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Research Article

Generalized CR-Iteration Scheme with Application in Textile Designing

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Abstract: In this manuscript, we develop a new iteration method that is generalized CR-iteration method. We generate the Julia and Mandelbrot set fractals for a complex function $\zeta_c(z) = z^p + \log c^t$ where $c \in \mathbb{C} \setminus \{0\}$, $p \in \mathbb{N}$, $p \geq 2$, and $t \in \mathbb{R}$, $t \geq 1$ by using the generalized CR-iteration process. We discuss the escape criterion by using the generalized CR-iteration process for Julia and Mandelbrot set fractals generation. We generate the quadratic Julia set fractals, cubic Julia set fractals, quadratic Mandelbrot set fractals, and cubic Mandelbrot set fractals with MATLAB R2024a by using the generalized CR-iteration process with variation of the parameters α, γ, t, η, and c. Subtle variations in iteration parameters affect the structure, density, and symmetry of the fractals, as seen by a succession of graphical images. We generate the quadratic Julia and Mandelbrot set by using CR-iteration method and generalized CR-iteration method. We use Julia and Mandelbrot set patterns and repeat it to generate some beautiful batik designs. At the end, we compare this research with some existing literature.

***Keywords*:** complex function, escape criterion, Mandelbrot set fractal, iteration scheme, batik design

MSC: 39B12, 37F10

1. Introduction

The pioneering work of Mandelbrot [1], whose seminal book The Fractal Geometry of Nature introduced the mathematical community to the idea of self-similarity and complexity arising from simple iterative rules, has been profoundly shaped the exploration of fractal structures. Due in great part to their recursive boundary structures and sensitivity to initial conditions, Mandelbrot sets and Julia sets remain central objects in complex dynamics have been extensively studied using this basic framework. Extending classical models using contemporary iterative techniques has been a recent emphasis of attention. By improving convergence control, Kumari et al. [2] presented a viscosity approximation method for the synthesis of Mandelbrot sets, Julia sets, and biomorphs, so augmenting the structural richness of the resultant fractals.

Their method prepared the way for later studies on the behavior of intricate dynamical systems under perturbations including logarithmic and transcendental terms. Complementing this work, Gdawiec and Kotarski used Kalantari's

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polynomiograph framework to be complex using the infinity norm and nonstandard iterations, so illustrating how iterative visualization techniques can generate aesthetically beautiful and mathematically strong fractal images [3]. Their results also shed light on the graphical encoding of root-finding techniques, which naturally relate to the recursively structured Julia sets exhibit. A rigorous mathematical foundation for iterated function systems and a theoretical underpinning supporting fractal generation in both deterministic and random frameworks, Barnsley's influential book Fractals Everywhere helped His work finds uses in both natural simulations and textile design since it bridges abstract mathematical theory with the useful creation of fractal images.

Draves and Reckase's development of the fractal flame algorithm-a refinement of iterated function systems meant to generate vivid and smooth images through nonlinear color mappings [4] drove aesthetic innovation in fractal design even more. Particularly important in creative fields like digital Batik patterning and generative textile design, their work highlights the mix of algorithmic artistry with dynamic complexity. From a dynamical systems point of view, Peitgen and Stroh examined the connectedness of Julia sets for rational functions, stressing how modest parameter changes might produce quite different topologies, including disconnected or filamented Julia sets [5]. Domínguez and Fagella, who investigated residual Julia sets resulting from rational and transcendental functions, so providing a refined classification of sets exhibiting nontrivial dynamical boundaries [6], extended this analytical viewpoint. Building on this, Koss investigated elliptic functions with disconnected Julia sets, finding how transcendental functions produce more exotic fractal structures, generally deviating from the classical connected Mandelbrot-Julia dichotomy [7].

These results complement the work of Crowe et al. on the Mandelbrot set; a variation of the Mandelbrot set under complex conjugation that exposes rich bifurcation behavior in the dynamical plane [8]. Another important advance in controlling the geometry of fractals through modified iteration rules, enabling tighter bounds and improved symmetry, came from Rani and Kumar's introduction of superior Mandelbrot sets [9]. Prajapati et al., who investigated Julia sets of whole transcendental functions using the Mann iterative scheme, confirmed its efficacy in navigating infinite-dimensional function spaces and capturing subtle escape dynamics, so supporting this theoretical development [10]. Analyzing Mandelbrot and Julia sets in the Picard-Mann orbit, Zou et al. presented a fresh viewpoint showing the advantages of combining the stability of Picard iteration with the flexibility of Mann's averaging process [11]. Hamada and Khbarat expanded their work by including sine and cosine functions into the generating polynomial, so highlighting the interaction between periodic functions and fractal geometry in the Picard-Mann framework [12].

Abbas et al. turned the emphasis to fixed point theory, stressing that in appropriate function spaces, where iterative convergence to non-escaping points defines the filled Julia set, the generation of fractals can be seen as a fixed-point problem [13]. Relatedly, Tassaddiq et al. presented a four-step feedback iteration showing enhanced convergence speed and resolution in produced fractals, especially in terms of fine boundary detail [14]. In the framework of Mandelbrot and Julia generation, Zhou, Tanveer, and Li performed a comparative analysis of several fixed-point techniques providing important insight on the performance and graphical results of many iteration types [15]. Among the more specialized contributions, Özgür et al. proposed the Fibonacci-Mann iteration for transcendental functions, mixing number-theoretic sequences with iterative dynamics to produce fractals with unique combinatorial symmetry [16].

Li and colleagues established general conditions under which Mandelbrot-like structures can develop and produced fixed-point results in the extended Jungck-SP orbit, so offering a larger theoretical framework for understanding convergence in iterative processes [17]. By means of implicit iterative schemes appropriate for highly nonlinear systems, Zhang et al. extended this line of research and illustrated their application in producing both limited and fragmented fractal sets [18]. Tassaddiq formalized conditions that define whether orbits tend toward infinity-a fundamental feature of the escape-time algorithm extensively used in fractal visualization [19], so establishing general escape criteria for extended Jungck-Noor iterations. Building on this, Guran et al. investigated stability, convergence, and data dependency using the Jungck-DK iterative scheme, then validated their results using computational simulations of fractal generation [20]. Investigating fractal variants for higher-order complex polynomial, Tomar et al. showed how degrees outside the quadratic case add extra bifurcation layers and influence Julia set global structure [21].

Panwar et al. meanwhile presented a targeted investigation on Mandelbrot and Julia sets affected by logarithmic functions, showing how such perturbations change the escape basin geometry and fractal symmetry [22]. By means of Mann and Picard-Mann iterations, Tanveer, Nazeer, and Gdawiec advanced the theory by providing computational insights on how iteration parameters control the evolution and complexity of both Mandelbrot and Julia sets [23]. This resulted in general form $z^p + \log c^t$.

Devaney's modern textbook on chaotic dynamical systems enriches this thorough basis by offering theoretical rigor for orbit behavior, bifurcations, and symbolic dynamics supporting fractal generation [24]. Xiangdong et al. showed further algorithmic creativity by creating an accelerated escape-time algorithm meant to build generalized Mandelbrot sets with better computational efficiency [25]. Last but not least, fundamental iterative ideas from Picard [26], Mann [27], and their contemporary hybrids such as Khan's Picard-Mann scheme [28], Gursoy and Karakaya's Picard-S hybrid method [29], and Kumar et al.'s three-step iterative process [30] have been absolutely crucial in defining how convergence behavior translates into graphical fractal forms. Recent s-convexity-based Picard-Mann iteration work by Shahid, Nazeer, and Gdawiec shows its ability in producing Julia and Mandelbrot sets with sharper detail and controlled divergence [31].

This manuscript is organized as:

• Section 2 will examine the basic definitions including Julia set, Mandelbrot set, Mann iteration, Picard-Mann iteration, and CR-iteration from current literature.

• In section 3 we formulate the escape criteria for the generalized CR-iteration procedure.

• Section 4 describes and illustrates the pseudocode for Mandelbrot set fractals and generate the quadratic Mandelbrot set fractals, and cubic Mandelbrot set fractals.

• Section 5 describes and illustrates the pseudocode for Julia set fractals and shows the graphical behavior of the quadratic Julia set fractals, and cubic Julia set fractals.

• In section 6, we generate the Julia and Mandelbrot set fractals by using the CR-iteration method and generalized CR-iteration process.

• In section 7, we repeat the Julia and Mandelbrot set pattern and make some beautiful Batik design.

• In section 8, we discuss the comparison with existing literature.

• Section 9, is the conclusion of our manuscript.

2. Basic definitions

In this section, we discuss essential definitions and basic ideas from existing literature. Throughout in this manuscript, set of complex number is represented with $\mathbb{C}$, set of netural number is denoted by $\mathbb{N}$, and $\mathbb{R}$ is the set of real numbers.

Definition 1 [32] Assume that ζ_r is a self-mapping on complex set, where $r \in \mathbb{C}$, and the collection of the set of points are

$$K_{\zeta_r} = \left\{ z \in \mathbb{C} : \{|\zeta_r^p(z)|\}_{p=0}^{\infty} \text{ is bounded} \right\}. \tag{1}$$

Then, the filled Julia set is denoted by K_{ζ_r}, that is the p^{th} iteration of ζ_r. If $(p \geq 2)$, then the Julia set refers to the boundary of the filled Julia set ∂K_{ζ_r}.

Definition 2 [24] The Mandelbrot set M includes all parameter values β that are associated to the filled-in Julia set of $\zeta_a(x) = x^2 + a$. That is,

$$M = \left\{ a \in \mathbb{C} : \partial K_{\zeta_a} \text{ is connected} \right\}. \tag{2}$$

The element x is a starting element in which $\zeta_a(x) = 0$.

Definition 3 Let $\zeta : \mathbb{C} \to \mathbb{C}$ be a complex function. Then, for any initial point $z_0 \in \mathbb{C}$. The Picard iteration [26] is defined as:

$$z_{k+1} = \zeta(z_k), \tag{3}$$

where $k \in \mathbb{N}$. The Mann iteration [27] is defined as:

$$z_{k+1} = (1 - \alpha_k) z_k + \alpha_k \zeta(z_k), \tag{4}$$

where $\alpha_k \in (0, 1]$ and $k \in \mathbb{N}$.

The Picard-Mann iteration [28] is defined as:

$$\begin{cases} z_{k+1} = \zeta(y_k), \\ y_k = (1 - \alpha_k) z_k + \alpha_k \zeta(z_k), \end{cases} \tag{5}$$

where $\alpha_k \in (0, 1]$ and $k \in \mathbb{N}$.

