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Research Article

Marczewski-Burstin Representation of Soft Algebras

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Abstract: Using Marczewski and Burstin's ideas, we present a general scheme for defining the concepts of (s)-sets and (s^0)-sets in soft environments. We show that the classes of all soft (s)-sets and soft (s^0)-sets are a soft algebra and a soft ideal, respectively. The soft (s)-sets and soft (s^0)-sets are defined with respect to a family of non-null soft sets, which is known as a basis. A soft algebra is said to be a soft Marczewski-Burstin algebra if it is equal to the class of (s)-sets for some basis. The same is true for the respective soft ideal. Essential properties and characterizations of soft Marczewski-Burstin algebras are studied. As an application to the scheme, if the basis of the classes of soft (s)-sets and soft (s^0)-sets is a soft topology, we show that soft Marczewski-Burstin algebra and soft Marczewski-Burstin ideal are respectively equal to the soft nowhere dense boundary algebra and the soft nowhere dense ideal. In this case, such a soft algebra and a soft ideal are called soft topological, which means each soft topological Marczewski-Burstin algebra is a soft Marczewski-Burstin algebra. The reverse is generally incorrect, which is verified by an example. In addition, some properties of soft topological Marczewski-Burstin algebras are also obtained.

Keywords: soft set, soft algebra, Marczewski-Burstin representation, soft topological algebra, soft Marczewski-Burstin algebra, inner soft MB-algebra

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1. Introduction

In the modern world, it is increasingly essential to mathematically characterize and manage various forms of uncertainty to address complex issues across multiple fields, such as engineering, economics, environmental science, medicine, and social sciences. While fuzzy and rough set theories, and probability are recognized and effective tools for handling unpredictability and vagueness, every theory comes with its own limitations. A significant drawback that these mathematical approaches share is the absence of parameterization tools.

In 1999, Molodtsov [1] introduced the soft set theory to deal with uncertainty, which has difficulties in the mentioned theories. A proposal was made which define soft sets as a collection of parameterized universes of possible universes. If used with soft sets, parameter sets are an effective way to model uncertain information in a standardized manner. This leads to rapid growth of soft set theory and its associated disciplines in a brief span of time and provides a lot of applications of soft sets in real life situations [2–4].

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Many mathematical structures, including soft topology [5, 6], soft group theory [7], and soft ring theory [8], have been extensively studied within the framework of soft set theory by numerous researchers.

The findings of [9–13], which explored diverse approaches to generating soft topologies, have garnered significant scholarly attention, thereby stimulating further research on various classes of soft topologies [14–16].

The advancement of various fields, such as analysis, theory of probability, economics, and finance, relies on the concept of an algebra or a σ-algebra of subsets of a classical universal set. The ideas of a σ-algebra of soft sets and an algebra of soft sets were established by Khameneh and Kilicman [17] and Riaz et al. [18] with some basic properties. These two concepts have been fully investigated and implemented in [19, 20].

Burstin [21] was the first to propose defining (s)-sets and (s^0)-sets in the context of measure theory. He showed that the class of Lebesgue measurable subsets of $\mathbb{R}$ is a Marczewski-Burstin algebra with a basis that contains perfect sets of positive Lebesgue measures. Later on, (s)-sets and (s^0)-sets are appeared in the paper of Marczewski [22], where he proved that the classes of (s)-sets and (s^0)-sets are respectively a σ-algebra and a σ-ideal whenever the basis is the family of perfect subsets of $\mathbb{R}$. These classes have been investigated by many researchers under different names and circumstances [23–25]. This topic of research is very rich; however, it does not exist in other set theoretical structures. Hence, we open the door for researchers and introduce the classes of (s)-sets and (s^0)-sets in soft settings. The concept of soft Marczewski-Burstin algebras and soft Marczewski-Burstin ideal are considered via classes of soft (s)-sets and soft (s^0)-sets, respectively. We examine that this scheme works well without imposing any assumptions on the so-called basis.

This is the rest of the paper: Section 2 provides the necessary definitions, notations, and conclusions on soft set theory. Section 3 focuses on the investigation of soft nowhere dense, soft nowhere dense boundary sets, and their consequences. Section 4 defines the concepts of soft (s)-sets and soft (s^0)-sets. We show that the families of these soft sets form a soft algebra and a soft ideal, respectively. Then, we introduce the type of soft Marczewski-Burstin algebras and obtain the essential characterizations. Section 5 is devoted to a subclass of soft Marczewski-Burstin algebras called soft topological. Some findings on soft topological Marczewski-Burstin algebras are also achieved in this part. Section 6 finishes with a short analysis and debate.

2. Preliminaries

In order to bolster our assertions in the subsequent sections, this portion offers fundamental definitions and set theoretical and topological proprieties in soft structures. In the following discussion, we will consider X and $\boldsymbol{\Pi}$ to represent a specified universal set and a nonempty collection of parameters, respectively.

Definition 1 [1] For any nonempty subset $\Pi \subseteq \boldsymbol{\Pi}$, a soft set defined over X (with respect to Π) is represented as an ordered pair (L, Π), where L is a mapping from Π into the power set 2^X of X.

Notice that any (L, Π) can be extended to the soft set $(L, \boldsymbol{\Pi})$ by taking that $L(u) = \emptyset$ for all $u \in \boldsymbol{\Pi} - \Pi$.

Definition 2 [3] The complement of (L, Π), denoted by $(L, \Pi)^c$, is a soft set (L^c, Π), where $L^c : \Pi \to 2^X$ is a mapping having the property that $L^c(u) = X - L(u)$ for all $u \in \Pi$.

Definition 3 [26] A finite soft set is the soft set (L, Π) for which $L(u)$ is finite for all $u \in \Pi$. Alternatively, it is referred to as infinite.

Definition 4 [27] A soft set (L, Π) defined over X is referred to as null with respect to Π, denoted as Φ_{Π}, if $L(u) = \emptyset$ for every $u \in \Pi$. It is termed absolute with respect to Π, labeled as X_{Π}, when $L(u) = X$ holds for all $u \in \Pi$. The null and absolute soft sets are symbolized as $\Phi_{\boldsymbol{\Pi}}$ and $X_{\boldsymbol{\Pi}}$, respectively.

The collection of all soft sets over X with respect to Π is denoted by $\mathfrak{S}(X_{\Pi})$.

Definition 5 A soft element [28], denoted by x_u, is a soft set (L, Π) over X whenever $L(u) = \{x\}$ and $L(q) = \emptyset$ for all $q \in \Pi$ with $q \neq u$, where $u \in \Pi$ and $x \in X$. The soft element is known as a soft point in [29]. We choose to utilize the idea of soft point moving forward. Moreover, we denote by $\mathfrak{P}(X_{\Pi})$ the set of all soft points in X.

Definition 6 [3, 30] Let $\Pi_1, \Pi_2 \subseteq \boldsymbol{\Pi}$. It is said that (L_1, Π_1) is a soft subset of (L_2, Π_2) (written by $(L_1, \Pi_1)\tilde{\subseteq}(L_2, \Pi_2)$) if $\Pi_1 \subseteq \Pi_2$ and $F_1(u) \subseteq F_2(u)$ for all $u \in \Pi_1$. And (L_1, Π_1) is soft equal to (L_2, Π_2) if $(L_1, \Pi_1)\tilde{\subseteq}(L_2, \Pi_2)$ and $(L_2, \Pi_2)\tilde{\subseteq}(L_1, \Pi_1)$.

Definition 7 [13, 27] The union of any indexed family of soft sets $\{(L_i, \Pi) : i \in I\}$ is a soft set (L, Π) for which $L(u) = \bigcup_{i \in I} F_i(u)$ for each $u \in \Pi$ and is labeled as $(L, \Pi) = \tilde{\bigcup}_{i \in I}(L_i, \Pi)$. The intersection of $\{(L_i, \Pi) : i \in I\}$ is a soft set (L, Π) for which $L(u) = \bigcap_{i \in I} F_i(u)$ for each $u \in \Pi$ and is labeled as $(L, \Pi) = \tilde{\bigcap}_{i \in I}(L_i, \Pi)$.

Definition 8 [31] Let (L, Π), (G, Π) be soft subsets over X. The soft difference between (L, Π), (G, Π) is the soft set $(H, \Pi) = (L, \Pi)\tilde{-}(G, \Pi)$, where $H(u) = L(u) - G(u)$ for all $u \in \Pi$.

For any collection $\mathscr{C} \tilde{\subseteq} \mathfrak{S}(X_\Pi)$, we mean by $\mathscr{C}^* = \mathscr{C} \tilde{-} \Phi_\Pi$.

