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A Weibull-Based Critique of the Uniform Distribution in Interval Data Analysis

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Abstract: In recent years, there has been a growing interest in neutrosophic probability distributions as effective tools for modeling data that involve uncertainty, ambiguity, or vagueness—limitations that classical probability models often fail to address. In addition, the simulation of interval data has been misapplied in neutrosophic analysis by assuming a uniform distribution over the interval. In this study, a neutrosophic extension of the Weibull distribution is used to generate neutrosophic data. From this data, the indeterminacy component, referred to as "indeterminacy factor," is extracted and estimated. To understand the behavior of this indeterminacy factor, several continuous probability distributions are fitted to its values. This paper makes three main contributions: (1) it presents a novel Neutrosophic Weibull distribution that can capture non-uniform indeterminacy patterns; (2) it offers a comparative analysis of several candidate distributions to assess the probabilistic framework of indeterminacy; and (3) it supports the suggested model using simulations and real-life data sets, proving its outstanding goodness-of-fit and practical importance. These findings emphasize the need for more suitable probabilistic models when dealing with neutrosophic data. Finally, the proposed neutrosophic Weibull distribution is applied to two real-world datasets containing uncertain observations. In both cases, the Weibull model shows the best fit. The corresponding indeterminacy values are then modeled using different probability distributions, and the results reflect similar patterns to those observed in the simulated neutrosophic data. Based on the analysis, it is concluded that the existing simulation—originally developed for interval analysis under a uniform distribution assumption—is not suitable for neutrosophic analysis.

***Keywords*:** neutrosophic probability, neutrosophic Weibull distribution, indeterminacy measure, uncertain data, uncertainty quantification, probability distribution fitting, reliability analysis, uniform distribution, beta distribution, Kumaraswamy distribution, real-life applications

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Abbreviation

PDF	Probability Density Function
CDF	Cumulative Distribution Function
NWD	Neutrosophic Weibull Distribution
RV	Random Variable
NRV	Neutrosophic Random Variable
KS	Kolmogorov Smirnov
ML	Maximum Likelihood
MLE	Maximum Likelihood Estimation
SE	Standard Error
AIC	Akaike Information Criterion
BIC	Bayesian Information Criterion
CAIC	Consistent Akaike Information Criterion
HQIC	Hannan-Quinn Information Criterion

1. Introduction

For many years, statistical modeling and probability theory have been essential to comprehending randomness, uncertainty and inconsistency in a variety of disciplines, such as engineering, medicine, natural sciences and economics. Conventional probability distributions that depend on precise data and well-defined parameters include the Weibull, exponential and normal distributions. However, in real-world scenarios, information is often confusing, conflicting, or insufficient due to measurement errors, inconsistent sources, or subjective evaluation. Such complex and unexpected situations are often not captured by classical models. To overcome these limitations, neutrosophic theory first introduced by [1] offers a more comprehensive framework for modeling uncertainty. Unlike classical logic, which only considers truth and falsehood, neutrosophic logic adds indeterminacy as a third dimension. A neutrosophic component is often represented as a triplet (T, I, F), where T stands for truth, I for indeterminacy, and F for falsehood, all of which fall within the interval [0, 1]. This powerful concept enables the modeling of scenarios in which truth and untruth are not necessarily complimentary, the degrees of ambiguity and partial knowledge can coexist [2]. Recently, Al-Omeri [3] introduced the idea of fuzzy continuous mappings based on fuzzy α^m-open sets in Šostak's sense. This makes it easier to develop neutrosophic-driven continuity models and broadens the theoretical context of fuzzy topological domains. By presenting new fixed-point results on ordered metrics inside neutrosophic theory, Al-Omeri [4] extended the theoretical foundations of neutrosophic topological theory and its application in nonlinear analysis. Recent developments such as the work on Fermatean Neutrosophic Fuzzy Graphs and the development of the Winner Index to enhance election analysis demonstrate the growing significance of neutrosophic models in handling uncertainty and making decisions [5].

According to [6], the classical statistics are expanded upon by neutrosophic statistics. The data in neutrosophic statistics contains some indeterminacy, whereas the data in classical statistics is known and composed of distinct numbers. The data in neutrosophic statistics could be unclear, inaccurate, partial, confusing, or even unknown. In neutrosophic statistics, sets that roughly correspond to the crisp numbers used in classical statistics are employed in place of the crisp numbers. Patro and Smarandache [7] investigate that the neutrosophic concept is an extended form of the classical distribution. This indicates that the probability experiment has some degree of uncertainty. Every trial's experimental data may yield an outcome that is failure (F) or some indeterminacy (I). Recent advances in statistical modeling show that neutrosophic distributions offer a more flexible and dependable choice than classical distributions when dealing with data that contains indeterminacy, ambiguity, or inadequate information. In order to better correctly depict uncertainty across a range of domains, researchers have proposed neutrosophic equivalents of common distributions. For example, compared to its classical version, the Neutrosophic Burr-III distribution proposed by [8] demonstrated superior fitting capabilities on COVID-19 and reliability datasets because it included neutrosophic parameters that allowed modeling of parameter and observation uncertainty in the form of intervals. Similarly, Mansour [9] presented a Neutrosophic Size-

Biased Exponential distribution, which demonstrated superior moment-based characterizations, a more precise hazard function with indeterminacy, and more flexibility in the modeling of skewed data. Al-Omeri and Kaviyarasu [10] applied neutrosophic graph theory to model decision-making and disaster-response systems for earthquake management in Japan, demonstrating the growing practical potential of neutrosophic frameworks in real-world problem solving. In addition, some of the developed neutrosophic distributions are neutrosophic beta distribution developed by [11]; Neutrosophic exponential distribution [12]; neutrosophic Burr-XII distribution [13]; neutrosophic Lindley distribution [14]; Neutrosophic marshal Olkin extended burr-XII distribution [15].

In real-world scenarios traditional probability distributions may not capture the data which is often subject to various forms of imprecision, ambiguity, and incomplete information. Classical statistical methods usually rely on the assumption of precise and well-defined data, making them inadequate for handling situations where uncertainty is an inherent part of the dataset. To address this constraint, neutrosophic probability theory has appeared as a powerful framework capable of modeling indeterminacy explicitly. The necessity to apply classical probability distributions to the neutrosophic field to describe unclear or ambiguous data more accurately is the driving force behind this study. Because of its adaptability in representing life statistics and failure durations, the Weibull distribution is one of the most extensively used continuous distributions. The purpose of this study is to create neutrosophic data by creating a neutrosophic version of the Weibull distribution and analyze the behavior of the resulting indeterminacy component I_N, which indicates the degree of uncertainty related to the data.

Understanding the probabilistic nature of I_N by modeling various probability distributions to its values and assessing their performance is a major goal. The research is particularly motivated by the proposition that commonly assumed distributions like uniform distribution may not be appropriate for modeling indeterminacy. Identifying a more suitable model for I_N can contribute significantly to the theoretical development of neutrosophic statistics and enhance its applicability to composite real-world problems where vagueness and ambiguity plays a central role. The article is resented in the following manner: Section 2 presents the Neutrosophic Weibull Distribution (NWD) Section 3 proposes extensive simulation study, Section 4 displays the comparative study, Section 5 represents applications of NWD, and Section 6 depicts the conclusion.

2. Neutrosophic random variable

In this section neutrosophic random variable and neutrosophic Weibull distribution has been discussed.

Classical Random Variables (RVs) assume complete precision and accuracy in outcomes, while Neutrosophic Random Variables (NRVs) extend this concept by incorporating indeterminacy and uncertainty. NRV is particularly useful in situations where data is imprecise, vague, or incomplete, such as in expert opinion, medical diagnosis, or survival studies etc. A neutrosophic random variable is a function of random variable and is often represented as: $X_N = (X_L,\ X_U)$.

But in another way, it can be explained as: While recording the temperature of a particular location over several time intervals, it was observed that due to limitations in the measuring instrument, only the minimum or lower bound of the temperature could be accurately recorded beyond a certain threshold. For instance, the first recorded value indicated that the temperature was 30 °C or above, the second value was 33 °C or above, and so on for the subsequent time intervals. Although the exact upper values of the temperature were not known, the data clearly showed that the temperature did not fall below the given lower bounds. This introduces indeterminacy in the upper observations, where the lower bound is certain but the true upper value remains ambiguous or imprecise. A neutrosophic random variable (i.e. function of random variable) in this case is represented as:

$$X_N = X_L + I_N X_L = (1 + I_N) X_L.$$

Here, the classical random variable is extended to a neutrosophic random variable as above and I_N is the indeterminate factor which shows the amount of uncertainty and ambiguity between lower (X_L) and upper values of each observation. The neutrosophic statistical framework, which can handle partial or ambiguous data by explicitly including the indeterminacy included in the observations, is a good fit for this kind of circumstance. In these situations, neutrosophic probability distributions that are especially made to account for uncertainty and indeterminacy in observations can be used to simulate this kind of neutrosophic data.

The expectation and variance on neutrosophic random variable can be applied as:

$$E\left(X_N\right)^r = E\left[\left(1+I_N\right)X_L\right]^r = \left(1+I_N\right)^r E\left(X_L\right)^r \tag{1}$$

and

$$Var\left(X_N\right) = Var\left[\left(1+I_N\right)X_L\right] = \left(1+I_N\right)^2 Var\left(X_L\right). \tag{2}$$

$E\left(X_L\right)^r$ are the rth moments of the classical random variable in Equation (2) and $Var\left(X_L\right)$ in Equation (3) is the variance of the classical random variable.