The Picard-S iteration [29] is defined as:

$$\begin{cases} z_{k+1} = \zeta(y_k), \\ y_k = (1 - \alpha_k) \zeta(z_k) + \alpha_k \zeta(x_k), \\ x_k = (1 - \beta_k) z_k + \beta_k \zeta(z_k), \end{cases} \tag{6}$$

where $\alpha_k \in (0, 1]$, $\beta_k \in [0, 1]$, and $k \in \mathbb{N}$. The CR iteration [30] is defined as

$$\begin{cases} z_{k+1} = (1 - \alpha_k) y_k + \alpha_k \zeta(y_k), \\ y_k = (1 - \beta_k) z_k + \beta_k \zeta(x_k), \\ x_k = (1 - \gamma_k) z_k + \gamma_k \zeta(z_k), \end{cases} \tag{7}$$

where $\alpha_k, \beta_k, \gamma_k \in (0, 1]$ and $k \in \mathbb{N}$.

The four-step generalized CR-iteration process as given below:

$$\begin{cases} z_{k+1} = (1 - \eta) \zeta_c(t_k) + \eta \zeta_c(t_k) \\ t_k = (1 - \alpha) y_k + \alpha \zeta_c(y_k), \\ y_k = (1 - \beta) \zeta_c(z_k) + \beta \zeta_c(x_k), \\ x_k = (1 - \gamma) z_k + \gamma \zeta_c(z_k). \end{cases} \tag{8}$$

Where $\alpha_k,\ \beta_k,\ \gamma_k,\ \eta_k \in (0,\ 1)$ for all $k \in \mathbb{N}$.

3. Escape criterion

In this section, we discuss the four-step generalized CR iteration method, and establish the escape criterion for the complex-valued function $\zeta_c(z) = z^p + \log c^t$ where $c \in \mathbb{C}\setminus\{0\}$, $p \in \mathbb{N},\ p \geq 2$, and $t \in \mathbb{R}, t \geq 1$. We assume that $\lambda = \dfrac{\log c^t}{c}$, and $\log c^t = c\,\lambda$, where $c \in \mathbb{C}$, $t \in \mathbb{R}$, and $c^t \neq 1$.

Next, we establish the escape criterion for the generalized CR-iteration process.

Theorem 1 Assume that $\zeta_c(z) = z^p + \log c^t$, where $p \in \mathbb{N},\ p \geq 2,\ t \in \mathbb{R},\ t \geq 1$, and $c \in \mathbb{C}\setminus\{0\}$, be a complex function and the following inequalities are satisfied:

$$|c| \leq |z_0|,\ (2+|\lambda|)^{\frac{1}{p-1}} \leq |z_0|,\ \left(\frac{2+\alpha(|\lambda|+1)}{\alpha}\right)^{\frac{1}{p-1}} \leq |z_0|,$$

$$\left(\frac{2+|\lambda|}{\beta}\right)^{\frac{1}{p-1}} \leq |z_0|,\ \left(\frac{2+\gamma(|\lambda|-1)}{\gamma}\right)^{\frac{1}{p-1}} \leq |z_0|,\ \left(\frac{2+|\lambda|}{\eta}\right)^{\frac{1}{p-1}} \leq |z_0|,$$

where $\alpha,\ \beta,\ \gamma \in (0,\ 1]$ and $z_0 \in\ \mathbb{C}$. The sequence $\{z_k\}_{k\in\mathbb{N}}$ is define in equation (8). Then, $|z_k| \to \infty$, as $k \to \infty$.

Proof. Assume that the value of $k = 0$, in first step of generalized CR iteration process. We get

$$\begin{aligned}
|x_0| &= |(1-\gamma)z_0 + \gamma\zeta_c(z_0)| \\
&= \left|(1-\gamma)z_0 + \gamma\left(z_0^p + \log c^t\right)\right| \\
&= \left|(1-\gamma)z_0 + \gamma\left(z_0^p + c\lambda\right)\right| \\
&\geq \left|\gamma z_0^p\right| - |\gamma c\lambda| - (1-\gamma)|z_0| \\
&\geq \left|\gamma z_0^p\right| - \gamma|\lambda|\,|z_0| - (1-\gamma)|z_0| \\
&= |z_0|\left[\gamma\left|z_0^{p-1}\right| - \gamma|\lambda| - (1-\gamma)\right].
\end{aligned}$$

Since $|z_0| > \left(\dfrac{2+\gamma(|\lambda|-1)}{\gamma}\right)^{\frac{1}{p-1}}$, thus, $|x_0|\ >\ |z_0|$. The second step of generalized CR iteration process for $k = 0$. We have

$$
\begin{aligned}
|y_0| &= |(1-\beta)\zeta_c(z_0) + \beta\zeta_c(x_0)| \\
&= |\,(1-\beta)\left(z_0^p + \log c^t\right) + \beta\left(x_0^p + \log c^t\,\right| \\
&= \left|(1-\beta)\left(z_0^p + c\lambda\right) + \beta\left(x_0^p + c\lambda\right)\right| \\
&= \left|\left(z_0^p + c\lambda\right) - \beta\left(z_0^p + c\lambda\right) + \beta\left(x_0^p + c\lambda\right)\right| \\
&\geq \left|z_0^p\right| - |c\lambda| \\
&\geq \left|\beta z_0^p\right| - |z_0\lambda|, \quad \because 1 \geq \beta, \quad |z_0| \geq |c| \\
&\geq |z_0|\left(\beta\left|z_0^{p-1}\right| - (1+|\lambda|)\right) \because 1+|\lambda| > |\lambda|.
\end{aligned}
$$

Our assumption $|z_0| > \left(\dfrac{2+|\lambda|}{\beta}\right)^{\frac{1}{p-1}}$. Thus, $|y_0| > |z_0|$. Now, the third step of generalized iteration process for $k = 0$. We obtain

$$
\begin{aligned}
|t_0| &= |(1-\alpha)y_0 + \alpha\zeta_c(y_0)| \\
&= \left|(1-\alpha)y_0 + \alpha\left(y_0^p + \log c^t\right)\right| \\
&= \left|(1-\alpha)y_0 + \alpha\left(y_0^p + c\lambda\right)\right| \\
&\geq \left|\alpha y_0^p\right| - |\alpha c\lambda| - (1-\alpha)|y_0| \\
&\geq \left|\alpha y_0^p\right| - |\alpha z_0\lambda| - (1-\alpha)|y_0| \\
&\geq \left|\alpha z_0^p\right| - \alpha|\lambda|\,|z_0| - (1-\alpha)|z_0| \\
&= |z_0|\left[\alpha\left|z_0^p\right| - \alpha|\lambda| - (1-\alpha)\right].
\end{aligned}
$$

Since $|z_0| > \left(\dfrac{2+\alpha(|\lambda|-1)}{\alpha}\right)^{\frac{1}{p-1}}$, then $|t_0| > |z_0|$ and the fourth step of generalized CR iteration process for $k = 0$, we have

$$
\begin{aligned}
|z_1| &= |(1-\eta)\zeta_c(t_0) + \eta\zeta_c(t_0)| \\
&=|\ (1-\eta)\left(t_0^p + \log c^t\right) + \eta\left(t_0^p + \log c^t\right) \\
&= \left|(1-\eta)\left(t_0^p + c\lambda\right) + \eta\left(t_0^p + c\lambda\right)\right| \\
&\geq \left|z_0^p\right| - |c\lambda| \\
&\geq \left|\eta z_0^p\right| - |z_0\lambda|, \quad \because 1 \geq \eta, \quad |z_0| \geq |c| \\
&\geq |z_0|\left(\eta\left|z_0^{p-1}\right| - (1+|\lambda|)\right) \because 1+|\lambda| > |\lambda|.
\end{aligned}
$$

Since $|z_0| > \left(\frac{2+|\lambda|}{\eta}\right)^{\frac{1}{p-1}}$ then $\eta\left|z_0^{p-1}\right| - (1+|\lambda|) > 1$. Thus, $|z_1| > |z_0|$. There exists $\mu > 0$ such that $\eta|z_0^{p-1}| - (1+|\lambda|) > 1+\mu$. Thus, $|z_1| > (1+\mu)|z_0|$. Continuing this process we have, $|z_k| > (1+\mu)^k |z_0|$. Hence $\lim_{k\to\infty}|z_k| = \infty$. □

The result from Theorem 1 can be refined in the following corollary.

Corollary 1 Assume that

$$
|z_0| > \max\left\{|c|,\ \left(\frac{2+\alpha(|\lambda|+1)}{\alpha}\right)^{\frac{1}{p-1}},\ \left(\frac{2+|\lambda|}{\beta}\right)^{\frac{1}{p-1}},\ \left(\frac{2+\gamma(|\lambda|-1)}{\gamma}\right)^{\frac{1}{p-1}},\ \left(\frac{2+|\lambda|}{\eta}\right)^{\frac{1}{p-1}}\right\},
$$

where $p \in \mathbb{N}$, $p \geq 2$, $t \in \mathbb{R}$, $t \geq 1$, $c \in \mathbb{C}\backslash\{0\}$, and $\alpha,\ \beta,\ \gamma,\ \eta \in (0,\ 1)$. Then, for the generalized CR orbit of z_0, we have $\lim_{k\to\infty}|z_k| = \infty$.

We derive the escape criterion for generalized CR iteration by applying the same logic as in Theorem 1 and Corollary 1.

Corollary 2 Suppose that for the new CR orbit of z_0, we have

$$
\left|z_j\right| > \max\left\{|c|,\ \left(\frac{2+\alpha(|\lambda|+1)}{\alpha}\right)^{\frac{1}{p-1}},\ \left(\frac{2+|\lambda|}{\beta}\right)^{\frac{1}{p-1}},\ \left(\frac{2+\gamma(|\lambda|-1)}{\gamma}\right)^{\frac{1}{p-1}},\ \left(\frac{2+|\lambda|}{\eta}\right)^{\frac{1}{p-1}}\right\},
$$

for some $j \geq 0$, where $p \in \mathbb{N}$, $p \geq 2$, $t \in \mathbb{R}$, $t \geq 1$, $c \in \mathbb{C}\backslash\{0\}$, and $\alpha,\ \beta,\ \gamma \in (0,\ 1)$. Then, there exists $\eta > 0$ such that $|z_{j+k}| > (1+\eta)^k |z_j|$ and we have $\lim_{k\to\infty}|z_k| = \infty$.

4. Generation of Mandelbrot set fractals

We present pseudocode for the escape-time algorithm of Mandelbrot set fractals by using the generalized CR orbit for the complex function $\zeta_c(z) = z^p + \log c^t$, where $p \geq 2$, $t \in \mathbb{R}$, and $c^t \neq 1$.

Algorithm 1 Mandelbrot set generation

Input: $\zeta_{\mathbf{c}}(\mathbf{z}) = \mathbf{z}^{\mathbf{p}} + \log \mathbf{c}^{\mathbf{t}}$ a complex function; $\mathbf{p} \in \mathbb{N}$, $\mathbf{p} \geq 2$, and $\mathbf{t} \in \mathbb{R}$, $\mathbf{t} \geq 1$—parameters for $\zeta_{\mathbf{c}}$; **A**—area; **K**—the maximal number of iterations; $\alpha, \beta, \gamma, \eta \in (0, 1]$—the parameter for the generalized CR iteration; colourmap $[\mathbf{0}..\mathbf{K}]$—colour map with $\mathbf{K}+\mathbf{1}$ colours.

