	25
4	5	5	6	6	6	7	7	7	8	8
13	14	15	16	17	19	20	21	22	23	25
2	2	2	2	2	2	3	3	3	3	3
19	21	22	24	26	28	30	31	33	35	37
20	22	24	26	27	29	31	33	35	37	39
22	24	26	28	30	32	34	36	39	41	43
18	20	22	24	26	27	29	31	33	35	36

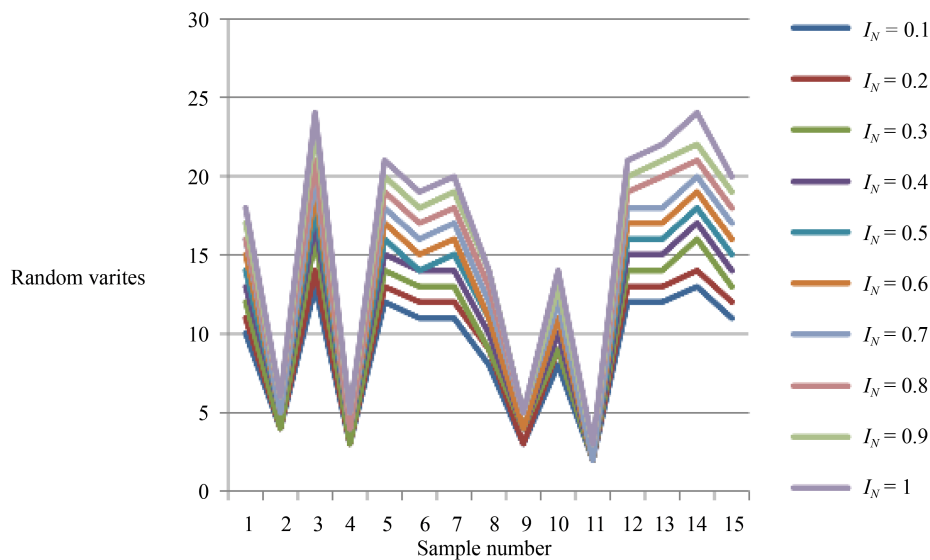


Figure 5. Random variates covers when $a = 10, b = 20$ s

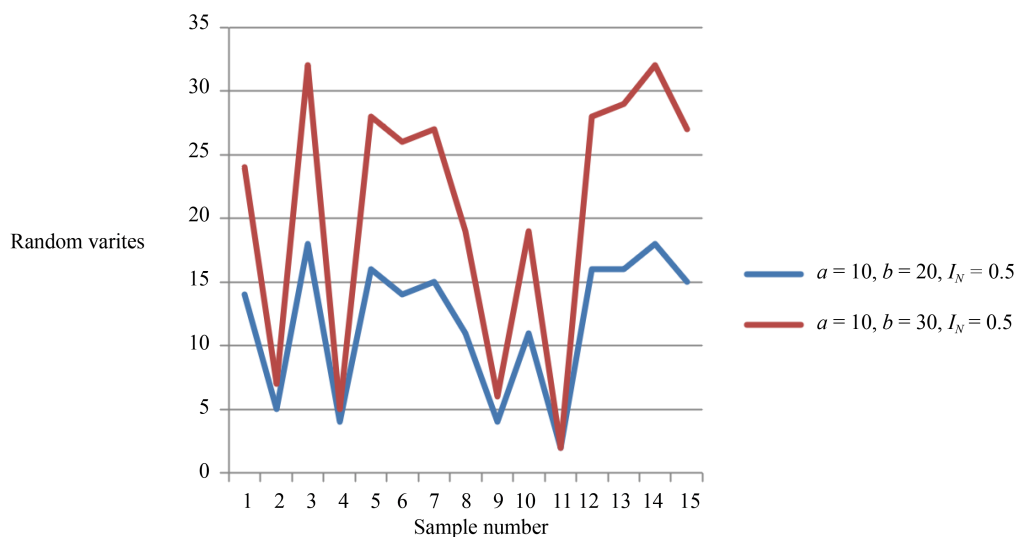


Figure 6. Random variates covers when b changes

8. Comparative studies

In this section, we compare the performance of the proposed algorithms with a classical statistical algorithm mentioned in Thomopoulos [28]. The proposed algorithms, as previously mentioned, extend the classical statistical algorithms. Specifically, the proposed algorithms converge to the existing ones under classical statistics when $I_L = 0$. Random variates were generated from both the existing and proposed algorithms. The first column of Tables 1-8 displays random variates from the existing algorithms. Tables 1-4 show random variates from both the existing and proposed Algorithm 1, while Tables 5-8 present random variates from the existing and proposed Algorithm 2. Analyzing Tables 1-4 reveals that the random variates generated by the existing algorithm from Thomopoulos [28] produce smaller values when $I_U > 0$. Similarly, examining Tables 5-8 for data generated using Algorithm 2 highlights the consistent behavior that the existing algorithm yields smaller values compared to the proposed Algorithm 2. To visually depict these

differences, Figure 7 illustrates the contrast between the existing algorithm and proposed Algorithm 1, while another Figure 8 showcases the distinction between the existing algorithm and proposed Algorithm 2. Both figures demonstrate an increasing trend in data as the degree of indeterminacy rises. This comparative study concludes that under uncertainty, the existing algorithms generate higher values of data. Consequently, relying on data from existing algorithms in uncertain scenarios may mislead decision-makers. Therefore, based on this analysis, it is recommended to use the proposed algorithms to generate data from the NDUD under conditions of uncertainty.