Definition 9 [32, 33] A subcollection $\mathscr{C}$ of $\mathfrak{S}(X_\Pi)$ is known as a soft ideal on X if $\mathscr{C}$ satisfies the following axioms:

(1) $X_\Pi \tilde{\notin} \mathscr{C}$;

(2) $(L, \Pi) \tilde{\in} \mathscr{C}$ and $(G, \Pi) \tilde{\subseteq} (L, \Pi)$ implies $(G, \Pi) \tilde{\in} \mathscr{C}$; and

(3) $(L, \Pi), (G, \Pi) \tilde{\in} \mathscr{C}$ implies $(L, \Pi) \tilde{\cup} (G, \Pi) \tilde{\in} \mathscr{C}$.

Definition 10 [34] A subcollection $\mathscr{B}$ of $\mathfrak{S}(X_\Pi)$ is known as a soft filter on X if $\mathscr{B}$ satisfies the following axioms:

(1) $\Phi_\Pi \tilde{\notin} \mathscr{B}$;

(2) $(L, \Pi) \tilde{\in} \mathscr{B}$ and $(L, \Pi) \tilde{\subseteq} (G, \Pi)$ implies $(G, \Pi) \tilde{\in} \mathscr{B}$; and

(3) (L, Π), $(G, \Pi) \tilde{\in} \mathscr{B}$ implies $(L, \Pi) \tilde{\cap} (G, \Pi) \tilde{\in} \mathscr{B}$.

A soft filter $\mathscr{B}$ is known as a soft ultrafilter if either (L, Π) or (L^c, Π) is in $\mathscr{B}$ for each $(L, \Pi) \tilde{\in} \mathfrak{S}(X_\Pi)$.

Definition 11 [18] A subcollection $\mathscr{S} \tilde{\subseteq} \mathfrak{S}(X_\Pi)$ is said to be a soft algebra if it meets the following axioms:

(1) $\Phi_\Pi \tilde{\in} \mathscr{S}$;

(2) $(L, \Pi) \tilde{\in} \mathscr{S} \implies (L, \Pi)^c \tilde{\in} \mathscr{S}$; and

(3) $(L_n, \Pi) \tilde{\in} \mathscr{S}$, for $n = 1, 2, \ldots, k$, $\implies \tilde{\bigcup}_{n=1}^{k}(L_n, \Pi) \tilde{\in} \mathscr{S}$.

For any soft algebra $\mathscr{S}$ on X, the hereditary soft ideal is given by $\mathscr{H}(\mathscr{S}) = \{(F, \Pi) \tilde{\in} \mathscr{S} : \mathfrak{S}(F_\Pi) \tilde{\subseteq} \mathscr{S}\}$, where $\mathfrak{S}(F_\Pi)$ means the family of all soft subsets of (F, Π).

Theorem 1 [20] Let $\Pi = \{u\}$ and let $\mathscr{A}$ be a family of subsets of X. Then $\mathscr{S} = \{(u, L(u)) : L(u) \in \mathscr{A}\}$ is a soft algebra if and only if $\mathscr{S}_u = \{L(u) : (u, L(u)) \tilde{\in} \mathscr{S}\}$ is an ordinary algebra.

Definition 12 [20] Let $\mathscr{C}$ be a subcollection of $\mathfrak{S}(X_\Pi)$. The soft intersection of all soft algebras on X containing $\mathscr{C}$ is known as the soft algebra generated by $\mathscr{C}$ and is denote it by $\mathscr{A}(\mathscr{C})$.

Definition 13 [5, 6] A subfamily $\mathscr{T}$ of $\mathfrak{S}(X_\Pi)$ is known as a soft topology on X if the following conditions are satisfied:

(1) Φ_Π, $X_\Pi \tilde{\in} \mathscr{T}$;

(2) If $(Y_1, \Pi), (Y_2, \Pi) \tilde{\in} \mathscr{T}$, then $(Y_1, \Pi) \tilde{\cap} (Y_2, \Pi) \tilde{\in} \mathscr{T}$; and

(3) If any $\{(Y_i, \Pi) : i \in I\} \tilde{\subseteq} \mathscr{T}$, then $\tilde{\bigcup}_{i \in I}(Y_i, \Pi) \tilde{\in} \mathscr{T}$.

$(X, \mathscr{T}, \Pi)$ is called a soft topological space on X. The members of $\mathscr{T}$ are referred to as soft $\mathscr{T}$-open sets (or soft open sets if no confusion can arise), and their complements are the soft $\mathscr{T}$-closed sets (or soft closed sets). We denote by $\mathscr{T}^c$ the collection of all soft $\mathscr{T}$-closed sets.

Definition 14 [5] If every member of $\mathscr{T}$ is a union of elements of the family $\mathscr{B} \subseteq \mathscr{T}$, then $\mathscr{B}$ is referred to as a soft base for the soft topology $\mathscr{T}$.

From their definitions, we can easily check the following conclusion:

Remark 1 If $\mathscr{F}$ is a filter on X, then $\mathscr{F}$ forms a base for a soft topology on X.

Definition 15 [6] For a soft subset (L, Π) of $(X, \mathscr{T}, \Pi)$, the soft interior of (L, Π) is denoted by $\mathrm{Int}_X(L, \Pi)$ (or simply $\mathrm{Int}(L, \Pi)$) and defined by

$$\mathrm{Int}(L, \Pi) = \tilde{\cup}\{(G, \Pi) : (G, \Pi) \tilde{\subseteq} (L, \Pi), (G, \Pi) \in \mathscr{T}\}.$$

The soft closure of (L, Π) is denoted by $\mathrm{Cl}_X(L, \Pi)$ (or simply $\mathrm{Cl}(L, \Pi)$) and defined by

$$\mathrm{Cl}(L, \Pi) = \tilde{\cap}\{(L, \Pi) : (L, \Pi) \tilde{\supseteq} (L, \Pi), (G, \Pi) \in \mathscr{T}^c\}.$$

The soft boundary of (L, Π) is denoted by $\mathrm{Bd}(L, \Pi)$ and defined by

$$\mathrm{Bd}(L, \Pi) = \mathrm{Cl}(L, \Pi) \tilde{\cap} \mathrm{Cl}((L, \Pi)^c).$$

Definition 16 Let (L, Π) be a soft subset of $(X, \mathscr{T}, \Pi)$. Then (L, Π) is known as soft nowhere dense [26] if $\mathrm{Int}(\mathrm{Cl}(L, \Pi)) = \Phi_\Pi$; soft semiopen [35] if $(L, \Pi) \tilde{\subseteq} \mathrm{Cl}(\mathrm{Int}(L, \Pi))$. The family of all soft nowhere dense (resp. soft semiopen) sets over $(X, \mathscr{T}, \Pi)$ is denoted by $\mathscr{N}_{\mathscr{T}}$ (resp. $SSO(\mathscr{T})$).

3. Soft nowhere dense boundary sets

We define "soft nowhere dense boundary sets" and establish their basic properties, which will be used later.

Lemma 1 Let $(X, \mathscr{T}, \Pi)$ be a soft topological space and let $(L, \Pi) \tilde{\in} \mathfrak{S}(X_\Pi)$. Then (L, Π) is soft nowhere dense if and only if for each $(G, \Pi) \tilde{\in} \mathscr{T}^*$, there exists $(H, \Pi) \tilde{\in} \mathscr{T}^*$ with $(H, \Pi) \tilde{\subseteq} (G, \Pi)$ such that $(H, \Pi) \tilde{\cap} (L, \Pi) = \Phi_\Pi$.

Proof. If (L, Π) is soft nowhere dense, then $\mathrm{Cl}(L, \Pi)$ does not contain any $(K, \Pi) \tilde{\in} \mathscr{T}^*$. Therefore, if $(G, \Pi) \tilde{\in} \mathscr{T}^*$, then $(H, \Pi) = (G, \Pi) \simeq \mathrm{Cl}(L, \Pi) \tilde{\in} \mathscr{T}^*$ such that $(H, \Pi) \tilde{\subseteq} (G, \Pi)$ and $(H, \Pi) \tilde{\cap} (L, \Pi) = \Phi_\Pi$.

Conversely, if (L, Π) is not soft nowhere dense, then there exists $(G, \Pi) \tilde{\in} \mathscr{T}^*$ such that $(G, \Pi) \tilde{\subseteq} \mathrm{Int}(\mathrm{Cl}(L, \Pi)) \tilde{\subseteq} \mathrm{Cl}(L, \Pi)$. Therefore, $(G, \Pi) \tilde{\cap} \mathrm{Cl}(L, \Pi) \neq \Phi_\Pi$ and so $(G, \Pi) \tilde{\cap} (L, \Pi) \neq \Phi_\Pi$. Thus, there exists no $(H, \Pi) \tilde{\in} \mathscr{T}^*$ with $(H, \Pi) \tilde{\subseteq} (G, \Pi)$ such that $(H, \Pi) \tilde{\cap} (L, \Pi) = \Phi_\Pi$.