Output: Mandelbrot set for area A.

for $\mathbf{c} \in \mathbf{A}$ **do**
    if $\mathbf{c} = \mathbf{0}$ **then**
        discard the point
        $\lambda = \dfrac{\mathbf{logc^t}}{\mathbf{C}}$,
    end if
    $\mathbf{R_1} = \left(\dfrac{\mathbf{2} + \alpha(|\lambda| + \mathbf{1})}{\alpha}\right)^{\frac{\mathbf{1}}{\mathbf{p-1}}}$,
    $\mathbf{R_2} = \left(\dfrac{\mathbf{2} + \gamma(|\lambda| + \mathbf{1})}{\gamma}\right)^{\frac{\mathbf{1}}{\mathbf{p-1}}}$,
    $|\mathbf{R}| > \max\left\{|\mathbf{c}|, \mathbf{R_1}, \mathbf{R_2}, \left(\dfrac{\mathbf{2}+|\lambda|}{\beta}\right)^{\frac{\mathbf{1}}{\mathbf{p-1}}}, \left(\dfrac{\mathbf{2}+|\lambda|}{\eta}\right)^{\frac{\mathbf{1}}{\mathbf{p-1}}}\right\}$,
    $\mathbf{k} = \mathbf{0}$,
    $\mathbf{z} = \mathbf{0}$,
    while $|\mathbf{z}| < \mathbf{R}$ **and** $\mathbf{k} < \mathbf{K}$ **do**
        $\mathbf{x_k} = (\mathbf{1} - \gamma)\mathbf{z_k} + \gamma\, \zeta_{\mathbf{c}}(\mathbf{z_k})$
        $\mathbf{y_k} = (\mathbf{1} - \beta)\zeta_{\mathbf{c}}(\mathbf{z_k}) + \beta\, \zeta_{\mathbf{c}}(\mathbf{x_k})$,
        $\mathbf{t_k} = (\mathbf{1} - \alpha)\mathbf{y_k} + \alpha\, \zeta_{\mathbf{c}}(\mathbf{y_k})$,
        $\mathbf{z_{k+1}} = (\mathbf{1} - \eta)\zeta_{\mathbf{c}}(\mathbf{t_k}) + \eta\, \zeta_{\mathbf{c}}(\mathbf{t_k})$
        $\mathbf{k} = \mathbf{k} + \mathbf{1}$
    end while
    color **c** with colourmap [k]
end for.

4.1 *Quadratic Mandelbrot set fractals*

In this section, we generate the quadratic Mandelbrot set fractals with MATLAB R2024a by using generalized CR-iteration process. The value of $p = 2$, then the quadratic complex function is $\zeta_c(z) = z^2 + \log c^t$, where $z \in \mathbb{C}$, $t \in \mathbb{R}$ and $c^t \neq 1$. We see the behavior of quadratic Mandelbrot set fractals in Figure 1, for variation of the parameter t from 1 to 10, and the values of the other parmeters $\alpha, \beta, \gamma, \eta$, are fixed. The values of the parametes $\alpha, \beta, \gamma, \eta$, and t are shown in Table 1.

Table 1. Variation of the parameter p

p	t	α	β	γ	η
2	1	0.0001	0.0002	0.0009	0.0006
2	2	0.0001	0.0002	0.0009	0.0006
2	3	0.0001	0.0002	0.0009	0.0006
2	4	0.0001	0.0002	0.0009	0.0006
2	5	0.0001	0.0002	0.0009	0.0006
2	10	0.0001	0.0002	0.0009	0.0006

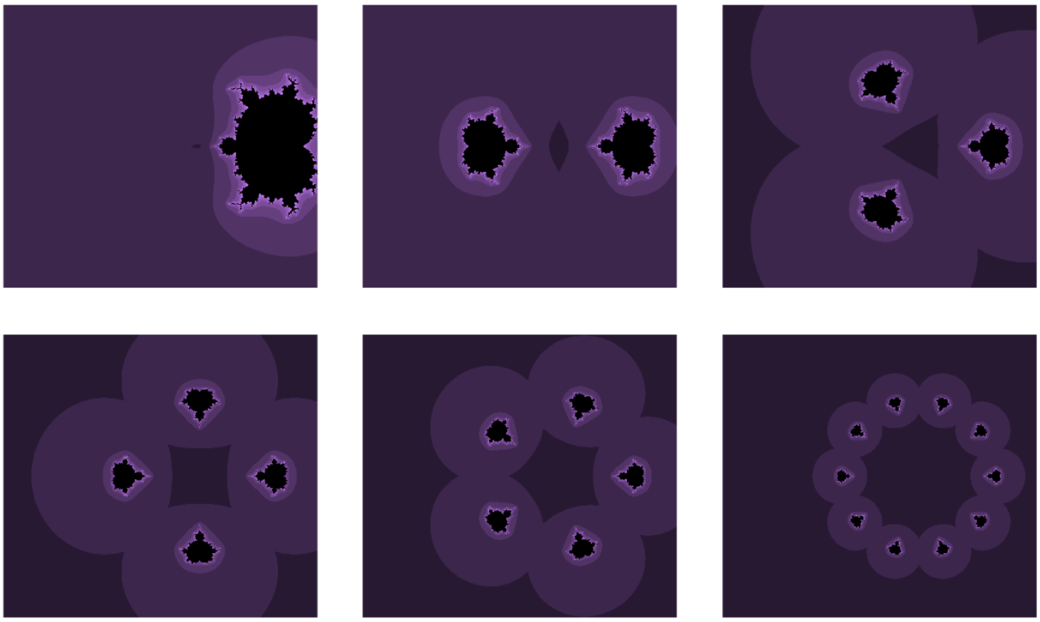

Figure 1. Depict the quadratic Mandelbrot set fractal with variation of t

In Figure 1, the parameter t is varies from 1 to 10. See subsection one to six when we increase the value of the parameter t then the same number of images are repeated.

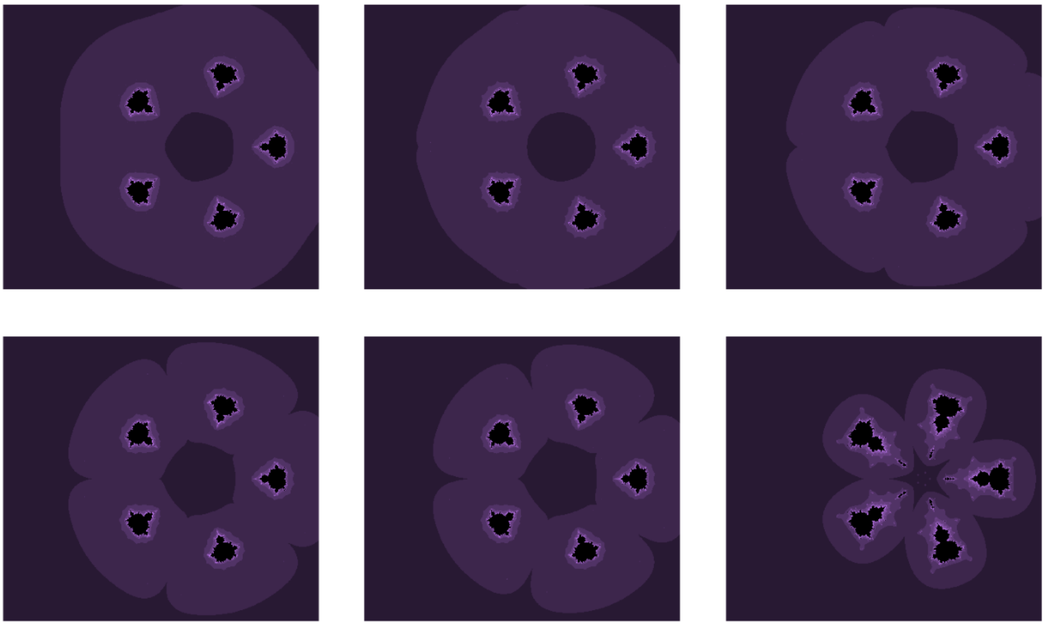

Figure 2. Depict the quadratic Mandelbrot set fractals by variation of the α

In Figure 2, the value of the parameter $p = 2$ then the function $\zeta_c(z) = z^2 + \log c^t$, where $z \in \mathbb{C}$, is a quadratic complex function. The values of the parametes t, β, γ and η are fixed and the values of of the parameter α are varies from 0.001

to 0.1 see Table 2. See in this figure when change the value of the parameter α the centeral part of the fractals convert circle to star shape.

Table 2. Variation of the parameter α

p	t	α	β	γ	η
2	5	0.001	0.0001	0.0002	0.0006
2	5	0.003	0.0001	0.0002	0.0006
2	5	0.005	0.0001	0.0002	0.0006
2	5	0.007	0.0001	0.0002	0.0006
2	5	0.009	0.0001	0.0002	0.0006
2	5	0.1	0.0001	0.0002	0.0006

Figure 3. Depict the quadratic Mandelbrot set fractals with variation of η

Figure 3, shows the quadratic Mandelbrot set fractals behavior by the variation of the parameter η see Table 3, while the other parameters are fixed. When we change the value of the parameter η from 0.001 to 0.1 the Patels are closer to the central part. Central part is converting circle to star and the patels are connected when the value of the parameter is 0.1.

Table 3. Variation of the parameter η

p	t	α	β	γ	η
2	5	0.0001	0.0002	0.0009	0.001
2	5	0.0001	0.0002	0.0009	0.003
2	5	0.0001	0.0002	0.0009	0.005
2	5	0.0001	0.0002	0.0009	0.007
2	5	0.0001	0.0002	0.0009	0.009
2	5	0.0001	0.0002	0.0009	0.1

4.2 *Cubic Mandelbrot set fractals*

In this the value of the parameter p is 3, then the cubic complex function is $\zeta_c(z) = z^3 + \log\, c^t$, where $z \in \mathbb{C}$. We see the behavior of Mandelbrot set fractals for variation of the parameter t from 1 to 10, and the values of the other parmeters α, β, γ, η, are fixed. The values of the parametes α, β, γ, η, and t are shown in Table 4.