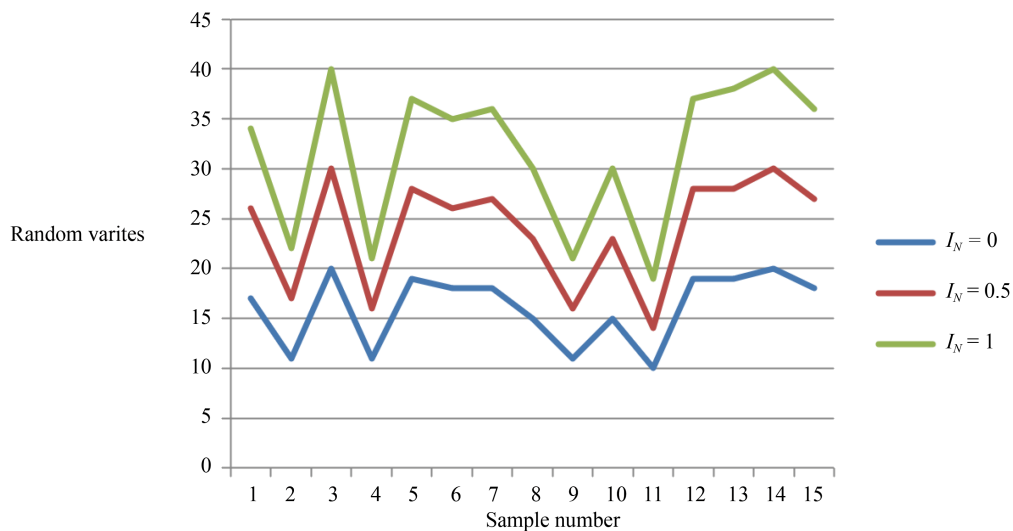


Figure 7. Random variates from the existing and proposed Algorithm 1

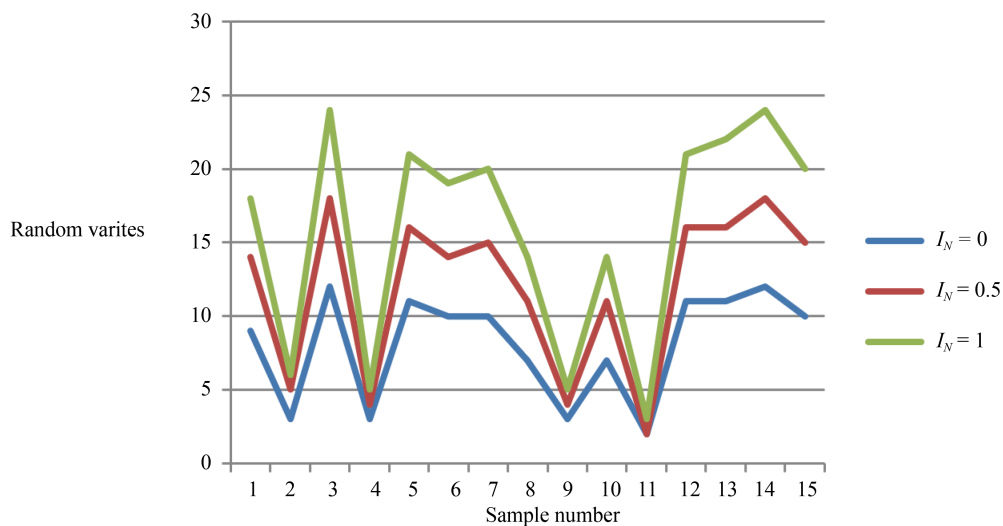


Figure 8. Random variates from the existing and proposed Algorithm 2

9. Application

In this section, we discuss the application of data generated using the proposed algorithms for acceptance sampling plans, assuming the product lifetime follows a Weibull distribution. An acceptance sampling plan is defined by two key

parameters: the sample size n and the acceptance number c . The product lot is tested for a specified duration, t_0 . A sample of size n is drawn from the lot and subjected to testing. If the number of failures observed is less than c by the end of t_0 , the lot is accepted; otherwise, it is rejected, see AL-Marshadi et al. [30]. As noted in Hsu et al. [31], the lifetime data is imprecise and recorded under conditions of uncertainty. To implement the sampling plan, data from Table 1 with parameters $a = 10$, $b = 20$, and $I_N = 0.1$ is utilized and presented in Table 9. Before using this data for product inspection, it was normalized as follows:

$$X_{normalized} = \frac{u}{Maximum\ value} + \varepsilon. \quad (12)$$

Here, ε represents a very small addition to the normalized values and u represents the uniform value. The neutrosophic data (X_N) follows a Weibull distribution with the neutrosophic shape parameter $k_N = (1 + I_N)k_L$ and the neutrosophic scale parameter $\lambda_N = (1 + I_N)\lambda_L$, which can be determined as follows:

$$X_N = \lambda_N [-\ln(1 - u)]^{\frac{1}{k_N}}. \quad (13)$$

Suppose $k_L = 1.5$, $\varepsilon = 0.01$ and $\lambda_L = 2$. Using these values, the data follows the Weibull distribution under classical statistics, while the neutrosophic Weibull distribution values are reported in Table 9. To implement the single sampling plan, we assume $n = 15$, $c = 0$, and $t_0 = 1.60$ h. From the ordered neutrosophic data X_N , we observe no failures before the specified time, and therefore, the product lot is accepted. Conversely, from the ordered data X_L , we note one failure before the specified time, resulting in the rejection of the product lot. This analysis clearly demonstrates that incorporating degrees of uncertainty into the sampling plan can result in different decisions regarding the acceptance or rejection of the product lot.

