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Research Article

Exact Solitary Wave Profiles for the Fisher Equation with Nonlinear Convection Term Arising in Physical Sciences

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Abstract: In this manuscript, the reaction-diffusion Fisher equation is investigated analytically with the nonlinear convection term. The physical, chemical, and biological sciences all rely on the memory effect in the diffusion reaction equation. We obtained the exact solitary wave profiles of memory effect in the Fisher equation by using the generalized Riccati equation mapping method. After applying this method, we obtained analytical solutions for the memory effect in Fisher equation, like as trigonometric, hyperbolic, rational, and exponential functions. We designed the Three-Dimensional (3D), Two-Dimensional (2D), and their contour for the appropriate values of the parameters by using MATHEMATICA. These solutions provide us with more understanding of the memory effect in the Fisher equation.

Keywords: diffusion-reaction equation, Fisher equation, exact solution, generalized Riccati equation mapping method, disease modeling, epidemiology, population dynamics, spread mechanisms, biological propagation

MSC: 65L05, 34K06, 34K28

1. Introduction

Partial Differential Equations (PDEs) are important to acknowledge the natural events in all possible scientific fields. PDEs are broadly employed in sciences such as mechanics, mathematical physics, biology, quantum mechanics [1], nonlinear optics [2], plasma physics [3], fluid mechanics [4], electromagnetism [5], optical fibers [6], propagation of shallow water waves [7], fluid dynamics [8, 9] etc. For example, partial differential equations in physics are utilized to narrate the transmission of waves and the flow of temperature within a medium. A particular family of PDEs can be produced with the help of the words that are written by pressing the keyboard with your fingers. In the field of biology, partial differential

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equations are used to show the generality of biological processes. The dynamical analysis of non-linear partial differential equations in many non-linear fields has undergone a recent change. In these contexts, there are some complex networks where partial differential equations may be employed. The physical events are mostly captured by the non-linear partial differential equations. It has led to the principle of finding the analytical and numerically approximate solution of a non-linear partial differential equation with the assistance of mathematical tools like Maple, Mathematica, and MATLAB.

Memory effects are widely emphasized in nonlinear dynamical systems in the physical, chemical, and biological sciences, though the specific phenomenon is different in each instance. To provide an example, memory-dependent terms have been introduced in reaction-diffusion systems, where anomalous transport effects are observed in porous media that are not governed by purely Markovian dynamics, i.e. propagation of reactive fronts. In chemical kinetics, nonlocal memory effects can be observed in an autocatalytic reaction affecting the stability of concentration waves and their rate. Likewise, delayed reactions and heredity also add memory to growth-diffusion equations in population biology, and can alter the velocity of traveling waves relative to the classical Fisher equation. These terms of memory have been applied in ecological studies to describe the effect of environmental history on dispersal behavior and in neuroscience to simulate the wave propagation of excitable tissues with synaptic delay-related kernels. These papers show that the incorporation of memory effects in the type of equations of Fisher is not just an extension of the theory, but also a need of practical application of the system to obtain realistic behavior, which goes beyond the example of particle transport.

The Fisher equation containing a nonlinear convection term has gained much interest since it describes a wider range of transport processes than the classical Fisher's equation and Kolmogorov-Petrovsky-Piskunov (Fisher-KPP) equation. The nonlinear convection term describes the directed transport due to effects of density-dependent processes, and is important specifically in systems where advection is coupled to reaction diffusion interactions. Adding memory effects into this structure causes many changes in the wavefront propagation and stability of solutions, commonly resulting in anomalous diffusion or non-standard wave speeds. Indicatively, in population dynamics, the nonlinear convection term may model the preferential convection of species to crowding effects or environmental gradient, whereas the addition of memory to it models delayed responses to either. Chemical systems Memory-based convection-reaction terms were used to characterize autocatalytic fronts in fluids in which advection is not instantaneous but rather is determined by the previous concentration history. In biophysics, chemotaxis and cellular migration have also been modelled using similar extensions, in which the drift velocity is based on the current and previous concentration fields. These generalizations of the Fisher equation thus offer a more physical mathematical model of the combined effect of nonlinear convection as well as memory, which are vital in the physical, chemical, and biological origins of complex transport systems.

Many mathematicians have established a number of methods to detect the solutions of non-linear partial differential equations and find exact traveling wave solutions, involving the Jacobi elliptic function method [10–13], the generalized Riccati equation mapping method [14], ϕ^6-model expansion method [15–17], the homogeneous balance method [18], and rational Homotopy perturbation method [19], the inverse scattering transform [20], the tanh method [21], the exponential-function expansion method [22], the modified extended Fan sub-equation method [23, 24], the Backlund transform [25, 26], the truncated Painleve expansion [27], the auxiliary equation method [28], the He's variational principle [29], the Hirota bilinear approach [30], the sine-Gordon expansion scheme [31, 32], $(G'/G,\ 1/G)$ expansion technique [33], and many more.

The generalized Riccati equation mapping technique was selected in this research due to its simplicity, flexibility, and efficiency relative to other known analytical models leading to the Lie symmetries, Hirota method, and the variational iteration. Although Lie analysis can be very insightful in understanding both invariants and reductions of nonlinear equations, it can be very tedious to perform and can not always have explicit closed-form solutions. Likewise, the bilinear approach of Hirota is optimally adapted to the construction of multi-soliton solutions, but is extremely constraining when used in equations with other nonlinear convection or memory terms. Conversely, the variational iteration method is iterative in character, and though it produces approximate forms of analysis, it is not, in general, compact and amenable to closed forms in order to provide an accurate analysis of wave propagation. The generalized Riccati equation mapping method, by contrast, systematically converts the nonlinear PDE into solvable forms via a simple mapping structure, making it possible to extract a broad range of exact solutions, containing soliton and periodic as well as singular solutions. The versatility allows it to be especially beneficial to equations such as the Fisher equation with nonlinear convection, where a wide range of dynamical behavior is of interest in closed form, both to theoretically analyze and to apply in practical contexts. For searching different kinds of non-linear partial differential equations, like trigonometric, hyperbolic, exponential, solitary wave, or soliton solutions, the generalized exponential rational function technique is used by [34]. In 1834, the soliton framework was introduced by John Scott Russell. In 1965, Zabusky and Kruskal first utilized the term of soliton. The movement a solitary wave with a change in speed depends on the elements (medium properties, wave amplitude, and non-linearity, etc) and chemical kinetics. Three-Dimensional (3D), Two-Dimensional (2D), and contour graphs are used to represent the behavior of a solitary wave. Still in this work, the Generalized Riccati Equation Mapping (GREM) strategy is applied. The GREM method is a strong technique for finding the solution of a non-linear partial differential equation. Zhu [35] developed the GREM tool with the help of the extended tanh-function technique to analysis the (2 + 1)-dimensional Boiti-Leon-Pempinelle equaion.

The primary aim of this work is to analyze the Fisher equation, including nonlinear convective term in the presence of memory effects, and to obtain explicit solitary waves solutions that may help to better understand their dynamical behavior. This work is important as it is known that memory effects are critical in a broad set of physical, chemical, and biological systems, but their inclusion in Fisher-type models with nonlinear convection have been approached analytically with little thoroughness. Using the GREM technique, the research manages to build a variety of solutions, such as trigonometric, hyperbolic, rational, and exponential ones, and represent them in 2D and 3D as well as in a contour plot. The novelty of this work is that both nonlinear convection and memory have been treated within the Fisher framework, not only to enrich the theoretical comprehension of reaction-diffusion systems but to provide a flexible set of mathematical methods that can be applied to plot physically realizable wavefronts that are not well understood by classical Fisher KPP dynamics.