Lemma 2 Suppose $(X, \mathscr{T}, \Pi)$ is a soft topological space and (N, Π), (N_1, Π), $(N_2, \Pi) \tilde{\in} \mathfrak{S}(X_\Pi)$. The following properties hold:

(1) If $(N_1, \Pi) \tilde{\subseteq} (N_2, \Pi)$ and $(N_2, \Pi) \tilde{\in} \mathscr{N}_{\mathscr{T}}$, then $(N_1, \Pi) \tilde{\in} \mathscr{N}_{\mathscr{T}}$.

(2) If (N_1, Π), $(N_2, \Pi) \tilde{\in} \mathscr{N}_{\mathscr{T}}$, then $(N_1, \Pi) \tilde{\cup} (N_2, \Pi) \tilde{\in} \mathscr{N}_{\mathscr{T}}$.

(3) If $(N, \Pi) \tilde{\in} \mathscr{N}_{\mathscr{T}}$, then $\mathrm{Bd}(N, \Pi) \tilde{\in} \mathscr{N}_{\mathscr{T}}$.

(4) If $(N, \Pi) \tilde{\in} \mathscr{T}$, then $\mathrm{Bd}(N, \Pi) \tilde{\in} \mathscr{N}_{\mathscr{T}}$.

(5) If $(N, \Pi) \tilde{\in} \mathscr{T}^c$, then $\mathrm{Bd}(N, \Pi) \tilde{\in} \mathscr{N}_{\mathscr{T}}$.

Proof. (1) is obvious, and (2) is Theorem 2 in [36].

(3) If $\mathrm{Int}(\mathrm{Cl}(N, \Pi)) = \Phi_\Pi$, then $\mathrm{Int}(\mathrm{Bd}(N, \Pi)) \tilde{\subseteq} \mathrm{Int}(\mathrm{Cl}(N, \Pi)) = \Phi_\Pi$ and hence, $\mathrm{Bd}(N, \Pi) \tilde{\in} \mathscr{N}_{\mathscr{T}}$.

(4) Let $(N, \Pi) \tilde{\in} \mathscr{T}$. Since $\mathrm{Bd}(N, \Pi)$ is soft closed, so it suffices to show that $\mathrm{Int}(\mathrm{Bd}(N, \Pi)) = \Phi_\Pi$. Now,

$$\begin{aligned}
\mathrm{Int}(\mathrm{Bd}(N, \Pi)) &= \mathrm{Int}(\mathrm{Cl}(N, \Pi) \tilde{\cap} \mathrm{Cl}((N, \Pi)^c)) \\
&= \mathrm{Int}(\mathrm{Cl}(N, \Pi) \tilde{\cap} (N, \Pi)^c) \qquad (\text{Since } (N, \Pi) \tilde{\in} \mathscr{T}) \\
&= \mathrm{Int}(\mathrm{Cl}(N, \Pi)) \tilde{\cap} \mathrm{Int}((N, \Pi)^c) \\
&= \mathrm{Int}(\mathrm{Cl}(N, \Pi)) \tilde{\cap} (\mathrm{Cl}(N, \Pi))^c \\
&\tilde{\subseteq} \mathrm{Cl}(N, \Pi) \tilde{\cap} (\mathrm{Cl}(N, \Pi))^c = \Phi_\Pi.
\end{aligned}$$

(5) Since, by Property B*.2 in [11], $\mathrm{Bd}(N, \Pi) = \mathrm{Bd}((N, \Pi)^c)$, the rest follows from (4).

Lemma 3 [37] Assume $(X, \mathscr{T}, \Pi)$ is a soft topological space and $(L, \Pi) \tilde{\in} \mathfrak{S}(X_\Pi)$. Then (L, Π) is soft semiopen if and only if for each $(G, \Pi) \tilde{\in} \mathscr{T}$, there exists $(H, \Pi) \tilde{\in} \mathscr{T}^*$ such that $(H, \Pi) \tilde{\subseteq} (L, \Pi) \tilde{\cap} (G, \Pi)$.

Lemma 4 Let $(X, \mathscr{T}, \Pi)$ be a soft topological space and let $(L, \Pi) \tilde{\in} \mathfrak{S}(X_\Pi)$. If (L, Π) is soft semiopen, then $(L, \Pi) = (G, \Pi) \tilde{\cup} (N, \Pi)$ for some $(G, \Pi) \tilde{\in} \mathscr{T}$ and $(N, \Pi) \tilde{\in} \mathscr{N}_{\mathscr{T}}$.

Proof. If (L, Π) is a soft semiopen set, then $(G, \Pi) \tilde{\subseteq} (L, \Pi) \tilde{\subseteq} \mathrm{Cl}((G, \Pi))$ for some $(G, \Pi) \tilde{\in} \mathscr{T}$. Clearly, $(L, \Pi) = (G, \Pi) \tilde{\cup} \big((L, \Pi) \tilde{-} (G, \Pi)\big)$. Since $(G, \Pi) \tilde{\in} \mathscr{T}$, then $\mathrm{Cl}(L, \Pi) \tilde{-} (G, \Pi)$ is soft nowhere dense. But $(L, \Pi) \tilde{-} (G, \Pi) \tilde{\subseteq} \mathrm{Cl}(L, \Pi) \tilde{-} (G, \Pi)$, so $(L, \Pi) \tilde{-} (G, \Pi)$ is soft nowhere dense. Set $(N, \Pi) = (L, \Pi) \tilde{-} (G, \Pi)$. Therefore, $(L, \Pi) = (G, \Pi) \tilde{\cup} (N, \Pi)$.

Definition 17 Let $(X, \mathscr{T}, \Pi)$ be a soft topological space. A soft set $(L, \Pi) \tilde{\in} \mathfrak{S}(X_\Pi)$ is known as soft nowhere dense boundary (shortly, a soft NB-set) if the soft boundary of (L, Π) is soft nowhere dense.

Lemma 5 Let $(X, \mathscr{T}, \Pi)$ be a soft topological space and let $(L, \Pi) \tilde{\in} \mathfrak{S}(X_\Pi)$. Then (L, Π) is a soft NB-set if and only if $(L, \Pi) = (G, \Pi) \tilde{\cup} (N, \Pi)$ for some $(G, \Pi) \tilde{\in} \mathscr{T}$ and $(N, \Pi) \tilde{\in} \mathscr{N}_{\mathscr{T}}$.

Proof. Let (L, Π) be a soft NB-set. Set $(G, \Pi) = \mathrm{Int}(L, \Pi)$ and $(N, \Pi) = (L, \Pi) \tilde{-} \mathrm{Int}(L, \Pi)$. Therefore, $(N, \Pi) \tilde{\subseteq} \mathrm{Bd}(L, \Pi)$. Thus, by assumption, $\mathrm{Int}(\mathrm{Cl}(N, \Pi)) \tilde{\subseteq} \mathrm{Int}(\mathrm{Bd}(L, \Pi)) = \Phi_\Pi$. Hence, $(N, \Pi) \tilde{\in} \mathscr{N}_{\mathscr{T}}$.

Conversely, suppose $(L, \Pi) = (G, \Pi) \tilde{\cup} (N, \Pi)$, where $(G, \Pi) \tilde{\in} \mathscr{T}$ and $(N, \Pi) \tilde{\in} \mathscr{N}_{\mathscr{T}}$. From Lemma 4 (7) [11], we have $\mathrm{Bd}(L, \Pi) \tilde{\subseteq} \mathrm{Bd}(G, \Pi) \tilde{\cup} \mathrm{Bd}(N, \Pi)$. Therefore, Lemma 2 establishes that $\mathrm{Bd}(L, \Pi) \tilde{\in} \mathscr{N}_{\mathscr{T}}$.

Remark 2 The family $\mathscr{N}_{\mathscr{T}}$ of all soft nowhere dense sets over a soft topological space $(X, \mathscr{T}, \Pi)$ constitutes a soft ideal and is known as a soft nowhere dense ideal. The conclusion follows from (1) and (2) in Lemma 2.

Lemma 6 The family of all soft NB-sets over a soft topological space $(X, \mathscr{T}, \Pi)$ forms a soft algebra.