Table 4. Variation of the parameter t

p	t	α	β	γ	η
3	1	0.00001	0.02	0.09	0.06
3	2	0.00001	0.02	0.09	0.06
3	3	0.00001	0.02	0.09	0.06
3	4	0.00001	0.02	0.09	0.06
3	5	0.00001	0.02	0.09	0.06
3	10	0.00001	0.02	0.09	0.06

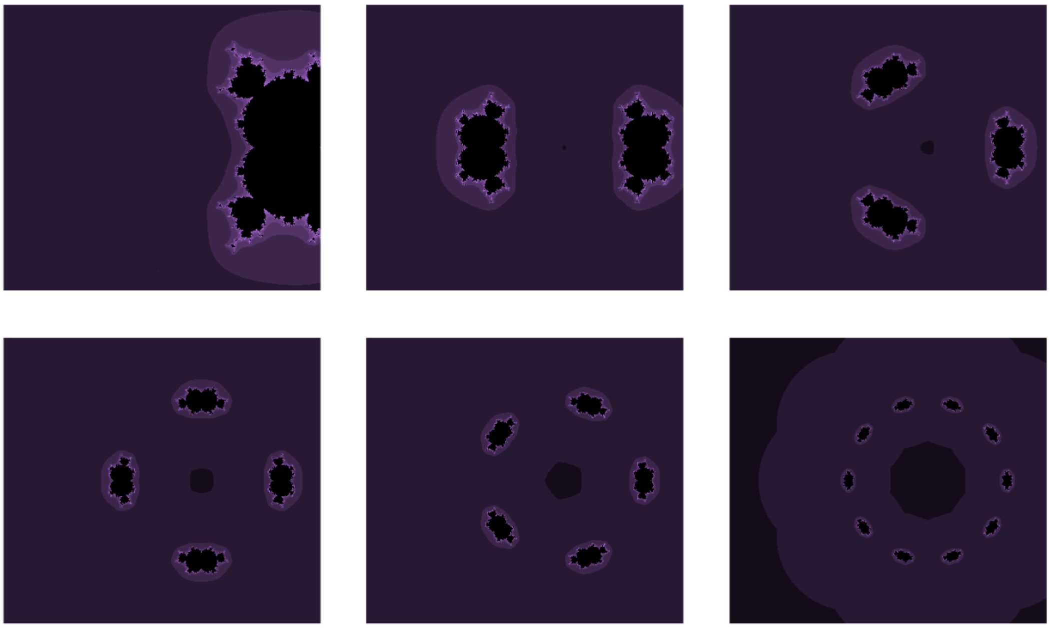

Figure 4. Depict the cubic Mandelbrot set fractals by variation of t

Figure 4, shows the cubic Mandelbrot set fractals in which when the value of the parameter t increase the fractals repeat in a circular shape.

Table 5. Variation of the parameter α

p	t	α	β	γ	η
3	5	0.01	0.0001	0.002	0.0006
3	5	0.03	0.0001	0.002	0.0006
3	5	0.05	0.0001	0.002	0.0006
3	5	0.07	0.0001	0.002	0.0006
3	5	0.09	0.0001	0.002	0.0006
3	5	0.1	0.0001	0.002	0.0006

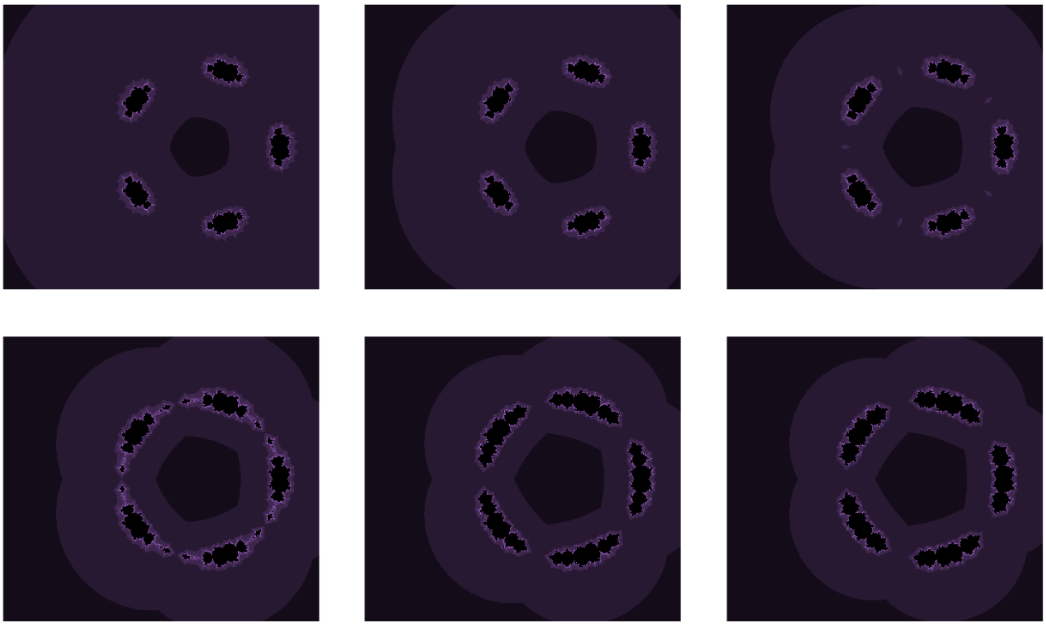

Figure 5. Depict the cubic Mandelbrot set fractals with variation of α

Figure 5, shows the behavior of cubic Mandelbrot set fractals for different values of α. In which the when we increase the value of α the fractals segment closer to each other and the size of the central part of the fractals are increase. The values of α are 0.01, 0.03, 0.05, 0.07, 0.09, and 0.1 while the other parameters are fixed see Table 5.

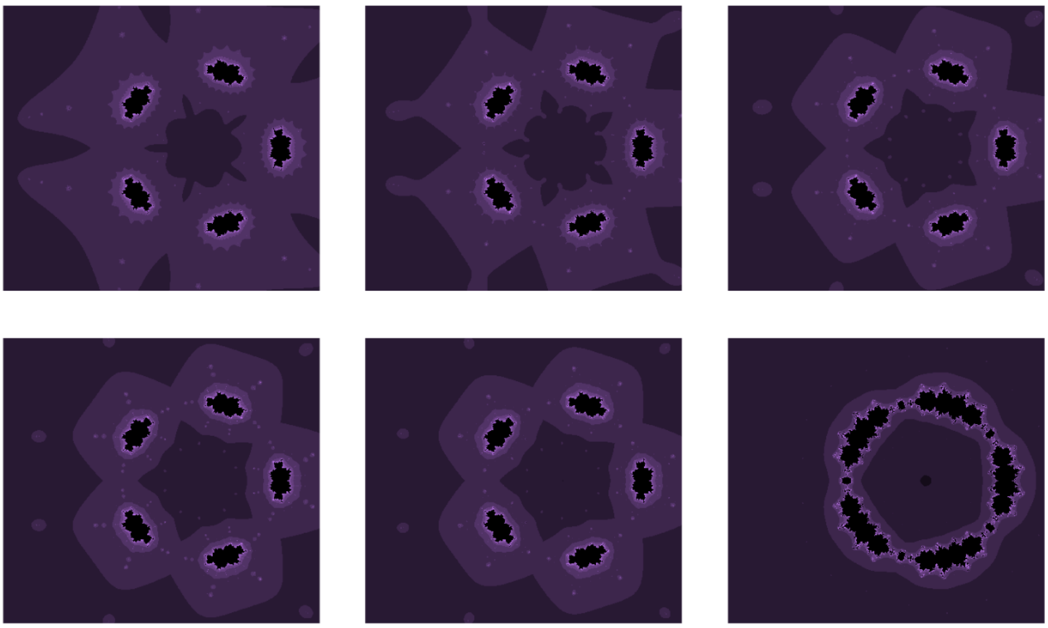

Figure 6. Depict the cubic Mandelbrot set fractals by the variation of η

Figure 6, shows the behavior of the cubic Mandelbrot set fractals with different values of η. The values of η are varies from 0.001 to 0.1 which are shown in Table 6.

Table 6. Variation of the parameter η

p	t	α	β	γ	η
3	5	0.0001	0.0002	0.0009	0.001
3	5	0.0001	0.0002	0.0009	0.003
3	5	0.0001	0.0002	0.0009	0.005
3	5	0.0001	0.0002	0.0009	0.007
3	5	0.0001	0.0002	0.0009	0.009
3	5	0.0001	0.0002	0.0009	0.1

5. Generation of Julia set fractals

In this section, we generate the Julia set fractals by using the generalized CR-iteration process for complex function $\zeta_c(z) = z^p + \log c^t$ a complex function; $p \in \mathbb{N}$, $p \geq 2$, and $t \in \mathbb{R}$, $t \geq 1$. We use the MATLAB R2024a to generate the quadratic Julia set fractals and cubic Julia set fractals for the variation of the parameters α, t, β, γ and η.

Algorithm 2 Julia set generation

Input: $\zeta_c(z) = z^p + \log c^t$ a complex function; $p \in \mathbb{N}$, $p \geq 2$, and $t \in \mathbb{R}$, $t \geq 1$—parameters for ζ_c; **A**—area; **K**—the maximal number of iterations; $\alpha, \beta, \gamma, \eta \in (0, 1]$—the parameter for the generalized CR iteration; colourmap $[0..K]$—colour map with $K+1$ colours.

Output: Julia set for area A.

1. **for** $z_0 \in A$ **do**
2. **if** $c = 0$ **then**
3. discard the point
4. $\lambda = \dfrac{\log c^t}{C}$,
5. **end if**
6. $R_1 = \left(\dfrac{2 + \alpha(|\lambda|+1)}{\alpha}\right)^{\frac{1}{p-1}}$,
7. $R_2 = \left(\dfrac{2 + \gamma(|\lambda|+1)}{\gamma}\right)^{\frac{1}{p-1}}$,
8. $|R| > \max\left\{|c|, R_1, R_2, \left(\dfrac{2+|\lambda|}{\beta}\right)^{\frac{1}{p-1}}, \left(\dfrac{2+|\lambda|}{\eta}\right)^{\frac{1}{p-1}}\right\}$,
9. $k = 0$,
10. **while** $|z| < R$ **and** $k < K$ **do**
11. $x_k = (1-\gamma) z_k + \gamma\, \zeta_c(z_k)$
12. $y_k = (1-\beta)\zeta_c(z_k) + \beta\, \zeta_c(x_k)$,
13. $t_k = (1-\alpha) y_k + \alpha\, \zeta_c(y_k)$,
14. $z_{k+1} = (1-\eta)\zeta_c(t_k) + \eta\, \zeta_c(t_k)$
15. $k = k+1$
16. **end while**
17. color **c** with colourmap [k]
18. **end for**.

5.1 *Quadratic Julia set fractals*

In this section, we generate the Julia set fractals by using generalized CR-iteration method for quadratic complex function $\zeta_c(z) = z^2 + \log c^t$ where $t \in \mathbb{R}$, $t \geq 1$.