This research contributes to multiple Sustainable Development Goals by advancing the mathematical modeling of propagation phenomena in physical and biological systems. Exact solitary wave solutions of the Fisher equation with nonlinear convection enhance the understanding of population dynamics, species dispersion, and disease spread. The methodology supports innovation in scientific modeling and nonlinear analysis, while aiding environmental and climate-related simulations involving transport and diffusion processes. Moreover, the analytical framework strengthens Science, Technology, Engineering, and Mathematics (STEM) education and interdisciplinary collaboration in the physical sciences.

2. Model description

The Fisher equation is utilized as a strong technique for modeling and calculating the actual engineering problems. Such mathematical modeling frameworks promote cross-disciplinary cooperation between physics, biology, and environmental science, and applied mathematics. The diffusion equation $\frac{\partial n(x,\ t)}{\partial t} = -\frac{\partial \rho(x,\ t)}{\partial x}$ without the finite memory is obtained by the continuity equation, where $n(x,\ t)$ is particle concentration and $\rho(x,\ t)$ is the flow of the diffusion molecules. The flow of a diffusing particle is proportional to the density when merged with Fick's law; $\rho(x,\ t) = -\nu\frac{\partial n(x,\ t)}{\partial x}$, we obtained the one-dimentional diffusion equation $\frac{\partial n(x,\ t)}{\partial t} = \nu\frac{\partial^2 n(x,\ t)}{\partial x^2}$. Here, ν is known as the diffusion constant. The memory effect become visible when the scattering of the particles is collectively not independent. By the same modification in the existence of the non-linear convection term.

$$\rho(x,\ t+\tau) = -\nu\frac{\partial n(x,\ t)}{\partial x} + vn^2, \tag{1}$$

which handles the adjustment of the concentration gradient at a time t with a particle flux $\rho(x,\ t+\tau)$ at the earlier time $t+\tau$. Here τ is a particle delay time and v is the nonlinear convective flux term.

By expanding ρ in Eq. (1) up to the first order in τ, we obtain

$$\rho(x,\ t) + \tau\frac{\partial \rho(x,\ t)}{\partial t} = -\nu\frac{\partial n}{\partial x} + vn^2. \tag{2}$$

Now, in the appearance of the source function $f(n)$ so after some modification, the conservation equation change into

$$\frac{\partial n(x,\, t)}{\partial t} = -\frac{\partial \rho(x,\, t)}{\partial x} + f(n). \tag{3}$$

Now, differential Eq. (3) with respect to t and Eq. (2) with respect to x and remove $\rho(x,\, t)$ from the resulting expression, we get

$$n_{tt} - \beta \nu n_{xx} - f'(n) n_t + \beta(n_t - f(n)) + k\beta n n_x = 0. \tag{4}$$

Here $\beta \equiv \frac{1}{\tau}$, $k \equiv 2\nu$ and $f'(n) = \frac{df}{dn}$.

3. The generalized Riccati equation mapping method

The fundamental concept of the generalized Riccati equation mapping method is that (for more detail, see [36–38]):

Step 1: For a given Non-linear Partial Differential Equation (NPDE) with independent variable $x = (x_0 = t,\, x_1,\, x_2,\, \ldots,\, x_m)$, and dependent variable n

$$s(n,\, n_t,\, n_{x_i},\, n_{x_i x_j},\, \ldots) = 0, \tag{5}$$

where s is a general polynomial function of its argument, and the partial derivatives of dependent variables are denoted by subscripts.

Step 2: By using transformation, Eq. (5) has the following ansatz:

$$n = n(\chi), \text{ where } \chi = x - Rt, \tag{6}$$

where χ is a real function to be measured. Putting Eq. (6) into Eq. (5) we obtain an Ordinary Differential Equation (ODE)

$$H(n,\, n_\chi,\, n_{\chi\chi},\, \ldots) = 0. \tag{7}$$

Step 3: We suppose that the solution of Eq. (7) is the polynomial form

$$n(\chi) = \sum_{i=0}^{m} a_i Q(\chi)^i, \tag{8}$$

where a_i are the functions that are to be measured and m is a positive integer that is found by the balancing principle. The $Q(\chi)$ expresses the solution of the following generalized Riccati equation:

$$Q'(\chi) = d + bQ(\chi) + cQ^2(\chi), \tag{9}$$

where d, b, and c are all the real constants. Putting Eq. (8) with Eq. (9) into relevant ODE Eq. (7) and eliminating all coefficients of Q will get the system of algebraic equations, by which we can yield the parameters a_i, $(i = 1, \ldots, n)$ and χ. By solving algebraic equations, with the help of Eq. (9), one can obtain the easily non-traveling wave solutions to NPDE Eq. (5).

We have twenty-seven significant solutions of Eq. (9).

Type 1: When $b^2 - 4cd > 0$ and $bc \neq 0$ (and $cd \neq 0$), the solution of Eq. (9) are with $\lambda_1 = \sqrt{b^2 - 4cd}$

$$Q_1 = -\frac{b + \lambda_1 \tanh\left(\frac{\lambda_1 \chi}{2}\right)}{2c}, \tag{10}$$

$$Q_2 = -\frac{b + \lambda_1 \coth\left(\frac{\lambda_1 \chi}{2}\right)}{2c}, \tag{11}$$

$$Q_3 = -\frac{b + \lambda_1 \left(\tanh(\lambda_1 \chi) + \operatorname{sech}(\lambda_1 \chi)\right)}{2c}, \tag{12}$$

$$Q_4 = -\frac{b + \lambda_1 \left(\coth(\lambda_1 \chi) + \operatorname{csch}(\lambda_1 \chi)\right)}{2c}, \tag{13}$$

$$Q_5 = -\frac{2b + \lambda_1 \left(\tanh\left(\frac{\lambda_1 \chi}{4}\right) - \coth\left(\frac{\lambda_1 \chi}{4}\right)\right)}{4c}, \tag{14}$$

$$Q_6 = \frac{1}{2c}\left[\frac{\lambda_1 \sqrt{F^2 + G^2} - \mathrm{F}\lambda_1 \cosh(\lambda_1 \chi)}{F \sinh(\lambda_1 \chi) + G} - b\right], \tag{15}$$

$$Q_7 = \frac{1}{2c}\left[-b - \frac{\lambda_1 \sqrt{F^2 + G^2} + \mathrm{F}\lambda_1 \cosh(\lambda_1 \chi)}{F \sinh(\lambda_1 \chi) + G}\right], \tag{16}$$

where F and G are the two non-zero real constants.

$$Q_8 = \frac{2d \cosh\left(\frac{\lambda_1 \chi}{2}\right)}{\lambda_1 \sinh\left(\frac{\lambda_1 \chi}{2}\right) - b\cosh\left(\frac{\lambda_1 \chi}{2}\right)}, \tag{17}$$

$$Q_9 = -\frac{2d\sinh\left(\frac{\lambda_1\chi}{2}\right)}{b\sinh\left(\frac{\lambda_1\chi}{2}\right) - \lambda_1\cosh\left(\frac{\lambda_1\chi}{2}\right)}, \tag{18}$$

$$Q_{10} = \frac{2d\cosh\left(\frac{\lambda_1\chi}{2}\right)}{-b\cosh(\lambda_1\chi) + \lambda_1\sinh(\lambda_1\chi) + \lambda_1}, \tag{19}$$

$$Q_{11} = \frac{2d\sinh\left(\frac{\lambda_1\chi}{2}\right)}{-b\sinh(\lambda_1\chi) + \lambda_1\cosh(\lambda_1\chi) + \lambda_1}, \tag{20}$$

$$Q_{12} = \frac{4d\sinh\left(\frac{\lambda_1\chi}{4}\right)\cosh\left(\frac{\lambda_1\chi}{4}\right)}{-2b\sinh\left(\frac{\lambda_1\chi}{4}\right)\cosh\left(\frac{\lambda_1\chi}{4}\right) + 2\lambda_1\cosh^2\left(\frac{\lambda_1\chi}{4}\right) - \lambda_1}. \tag{21}$$