2.1 *Neutrosophic Weibull distribution*

Alhasan and Smarandache [16] introduced the Neutrosophic Weibull Distribution (NWD) and its family of related distributions by employing a different approach, where the neutrosophic random variable was defined as $X_N = (X_L,\ X_U)$. However, their work provided limited discussion on the statistical properties and practical applications of the NWD. Subsequently, Sherwani et al. [17] explored neutrosophic entropy measures for the Weibull distribution, yet they also adopted the same earlier concept of the neutrosophic variable. Khan et al. [18] discussed the neutrosophic Weibull model in the context of survival studies. More recently, Albassam et al. [19] advanced the development of the NWD by introducing a novel formulation of the neutrosophic random variable such as: $X_N = X_L + I_N X_L = \left(1+I_N\right)X_L$ characterized by the following Probability Density Function (PDF) and Cumulative Distribution Function (CDF).

$$f\left(x_N\right) = \frac{k}{\lambda}\left(\frac{x_N\left(1+I_N\right)}{\lambda}\right)^{k-1} e^{-\left(\frac{x_N\left(1+I_N\right)}{\lambda}\right)^k},\ x_N > 0;\ \lambda,\ k > 0. \tag{3}$$

$$F\left(x_N\right) = 1 - e^{-\left(\frac{x_N\left(1+I_N\right)}{\lambda}\right)^k}. \tag{4}$$

Quantile function for the NWD is

$$Q = \frac{\lambda}{1+I_N}\left(-\ln(1-p)\right)^{1/k}. \tag{5}$$

The mean of the NWD is

$$E\left(X_N\right) = \left(1+I_N\right)\lambda\Gamma(1+1/k). \tag{6}$$

The variance of the NWD is

$$Var(X_N) = (1+I_N)^2 \lambda^2 \left[\Gamma(1+2/k) - (\Gamma(1+1/k))^2\right]. \tag{7}$$

Equation (3) is the PDF and Equation (4) is the CDF of the neutrosophic Weibull distribution while the quantile function, mean, and variance of the NWD are presented in the Equations (5), (6), and (7) respectively.

3. Simulation study from neutrosophic Weibull distribution

In this section a simulation study has been conducted by using the neutrosophic Weibull distribution. data has been generated by taking different samples sizes then neutrosophic factor is calculated and various classical distributions has been applied on it. Following is the algorithm for the process:

Algorithm Modeling indeterminacy from Neutrosophic Weibull distribution.

Equations (3), (4) and (5) are used for the following procedure.

Step 1: Initialize Parameters

• Define sample sizes: $n = 10,\ 50,\ 100$.

• Define indeterminacy levels: $I_N = 0.01,\ 0.03,\ 0.05,\ 0.07,\ 0.09$.

• Total combinations: 5 levels of $I_N \times 3$ sample sizes $= 15$ scenarios.

Step 2: Data Generation

For each value of I_N in the set $\{0.01,\ 0.03,\ 0.05,\ 0.07,\ 0.09\}$:

For each sample size $n \in \{10,\ 50,\ 100\}$:

i. Make a foundation random sample, $X_L = X_{L1},\ X_{L2},\ \ldots,\ X_{Ln}$ from the Weibull model using the quantile function.

ii. For every observation, construct the upper limit as: $X_{Ui} = X_{Li}(1+I_N)$.

iii. Show an interval for every neutrosophic observation as: $X_N = (X_L,\ X_U)$.

iv. Calculate the resulting indeterminacy factor $I_N = \dfrac{X_U - X_L}{X_L}$ from the data generated.

v. To create numerous neutrosophic datasets, repeat steps i. to iv. with different sample sizes for each indeterminacy level.

Step 3: Total Data Sets

• A total of 15 neutrosophic datasets are obtained (one for each I_N, n combination).

Step 4: Model Indeterminacy Factor

• For each of the 15 datasets of I_N, fit the following continuous probability distributions: Beta; Kumaraswamy; Triangular; Normal; Gamma; Log-logistic; Exponential; Uniform.

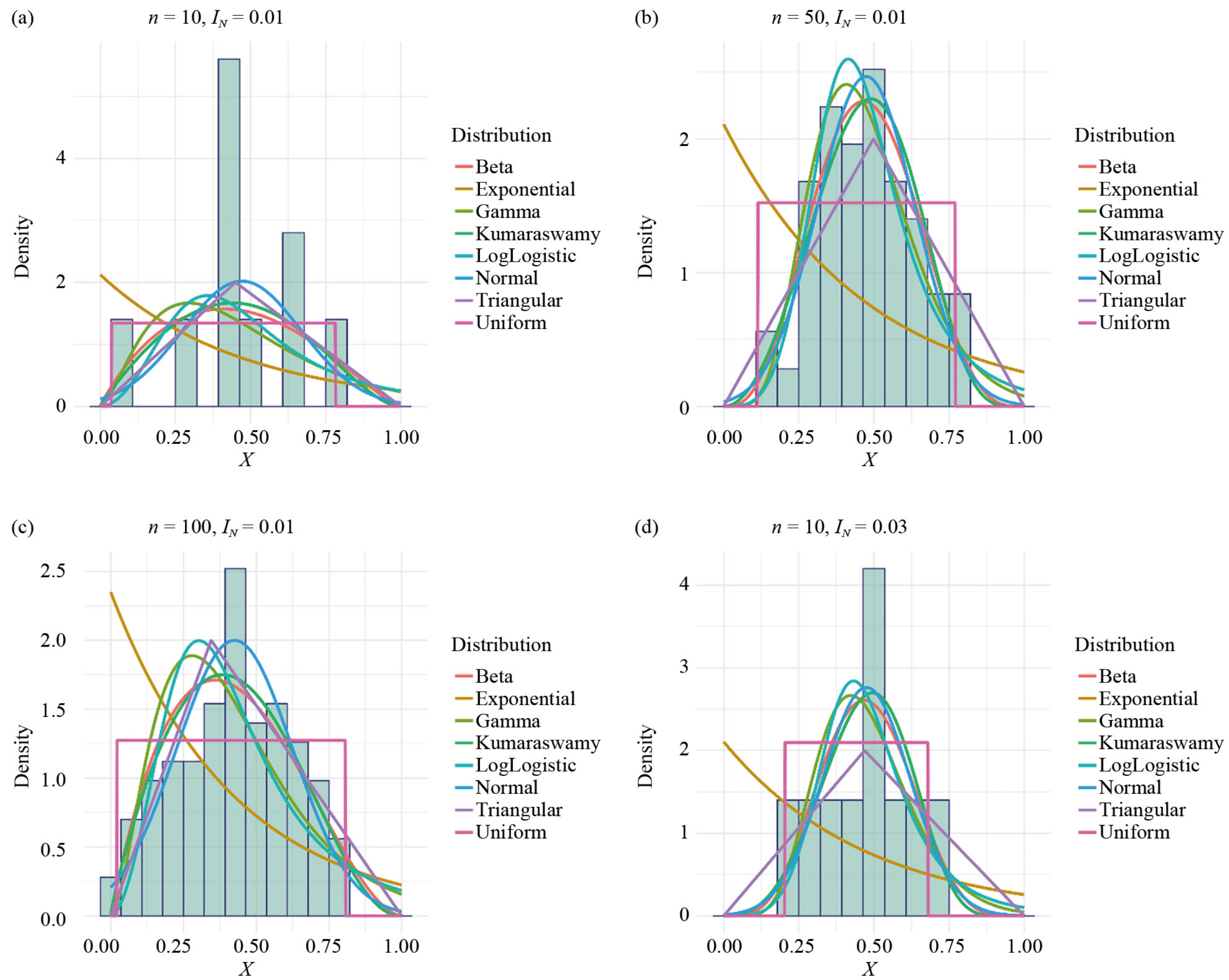
Step 5: Goodness-of-Fit Evaluation

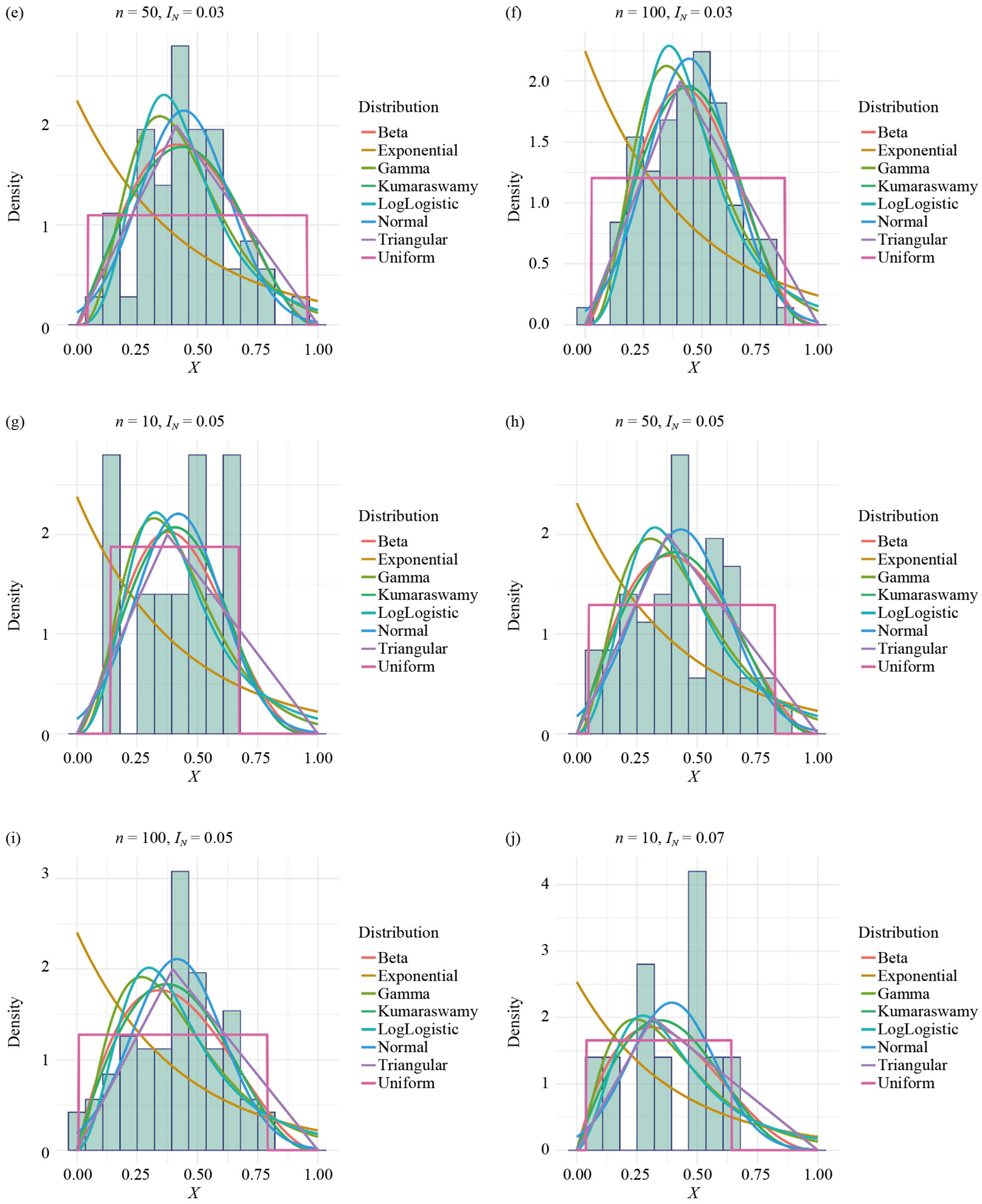
• Apply the Kolmogorov-Smirnov (KS) test to evaluate the goodness-of-fit for each distribution.