Table 9. The data from Weibull distribution

$I_N = 0$	$I_N = 0.1$	X_L	X_N	Ordered data	Ordered data
u	u			X_L	X_L
17	19	3.0619	3.3380	1.5657	1.7611
11	12	1.7205	1.9041	1.7205	1.9041
20	22	7.7320	7.5784	1.7205	1.9041
11	12	1.7205	1.9041	1.7205	1.9041
19	21	4.1475	4.3516	2.4848	2.7901
18	20	3.4829	3.7336	2.4848	2.7901
18	20	3.4829	3.7336	3.0619	3.338
15	17	2.4848	2.7901	3.4829	3.7336
11	12	1.7205	1.9041	3.4829	3.7336
15	17	2.4848	2.7901	3.4829	3.7336
10	11	1.5657	1.7611	4.1475	3.7336
19	20	4.1475	3.7336	4.1475	4.3516
19	21	4.1475	4.3516	4.1475	4.3516
20	22	7.7320	7.5784	7.732	7.5784
18	20	3.4829	3.7336	7.732	7.5784

10. Updating computer software

Based on extensive simulation and comparative studies, it is evident that discrete data generated from a uniform distribution under specific conditions differs from data generated when accounting for the degree of indeterminacy. To the best of our knowledge, there is currently no computer software or algorithm designed for generating data while considering the degree of indeterminacy. Consequently, based on these findings, we recommend updating existing software with the proposed algorithms to generate imprecise data in complex or uncertain environments. These algorithms are capable of producing imprecise discrete data from a uniform distribution. Incorporating imprecise data generation into the decision-making process enhances the ability to make better-informed decisions regarding the studied problem. The integration of the proposed algorithms into existing statistical software can be achieved by modifying the code and updating the current data generation modules to account for the degree of indeterminacy. However, this integration presents several technical challenges, such as enhancing the data generation modules to incorporate imprecision, ensuring computational efficiency when handling imprecise data, and thoroughly testing the software to verify that the generated data maintains accuracy across various scenarios. It is crucial to ensure that the degree of indeterminacy does not compromise the reliability of downstream analyses.

11. Limitations and utilization

The utilization of the uniform distribution is prevalent in generating random data, especially in computer systems and gaming applications. Consequently, the suggested algorithms find applicability in computer-generated data when uncertainty is a factor. Moreover, these algorithms can be effectively employed in reliability analysis for generating count data. It is important to note that the proposed algorithms come with limitations—they are not applicable in situations where there is no uncertainty in the data. Moreover, determining the degree of uncertainty can be challenging in certain cases. The uniqueness of these algorithms lies in their incorporation of the degree of indeterminacy, making them well-suited for application in complex and uncertain environments.

12. Concluding remarks

This paper introduces the concept of the neutrosophic discrete uniform distribution and subsequently incorporates it into the design of two algorithms. These algorithms serve the purpose of generating imprecise data by accounting for varying degrees of indeterminacy. Notably, the proposed algorithms extend the scope of existing classical statistical algorithms. Simulation studies reveal a significant impact of the degree of indeterminacy on the generation of random data. As a result, it is suggested to update computer software by integrating the proposed algorithms, presenting a promising avenue for future research. Furthermore, the application of these algorithms in generating data within complex or uncertain environments is recommended. To enhance the usefulness and persuasiveness of the proposed algorithms, they can be applied in areas such as reliability analysis, statistical process control, finance, environmental studies, healthcare, machine learning, and engineering, where uncertainty and imprecision are common in the data. The proposed algorithm, employing the accept-reject method, holds potential for further exploration in future research endeavors. The proposed simulation following Mohamad et al. [29] can be extended in future research by incorporating metrics such as computational time. We proposed the discrete distribution under neutrosophic statistics, including its basic properties such as the mean and variance. Other properties of the proposed distribution can be explored in future research.

Acknowledgement

The authors sincerely thank the editor and reviewers for their valuable suggestions, which helped improve the quality, novelty, and presentation of the paper. We would like to acknowledge the use of OpenAI's ChatGPT for assisting in improving the grammar.

Conflict of interest

The authors declare no competing financial interest.