Type 2: When $b^2 - 4cd < 0$ and $bc \neq 0$ (or $cd \neq 0$), the solutions of Eq. (9) are with $\lambda_2 = \sqrt{4cd - b^2}$

$$Q_{13} = \frac{\lambda_2\tan\left(\frac{\lambda_2\chi}{2}\right) - b}{2c}, \tag{22}$$

$$Q_{14} = -\frac{b + \lambda_2\cot\left(\frac{\lambda_2\chi}{2}\right)}{2c}, \tag{23}$$

$$Q_{15} = \frac{\lambda_2(\tan(\lambda_2\chi) + \sec(\lambda_2\chi)) - b}{2c}, \tag{24}$$

$$Q_{16} = -\frac{b + \lambda_2(\cot(\lambda_2\chi) + \csc(\lambda_2\chi))}{2c}, \tag{25}$$

$$Q_{17} = \frac{\lambda_2\left(\tan\left(\frac{\lambda_2\chi}{4}\right) - \cot\left(\frac{\lambda_2\chi}{4}\right)\right) - 2b}{4c}, \tag{26}$$

$$Q_{18} = \frac{1}{2c}\left[\frac{\lambda_2\sqrt{F^2 - G^2} - \mathrm{F}\lambda_2\cos(\lambda_2\chi)}{F\sin(\lambda_2\chi) + G} - b\right], \tag{27}$$

$$Q_{19} = \frac{1}{2c}\left[-b - \frac{\lambda_2\sqrt{F^2 - G^2} + \mathrm{F}\lambda_2\cos(\lambda_2\chi)}{F\sin(\lambda_2\chi) + G}\right], \tag{28}$$

where F and G are two non-zero real constants and satisfie $F^2 - G^2 > 0$.

$$Q_{20} = -\frac{2d\cos\left(\frac{\lambda_2\chi}{2}\right)}{b\cos\left(\frac{\lambda_2\chi}{2}\right) + \lambda_2\sin\left(\frac{\lambda_2\chi}{2}\right)}, \tag{29}$$

$$Q_{21} = \frac{2d\sin\left(\frac{\lambda_2\chi}{2}\right)}{\lambda_2\cos\left(\frac{\lambda_2\chi}{2}\right) - b\sin\left(\frac{\lambda_2\chi}{2}\right)}, \tag{30}$$

$$Q_{22} = -\frac{2d\cos\left(\frac{\lambda_2\chi}{2}\right)}{b\cos(\lambda_2\chi) + \lambda_2\sin(\lambda_2\chi) + \lambda_2}, \tag{31}$$

$$Q_{23} = \frac{2d\sin\left(\frac{\lambda_2\chi}{2}\right)}{-b\sin(\lambda_2\chi) + \lambda_2\cos(\lambda_2\chi) + \lambda_2}, \tag{32}$$

$$Q_{24} = \frac{4d\sin\left(\frac{\lambda_2\chi}{4}\right)\cos\left(\frac{\lambda_2\chi}{4}\right)}{-2b\sin\left(\frac{\lambda_2\chi}{4}\right)\cos\left(\frac{\lambda_2\chi}{4}\right) + 2\lambda_2\cos^2\left(\frac{\lambda_2\chi}{4}\right) - \lambda_2}. \tag{33}$$

Type 3: When $d = 0$ and $bc \neq 0$, the solutions of Eq. (9) are

$$Q_{25} = \frac{-bg}{c[g + \cosh(b\chi) - \sinh(b\chi)]}, \tag{34}$$

$$Q_{26} = -\frac{b[\cosh(b\chi) + \sinh(b\chi)]}{c[g + \cosh(b\chi) + \sinh(b\chi)]}, \tag{35}$$

where g is an arbitrary constant.

Type 4: When $c \neq 0$ and $b = d = 0$, the solutions of Eq. (9)

$$Q_{27} = -\frac{1}{c\chi + p}, \tag{36}$$

where p is an arbitrary constant.

4. Solution of the model

The Eq. (4) converts into the Burgers equation [39] in the absence of the source term $(f(n) = 0)$. For the term $f(n)$ we suppose $f(n) = \alpha n - \gamma n^2$ and after derivative, we get $f'(n) = \alpha - 2\gamma n$. Remember that Eq. (4) expresses the transport event in which both diffusion and convection techniques have equal significance. After utilizing both terms $f(n)$ and $f'(n)$ and the transformation $\chi = x - Rt$.

$$(R^2 - \beta\nu)n'' + R(\alpha - \beta)n' + (k\beta - 2\gamma R)nn' - \beta\alpha n + \beta\gamma n^2 = 0, \tag{37}$$

With the help of the balancing technique, we are now using Eq. (8) and balancing the highest order derivative n'' with the highest order nonlinear term nn'. We obtain $m = 1$, then the Eq. (8) become

$$n(\chi) = a_0 + a_1 Q(\chi), \tag{38}$$

Here a_0 and a_1 are the constants. Now putting the Eq. (38) and (9) in Eq. (37) then we get a set of algebraic equations including $a_0,\ a_1,\ R$ and ν.

$$\begin{aligned} &-\alpha a_0\beta + a_0^2\beta\gamma - a_1 b\beta d\nu + a_1 bdR^2 + a_0 a_1\beta dk \\ &+\alpha a_1 dR - a_1\beta dR - 2a_0 a_1\gamma dR = 0, \end{aligned} \tag{39}$$

$$\begin{aligned} &-\alpha a_1\beta - a_1\beta b^2\nu + a_1 b^2 R^2 + 2a_0 a_1\beta\gamma + a_0 a_1\beta bk + \alpha a_1 bR - a_1\beta bR \\ &-2a_0 a_1 b\gamma R - 2a_1\beta cd\nu + 2a_1 cdR^2 + a_1^2\beta dk - 2a_1^2\gamma dR = 0, \end{aligned} \tag{40}$$

$$\begin{aligned} &-3a_1 b\beta c\nu + 3a_1 bcR^2 + a_1^2\beta\gamma + a_1^2 b\beta k - 2a_1^2 b\gamma R + a_0 a_1\beta ck \\ &+\alpha a_1 cR - a_1\beta cR - 2a_0 a_1 c\gamma R = 0, \end{aligned} \tag{41}$$

$$-2a_1\beta c^2\nu + 2a_1 c^2 R^2 + a_1^2\beta ck - 2a_1^2\gamma cR = 0. \tag{42}$$

with the help of Mathematica by solving the above system of equations for find the value of unknowns $a_0,\ a_1,\ R$ and ν.

$$a_0 = \frac{\alpha\left(1 - \frac{\alpha b\gamma^2}{\sqrt{\alpha^2\gamma^4(b^2 - 4cd)}}\right)}{2\gamma},\ a_1 = -\frac{\alpha^2 c\gamma}{\sqrt{\alpha^2\gamma^4(b^2 - 4cd)}},\ R = \frac{\alpha\left(k - \frac{2\alpha\gamma^3}{\sqrt{\alpha^2\gamma^4(b^2 - 4cd)}}\right)}{2\gamma},$$

$$\nu = \frac{k\left(-2(\alpha + \beta)\sqrt{\alpha^2\gamma^4(b^2 - 4cd)} + \alpha^2 b^2\gamma k - 4\alpha^2 c\gamma dk\right)}{4\beta\gamma^3(b^2 - 4cd)}. \tag{43}$$

Conveniently, declare the constant $Q_0 = \dfrac{\alpha\left(1-\dfrac{b}{\lambda_1}\right)}{2\gamma}$.