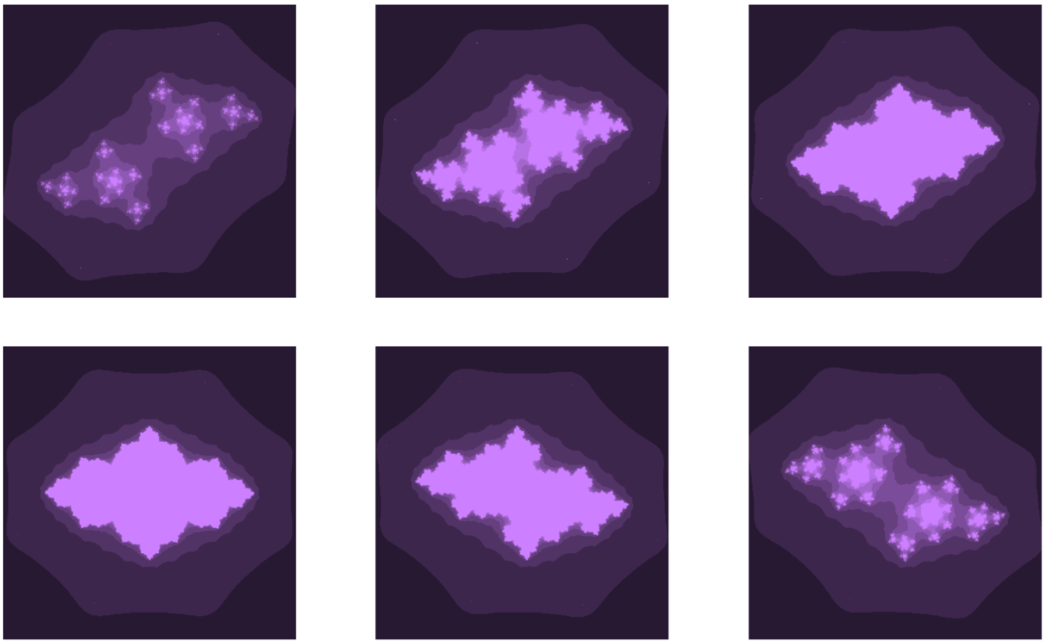

Figure 7. Depict the quadratic Julia set fractals by variation of the parameter t

Figure 7, shows the behavior of quadratic Julia set fractals by using the generalized CR-iteration method for quadratic complex function $\zeta_c(z) = z^2 + \log c^t$ where $t \in \mathbb{R}, t \geq 1$, with variation of the parameter t while other parameters are constant. The values of the parameter t changes from 2 to 7 that are shown in Table 7.

Table 7. Variation of the parameter t

p	t	α	β	γ	η	c
2	2	0.01	0.0001	0.009	0.006	$-0.7+0.3i$
2	3	0.01	0.0001	0.009	0.006	$-0.7+0.3i$
2	4	0.01	0.0001	0.009	0.006	$-0.7+0.3i$
2	5	0.01	0.0001	0.009	0.006	$-0.7+0.3i$
2	6	0.01	0.0001	0.009	0.006	$-0.7+0.3i$
2	7	0.01	0.0001	0.009	0.006	$-0.7+0.3i$

Table 8. Variation of the parameter α

p	t	α	β	γ	η	c
2	2	0.1	0.8	0.9	0.6	$-0.7+0.3i$
2	2	0.3	0.8	0.9	0.6	$-0.7+0.3i$
2	2	0.5	0.8	0.9	0.6	$-0.7+0.3i$
2	2	0.7	0.8	0.9	0.6	$-0.7+0.3i$
2	2	0.9	0.8	0.9	0.6	$-0.7+0.3i$
2	2	0.99	0.8	0.9	0.6	$-0.7+0.3i$

Figure 8, shows the behavior of the quadratic Julia set fractals by using the generalized CR-iteration process with variation of the parameter α varies from 0.1 to 0.99 while the other parameters are fixed. The values of the parameters p, t, α, β, γ, η and c are given in Table 8.

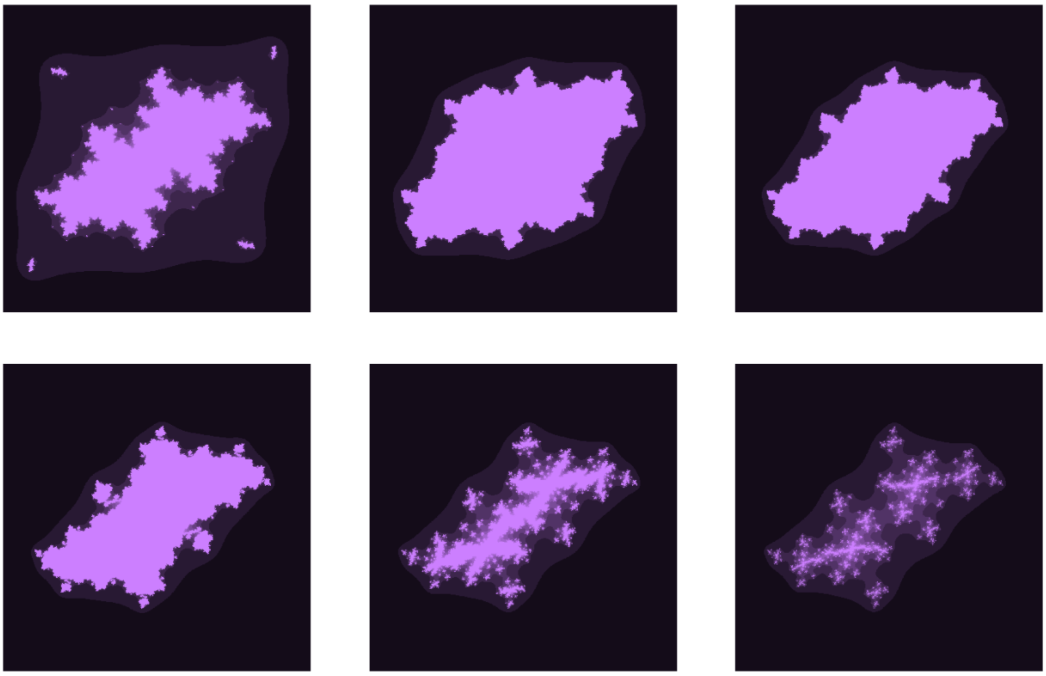

Figure 8. Depict the quadratic Julia set fractals with variation of the parameter α

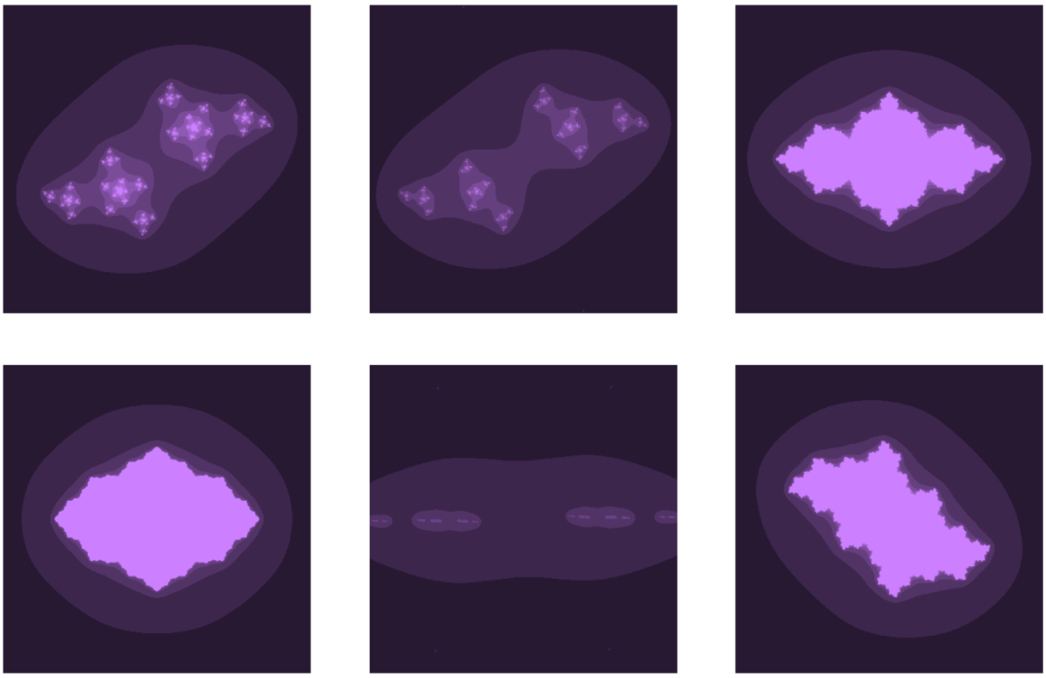

Figure 9. Depict the quadratic Julia set fractals by variation of the parameter c

In Figure 9, the parameter c has the values $-0.7+0.3i,\ -0.6+0.3i,\ -0.7,\ -0.8,\ -0.285+0.1i$ and $-0.83+0.23i$ and all other parameters are constant. We generate the quadratic Julia set fractals by using generalized CR-iteration method for quadratic complex polynomial in which the value of p is 2 see Table 9.

Table 9. Variation of parameter c

p	t	α	β	γ	η	c
2	2	0.001	0.002	0.003	0.6	$-0.7+0.3i$
2	2	0.001	0.002	0.003	0.6	$-0.6+0.3i$
2	2	0.001	0.002	0.003	0.6	-0.7
2	2	0.001	0.002	0.003	0.6	-0.8
2	2	0.001	0.002	0.003	0.6	$-0.285+0.1i$
2	2	0.001	0.002	0.003	0.6	$-0.83+0.23i$

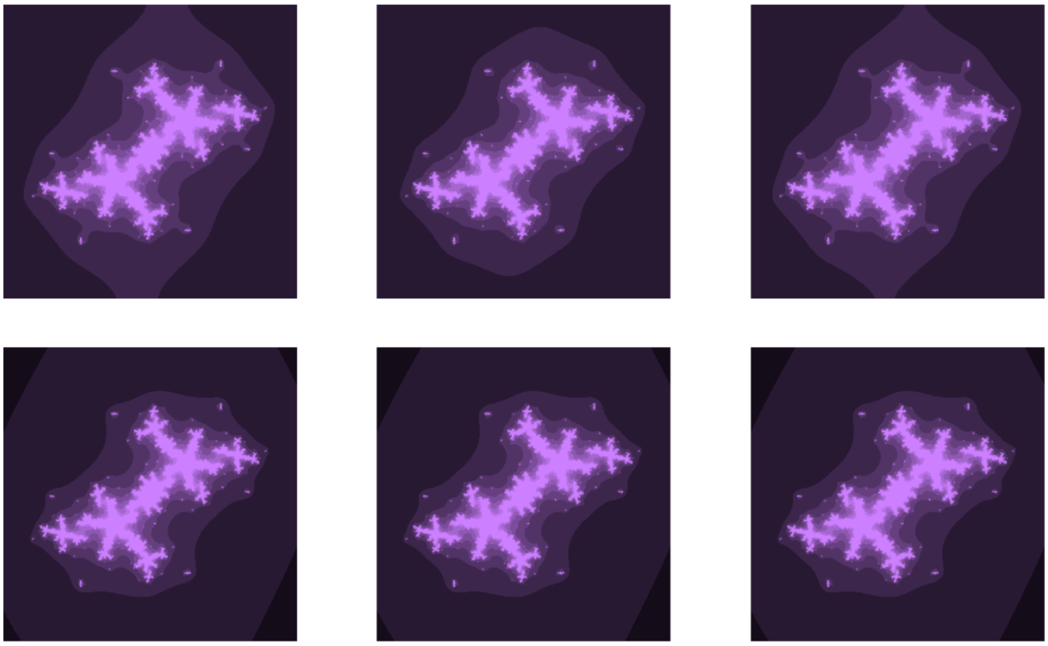

Figure 10. Depict the quadratic Julia set fractals by variation of the parameter η

In Figure 10, shows the behavior of quadratic Julia set fractals by variation of the parameter η, while other parameters are fixed. The values are shown in Table 10.