Proof. Let $\mathscr{A}$ be the family of soft NB-sets in $(X, \mathscr{T}, \Pi)$. Evidently, $\Phi_\Pi \tilde{\in} \mathscr{A}$ as $\mathrm{Bd}(\Phi_\Pi)$ is a soft nowhere dense set. The property B.2 in [11] states that $\mathrm{Bd}(L, \Pi) = \mathrm{Bd}((L, \Pi)^c)$, so $\mathscr{A}$ is closed under soft complements. Let $(L, \Pi), (G, \Pi) \tilde{\in} \mathscr{A}$. Then $\mathrm{Bd}[(L, \Pi) \tilde{\cup} (G, \Pi)] \tilde{\subseteq} \mathrm{Bd}(L, \Pi) \tilde{\cup} \mathrm{Bd}(G, \Pi)$. By Lemma 2 (2), $(L, \Pi) \tilde{\cup} (G, \Pi) \tilde{\in} \mathscr{A}$ and hence, $\mathscr{A}$ is a soft algebra.

The soft algebra mentioned above is known as a soft nowhere dense boundary algebra and denoted by $\mathrm{Alg}_{\mathrm{NB}}(\mathscr{T})$.

From Lemmas 4 and 5, one can conclude the following:

Remark 3 For a soft topological space $(X, \mathscr{T}, \Pi)$, $\mathrm{Alg}_{\mathrm{NB}}(\mathscr{T})$ is the soft algebra on X generated by $\mathscr{T} \tilde{\cup} \mathscr{N}_{\mathscr{T}}$ or equivalently by $SSO(\mathscr{T}) \tilde{\cup} \mathscr{N}_{\mathscr{T}}$.

4. Soft MB-algebras

In this section, we first introduce the notions of soft (s)-sets and soft (s^0)-sets, which constitute the fundamental building blocks of the soft MB-algebras.

Definition 18 Let $\mathscr{C} \tilde{\subseteq} \mathfrak{S}^*(X_\Pi)$. A soft subset (L, Π) over X is said to be an (s)-set (or simply s-set) if for each $(C, \Pi) \tilde{\in} \mathscr{C}$, there exists $(D, \Pi) \tilde{\in} \mathscr{C}$ with $(D, \Pi) \tilde{\subseteq} (C, \Pi)$ such that $(D, \Pi) \tilde{\subseteq} (L, \Pi)$ or $(D, \Pi) \tilde{\cap} (L, \Pi) = \Phi_\Pi$. The family of all soft s-sets over X with respect to $\mathscr{C}$ is denoted by $\mathscr{A}[\mathscr{C}]$.

Definition 19 Let $\mathscr{C} \tilde{\subseteq} \mathfrak{S}^*(X_\Pi)$. A soft subset (L, Π) over X is said to be an (s^0)-set (or simply s^0-set) if for each $(C, \Pi) \tilde{\in} \mathscr{C}$, there exists $(D, \Pi) \tilde{\in} \mathscr{C}$ with $(D, \Pi) \tilde{\subseteq} (C, \Pi)$ such that $(D, \Pi) \tilde{\cap} (L, \Pi) = \Phi_\Pi$.

The collection of all soft s^0-sets over X related to $\mathscr{C}$ is labeled as $\mathscr{I}_0[\mathscr{C}]$.

Remark 4 Let $\mathscr{C} \tilde{\subseteq} \mathfrak{S}^*(X_\Pi)$. A soft set (L, Π) is in $\mathscr{A}[\mathscr{C}]$ if and only if for each $(C, \Pi) \tilde{\in} \mathscr{C}$, there exists $(D, \Pi) \tilde{\in} \mathscr{C}$ such that either $(D, \Pi) \tilde{\subseteq} (C, \Pi) \tilde{\cap} (L, \Pi)$ or $(D, \Pi) \tilde{\subseteq} (C, \Pi) \tilde{-} (L, \Pi)$. And, (L, Π) is in $\mathscr{I}_0[\mathscr{C}]$ if and only if for each $(C, \Pi) \tilde{\in} \mathscr{C}$, there exists $(D, \Pi) \tilde{\in} \mathscr{C}$ such that $(D, \Pi) \tilde{\subseteq} (C, \Pi) \tilde{-} (L, \Pi)$.

Proposition 1 For any $\mathscr{C} \tilde{\subseteq} \mathfrak{S}^*(X_\Pi)$, the following properties hold:

(1) $\mathscr{A}[\mathscr{C}]$ is a soft algebra.

(2) $\mathscr{I}_0[\mathscr{C}]$ is a soft ideal.

(3) $\mathscr{C} \tilde{\cap} \mathscr{I}_0[\mathscr{C}] = \Phi_\Pi$.

(4) For each $(C, \Pi) \tilde{\in} \mathscr{A}[\mathscr{C}] \tilde{-} \mathscr{I}_0[\mathscr{C}]$, there exists $(D, \Pi) \tilde{\in} \mathscr{C}$ such that $(D, \Pi) \tilde{\subseteq} (C, \Pi)$.

Proof. (1) Clearly, $X_\Pi \tilde{\in} \mathscr{A}[\mathscr{C}]$ as for each $(C, \Pi) \tilde{\in} \mathscr{C}$, $(C, \Pi) \tilde{\subseteq} (C, \Pi) \tilde{\cap} X_\Pi$. If $(C, \Pi) \tilde{\in} \mathscr{A}[\mathscr{C}]$, directly by Definition 18, $(C, \Pi)^c \tilde{\in} \mathscr{A}[\mathscr{C}]$. Let $(C, \Pi), (D, \Pi) \tilde{\in} \mathscr{A}[\mathscr{C}]$ and let $(L, \Pi) \tilde{\in} \mathscr{C}$. We consider the following two cases:

Case 1: If there exists $(B, \Pi)\tilde{\in}\mathscr{C}$ such that $(B, \Pi)\tilde{\subseteq}(C, \Pi)\tilde{\cap}(L, \Pi)$ or $(B, \Pi)\tilde{\subseteq}(D, \Pi)\tilde{\cap}(L, \Pi)$, then $(B, \Pi)\tilde{\subseteq}[(C, \Pi)\tilde{\cup}(D, \Pi)]\tilde{\cap}(L, \Pi)$.

Case 2: If there does not exist such (B, Π), there exists $(B_0, \Pi)\tilde{\in}\mathscr{C}$ such that $(B_0, \Pi)\tilde{\subseteq}(L, \Pi)\tilde{-}(C, \Pi)$. Since $(D, \Pi)\tilde{\in}\mathscr{A}[\mathscr{C}]$, so there exists $(B_1, \Pi)\tilde{\subseteq}(B_0, \Pi)\tilde{-}(D, \Pi)$. This implies

$$(B_1, \Pi)\tilde{\subseteq}(L, \Pi)\tilde{-}[(C, \Pi)\tilde{\cup}(D, \Pi)].$$

Thus, $(C, \Pi)\tilde{\cup}(D, \Pi)\tilde{\in}\mathscr{A}[\mathscr{C}]$ and hence, $\mathscr{A}[\mathscr{C}]$ is a soft algebra.

(2) Since for each $(C, \Pi)\tilde{\in}\mathscr{C}$, we have $(C, \Pi)\tilde{\subseteq}(C, \Pi)\tilde{-}\Phi_\Pi$, so $\Phi_\Pi\tilde{\in}\mathscr{I}_0[\mathscr{C}]$. Let $(L_1, \Pi)\tilde{\in}\mathscr{I}_0[\mathscr{C}]$ and $(L_0, \Pi)\tilde{\subseteq}(L_1, \Pi)$. Suppose $(C, \Pi)\tilde{\in}\mathscr{C}$. Then, there exists $(D, \Pi)\in\mathscr{C}$ with $(D, \Pi)\tilde{\subseteq}(C, \Pi)$ such that $(D, \Pi)\tilde{\subseteq}(C, \Pi)\tilde{-}(L_1, \Pi)$. Since $(L_0, \Pi)\tilde{\subseteq}(L_1, \Pi)$, we have

$$(D, \Pi)\tilde{\subseteq}(C, \Pi)\tilde{-}(L_0, \Pi).$$

Therefore, $(L_0, \Pi)\tilde{\in}\mathscr{I}_0[\mathscr{C}]$. The Case 2 guarantees that $\mathscr{I}_0[\mathscr{C}]$ is closed under finite unions.

(3) It follows from Definition 19.

(4) Suppose, to the contrary, that there exists $(L, \Pi)\tilde{\in}\mathscr{A}[\mathscr{C}]\tilde{-}\mathscr{I}_0[\mathscr{C}]$ such that $(C, \Pi)\tilde{-}(L, \Pi)\neq\Phi_\Pi$ for each $(C, \Pi)\tilde{\in}\mathscr{C}$. Since $(L, \Pi)\tilde{\notin}\mathscr{I}_0[\mathscr{C}]$, there exists $(B, \Pi)\tilde{\in}\mathscr{C}$ such that $(D, \Pi)\tilde{\cap}(L, \Pi)\neq\Phi_\Pi$ for each $(D, \Pi)\tilde{\in}\mathscr{C}$ with $(D, \Pi)\tilde{\subseteq}(B, \Pi)$. Moreover, since $(L, \Pi)\tilde{\in}\mathscr{A}[\mathscr{C}]$, there exists $(W, \Pi)\tilde{\in}\mathscr{C}$ such that $(W, \Pi)\tilde{\subseteq}(B, \Pi)\tilde{\cap}(L, \Pi)$. But $(W, \Pi)\tilde{-}(L, \Pi)=\Phi_\Pi$, which contradicts the assumption. Hence, (4) must hold.