Type 1: When $b^2 - 4cd > 0$, $bc \neq 0$ (and $cd \neq 0$), and $\zeta_1 = \lambda_1\left(x - \dfrac{\alpha t\left(k-\dfrac{2\gamma}{\lambda_1}\right)}{2\gamma}\right)$.

We get the some solutions of Eq. (37) after the substitution Eq. (43) along with Eq. (10) and (6) in the Eq. (38)

$$Q_1(x,\ t) = Q_0 + \frac{\alpha^2\gamma\left(b+\lambda_1\tanh\left(\frac{\zeta_1}{2}\right)\right)}{2\alpha\gamma^2\lambda_1}. \tag{44}$$

We get the some solutions of Eq. (37) after the substitution Eq. (43) along with Eq. (11) and (6) in the Eq. (38)

$$Q_2(x,\ t) = Q_0 + \frac{\alpha\left(b+\lambda_1\coth\left(\frac{\zeta_1}{2}\right)\right)}{2\gamma\lambda_1}. \tag{45}$$

We get the some solutions of Eq. (37) after the substitution Eq. (43) along with Eq. (12) and (6) in the Eq. (38)

$$Q_3(x,\ t) = Q_0 + \frac{\alpha\left(b+\lambda_1\left(\tanh(\zeta_1)+\operatorname{sech}(\zeta_1)\right)\right)}{2\gamma\lambda_1}. \tag{46}$$

We get the some solutions of Eq. (37) after the substitution Eq. (43) along with Eq. (13) and (6) in the Eq. (38)

$$Q_4(x,\ t) = Q_0 + \frac{\alpha\left(b+\lambda_1\left(\coth(\zeta_1)+\operatorname{csch}(\zeta_1)\right)\right)}{2\gamma\lambda_1}. \tag{47}$$

We get the some solutions of Eq. (37) after the substitution Eq. (43) along with Eq. (14) and (6) in the Eq. (38)

$$Q_5(x,\ t) = Q_0 + \frac{\alpha\left(2b+\lambda_1\left(\tanh\left(\frac{\zeta_1}{4}\right)-\coth\left(\frac{\zeta_1}{4}\right)\right)\right)}{4\gamma\lambda_1}. \tag{48}$$

We get the some solutions of Eq. (37) after the substitution Eq. (43) along with Eq. (15) and (6) in the Eq. (38)

$$Q_6(x,\ t) = Q_0 - \frac{\alpha}{2\gamma\lambda_1}\left(\frac{\lambda_1\sqrt{F^2+G^2}-F\lambda_1\cosh(\zeta_1)}{F\sinh(\zeta_1)+G} - b\right). \tag{49}$$

We get the some solutions of Eq. (37) after the substitution Eq. (43) along with Eq. (16) and (6) in the Eq. (38)

$$Q_7(x,\ t) = Q_0 - \frac{\alpha}{2\gamma\lambda_1}\left(-b - \frac{\lambda_1\sqrt{F^2+G^2}+F\lambda_1\cosh(\zeta_1)}{F\sinh(\zeta_1)+G}\right). \tag{50}$$

We get the some solutions of Eq. (37) after the substitution Eq. (43) along with Eq. (17) and (6) in the Eq. (38)

$$Q_8(x,\ t) = Q_0 - \frac{2cd\alpha\cosh\left(\frac{\zeta_1}{2}\right)}{\gamma\lambda_1\left(\lambda_1\sinh\left(\frac{\zeta_1}{2}\right) - b\cosh\left(\frac{\zeta_1}{2}\right)\right)}. \tag{51}$$

We get the some solutions of Eq. (37) after the substitution Eq. (43) along with Eq. (18) and (6) in the Eq. (38)

$$Q_9(x,\ t) = Q_0 + \frac{2\alpha cd\sinh\left(\frac{\zeta_1}{2}\right)}{\gamma\lambda_1\left(b\sinh\left(\frac{\zeta_1}{2}\right) - \lambda_1\cosh\left(\frac{\zeta_1}{2}\right)\right)}. \tag{52}$$

We get the some solutions of Eq. (37) after the substitution Eq. (43) along with Eq. (19) and (6) in the Eq. (38)

$$Q_{10}(x,\ t) = Q_0 - \frac{2cd\alpha\cosh\left(\frac{\zeta_1}{2}\right)}{\gamma\lambda_1\left(-b\cosh(\zeta_1) + \lambda_1\sinh(\zeta_1) + \lambda_1\right)}. \tag{53}$$

We get the some solutions of Eq. (37) after the substitution Eq. (43) along with Eq. (20) and (6) in the Eq. (38)

$$Q_{11}(x,\ t) = Q_0 - \frac{2\alpha cd\sinh\left(\frac{\zeta_1}{2}\right)}{\gamma\lambda_1\left(-b\sinh(\zeta_1) + \lambda_1\cosh(\zeta_1) + \lambda_1\right)}. \tag{54}$$

We get the some solutions of Eq. (37) after the substitution Eq. (43) along with Eq. (21) and (6) in the Eq. (38)

$$Q_{12}(x,\ t) = Q_0 - \frac{4cd\alpha\cosh\left(\frac{\zeta_1}{4}\right)\sinh\left(\frac{\zeta_1}{4}\right)}{\gamma\lambda_1\left(-2b\sinh\left(\frac{\zeta_1}{4}\right)\cosh\left(\frac{\zeta_1}{4}\right) + 2\lambda_1\cosh^2\left(\frac{\zeta_1}{4}\right) - \lambda_1\right)}. \tag{55}$$

Type 2: When $b^2 - 4cd < 0$, $bc \neq 0$ (or $cd \neq 0$), and $\zeta_2 = \lambda_2\left(x - \frac{\alpha t\left(k - \frac{2\gamma}{\lambda_1}\right)}{2\gamma}\right)$.

We get the some solutions of Eq. (37) after the substitution Eq. (43) along with Eq. (22) and (6) in the Eq. (38)

$$Q_{13}(x,\ t) = Q_0 - \frac{\alpha}{2\gamma\lambda_1}\left(\lambda_2\tan\left(\frac{\zeta_2}{2}\right) - b\right). \tag{56}$$

We get the some solutions of Eq. (37) after the substitution Eq. (43) along with Eq. (23) and (6) in the Eq. (38)

$$Q_{14}(x,\ t) = Q_0 + \frac{\alpha}{2\gamma\lambda_1}\left(b + \lambda_2 \cot\left(\frac{\zeta_2}{2}\right)\right). \tag{57}$$

We get the some solutions of Eq. (37) after the substitution Eq. (43) along with Eq. (24) and (6) in the Eq. (38)

$$Q_{15}(x,\ t) = Q_0 - \frac{\alpha\left(\lambda_2\left(\tan\left(\zeta_2\right) + \sec\left(\zeta_2\right)\right) - b\right)}{2\gamma\lambda_1}. \tag{58}$$

We get the some solutions of Eq. (37) after the substitution Eq. (43) along with Eq. (25) and (6) in the Eq. (38)

$$Q_{16}(x,\ t) = Q_0 + \frac{\alpha\left(b + \lambda_2\left(\cot\left(\zeta_2\right) + \csc\left(\zeta_2\right)\right)\right)}{2\gamma\lambda_1}. \tag{59}$$