• Record the KS Statistic and associated p-values for comparison.

4. Discussion

Figure 1 presents the fitted distributions overlaid on histograms of the simulated data for different values of I_N and varying sample sizes. Figure 2 displays the fitted distributions using the CDF plots under the same conditions. Figure 3 provides a comparison of the KS statistics for the fitted models. Figure 4 helps visually compare the goodness-of-fit of various distributions across different sample sizes and levels of indeterminacy. Rows: Different combinations of sample size n and indeterminacy level I_N. Columns: Different continuous probability distributions (Beta, Kumaraswamy, Triangular, Uniform, Normal, Gamma, Log-logistic, Exponential) Color Scale: Indicates p-values from the KS test, where green implies better fit (higher p-values) and yellow implies poorer fit (lower p-values). Figure 5 shows that the calculated I_N values tend to stabilize around their levels as sample size increases. Though, for smaller samples ($n = 10$), the spread is larger and the calculated I_N values often deviate more from their proposed values. For $n = 50$ and $n = 100$, the variability reduces slightly which indicates improved estimation accuracy with larger sample sizes. However, some degree of bias or spread remains for all settings, mainly at higher I_N levels like 0.09. The violin plots in Figure 6 show that the spread and shape of the calculated I_N values vary across sample sizes and selected I_N levels. The distributions are more variable and wider, particularly at lower I_N levels and for small sample sizes ($n = 10$). The distributions become more symmetrical and concentrate around their respective selected I_N values as the sample size increases to $n = 50$ and $n = 100$, which indicates improved accuracy and stability of estimation. Figure 7 illustrates the empirical and theoretical Quantile-Quantile (QQ) plots for the fitted distributions across different sample sizes and values of I_N.




(e)
n = 50, I_N = 0.03
(f)
n = 100, I_N = 0.03
(g)
n = 10, I_N = 0.05
(h)
n = 50, I_N = 0.05
(i)
n = 100, I_N = 0.05
(j)
n = 10, I_N = 0.07
Density
X
0.00
0.25
0.50
0.75
1.00
Distribution
Beta
Exponential
Gamma
Kumaraswamy
LogLogistic
Normal
Triangular
Uniform

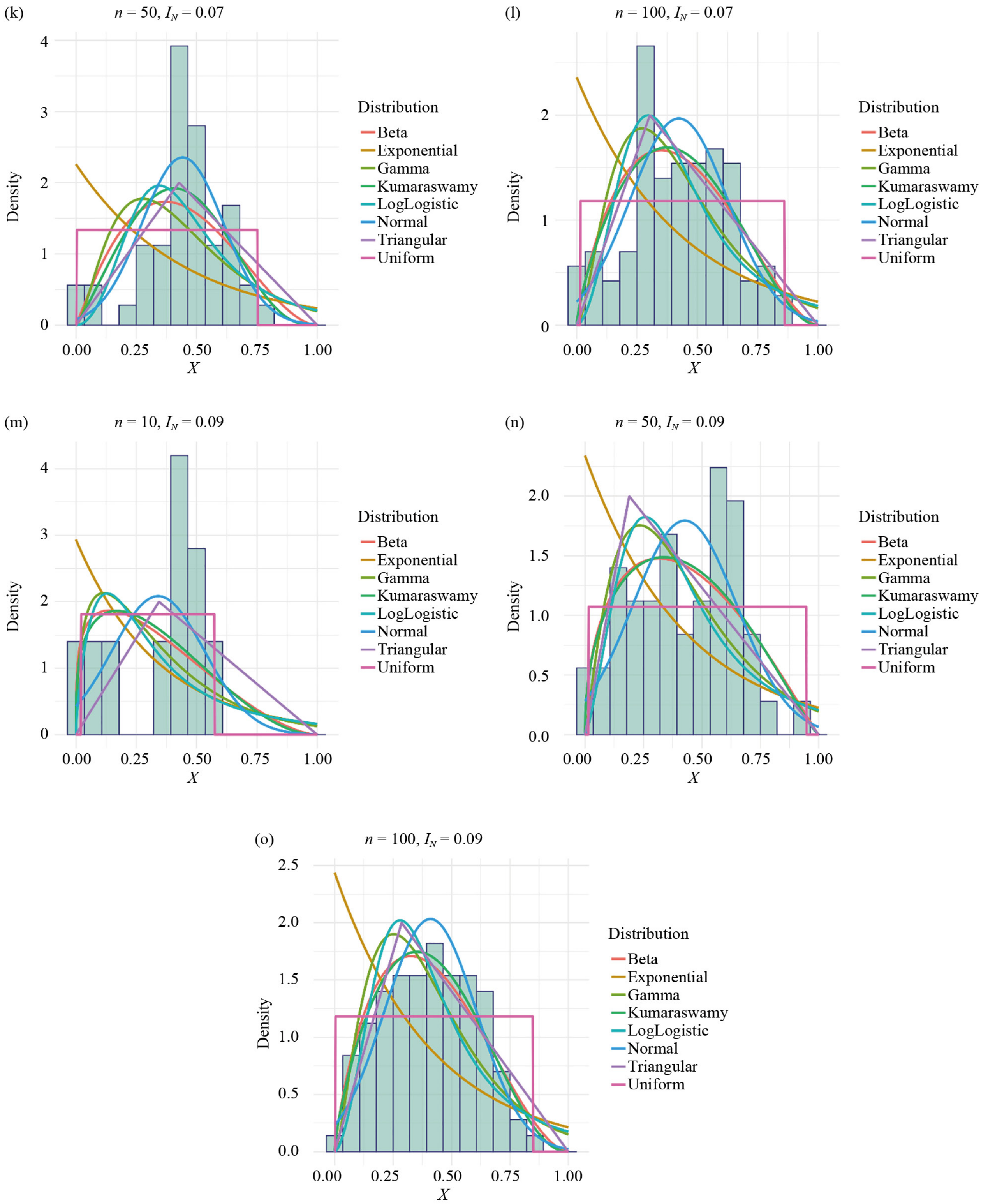
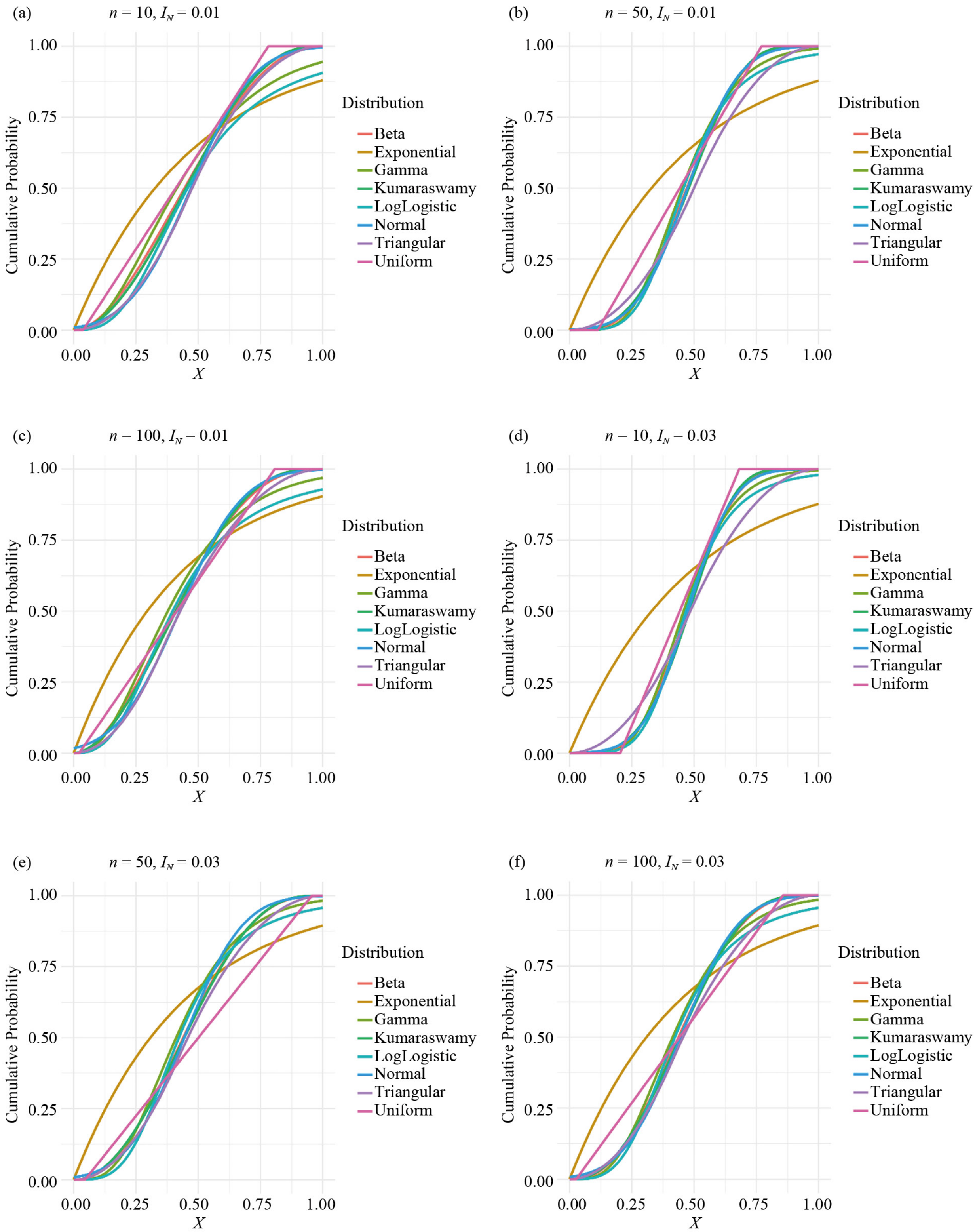
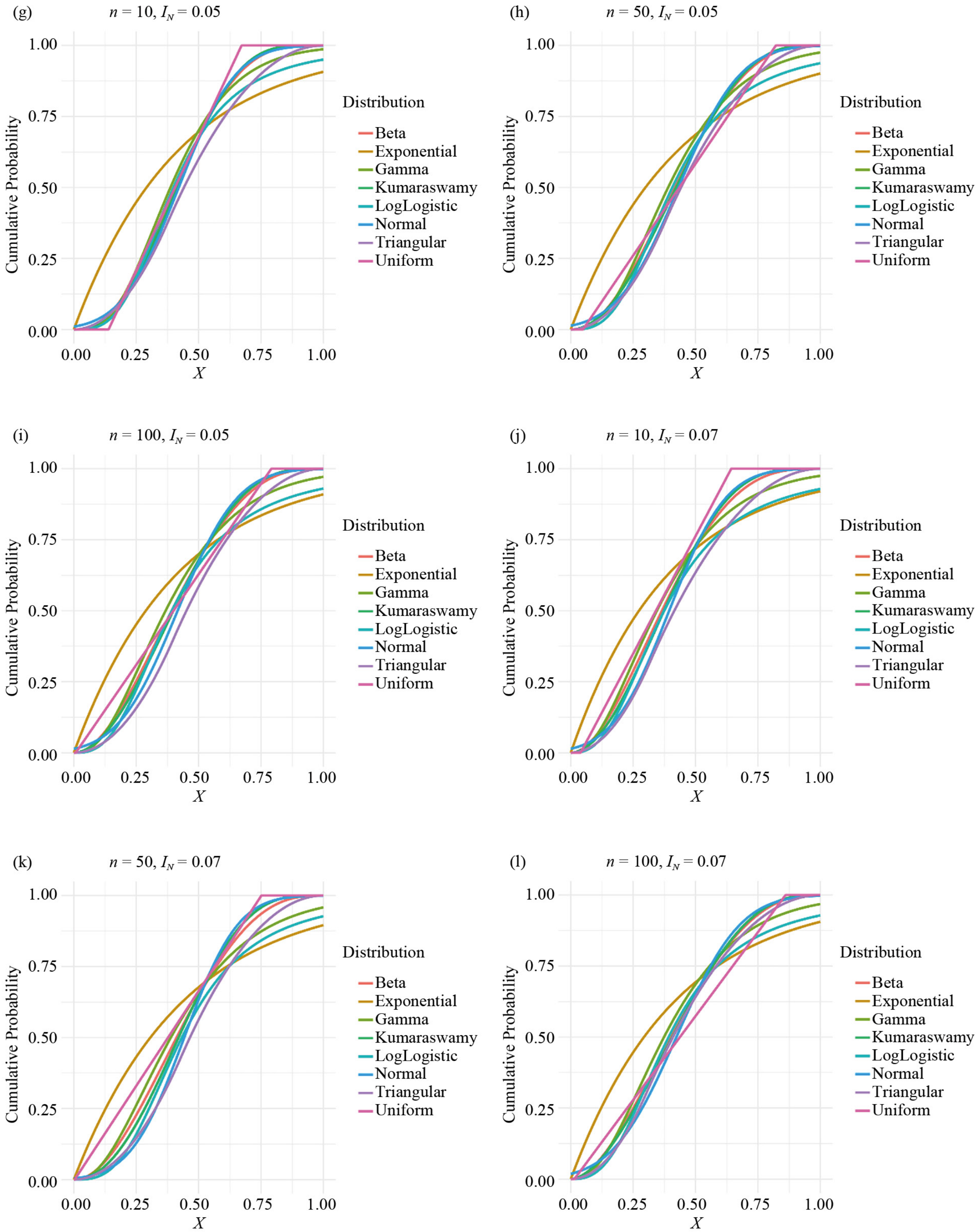