Table 10. Variation of the parameter η

p	t	α	β	γ	η	c
2	2	0.1	0.08	0.09	0.001	$-0.7+0.3i$
2	2	0.1	0.08	0.09	0.007	$-0.7+0.3i$
2	2	0.1	0.08	0.09	0.002	$-0.7+0.3i$
2	2	0.1	0.08	0.09	0.7	$-0.7+0.3i$
2	2	0.1	0.08	0.09	0.9	$-0.7+0.3i$
2	2	0.1	0.08	0.09	0.099	$-0.7+0.3i$

5.2 *Cubic Julia set fractals*

In this section, we generate the cubic Julia set fractals by using the generalized CR-iteration method for cubic complex polynomial $\zeta_c(z) = z^2 + \log c^t$, where $t \in \mathbb{R}$, $t \geq 1$, in which the value of p is 3.

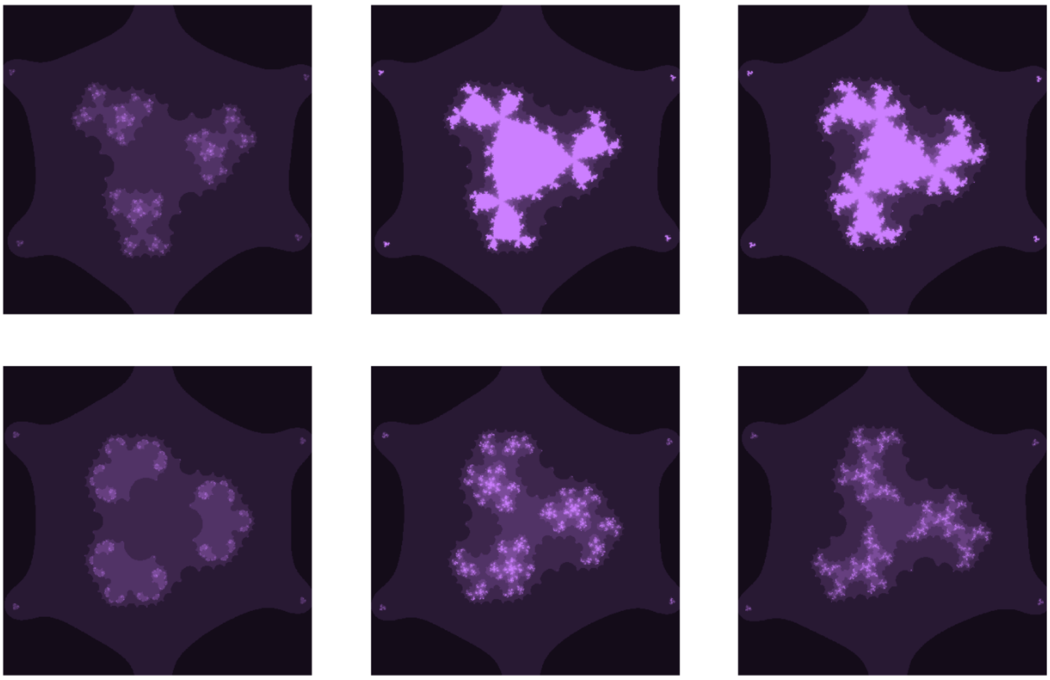

Figure 11. Depict the cubic Julia set fractals by variation of the parameter t

In Figure 11, shows the behavior of cubic Julia set fractals by variation of the parameter t, while other parameters are fixed. The values of the parameter t are 2, 3, 4, 5, 6 and 7 which are shown in Table 11.

Table 11. Variation of the parameter t

p	t	α	β	γ	η	c
3	2	0.01	0.0001	0.009	0.006	$-0.7+0.3i$
3	3	0.01	0.0001	0.009	0.006	$-0.7+0.3i$
3	4	0.01	0.0001	0.009	0.006	$-0.7+0.3i$
3	5	0.01	0.0001	0.009	0.006	$-0.7+0.3i$
3	6	0.01	0.0001	0.009	0.006	$-0.7+0.3i$
3	7	0.01	0.0001	0.009	0.006	$-0.7+0.3i$

Table 12. Variation of the parameter α

p	t	α	β	γ	η	c
3	2	0.09	0.008	0.009	0.6	$-0.7+0.3i$
3	2	0.099	0.008	0.009	0.6	$-0.7+0.3i$
3	2	0.2	0.008	0.009	0.6	$-0.7+0.3i$
3	2	0.3	0.008	0.009	0.6	$-0.7+0.3i$
3	2	0.5	0.008	0.009	0.6	$-0.7+0.3i$
3	2	0.7	0.008	0.009	0.6	$-0.7+0.3i$

In Figure 12, the parameter α has the values 0.09, 0.099, 0.2, 0.3, 0.5, and 0.7 and all other parameters are constant. We generate the cubic Julia set fractals by using generalized CR-iteration method for quadratic complex polynomial in which the value of p is 3 see Table 12.

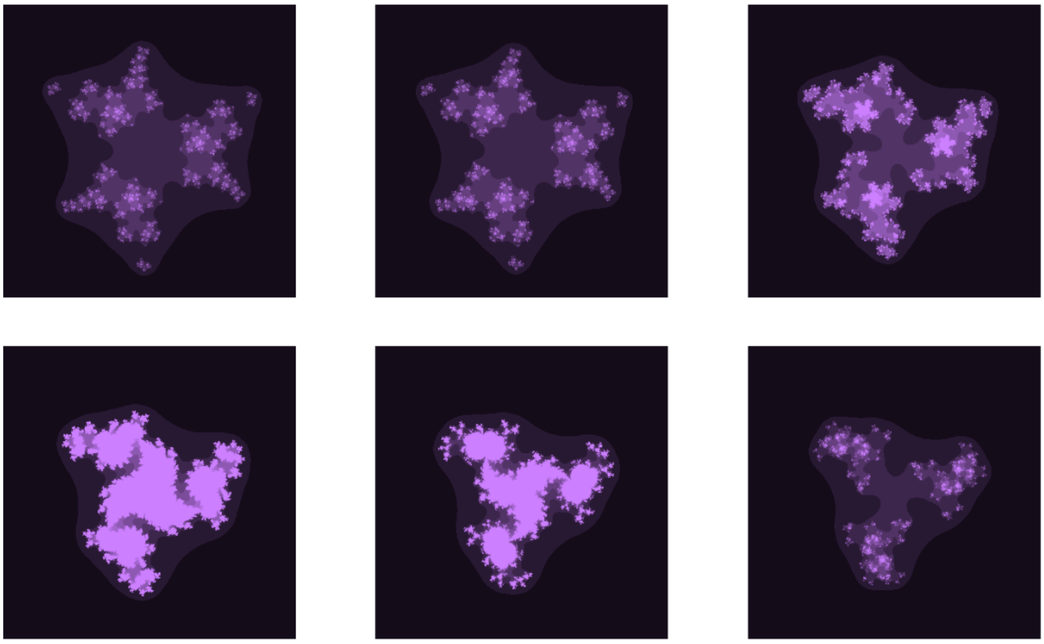

Figure 12. Depict the cubic Julia set fractals with variation of the parameter α

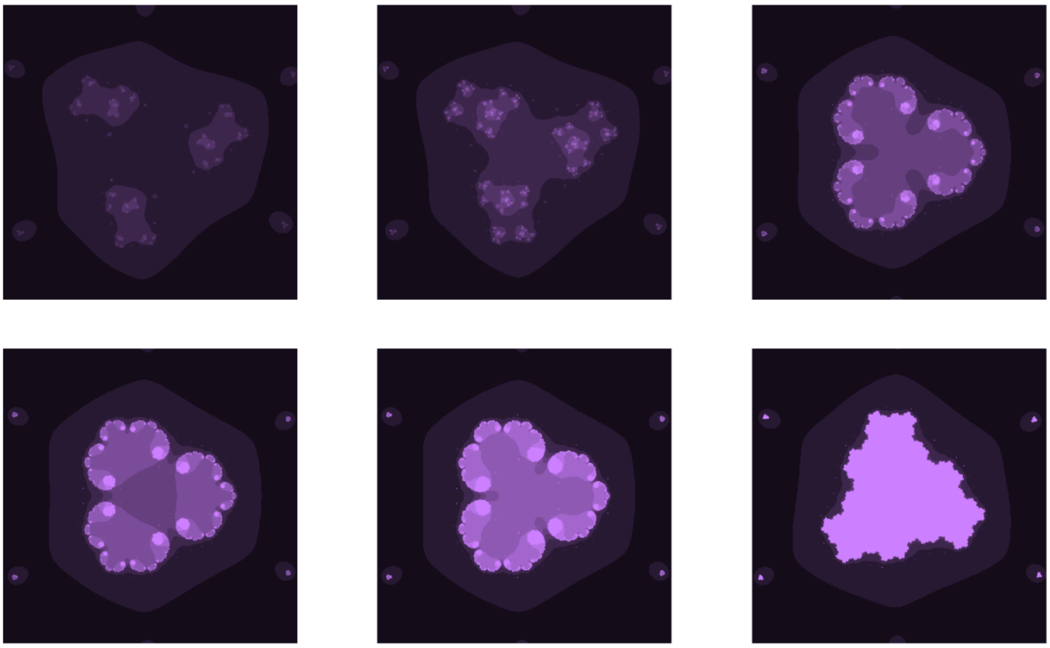

Figure 13. Depict the cubic Julia set fractals by variation of the parameter c

Figure 13, shows the behavior of the cubic Julia set fractals by using the generalized CR-iteration process with variation of the parameter c varies and the values of the parameter c are $-0.7+0.3i$, $-0.6+0.3i$, -0.7, -0.8, $-0.285+0.1i$ and $-0.83+0.23i$, while the other parameters are fixed. The values of the parameters p, t, α, β, γ, η and c are given in Table 13.