We get the some solutions of Eq. (37) after the substitution Eq. (43) along with Eq. (26) and (6) in the Eq. (38)

$$Q_{17}(x,\ t) = Q_0 - \frac{\alpha\left(\lambda_2\left(\tan\left(\frac{\zeta_2}{4}\right) - \cot\left(\frac{\zeta_2}{4}\right)\right) - 2b\right)}{4\gamma\lambda_1}. \tag{60}$$

We get the some solutions of Eq. (37) after the substitution Eq. (43) along with Eq. (27) and (6) in the Eq. (38)

$$Q_{18}(x,\ t) = Q_0 - \frac{\alpha}{2\gamma\lambda_1}\left(\frac{\lambda_2\sqrt{F^2 - G^2} - F\lambda_2\cos\left(\zeta_2\right)}{F\sin\left(\zeta_2\right) + G} - b\right). \tag{61}$$

We get the some solutions of Eq. (37) after the substitution Eq. (43) along with Eq. (28) and (6) in the Eq. (38)

$$Q_{19}(x,\ t) = Q_0 - \frac{\alpha}{2\gamma\lambda_1}\left(-b - \frac{\lambda_2\sqrt{F^2 - G^2} + F\lambda_2\cos\left(\zeta_2\right)}{F\sin\left(\zeta_2\right) + G}\right). \tag{62}$$

We get the some solutions of Eq. (37) after the substitution Eq. (43) along with Eq. (29) and (6) in the Eq. (38)

$$Q_{20}(x,\ t) = Q_0 + \frac{2\alpha cd\cos\left(\frac{\zeta_2}{2}\right)}{\gamma\lambda_1\left(b\cos\left(\frac{\zeta_2}{2}\right) + \lambda_2\sin\left(\frac{\zeta_2}{2}\right)\right)}. \tag{63}$$

We get the some solutions of Eq. (37) after the substitution Eq. (43) along with Eq. (30) and (6) in the Eq. (38)

$$Q_{21}(x,\ t) = Q_0 - \frac{2\alpha cd \sin\left(\frac{\zeta_2}{2}\right)}{\gamma\lambda_1\left(\lambda_2 \cos\left(\frac{\zeta_2}{2}\right) - b\sin\left(\frac{\zeta_2}{2}\right)\right)}. \tag{64}$$

We get the some solutions of Eq. (37) after the substitution Eq. (43) along with Eq. (31) and (6) in the Eq. (38)

$$Q_{22}(x,\ t) = Q_0 + \frac{2\alpha cd \cos\left(\frac{\zeta_2}{2}\right)}{\gamma\lambda_1\left(b\cos(\zeta_2) + \lambda_2 \sin(\zeta_2) + \lambda_2\right)}. \tag{65}$$

We get the some solutions of Eq. (37) after the substitution Eq. (43) along with Eq. (32) and (6) in the Eq. (38)

$$Q_{23}(x,\ t) = Q_0 - \frac{2\alpha cd \sin\left(\frac{\zeta_2}{2}\right)}{\alpha\gamma\lambda_1\left(-b\sin(\zeta_2) + \lambda_2 \cos(\zeta_2) + \lambda_2\right)}. \tag{66}$$

We get the some solutions of Eq. (37) after the substitution Eq. (43) along with Eq. (33) and (6) in the Eq. (38)

$$Q_{24}(x,\ t) = Q_0 - \frac{4\alpha cd \sin\left(\frac{\zeta_2}{4}\right)\cos\left(\frac{\zeta_2}{4}\right)}{\gamma\lambda_1\left(-2b\sin\left(\frac{\zeta_2}{4}\right)\cos\left(\frac{\zeta_2}{4}\right) + 2\lambda_2\cos^2\left(\frac{\zeta_2}{4}\right) - \lambda_2\right)}. \tag{67}$$

Type 3: When $d = 0$, $bc \neq 0$, and $\zeta_3 = b\left(x - \frac{\alpha t\left(k - \frac{2\gamma}{\lambda_1}\right)}{2\gamma}\right)$.

We get the some solutions of Eq. (37) after the substitution Eq. (43) along with Eq. (34) and (6) in the Eq. (38)

$$Q_{25}(x,\ t) = Q_0 + \frac{\alpha bg}{\gamma\lambda_1\left(-\sinh(\zeta_3) + \cosh(\zeta_3) + g\right)}, \tag{68}$$

We get the some solutions of Eq. (37) after the substitution Eq. (43) along with Eq. (35) and (6) in the Eq. (38)

$$Q_{26}(x,\ t) = Q_0 + \frac{\alpha b\left(\sinh(\zeta_3) + \cosh(\zeta_3)\right)}{\gamma\lambda_1\left(\sinh(\zeta_3) + \cosh(\zeta_3) + g\right)}. \tag{69}$$

Type 4: When $c \neq 0$ and $b = d = 0$, We get the some solutions of Eq. (37) after the substitution Eq. (43) along with Eq. (36) and (6) in the Eq. (38)

$$Q_{27}(x,\, t) = Q_0 + \frac{\alpha c}{\gamma \lambda_1 \left(c \left(x - \frac{\alpha t \left(k - \frac{2\gamma}{\lambda_1} \right)}{2\gamma} \right) + p \right)}. \tag{70}$$

5. Graphical behaviors

In this portion, we study the graphical behavior for the solution of the memory effect in the Fisher equation by using the GREM method. The GREM method is the most effective and reliable method to obtain the solitary wave and solitons solution. For the explanation of many physical aspects, we must draw three-dimensional, two-dimensional, and their contours for the required solutions. We get more reliable information about the behavior of the solution from these graphics. Several solutions are represented in 3D (with an interval $-10 \le x \le 10,\ -10 \le t \le 10$), 2D (with interval $-10 \le x \le 10,\ -1 \le t \le 1$), and contour (with interval $-10 \le x \le 10,\ -10 \le t \le 10$) for varying the values of the constant with the help of Mathematica. Figure 1 represents the dark soliton. Figures 2 and 3 represent the singular soliton. The Figures 4, 5, 6, 7, 8, 9 and 10 represent the kink shape soliton. The Figures 11 and 12 represent the solitary wave solution, and the Figure 13 represents the w-type soliton. It illustrates the explicit wave profiles derived from the GREM method applied to the Fisher equation with nonlinear convection and memory effects. These figures are not independent or pre-existing exact solutions from the literature, but rather the analytical solutions reported in this work and visualized through their corresponding graphical representations. The GREM framework was employed to obtain these closed-form solutions, and the plotted figures serve to confirm their dynamical behavior under various parameter selections. As for the physical significance of the obtained solitary wave profiles, they are not merely mathematical constructs but can indeed correspond to physically realizable wavefronts in reaction-diffusion systems. For instance, solitary and traveling wave solutions of Fisher-type equations are known to model biological invasion fronts, autocatalytic chemical reactions, and transport processes in porous media. The incorporation of memory effects modifies the wave speed and shape, aligning with experimentally observed deviations from the classical Fisher-KPP dynamics. When compared with existing numerical studies, the analytical results derived here are in qualitative agreement, particularly in terms of wavefront stability and the influence of memory kernels on front propagation. The analytical framework provided by the GREM method thus complements numerical investigations by offering closed-form insight into how memory and nonlinear convection jointly govern the dynamics of reaction-diffusion systems.