Figure 1. Histogram of I_N with fitted distributions


(a)
n = 10, I_N = 0.01
(b)
n = 50, I_N = 0.01
(c)
n = 100, I_N = 0.01
(d)
n = 10, I_N = 0.03
(e)
n = 50, I_N = 0.03
(f)
n = 100, I_N = 0.03
Cumulative Probability
0.00
0.25
0.50
0.75
1.00
X
Distribution
Beta
Exponential
Gamma
Kumaraswamy
LogLogistic
Normal
Triangular
Uniform


(g)
n = 10, I_N = 0.05
(h)
n = 50, I_N = 0.05
(i)
n = 100, I_N = 0.05
(j)
n = 10, I_N = 0.07
(k)
n = 50, I_N = 0.07
(l)
n = 100, I_N = 0.07
Cumulative Probability
0.00
0.25
0.50
0.75
1.00
X
Distribution
Beta
Exponential
Gamma
Kumaraswamy
LogLogistic
Normal
Triangular
Uniform

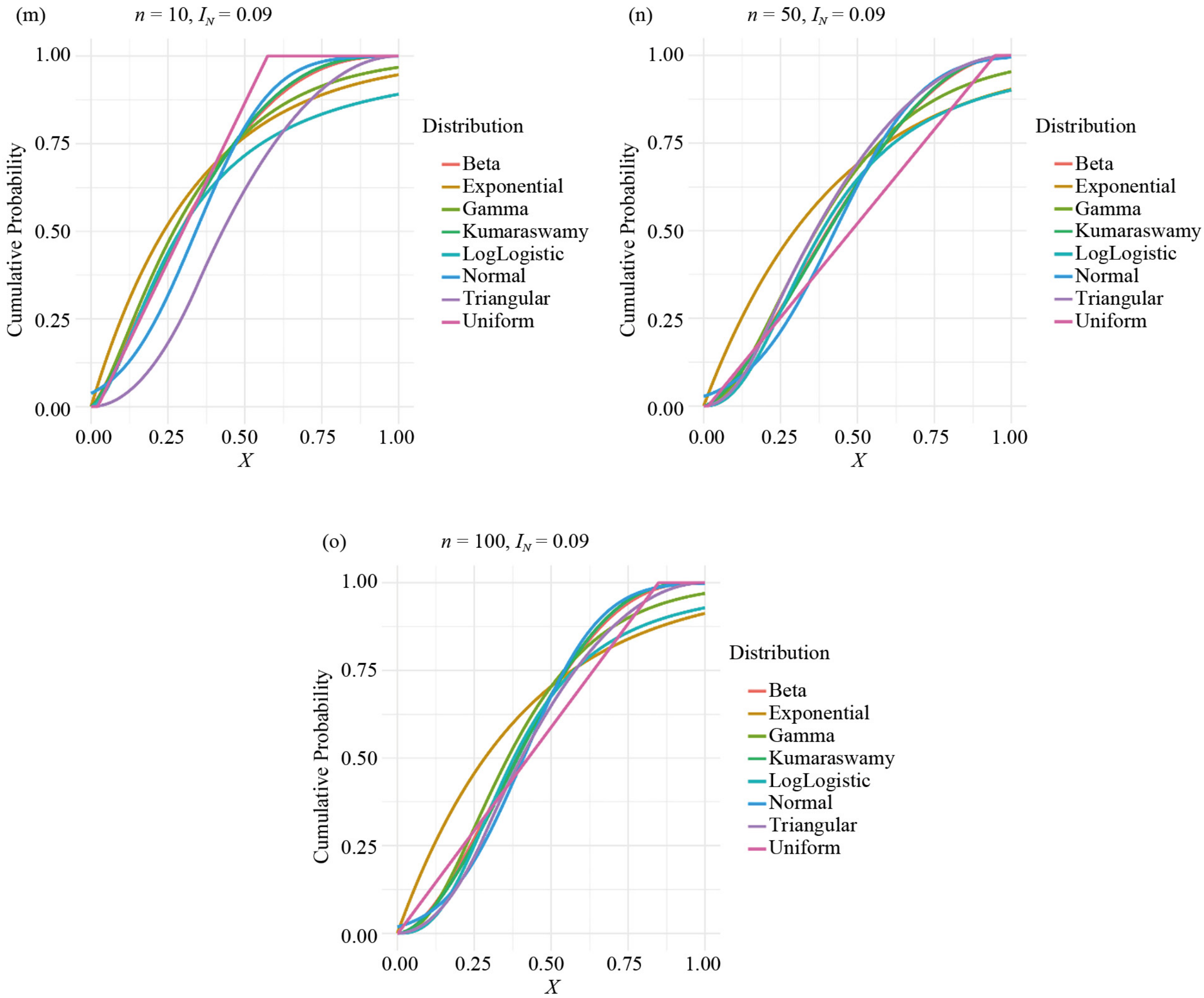
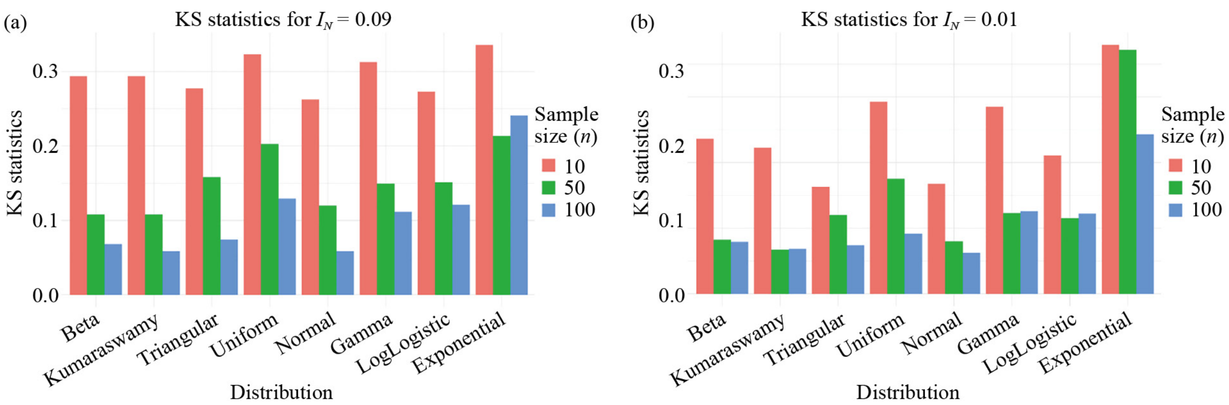