Table 13. Variation of the parameter c

p	t	α	β	γ	η	c
2	2	0.001	0.002	0.003	0.6	$-0.7+0.3i$
2	2	0.001	0.002	0.003	0.6	$-0.6+0.3i$
2	2	0.001	0.002	0.003	0.6	-0.7
2	2	0.001	0.002	0.003	0.6	-0.8
2	2	0.001	0.002	0.003	0.6	$-0.285+0.1i$
2	2	0.001	0.002	0.003	0.6	$-0.83+0.23i$

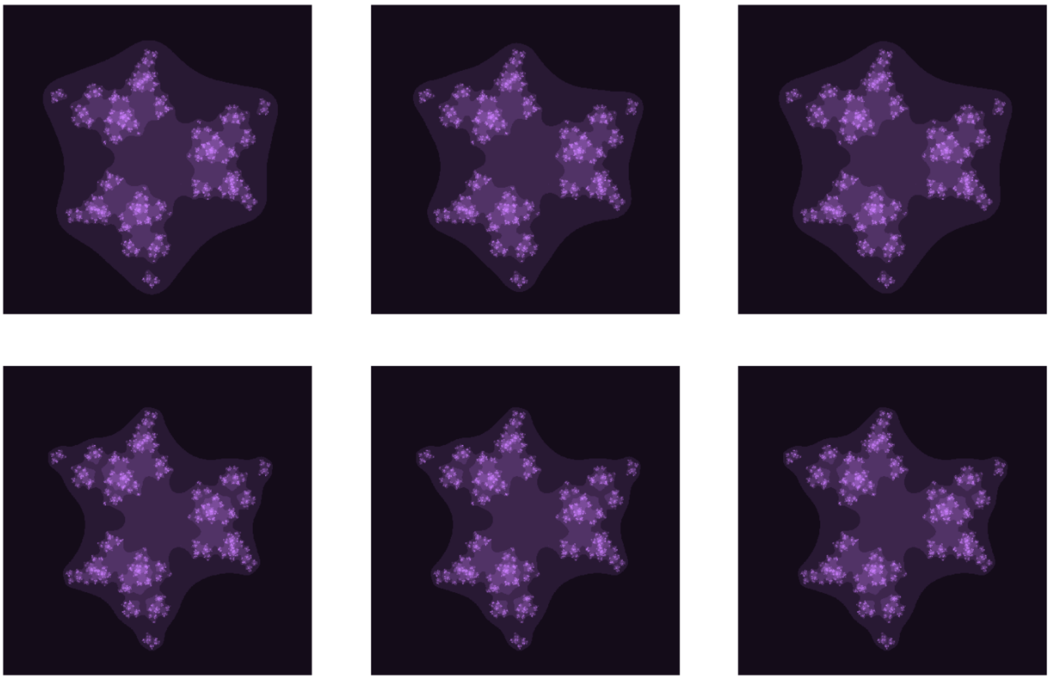

Figure 14. Depict the cubic Julia set fractals with different values of the parameter η

Table 14. Variation of the parameter η

p	t	α	β	γ	η	c
3	2	0.1	0.08	0.09	0.001	$-0.7+0.3i$
3	2	0.1	0.08	0.09	0.007	$-0.7+0.3i$
3	2	0.1	0.08	0.09	0.002	$-0.7+0.3i$
3	2	0.1	0.08	0.09	0.7	$-0.7+0.3i$
3	2	0.1	0.08	0.09	0.9	$-0.7+0.3i$
3	2	0.1	0.08	0.09	0.099	$-0.7+0.3i$

Figure 14, shows the cubic Julia set fractals for cubic complex function by using the generalized CR-iteration process with the variation of the parameter η and the other parameters are constant. The values of the parameter η are 0.001, 0.007, 0.002, 0.7, 0.9, and 0.099 that are given in Table 14.

6. Comparison with CR-iteration method

In this section, we will compare the generalized CR-iteration process and CR-iteration method for a complex function $\zeta_c(z) = z^p + \log\, c^t$ a complex function; $p \in \mathbb{N}$, $p \geq 2$, and $t \in \mathbb{R}$, $t \geq 1$. We use the MATLAB R2024a to generate the Julia and Mandelbrot set fractals by using CR-iteration process and generalized CR-iteration method.

6.1 *Julia set fractals for CR-iteration and generalized CR-iteration*

In this section we discuss the Julia set fractals for complex function $\zeta_c(z) = z^p + \log\, c^t$ a complex function; $p \in \mathbb{N}$, $p \geq 2$, and $t \in \mathbb{R}$, $t \geq 1$. We generate the Julia set fractals with MATLAB R2024a for CR-iteration method and generalized CR-iteration process.

Algorithm 3 Julia set generation

Input: $\zeta_\mathbf{c}(\mathbf{z}) = \mathbf{z}^\mathbf{p} + \log\, \mathbf{c}^\mathbf{t}$ a complex function; $\mathbf{p} \in \mathbb{N}$, $\mathbf{p} \geq \mathbf{2}$, and $\mathbf{t} \in \mathbb{R}$, $\mathbf{t} \geq \mathbf{1}$—parameters for $\zeta_\mathbf{c}$; **A**—area; **K**—the maximal number of iterations; $\alpha, \beta, \gamma \in (\mathbf{0}, \mathbf{1}]$—the parameter for the generalized CR iteration; colourmap $[\mathbf{0}..\mathbf{K}]$—colour map with $\mathbf{K}+\mathbf{1}$ colours.

Output: Juia set for area A.

for $\mathbf{z_0} \in \mathbf{A}$ **do**
  if $\mathbf{c} = \mathbf{0}$ **then**
    discard the point
    $\lambda = \dfrac{\mathbf{logc^t}}{\mathbf{C}}$,
  end if
  $\mathbf{R_1} = \left(\dfrac{\mathbf{2} + \alpha(|\lambda| + \mathbf{1})}{\alpha}\right)^{\frac{\mathbf{1}}{\mathbf{p}-\mathbf{1}}}$,
  $\mathbf{R_2} = \left(\dfrac{\mathbf{2} + \gamma(|\lambda| + \mathbf{1})}{\gamma}\right)^{\frac{\mathbf{1}}{\mathbf{p}-\mathbf{1}}}$,
  $|\mathbf{R}| > \max\left\{|\mathbf{c}|, \mathbf{R_1}, \mathbf{R_2}, \left(\frac{\mathbf{2}+|\lambda|}{\beta}\right)^{\frac{\mathbf{1}}{\mathbf{p}-\mathbf{1}}}\right\}$,
  $\mathbf{k} = \mathbf{0}$,
  while $|\mathbf{z}| < \mathbf{R}$ **and** $\mathbf{k} < \mathbf{K}$ **do**
    $\mathbf{x_k} = (\mathbf{1} - \gamma)\mathbf{z_k} + \gamma\, \zeta_\mathbf{c}(\mathbf{z_k})$
    $\mathbf{y_k} = (\mathbf{1} - \beta)\, \zeta_\mathbf{c}(\mathbf{z_k}) + \beta\, \zeta_\mathbf{c}(\mathbf{x_k})$,
    $\mathbf{z_{k+1}} = (\mathbf{1} - \alpha)\mathbf{y_k} + \alpha\, \zeta_\mathbf{c}(\mathbf{y_k})$,
    $\mathbf{k} = \mathbf{k} + \mathbf{1}$
  end while
  color **c** with colourmap [k]
end for.

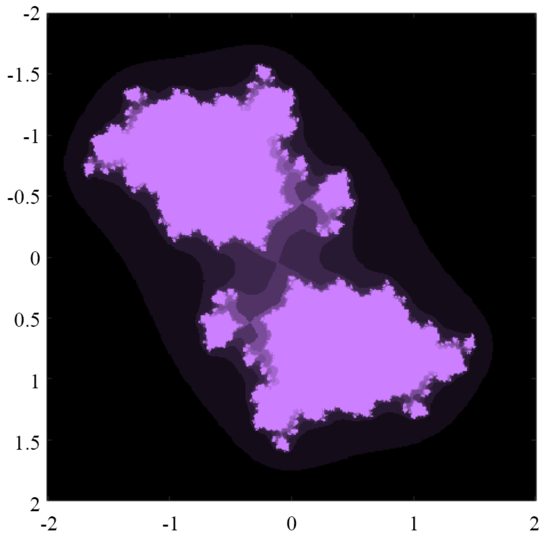

Figure 15. Quadratic Julia set for CR-iteration method

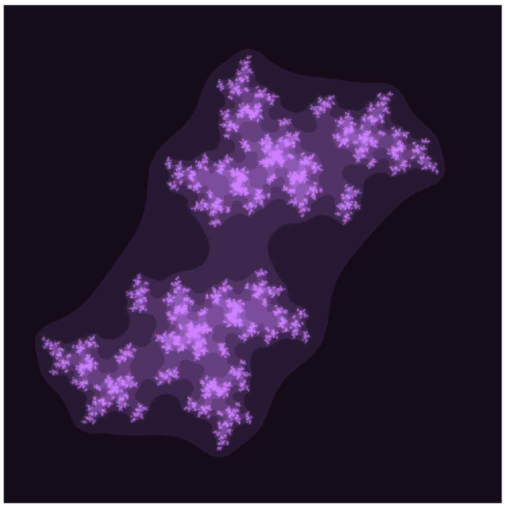

Figure 16. Quadratic Julia set for generalized CR-iteration method

In Figure 15, the values of the parameters $c = -0.7+0.5i$, $\alpha = 0.7$, $\beta = 0.6$, $\gamma = 0.8$, $t = 2$, and $p = 2$. In Figure 16, are the same as $c = -0.7+0.5i$, $\alpha = 0.7$, $\beta = 0.6$, $\gamma = 0.8$, $t = 2$, $p = 2$, and $\eta = 0.6$.

6.2 *Mandelbrot set fractals for CR-iteration and generalized CR-iteration*

In this section we discuss the Julia set fractals for complex function $\zeta_c(z) = z^p + \log\ c^t$ a complex function; $p \in \mathbb{N}$, $p \geq 2$, and $t \in \mathbb{R}$, $t \geq 1$. We generate the Mandelbrot set fractals with MATLAB R2024a for CR-iteration method and generalized CR-iteration process.

Algorithm 4 Mandelbrot set generation

Input: $\mathbf{\zeta_c(z)} = \mathbf{z^p} + \log\ \mathbf{c^t}$ a complex function; $\mathbf{p} \in \mathbb{N}, \mathbf{p} \geq \mathbf{2}$, and $\mathbf{t} \in \mathbb{R}$, $\mathbf{t} \geq \mathbf{1}$-parameters for $\zeta_\mathbf{c}$; **A**-area; **K**-the maximal number of iterations; α, β, $\gamma \in (0, 1]$-the parameter for the generalized CR iteration; colourmap $[\mathbf{0}..\mathbf{K}]$-colour map with $\mathbf{K}+\mathbf{1}$ colours.