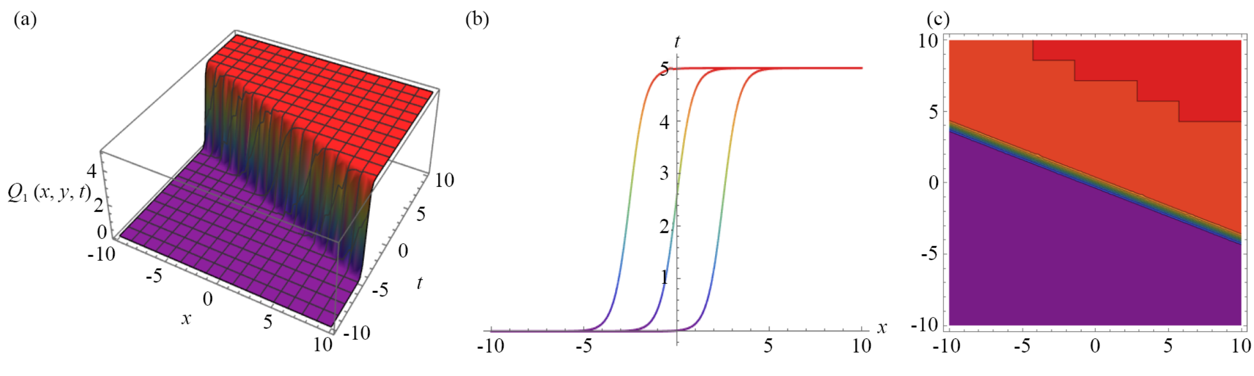

Figure 1. Above 3D, 2D, and contour figures represent the graphically behavior of Eq. (44) with parameters $\alpha = 0.05,\ b = -1.3,\ \gamma = -0.01,\ c = 0.95,\ d = -1,\ k = 1$

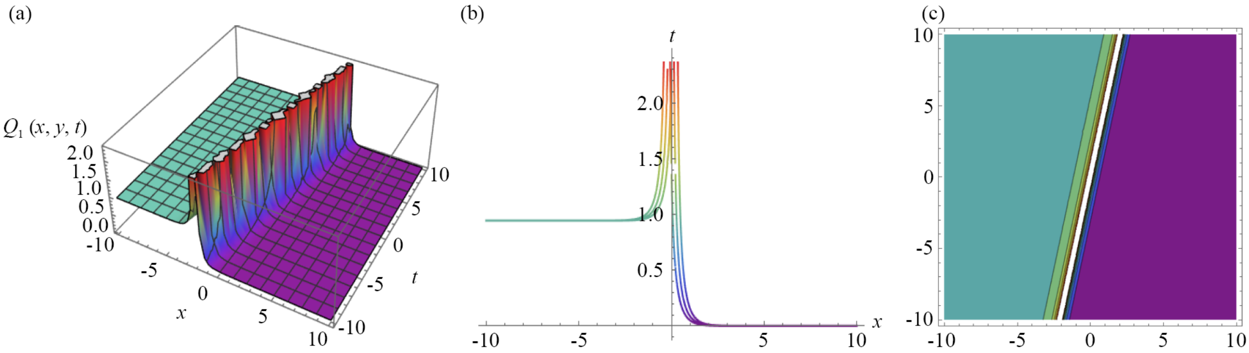

Figure 2. Above 3D, 2D, and contour figures represent the graphically behavior of Eq. (45) with parameters $\alpha = -1$, $b = -1.45$, $\gamma = 1.06$, $c = 0.95$, $d = -1.05$, $k = -1.3$

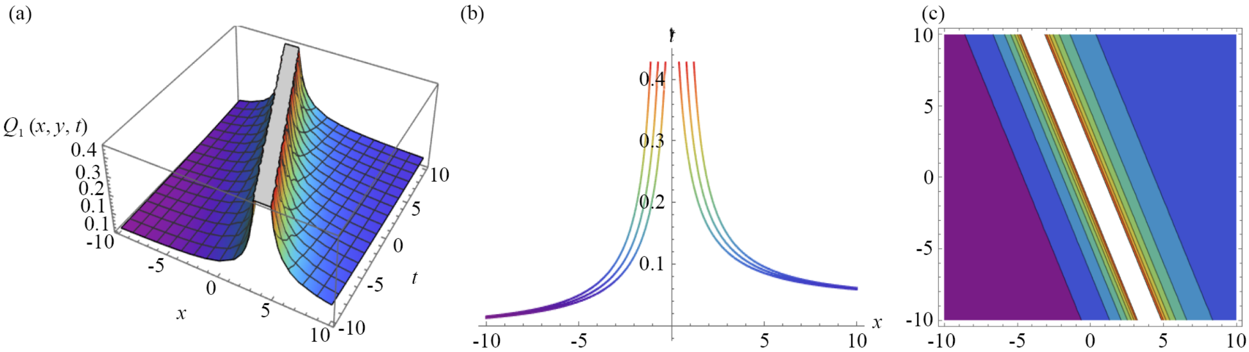

Figure 3. Above 3D, 2D, and contour figures represent the graphically behavior of Eq. (47) with parameters $\alpha = 0.06$, $b = 0.01$, $\gamma = 1.3$, $c = -0.1$, $d = 0.05$, $k = 1$

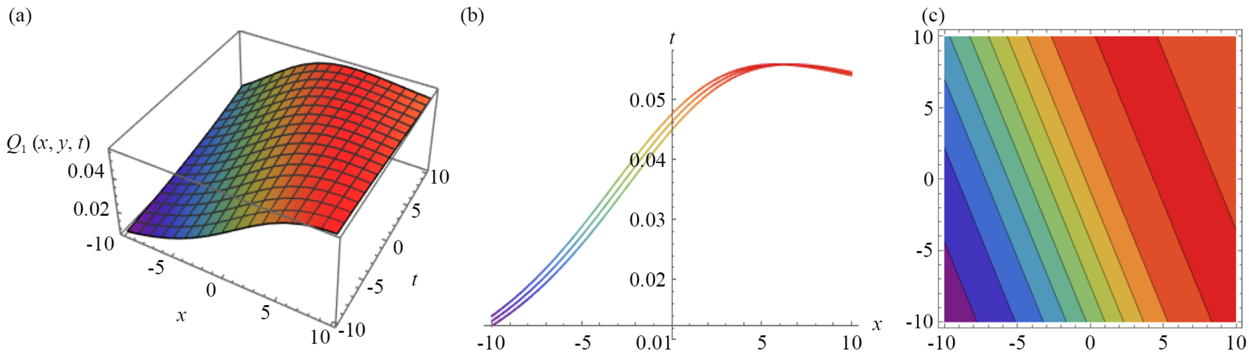

Figure 4. Above 3D, 2D, and contour figures represent the graphically behavior of Eq. (46) with parameters $\alpha = 0.06$, $b = 0.01$, $\gamma = 1.3$, $c = -0.1$, $d = 0.05$, $k = 1$

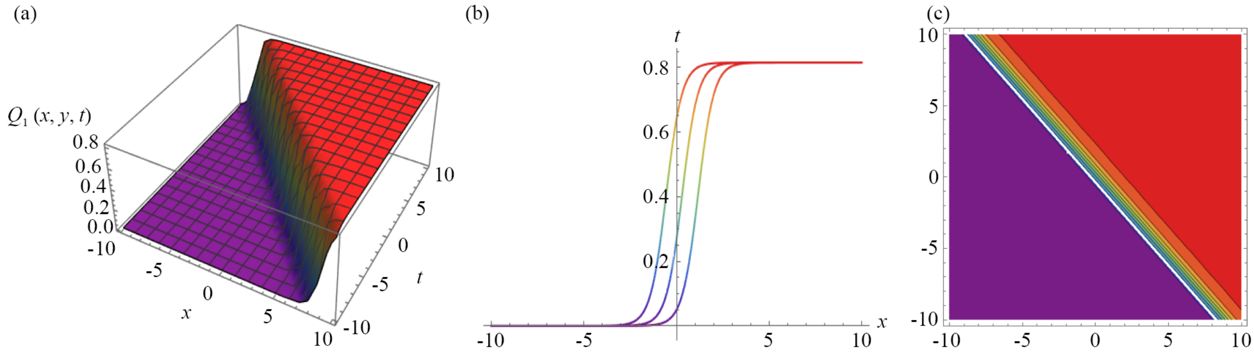

Figure 5. Above 3D, 2D, and contour figures represent the graphically behavior of Eq. (49) with parameters $\alpha = 1.06$, $b = 1.1$, $\gamma = 1.3$, $c = 1.02$, $d = -1.05$, $F = 0.95$, $G = 0.75$, $k = -1$