Figure 2. Fitted distributions using CDF with different values of n and I_N



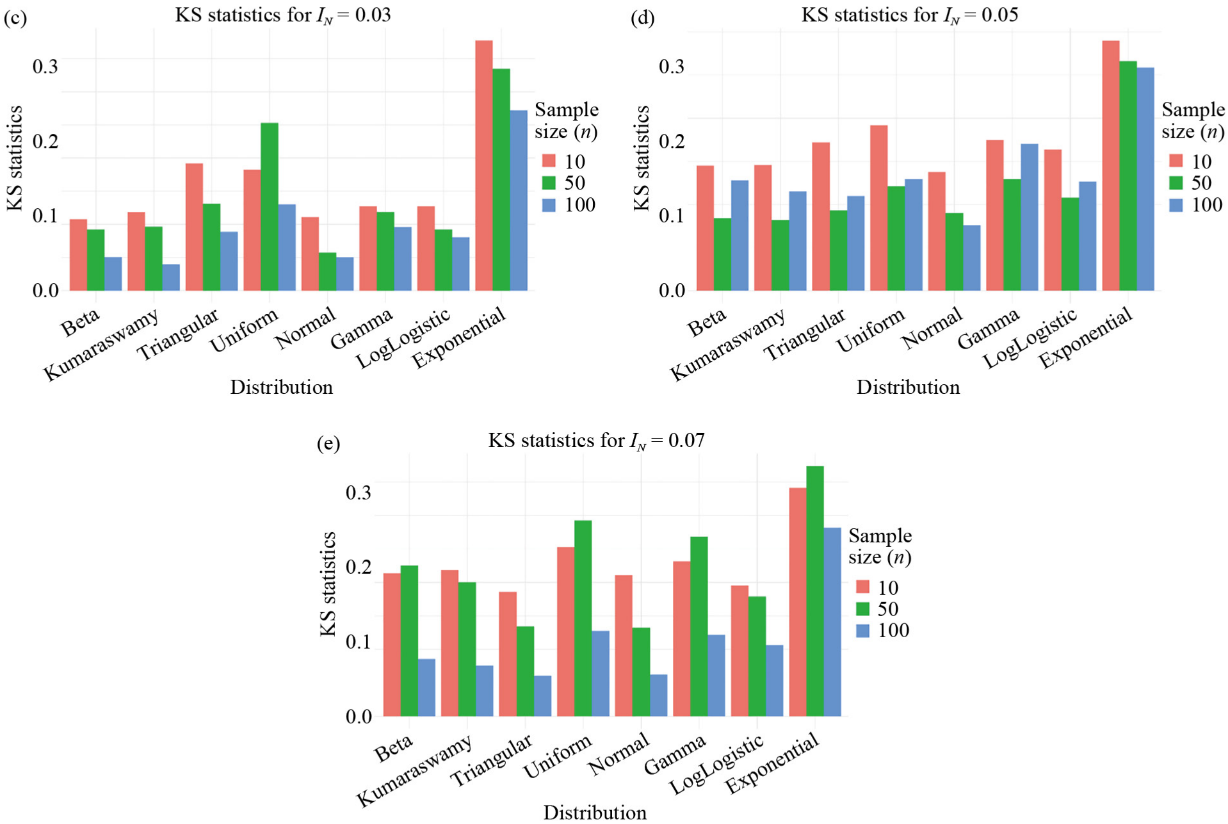


Figure 3. Multiple bar plots for KS statistics with different values of n and I_N

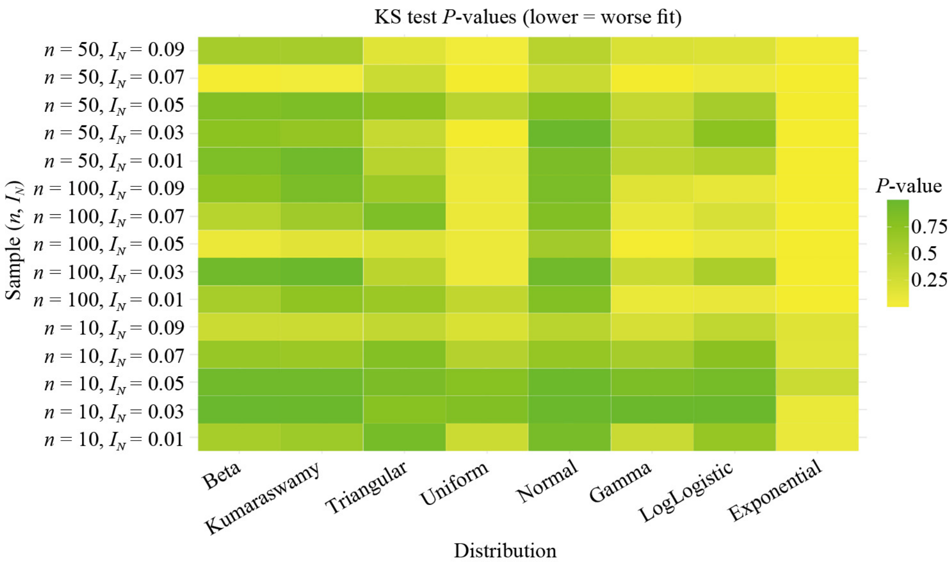


Figure 4. Heatmap of Kolmogorov-Smirnov (KS) test p-values for different distributions