References

- [1] Çalik S, Güngör M. On the expected values of the sample maximum of order statistics from a discrete uniform distribution. *Applied Mathematics and Computation*. 2004; 157(3): 695-700. Availavle from: <https://doi.org/10.1016/j.amc.2003.08.056>.
- [2] Lumbroso J. Optimal discrete uniform generation from coin flips, and applications. *arXiv: 1304.1916*. 2013. Availavle from: <https://doi.org/10.48550/arXiv.1304.1916>.
- [3] Papatsouma I, Mahmoudvand R, Farmakis N. Evaluating the goodness of the sample coefficient of variation via discrete uniform distribution. *Statistics, Optimization and Information Computing*. 2019; 7(4): 642-652. Availavle from: <https://doi.org/10.19139/soic-2310-5070-798>.
- [4] Alrumayh A, Khogeer HA. A new two-parameter discrete distribution for overdispersed and asymmetric data: its properties, estimation, regression model, and applications. *Symmetry*. 2023; 15(6): 1289. Availavle from: <https://doi.org/10.1016/j.heliyon.2024.e36764>.
- [5] Lei X. An efficient method for generating a discrete uniform distribution using a biased random source. *Journal of Applied Probability*. 2023; 60(3): 1-10. Availavle from: <https://doi.org/10.1017/jpr.2022.111>.
- [6] Mattner L, Roos B. Maximal probabilities of convolution powers of discrete uniform distributions. *Statistics and Probability Letters*. 2008; 78(17): 2992-2996. Availavle from: <https://doi.org/10.1016/j.spl.2008.05.005>.
- [7] Comoglio F, Fracchia L, Rinaldi M. Bayesian inference from count data using discrete uniform priors. *PLoS One*. 2013; 8(10): e74388. Availavle from: <https://doi.org/10.1371/journal.pone.0074388>.
- [8] Roychowdhury MK. Center of mass and the optimal quantizers for some continuous and discrete uniform distributions. *Journal of Interdisciplinary Mathematics*. 2019; 22(4): 451-471.
- [9] Pepelyshev A, Zhigljavsky A. Discrete uniform and binomial distributions with infinite support. *Soft Computing*. 2020; 24(23): 17517-17524. Availavle from: <https://doi.org/10.1007/s00500-020-05190-2>.
- [10] Noori NA, Khaleel MA, Salih AM. Some expansions to the Weibull distribution families with two parameters: a review. *Babylonian Journal of Mathematics*. 2025; 2025: 61-87. Availavle from: <https://doi.org/10.58496/BJM/2025/008>.
- [11] Sarwar M, Jamal F, Hussain Z, Iqbal S, Aslam R. A comprehensive study of the extended JCA distribution with properties and applications. *SCOPUA Journal of Applied Statistical Research*. 2025; 1(1): 1-24. Availavle from: <https://doi.org/10.64060/jasr.v1.i1.5>.
- [12] Idrees R, Bashir S. A new generalization of exponentiated exponential distribution using quantile functions. *SCOPUA Journal of Applied Statistical Research*. 2025; 1(1): 1-29. Availavle from: <https://doi.org/10.64060/jasr.v1.i1.3>.
- [13] Smarandache F. *Introduction to Neutrosophic Statistics*. Romania: Educational Publisher; 2014.
- [14] Chen J, Ye J, Du S. Scale effect and anisotropy analyzed for neutrosophic numbers of rock joint roughness coefficient based on neutrosophic statistics. *Symmetry*. 2017; 9(10): 208. Availavle from: <https://doi.org/10.3390/sym9100208>.
- [15] Chen J, Ye J, Du S, Yong R. Expressions of rock joint roughness coefficient using neutrosophic interval statistical numbers. *Symmetry*. 2017; 9(7): 123. Availavle from: <https://doi.org/10.3390/sym9070123>.
- [16] Alhabib R, Ranna MM, Farah H, Salama AA. Some neutrosophic probability distributions. *Neutrosophic Sets and Systems*. 2018; 22: 30-38. Availavle from: <https://doi.org/10.5281/zenodo.2160478>.