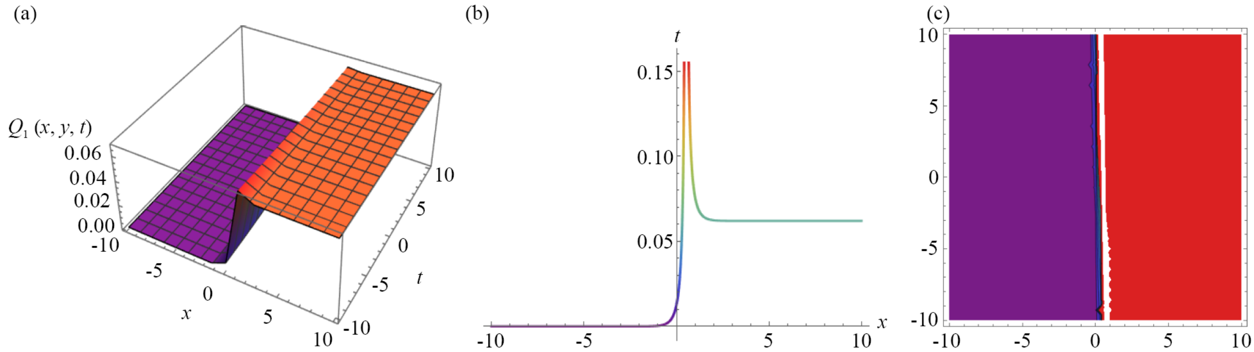

Figure 6. Above 3D, 2D, and contour figures represent the graphically behavior of Eq. (51) with parameters $\alpha = 0.07$, $b = 2.45$, $\gamma = 1.13$, $c = 1.02$, $d = -1.45$, $k = -0.05$

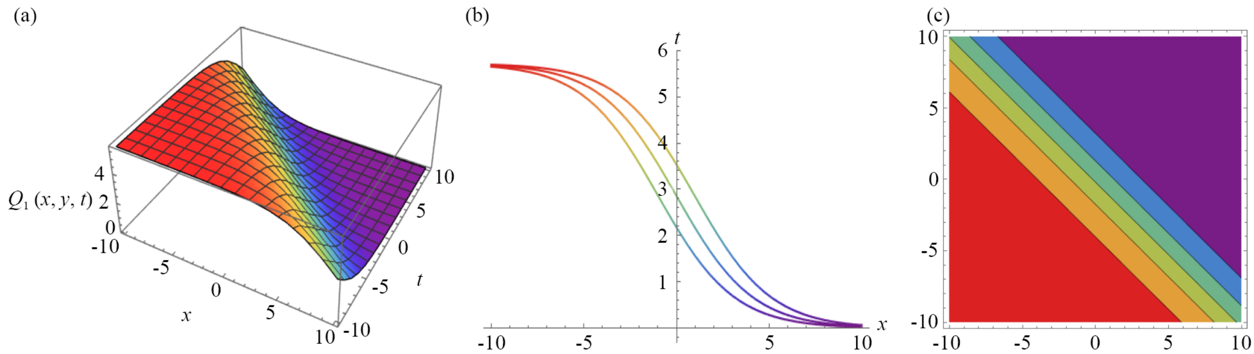

Figure 7. Above 3D, 2D, and contour figures represent the graphically behavior of Eq. (56) with parameters $\alpha = -0.4$, $b = 1$, $\gamma = 0.07$, $c = 0.2$, $d = 0.95$, $k = 0.06$

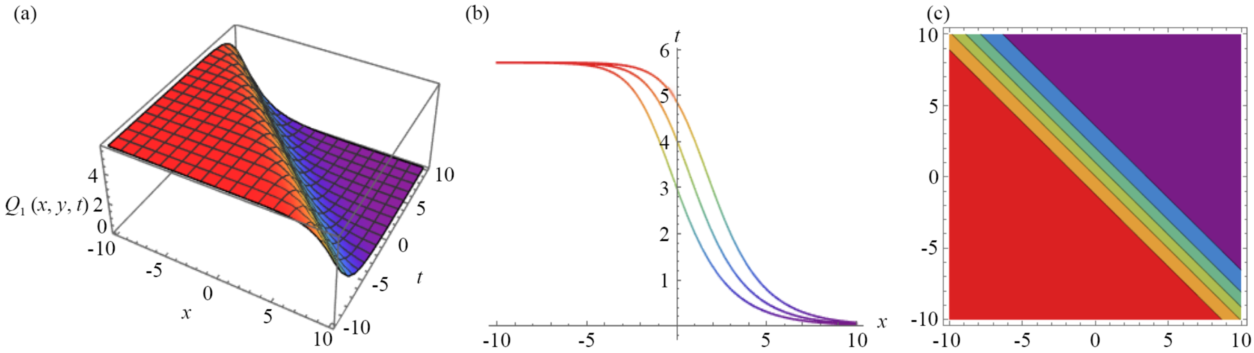

Figure 8. Above 3D, 2D, and contour figures represent the graphically behavior of Eq. (58) with parameters $\alpha = -0.4,\ b = 1,\ \gamma = 0.07,\ c = 0.2,\ d = 0.95,\ k = 0.06$

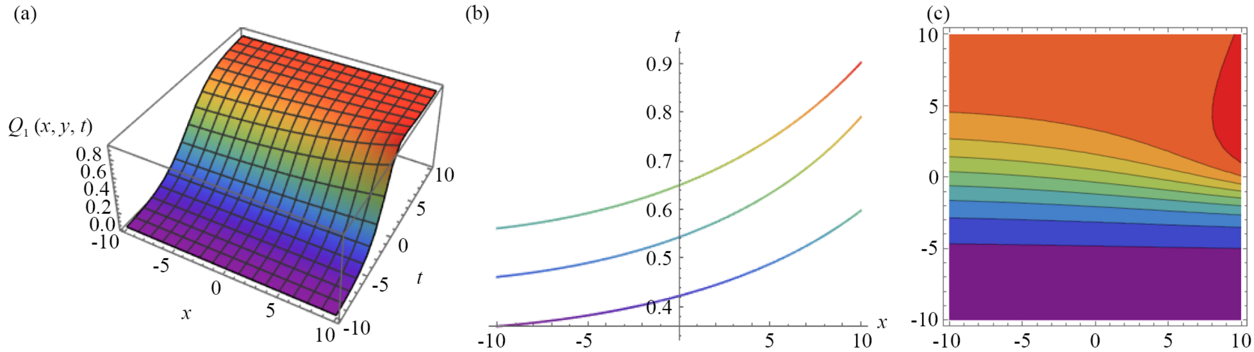

Figure 9. Above 3D, 2D, and contour figures represent the graphically behavior of Eq. (67) with parameters $\alpha = 0.45,\ b = 0.05,\ \gamma = 0.5,\ c = 0.02,\ d = 0.1,\ k = 0.95$