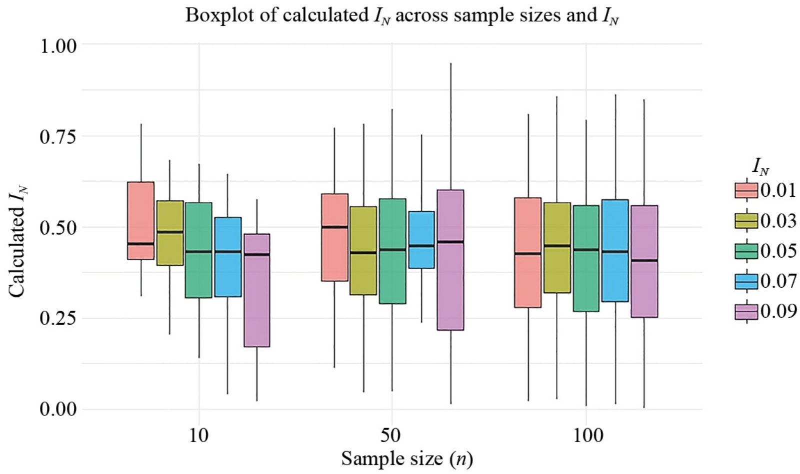


Figure 5. Boxplot for different values of I_N

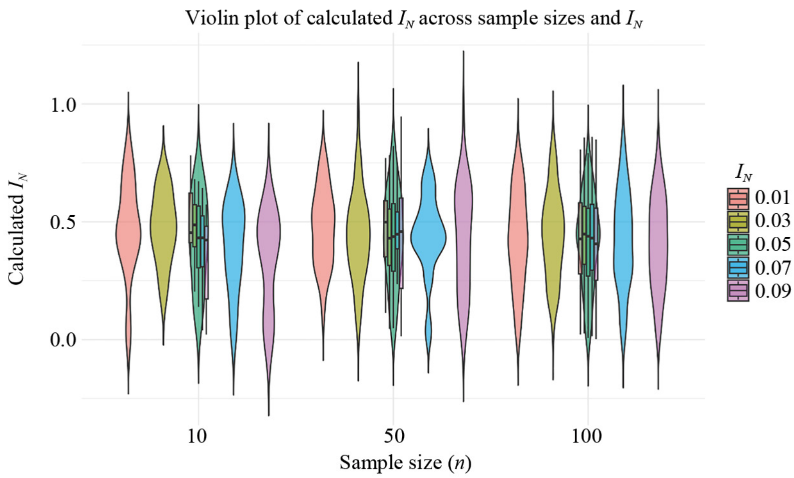
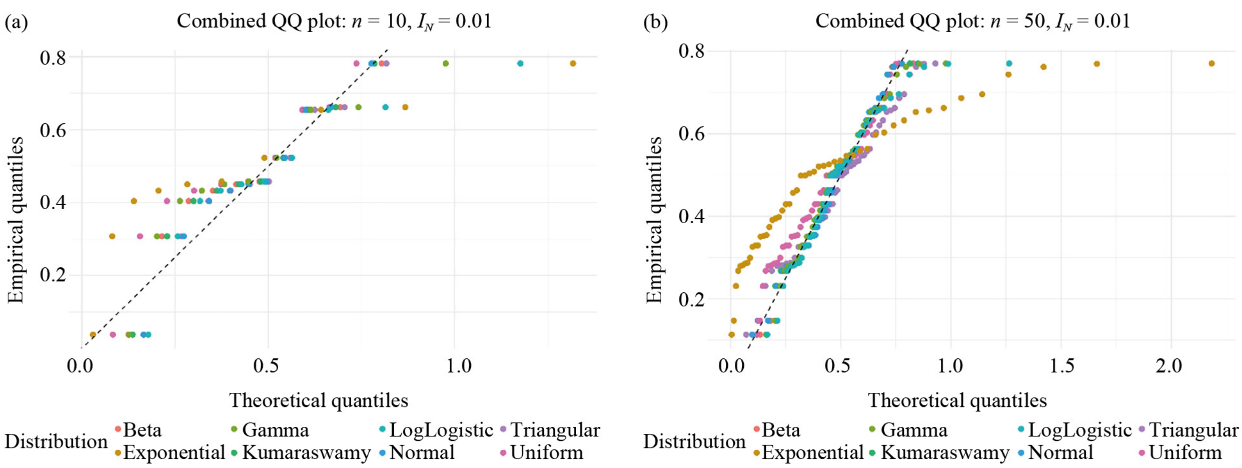
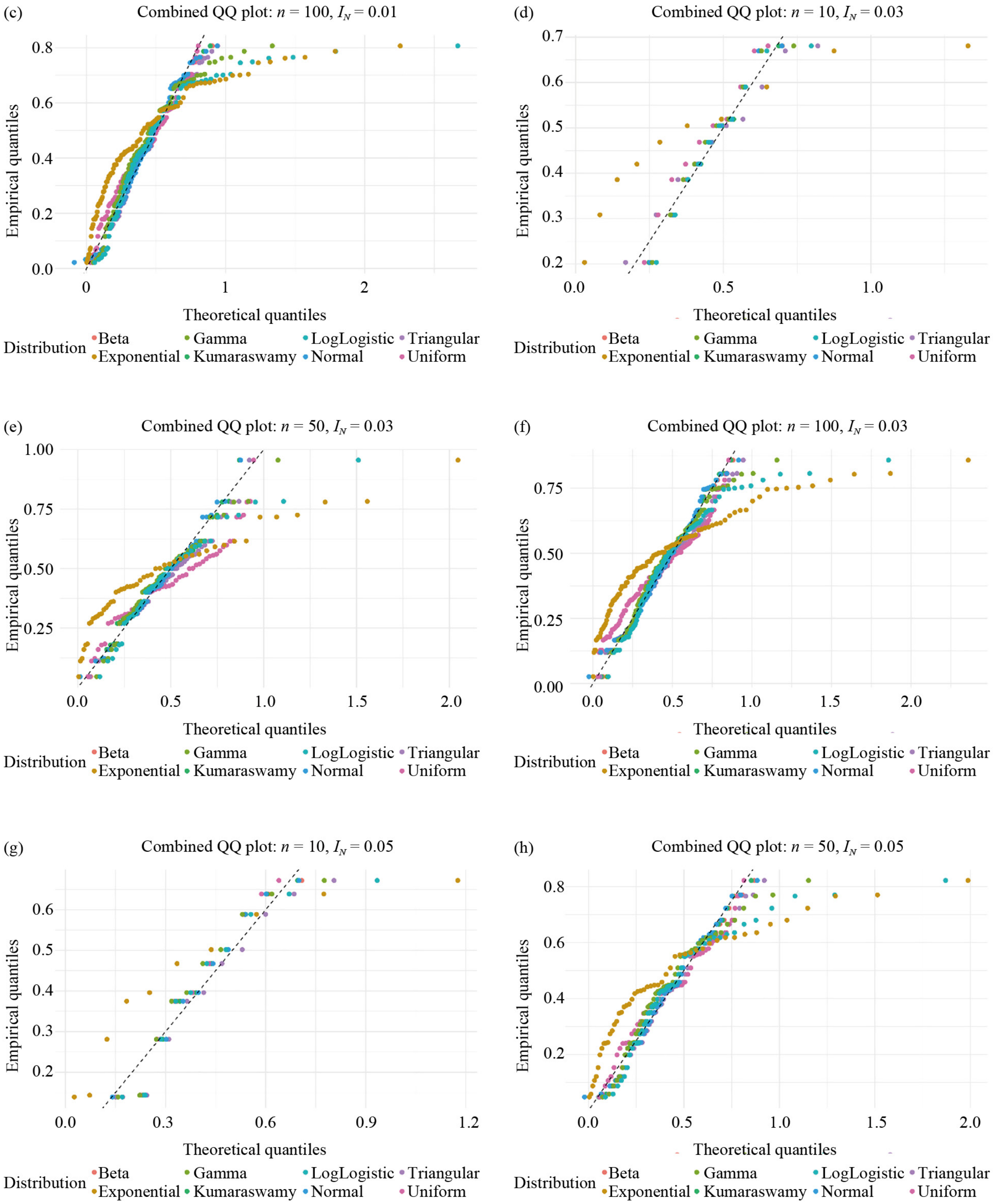
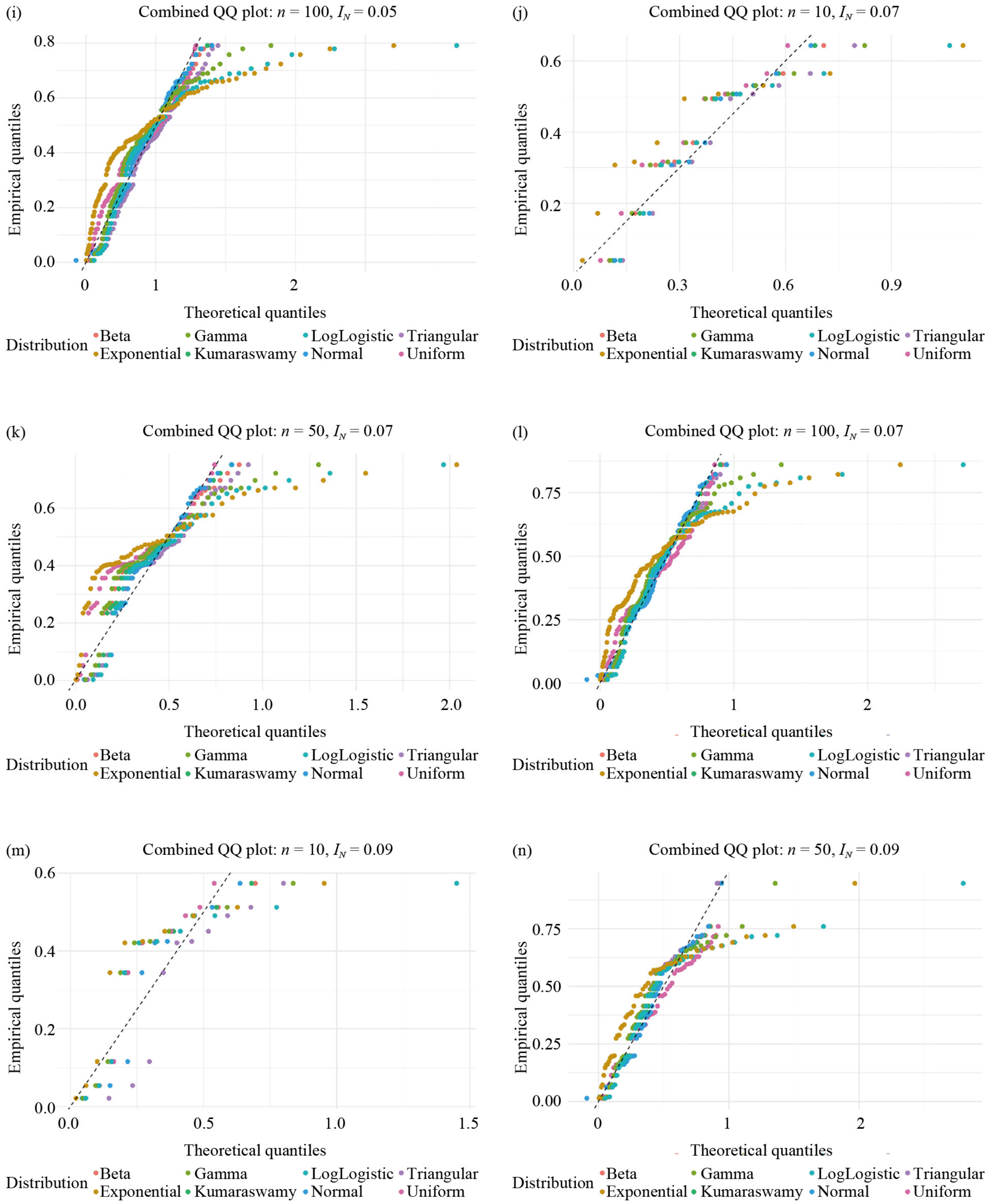


Figure 6. Violin plot for different values of I_N




(c)
Combined QQ plot: n = 100, I_N = 0.01
(d)
Combined QQ plot: n = 10, I_N = 0.03
(e)
Combined QQ plot: n = 50, I_N = 0.03
(f)
Combined QQ plot: n = 100, I_N = 0.03
(g)
Combined QQ plot: n = 10, I_N = 0.05
(h)
Combined QQ plot: n = 50, I_N = 0.05
Empirical quantiles
Theoretical quantiles
Distribution
Beta
Gamma
LogLogistic
Triangular
Exponential
Kumaraswamy
Normal
Uniform


(i)
Combined QQ plot: n = 100, I_N = 0.05
(j)
Combined QQ plot: n = 10, I_N = 0.07
(k)
Combined QQ plot: n = 50, I_N = 0.07
(l)
Combined QQ plot: n = 100, I_N = 0.07
(m)
Combined QQ plot: n = 10, I_N = 0.09
(n)
Combined QQ plot: n = 50, I_N = 0.09
Empirical quantiles
Theoretical quantiles
Distribution
Beta
Gamma
LogLogistic
Triangular
Exponential
Kumaraswamy
Normal
Uniform

(o)

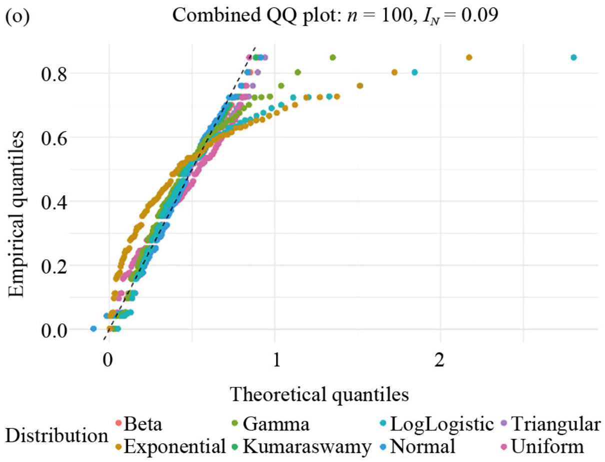


Figure 7. Empirical and theoretical QQ plot with different values of n and I_N

5. Comparative study

In this section, we present a comparative analysis using various statistical distributions fitted to different values of the degree of indeterminacy. The objective is to identify the most suitable distribution for modeling the indeterminacy component across a range of values. The results of this analysis are summarized in Tables 1-5, which highlight the best-fitting distributions corresponding to each level of indeterminacy.