At this point, some examples on the preceding concepts may be needed to show that the output of this scheme is not trivial.

Example 1 Let X be an infinite set and let Π be a parametric set. For $\mathscr{C}=\{(L, \Pi)\tilde{\in}\mathfrak{S}(X_\Pi):(L, \Pi)^c \text{ is finite}\}$, the soft algebra $\mathscr{A}[\mathscr{C}]$ and soft ideal $\mathscr{I}_0[\mathscr{C}]$ are as follows:

$$\mathscr{A}[\mathscr{C}]=\{(L, \Pi)\tilde{\in}\mathfrak{S}(X_\Pi):\text{either } (L, \Pi) \text{ or } (L, \Pi)^c \text{ is finite}\},$$

and

$$\mathscr{I}_0[\mathscr{C}]=\{(L, \Pi)\tilde{\in}\mathfrak{S}(X_\Pi):(L, \Pi) \text{ is finite}\}.$$

Example 2 Let X be an infinite set and let Π be a parametric set. For $\mathscr{C}=\{(L, \Pi)\tilde{\in}\mathfrak{S}(X_\Pi):$ either (L, Π) or $(L, \Pi)^c$ is finite, the soft algebra $\mathscr{A}[\mathscr{C}^*]$ and soft ideal $\mathscr{I}_0[\mathscr{C}^*]$ are

$$\mathscr{A}[\mathscr{C}^*]=\mathfrak{S}(X_\Pi) \text{ and } \mathscr{I}_0[\mathscr{C}^*]=\{\Phi_\Pi\}.$$

Example 3 Let $\mathscr{C}$ be the soft algebra on the set of reals $\mathbb{R}$ generated by

$$\mathscr{B}=\{\{(u_1, (a, b)), (u_2, [c, d)\}:a, b, c, d\in\mathbb{R}; u_1, u_2\in\Pi\}.$$

The soft algebra $\mathscr{A}[\mathscr{C}^*]$ and soft ideal $\mathscr{I}_0[\mathscr{C}^*]$ are obtained as:

$$\mathscr{A}[\mathscr{C}^*] = \mathrm{Alg}_{\mathrm{NB}}(\mathscr{T}) \text{ and } \mathscr{I}_0[\mathscr{C}^*] = \{\{(u_1,\ L(u_1)),\ (u_2,\ \emptyset)\} : L(u_1) \text{ is finite}\},$$

where $\mathscr{T}$ is the soft topology on $\mathbb{R}$ generated by

$$\mathscr{B}' = \{\{(u_1,\ (a,\ b)),\ (u_2,\ (c,\ d)\} : a,\ b,\ c,\ d \in \mathbb{R};\ u_1,\ u_2 \in \Pi\}.$$

Remark 5 For some collection $\mathscr{C} \tilde{\subseteq} \mathfrak{S}^*(X_\Pi)$, we may not generally have $\mathscr{A}[(\mathscr{A}[\mathscr{C}])^*] = \mathscr{A}[\mathscr{C}]$ (see, Examples 1-2).

The following is an immediate consequence of Remark 4.

Proposition 2 Let $\mathscr{C} \tilde{\subseteq} \mathfrak{S}^*(X_\Pi)$. Then $\mathscr{C} \tilde{\subseteq} \mathscr{A}[\mathscr{C}]$ if and only if for each $(C,\ \Pi),\ (D,\ \Pi) \tilde{\in} \mathscr{C}$, there exists $(B,\ \Pi) \tilde{\in} \mathscr{C}$ such that $(B,\ \Pi) \tilde{\subseteq} (C,\ \Pi) \tilde{\cap} (D,\ \Pi)$ or $(B,\ \Pi) \tilde{\subseteq} (C,\ \Pi) \tilde{-} (D,\ \Pi)$.

Proposition 3 Let $\mathscr{C} \tilde{\subseteq} \mathfrak{S}^*(X_\Pi)$. If $\{x_u\} \tilde{\in} \mathscr{C}$ for each $x_u \tilde{\in} \mathfrak{P}(X_\Pi)$, then $\mathscr{A}[\mathscr{C}] = \mathfrak{S}(X_\Pi)$ and $\mathscr{I}_0[\mathscr{C}] = \{\Phi_\Pi\}$.

Proof. Let $(L,\ \Pi) \tilde{\in} \mathscr{I}_0[\mathscr{C}]$ and let $\{x_u\} \tilde{\in} \mathscr{C}$. Then either $\{x_u\} \tilde{\subseteq} \{x_u\} \tilde{\cap} (L,\ \Pi)$ or $\{x_u\} \tilde{\subseteq} \{x_u\} \tilde{-} (L,\ \Pi)$. Therefore, $(L,\ \Pi) \tilde{\in} \mathscr{A}[\mathscr{C}]$ and so $\mathscr{A}[\mathscr{C}] = \mathfrak{S}(X_\Pi)$.

Suppose $(L,\ \Pi) \tilde{\in} \mathfrak{S}(X_\Pi)$ and $(L,\ \Pi) \neq \Phi_\Pi$. Since $\{x_u\} \tilde{\in} \mathscr{C}$ for each $x_u \tilde{\in} \mathfrak{P}(X_\Pi)$, then $\{x_u\} \tilde{\subseteq} \{x_u\} \tilde{-} (L,\ \Pi)$ for each $x_u \tilde{\in} \mathfrak{P}(X_\Pi)$. This means that $(L,\ \Pi) = \Phi_\Pi$, a contradiction. Thus, $\mathscr{I}_0[\mathscr{C}] = \{\Phi_\Pi\}$.

A direct consequence of Propositions 1 (3) and 2 is the following result.

Proposition 4 Let $\mathscr{C} \tilde{\subseteq} \mathfrak{S}^*(X_\Pi)$. If $\mathscr{C}$ a soft algebra, then $\mathscr{C} \tilde{\subseteq} \mathscr{A}[\mathscr{C}] \tilde{-} \mathscr{I}_0[\mathscr{C}]$.

Remark 6 Given a soft filter $\mathscr{B}$ on X. If $\mathscr{B}^c = \{(B,\ \Pi)^c : (B,\ \Pi) \tilde{\in} \mathscr{B}\}$, by Definition 10, one can verify that $\mathscr{B} \tilde{\cup} \mathscr{B}^c$ is a soft algebra on X. Furthermore, $\mathscr{B} \tilde{\cup} \mathscr{B}^c$ is the soft algebra generated by $\mathscr{B}$, i.e., $\mathscr{A}(\mathscr{B}) = \mathscr{B} \tilde{\cup} \mathscr{B}^c$.

Proposition 5 Let $\mathscr{B}$ be a soft filter on X. Then $\mathscr{A}[\mathscr{B}] = \mathscr{A}(\mathscr{B})$ and $\mathscr{I}_0[\mathscr{B}] = \mathscr{B}^c$.

Proof. From Proposition 2, one can see that $\mathscr{B} \tilde{\subseteq} \mathscr{A}[\mathscr{B}]$. By Remark 6, $\mathscr{A}(\mathscr{B}) \tilde{\subseteq} \mathscr{A}[\mathscr{B}]$. On the other hand, if $(L,\ \Pi) \tilde{\in} \mathscr{A}[\mathscr{B}]$, then one can find $(B,\ \Pi) \tilde{\in} \mathscr{B}$ such that $(B,\ \Pi) \tilde{\subseteq} (L,\ \Pi)$ or $(B,\ \Pi) \tilde{\subseteq} X_\Pi \tilde{-} (L,\ \Pi) = (L,\ \Pi)^c$ because $X_\Pi \tilde{\in} \mathscr{B}$. This implies that either $(L,\ \Pi) \tilde{\in} \mathscr{B}$ or $(L,\ \Pi)^c \tilde{\in} \mathscr{B}$, and so, $(L,\ \Pi) \tilde{\in} \mathscr{A}(\mathscr{B})$. Hence, $\mathscr{A}[\mathscr{B}] = \mathscr{A}(\mathscr{B})$.

By the last steps in the above part, we can show that $\mathscr{I}_0[\mathscr{B}] = \mathscr{B}^c$.