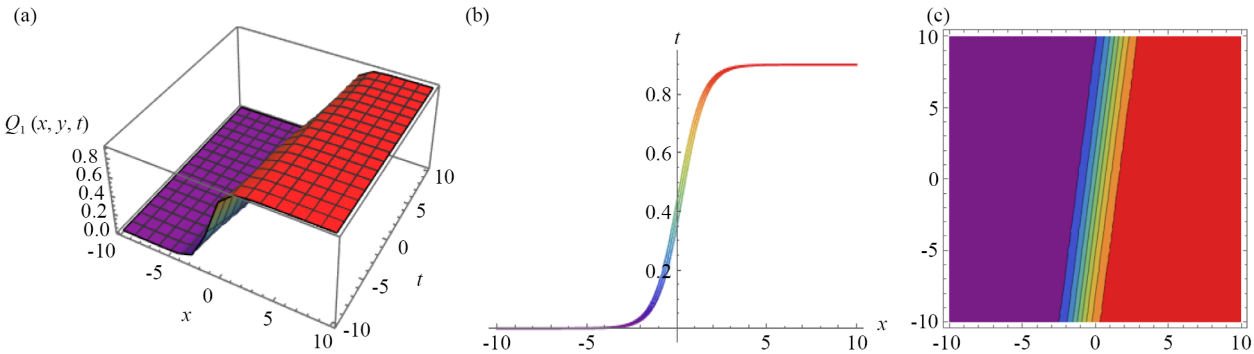

Figure 10. Above 3D, 2D, and contour figures represent the graphically behavior of Eq. (68) with parameters $\alpha = 0.45,\ b = 1.5,\ \gamma = 0.5,\ c = 0.02,\ d = 0,\ g = 0.75,\ k = 0.95$

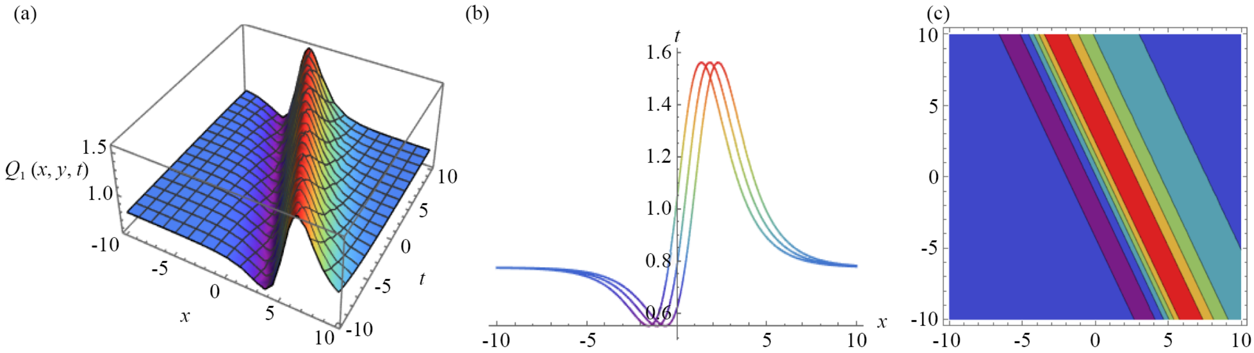

Figure 11. Above 3D, 2D, and contour figures represent the graphically behavior of Eq. (54) with parameters $\alpha = 0.4$, $b = 1$, $\gamma = 0.07$, $c = -0.2$, $d = 1.1$, $k = -0.06$

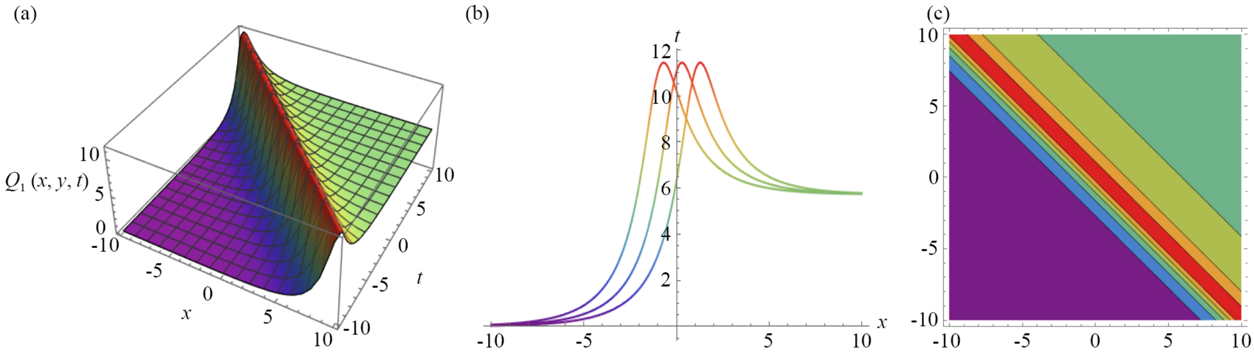

Figure 12. Above 3D, 2D, and contour figures represent the graphically behavior of Eq. (62) with parameters $\alpha = 0.4$, $b = 1$, $\gamma = 0.07$, $c = 0.2$, $d = 0.95$, $F = 2$, $G = 1$, $k = -0.06$

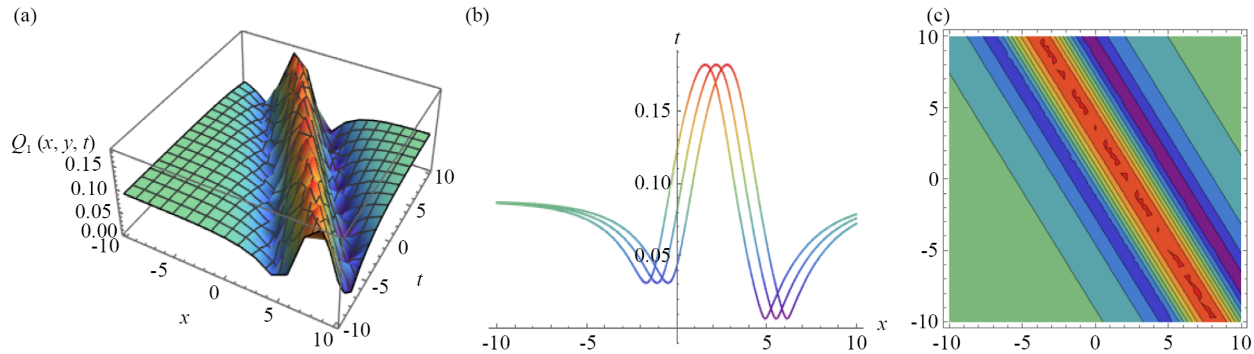

Figure 13. Above 3D, 2D, and contour figures represent the graphically behavior of Eq. (65) with parameters $\alpha = 0.95$, $b = 1.06$, $\gamma = -1.03$, $c = 1.65$, $d = 0.05$, $k = -1$

6. Conclusions

In this work, twenty-seven different types of wave solutions, like hyperbolic and trigonometric, were obtained for the memory effect in the Fisher equation by using the generalized Riccati equation mapping method. The GREM method is a more reliable and effective method to obtain the analytical solutions of different nonlinear differential equations. The GREM procedure is used for finding the analytical solutions. The solitary wave solutions of the required model are found by using Mathematica. Much physical importance is described by sketching some 3D, 2D, and contour graphs for the solutions. These graphs provide us with better information about the behavior of solutions. These results demonstrate how memory and nonlinear convection significantly influence the structure, propagation speed, and stability of solitary wave profiles, thereby offering deeper theoretical insight into reaction-diffusion processes observed in physical, chemical, and biological systems. The findings not only extend the mathematical understanding of Fisher-type equations, but also provide a foundation for future research aimed at connecting such analytical solutions with experimental and numerical studies of memory-driven wavefront phenomena.

Data availability

Data will be provided by corresponding author on a reasonable request.

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Conflict of interest

The authors declare no competing financial interest.

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