Table 1. Comparison of different distributions by KS test statistics with p-values (in parenthesis) for different values of n and $I_N = 0.01$

Sample size		10	50	100
Distributions	Beta	0.2361 (0.5562)	0.0825 (0.8577)	0.0794 (0.5540)
	Kumaraswamy	0.2223 (0.6300)	0.0675 (0.9650)	0.0686 (0.7346)
	Triangular	0.1630 (0.9159)	0.1200 (0.4335)	0.0737 (0.6482)
	Normal	0.1674 (0.8998)	0.0799 (0.8815)	0.0627 (0.8269)
	Gamma	0.2855 (0.3242)	0.1225 (0.4081)	0.1257 (0.0850)
	Log Logistic	0.2107 (0.6928)	0.1148 (0.4897)	0.1225 (0.0996)
	Exponential	0.3793 (0.0840)	0.3716 (0.0000)	0.2430 (0.0000)
	Uniform	0.2925 (0.2972)	0.1752 (0.0820)	0.0914 (0.3733)

Table 2. Comparison of different distributions by KS test statistics with p-values (in parenthesis) for different values of n and $I_N = 0.03$

Sample size		10	50	100
Distributions	Beta	0.1075 (0.9988)	0.0923 (0.7527)	0.0509 (0.9582)
	Kumaraswamy	0.1185 (0.9956)	0.0964 (0.7048)	0.0394 (0.9977)
	Triangular	0.1919 (0.7899)	0.1310 (0.3288)	0.0888 (0.4092)
	Normal	0.1109 (0.9981)	0.0572 (0.9936)	0.0502 (0.9623)
	Gamma	0.1274 (0.9899)	0.1188 (0.4462)	0.0963 (0.3120)
	Log Logistic	0.1275 (0.9898)	0.0924 (0.7521)	0.0805 (0.5353)
	Exponential	0.3774 (0.0867)	0.3347 (0.0000)	0.2720 (0.0000)
	Uniform	0.1822 (0.8371)	0.2532 (0.0026)	0.1302 (0.0672)

Table 3. Comparison of different distributions by KS test statistics with p-values (in parenthesis) for different values of n and $I_N = 0.05$

Sample size		10	50	100
Distributions	Beta	0.1451 (0.9649)	0.0841 (0.8423)	0.1279 (0.0758)
	Kumaraswamy	0.1455 (0.9641)	0.0820 (0.8626)	0.1149 (0.1424)
	Triangular	0.1721 (0.8816)	0.0930 (0.7449)	0.1099 (0.1788)
	Normal	0.1374 (0.9784)	0.0898 (0.7812)	0.0761 (0.6095)
	Gamma	0.1746 (0.8711)	0.1297 (0.3403)	0.1700 (0.0062)
	Log Logistic	0.1639 (0.9126)	0.1076 (0.5713)	0.1263 (0.0825)
	Exponential	0.2900 (0.3066)	0.2658 (0.0013)	0.2587 (0.0000)
	Uniform	0.1919 (0.7899)	0.1209 (0.4247)	0.1293 (0.0707)

Table 4. Comparison of different distributions by KS test statistics with p-values (in parenthesis) for different values of n and $I_N = 0.07$

Sample size		10	50	100
Distributions	Beta	0.2138 (0.6762)	0.2252 (0.0104)	0.0859 (0.4514)
	Kumaraswamy	0.2185 (0.6508)	0.2003 (0.0311)	0.0756 (0.6165)
	Triangular	0.1859 (0.8194)	0.1339 (0.3034)	0.0603 (0.8599)
	Normal	0.2104 (0.6943)	0.1321 (0.3186)	0.0623 (0.8327)
	Gamma	0.2314 (0.5812)	0.2681 (0.0012)	0.1215 (0.1044)
	Log Logistic	0.1954 (0.7721)	0.1788 (0.0718)	0.1066 (0.2062)
	Exponential	0.3412 (0.1530)	0.3736 (0.0000)	0.2818 (0.0000)
	Uniform	0.2528 (0.4704)	0.2923 (0.0003)	0.1277 (0.0765)

Table 5. Comparison of different distributions by KS test statistics with p-values (in parenthesis) for different values of n and $I_N = 0.09$

Distributions	$I_N = 0.09$		
	Sample size: 10	Sample size: 50	Sample size: 100
Beta	0.2934 (0.2942)	0.1079 (0.5680)	0.0678 (0.7475)
Kumaraswamy	0.2936 (0.2934)	0.1079 (0.5688)	0.0583 (0.8862)
Triangular	0.2770 (0.3586)	0.1578 (0.1485)	0.0744 (0.6373)
Normal	0.2622 (0.4245)	0.1196 (0.4383)	0.0584 (0.8846)
Gamma	0.3128 (0.2288)	0.1493 (0.1944)	0.1118 (0.1640)
Log Logistic	0.2729 (0.3763)	0.1514 (0.1823)	0.1212 (0.1058)
Exponential	0.3357 (0.1658)	0.2136 (0.0176)	0.2412 (0.0000)
Uniform	0.3233 (0.1981)	0.2025 (0.0283)	0.1292 (0.0708)

From Tables 1 to 6 the following results are observed:

For $I_N = 0.01$:

• $n = 10$: The Triangular distribution shows the best fit, followed by Normal, Log-logistic, Kumaraswamy, and Beta.

• $n = 50$: The ranking changes, with Kumaraswamy performing best, followed by Beta, Normal, Log-logistic, Triangular, and Gamma.

• $n = 100$: The order of fit becomes Normal, Kumaraswamy, Triangular, and Beta.

For $I_N = 0.03$:

• $n = 10$: The Beta distribution performs best, followed by Normal, Kumaraswamy, Gamma, Log-logistic, and Triangular.

• $n = 50$: The Normal distribution shows the best performance, followed by Beta, Log-logistic, and Kumaraswamy.

• $n = 100$: The order of best fit is Kumaraswamy, followed by Normal, Beta, Log-logistic, Triangular, and Gamma.

For $I_N = 0.05$:

• $n = 10$: The Normal distribution performs best, followed by Beta, Kumaraswamy, Log-logistic, Triangular, and Gamma.

• $n = 50$: Kumaraswamy ranks highest, followed by Beta, Normal, Triangular, Log-logistic, and Gamma.

• $n = 100$: The best fit is shown by Normal, followed by Triangular, Kumaraswamy, Log-logistic, and Beta.

For $I_N = 0.07$:

• $n = 10$: Triangular distribution is best, followed by Log-logistic, Normal, Beta, Kumaraswamy, and Gamma.

• $n = 50$: Normal performs best, followed by Triangular and Log-logistic.

• $n = 100$: Again, Triangular performs best, followed by Normal, Kumaraswamy, Beta, Log-logistic, and Gamma.

For $I_N = 0.09$:

• $n = 10$: Normal ranks first, followed by Log-logistic, Triangular, Beta, Kumaraswamy, and Gamma.

• $n = 50$: Kumaraswamy performs best, followed by Beta, Normal, Gamma, Log-logistic, and Triangular.

• $n = 100$: The top performer remains Kumaraswamy, followed by Normal, Beta, Triangular, Gamma, and Log-logistic.

These rankings are also visually supported by Figures 1, 2, 3 and 7, which illustrates the comparative performance of the selected distributions across the different I_N values and sample sizes. The comparative analysis of distributional fits across varying values of I_N and sample sizes, shows that no single distribution consistently outperforms others in all scenarios. However, certain trends can be observed such as: Normal and Kumaraswamy distributions frequently appear as top performers, specifically as the sample size increases. The Triangular distribution shows good results for small sample sizes but loses its performance with larger samples. The Beta and Log-logistic distributions also show good fit in explicit conditions, while the Gamma distribution consistently ranks lower in performance across most settings as compared to above said distributions. Although in some cases uniform distribution yields significant p-valuess, it generally performs poorly compared to the other competitive distributions. Its p-valuess are often lower in comparison and fall below the 0.05 threshold, demonstrating a lack of fit. Likewise, the exponential distribution fails to demonstrate good fit crosswise all scenarios. These descriptions support the conclusion that distributions such as normal, Kumaraswamy, beta, and log-logistic are more reliable choices for modeling indeterminate factors, as evidenced in the results and visualized in Figure 1 highlight the importance of considering both the level of imprecision (I_N) and sample size when selecting an appropriate distribution for modeling purposes. These findings do not support the simulation proposed by Woodall et al. [20, 21], who employed the uniform distribution over each interval. Based on the analysis, it can be concluded that the simulation put forward by Woodall et al. [20, 21] fails to incorporate an appropriate distribution of the degree of uncertainty. By neglecting or overlooking this factor, their simulation does not accurately represent neutrosophic analysis.

6. Application using real-data sets

In this section two real-world datasets are examined to assess the performance of the developed neutrosophic Weibull distribution by [19].

The endurance test results of around 5,000 deep-groove ball bearings, which indicate the number of rotations till failure, are included in the first dataset. The failure times of 25 light bulbs evaluated in accelerated settings make up the second dataset. The suggested distribution is used to represent the two datasets, which involve imprecision and uncertainty. Its usefulness for modeling lifetime data under neutrosophic circumstances is supported by the results. Some model selection criterions and the KS test are used to check the reliability of the proposed model.