Output: Mandelbrot set for area A.

for $\mathbf{c} \in \mathbf{A}$ **do**
    if $\mathbf{c} = \mathbf{0}$ **then**
        discard the point
        $\lambda = \dfrac{\mathbf{log c^t}}{\mathbf{C}}$,
    end if
    $\mathbf{R_1} = \left(\dfrac{\mathbf{2} + \alpha(|\lambda| + \mathbf{1})}{\alpha} \right)^{\frac{1}{p-1}}$,
    $\mathbf{R_2} = \left(\dfrac{\mathbf{2} + \gamma(|\lambda| + \mathbf{1})}{\gamma} \right)^{\frac{1}{p-1}}$,
    $|\mathbf{R}| > \max \left\{ |\mathbf{c}|, \mathbf{R_1}, \mathbf{R_2}, \left(\dfrac{\mathbf{2} + |\lambda|}{\beta} \right)^{\frac{1}{p-1}} \right\}$,
    $\mathbf{k} = \mathbf{0}$,
    $\mathbf{z} = \mathbf{0}$,
    while $|\mathbf{z}| < \mathbf{R}$ **and** $\mathbf{k} < \mathbf{K}$
        $\mathbf{x_k} = (\mathbf{1} - \gamma)\mathbf{z_k} + \gamma\, \zeta_{\mathbf{c}}(\mathbf{z_k})$
        $\mathbf{y_k} = (\mathbf{1} - \beta)\zeta_{\mathbf{c}}(\mathbf{z_k}) + \beta\, \zeta_{\mathbf{c}}(\mathbf{x_k})$,
        $\mathbf{z_{k+1}} = (\mathbf{1} - \alpha)\mathbf{y_k} + \alpha\, \zeta_{\mathbf{c}}(\mathbf{y_k})$,
        $\mathbf{k} = \mathbf{k} + \mathbf{1}$
    end while
    color **c** with colourmap [k]
end for.

In Figure 17, the values of the parameters $\alpha = 0.1$, $\beta = 0.6$, $\gamma = 0.8$, $t = 5$, and $p = 2$. In Figure 18, are the same as $\alpha = 0.1$, $\beta = 0.6$, $\gamma = 0.8$, $t = 5, p = 2$, and $\eta = 0.6$.

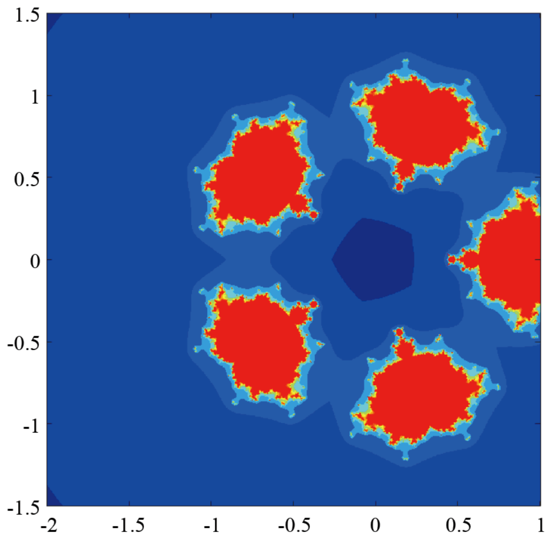

Figure 17. Depict the quadratic Mandelbrot set fractals via CR-iteration method

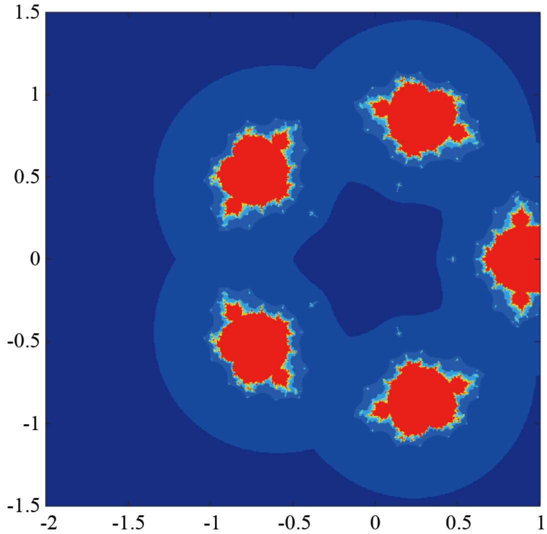

Figure 18. Depict the quadratic Mandelbrot set fractals via generalized CR-iteration method

7. Application for batik design

In this section, we use the reptation of Julia and Mandelbrot set fractals to generate a beautiful batik design. In Figure 19, we repeat the two Mandelbrot set pattern and make a batik design but in Figure 20, we repeat a single cubic Julia set pattern for batik design.

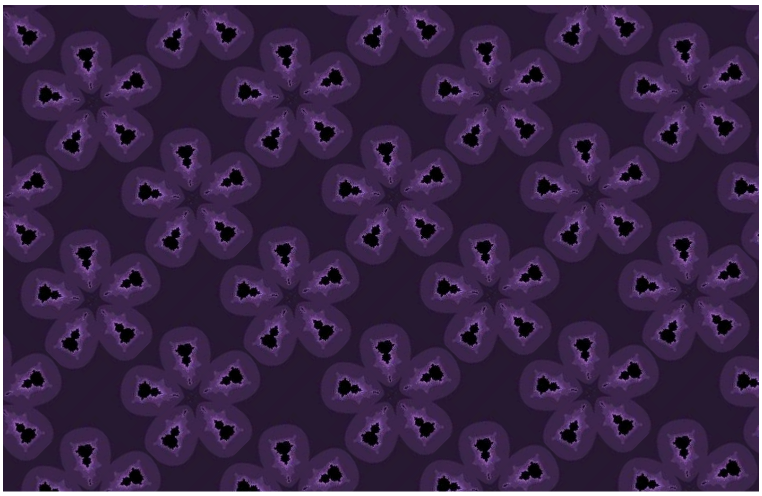

Figure 19. Depict the batik design

In Figure 19, we used the sixth subsection of Figure 2, for batik design.

In Figure 20, we use the fourth subsection of Figure 12, and repeat the cubic Julia set fractals for a beautiful batik design.

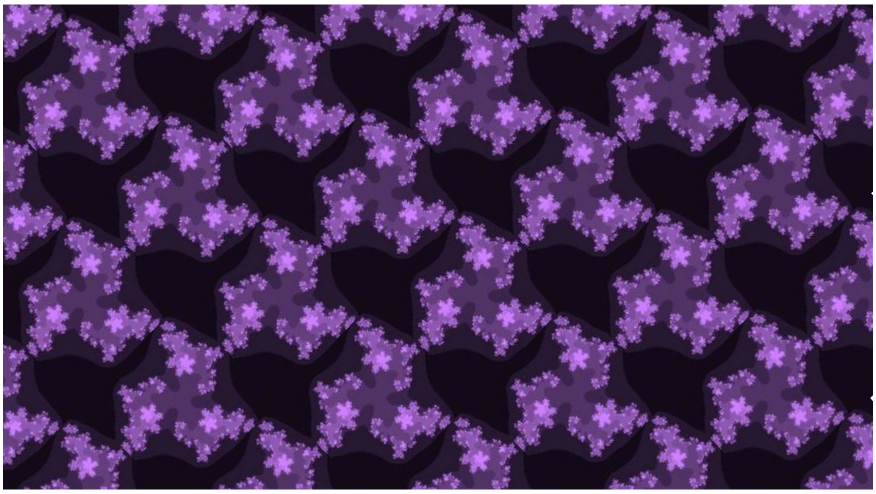

Figure 20. Depict the batik design

8. Comparative analysis

In this section, we will compare our manuscript with Shahid et al. [31], Tanveer et al. [23], Guran et al. [20], and Tomar et al. [21] in Table 15.

Table 15. Comparison with existing literature

Feature	Our work	Shahid et al. [31]	Tanveer et al. [23]	Guran et al. [20]	Tomar et al. [21]
Iteration scheme	Generalized CR-iteration	Picard-Mann iteration	Mann & Picard-Mann	Jungck-DK iteration	Classical and variant iterations
Escape criterion	Newly formulated, specific to CR	s-convexity-based	Power-log enhanced	Stability-based	Classical escape-time
Function Type	$z^p + \log c^t$ and complex variants	Polynomial mappings	$z^p + \log c^t$	General operators	Higher-degree polynomials
Parameter sensitivity	Explored extensively for α, β, γ	Moderate focus on parameters	High impact of p and t	Parameters tied to convergence analysis	Focus on degree variation
Graphical results	Extensive Mandelbrot images; Batik integration	Julia and Mandelbrot visualized	Multiple fractal morphologies	Engineering simulations	Variants of standard fractals
Applications	Textile design (Batik)	Theoretical dynamics	Fractal behavior under transcendental terms	Engineering design & numerical modeling	Aesthetic and geometric exploration
Contribution uniqueness	Visual-textile pipeline; escape-time + symmetry + motif tiling	Precision in detail via s-convexity	Hybrid dynamics in log-polynomial domain	Theoretical rigor in numerical methods	Enriched pattern diversity via function degree

9. Conclusion

In this work, we discussed the Julia and Mandelbrot set fractals by using the generalized CR-iteration method. Designed for the generalized CR-iteration process, a new escape criterion provides exact identification of divergence in complex dynamical systems. This theoretical development allowed the construction of complex fractal designs by means of a tailored escape-time approach. We generated the quadratic Mandelbrot set fractals, cubic Mandelbrot set fractals, quadratic Julia set fractals, and cubic Julia set fractals by using generalized CR-iteration method for complex function

$\zeta_c(z) = z^p + \log c^t$ a complex function; $p \in \mathbb{N}$, $p \geq 2$, and $t \in \mathbb{R}$, $t \geq 1$. We use MATLAB R2024a to generate Julia and Mandelbrot set fractals with variation of the parameters t, α, γ, η, and c. We generated beautiful batik designs by using these Julia and Mandelbrot set fractals. In future, we will generate 3D fractals with generalized iteration schemes.

Author contributions

Conceptualization, K.A., U.I., M.A. and I.L.P.; methodology, K.A., U.I., M.A. and I.L.P.; software, K.A., U.I., M.A. and I.L.P.; validation, K.A., U.I., M.A. and I.L.P.; formal analysis, K.A., U.I., M.A. and I.L.P.; investigation, K.A., U.I., M.A. and I.L.P.; resources, K.A., U.I., M.A. and I.L.P.; data curation, K.A., U.I., M.A. and I.L.P.; writing—original draft preparation, K.A., U.I., M.A. and I.L.P.; writing—review and editing, K.A., U.I., M.A. and I.L.P.; visualization, K.A., U.I., M.A. and I.L.P.; supervision, K.A., U.I., M.A. and I.L.P.; project administration, K.A., U.I., M.A. and I.L.P.; funding acquisition, K.A., U.I., M.A. and I.L.P. All authors have read and agreed to the published version of the manuscript.

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Availability of data and material

The data that supports the findings of this study is available from the corresponding author, upon reasonable request

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Conflict of interest

The authors declare that they have no conflict of interest.

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