Definition 20 [13] Let $\mathscr{C},\ \mathscr{F} \tilde{\subseteq} \mathfrak{S}^*(X_\Pi)$. It is said that $\mathscr{C}$ and $\mathscr{F}$ are mutually co-initial, denoted by $\mathscr{C} \leftrightarrows \mathscr{F}$, if for each $(C,\ \Pi) \tilde{\in} \mathscr{C}$, one can find $(F,\ \Pi) \tilde{\in} \mathscr{F}$ provided that $(C,\ \Pi) \tilde{\subseteq} (F,\ \Pi)$ and for each $(F,\ \Pi) \tilde{\in} \mathscr{F}$, one can find $(C,\ \Pi) \tilde{\in} \mathscr{C}$ provided that $(F,\ \Pi) \tilde{\subseteq} (C,\ \Pi)$.

Example 11 in [13] shows that two co-initial families are not always comparable.

Theorem 2 For any families $\mathscr{C},\ \mathscr{F} \tilde{\subseteq} \mathfrak{S}^*(X_\Pi)$, if $\mathscr{C} \leftrightarrows \mathscr{F}$, then $\mathscr{A}[\mathscr{C}] = \mathscr{A}[\mathscr{F}]$ and $\mathscr{I}_0[\mathscr{C}] = \mathscr{I}_0[\mathscr{F}]$.

Proof. Let $\mathscr{C},\ \mathscr{F} \tilde{\subseteq} \mathfrak{S}^*(X_\Pi)$. We first show that $\mathscr{A}[\mathscr{C}] \tilde{\subseteq} \mathscr{A}[\mathscr{F}]$. Let $(L,\ \Pi) \tilde{\in} \mathscr{A}[\mathscr{C}]$ and let $(H,\ \Pi) \tilde{\in} \mathscr{F}$. Since $\mathscr{C} \leftrightarrows \mathscr{F}$, then there exists $(G,\ \Pi) \tilde{\in} \mathscr{C}$ such that $(G,\ \Pi) \tilde{\subseteq} (H,\ \Pi)$. Since $(L,\ \Pi) \tilde{\in} \mathscr{A}[\mathscr{C}]$, so there exists $(G_0,\ \Pi) \tilde{\in} \mathscr{C}$ with $(G_0,\ \Pi) \tilde{\subseteq} (G,\ \Pi)$ such that

$$(G_0,\ \Pi) \tilde{\subseteq} (G,\ \Pi) \tilde{\cap} (L,\ \Pi) \tilde{\subseteq} (H,\ \Pi) \tilde{\cap} (L,\ \Pi) \text{ or } (G_0,\ \Pi) \tilde{\subseteq} (G,\ \Pi) \tilde{-} (L,\ \Pi) \tilde{\subseteq} (H,\ \Pi) \tilde{-} (L,\ \Pi).$$

Since $\mathscr{C} \leftrightarrows \mathscr{F}$, one can find $(H_0,\ \Pi) \tilde{\in} \mathscr{F}$ such that $(H_0,\ \Pi) \tilde{\subseteq} (G_0,\ \Pi)$. This implies that

$$(H_0,\ \Pi) \tilde{\subseteq} (H,\ \Pi) \tilde{\cap} (L,\ \Pi) \text{ or } (H_0,\ \Pi) \tilde{\subseteq} (H,\ \Pi) \tilde{-} (L,\ \Pi).$$

Consequently, $(L,\ \Pi) \tilde{\in} \mathscr{A}[\mathscr{F}]$. The reverse of the inclusion is similar. Using a similar approach, one may demonstrate that $\mathscr{I}_0[\mathscr{C}] = \mathscr{I}_0[\mathscr{F}]$.

The converse of the above result is not true as the following example shows.

Example 4 Let $X = \{x_1, x_2\}$ and $\Pi = \{u_1, u_2\}$. Assume that $\mathscr{C} = \{(u_1, \{x_1\}), (u_2, \emptyset)\}$ and $\mathscr{F} = \{(u_1, \{x_2\}), (u_2, \emptyset)\}$. Following the construction stated in Definitions 18-19, one can easily check that $\mathscr{A}[\mathscr{C}] = \mathscr{A}[\mathscr{F}]$. On the other hand, $\mathscr{C} \not\leftrightarrows \mathscr{F}$.

However, we have the following result as a possible converse of Theorem 2 under certain assumptions:

Theorem 3 Let $\mathscr{C}, \mathscr{F} \tilde{\subseteq} \mathfrak{S}^*(X_\Pi)$ such that $\mathscr{C} \tilde{\subseteq} \mathscr{I}_0[\mathscr{C}]$ and $\mathscr{F} \tilde{\subseteq} \mathscr{I}_0[\mathscr{F}]$. If $\mathscr{A}[\mathscr{C}] = \mathscr{A}[\mathscr{F}]$ and $\mathscr{I}_0[\mathscr{C}] = \mathscr{I}_0[\mathscr{F}]$, then $\mathscr{C} \leftrightarrows \mathscr{F}$.

Proof. Let $(C, \Pi) \tilde{\in} \mathscr{C}$. By Proposition 1 (3), $(C, \Pi) \tilde{\notin} \mathscr{I}_0[\mathscr{C}]$. Therefore, $(C, \Pi) \tilde{\in} \mathscr{A}[\mathscr{C}] \tilde{-} \mathscr{I}_0[\mathscr{C}] = \mathscr{A}[\mathscr{F}] \tilde{-} \mathscr{I}_0[\mathscr{F}]$. Proposition 1 (4) guarantees that there exists $(F, \Pi) \tilde{\in} \mathscr{F}$ such that $(F, \Pi) \tilde{\subseteq} (C, \Pi)$. By the same technique, one can prove that for each $(F, \Pi) \tilde{\in} \mathscr{F}$, there exists $(C, \Pi) \tilde{\in} \mathscr{C}$ such that $(C, \Pi) \tilde{\subseteq} (F, \Pi)$. Thus, $\mathscr{C} \leftrightarrows \mathscr{F}$.

Definition 21 Let Σ and $\mathscr{I}$ be a soft algebra and a soft ideal on X such that $\mathscr{I} \tilde{\subseteq} \Sigma$, respectively. It is said that Σ (resp. $\mathscr{I}$) is a Marczewski-Burstin soft algebras (resp. Marczewski-Burstin soft ideal) if there exists a collection $\mathscr{C} \tilde{\subseteq} \mathfrak{S}^*(X_\Pi)$ such that $\Sigma = \mathscr{A}[\mathscr{C}]$ (resp. $\mathscr{I} = \mathscr{I}_0[\mathscr{C}]$).

From now on, we respectively call Marczewski-Burstin soft algebras and Marczewski-Burstin soft ideals "soft MB-algebras" and "oft MB-ideals." The collection $\mathscr{C}$ is known as a basis for Σ (resp. $\mathscr{I}$).

Theorem 4 Let $\mathscr{I}$ be a soft ideal in a soft algebra Σ on X. The Σ (resp. $\mathscr{I}$) is a soft MB-algebra (resp. soft MB-ideal) with a basis $\mathscr{B} \tilde{\subseteq} \Sigma$ if and only if $\Sigma = \mathscr{A}[\Sigma \tilde{-} \mathscr{I}]$.

Proof. Suppose $\Sigma = \mathscr{A}[\mathscr{B}]$ and $\mathscr{I} = \mathscr{I}_0[\mathscr{B}]$ for a basis $\mathscr{B} \tilde{\subseteq} \Sigma$. We need to show that $\mathscr{A}[\Sigma \tilde{-} \mathscr{I}] = \mathscr{A}[\mathscr{B}]$. According to Theorem 2, it suffices to prove that $\Sigma \tilde{-} \mathscr{I} \leftrightarrows \mathscr{B}$. Evidently, $\mathscr{B} \tilde{\subseteq} \Sigma \tilde{-} \mathscr{I}_0[\mathscr{B}] = \Sigma \tilde{-} \mathscr{I}$. Therefore, for each $(B, \Pi) \tilde{\in} \mathscr{B}$, there exists $(L, \Pi) \tilde{\in} \Sigma \tilde{-} \mathscr{I}$ such that $(L, \Pi) \tilde{\subseteq} (B, \Pi)$. On the other hand, if $(L, \Pi) \tilde{\in} \Sigma \tilde{-} \mathscr{I}$, then there exists $(B, \Pi) \tilde{\in} \mathscr{B}$ such that $(B, \Pi) \tilde{\subseteq} (L, \Pi)$ as $(L, \Pi) \tilde{\notin} \mathscr{I} = \mathscr{I}_0[\mathscr{B}]$. Thus, $\mathscr{A}[\Sigma \tilde{-} \mathscr{I}] = \mathscr{A}[\mathscr{B}]$.