The model selection criteria and goodness-of-fit findings for the proposed Neutrosophic Weibull Distribution (NWD) are shown in Tables 7 and 9 while Table 6 and 8 shows the estimated values of the parameters and their standard errors.

The information criteria's continually lower values imply that NWD offers a competitive match to the data. Furthermore, the model fits both data sets well and with no significant variation, according to the KS test findings with strong p-values.

Data 1:

(17.88, 33.67) (28.92, 39.88) (33, 41.14) (41.52, 45.55) (42.12, 56.25) (45.6, 57.01) (48.48, 70.04) (51.84, 72.16) (51.96, 73.93) (54.12, 80.65) (55.56, 83.72) (67.8, 85.78) (68.64, 89.43) (68.64, 94.09) (68.88, 95.29) (84.12, 115.13) (93.12, 116.37) (98.64, 116.72) (105.12, 117.3) (127.92, 120.15) (128.04, 123.23) (173.4, 132.22).

Data 2:

(1.25, 0.95) (1.37, 1.17) (0.28, 0.49) (0.53, 0.78) (0.98, 0.47) (1.17, 0.96) (0.65, 0.82) (1.00, 1.02) (0.66, 0.95) (1.76, 1.07) (0.42, 0.31) (1.39, 0.77) (0.25, 1.18) (0.82, 0.41) (0.57, 0.71) (1.71, 1.16) (0.96, 0.59) (0.45, 0.4) (1.61, 0.53) (0.31, 0.76) (0.95, 1.07) (1.03, 0.72) (0.67, 0.5) (0.48, 0.75) (0.29, 0.53).

Results from Data 1:

Table 6. ML estimates and the standard error for the deep-groove ball bearings

Distribution		MLE	SE
NWD	λ	[95.9400, 114.8830]	[10.5534, 12.6374]
	k	[2.0536, 2.0539]	[0.3252, 0.3252]

Table 7. Model Selection criteria of the proposed model

Model	$-$ll	AIC	BIC	CAIC	HQIC	KS	p-values
NWD	[$-$112.531, $-$116.489]	[229.063, 236.978]	[231.245, 239.160]	[229.694, 237.609]	[229.577, 237.492]	[0.162, 0.163]	[0.607, 0.605]

Results from Data 2:

Table 8. ML estimates and the standard error for the failure times of light bulbs

Distribution		MLE	SE
NWD	λ	[0.7879, 0.6351]	[0.0818, 0.0660]
	k	[2.0331, 2.0331]	[0.3216, 0.3216]

Table 9. Model Selection criteria of the proposed model

Model	$-$ll	AIC	BIC	CAIC	HQIC	KS	p-values
NWD	[$-$8.551, $-$3.163]	[21.101, 10.327]	[23.539, 12.765]	[21.647, 10.872]	[21.778, 11.003]	[0.109, 0.109]	[0.899, 0.899]

We used the following formula to get the estimated indeterminacy for each observation in order to examine the uncertainty between the lower and upper interval bounds:

$$I_N = \frac{X_U - X_L}{X_L}$$

where X_L and X_U stand for the lower and upper boundaries of the observed intervals, respectively. Eight continuous probability distributions are fitted to describe the distributional behavior of the I_N values: Exponential, Log-Logistic, Gamma, Normal, Triangular, Beta, Kumaraswamy, and Uniform. Each model's goodness-of-fit is evaluated using the Kolmogorov-Smirnov (KS) test, and the associated p-valuess and KS statistics are noted. For the I_N data, the distribution with the greatest p-values is deemed to be the best-fitting model.

According to the findings of the Kolmogorov-Smirnov (KS) test in Tables 10 and 11, the Beta distribution demonstrated the best match for the light bulb failure time data (Table 11), while the Normal distribution offered the greatest fit obtained from the deep-groove ball bearing data (Table 10). The highest p-valuess and lowest KS statistics found in each instance, which show no difference between the theoretical and actual distributions. Compared to the triangular and exponential distributions, which showed poor fits, other appropriate models for both datasets were the Kumaraswamy, Log-Logistic, and Normal distributions.

Table 10. KS test statistics with p-valuess for the deep-groove ball bearings

Models	KS	p-valuess
Beta	0.2039	0.2797
Kumaraswamy	0.1701	0.4946
Triangular	0.5374	0.0000
Normal	0.1391	0.7376
Gamma	0.2209	0.2007
Log Logistic	0.1722	0.4796
Exponential	0.3478	0.0070
Uniform	0.2619	0.0805

Table 11. KS test statistics with p-valuess for the failure times of light bulbs

Models	KS	p-valuess
Beta	0.0684	0.9992
Kumaraswamy	0.0719	0.9983
Triangular	0.2406	0.0929
Normal	0.1019	0.9340
Gamma	0.1026	0.9307
Log Logistic	0.0899	0.9767
Exponential	0.2497	0.0736
Uniform	0.2363	0.1035

From Figure 8, the Deep-groove Ball Bearings dataset is represented by Figure 8a, where the Normal and Kumaraswamy distributions closely resemble the empirical distribution, indicating a strong match. On the other hand, the exponential and triangular curves differ greatly, especially in the tails. Several distributions, including Normal, Beta, Gamma, and Log-Logistic, match the histogram nicely in Figure 8b, which shows the light bulb failure times. Since it is unable to reflect the shape and distribution of the observed data, the uniform distribution offers the worse fit in both datasets.

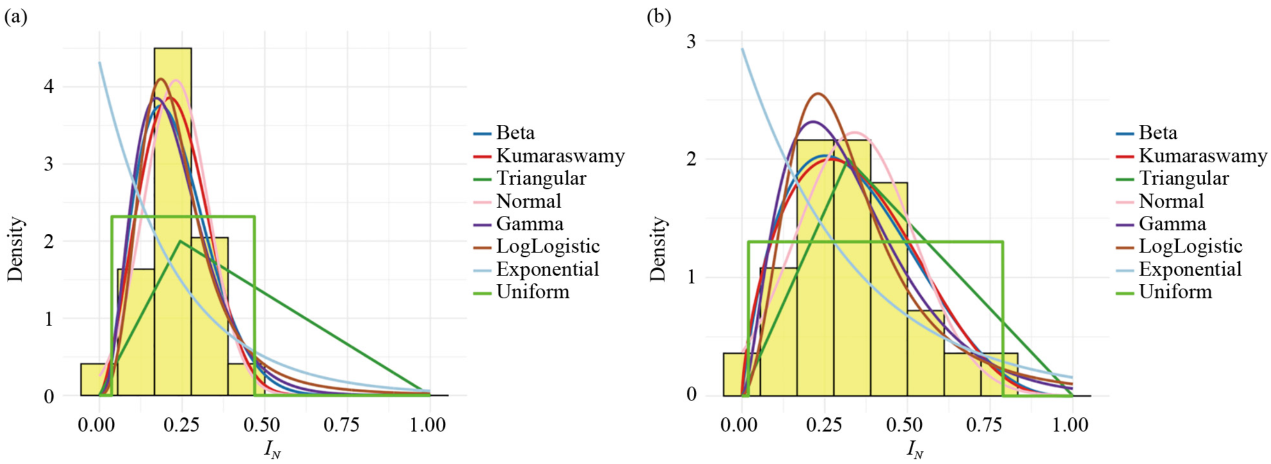


Figure 8. Histogram of I_N with fitted distributions

7. Concluding remarks

Comparative analysis demonstrated that the uniform distribution did not provide an adequate fit to the observed indeterminacy values, suggesting that the underlying pattern of indeterminacy in the generated neutrosophic data is non-uniform and likely governed by a more complex probabilistic structure. This result highlights the necessity for employing more appropriate and tailored probability models when analyzing neutrosophic data, particularly due to its inherent uncertainty. Based on the tabulated results and graphical visualizations, it is evident that the Beta, Normal, Kumaraswamy, Log-Logistic, Triangular, and Gamma distributions generally exhibit better performance in modeling the indeterminacy values, though the order of their effectiveness varies depending on the sample size and level of I_N. In contrast, the Exponential distribution consistently shows a poor fit, and more notably, the Uniform distribution fails to provide flexible or consistently acceptable p-valuess compared to the other distributions. In several cases, it also demonstrates a clear lack of fit. This overall trend is further supported by the visual representations in the graphs. Finally, the proposed neutrosophic Weibull distribution is applied to two real-life neutrosophic data sets to evaluate its performance. The analysis demonstrated that the Weibull model provided the best fit for both data sets. Based on the calculated indeterminacy measure (I_N), several well-known distributions discussed above are model and compared for I_N data. For the first data, the order of best fit following is Normal, Kumaraswamy, Log-Logistic, Beta, and Gamma. In contrast, for the second data set, the beta distribution showed the best fit, followed by Kumaraswamy, log-logistic, normal, and gamma. Moreover, these findings contrast to the simulations of Woodall et al. [20, 21], who supported the use of interval-analysis based simulation with a uniform distribution for the neutrosophic analysis. Furthermore, the mere use the term "neutrosophic" in their simulation, despite it not being specifically designed for neutrosophic analysis, lead to inaccurate and flawed results. The importance of the suggested neutrosophic Weibull distribution as a versatile and effective model for examining data with inherent uncertainty and indeterminacy is highlighted in this work. The results show that indeterminacy values frequently follow intricate probabilistic patterns, in contrast to the conventional uniform assumption, highlighting the need for more flexible models in neutrosophic research. The robustness and practical application of the proposed model are confirmed by its good performance on both simulated and real-world datasets. These findings offer a useful basis for applying neutrosophic models to more general situations including risk assessment, reliability engineering, and uncertain decision-making.

AI tool declaration

The authors acknowledge the use of ChatGPT (GPT-5 mini, OpenAI) solely for improving the English language, clarity, and readability of our own writing in the manuscript. All scientific content, mathematical derivations, data analyses, simulations, and conclusions were developed, verified, and approved exclusively by the authors.

Conflict of interest

The authors declare no competing financial interest.

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