- [17] Duan WQ, Khan Z, Gulistan M, Khurshid A. Neutrosophic exponential distribution: modeling and applications for complex data analysis. *Complexity*. 2021; 2021: 1-8. Availavle from: <https://doi.org/10.1155/2021/5970613>.
- [18] Khan Z, Al-Bossly A, Almazah MM, Alduais FS. On statistical development of neutrosophic gamma distribution with applications to complex data analysis. *Complexity*. 2021; 2021(1): 3701236. Availavle from: <https://doi.org/10.1155/2021/3701236>.
- [19] Smarandache F. Neutrosophic statistics is an extension of interval statistics, while plithogenic statistics is the most general form of statistics (second version). *Infinite Study*. 2022. Availavle from: <https://doi.org/10.54216/IJNS.190111>.
- [20] Granados C. Some discrete neutrosophic distributions with neutrosophic parameters based on neutrosophic random variables. *Hacettepe Journal of Mathematics and Statistics*. 2022; 51(5): 1442-1457. Availavle from: <https://doi.org/10.15672/hujms.1099081>.
- [21] Granados C, Das AK, Birojit DAS. Some continuous neutrosophic distributions with neutrosophic parameters based on neutrosophic random variables. *Advances in the Theory of Nonlinear Analysis and its Application*. 2022; 6(3): 380-389. Availavle from: <https://doi.org/10.31197/atnaa.1056480>.
- [22] Guo Y, Sengur A. NCM: Neutrosophic c-means clustering algorithm. *Pattern Recognition*. 2015; 48(8): 2710-2724. Availavle from: <https://doi.org/10.1016/j.patcog.2015.02.018>.
- [23] Garg H, Nancy. Algorithms for single-valued neutrosophic decision making based on TOPSIS and clustering methods with new distance measure. *AIMS Mathematics*. 2020; 5(3): 2671-2693. Availavle from: <https://doi.org/10.1186/s40537-023-00855-9>.
- [24] Aslam M, Alamri FS. Algorithm for generating neutrosophic data using accept-reject method. *Journal of Big Data*. 2023; 10(1): 175. Availavle from: <https://doi.org/10.1186/s40537-023-00855-9>.
- [25] Aslam M, Albassam M. Neutrosophic geometric distribution: data generation under uncertainty and practical applications. *AIMS Mathematics*. 2024; 9(6): 16436-16452. Availavle from: <https://doi.org/10.3934/math.2024796>.
- [26] El-latif AMA, Almulhim FA, Noori NA, Khaleel MA, Alsaedi BSO. Properties with application to medical data for new inverse Rayleigh distribution utilizing neutrosophic logic. *Journal of Radiation Research and Applied Sciences*. 2025; 18(2): 101391. Availavle from: <https://doi.org/10.1016/j.jrras.2025.101391>.
- [27] Aslam M. The neutrosophic negative binomial distribution: algorithms and practical application. *REVSTAT-Statistical Journal*. 2024. Availavle from: <https://doi.org/10.57805/revstat.vi.781>.
- [28] Thomopoulos NT. *Essentials of Monte Carlo Simulation: Statistical Methods for Building Simulation Models*. Germany: Springer; 2014.
- [29] Mohamad SK, Kahleel AH. Optimizing network reliability in dynamic or rapidly-changing topologies. *Babylonian Journal of Mathematics*. 2023; 2023: 98-104. Availavle from: <https://doi.org/10.58496/BJM/2023/019>.
- [30] AL-Marshadi AH, Aslam M, Alharbey A. Design of double acceptance sampling plan for Weibull distribution under indeterminacy. *AIMS Mathematics*. 2023; 8(6): 13294-13305. Availavle from: <https://doi.org/10.3934/math.2023672>.
- [31] Hsu BM, Shu MH, Chen BS. Evaluating lifetime performance for the Pareto model with censored and imprecise information. *Journal of Statistical Computation and Simulation*. 2011; 81(12): 1817-1833. Availavle from: <https://doi.org/10.1080/00949655.2010.506439>.