Conversely, assume that $\Sigma = \mathscr{A}[\Sigma \tilde{-} \mathscr{I}]$. Obviously, $\Sigma \tilde{-} \mathscr{I} \tilde{\subseteq} \Sigma$. By assumption. We have $\Sigma \tilde{-} \mathscr{I} \tilde{\subseteq} \Sigma \tilde{-} \mathscr{I}_0[\Sigma \tilde{-} \mathscr{I}]$. Therefore, by Proposition 1 (3), $\mathscr{I}_0[\Sigma \tilde{-} \mathscr{I}] \tilde{\subseteq} \mathscr{I}$. And, $\mathscr{I} \tilde{\subseteq} \mathscr{I}_0[\Sigma \tilde{-} \mathscr{I}]$ is always the case.

Remark 7 If $\mathscr{C}, \mathscr{F}$ are families of non-null soft sets over X having the property $\mathscr{C} \tilde{\subseteq} \mathscr{F}$. Ten $\mathscr{A}[\mathscr{C}]$ may not be a soft subalgebra of $\mathscr{A}[\mathscr{F}]$. The same holds for $\mathscr{I}_0[\mathscr{C}]$ and $\mathscr{I}_0[\mathscr{F}]$. It is explained in the following example.

Example 5 Let $X = \{x_1, x_2, x_3\}$ and $\Pi = \{u_1, u_2\}$. Suppose that $\mathscr{C} = \{\{(u_1, \{x_1\}), (u_2, \emptyset)\}\}$ and $\mathscr{F} = \{\{(u_1, \{x_1\}), (u_2, X)\}, \{(u_1, \{x_2, x_3\}), (u_2, \{x_1\})\}\}$. Clearly, $\mathscr{C} \tilde{\subseteq} \mathscr{F}$ and $\{(u_1, \{x_2\}), (u_2, \{x_3\})\} \tilde{\in} \mathscr{A}[\mathscr{C}]$, but $\{(u_1, \{x_2\}), (u_2, \{x_3\})\} \tilde{\notin} \mathscr{A}[\mathscr{F}]$.

However, the next conclusion is possible:

Proposition 6 Let $\mathscr{I}, \mathscr{J}$ be soft ideals in a soft algebra Σ on X. If $\mathscr{I} \tilde{\subseteq} \mathscr{J}$, then $\mathscr{A}[\Sigma \tilde{-} \mathscr{J}] \tilde{\subseteq} \mathscr{A}[\Sigma \tilde{-} \mathscr{I}]$.

Proof. Suppose $(L, \Pi) \tilde{\in} \mathscr{A}[\Sigma \tilde{-} \mathscr{J}]$. Let $(C, \Pi) \tilde{\in} \Sigma \tilde{-} \mathscr{I}$. We shall find $(F, \Pi) \tilde{\in} \Sigma \tilde{-} \mathscr{I}$ such that

$$\text{either } (F, \Pi) \tilde{\subseteq} (C, \Pi) \tilde{\cap} (L, \Pi) \text{ or } (F, \Pi) \tilde{\subseteq} (C, \Pi) \tilde{-} (L, \Pi). \tag{1}$$

Here, we examine two situations. If $(C, \Pi) \tilde{\in} \Sigma \tilde{-} \mathscr{J}$, then exists always $(F, \Pi) \tilde{\in} \Sigma \tilde{-} \mathscr{J} \tilde{\subseteq} \Sigma \tilde{-} \mathscr{I}$ that satisfies (1) as $(L, \Pi) \tilde{\in} \mathscr{A}[\Sigma \tilde{-} \mathscr{J}]$. If $(C, \Pi) \tilde{\notin} \Sigma \tilde{-} \mathscr{J}$, then $(C, \Pi) \tilde{\in} \mathscr{J} \tilde{-} \mathscr{I}$. Therefore, by Definition 9, both $(C, \Pi) \tilde{\cap} (L, \Pi)$, $(F, \Pi) \tilde{\subseteq} (C, \Pi) \tilde{-} (L, \Pi) \tilde{\in} \mathscr{J}$. On the other hand, only $(C, \Pi) \tilde{\cap} (L, \Pi)$ or $(F, \Pi) \tilde{\subseteq} (C, \Pi) \tilde{-} (L, \Pi)$ can be in $\mathscr{I}$. Let (F, Π) be the one that is not in $\mathscr{I}$. This is possible since $\mathscr{J} \tilde{-} \mathscr{I} \tilde{\subseteq} \Sigma \tilde{-} \mathscr{I}$. This shows that $(L, \Pi) \tilde{\in} \mathscr{A}[\Sigma \tilde{-} \mathscr{I}]$.

Proposition 7 Let $\mathscr{I}$ be a soft ideal in a proper soft algebra Σ on X such that $\Sigma = \mathscr{A}[\Sigma \tilde{-} \mathscr{I}]$. Then $\Sigma = \mathscr{A}[\Sigma \tilde{-} \mathscr{J}]$ for each soft ideal $\mathscr{J}$ with $\mathscr{I} \tilde{\subseteq} \mathscr{J} \tilde{\subseteq} \mathscr{H}(\Sigma)$.

Proof. First, we shall note that for any soft ideal $\mathscr{J}$, we have $\mathscr{J} \tilde{\subsetneq} \Sigma$ as $\Sigma \neq \mathfrak{S}(X_\Pi)$. We claim that $\Sigma \tilde{\subseteq} \mathscr{A}[\Sigma \tilde{-} \mathscr{J}]$ for each soft ideal $\mathscr{J}$ of that kind. Let $(L, \Pi) \tilde{\in} \Sigma$ and $(C, \Pi) \tilde{\in} \Sigma \tilde{-} \mathscr{J}$. Evidently, either $(F, \Pi) = (L, \Pi) \tilde{\cap} (C, \Pi)$ or $(F, \Pi) = (C, \Pi) \tilde{-} (L, \Pi)$ is in $\Sigma \tilde{-} \mathscr{J}$, and so $\Sigma \tilde{\subseteq} \mathscr{A}[\Sigma \tilde{-} \mathscr{J}]$. Now, applying Proposition 6, we obtain $\Sigma \tilde{\subseteq} \mathscr{A}[\Sigma \tilde{-} \mathscr{H}(\Sigma)] \tilde{\subseteq} \mathscr{A}[\Sigma \tilde{-} \mathscr{J}] \tilde{\subseteq} \mathscr{A}[\Sigma \tilde{-} \mathscr{I}] = \Sigma$. Thus, $\Sigma = \mathscr{A}[\Sigma \tilde{-} \mathscr{J}]$.

From Theorem 4 and Proposition 7 one obtains.

Theorem 5 A proper soft algebra Σ is an inner soft MB-algebra with a basis $\mathscr{B} \tilde{\subseteq} \Sigma$ if and only if $\Sigma = \mathscr{A}[\Sigma \tilde{-} \mathscr{H}(\Sigma)]$.

5. Soft topological MB-algebras

Definition 22 Let Σ and $\mathscr{I}$ be respectively a soft algebra and a soft ideal on X such that $\mathscr{I}\tilde{\subseteq}\Sigma$. It is said that Σ (resp. $\mathscr{I}$) is a soft topological MB-algebras (resp. soft topological MB-ideal) if there exists a soft topology $\mathscr{T}$ on X such that $\Sigma=\mathscr{A}[\mathscr{T}]$ (resp. $\mathscr{I}=\mathscr{I}_0[\mathscr{T}]$).

We begin with the following result which follows from Lemmas 1 & 5 and Remark 3.

Theorem 6 If $\mathscr{T}$ is a soft topology on X parameterized with Π, then $\mathscr{A}[\mathscr{T}^*]=\mathrm{Alg}_{\mathrm{NB}}(\mathscr{T})$ and $\mathscr{I}_0[\mathscr{T}^*]=\mathscr{N}_{\mathscr{T}}$.

Also, the following result is a consequence of Theorem 2.

Theorem 7 Let $\mathscr{C}\tilde{\subseteq}\mathfrak{S}^*(X_\Pi)$. If $\mathscr{C}\leftrightarrows\mathscr{B}$, where $\mathscr{B}$ is a soft base of some topology on X, then $\mathscr{A}[\mathscr{C}]$ and $\mathscr{I}_0[\mathscr{C}]$ are respectively soft topological MB-algebra and soft topological MB-ideal.

Lemma 7 Let Σ be a soft algebra on X. For any $(G,\ \Pi)\tilde{\notin}\Sigma$, there exists a soft ultrafilter $\mathscr{F}$ in Σ such that

$$\mathfrak{S}(G_\Pi)\tilde{\cap}(L,\ \Pi)=\mathfrak{S}(G^c_\Pi)\tilde{\cap}(L,\ \Pi)=\Phi_\Pi,\ \text{ for all } (L,\ \Pi)\tilde{\in}\mathscr{F}.$$

Proof. Suppose otherwise that for each soft ultrafilter $\mathscr{F}$ in Σ, we get $\mathfrak{S}(G_\Pi)\tilde{\cap}(L,\ \Pi)\neq\Phi_\Pi$ or $\mathfrak{S}(G^c_\Pi)\tilde{\cap}(L,\ \Pi)\neq\Phi_\Pi$. For each $\mathscr{F}$ in Σ, we choose a member $(H,\ \Pi)_{\mathscr{F}}\tilde{\in}\mathscr{F}$ such that $(H,\ \Pi)_{\mathscr{F}}\tilde{\subseteq}(G,\ \Pi)$ or $(H,\ \Pi)_{\mathscr{F}}\tilde{\subseteq}(G^C,\ \Pi)$. Let $\mathscr{H}$ be the collection of all such soft sets $(H,\ \Pi)_{\mathscr{F}}$. We claim that $\mathscr{H}$ is a proper soft ideal in Σ. It is not difficult to check that $\mathscr{H}$ is a soft ideal. We now show that it is proper. It needs to prove that the soft union of any finite subcollection of $\mathscr{H}$ is not absolute. Suppose $\tilde{\bigcup}\mathscr{H}_0=X_\Pi$ for some finite $\mathscr{H}_0\tilde{\subseteq}\mathscr{H}$, where $\tilde{\bigcup}\mathscr{H}_0=\tilde{\cup}\{(H,\ \Pi):(H,\ \Pi)\tilde{\in}\mathscr{H}_0\}$. Set $\mathscr{H}_1=\mathfrak{S}(G_\Pi)\tilde{\cap}\mathscr{H}_0$ and $\mathscr{H}_2=\mathfrak{S}(G^c_\Pi)\tilde{\cap}\mathscr{H}_0$. Since $\mathscr{H}_0\tilde{\subseteq}\mathfrak{S}(G_\Pi)\tilde{\cup}\mathfrak{S}(G^c_\Pi)$, then $\mathscr{H}_0=\mathscr{H}_1\tilde{\cup}\mathscr{H}_2$ and so $X_\Pi=\tilde{\bigcup}\mathscr{H}_0=(\tilde{\bigcup}\mathscr{H}_1)\tilde{\cup}(\tilde{\bigcup}\mathscr{H}_2)$, whereas $\tilde{\bigcup}\mathscr{H}_1\tilde{\subseteq}(G,\ \Pi)$ and $\tilde{\bigcup}\mathscr{H}_2\tilde{\subseteq}(G^C,\ \Pi)$. Now, consider

$$\begin{aligned}(G,\ \Pi)\tilde{-}\tilde{\cup}\mathscr{H}_1 &=[(G,\ \Pi)\tilde{-}\tilde{\cup}\mathscr{H}_1]\tilde{\cap}[(\tilde{\cup}\mathscr{H}_1)\tilde{\cup}(\tilde{\cup}\mathscr{H}_2)]\\ &=\Big([(G,\ \Pi)\tilde{-}\tilde{\cup}\mathscr{H}_1]\tilde{\cap}(\tilde{\cup}\mathscr{H}_1)\Big)\tilde{\bigcup}\Big([(G,\ \Pi)\tilde{-}\tilde{\cup}\mathscr{H}_1]\tilde{\cap}(\tilde{\cup}\mathscr{H}_2)\Big)\\ &\leq[(G,\ \Pi)\tilde{-}\tilde{\cup}\mathscr{H}_1]\tilde{\cap}(G^C,\ \Pi)=\Phi_\Pi.\end{aligned}$$

This implies that $\tilde{\cup}\mathscr{H}_1=(G,\ \Pi)$, a contradiction, as $\mathscr{H}_1\tilde{\subseteq}\Sigma$ and $(G,\ \Pi)\tilde{\notin}\Sigma$. We have proved that the soft union of any finite subcollection of $\mathscr{H}$ is not absolute which means, by Theorem 3.27 [34], that the family $\mathscr{H}^*=\{(H^C,\ \Pi):(H,\ \Pi)\tilde{\in}\mathscr{H}\}$ can be extended to a soft ultrafilter in Σ, a contradiction to the conclusion that each soft ultrafilter in Σ shall include the complement of an element in $\mathscr{H}^*$ according to the definition of $\mathscr{H}$.

Theorem 8 Let Σ be a soft algebra on X and let $\{\Sigma^i:i\in I\}$ be any family of soft topological MB-algebras with $\Sigma\tilde{\subseteq}\Sigma^i$ for each i. Then

$$\Sigma=\tilde{\bigcap}_{i\in I}\Sigma^i.$$

Proof. Suppose that $\Sigma\neq\mathfrak{S}(X_\Pi)$, otherwise, the conclusion is clear. It is enough to prove that for each $(L,\ \Pi)\tilde{\notin}\Sigma$, there exists a soft topological MB-algebra $\Sigma^i_{(L,\ \Pi)}$ such that $\Sigma\tilde{\subseteq}\Sigma^i_{(L,\ \Pi)}$ and $(L,\ \Pi)\tilde{\notin}\Sigma^i_{(L,\ \Pi)}$. Therefore, if $(L,\ \Pi)\tilde{\notin}\Sigma$, by Lemma 7, one can find a soft ultrafilter $\mathscr{F}_{(L,\ \Pi)}$ in Σ such that $\mathfrak{S}(F_\Pi)\tilde{\cap}(H,\ \Pi)=\mathfrak{S}(F^c_\Pi)\tilde{\cap}(H,\ \Pi)=\Phi_\Pi$ for all $(H,\ \Pi)\tilde{\subseteq}\mathscr{F}_{(L,\ \Pi)}$. This means that $(L,\ \Pi)\tilde{\notin}\mathscr{A}\left[\mathscr{F}_{(L,\ \Pi)}\right]$. Set $\Sigma^i_{(L,\ \Pi)}=\mathscr{A}\left[\mathscr{F}_{(L,\ \Pi)}\right]$ for some i. By Remark 1 and Theorem 7, $\Sigma^i_{(L,\ \Pi)}$ is a soft topological MB-algebra. On the other hand, since $\mathscr{F}_{(L,\ \Pi)}$ is a soft ultrafilter in Σ, then $\Sigma\tilde{\subseteq}\mathscr{A}\left[\mathscr{F}_{(L,\ \Pi)}\right]=\Sigma^i_{(L,\ \Pi)}$. Hence, we shall have $\Sigma=\tilde{\bigcap}_{i\in I}\Sigma^i_{(L,\ \Pi)}$.

We end this section by the relationships between the types of soft algebras defined earlier.

$$\text{soft topological MB-algebra} \longrightarrow \text{soft MB-algebra.}$$

The arrow in the above implication is not reversible. By the use of Theorem 1, we can recall some counterexamples from the literature on ordinary algebras. The interval algebra defined on $[0, 1)$ is a soft MB-algebra but not soft topological (see, Corollary 2.1 in [23]).

6. Conclusion

The extensive literature on (s)-sets and (s^0)-sets demonstrates that investigating these classes of sets becomes attractive. In this paper, we have generalized the ideas of studying (s)-sets and (s^0)-sets from ordinary set theory to soft set theory. That is, we have defined the classes of soft (s)-sets and soft (s^0)-sets, and shown that they respectively form a soft algebra and a soft ideal with respect to a family of soft sets called a basis. Several examples are presented to show that the approach for defining such soft sets produces non-trivial results. Then, we have introduced the concepts of Marczewski-Burstin soft algebras and Marczewski-Burstin soft ideals. A Marczewski-Burstin soft algebra (ideal) is a soft algebra (ideal) that is identical to the class of soft (s)-sets ((s^0)-sets) for some basis. Essential properties and characterizations of the latter concepts have been obtained. We have further defined a soft topological Marczewski-Burstin algebra, which is a Marczewski-Burstin soft algebra whenever the basis is some soft topology. We have proved that soft topological Marczewski-Burstin algebras (ideals) are identical to soft nowhere dense boundary algebras (soft nowhere dense ideals). Clearly, a soft topological Marczewski-Burstin algebra is a Marczewski-Burstin soft algebra; however, the other way round need not be true. We have established that any soft algebra can be reperesented by a family of soft topological Marczewski-Burstin algebras.

The findings in this publication are preliminary, and much more research is expected. For example, one can investigate this topic by imposing particular requirements on the basis of a Marczewski-Burstin soft algebra. We obtained some results on soft topological Marczewski-Burstin algebras for any soft topology; nevertheless, one may investigate the topic using a specific soft topology.

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Data availability

Data sharing is not applicable to this article as no data sets were generated or analyzed during the current study.

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Conflict of interest

The authors declare no conflict of interests.

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